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Using wavelets to measure core inflation: the case of New Zealand*

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Abstract

This paper uses wavelets to develop a core inflation measure for inflation targeting central banks. The analysis is applied to the case of New Zealand – the country with the longest history of explicit inflation targeting. We compare the performance of our proposed measure against some popular alternatives. Our measure does well at identifying a reliable medium-term trend in inflation. It also has comparable forecasting performance to standard benchmarks.

* The views expressed in this paper are those of the author(s) and do not necessarily reflect the views of the Reserve Bank of New Zealand. Many thanks to Kirdan Lees and Troy Matheson for comments on earlier drafts.

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1 Introduction

Over the past two decades, inflation targeting has become the primary objective of many central banks. Thus, accurately measuring and reliably forecasting inflation has become increasingly important.

Cecchetti (1997) notes that, from a central bank's point of view, one of the main problems associated with measuring inflation concerns the presence of short-lived shocks that should not influence policy makers' actions. He refers to such shocks to headline inflation as *noise*. Cecchetti identifies the causes of such noise to include: "changing seasonal patterns, broad-based resource shocks, exchange-rate changes, changes in indirect taxes and asynchronous price adjustment."

In most situations, noise is defined in terms of its cause rather than content. However, in the context of inflation targeting, the main feature of noise is not its cause but its duration. Short-lived shocks in inflation, regardless of their cause, are unlikely to affect inflation expectations and will disappear before a response in monetary policy takes effect. So, such shocks can, and should, be ignored by central bankers when measuring inflation.

Core inflation measures are an attempt to strip short-lived shocks from longer term trends in headline inflation. There is no universally accepted definition of core inflation in the literature. Most authors, however, agree that there are two key attributes that an ideal core inflation measure should have (Silver 2007). Firstly, a good core measure should reduce volatility in its parent series so that central banks do not overreact to transitory shocks. Secondly, a core inflation measure should be a useful predictor of future inflation. Some authors like Cecchetti (1997) or Bryan, Cecchetti, and II (1997) use the former property to define core inflation, while others like Blinder (1997) use the latter. In practice, we would like our measure to have both these properties.

In this paper, we construct a core inflation measure from annual inflation in New Zealand's Consumer Price Index (CPI). New Zealand is a good test case for this analysis because the Reserve Bank of New Zealand (RBNZ) operates an explicit medium-term inflation targeting regime. Targeting inflation in the medium term indicates that the RBNZ wishes to isolate short-lived shocks from longer trends in inflation – precisely the objective we have in this paper. Further, the RBNZ already uses and publishes a range of core inflation measures in its Monetary Policy Statements (MPS) against which we can compare the performance of our proposed measure.

A basic mathematical representation of the problem of measuring core inflation is:

$$\pi_t \;=\; \pi_t^* + \varepsilon_t,$$

where π_t is headline inflation at time t, the core or trend inflation is π_t^* and ε_t is the volatile, non-smooth portion of the signal that we have defined to be noise.[1] We would like to isolate short-lived, self-reverting, transitory phenomena from longer term trend inflation. In this paper, we examine a core measure constructed using wavelet analysis.

Wavelets were specifically designed for isolating short-lived phenomena from long term trends in a signal. Since their inception the early 1980s, wavelet research has exploded. According to Crowley (2007), in the past 15 years, 1600 articles and papers have been published using wavelet methods in a wide range of disciplines including: acoustics, astronomy, engineering, forensics, geology, medicine, meteorology, oceanography and physics.

Wavelet methods have been popular due to their computational efficiency, flexibility and overall superiority to already established techniques in analysing and transforming data. Wavelet methods have lead to paradigm shifts in many disciplines, sometimes replacing tried and tested methods like the immensely popular Fourier transform. One of the greatest strengths of wavelets over conventional frequency-domain techniques is their ability to deal with non-stationary, badly behaved data.

Wavelets have not been used as extensively in economics as they have been in the hard sciences, but they are beginning to enter into the mainstream. Cotter and Dowd (2006) have investigated the use of wavelets for measuring U.S. core inflation. Our analysis expands upon their work by carrying out a more exhaustive analysis and by addressing some of the practical issues that would potentially stop a wavelet measure from being adopted by a central bank or other policy maker. Essentially, compared with Cotter and Dowd (2006), we use a more flexible non-orthogonal wavelet transform, we consider a range of thresholding algorithms, we investigate the robustness of our wavelet measure to boundary conditions, we examine the real time properties of our wavelet measure and we explore its forecasting properties.

The outline of this paper is as follows. In section 2 we describe the diagnostics

[1] As Ramsey (2002) points out there is a distinction to be made between denoising and smoothing a signal. In general denoising and smoothing have different goals. However, in applications where the underlying signal is assumed to be smooth and the noisy process is assumed to be non-smooth, the distinction is immaterial. Given that we have defined our "noise" in terms of its duration and volatility, we can legitimately refer to our construction of a measure for core inflation as a denoising exercise.

we use to assess the performance of core inflation measures. In section 3 we provide a quick primer on wavelets and introduce some of the basic ideas used in our analysis. Section 4 describes our methodology, and section 5 presents our denoising results and describes some of the real time properties of our proposed measure. In section 6, we turn our attention to forecasting. Finally, we make some concluding remarks in section 7.

2 Definitions and Diagnostics

We use two sets of diagnostics to assess the performance of various measures of core inflation. The first set of diagnostics relates to a measure's ability to denoise headline inflation. The second set of diagnostics relates to a measure's usefulness in forecasting.

Following Cotter and Dowd (2006), to gauge a measure's denoising properties we perform the following checks: (i) we check to see if it is unbiased, since bias in a core inflation measure hampers its credibility; (ii) we compare its variance to the variance in CPI inflation, since a lower variance indicates less volatility; (iii) we compare its number of turning points to that of CPI inflation, since fewer turning points indicates lower sensitivity to shocks; and, (iv) we test the stationarity of its residuals, since we want a core measure which is co-integrated with its parent series.

These tests provide information about a measure's ability to reduce volatility. Unfortunately, these objective measures are not sufficient for assessing the performance of a core measure; for example, according to such diagnostics the optimum measure for core inflation is a constant average. Clearly, we want *some* variation to be preserved – namely the type of long-term variation that merits a response in monetary policy. To this end, we use two tests to check for oversmoothing.

First, we compare the variance of core measures to that of a 7 quarter centred moving average (CMA). The reason for adopting this diagnostic is that the RBNZ explicitly targets medium-term inflation. According to the RBNZ, changes in monetary policy can take up to 8 quarters to take effect.[2] We use a CMA in lieu of other appropriate statistical diagnostics so that the decision making process is transparent and objective. Second, we regress inflation at time t against core measures at time $t-1$. If a core measure is less correlated

[2] See, for example, http://www.rbnz.govt.nz/publications/3064172.pdf

with future inflation than headline inflation itself, it is likely to have removed useful information from the trend.

In addition to reducing volatility, some researchers have emphasised credibility as an important property of the ideal core inflation measure. For example, Roger (1998), Wynne (1999) and Rich and Steindel (2005) argue that an ideal measure of core inflation should: (i) track changes in headline inflation, (ii) not be subject to revisions, (iii) be robust and unbiased and (iv) be understandable by the public.

As Wynne (1999) emphasises, however, such properties are only relevant to the extent that a central bank uses a core inflation measure as an important part of its routine communication with the public to explain policy decisions. Since the RBNZ currently publishes its core inflation measures for the public, we also assess our core inflation measure based on these additional criteria.

To gauge a measure's usefulness in forecasting, following Laflèche (1997), we use prediction errors from an AR model to judge a measure's predictive content. This diagnostic should be treated cautiously since it is not just influenced by the core measure but also by the particular forecasting model we choose to employ. Despite this shortcoming, such a diagnostic is likely to provide guidance about the predictive content of a core measure.

3 A Primer on Wavelets

To understand wavelets, we first need to define some concepts.[3] Consider the inner-product space:

$$L_2(\mathbb{R}) \equiv \left\{ f : \int_{-\infty}^{\infty} |f(t)|^2 \, \mathrm{d}t < \infty \right\},$$

with the inner-product:

$$\langle f, g \rangle \equiv \int_{-\infty}^{\infty} f(t) g(t) \, \mathrm{d}t.$$

[3] In the past decade, many excellent expositions of wavelet theory have been written from different points of view and catering to different audiences. Schleicher (2002) provides an excellent introduction to wavelets for economists which can serve as an entry point for the uninitiated. Percival and Walden (2000) is a thorough and technically sound text geared towards the use of wavelets in statistical applications, and Nason (2008) is a recent and accessible monograph with an emphasis on the details of wavelet implementation using freely available software.

In its most general form, our analysis is confined to this space. This gives us the broadest understanding of wavelets and their usefulness; we shall migrate to less general, discrete spaces as the need arises.

Wavelets are mathematical functions which satisfy a few conditions: If $\psi(t)$ is a wavelet, then:

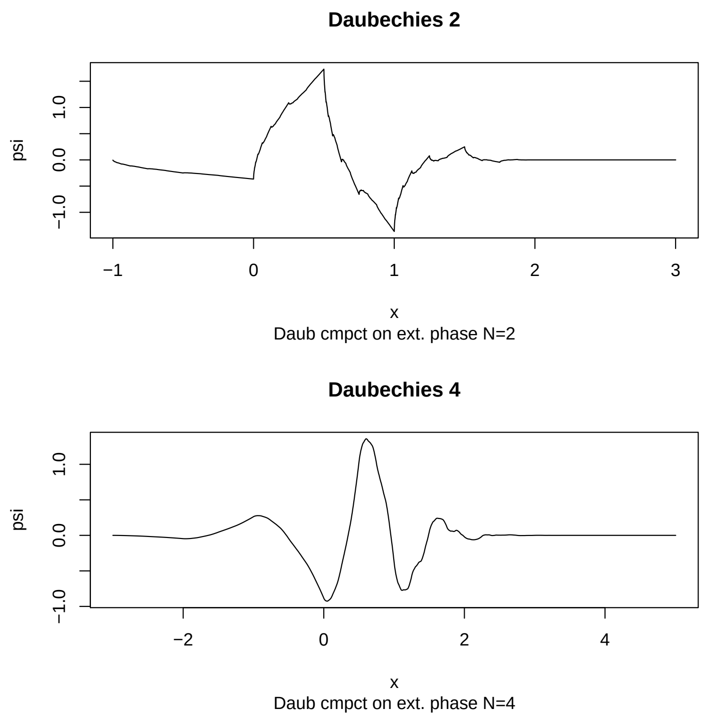
$$\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt = 1$$

and:

$$\int_{-\infty}^{\infty} \psi(t) \, dt = 0.$$

The first condition ensures that the function is "little", while the second ensures that it is a "wave". Basically, wavelets are oscillatory functions which decay quickly.[4] Some example wavelets are presented in figure 1.

Figure 1
Example $\psi(t)$ functions, Daubechies 2 and Symmlets 8



Given a wavelet $\psi(t)$, which we shall call the mother wavelet, we can generate

[4] There are other conditions that can be placed on $\psi(t)$ depending on the application.

daughter wavelets $\psi_{j,k}(t)$ defined as

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k).$$

Daughter wavelets are translations and dilations of their mother, and they are indexed by two integers, j and k. The first index j refers to scale – that is, the length of a wavelet in time. The second index k refers to position – that is, the location of a wavelet in time. Note that with this notation, $\psi_{0,0}(t)$ corresponds to our initial mother wavelet $\psi(t)$. The set consisting of all $\psi_{j,k}(t)$ is an orthonormal basis for $L_2(\mathbb{R})$.

Wavelet families allow us to perform multiresolution analyses (MRAs) of functions belonging to $L_2(\mathbb{R})$. Such analyses give us wavelet series, which are decompositions of square integrable functions into components of different volatility (or, as it is called in the wavelet literature, scale). The idea is similar to the one behind the construction of Fourier series. Standard Fourier series are representations of periodic functions in terms of sinusoidal components of differing frequency. Wavelet series, on the other hand, are representations of square integrable functions in terms of components of differing scale.

Since wavelets are localised in both time and frequency, they are far better than Fourier series at representing non-stationary signals. This is because wavelets are designed to pick up one-time shocks, discontinuities and structural breaks. For more information on the relationship between Fourier analysis and Wavelet analysis see Priestly (1996).

Since a family of wavelets forms an orthonormal basis for $L_2(\mathbb{R})$, if $f(t)$ belongs to $L_2(\mathbb{R})$, then it has the wavelet representation:

$$f(t) = \sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}}\langle f, \psi_{j,k}\rangle\psi_{j,k}(t). \tag{1}$$

Associated with $\psi(t)$ is a related function $\phi(t)$ called the scaling function. We denote the translations $\phi(t-l)$ by $\phi_l(t)$. The scaling function is intricately linked to the mother wavelet by way of a dilation equation. For our purposes, it suffices to think of the scaling function as an averaging function and the mother wavelet as a differencing function.

The scaling function is useful because its translations span the same space as the one spanned by all daughter wavelets whose scales are negative. That is:

$$\text{span}\left(\{\psi_{-j,k}(t) : k \in \mathbb{Z}, j \in \mathbb{N}\}\right) = \text{span}\left(\{\phi(t-l) : l \in \mathbb{Z}\}\right).$$

This fact allows us to rewrite (1) as:

$$f(t) = \sum_{l \in \mathbb{Z}} \langle f, \phi_l \rangle \phi_l(t) + \sum_{j=0}^{\infty} \sum_{k \in \mathbb{Z}} \langle f, \psi_{j,k} \rangle \psi_{j,k}(t).$$

This decomposition has a very intuitive interpretation. The first sum picks out the underlying smooth trend in the data by projecting f onto translations of the scaling function. The second double sum picks out varying levels of detail: The inner sum translates a wavelet of scale j, while the outer sum changes the scale of the translations. An example decomposition using Haar wavelets (Haar wavelets are step functions) can be seen in figure 2.

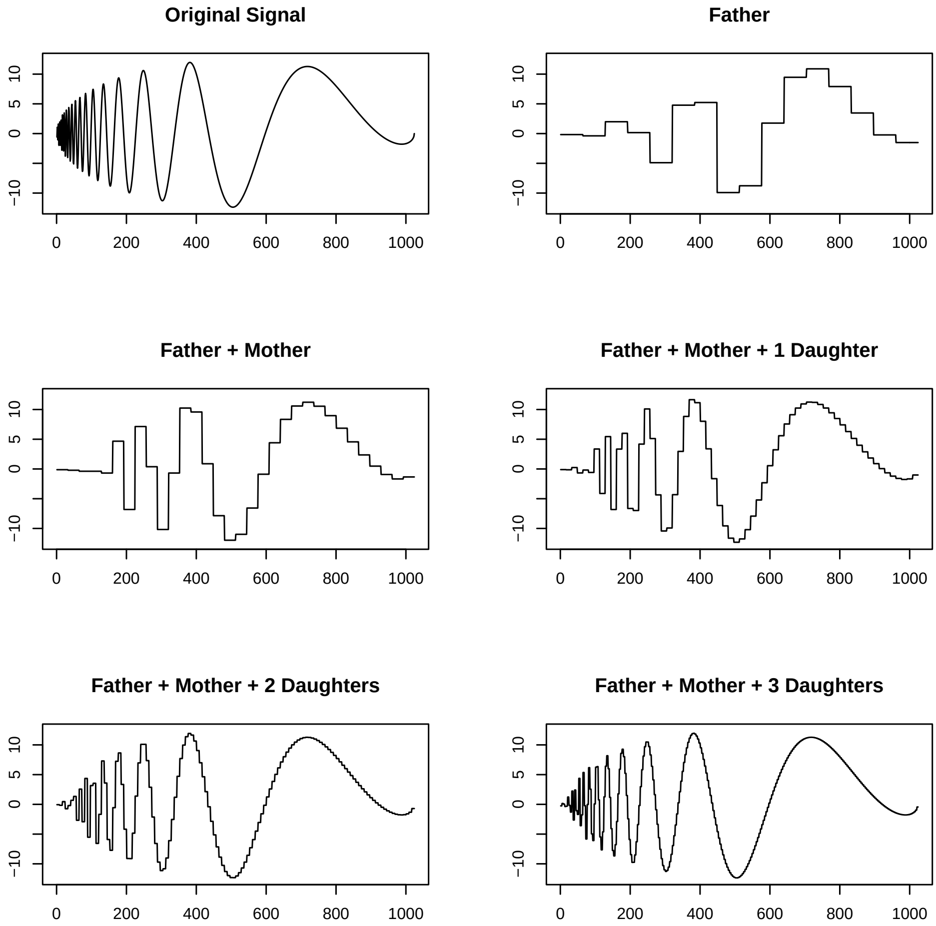
In keeping with the family motif, the function $\phi(t)$ is sometimes referred to as the father wavelet. However, the "father wavelet", otherwise known as the scaling function, is not a wavelet because it does not integrate to zero. Regardless, this is a convenient name that we shall use interchangeably with scaling function.

It should be noted that the father wavelet can be scaled in exactly the same way as the mother wavelet. This produces "son" wavelets, which are dilated copies of their father. Through dilations and subsequent translations, we obtain two doubly-infinite double sequences of functions $\{\phi_{j,k}(t)\}_{j,k \in \mathbb{Z}}$ and $\{\psi_{j,k}(t)\}_{j,k \in \mathbb{Z}}$. Given the nature of these sequences, the choice of father and mother wavelet (corresponding to $j = k = 0$ in both sequences) is arbitrary; once we have these sequences, we can pick functions of any scale level to be our father and mother wavelets.

When dealing with discrete time processes, such as those commonly seen in economics, we move from the function space of $L_2(\mathbb{R})$ to the sequence space of l_2, with the implication that integrals are replaced by sums; the underlying idea of wavelet decompositions, however, remains the same. Further, when dealing with finite discrete data, as will always be the case in practice, we can go even further, migrating from l_2 space to the friendly and familiar finite dimensional Euclidean space $\mathbb{R}^n$. This again simplifies matters and allows us to think of a wavelet family as simply a basis for $\mathbb{R}^n$. In this case, due to the fact that there is only a finite amount of information contained in a signal, only a finite number of scales are needed for a full wavelet decomposition. More precisely, if x belongs to $\mathbb{R}^n$, then:

$$x = \sum_{l=0}^{n} (x \cdot \phi_l) \phi_l(t) + \sum_{j=0}^{\log_2 n} \sum_{k=0}^{n} (x \cdot \psi_{j,k}) \psi_{j,k}(t).$$

Figure 2
An example additive decomposition in scale



We can rewrite any $n \times 1$ vector in terms of elements of the wavelet basis using simple dot products; the interpretation of this transformation remains the same as before, so that we still have a decomposition in terms of volatility.

The coordinates of a vector in the wavelet basis are collectively called the Discrete Wavelet Transform (DWT) of the vector.[5] Unfortunately, the DWT has some critical shortcomings that can vastly reduce its usefulness to economists.

[5] A worked example of a DWT can be found in Schleicher (2002).

The biggest of these flaws is that the DWT imposes stringent requirements on the length of input vectors. For a full decomposition, the use of the DWT requires the input vector to be of length 2^n, where n is a natural number. This requirement is somewhat relaxed in the case of a partial decomposition, where the vector length needs to be an integer multiple of 2^J, where J is the maximum scale. Such restrictions have grave implications for economic applications where data is often scarce and valuable.[6]

The second problem is that the DWT is very sensitive to the origin of the time series. Wavelet coefficients and decompositions will change in real time as new information becomes available. Somewhat related to this are artefacts that can be introduced into the transform if wavelets fail to line up with the important features of the data properly.

Fortunately, there is an alternative non-orthogonal transform, called the Maximal Overlap DWT (MODWT), which retains the desirable properties of the DWT that we are interested in, but satisfactorily deals with these problems.[7] We will provide more details of this transform in the next section.

Once a wavelet transform has been computed, a raft of different shrinkage, or thresholding, algorithms exist to denoise the transform. We mention some of these algorithms in the next section, but the basic idea behind most of them is to set some of the wavelet coefficients to zero. The choice of which wavelet coefficients should be zeroed is dictated by a thresholding rule that produces the optimum outcome given certain assumptions about the nature of the noise.

While much work has been done on the subject of wavelet thresholding, relatively little work has been done on wavelet forecasting. Ramsey (2002) discusses the fundamental theoretical problem of extrapolatory forecasting in general – that local fits need not yield good global forecasts.

Notwithstanding this pessimistic outlook, tentative efforts have been made at using wavelets in forecasting and the results have been promising. The use of wavelet methods in conjunction with normal forecasting methods seems to produce forecasts that are at least as good as the regular, non-wavelet versions (Crowley 2007). Specialised forecasting procedures that take advan-

[6] This corresponds to a decomposition where the length of the father wavelet is shorter than the length of the time series, which is achieved by shifting the scale index j.

[7] Wavelet transforms very similar (or even identical) to MODWT go by many different names in the literature, such as, the "Stationary wavelet transform", the "Translation-invariant wavelet transform", the "Undecimated wavelet transform", the "à trous wavelet transform" or the "Shift invariant wavelet transform".

tage of the multiscale nature of wavelet transforms have also been developed in recent years, with an emphasis on forecasting non-stationary series; examples can be found in Arino, Morettin, and Vidakovic (2004), Renaud and Murtagh (2002), Ahmad, Popoola, and Ahmad (2005) and Zhang, Coggins, Jabri, Dersch, and Flower (2001).

In our case, it is clear why wavelets should be able to help remove shocks from headline inflation ex-ante. However, there is no ex-ante evidence that this should then produce better forecasts of future headline inflation. As Marques, Neves, and da Silva (2002) point out, there is no reason why a good core inflation measure, stripped of noise, should be able to forecast future transitory changes in headline inflation better. In fact, if the noise is serially correlated, then a core measure is likely to be a worse forecaster than headline inflation itself. It turns out that the wavelet measure performs relatively well in forecasting. However, this result is not supported by a strong theoretical justification.

4 Methodology

We began by computing the MODWT of headline inflation using different wavelet families. There are many different treatments of the details of the MODWT available and the particular implementation used in this analysis is due to Percival and Walden (2000).

We chose the MODWT over the more conventional orthogonal DWT because, by giving up orthogonality, the MODWT gains attributes that are far more desirable in economic applications. For example, the MODWT can handle input data of any length, not just powers of two; it is translation invariant – that is, a shift in the time series results in an equivalent shift in the transform; it also has increased resolution at lower scales since it oversamples data (meaning that more information is captured at each scale); and, finally, excepting the last few coefficients, the MODWT is not affected by the arrival of new information.

We initially restricted our analysis to the most popular wavelet families. We analysed the Haar Wavelet family, the Daubechies wavelet family with vanishing moments 2, 4, 6 and 8 and the Symmlets wavelet family with vanishing moments 4, 6, 8 and 10 (Daubechies 1992). We also analysed the Complex Daubechies Wavelet family (Lina and Mayrand 1995).

Since the MODWT oversamples data, the choice of wavelet family did not affect the decomposition appreciably. So, the Haar wavelet family (or equivalently Daubechies wavelet family with one vanishing moment) was chosen as the ideal orthonormal basis for the decomposition. The Haar wavelets were chosen, because while they captured pertinent features of the data as well as other wavelet families did, they were the shortest and therefore the least susceptible to retrospective revisions and boundary conditions. The fact that they were symmetric, intuitive and had simple closed form expressions also weighed in the decision to select them.

In order to deal with boundary conditions, the time series was padded with the average inflation rate of the previous 8 quarters. Unfortunately, theory gives us no useful insight into extending series for the coefficients at the end, so the extension method was chosen based on empirical performance. Ideally, we would like to pad the series with our best forecast of future headline inflation. While we find that our denoising results are robust to different padding methods, our forecasting results are more sensitive. More information about this can be found in section 6, where we present robustness checks.

Furthermore, the decompositions were carried out with four wavelet scales (the maximum possible scale for our series was 6), in order to minimize boundary effects on the wavelet coefficients while still capturing relevant features in the data. More information about boundary effects can be found in Percival and Walden (2000).

Once the transforms were computed, we ran thresholding procedures to shrink wavelet coefficients. The thresholding algorithms used were: Universal Thresholding (Donoho and Johnstone 1994), SURE Thresholding (Donoho and Johnstone 1995), Bayesian Thresholding (Johnstone and Silverman 2005), Complex Universal Thresholding (Barber and Nason 2004) and linear thresholding.

Our results showed that the best performing thresholding algorithms performed similarly to simple linear thresholding. Linear thresholding is where we simply discard noisy daughter wavelets, leaving behind a smoothed trend line. This is justifiable theoretically because we have defined our noise to be short-term fluctuations in headline inflation that do not last into the medium term. Since the last two daughter wavelets are picking up exactly these fluctuations in the data, we can safely discard them. The fact that these daughter wavelets also pass Shapiro-Wilk tests for normality, indicating that they could have been drawn from a random normal distribution, gives us some statistical reassurances about removing them.

To a large extent, the number of daughter wavelets that we choose to discard is dictated by our ex-ante definition of noise. We discarded the last two daughters because we were aiming to estimate a medium term (approximately 2 year) measure of inflation; if we modify this objective, the number of discarded daughter wavelets changes.

Since linear thresholding is intuitive, readily justifiable and has well-behaved real time properties (we simply discard the last two scales for all time), we chose it as our optimum wavelet measure of core inflation. We refer to this optimum measure of core inflation as the Wavelet Inflation Measure (WIM).

This analysis was carried out using the freely available WaveThresh and Waveslim packages for the open source statistical software environment, R. We hope that the availability of these tools makes our results easily reproducible and leads to further developments in the use of wavelet techniques for measuring and forecasting inflation.

4.1 Existing Measures of Core Inflation

Some of the methods currently in vogue can be classed into several different categories:

Exclusion Measures: These measures remove some components of the CPI, such as food and energy prices. These measures are easy to implement and are easily understood by the public. They are not sensitive to statistical assumptions and are not subject to frequent revisions. But, a large number analyses, starting with Cecchetti (1997) and confirmed by Armour (2006) and Heath, Roberts, and Bulman (2004) among others, report that these measures are actually worse than headline inflation when judged on the two criteria we have outlined (reduction in volatility and usefulness in forecasting). In other words, they can strip the signal of useful information without reducing volatility.

Limited Influence Measures: These measures are a less drastic version of exclusion measures, where instead of excluding pre-determined factors outright, factors are assigned a weighting in accordance to some criteria (usually one or both of the two we have outlined). Such measures include the trimmed mean measure, the weighted median measure and the double weighting measure.[8] These measures are more robust than exclusion measures but are sensitive to the distribution of prices. Trimmed mean and weighted median measures

[8] The double weighted measure weights items firstly by their expenditure share and secondly by the reciprocal of the standard deviation of relative price changes.

have been shown to be vulnerable to bias (Armour 2006). Further, according to Armour (2006), these measures have only limited use in reducing volatility and are not especially useful for forecasting.

Dynamic Factor Models: These use disaggregated data and a factor model to measure core inflation. The implementation we use in this paper is due to Giannone and Matheson (2007) and is currently reported by the RBNZ in the MPS.

For further discussion and evaluation of these and other core measures see Silver (2007), Armour (2006) or Heath, Roberts, and Bulman (2004). In this paper, we compare the performance of our proposed core measure to those measures that are currently published by the RBNZ – that is, the weighted median, the trimmed mean and the dynamic factor model. Our measure differs from these measures in that it only uses aggregated prices; the rest rely on a much bigger disaggregated data set.

5 Denoising Results

The denoising test results are displayed in table 1.[9] The WIM had significantly fewer turning points than the alternatives. It is also worth pointing out that the WIM did not introduce any turning points that it did not keep for all time. Thus, the appearance of a turning point in the WIM, in real time, appears to be a reliable indication of a change in the trend for medium term inflation. This property makes the WIM very useful to a policy maker that is trying to react to changes in underlying medium term inflation in real time.

The WIM, by virtue of construction, will never be very vulnerable to bias. Any bias that appears in the data is a result of how we choose to deal with boundary conditions. These boundary conditions will only affect the last few wavelet coefficients and their influence on the overall mean of the series will always be extremely limited.

The results of the tests for over-smoothing are displayed in table 2. The correlation between the core measure and headline inflation is in the first column, and the ratio of the variance of core inflation to a 7-quarter CMA is in the second column. If a core measure has a ratio which significantly differs from unity, then it is likely to have either retained too much volatility

[9] The residuals from all core measures easily passed stationarity tests, so these results have been omitted.

Table 1
Table of Smoothing Results: 1992Q4-2008Q3

Core measure	No. of Turning points	Mean	Variance
Headline	28	2.325	0.639
WIM	**9**	2.300	0.378
Trimmed Mean	20	2.251	0.650
Weighted Median	20	1.814	0.426
Factor Core (last vintage)	25	2.294	0.237

Table 2
Results for over-smoothing: 1992Q4-2008Q3

Core Measure	Correlation	Variance Ratio	R^2
Headline	1.000	1.214	0.669
WIM	**0.933**	**1.120**	**0.776**
Trimmed Mean	0.922	1.829	0.646
Factor	0.889	0.790	0.599
Weighted Median	0.729	1.256	0.398

or stripped too much information from the signal for the medium-term time frame that we are interested in. In the final column, we display the coefficient of determination from a regression of headline inflation on lagged core inflation.

The WIM performs well on these tests and seems to have struck the right balance between reducing volatility but retaining important information. The WIM exhibited a significantly higher R^2 than other core measures of inflation. This test is a traditional pitfall for most core inflation measures because they fail to provide better R^2 values than headline inflation itself; the WIM was the only core measure in our analysis that managed this.

Lastly, we also visually inspected the various measures of core inflation. As can clearly be seen in the appendix, the WIM provides a very intuitive, smooth trend line with very few turning points. It picks out movements in inflation that appear more likely to require the attention of policy makers but filters out short-term shocks.

5.1 Real Time Properties

By construction, the WIM is robust to real time changes. It is theoretically impossible for new information to change core inflation values beyond four years and to affect a four year revision infeasibly volatile data would need to be inserted. In practice, only the last few quarters are ever revised. In our empirical analysis, the average change made to the previous quarter as new information became available was 0.162, the average change made to two quarters ago was 0.102 and the average change to three quarters ago was 0.048; beyond this, no changes occurred.[10]

Furthermore, these revisions were generally corrections in magnitude, not sign. This meant that the WIM tracked changes in headline inflation as new information became available. As Rich and Steindel (2005) point out, the ability of a core measure to track headline inflation enhances its credibility for use by policy-makers. As such, the close coherence the WIM shows to headline inflation, coupled with its robust real time properties, imbue it with a degree of credibility and transparency that increase its usefulness to central banks.

6 Forecasting Results

To measure predictive usefulness, we compared the mean absolute errors of AR forecasts computed from different core measures.[11] We point out that we used future headline inflation, not future core inflation, to compute forecast errors. For completeness, we also included the forecast errors from the RBNZ's official forecasts (MPS forecasts) in our tables. In the case of the WIM, we took advantage of the wavelet decomposition by forecasting each scale separately and adding the results to obtain an overall forecast.[12] It should be noted that the MPS forecasts are performed using different forecasting methods than the rest and incorporate a considerable amount of judgement. For this reason, it is difficult to compare the MPS forecasts directly with mechanical AR forecasts from core measures.

[10] These are averages of magnitude, so sign changes do not cancel out. The regular averages are almost zero.

[11] An AR(4) process yielded the best forecasts for all core measures, with the exception of the WIM.

[12] This means that an AR(1), AR(2) and AR(2) process were used for the first, second and third scales respectively.

Table 3
Forecast error comparisons for different padding methods: 2001Q1-2006Q4

Padding Method	Mean Absolute Error over 8 quarters
MPS forecasts	0.536
Last factor model value	0.592
Last trimmed mean value	0.651
AR(4)	0.664
Mean padding	0.673
8-quarter mean padding	0.717
Last headline value	0.738

Table 4
Forecast error comparisons: 2001Q1-2006Q4

Core measure	Mean Absolute Error over 8 quarters
Trimmed mean AR(4)	0.654
Factor model AR(4)	0.671
Headline AR(4)	0.734
Weight median AR(4)	0.889
MPS forecasts	0.516

We present forecast errors from the WIM based on different padding methods in table 3, and we present forecast errors from other measures of core inflation in table 4. Unlike our denoising results, the forecasting results from the WIM vary noticeably depending on our choice of padding method. As can be seen, the forecasting performance of the WIM is similar to that of the method we pad the data with.

7 Concluding Remarks

We used wavelets to decompose headline NZ inflation in terms of volatility and discarded the most volatile components. We found that this yielded a measure of core inflation which outperformed the alternatives currently used by the RBNZ in terms of nowcasting medium term inflation. Our wavelet measure also had excellent real time properties and was highly coherent with headline inflation. We also found that, depending on the padding method, the wavelet measure produced competitive forecasts.

We conclude that our wavelet measure has the performance, credibility and perspicuity needed for it to be a suitable tool for central banks and other policy makers. We believe that wavelets are a very promising avenue for further research into the analysis and forecasting of economic and financial data.

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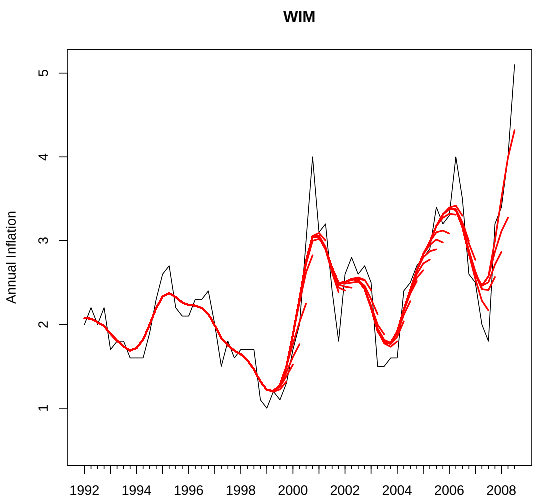
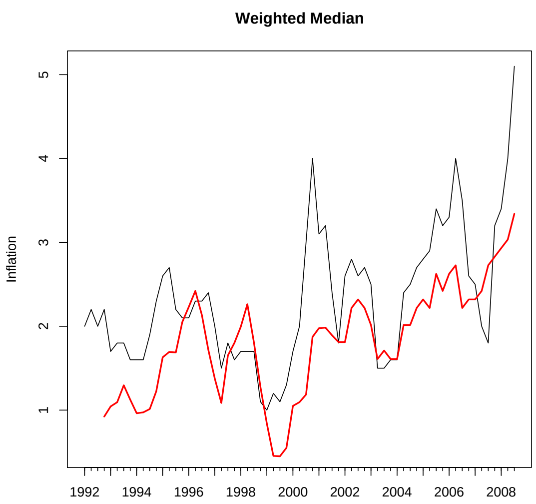
Appendix

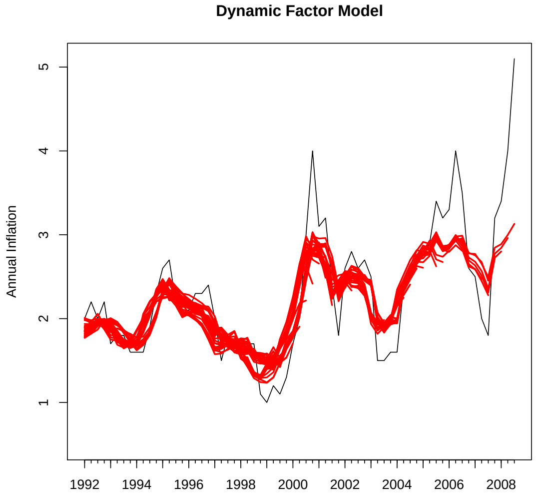
Headline inflation and the WIM (35 vintages)



Headline inflation and the weighted median



Headline inflation and the dynamic factor model (35 vintages)



Headline inflation and the trimmed mean