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A model of spatial arbitrage with transport capacity constraints and endogenous transport prices

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Abstract

This article solves a high frequency model of price arbitrage incorporating storage and trade when the amount of trade is limited by transport capacity constraints. In equilibrium there is considerable variation in transport costs, because transport costs rise when the demand to ship goods exceeds the capacity limit. This variation is necessary to attract shipping capacity into the industry. In turn, prices in different locations differ by a time varying amount. Thus while the law of one price holds, it holds because of endogenous variation in transport costs.

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[1] The views expressed in this paper are those of the author(s) and do not necessarily reflect the views of the Reserve Bank of New Zealand. This paper was begun while I was a faculty member in the Department of Economics at the University of Michigan. I would like to thank my colleagues there for their helpful comments during the time that this article was written. I would also like to thank Shaun Vahey, and seminar participants at the University of Indiana. Contact information: Economics Department, Reserve Bank of New Zealand, 2 The Terrace, PO Box 2498, Wellington; phone +64 471 3778; fax +64 4 473 1209; email Andrew.Coleman@rbnz.govt.nz.
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1 Introduction

Economic models of the law of one price have a long history, traceable to Cournot (1838). Much of this literature has developed and estimated empirical models that examine how the law of one price has held in practice. A smaller strand has developed models that examine how the structure of the transport industry affects prices. This article is in the latter tradition. It develops a model that examines how commodity prices in two locations are determined when there is a fixed quantity of transport equipment and agents use logistics management techniques to arbitrage prices across space and time.

Formally, the model is a rational expectations model of trade between two centres that incorporates transport system capacity constraints, transportation time, storage, and uncertainty. The two centres have separate demand functions and stochastic supply functions, and forward looking, risk neutral agents store or ship goods to take advantage of arbitrage possibilities. The key feature of the model is the behaviour of transport prices when the demand to ship goods is large. Because transport prices are determined competitively and there is only limited shipping capacity, transport prices have peak-pricing patterns similar to those exhibited by other capital intensive and capital constrained industries, such as the electricity industry (Williamson 1966). In particular, the transport price equals the marginal transport cost when the amount shipped is less than full capacity, and equals the difference between the spot price in the exporting city and the discounted expected future price in the importing city when the amount shipped is equal to full capacity. As such, demand or supply shocks in the commodity markets cause endogenous variation in transport prices, and, in parallel with other "peak-pricing" industries, this variation provides shipping companies with a return on their capital. Since transport price variation also causes spatial price variation, in this model time varying transport prices and spatial price differences are not simply nuisance issues that complicate the empirical analysis of the law of one price, but crucial parts of the arbitrage mechanism.

Storage is an essential part of the model. Storage ensures the prices are dynamically consistent, and it means that arbitrageurs facing high transport prices recognize the opportunity cost of not shipping, namely storing and shipping at a cheaper time. Because inventories cannot be negative, an analytic solution to the model cannot be found, but a numerical solution is found using techniques developed by Williams and Wright (1991) and

Deaton and Laroque (1992, 1996). The solution comprises the optimal quantities that are shipped, stored, and consumed, along with the equilibrium distributions of commodity and shipping prices. These distributions depend on the primitives of the model, such the distribution of output shocks.

The solution reveals the inter-relationship between commodity and transport prices. Ordinarily, both centres have sufficient inventories to smooth small fluctuations in the local supply of the commodity. When a centre has a small supply shock, arbitrageurs adjust their inventories to compensate for the shock, and the difference between prices in the two cities is less than or equal to the marginal cost of shipping, adjusted for storage costs. If a city has a large negative supply shock, however, local inventories are run down, and new supplies are ordered from the other city. Because the imported goods do not arrive immediately, the spot price in the importing centre temporarily exceeds the spot price in the exporting centre plus the transport cost. The transport cost depends on the amount shipped. If it is only necessary to ship small amounts, the transport cost will be the marginal transport cost. If the shortage in the importing city is sufficiently severe, shippers attempting to take advantage of the expected high prices in the importing city will drive up the transport price until it just equals the difference between the export price in the export centre and the discounted expected future price in the importing centre.

The model is initially solved for a fixed shipping capacity constraint, and is used to calculate the average earnings of the transport sector. As the shipping capacity increases, shipping constraints bind less frequently, so the transport price exceeds the marginal transport cost less frequently and average earnings decrease. In turn, the lower variability of transport prices means that the variance of the spatial price difference decreases.

Because entry into the shipping industry is assumed to be competitive, the model is iterated to find the shipping capacity that generates a competitive rate of return. This means the effect of endogenously determined shipping constraints can be examined. For example, the article examines how an increase in the variance of the underlying output shocks affects prices. As the underlying output shocks in the model are increased, the variance of shipping prices, the profitability of the shipping industry, and the variance of the spatial price difference increases for any level of shipping capacity. The additional profitability of shipping attracts more shipping capacity into the market. Nonetheless, if shipping capacity increases until the return equals the competitive rate of return, both the variance of shipping prices and the variance of the spatial price difference will be higher than the initial levels. Shipping companies increase their capacity in response to more volatile output, as their peak demands are higher; but they operate below capacity for a greater fraction of the time, so their earnings are more volatile.

The focus of this article is theoretical. Nonetheless, the article is inspired by descriptions of spatial arbitrage in markets where transport capacity constraints have been important, such as the corn market in north-east United States in the late nineteenth century (US Congress 1874; Coleman 2004) and the wheat market in Western Australia in the late twentieth century (Brennan, Williams and Wright 1997). Transport capacity constraints will not be important in all markets, although in most markets one would expect owners of transportation and distribution systems to restrict their purchases of capital intensive transport machinery and adopt logistics management practices to ensure their machinery is utilized as much as possible. Thus while the model is an abstraction, it provides some insight into the way commodity prices and transport costs are simultaneously determined in circumstances where transport capacity is not perfectly elastic, and transport markets are equilibrated through variable transport prices.

2 Previous literature

In recent years, several authors have demonstrated that empirical analyses of spatial arbitrage must take into account transport cost variation. For example, Goodwin, Grennes, and Wohlgenant (1990) estimated a model of the grain trade between the United States, Japan, and Europe that allowed for time varying transport costs and shipping times and showed that both factors were important in explaining price differences between the regions. Using a different approach, Spiller and Wood (1988) developed a switching regime model that estimated the distribution of transport costs from spatial price differences under the assumptions that arbitrage takes place instantaneously and that transport costs follow a parametric distribution such as a truncated normal or a gamma distribution. Their estimates of transport costs revealed considerable time-variation. In turn, this framework was significantly expanded by Baulch (1997) and Barrett and Li (2002), who developed parity bound models that directly incorporate transport cost information into the estimation procedure.

From a theoretical perspective, these approaches have two potential limitations. First, they are designed to examine prices conditional on transport prices and trade flows, rather than to examine the potentially endogenous determinants of transport prices and trade flows. Thus while they show that spatially separated markets tend to be well linked through arbitrage, conditional on transport costs, they do not explain why the variation in transport prices occurs. Second, these models ignore how storage affects the arbitrage process, not least by enabling shippers to substitute away from high cost shipping times to low cost shipping times. To this extent, the literature on spatial price arbitrage has evolved somewhat separately from the literature on logistics management, where the interaction of inventory management and transport systems is central, as well as much of the transport economics literature, where it is commonly assumed that transport costs vary endogenously because of capacity constraints (Tyworth 1991; Stopford 1988).

The model in this article is designed to integrate these different approaches. In the taxonomy developed by Fackler and Goodwin (2001), the model is a dynamic point-location model of spatial arbitrage that incorporates uncertainty and rational expectations, transport delays, capacity constraints, storage, and endogenously determined transport costs. As Fackler and Goodwin note, models of spatial price determination with some of these features already exist, so some of the results of this article are not new. What is distinctive about the article, however, is the use of rational storage behaviour to generate the intertemporal price dynamics, while other features of the transport system such as time delays and capacity constraints are also modeled.

Rational storage models have a long history, including Williams (1936), Gustafson (1958), Williams and Wright (1991), and Deaton and Laroque (1992, 1996). There are two small literatures that simultaneously analyse trade and storage behaviour. The first, including Williams and Wright (1991) and Coleman (2004), explicitly examines the implications for prices in different locations when agents use both storage and transport technologies to arbitrage prices. Both of these articles examine trade under uncertainty when the supply of transport services is infinitely elastic and the price is fixed. The second, including Wright and Williams (1989), Benirschka and Binkley (1995), and Frechette and Fackler (1999), examines why storage under backwardation is frequently observed. Technically, the model in this article is quite similar to Coleman (2004), except it relaxes the assumption that the supply of transport services is perfectly elastic, and thus allows transport prices to be determined endogenously. Conceptually, the model is more closely related to the Wright and Williams (1989) model that offers an explanation for storage under price backwardation.

Wright and Williams observed that most inventories are held in locations distant from the primary market for which spot and futures prices exist. They argued that when the spot price exceeds the future price, inventories in the primary market are run down and then attempts are made to obtain inventories held elsewhere. Some inventories are moved, but most are not because there are only limited transport facilities; rather, transport costs are bid up to unusually high levels, making it unprofitable to ship most of the inventories to the primary market to take advantage of the unusually high spot prices. The model in this article has the same basic idea, and can be considered a formalization of their approach in which equilibrium prices, storage quantities, shipping volumes and transport capacity are determined endogenously. It extends the approaches of Benirschka and Binkley (1995), and Frechette and Fackler (1999) by allowing for uncertainty and by relaxing their assumptions that transport is instantaneous and infinitely elastic.

3 The model[2]

The model is an extension of the rational expectations models of storage and trade developed by Coleman (2004) and Williams and Wright (1991). Following Samuelson (1952), the model links a set of equations representing supply and demand curves in different locations with a set of no-arbitrage conditions that preclude excess profits from either the storage of goods at a location or the shipment of goods between locations.

There are two centres, A and B, each with a separate inverse demand function for a commodity:

$$P_t^i = D_i^{-1}(Q_t^i) : \qquad D_i^{-1}(0) < \infty, \quad \lim_{Q \to \infty} D_i^{-1}(Q) = 0 \qquad (1)$$

where Q_t^i is the amount purchased for final use at time t and i = A, B. The output produced each period is stochastic but price inelastic, because it has a

2 The following paragraphs are adapted from Coleman (2004).

long gestation period. In the literature, output is usually modeled as a serially autocorrelated or seasonal stochastic process. Since the analytical structure of the model does not depend on this choice, in this article I follow Deaton and Laroque (1996) and assume that output in each centre follows an independent first order autoregressive process around a constant mean:

$$(X_t^i - \bar{X}^i) = \rho^i (X_{t-1}^i - \bar{X}^i) + e_t^i \qquad i = A, B \qquad (2)$$

where e_t^i is a white noise process and $|\rho^i|<1$.

All production, consumption, storage and trade activity takes place at the beginning of the period. It is assumed that unlimited quantities of the good can be stored in either centre, and that goods produced at different times are indistinguishable and have the same price. The length of a period is the time that it takes to ship goods from one centre to another. Risk neutral arbitrageurs are assumed to predict future prices, and purchase and hold inventories until the expected price increase just offsets the cost of storage. Conversely, if the expected appreciation is less than the cost of storage, inventories will be zero. In keeping with the literature, there are three storage costs. First, there is an elevator charge K^S per unit to store goods each period; this charge is smaller than the marginal transport cost K^T. Second, the commodity depreciates at rate δ so if S_t is stored in period t, $(1-\delta)S_t$ will be available in period $t+1$. Thirdly, there is an interest cost r foregone when storage is undertaken. The total quantity of stored and imported goods in a centre at the beginning of the period is M_t^i,

$$M_t^i = (1-\delta)\left(S_{t-1}^i + T_{t-1}^j\right) \qquad (3)$$

where S_{t-1}^i is the non-negative quantity stored in centre i and T_{t-1}^j is the non-negative quantity exported from centre j. The quantities consumed, stored and exported from a centre at time t are such that $Q_t^i + S_t^i + T_t^i = X_t^i + M_t^i$.

When transport is capacity constrained and shipping takes time, it is necessary to trace the amount of transport equipment in each centre. To simplify the analysis, it is assumed all transport operators own two pieces of transport equipment, one in each centre, and that whenever one is dispatched full the other one is shipped back in the opposite direction. In this way, there is always the same amount of transport equipment in each centre at the beginning of each period, so the shipping capacity is the same every period.

The amount of transport capacity is T^*. If less than T^* is shipped, shippers pay the marginal transport cost, K^T.[3] If T^* is shipped, shippers bid competitively for capacity and the price equals the difference between the spot price in the originating city and the appropriately discounted expected future price in the other city. (See equations 4g – 4p for a formal definition.) When goods are transported, they also depreciate at the rate δ.

It is assumed that risk neutral, profit maximising, and rational speculators ship and store goods to take advantage of expected price differences. The speculators have expectations about future prices that incorporate all information about output, storage, and trade in both centres. The behavior of risk neutral speculators can be represented by four inequalities. Let y_t = $\left[M_t^a, M_t^b, X_t^a, X_t^b\right]$ be the vector of state variables. Let K_t^a and K_t^b be the variable transport costs of shipping from A and B respectively, and let the excess return to the carriers be $R_t^i = K_t^i - K^T$. Let $\Pi^{ij}(y_t) = \left(\frac{1-\delta}{1+r}\right) E[P_{t+1}^i \mid y_t] - P^j(y_t)$ be the difference between the expected future price in centre i and the price in centre j at the point y_t. Then, at each point y_t:

$$\Pi^{aa}(y_t) = \left(\frac{1-\delta}{1+r}\right) E[P_{t+1}^a \mid y_t] - P^a(y_t) \le K^S \qquad (4a)$$

$$\left[\Pi^{aa}(y_t) - K^S\right].S^a(y_t) \;\; = 0 \qquad (4b)$$

$$S^a(y_t) \ge 0 \qquad (4c)$$

$$\Pi^{bb}(y_t) = \left(\frac{1-\delta}{1+r}\right) E[P_{t+1}^b \mid y_t] - P^b(y_t) \le K^S \qquad (4d)$$

$$\left[\Pi^{aa}(y_t) - K^S\right].S^b(y_t) \;\; = 0 \qquad (4e)$$

$$S^b(y_t) \ge 0 \qquad (4f)$$

[3] Since there is only one good in the model, it is never profitable to simultaneously ship the good in both directions. The cost K^T is the cost of sending goods from one city to the other and simultaneously sending empty ships back in the other direction.

$$\Pi^{ba}(y_t) = \left(\frac{1-\delta}{1+r}\right) E[P^b_{t+1} \mid y_t] - P^a(y_t) \le R^a_t(y_t) + K^T \qquad (4g)$$

$$R^a_t(y_t) \ge 0 \qquad (4h)$$

$$0 \le T^a(y_t) \le T^* \qquad (4i)$$

$$\left[\Pi^{ba}(y_t) - (R^a_t(y_t) + K^T)\right].T^a(y_t) \;\; = 0 \qquad (4j)$$

$$\left[T^a(y_t) - T^*\right].\left[R^a_t(y_t)\right] = 0 \qquad (4k)$$

$$\Pi^{ab}(y_t) = \left(\frac{1-\delta}{1+r}\right) E[P^a_{t+1} \mid y_t] - P^b(y_t) \le R^b_t(y_t) + K^T \qquad (4l)$$

$$R^b_t(y_t) \ge 0 \qquad (4m)$$

$$0 \le T^b(y_t) \le T^* \qquad (4n)$$

$$\left[\Pi^{ab}(y_t) - (R^b_t(y_t) + K^T)\right].T^b(y_t) \;\; = 0 \qquad (4o)$$

$$\left[T^b(y_t) - T^*\right].\left[R^b_t(y_t)\right] = 0 \qquad (4p)$$

where

$$P^i(y_t) = D_i^{-1}(X^i_t + M^i_t - S^i(y_t) - T^i(y_t)),$$

$$M^i_{t+1}(y_t) = (1-\delta)\left(S^i(y_t) + T^j(y_t)\right),$$

and

$$E\left[P^i_{t+1} \mid y_t\right] = \iint_X D_i^{-1}(X^i_{t+1} + M^i_{t+1}(y_t) - S^i(y_{t+1}) - T^i(y_{t+1})) f\left(X^i_{t+1}, X^j_{t+1}\right) dX^i_{t+1} dX^j_{t+1} \qquad (5)$$

The inequalities 4a–4c and 4d–4f are the conditions for profitable storage in either centre. Inventories in each centre will be zero if the expected future price exceeds the current spot price by less than the costs of storage; otherwise, arbitrageurs hold a quantity that ensures the expected future price exactly equals the current price plus the storage costs. Equations 4g–4k are the conditions for trade from centre A to centre B. Equations 4g and 4j say that trade will be zero if the expected future price in centre B exceeds the spot price in centre A by less than the costs of trade; otherwise, arbitrageurs ship a quantity that ensures the expected future price exactly equals the current price plus the trade cost. Equations 4j and 4k say that the transport cost will equal K^T if the traded amount is less than the shipping capacity; otherwise, the transport price will exceed K^T by the difference between the expected future price in centre B (adjusted for the amount lost in transit and interest costs) and the spot price in centre A. The quintet 4l–4p are similar, but describe the conditions for trade from centre B to centre A.

The model solution comprises two parts. The first part is the set of optimal storage and trade functions [S^a(.), S^b(.), T^a(.), T^b(.)] that satisfy the inequalities 4a–4p. The second part is the equilibrium distribution of the state variables, which depends on the assumed stochastic process determining output and the optimal storage and trade functions. The solution fulfils two conditions: first, that storage and trading decisions are profit-maximising conditional on expectations of future prices; and, second, that price expectations are consistent with the storage and trading decisions and expectations of future output quantities.

A numerical solution is found using the methodology in Coleman (2004). The solution is calculated over a discrete four-dimensional grid corresponding to the four state variables. There are three steps. First, a discrete joint probability distribution over the grid values for the stochastic variables X^a and X^b is chosen, and the double integral formula in equation 5 is replaced by the equivalent summation formula. The joint probability density for X is chosen to mimic an autocorrelated process with normal innovations, and is represented by an $m_2 \times m_2$ Markov transition matrix Π specifying the probability of going from one point (X^a_{i1}, X^b_{j1}) to a second point (X^a_{i2}, X^b_{j2}). Second, an algorithm is used to calculate the optimal amounts of storage and trade in the two centres. The algorithm constructs a series of successive approximations to the optimal storage and trade functions, and is repeated until the difference between successive values of the control values is small. Piece-wise linear demand functions for each centre are specified:

$$D_i^{-1}(Q_t^i) = \begin{cases} \alpha^i & \text{if } \; Q_t^i = 0 \\ \alpha^i - \beta^i Q_t^i & \text{if } \; 0 < Q_t^i \le \alpha^i / \beta^i \\ 0 & \text{if } \; Q_t^i > \alpha^i / \beta^i \end{cases} \qquad (6)$$

Third, the invariant probability distribution – the unconditional probability of being at every grid point – is found.

Properties of the model

The solution depends on the basic parameters of the model including the demand and supply functions, the interest rate, and the marginal transport cost. An important parameter is the extent to which output is regionally specialised. When the centres are identical, trade occurs infrequently in either direction, and average trade flows are small; but when one centre produces a large fraction of output, it exports most of the time and average trade flows are larger. The solutions are sufficiently distinctive that numerical results corresponding to the two cases are shown.

Analytical results

Key results can be derived analytically by considering the combinations of the complementary conditions that indicate whether inventories and trade volumes were zero, positive, or equal to the capacity limit. These combinations are summarized in table 1. Under a wide range of parameters, the invariant probability distribution indicates that most of the time either (a) both centres had positive inventories but neither centre exported or (b) both centres had positive inventories and one centre exported, or (c) one centre had positive inventories and exported, while the other centre had zero inventories and imported. The other combinations occurred rarely.[4] The set of conditions 4a – 4p indicates how prices adjust in each of these cases.

First, consider a point y_t when inventories are positive in each centre, but there is no trade. Because inventories are positive, the price in each centre is expected to increase; more precisely, by equations 4a–4f

$$E[P_{t+1}^{i} \mid y_t] = \left(\frac{1+r}{1-\delta}\right)\left(P^{i}(y_t) + K^{S}\right) \qquad i = A, B \qquad (7)$$

and consequently

$$E[P_{t+1}^{a} - P_{t+1}^{b} \mid y_t] = \left(\frac{1+r}{1-\delta}\right)\left(P^{a}(y_t) - P^{b}(y_t)\right) \qquad (8a)$$

[4] As the stochastic process determining output is changed, the optimal storage and trade functions change but the solution still fulfills the set of conditions 4a-4p. Consequently, analytic statements about the solution will hold irrespective of the assumed stochastic process, but the distribution of the state variables in equilibrium will differ. As the modeling assumptions are changed, other combinations of the state variables might become more important.

Furthermore, equations 4e, 4f, 4h, and 4i imply the spatial price difference lies within the range

$$-(K^T - K^S) \le P_t^a - P_t^b \le (K^T - K^S) \qquad (8b)$$

Second, consider a point y_t at which there are positive inventories in centre B, and arbitrageurs export to centre A. If centre A also has positive inventories, equations 4a, 4c, and 4h hold with equality and imply

$$E[P_{t+1}^a - P_{t+1}^b \mid y_t] = \frac{1+r}{1-\delta}(K_t^b - K^S) \qquad (9a)$$

$$P^a(y_t) = P^b(y_t) + K_t^b - K^S \qquad (9b)$$

Equation 9b indicates that the law of one price holds exactly if centre A has positive inventories at time t when goods are exported from centre B: that is, the spatial price difference will equal the difference between the transport cost and the storage cost. The transport cost equals K^T if little is shipped; if the amount shipped equals T*, the transport cost will be bid up to the difference between the export price and the expected future import price, adjusted for storage costs.

Alternatively, if centre A has zero inventories, equations 4c and 4h hold and imply

$$E[P_{t+1}^a - P_{t+1}^b \mid y_t] = \frac{1+r}{1-\delta}(K_t^b - K^S) \qquad (10a)$$

$$P^a(y_t) = P^b(y_t) + K_t^b - K^S + \varepsilon_t, \qquad \varepsilon_t > 0 \qquad (10b)$$

In this case, the spatial price difference at time t exceeds the difference between the transport and storage costs, as prices are unusually high in the importing centre, for inventories have been exhausted and imports will not arrive until the next period. The price is expected to fall in the importing centre when the goods arrive, however. Once again, the transport cost equals K^T if $T_t^b < T^*$, and is greater than K^T if $T_t^b = T^*$.[5]

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[5] More precisely, $\left(\frac{1-\delta}{1+r}\right) E[P_{t+1}^a \mid y_t] < P_t^a + K^S$; this does not necessarily imply that prices will fall, although it does mean that the expected future price is less than the spot price plus fall carrying charges.

Equations 8b, 9b, and 10b describe the possible variation in the difference between the two spot prices. The absolute value of the price difference is usually less than or equal to $K^T - K^S$; but it can exceed this amount if the importing centre depletes its inventories, or the amount shipped is equal to T^* and shipping prices are high.

Equations 9a and 10a state that when goods are exported at time t, the expected price difference at $t+1$ equals $\frac{1+r}{1-\delta}(K_t^b - K^S)$. When the goods arrive at $t+1$, however, equations 8b, 9b, and 10b applied at time $t+1$ indicate the actual spot price differential will be:

(i) less than or equal to $(K^T - K^S)$ if there are no further exports at time $t+1$;
(ii) equal to $(K_{t+1}^b - K^S)$ if B exports at time $t+1$ and centre A has inventories at $t+1$; or
(iii) greater than $(K_{t+1}^b - K^S)$ if B exports at time $t+1$ and centre A has no inventories.

Consequently, in order to cover interest costs and depreciation on average, the goods exported from B at t must sometimes arrive when the spot price in centre A is unusually high relative to prices in centre B either because transport costs at time $t+1$ are unusually high (and A imports again at $t+1$) or centre A has a shortfall and inventories are zero. Put differently, arbitrageurs who ship goods only cover their costs on average by sometimes having goods arrive when they are unusually scarce in the destination market.

It is worth noting that there are times when an importing centre will import even though it has positive inventories and is certain to have positive inventories in the next period. This possibility is not possible in models without capacity constraints. If a centre has a particularly bad (negative) output shock, arbitrageurs will start to run down their inventories and simultaneously import. Because the amount they can import this period and subsequent periods is limited, they will not want to deplete their inventories too fast as they know they will need to use them to supplement the limited imports in the future.

Numerical results

In the rest of this section, numerical simulations are used to illustrate how transport capacity constraints affect trade flows, storage quantities, and prices. Simulations are presented for the case that the centres are the same, and for the case that output is regionally specialised. The parameters are chosen so that the centres have identical demand functions, the price elasticity in each centre at average consumption is one, the storage cost K^S is zero, marginal transport price K^T is 5 percent of average prices, the period is one week, and the annual interest and depreciation rates are five percent.[6] The simulations show how inventories, trade flows, and prices are distributed for different values of the capacity constraint (tables 2-3), and how they depend on the size of the underlying shocks to the economy (table 4). The capacity constraint is then endogenised to show how much capacity is needed to reach a target rate of return as the size of the shocks to the economy is varied (table 5).

Tables 2 and 3 show how trade flows, transport costs, storage quantities, and prices vary as transport capacity is increased. Table 2 describes the case when centre A produces 50 percent of total output on average; table 3 describes what happens when centre A produces 35 percent of output and frequently imports. The effects of increasing capacity on shipping prices and commodity prices are similar in each case. First, as transport capacity is increased the mean volume of trade increases and the fraction of occasions when the capacity constraint binds decreases. This lowers the mean and variance of transport costs. The effect on transport costs is slightly offset by a reduction in the size of inventories in the importing and exporting centres. Second, price dispersion, measured either by the variance of $P_t^a - P_t^b$ or the fraction of times that $P_t^a - P_t^b$ exceeds $K^T - K^S$, decreases. The effect of capacity constraints is particularly marked when one centre is the dominant producer, for then inventory management is less useful for smoothing prices than when both centres are the same.

Table 4 shows how the effects of capacity constraints depend on the size of the underlying shocks to the economy. In the simulations, centre A produces 45 percent of output and the standard deviation of the production shock is either 5 or 10 percent of output. As output volatility increases, inventory management becomes more important, and average inventory levels

[6] In the baseline case the following model parameters are used: the demand function $D_i^{-1}(Q) = \alpha - \beta Q = 200 - Q$; mean production is 100 in each centre; the production conditional variance $\sigma^2 = 100$; the production autocorrelation $\rho = 0.9$; the weekly interest rate $r = 0.001$; the weekly depreciation rate $\delta = 0.001$; $K^S = 0$; and $K^T = 5$. The mean price is 100.

increase markedly. Trade occurs less frequently, but the transport capacity is more likely to be fully utilized when trade does occur so the fraction of times that the transport cost is bid above the marginal cost increases.[7] This increases the mean and variance of transport costs, which in turn increases price dispersion, as measured either by the variance of $P_t^a - P_t^b$ or the fraction of times that $P_t^a - P_t^b$ exceeds $K^T - K^S$. For a given level of transport capacity, therefore, high output volatility generates large spatial price dispersion and large returns to transport operators.

Table 5 shows how transport capacity responds to an increase in output volatility, for various levels of output in centre A. It shows the transport capacity needed to obtain a target return in the transport sector, in this case an average transport price of 7, or 40 percent higher than marginal cost.[8] In turn various statistics about the prices, transport costs, storage quantities and trade volumes that occur in equilibrium are calculated. The table shows that as the variance of output shocks increases, the equilibrium amount of transport capacity increases. The increase in transport capacity reduces the fraction of time that capacity constraints bind. Yet the variance of transport costs increases overall, for even though capacity limits bind less often, when they do bind the transport price is very high. In turn, spatial price dispersion increases. Thus even when one takes into account the endogenous expansion of transport capacity, an economy with a high variance of output shocks has greater price dispersion and greater transport price volatility than an economy with a low variance of output shocks.

4 Empirical relevance of the model

How relevant is the model? The model describes arbitrage behaviour for commodities that are traded in competitive markets, and which are transported by competitive but capacity constrained shipping firms. These conditions may exist for commodities that are transported by pipelines. It is plausible that they are important in developing countries, where a scarcity of capital is an everyday occurrence. More generally, as Williamson (1966) suggested, it is possible that many transport markets have bottlenecks and

[7] Note that under this parameterization goods would always be sent from B to A if there were no uncertainty. Uncertainty increases the fraction of time that no trade occurs, and the fraction of time that A exports to B because it has a temporary surplus and B a temporary shortfall.

[8] The target return can be used to calculate the return to capital if the cost of a unit of capital is specified.

capacity constraints and are equilibrated using a mixture of peak load pricing and logistics management techniques because transport equipment is capital intensive. According to Stopford (1988) the maritime shipping firms that transport most bulk commodities are competitive, and shipping prices vary substantially in the short term in accordance with supply and demand.[9] Contemporary data on grain shipments between the United States and Japan, and the United States and Europe show that trade volumes and shipping prices are positively correlated, which is consistent with this notion.[10] Whether or not goods prices and transport prices in a particular industry can be appropriately described by this model needs to be determined on a case by case basis. However, there is some direct evidence that the model has direct applicability, at least in grain markets.

The first evidence concerns the grain market in Western Australia. Wright and Williams (1989) suggested that storage under price backwardation might occur because of localised capacity constraints in the transport sector. Brennan, Williams and Wright (1997) investigated this idea directly by examining the way that rail and road networks were used in conjunction with storage elevators to transport wheat from rural hinterlands to a port. They argued that empirical evidence strongly supported the thesis that the rail and elevator network was used to minimise total transport and storage costs. In essence, elevators in distant locations were used to store grain during the peak harvest period while the rolling stock was used to move grain to the port from closer locations. Grain was kept in the distant locations even when prices in the port were in backwardation because rolling stock was limited and the implicit transport cost was too high.

The second evidence concerns the operation of the grain transport market between Chicago and New York in the late nineteenth century. This market was described in detail by Coleman (2004). He demonstrated that localised stock-outs frequently occurred in New York when inventories fell to very low levels. He also showed the spot price in New York was close to the spot price in Chicago plus the transport cost when inventories were normal or large in New York, but that it exceeded the Chicago spot price plus the

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9 According to Stopford (p15, p229) 74 percent of total seaborne cargoes in 1985 were bulk goods, of which 36 percent was oil and 23 percent was iron ore, coal or grain.

10 For example, for 1995-2000, the correlation coefficient between monthly Gulf of Mexico-Antwerp grain shipping prices and the total volume of US grain shipments is 0.54. Data is from International Grains Council (1995-2000).

transport cost when inventories were very small. This confirms a major prediction of the current model. He further showed that transport costs between Chicago and New York varied substantially from week to week, but did not analyse why they varied. However, there is historical evidence that transport prices varied as a result of fluctuating demand for transport services and a fixed supply of transport equipment.

The evidence is mainly anecdotal, and comes from a Senate enquiry into the operation of the transport market between the mid-west and the eastern seaboard (US Congress 1874). As part of this enquiry, Congress interviewed shipping agents and commodity merchants involved in the trade. Several of these agents argued that transport costs fluctuated daily in response to the supply and demand of shipping, and that the high prices obtained when shipping capacity sparse were necessary to generate a reasonable average return to shipping companies.[11] For example, Mr Hayes, General Manager of Blue Line Fast Freight, Detroit, said

> "…the lake rates from Chicago to Buffalo depend on the fluctuating demand for transportation. They will sometimes not only vary day by day but hourly through the day. If there happens to be a large influx of vessels brought in by a favorable wind the rates will go down, and the reverse will take place when there is a reverse condition of things, and this action takes place instantaneously, and ordinarily without any combination on the part of the vessel owners. Last week there was a sudden call for much transportation, I suppose caused by some sudden foreign grain demand. It was in excess of the capacity of the lake to furnish, and vessel owners rapidly advanced their prices from 6 to 15 cents a bushel." (US Congress 1874, Part II 33-34).

Later in this interview, this time in the context of railways, he said:

> "…at no time in any year in my knowledge, whether the crop was large or small, was there a regular demand for transportation up to the amount that the various lines could supply. Even when the crop

[11] A very similar set of explanations for why cattle prices in New York and Liverpool varied so much were given to the 1890 US Congress Select Committee on the Transportation and Sale of Meat Products. (US Congress 1890.) See the testimony by T C Eastman pp 513 – 527, and Mr F W J Hurst pp 555-557.

> was large cars laid idle at certain seasons, and because many laid idle the service was performed at a loss. If there is to be a fair average annual result to the transporter, then, when the demand again picks up, there must be a sufficient increase of charges to make a good average price." p 37.

This testimonial is clearly in concordance with the way transport is modeled in the article, and supports the statistical findings on the inter-relationships between prices, transport costs, and inventories that are reported in Coleman (2004).

5 Discussion and conclusions

Most papers analyzing the law of one price have assumed transport costs are exogenously determined and transport is supplied elastically to move goods. This article suggests these assumptions may not be innocuous. Rather, if transport operators plan to use their capital intensive machinery as much as possible, transport systems will have capacity constraints, transport prices will have a peak-pricing pattern, and commodity prices will vary substantially between locations. This variation is not evidence against the law of one price but the mechanism by which transport operators earn their cost of capital.

The model suggests that prices in different locations adjust to shocks in a way quite different to that often postulated. In most models, a region experiencing a negative output shock will smooth consumption by importing sufficient quantities of goods to ensure the local price does not exceed the world price by more than an exogenously determined transport cost. In this model, the same region would not be able to fully smooth consumption because not enough goods can be imported; rather, the output shock causes temporarily large profit margins for the shipping companies, and spatial price dispersion.

The model assumes transport services are sold in competitive markets. Yet if the service was not sold – the shipping was done on own account, for example, or the transport service was broadly interpreted to include distribution services such as retailing – the same logic would remain. Capital intensive distribution services would have limited capacity, and the profit in a particular period would vary with the size of supply and demand shocks. There would be imperfect price pass-through from one location to another. Evidence that price differences between locations varied significantly over time would not be evidence that arbitrage was not working, but a necessary condition to attract arbitrageurs into the industry.

The article raises several issues for further enquiry. The first concerns the number of sectors and time periods for which this model is applicable. Whether or not goods prices and transport prices can be described by this model needs to be determined on a case by case basis. However, it is possible that the model may have many applications. If, as Wright and Williams (1989) suggest, storage under price backwardation occurs because of localised capacity constraints in the transport sector, the model may be relevant wherever storage under backwardation occurs.

Second, the model could be extended to allow for the possibility that capacity constraints vary systematically through time, possibly because transport equipment is moved from market to market in response to peak demand periods. Such adjustments occur in the passenger airline industry, for instance, although this does not eradicate seasonal peak pricing patterns. A more complex extension would be to allow the transport supply curve to have a more general upward sloping form.

Third, the model uses an autoregressive rather than a seasonal process to model output uncertainty. While the set of arbitrage conditions (equations 4) is the same in both cases, the relative importance of each of the complementary slackness conditions will change. While I expect the overall flavour of the results to be similar – one would expect localised stock-outs and variable transport costs under both assumptions – it may be of interest to explicitly model the seasonal case for commodities where seasonal patterns in either transport availability or production are important.

Lastly, the model suggests that the way in which the transport sector is incorporated into trade models is important. Seemingly small changes to a model such as the incorporation of transport capacity constraints have large effects on the properties of the model. It may be the case that more realistic modeling of the transport systems used in other trade models would also have large effects on the properties of these models. If so, it may be necessary to better understand the structure and pricing of the transport and logistics sector to better understand price determination in spatially separate markets.

Table 1
Analytical results corresponding to different complementary conditions

S_t^A	S_t^B	T_t^A	T_t^B	Prob(.) X[A]=50%	Prob(.) X[A]=35%	$\lvert P_t^A - P_t^B \rvert$	$E[\lvert P_{t+1}^A - P_{t+1}^B \rvert\rvert y_t]$
>0	>0	=0	=0	63.7%	2.4%	$<(K^T - K^S)$	$\frac{1+r}{1-\delta}(P_t^A - P_t^B)$
>0	>0	=0	$0 < T_t^B < T*$	5.9%	1.7%	$=(K^T - K^S)$	$\frac{1+r}{1-\delta}(K^T - K^S)$
>0	>0	=0	=T*	9.5%	85.6%	$=(K_t^B - K^S)$ $\geq (K^T - K^S)$	$=\frac{1+r}{1-\delta}(K_t^B - K^S)$ $\geq \frac{1+r}{1-\delta}(K^T - K^S)$
>0	>0	$0 < T_t^A < T*$	=0	5.9%	0.01%	$=(K^T - K^S)$	$\frac{1+r}{1-\delta}(K^T - K^S)$
>0	>0	=T*	=0	9.5%	0.01%	$=(K_t^A - K^S)$ $\geq (K^T - K^S)$	$=\frac{1+r}{1-\delta}(K_t^A - K^S)$ $\geq \frac{1+r}{1-\delta}(K^T - K^S)$
>0	=0	=0	=0	0.04%	1.5%	Uncertain	Uncertain
>0	=0	=0	$0 < T_t^B < T*$	0.27%	3.9%	$=(K^T - K^S)$	$> \frac{1+r}{1-\delta}(K^T - K^S)$
>0	=0	=0	=T*	0.00%	0.69%	$=(K_t^B - K^S)$ $\geq (K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K_t^B - K^S)$ □ $\frac{1+r}{1-\delta}(K^T - K^S)$
>0	=0	$0 < T_t^A < T*$	=0	0.36%	0.21%	$\geq (K^T - K^S)$	$\frac{1+r}{1-\delta}(K^T - K^S)$
>0	=0	=T*	=0	1.8%	0.17%	$\geq (K_t^A - K^S)$ □ $(K^T - K^S)$	$=\frac{1+r}{1-\delta}(K_t^A - K^S)$ $\geq \frac{1+r}{1-\delta}(K^T - K^S)$
=0	>0	=0	=0	0.04%	0%	Uncertain	Uncertain
=0	>0	=0	$0 < T_t^B < T*$	0.36%	0.01%	$\geq (K^T - K^S)$	$\frac{1+r}{1-\delta}(K^T - K^S)$
=0	>0	=0	=T*	1.8%	2.9%	$\geq (K_t^B - K^S)$ □ $(K^T - K^S)$	$=\frac{1+r}{1-\delta}(K_t^B - K^S)$ $\geq \frac{1+r}{1-\delta}(K^T - K^S)$
=0	>0	$0 < T_t^A < T*$	=0	0.27%	0%	$=(K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K^T - K^S)$
=0	>0	=T*	=0	0.00%	0%	$=(K_t^A - K^S)$ $\geq (K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K_t^A - K^S)$ □ $\frac{1+r}{1-\delta}(K^T - K^S)$
=0	=0	=0	=0	0.11%	0%	Uncertain	Uncertain
=0	=0	=0	$0 < T_t^B < T*$	0.30%	0.04%	$\geq (K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K^T - K^S)$
=0	=0	=0	=T*	0.00%	0.01%	$\geq (K_t^B - K^S)$ □ $(K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K_t^B - K^S)$ □ $\frac{1+r}{1-\delta}(K^T - K^S)$
=0	=0	$0 < T_t^A < T*$	=0	0.30%	0%	$\geq (K^T - K^S)$	$> \frac{1+r}{1-\delta}(K^T - K^S)$
=0	=0	=T*	=0	0.00%	0%	$\geq (K_t^A - K^S)$ □ $(K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K_t^A - K^S)$ □ $\frac{1+r}{1-\delta}(K^T - K^S)$

The table gives the absolute value of the price differential and the expected future price differential $E[\lvert P_{t+1}^A - P_{t+1}^B \rvert\rvert y_t]$ for different combinations of the complementary conditions 4c, f, i, n. The probabilities of each set of conditions occurring pertain to the cases that marginal transport costs equal 5, the capacity constraint equals 20, and A either produces 50 percent of output or 35 percent of output.

Table 2
Prices, storage, and transport cost statistics corresponding to the model (centre A produces 50% of output, σ=10, changing transport capacity)

Statistic	$T^*=5$	$T^*=10$	$T^*=20$	$T^*=30$	$T^*=40$	$T^*=50$	$T^*=\infty$
Prices							
Mean(P^a)	100.4	100.3	100.3	100.3	100.3	100.3	100.3
S. Dev.(P^a)	10.3	9.6	8.5	8.6	8.6	8.5	8.5
Mean(P^a-P^b)	0	0	0	0	0	0	0
S. Dev.(P^a-P^b)	13.0	10.5	7.3	5.8	5.1	4.8	4.6
% (| P^a-P^b| >K^T-K^S)	54.2%	40.2%	23.8%	13.8%	7.8%	4.4%	3.1%
Storage							
Mean(S^a)	361	325	288	273	265	269	261
S. Dev.(S^b)	305	275	252	253	253	257	251
% (S^a > 0)	95%	96%	97%	97%	97%	97%	97%
Trade							
Mean(T^a)	1.4	2.2	2.9	3.1	3.1	3.1	3.2
S. Dev.(T^a)	2.2	4.1	6.8	8.2	9.0	9.1	10.5
Mean(T^a)|T^a>0	4.9	9.4	16.7	19.4	19.7	20.0	20.0
% (T^a > 0)	28.4%	23.5%	17.5%	15.9%	15.9%	15.7%	16.0%
% (T^a=T*)	27.1%	20.1%	11.4%	6.1%	2.8%	0.8%	0%
Transport Cost							
Mean (K_t^a)	7.27	6.40	5.54	5.20	5.08	5.04	5
S. Dev(K_t^a)	5.9	4.5	2.5	1.5	0.9	0.6	0
Excess return	4.54	2.80	1.08	0.32	0.16	0.08	0

P^a: the price in centre A. S^a: storage in centre A. T^a: trade from centre A to centre B.

K_t^a : the transport cost incurred sending a ship from A to B.

% (|P^a-P^b| >K^T-K^S): the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. K^S=0 in these simulations.

% (S^a[T^a] > 0): the fraction of time storage [exports] > 0.

% ($T^a = \overline{T}$): the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^a > K^T$.

Table 3
Prices, storage, and transport cost statistics corresponding to the model (centre A produces 35% of output, σ=10, changing transport capacity)

Statistic	$T^*=10$	$T^*=20$	$T^*=30$	$T^*=40$	$T^*=50$	$T^*=\infty$
Prices						
Mean(P^a)	120	112	106	104	103	103
S. Dev.(P^a)	13	12	11	10	9	9
Mean (P^b)	80	89	95	97	98	98
S. Dev(P^b)	10	9	8	8	9	10
Mean(P^a-P^b)	40.3	22.6	10.8	6.6	5.5	5.1
S. Dev.(P^a-P^b)	16.4	14.4	9.8	5.7	3.2	1.9
%($\|P^a-P^b\|>K^T-K^S$)	98%	92%	76%	52%	31%	2%
Storage						
Mean(S^a)	405	417	357	297	213	91
S. Dev.(S^a)	332	330	286	242	173	81
Mean (S^b)	458	367	233	178	213	300
S.Dev(S^b)	399	350	252	184	227	304
Trade						
Mean(T^a)	0.0	0.1	0.1	0.1	0.0	0.0
S. Dev.(T^a)	0.2	1.0	1.4	1.2	0.8	0.4
Mean(T^a)|T^a>0	10	15	20	16.7	14.3	12.8
% (T^a > 0)	0.04%	0.4%	0.4%	0.3%	0.1%	0.1%
% (T^a =T*)	0.04%	0.2%	0.1%	0.0%	0.0%	0%
Mean(T^b)	9.8	18.8	24.7	26.8	27.3	27.4
S. Dev.(T^b)	1.3	4.4	11	17	21	21
Mean(T^a)|T^b>0	9.94	19.7	28.3	34.8	37.0	32.7
% (T^b > 0)	99%	96%	87%	77%	74%	84%
% (T^b =T*)	98%	91%	74%	54%	35%	0%
Transport Cost						
Mean (K_t^a)	5.00	5.00	5.00	5.00	5.00	5
S. Dev(K_t^a)	0.1	0.1	0.1	0.0	0.0	0
Mean (K_t^b)	40.18	22.58	10.96	6.60	5.42	5
S. Dev(K_t^b)	15.6	13.2	8.5	4.3	1.9	0
Excess return	35.18	17.58	5.96	1.60	0.42	0

P^a: the price in centre A. S^a: storage in centre A. T^a: trade from centre A to centre B.

K_t^a : the transport cost incurred sending a ship from A to B.

% ($|P^a-P^b| > K^T-K^S$): the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. K^S=0 in these simulations.

% ($S^a[T^a] > 0$): the fraction of time storage [exports] > 0.

% ($T^a = \overline{T}$): the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^a > K^T$.

Table 4
Prices, storage, and transport cost statistics corresponding to the model (centre A produces 45% of output, changing transport capacity, σ=5 or σ=10)

Statistics	$T^* = 5$ **σ = 5**	$T^* = 10$ **σ = 5**	$T^* = 20$ **σ = 5**	$T^* = \infty$ **σ = 5**	$T^* = 5$ **σ = 10**	$T^* = 10$ **σ = 10**	$T^* = 20$ **σ = 10**	$T^* = \infty$ **σ =10**
Prices								
Mean(P^a-P^b)	11.8	7.1	4.7	4.5	13.1	8.5	4.8	3.3
S. Dev.(P^a-P^b)	7.0	6.0	2.8	2.3	14.5	11.7	7.6	3.9
%($\|P^a$-$P^b\|$>K^T-K^S)	82%	61%	24%	6%	80%	66%	42%	3%
Storage								
Mean(S^a)	92	78	45	31	362	349	284	169
Mean (S^b)	88	63	65	74	381	338	273	297
Trade								
Mean(T^a)	0.05	0.07	0.05	0.04	0.3	0.6	0.8	0.8
Mean(T^b)	4.1	6.5	7.7	7.8	3.8	6.3	8.4	9.1
% (T^a +T^b> 0)	86%	72%	58%	65%	82%	71%	55%	43%
% (T^a or T^b=T^*)	81%	59%	21%	0%	81%	66%	40%	0%
Transport Costs								
Mean (K_t^a)	5.01	5.01	5.00	5	5.46	5.35	5.14	5
S. Dev(K_t^a)	0.2	0.1	0.0	0	2.5	2.1	1.2	0
Mean (K_t^b)	12.27	7.68	5.23	5	14.80	10.46	6.78	5
S. Dev(K_t^b)	7.1	4.4	1.0	0	11.1	8.5	4.7	0
Excess return	7.28	2.69	0.23	0	10.26	5.78	1.92	0

σ: the standard deviation of the shock hitting output in each centre.
P^a: the price in centre A. S^a: storage in centre A. T^a: trade from centre A to centre B.
K_t^a : the transport cost incurred sending a ship from A to B.
% ($|P^a$-$P^b|$ >K^T-K^S): the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. K^S=0 in these simulations.
% (S^a[T^a] > 0): the fraction of time storage [exports] > 0.
% ($T^a = \overline{T}$): the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^a > K^T$.

Table 5
Equilibrium transport capacity as a function of output shock variance

Statistics **Centre A output** **σ**	**50%** **σ=2.5**	**50%** **σ = 5**	**50%** **σ =10**	**45%** **σ =2.5**	**45%** **σ = 5**	**45%** **σ = 10**	**35%** **σ =2.5**	**35%** **σ = 5**	**35%** **σ = 10**
Capacity limit	0.01[a]	3.0	13.6	8.2	11.3	19.6	28.1	30.8	38.3
Prices									
Mean(P^a-P^b)	0	0	0	7.1	6.4	4.9	7.2	7.2	6.9
S. Dev.(P^a-P^b)	6.0	8.1	9.2	3.4	5.3	7.8	3.4	4.2	6.1
%($\|P^a$-$P^b\|$>K^T-K^S)	36%	41%	33%	61%	55%	43%	61%	58%	57%
Storage									
Mean(S^A)	25	86	308	21	73	286	25	86	323
Mean (S^B)	25	86	308	13	59	274	13	34	179
Trade									
Mean(T^A)	0.002	0.64	2.54	0.0	0.1	0.8	0.0	0.0	0.1
Mean(T^B)	0.002	0.64	2.54	6.5	6.9	8.4	26.4	26.4	26.7
% (T^A +T^B> 0)	36%	44%	42%	90%	68%	53%	100%	99%	80%
% (T^A or T^B=$\overline{T}$)	36%	41%	34%	61%	52%	41%	56%	55%	57%
Transport Costs									
Mean (K_t^A)	5.54	5.99	6.01	5.00	5.01	5.14	5.00	5.00	5.00
S. Dev(K_t^A)	1.6	2.8	3.7	0.0	0.1	1.3	0.0	0.0	0.1
Mean (K_t^B)	5.54	5.99	6.01	7.00	6.98	6.86	7.00	7.00	7.00
S. Dev(K_t^B)	1.6	2.8	3.7	2.9	3.8	4.9	2.9	3.7	4.8
Excess return	1.04a	1.98	2.01	2.00	1.99	2.00	2.00	2.00	2.00

The table shows equilibrium values of prices, transport costs, and trade when transport capacity is determined endogenously to generate an excess return of 2.00. Centre A output is the fraction of output made in centre A on average.
σ: the standard deviation of the shock hitting output in each centre.
P^a: the price in centre A. S^a: storage in centre A. T^a: trade from centre A to centre B.
K_t^a : the transport cost incurred sending a ship from A to B.
% ($|P^a$-$P^b|$ >K^T-K^S): the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. K^S=0 in these simulations.
% (S^a[T^a] > 0): the fraction of time storage [exports] > 0.
% ($T^a = \overline{T}$): the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^a > K^T$.
[a]The minimum return to capital is not earned in this case. Output has so little volatility that consumption is almost completely smoothed through inventory adjustment.

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