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# Uncovering the Hit-list for Small Inflation Targeters: A Bayesian Structural Analysis*

**Timothy Kam, Kirdan Lees and Philip Liu**†

## Abstract

We estimate underlying macroeconomic policy objectives of three of the earliest explicit inflation targeters – Australia, Canada and New Zealand – within the context of a small open economy DSGE model. We assume central banks set policy optimally, such that we can reverse engineer policy objectives from observed time series data. We find that none of the central banks show a concern for stabilizing the real exchange rate. However, all three central banks share a concern for minimizing the volatility in the change in the nominal interest rate. The Reserve Bank of Australia places the most weight on minimizing the deviation of output from trend. Tests of the posterior distributions of these policy preference parameters suggest that the central banks have very similar objectives.

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# 1 Introduction

Within modern dynamic stochastic general equilibrium (DSGE) models, consumers and firms are modelled as pursuing their objectives optimally, subject to the constraints that limit their choices. Rather surprisingly, central bank behavior is often described as a reduced-form monetary policy rule. One reason for the widespread use of ad hoc reduced-form policy rules is that they provide good empirical fit. For example, the Taylor rule has provided a remarkably useful description of US monetary policy *behaviour* after the Volcker disinflation. However, such reduced-form descriptions are not generally helpful when one wishes to understand *central banks' preferences* and their macroeconomic stabilization objectives. That requires specifying and comparing specific macroeconomic policy objectives since reduced-form policy rules are functions of both underlying macroeconomic policy objectives and other key structural (private agents' taste and technology) parameters.

In this paper, we seek to identify the macroeconomic objectives of three of the earliest explicit inflation targeters – Australia, Canada and New Zealand, over the period 1990Q1 - 2005Q3. We treat the central bank as an optimizing agent, thus placing the central bank on the same footing as the other optimizing agents in the economy. We assume that monetary policy is set optimally and in a time-consistent fashion because no precommitment device is available (see pp 5-6, Bernanke, Laubach, Mishkin, and Posen 1999). We estimate the same DSGE model for each country and reverse engineer stabilization objectives that are conditioned on the structure of each economy.

We find that none of the central banks show a concern for stabilizing the real exchange rate. However, all three central banks share a concern for minimizing the volatility in the change in the nominal interest rate. According to our analysis, the Reserve Bank of Australia places the most weight on minimizing the deviation of output from trend. In contrast to existing applications of Bayesian econometrics to the evaluation of DSGE models, we also compare the posterior distributions of the central banks' preference parameters. Tests of the posterior distributions of these policy preference parameters suggest that the central banks have very similar preferences.

Our results should help inform monetary policy experiments that seek optimal policy rules for open economy inflation targeters. A wealth of policy experiments seek optimal policy for specific loss function parameterizations (see for example Rudebusch and Svensson 1999, Levin and Williams 2003, Levin and Williams 2003 and Del Negro and Schorfheide 2005, amongst others). However, typical loss function parameterizations may be inconsistent with the data. Dennis (2006) points out that policy rules optimized on typical loss function parameterizations yield particularly aggressive policy rules that are inconsistent with the observed smoothing of interest rates (see Lowe and Ellis 1997). We contribute to this debate by explicitly identifying the loss function parameters for open economy inflation targeters conditioned on a microfounded DSGE model.

Estimates of macroeconomic policy objectives can potentially enhance both the transparency and accountability of the practical implementation of monetary policy. Most inflation targeting central banks describe themselves as “flexible” in their approach to inflation targeting, implying central banks objectives embody factors beyond simply inflation. However, while central banks are often explicit about the macroeconomic variables they are concerned with, the trade-offs across these macroeconomic objectives are never elucidated. We believe transparency is enhanced by providing explicit statements of how alternative stabilization objectives are weighted (see Svensson 2005) and our analysis provides such statements.

Finally, historical estimates of stabilization objectives (conditioned on an explicit structural and microfounded model) provide a framework for central bank boards or government agencies tasked with assessing central bank performance. For example, clause 4(b) of New Zealand’s 2002 Policy Targets Agreement (PTA), the agreement between the Governor of the Reserve Bank of New Zealand and the Minster of Finance, states that: “In pursuing its price stability objective, the Bank shall seek to avoid unnecessary instability in output, interest rates and the exchange rate”. Simply observing the unconditional volatilities of the goal variables in the PTA cannot provide an examination of monetary policy, since these volatilities are also affected by non-policy structural features of the economy.

Several authors report empirical estimates of the objectives of the US Federal Reserve system. Salemi (1995) provides the earliest estimates based on a VAR model. In contrast to the mandate of the Federal Reserve, Favero and Rovelli (2003), Castelnuovo and Surico (2004) and Dennis (2006) find either small or insignificant weights on output stabilization over the Volcker-Greenspan period. In addition, Ozlale (2005) and Dennis (2006) find a significant weight on interest rate smoothing in the context of aggregate empirical models. Nimark (2006) provides estimates of macroeconomic objectives for both the Reserve Bank of Australia (RBA) and the Federal Reserve that suggest the RBA puts more weight on output stabilization and interest smoothing than the US Federal Reserve. However, Nimark’s paper uses a closed economy model that is silent on any preference for mitigating exchange rate volatility. Given Australia’s degree of openness and the focus of this paper, an open economy model appears necessary to approximate the constraint the RBA faces in implementing monetary policy.

In contrast to Nimark (2006), we estimate central bank preferences for Australia, Canada and New Zealand, within an open economy DSGE model. Furthermore, the DSGE model provides an incomplete exchange rate pass-through channel in import prices such that deviations from the law of one price (or alternatively real exchange rate deviations) matter for the economies. Such a model allows us to incorporate central bank preferences over exchange rate movements, as indicated by New Zealand PTA for example.

Our DGSE model extends Monacelli (2005) by introducing endogenous persistence on both the aggregate demand and supply sides of the model and has similarities with Justiniano and Preston (2005). We use Bayesian methods to estimate the model and apply an identical prior to each of the countries in our sample. We make inference statements regarding preferences from draws from the posterior distribution. Our Bayesian methodology closely follows related papers in the literature (see Smets and Wouters 2003 and Rabanal and Rubio-Ramírez 2005 for example). Although we focus on policy objectives, the estimates from our DSGE model should also help inform a growing empirical open economy literature (see for example Justiniano and Preston 2005, Lubik and Schorfheide 2005a, Lubik and Schorfheide 2005b).

The paper is organized as follows. Section 2 sets out the model. Section 3 outlines the empirical methodology and describes the data we use. Section 4 presents our results and we make concluding comments in section 5.

# 2 The model

## 2.1 The average household

The stylized economy is similar to the open economy model in Monacelli (2005) and Justiniano and Preston (2005). The economy has identical households with a total population of measure 1. We assume the functional form for period utility:

$$U(C_t, H_t, N_t) = \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{1}$$

where $C_t$ is an index of consumption goods, $H_t = hC_{t-1}$ is an external habit stock, with $h \in (0,1)$ capturing the degree of habit persistence, and $N_t$ is labour hours.

Define the prices for each differentiated home and foreign good of type $i \in [0,1]$ and $j \in [0,1]$, respectively, as $P_{H,t}(i)$ and $P_{F,t}(j)$. Let $B_{t+1}$ be the risk-free nominal value of an internationally traded bond held at the end of period $t$, and $W_t N_t$ be the total wage income. The stochastic discount factor is $\mathbb{E}_t Q_{t,t+1}$ such that it will be inversely related to the gross return on a nominal riskless one-period bond, $\mathbb{E}_t Q_{t,t+1} = R_t^{-1}$.

The average household solves a recursive problem:

$$V(B_t, H_t) = \max_{C_t, N_t} U(C_t, H_t, N_t) + \beta \mathbb{E}_t \{V(B_{t+1}, H_{t+1})\}; \qquad \beta \in (0,1) \tag{2}$$

subject to the sequence of budget constraints

$$B_t \geq \int_0^1 \int_0^1 [P_{H,t}(i) C_{H,t}(i) + P_{F,t}(j) C_{F,t}(j)]\, di\, dj + \mathbb{E}_t Q_{t,t+1} B_{t+1} - W_t N_t. \tag{3}$$

in all states and at all dates $t \in \mathbb{N}$ with $B_0$ given.

The consumption index $C_t$ is linked to a continuum of domestic, $C_{H,t}(i)$, and foreign goods, $C_{F,t}(j)$, which exist on the interval of $[0,1]$ where:

$$C_t = \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta-1}{\eta}}, \tag{4}$$

and

$$C_{H,t} = \left[ \int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad C_{F,t} = \left[ \int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

The elasticity of substitution between home and foreign goods is given by $\eta > 0$ and the elasticity of substitution between goods within each goods category (home and foreign) is $\varepsilon > 0$. Optimal allocation of the household expenditure across each good type gives rise to the demand functions:

$$C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}, \qquad C_{F,t}(j) = \left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \tag{6}$$

for all $i, j \in [0,1]$, where the aggregate price levels are defined as

$$P_{H,t} = \left( \int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, \qquad P_{F,t} = \left( \int_0^1 P_{F,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}, \tag{7}$$

and optimal consumption demand of home and foreign goods can be derived, respectively, as

$$C_{H,t} = (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \qquad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t.$$

Substitution of these demand functions into (4) yields the consumer price index as

$$P_t = \left[(1-\alpha)\, P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{8}$$

The intratemporal condition relating labour supply (the marginal rate of substitution between consumption and leisure) to the real wage (the marginal product of labour) must also be satisfied:

$$(C_t - H_t)^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{9}$$

Finally, intertemporal optimality for the household decision problem must satisfy

$$\beta \left(\frac{C_{t+1} - H_{t+1}}{C_t - H_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}. \tag{10}$$

for all dates $t \in \mathbb{N}$ and states in $t+1$ reachable from the time-$t$ state. Taking conditional expectations yields the familiar stochastic Euler equation

$$\beta R_t \mathbb{E}_t \left\{ \left(\frac{C_{t+1} - H_{t+1}}{C_t - H_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right\} = 1.$$

## 2.2 International risk sharing and relative prices

The rest of the world, denoted by variables and parameters with an asterisk, solves a similar problem to the small open economy. Specifically, the rest of the world is the limiting case of a closed economy, where $\alpha^* \to 1$. First-order conditions for optimal labour supply and consumption, analogues of (9) and (10), also hold for the rest of the world. Given identical global preferences and complete international markets, we obtain perfect risk sharing,

$$\begin{aligned} \beta \left(\frac{C_{t+1} - H_{t+1}}{C_t - H_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) &= Q_{t,t+1} \\ &= \beta \left(\frac{C^*_{t+1} - H^*_{t+1}}{C^*_t - H^*_t}\right)^{-\sigma} \left(\frac{P^*_t}{P^*_{t+1}}\right) \left(\frac{\widetilde{e}_t}{\widetilde{e}_{t+1}}\right) \end{aligned} \tag{11}$$

for all dates and states, $\widetilde{e}_t$ is the nominal exchange rate. We also define the real exchange rate conventionally as

$$\mathscr{Q}_t = \widetilde{e}_t P^*_t / P_t. \tag{12}$$

Assuming ex ante identical countries, and no preference shocks to the rest of the world, this implies that

$$C_t - hC_{t-1} = \vartheta^* \left(C_t^* - hC_{t-1}^*\right) \mathscr{Q}_t^{-\frac{1}{\sigma}}. \tag{13}$$

where $\vartheta^* = 1$ imposes ex ante symmetry of countries and zero net foreign asset holdings.

Let $c_t := \ln(C_t/C_{ss}), y_t^* := \ln(Y_t^*/Y_{ss}^*) = \ln(C_t^*/C_{ss}^*)$, and $q_t := \ln(\mathscr{Q}_t/\mathscr{Q}_t^*)$, denote the percentage deviation of home consumption, foreign output and real exchange rate from their respective steady states, where $X_{ss}$ is the deterministic steady state value of a variable $X_t$. Then, a log-linear approximation of (13) is

$$c_t - hc_{t-1} = y_t^* - hy_{t-1}^* + \frac{1-h}{\sigma} q_t. \tag{14}$$

Complete markets thus imply that global consumption will be perfectly correlated in the absence of deviations in the real exchange rate.

From (11) we can also derive the no-arbitrage condition for exchange rates, or the uncovered interest parity condition

$$R_t - R_t^* \frac{\widetilde{e}_t}{\widetilde{e}_{t+1}} = 0, \tag{15}$$

which must hold for all states and dates. A log-linear approximation of this, and taking expectations with respect to the time-$t$ sigma algebra, yields the familiar nominal interest parity condition:

$$\mathbb{E}_t e_{t+1} - e_t = r_t - r_t^* \tag{16}$$

where $e_t := \ln(\widetilde{e}_t/e_{ss})$, and the domestic and foreign rates of return are, $r_t = R_t - 1$ and $r_t^* = R_t^* - 1$, respectively.

We can define the terms of trade as the ratio of the foreign goods price index to the home goods price index. In log-linear terms this is

$$s_t = p_{F,t} - p_{H,t}. \tag{17}$$

## 2.3 Production and optimal pricing

There exist continua of monopolistically competitive domestic producers $i \in [0,1]$ that produce differentiated goods and import retailers $j \in [0,1]$ that add markups to goods imported at world prices. We employ similar pricing assumptions as in Justiniano and Preston (2005) and Smets and Wouters (2003). In particular, the conventional Calvo-style optimal pricing models and partial inflation indexation for non-optimizing price setters. This allows inflation to be partly a jump variable and also partially backward-looking.

### Domestic goods firms

Domestic goods firms operate a linear production technology, $Y_{H,t}(i) = \varepsilon_{a,t} N_t(i)$ where $\varepsilon_{a,t}$ is an exogenous domestic technology shock. Domestic firms face an independent signal that allows them to set prices optimally each period with probability $1-\theta_H$. In each period $t$, the remaining fraction $\theta_H \in (0,1)$ of firms partially index their price to take into account of aggregate domestic inflation according to the simple rule

$$P_{H,t}(i) = P_{H,t-1}(i)\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H} \tag{18}$$

where $\delta_H \in [0,1]$ measures the degree of inflation indexation. Since all firms either receive the same signal to reset prices or do not receive any signal, they will choose the same pricing strategies. Given Calvo price setting it is straightforward to define the dynamics of the aggregate price level of the domestic goods. In particular, define the evolution of the aggregate home goods price index as

$$P_{H,t} = \left\{(1-\theta_H)\left(P_{H,t}^{new}\right)^{1-\varepsilon} + \theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H}\right]^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}} \tag{19}$$

Consider a candidate firm $i$ that had set its price optimally in time $t$ as $P_{H,t}(i)$. Suppose at some time $t+s$, $s \geq 0$, the price $P_{H,t}(i)$ still prevails. Then the firm will have to face the demand for its product given by the demand constraint

$$Y_{H,t+s}(i) = \left[\frac{P_{H,t}(i)}{P_{H,t+s}}\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H}\right]^{-\varepsilon}\left(C_{H,t+s} + C_{H,t+s}^{*}\right). \tag{20}$$

Note that market demand at time $t+s$ will take into account the inflation indexation between $t$ and $t+s$.

Firms that set optimal prices do so to maximize their present value of the stochastic stream of profits. A candidate firm $i$ solves

$$\max_{P_{H,t}(i)} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta_H^s Y_{H,t+s}(i)\left[P_{H,t}(i)\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H} - P_{H,t+s}MC_{H,t+s}\exp\left(\varepsilon_{H,t+s}\right)\right] \tag{21}$$

subject to (20) for $t, s \in \mathbb{N}$ and the technological constraint given by real marginal cost,

$$MC_{H,t+s} = \frac{W_{t+s}}{\varepsilon_{a,t+s} P_{H,t+s}} \tag{22}$$

Note that we also allow for a structural shock to real marginal cost given by $\varepsilon_{H,t} \sim i.i.d.(0, \sigma_H)$. This has the interpretation of an independent cost-push shock to domestic goods producers.

The first order necessary condition characterizing domestic firms' optimal pricing function in a symmetric equilibrium is

$$\mathbb{E}_t \sum_{s=0}^{\infty} \theta_H^s Q_{t,t+s} Y_{H,t+s}(i) \left[ \widetilde{P}_{H,t} \left( \frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} - \left( \frac{\varepsilon}{\varepsilon - 1} \right) P_{H,t+s}(i) MC_{H,t+s} \exp(\varepsilon_{H,t+s}) \right] = 0 \tag{23}$$

Let the home goods inflation rate be $\pi_{H,t} := \ln(P_{H,t}/P_{H,t-1})$, and let $y_t := \ln(Y_t/Y_{ss})$ be the percentage deviation of home output from steady state. Denote the real marginal cost in percentage deviation terms from its deterministic steady-state $mc_{H,ss} = [\varepsilon/(\varepsilon - 1)]^{-1}$ as $mc_{H,t}$. In appendix A we derive the log-linear approximation of the optimal pricing decision rule, which can easily be expressed as the following Phillips curve for domestic goods inflation:

$$\pi_{H,t} - \delta_H \pi_{H,t-1} = \beta (\mathbb{E}_t \pi_{H,t+1} - \delta_H \pi_{H,t}) + \lambda_H (mc_{H,t} + \varepsilon_{H,t}) \tag{24}$$

where $\lambda_H = (1 - \beta\theta_H)(1 - \theta_H)\theta_H^{-1}$ and

$$mc_{H,t} = \varphi y_t - (1 + \varphi)\varepsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}\left(y_t^* - h y_{t-1}^*\right) + q_t + \varepsilon_{c,t}. \tag{25}$$

### Import retail firms

Import retailers are assumed to purchase imported goods at competitive world prices. However, these firms act as monopolistically competitively re-distributors of these goods. This creates a gap between the price of imported goods in domestic currency terms and the domestic retail price of imported goods. Define this law of one price (LOP) gap in log-linear terms as:

$$\psi_{F,t} = e_t + p_t^* - p_{F,t}. \tag{26}$$

The pricing behavior for import retailers is similar to that of domestic goods producers. In short, the evolution of the imports price index is given by

$$P_{F,t} = \left\{ (1-\theta_F)\left(P_{F,t}^{new}\right)^{1-\varepsilon} + \theta_F \left[ P_{F,t-1}\left(\frac{P_{F,t-1}}{P_{F,t-2}}\right)^{\delta_F} \right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}} \quad (27)$$

An importing firm $j$ at some time $t+s$, $s \geq 0$, faces the demand for its product given by the demand constraint

$$Y_{F,t+s}(j) = \left[ \frac{P_{F,t}(j)}{P_{F,t+s}} \left( \frac{P_{F,t+s-1}}{P_{F,t-1}} \right)^{\delta_F} \right]^{-\varepsilon} C_{F,t+s}. \quad (28)$$

Note that market demand at time $t+s$ will take into account the inflation indexation between $t$ and $t+s$. A candidate firm $j$ solves

$$\max_{P_{F,t}(j)} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta_F^s Y_{F,t+s}(j) \left[ P_{F,t}(j) \left( \frac{P_{F,t+s-1}}{P_{F,t-1}} \right)^{\delta_F} - \widetilde{e}_{t+s} P_{F,t+s}^{*}(j) \exp(\varepsilon_{F,t+s}) \right] \quad (29)$$

subject to (28) for $t,s = 0,1,\ldots$. Here we also allow for a structural shock to marginal cost (world price of good $j$) given by $\varepsilon_{F,t} \sim i.i.d.(0,\sigma_F)$. This has the interpretation of an independent cost-push shock to import retailers.

The first order necessary condition characterizing the import retailers' optimal pricing function in the symmetric equilibrium is

$$\mathbb{E}_t \sum_{s=0}^{\infty} \theta_F^s Q_{t,t+s} Y_{F,t+s}(j) \left[ \widetilde{P}_{F,t} \left( \frac{P_{F,t+s-1}}{P_{F,t-1}} \right)^{\delta_F} - \left( \frac{\varepsilon}{\varepsilon - 1} \right) \widetilde{e}_{t+s} P_{F,t+s}(j) \exp(\varepsilon_{F,t+s}) \right] = 0 \quad (30)$$

Let $\pi_{F,t} := \ln(P_{F,t}/P_{F,t-1})$. Log-linearizing this around the non-stochastic steady state (see appendix A) yields

$$\pi_{F,t} = \beta \mathbb{E}_t (\pi_{F,t+1} - \delta_F \pi_{F,t}) + \delta_F \pi_{F,t-1} + \lambda_F (\psi_{F,t} + \varepsilon_{F,t}), \quad (31)$$

where $\lambda_F = (1-\beta\theta_F)(1-\theta_F)\theta_F^{-1}$.

## 2.4 Terms of trade, real exchange rate and market clearing

We can derive a relationship between the terms of trade, the real exchange rate and the LOP gap. Specifically, log-linearizing the real exchange rate definition

(12) around the deterministic steady state we have

$$q_t = e_t + p_t^* - p_t. \tag{32}$$

From (26) we can re-write this as

$$q_t = \psi_{F,t} + p_{F,t} - p_t \approx \psi_{F,t} - (1-\alpha)(p_{F,t} - p_{H,t}) = \psi_{F,t} - (1-\alpha)s_t. \tag{33}$$

where the last term is obtained by log-linearizing the CPI definition and then using (17).

The remaining market-clearing condition to consider is in the product markets. In the rest of the world we have the limit of a closed economy so that $y_t^* = c_t^*$ for all $t$. In the small open economy, this requires that domestic output equals total domestic and foreign demand for home produced goods. In log-linear terms this is

$$y_t = c_{H,t} + c_{H,t}^*.$$

Since the demand for home and foreign consumption goods can be written in log-linear form as $c_{H,t} = (1-\alpha)[\alpha\eta s_t + c_t]$ and $c_{H,t}^* = \alpha[\eta(s_t + \psi_{F,t}) + y_t^*]$, respectively, we can write

$$y_t = (2-\alpha)\alpha\eta s_t + (1-\alpha)c_t + \alpha\eta\psi_{F,t} + \alpha y_t^*. \tag{34}$$

## 2.5 Log-linear approximation of the model

In this section we summarize the log-linearized equilibrium conditions. The consumption Euler equation is obtained by log-linearizing (10) and taking expectations conditional on the time-$t$ sigma algebra:

$$c_t - hc_{t-1} = \mathbb{E}_t(c_{t+1} - hc_t) - \frac{1-h}{\sigma}(r_t - \mathbb{E}_t\pi_{t+1}). \tag{35}$$

Domestic goods inflation is given by (24) and substituting out the term $mc_{H,t}$:

$$\begin{aligned}\pi_{H,t} = {} & \beta\mathbb{E}_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \delta_H\pi_{H,t-1} \\ & + \lambda_H\left[\varphi y_t - (1+\varphi)\varepsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1})\right] + \lambda_H\varepsilon_{H,t}\end{aligned} \tag{36}$$

Imports inflation is given by (31) and substituting out the term $\psi_{F,t}$ with (33):

$$\pi_{F,t} = \beta\mathbb{E}_t(\pi_{F,t+1} - \delta_F\pi_{F,t}) + \delta_F\pi_{F,t-1} + \lambda_F[q_t - (1-\alpha)s_t] + \lambda_F\varepsilon_{F,t} \tag{37}$$

Leading (32) one period, first-differencing, taking the time-$t$ conditional expectations operator, and then combining with (38) yields the real interest parity condition,

$$\mathbb{E}_t\left(q_{t+1}-q_t\right)=\left(r_t-\mathbb{E}_t\pi_{t+1}\right)-\left(r_t^*-\mathbb{E}_t\pi_{t+1}^*\right)+\varepsilon_{q,t}. \tag{38}$$

First differencing the terms of trade equation (17) we have

$$s_t-s_{t-1}=\pi_{F,t}-\pi_{H,t}+\varepsilon_{s,t}. \tag{39}$$

Goods market clearing (34) in combination with (26) yields

$$y_t=(1-\alpha)\,c_t+\alpha\eta q_t+\alpha\eta s_t+\alpha y_t^* \tag{40}$$

First-differencing the CPI definition yields CPI inflation,

$$\pi_t=(1-\alpha)\,\pi_{H,t}+\alpha\pi_{F,t} \tag{41}$$

There are exogenous stochastic processes for terms-of-trade, technology and real-interest-parity shocks:

$$\varepsilon_{j,t}=\rho_j\varepsilon_{j,t-1}+\nu_{j,t};\ \ \rho_j\in(0,1)\,,\nu_j\sim i.i.d.\left(0,\sigma_j^2\right) \tag{42}$$

for $j=s,a,q$. Recall the marginal cost shocks in the home goods and import retailers profit functions are $\varepsilon_H\sim i.i.d.(0,\sigma_H)$ and $\varepsilon_F\sim i.i.d.(0,\sigma_F)$, respectively.

Finally, for simplicity we assume that the foreign processes $\{\pi^*,y^*,r^*\}$ are given by uncorrelated AR(1) processes:[1]

$$\begin{pmatrix}\pi_t^*\\ y_t^*\\ r_t^*\end{pmatrix}=\begin{pmatrix}a_1&0&0\\0&b_2&0\\0&0&c_3\end{pmatrix}\begin{pmatrix}\pi_{t-1}^*\\ y_{t-1}^*\\ r_{t-1}^*\end{pmatrix}+\begin{pmatrix}\sigma_{\pi^*}&0&0\\0&\sigma_{y^*}&0\\0&0&\sigma_{r^*}\end{pmatrix}\begin{pmatrix}\nu_{\pi^*,t}\\ \nu_{y^*,t}\\ \nu_{r^*,t}\end{pmatrix}, \tag{43}$$

where $\left(\nu_{\pi^*,t},\nu_{y^*,t},\nu_{r^*,t}\right)\sim N(0,I_3)$ with $I_3$ a 3-dimension identity matrix.

## 2.6 Central bank preferences

Since the model possesses incomplete exchange rate pass through in the short run (which gives rise to persistent gaps in the law of one price for imported goods)

[1] Our earlier estimates have also utilized assumptions on $\{\pi^*,y^*,r^*\}$ as being generated by a VAR(1) process and also a limiting closed-economy new Keynesian model under a first best fiscal-monetary policy arrangement. The former is statistically more flexible than our current assumption, and the latter is a stricter theoretical restriction on the data. We found that these assumptions do not matter very much. Thus, a reasonable middle ground for statistical flexibility and parsimony in our model parameterization would be to use our current assumption.

there is a potential role for the central bank to minimize these gaps. Alternatively, given the terms of trade $s_t$ from equation (33), the central bank can target the real exchange rate $q_t$ to stabilize these law of one price gaps, $\psi_{F,t}$. For computational and estimation purposes, we suppose the one-period general loss function for the central bank is quadratic:[2]

$$L(\widetilde{\pi}_t, y_t, q_t, r_t) = \frac{1}{2}\left[\widetilde{\pi}^2 + \mu_y y_t^2 + \mu_q q_t^2 + \mu_r (r_t - r_{t-1} + \varepsilon_{r,t})^2\right] \tag{44}$$

where $\Delta\widetilde{r}_t := r_t - r_{t-1} + \varepsilon_{r,t}$ represents the targeted change in actual short-term interest rate (e.g. some measured 90-day rate) which cannot be perfectly controlled. The random variable $\varepsilon_{r,t} \sim i.i.d.(0, \sigma_r^2)$ represents imperfect central-bank control of the short-term interest rate.[3] The parameters $\mu_y, \mu_q, \mu_r \in [0, +\infty)$ express the concern with output stabilization, real exchange rate stabilization and targeted interest rate smoothing respectively. These objectives are expressed relative to a concern for annual inflation ($\widetilde{\pi}_t := \sum_{i=0}^{3} \pi_{t-i}/4$) that is normalized to one. This specification of macroeconomic objectives captures the expressed goals of so-called "flexible" inflation targeting central banks.

From a public finance perspective, such assignments of policy objective functions are clearly ad hoc. For example, the literature following the method of deriving an approximate private-welfare-based loss function in Woodford (2003) would argue that the loss function parameters are not "free" but must be constrained by the preferences of the representative household. However, in defence of our approach we make three arguments.

First, it may be impossible to map a second-order approximation of the welfare maximizing central-bank loss function from household preferences (see Galí and Monacelli (2005) and Monacelli (2005)).[4] Second, from an empirical perspective our loss function captures the range of goal variables Australia, Canada and New Zealand have defined as monetary policy objectives.

In the case of New Zealand, there is a legislated set of policy objectives: price stability, output, interest rates and the exchange rate. Australia's monetary policy tries to achieve a target for inflation of 2-3 percent per annum but does so over the medium term, implying interest rates need not be moved around excessively in the

[2] Our aim is recover the macroeconomic objectives of open economy central banks. We take no stance on normative design aspects of what these objectives should be. Rather, we simply seek what our three open economy inflation targeters have tried to achieve over the sample period.

[3] In a linear-quadratic framework this will naturally show up as a linear perturbation to the reduced form optimal monetary policy rule.

[4] Galí and Monacelli (2005) showed that this is only possible in the case of complete pass through and under a restrictive parametric case, which in our notation is $\sigma = \alpha = \eta = 1$.

short run. Furthermore, subject to the inflation objective policy is partly motivated by output objectives and "seeks to encourage the strong and sustainable growth in the economy". It is worth noting that although Section 10(2) of the RBA act gives the stability of the currency as a goal, the inflation target is viewed as the vehicle through which this objectives are achieved. Interestingly the Bank of Canada is comparatively silent on objectives for output and describes a flexible exchange rate as a "shock absorber" that enables the setting of independent monetary policy.

Finally, our formulation of objectives nests the monetary policy literature that seeks to evaluate the efficacy of alternative monetary policy rules using quadratic loss functions (see Rudebusch and Svensson 1999 and Levin and Williams 2003, for example).

**Optimal time-consistent monetary policy**

We assume the central bank acts optimally under a time-consistent policy or what is often called discretionary policy. Define $W(\varepsilon_t, z_{t-1})$ as the value function of the central bank's optimal action at time $t$ given state $z_t := \left\{c_{t-1}, y^*_{t-1}, \pi_{H,t-1}, \pi_{F,t-1}\right\}$ and $\varepsilon_t := \left(\pi^*_t, y^*_t, r^*_t, \left\{\varepsilon_{j,t}\right\}\right)$, for $j = s, a, a^*, H, F, q$. Let the infinite sequence of central bank actions, $\{u_t\}_{t=0}^{\infty} := \{c_t, \pi_{H,t}, \pi_{F,t}, q_t, r_t, s_t, y_t\}_{t=0}^{\infty}$, define a central bank's strategy starting from the state vector at time $t$, $x_t$. The private sector forms rational expectations of the future path of these variables such that these expectations will be consistent with the equilibrium behavior as approximated in (35)-(42).

Because the constraints (35)-(42) arising from the private sector's recursive decentralized equilibrium response involve lagged endogenous variables, $z_{t-1}$, which affect the value of a future central bank, the discretion problem for a current central bank is still dynamic.

The sequence of discretionary problems can be written recursively as:

$$W(\varepsilon_t, z_{t-1}) = \min_{u_t} L(\pi_t, y_t, q_t, r_t, \varepsilon_{r,t}) + \beta \mathbb{E}_t W(\varepsilon_{t+1}, z_t) \tag{45}$$

subject to (35)-(42). The central bank takes private expectations as given when it sequentially optimizes. We restrict our attention to discretionary, time-consistent policy using the equilibrium concept of Markov perfection. A Markov perfect strategy for the central bank is a sequence of state-contingent prices and allocations from which the central bank will not deviate, and these allocations are also consistent with private sector equilibrium expectations.

We compute the optimal time-consistent policy (as a numerical fixed-point problem in the product space of central-bank-private-sector feasible actions) using the algorithm of Dennis (2004). In the existing Bayesian literature on such models, one often estimates a reduced form Taylor type rule. However, when the central bank optimizes under discretion, it can be shown, as in Dennis (2004), that policy preference parameters and deep parameters place non-linear constraints on a reduced-form policy feedback rule.

# 3 Empirical investigation

The first question we would like to ask is the following: do these flexible inflation-targeting central banks place much weight on exchange rate deviations? Existing papers have focused on whether and how much central banks *respond* to exchange rates at the level of reduced form interest-rate rules (see Lubik and Schorfheide 2005b and Justiniano and Preston 2005). The second question is whether the preferences of the central banks are "different" or "similar".

## 3.1 Estimation strategy

We proceed by estimating two versions of the model for each country. The first version utilizes the general one-period loss specification in (44). We will call this larger model $M_1$ in subsequent discussions. The set of parameters to be estimated are the following central back preference parameters, $\{\mu_y, \mu_r, \mu_q\}$, the private sector deep parameters, $\{h, \sigma, \phi, \eta, \delta_H, \delta_F, \theta_H, \theta_F\}$ and the list of parameters for exogenous processes $\{a_1, b_2, c_3, \rho_a, \rho_q, \rho_s, \sigma_H, \sigma_F, \sigma_a, \sigma_q, \sigma_s, \sigma_{\pi^*}, \sigma_{y^*}, \sigma_{r^*}, \sigma_r\}$.[5]

The second version, which we shall denote as model $M_2$, uses (44) but restricts $\mu_q = 0$. We can then address the second question in (b) by using Bayesian posterior odds comparisons to see if a model with or without $\mu_q = 0$ is more probable, all other things equal.

We are interested in estimating the structural or deep parameters of our model and variations of it. We classify a candidate model $M$ by its list of parameters, $\theta$. Our estimation procedure uses the random-walk-Metropolis Markov chain Monte Carlo (MCMC) method. We outline this popular algorithm in Appendix B. Table

[5] There is very little information in the data to help us pin down the discount factor $\beta$ and imports share in domestic consumption, $\alpha$, so we set these as 0.99 and 0.45, respectively.

1 summarizes the prior marginal density functions we use on each estimated parameter in the models. We use fairly agnostic or dispersed prior densities as evident in the wide 95 percent confidence intervals around the prior means. To ensure that theoretical restrictions on the parameter ranges are satisfied, we draw from prior densities that are restricted to the appropriate supports. For example, we define a prior density for the Calvo parameter $\theta_H$ to have the domain $(0,1)$.

For each candidate $\theta$, the linear rational expectations (RE) system including the optimal monetary policy problem is solved to obtain a general solution in terms of the endogenous state variables $y_t$ and the central-bank policy decision variables, $x_t$ (which is just the scalar $r_t$ in our case):

$$\xi_{t+1} = A(\theta)\,\xi_t + C(\theta)\,\varepsilon_{t+1} \tag{46}$$

where $\xi_t := (y_t, x_t)$. We can map some of the variables in $\xi_t$ to a vector of observable variables, $y_t^o$ using an observation equation:

$$y_t^o = G\xi_t. \tag{47}$$

We set the length of the parameters' Markov chain to be $N = 2 \times 10^6$ draws and remove the first half of the sample (the "burn-in" period) to remove any effect of the initial condition of the Markov chain $\{\theta_n\}$ and also perform some diagnostic tests to check that our MCMC procedure has converged to its stable, invariant distribution.

## 3.2 Data

Each model we consider has nine structural shocks. To avoid stochastic singularity, we match these to nine observable time series for each of our sample countries: Australia, Canada and New Zealand. We use quarterly data over the period 1990Q1 to 2005Q3.[6] The time series data are from the IMF's *International Financial Statistics* database, with the exception of Australian and New Zealand CPI inflation series, which were obtained from the Reserve Bank of Australia and

[6] The Reserve Bank of New Zealand officially began targeting inflation in February 1990 and Canada followed one year later. The Reserve Bank of Australia suggests that inflation targeting was officially adopted in the first half of 1993 (Stevens 2003). However, Bernanke, Laubach, Mishkin, and Posen (1999) note that Australian interest rates rose dramatically in the late 1980s with no noticeable increase in inflation and conclude the RBA possessed objectives for inflation that predate the announced adoption of inflation targeting. Since we seek to define preferences via the underlying interest rate rule, we define the inflation targeting period in Australia as beginning slightly earlier than some other commentators.

the Reserve Bank of New Zealand, respectively. The data we collect (with their corresponding theoretical counterparts in parentheses) are import price inflation in home currency as a proxy for foreign goods inflation ($\pi_{F,t} := p_{F,t} - p_{F,t-1}$), home-US real exchange rate ($q_t$), the 'terms of trade' constructed as the ratio of import prices to export prices ($s_t := p_{F,t} - p_{H,t}$), home real GDP ($y_t$), home CPI inflation ($\pi_t$), home nominal (overnight cash) interest rate ($r_t$), the US CPI inflation rate from FRED ($\pi_t^*$), US output ($y_t^*$), and the US federal funds rate ($r_t^*$).

We take an agnostic view of trends and cycles. We detrend using the Hodrick-Prescott (HP) filter and construct an output gap measured as deviations of output from this trend. We also filter the terms of trade and real exchange rate data using the HP filter for similar reasons.

# 4 Results

Before we turn to addressing our first empirical question of whether the central banks in question care explicitly about the exchange rate, we discuss the estimates of parameters in the models themselves and show that the estimates are quite plausible economically. In section 4.2 we take up the first main question. In section 4.3 we address the second question of whether these central banks are similar in their policy preferences.

## 4.1 Structural parameter estimates

The estimated prior and posterior density functions on the key structural model parameters for Australia, Canada and New Zealand are displayed in figures 1-3. Mean estimates, standard deviations, and 95 percent confidence intervals for the posterior estimates are reported in tables 3-5.

In addition, the tables report summaries of diagnostic tests for convergence of the Markov chains of the parameters. The convergence test statistics were computed by taking a subsequence of the total 2 million draws, with a length of 0.5 million draws, to reduce computational burden. We sample using the Metropolis-Hasting algorithm. The NSE in the fifth column refers to the numeric standard error as an approximation to the true posterior standard error described in Geweke (1999). The $p$-values in the sixth column refer to the equality test between the means calculated using the first and second half of the chain. In each of the models, there are only one or two parameters that did not satisfy the equality test at the 5 percent level. None of the test statistics are significant at the 1 percent level and there is no obvious pattern to which coefficients fail the equality test.

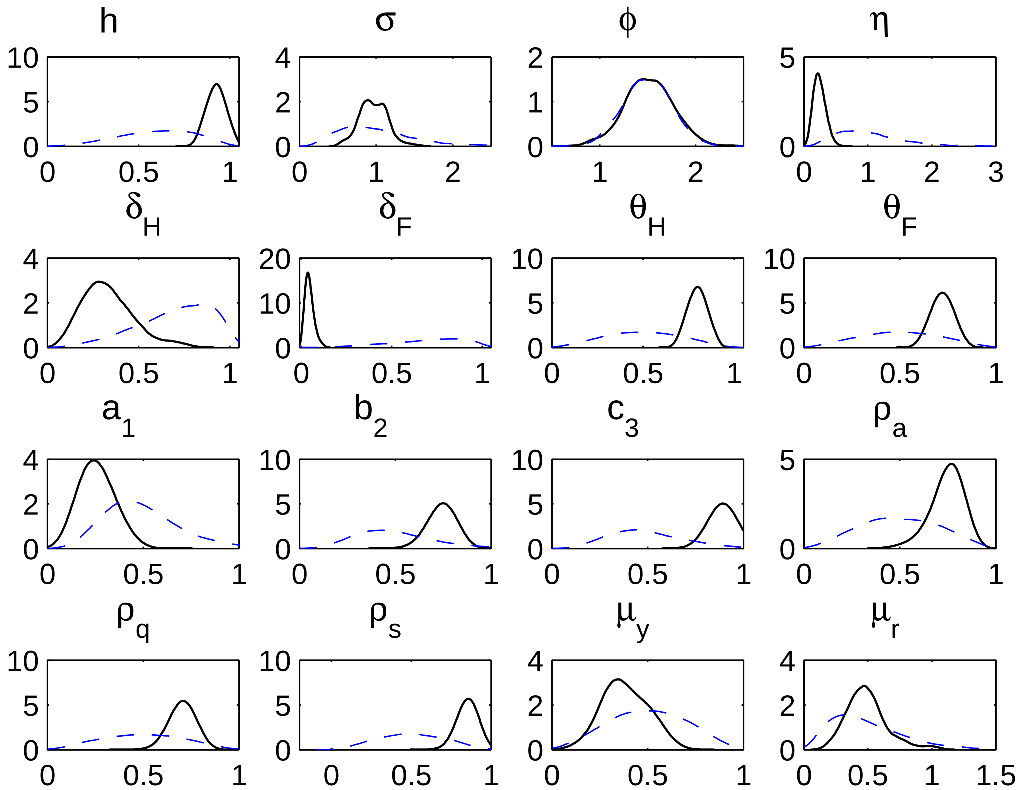
The seventh column shows the univariate "shrink factors" using the ratio of between and within variances as in Brooks and Gelman (1998). A shrink factor close to 1 is evidence of convergence to a stationary distribution. Almost all of the shrink factors were less than 1.1 and the maximum value across the six models is 1.26. The parameters with a shrink factor greater than 1.1 are those parameters that did not satisfy 5 percent equality test. Overall, the evidence suggests that the Markov chains have converged to their stationary distribution.

**Australia**

The posterior density estimates for the key parameters for Australia are displayed in figure 1 for the case where the central bank is restricted to put no weight on exchange rate variability. The full set of model estimates is reported in tables 2 for model $M_1$ and table 3 for model $M_2$. Our estimates of the Calvo-type frequency of price changes are $\widehat{\theta}_H \approx 0.77$ for $M_1$ and $\widehat{\theta}_H \approx 0.8$ for $M_2$,and, $\widehat{\theta}_F \approx 0.68$ for $M_1$ and $\widehat{\theta}_F \approx 0.72$ for $M_2$, respectively, in the home goods and imported goods sectors. This suggests that in the home goods sector, the average duration that prices remain fixed is between 4.3 to 5 quarters across the two models. Similarly, for the home goods sector, average prices stay the same for 3 to 3.6 quarters on average. The "backward-lookingness" in the Phillips curves, represnted by $\delta_H$ and $\delta_F$, is quite low, especially, for the imported goods sector. The high degrees of price stickiness imply that inflation is not very sensitive to changes in marginal cost (or LOP gap) movements, and therefore, a smaller and slower transmission of monetary policy to inflation.

In contrast, consumption is very sensitive to real-interest-rate changes because the estimate of $\sigma$, the coefficient of relative risk aversion, is quite close to 1. The degree of habit persistence is quite high, $\widehat{h} \approx 0.9$. This has the opposite effect on the sensitivity of consumption to real-interest-rate changes. The uniform within-sector demand elasticity of substitution estimate is $\widehat{\eta} \approx 0.36$ for model 1 and $\widehat{\eta} \approx 0.17$ for model 2. This is lower than typical calibrations. For example, Monacelli (2005) sets $\eta \approx 1.6$. A low $\eta$ implies $\pi_H$ or domestic output gap is not very sensitive to terms of trade movements compared to usual calibrations, all else equal. The inverse labour supply elasticity is $\widehat{\phi} \approx 1.5$.

**Figure 1**
**Posterior distribution of key parameters: Australia**



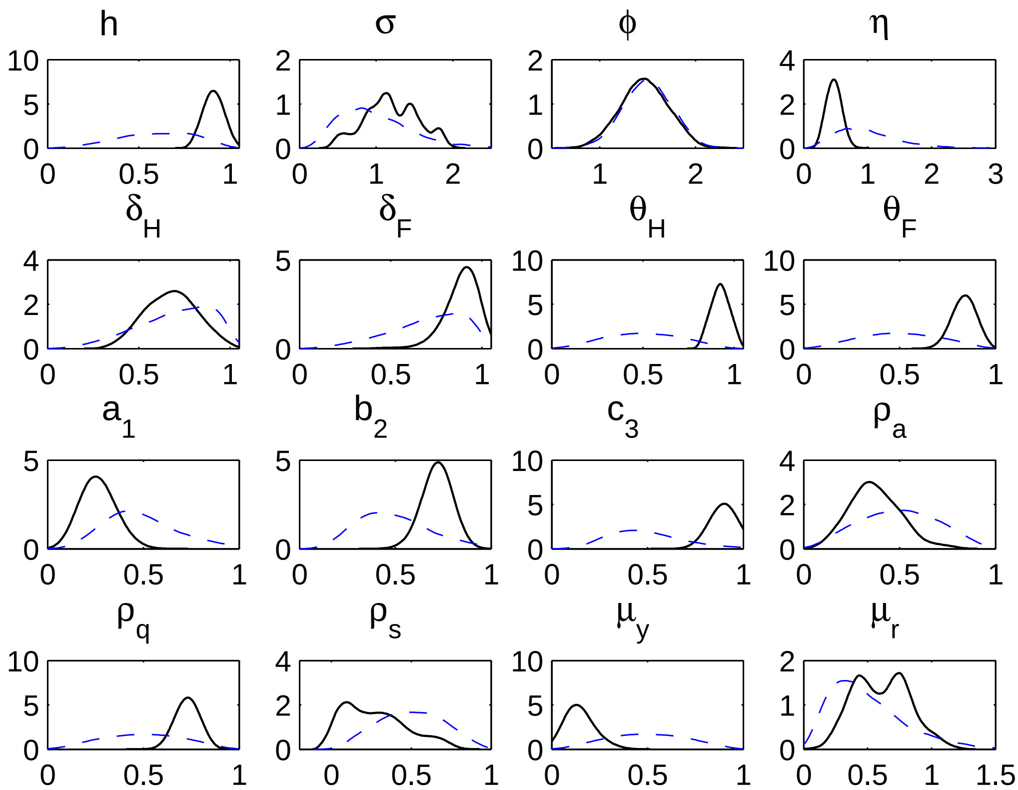

NB. Prior distributions are dashed lines, posterior distributions are solid lines.

**Canada**

The posterior density estimates for the key parameters for Canada are displayed in figure 2. The full set of model estimates is reported in table 4 for model $M_1$ and in table 5 for model $M_2$.

Notable exceptions for Canada's results are that the degrees of backward-looking behavior in firms' pricing are much higher than the estimates for Australia. Here we have $\delta_H$ and $\delta_F$ estimated in the order of 0.65 and 0.8 respectively.

**Figure 2**
**Posterior distribution of key parameters: Canada**



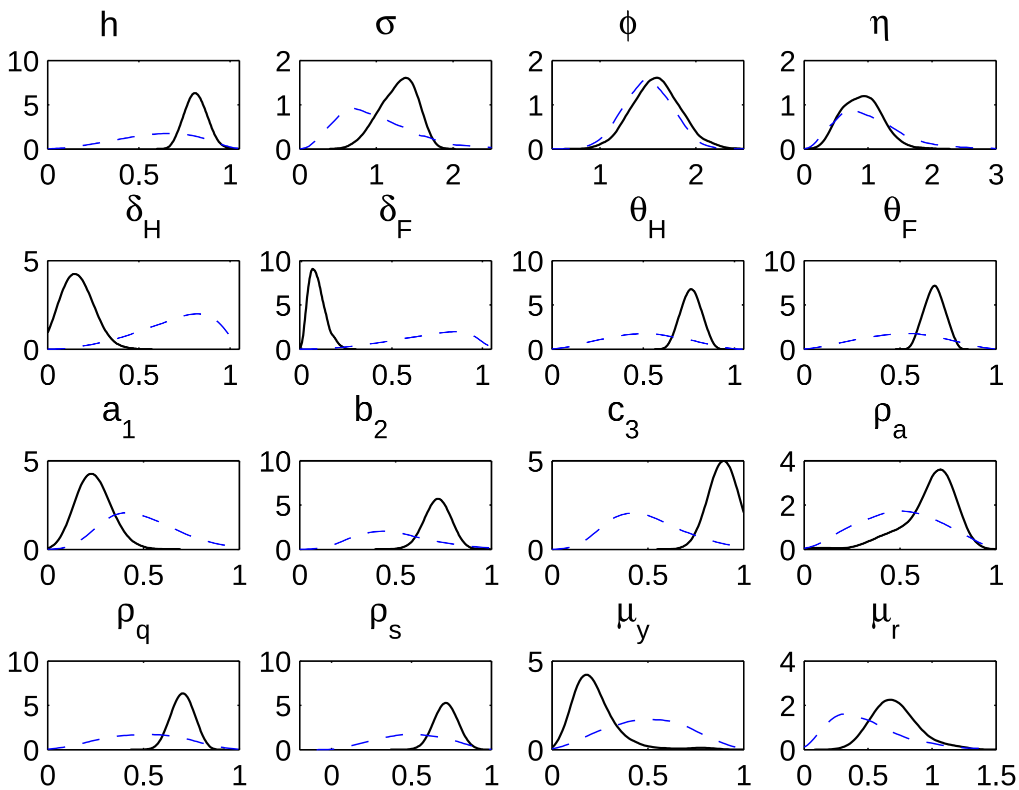

NB. Prior distributions are dashed lines, posterior distributions are solid lines.

**New Zealand**

The posterior density estimates for the key parameters for New Zealand are displayed in figure 3 for the case where $\mu_q = 0$. The full set of model estimates are reported in table 6 for model $M_1$ and table 7 for model $M_2$.

The private sector deep parameters in New Zealand are quite similar to Australia. The notable exception is that the uniform within-sector demand elasticity of substitution estimate of $\widehat{\eta} \approx 1$ is much higher than in Australia or Canada. This implies a greater elasticity of substitution of consumption between home and foreign goods in the model. It also implies that New Zealand's output gap will be very responsive to terms of trade movements.

**Figure 3**
**Posterior distribution of key parameters: New Zealand**




NB. Prior distributions are dashed lines, posterior distributions are solid lines.

## 4.2 Do central banks weight exchange rate volatility?

Our first empirical question asks whether these flexible inflation-targeting central banks care about the real exchange rate explicitly. Consider two competing models of central banks for a dataset $y$. Denote a flexible inflation targeter with one-period payoff summarized by (44) as $M_1 := \{\theta \in \Theta : 0 < \mu_q \in \theta\}$. Let the alternative central bank that does not target exchange rate deviations be given by $M_2 := \{\theta \in \Theta : 0 = \mu_q \in \theta\}$.

Table 8 summarizes our model comparison based on the posterior odds ratio or Bayes factor, which in our case, is the ratio of the marginal likelihoods of the two competing models.[7] For each of the three countries, there is a "better fit" of the data for model $M_2$ than $M_1$. For example, consider Canada which has the lowest Bayes factor of $2.97 \times 10^4$ across the three economies. In order to infer that the Bank of Canada explicitly targets exchange rate volatility ($M_1$), one would need to have a prior belief on $M_1$ which is $2,970$ times stronger than one's prior belief on $M_2$. Our results in favour of $M_2$ are corroborated by the observation that the posterior densities of $\mu_q$, in the case of model $M_1$, are very tightly centered around a positive number close to zero for all three countries. Our result suggests that these small open economy inflation targeters do not explicitly target exchange rates.

## 4.3 Central banks' objectives and similarities

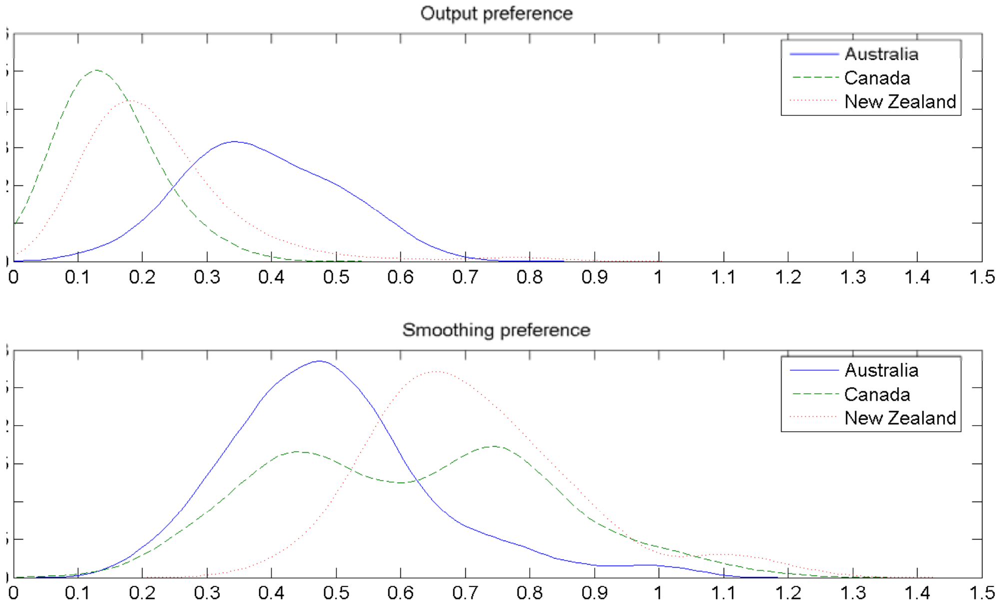
In this section we address the second empirical question of identifying what are the key features of central banks' preferences and whether they are similar in a statistical sense. More precisely, we will be looking at the "degree of overlap" between the marginal posterior distributions, and also the joint posterior distributions, on their preference parameters.

Inspection of the results displayed in tables 3-8, reveals that the following features drive central bank objectives in Australia, Canada and New Zealand. First, these central banks care a lot about smoothing interest rate movements. Second, there is not a lot of weight placed on the output gap, a result consistent with a strong inflation targeting focus for these central banks. Finally, these central banks place virtually no weight on exchange rates.

Cross-country comparisons of the preference parameters reveal whether our three

[7] The marginal likelihood of each model for a given data set is numerically computed using the modified harmonic mean estimator in Geweke (1999).

**Figure 4**
**Posterior comparison of loss function parameters**




open economy inflation targeters possess similar objectives. Figure 4 graphs the posterior distributions of both the output stabilization parameter and the interest rate smoothing parameter for each country on the same axes. The degree to which each country shares similar stabilization objectives is illustrated by the degree of similarity between the posterior distributions.

To measure the closeness of two distributions, DeJong, Ingram, and Whiteman (1996) construct a metric using the Confidence Interval Criterion (CIC). The CIC is:

$$CIC_{ij} = \frac{1}{1-\gamma}\int_a^b P_j(s_i)ds_i \tag{48}$$

where $P_j(s_i)$ is the distribution of the simulated model statistic and $s_i$, $i = 1,\cdots,n$ are the distributions of interest where $a = \frac{\gamma}{2}$ and $b = 1-a$ are particular quantiles of a reference distribution $D(s_i)$ the tails of which are truncated by the parameter $\gamma$. This implies that the CIC is in fact bounded by 0 and $(1-\gamma)^{-1}$ (such that the CIC is only bounded between 0 and 1 for the special case when $\gamma = 0$). The CIC statistic can be thought of as measuring the overlap in two distributions.

A CIC statistic close to the upper bound $(1-\gamma)^{-1}$, implies the distributions are very similar. A CIC close to zero implies the distributions are not particularly similar because either the location of the distributions is different or the reference distribution is particularly diffuse.

DeJong, Ingram, and Whiteman (1996) advocate using the following measure as a test for difference in location of the distributions:

$$d_{ji} = \frac{EP_j(s_i) - ED(s_i)}{\sqrt{var(D(s_i))}} \tag{49}$$

large differences in expected values (and hence expected location) generate large test statistics while diffusion in the reference distribution $D(S_i)$ reduces the test statistic.

Inspecting figure 4, the output stabilization parameter in the top half of the figure shows that all three countries place some weight on output stabilization. Canada appears to put the least weight on output stabilization with the left-most posterior distribution with a posterior mode of 0.147. The corresponding distribution for New Zealand is very similar in both shape and location, with a posterior mode of 0.217. With $\gamma = 0.1$ the CIC returns a value of 0.864, indicating that Canada and New Zealand share a similar concern for output stabilization. The Australian posterior distribution places a higher weight on output stabilization with a posterior mode of 0.384. The CIC between Australia and Canada is much smaller – 0.186 although the CIC returns a statistic of 0.475 for the overlap between output stabilization in Australia and New Zealand.

The graphic in the bottom half of figure 4 shows the overlap of the preference for interest rate smoothing across the three countries. All three countries show some interest rate smoothing behavior. Australia appears to place the least weight on smoothing the interest rate, returning a posterior model of 0.493 while the corresponding parameter is 0.647 for Canada and 0.732 for the case of New Zealand. However, the CIC statistics emphasize similarities rather than differences. The overlap in preferences for smoothing interest rates is 0.830 between Australia and Canada, 0.829 for Australia and New Zealand, and 1.0364 for Canada and New Zealand (which is greater than one since $\gamma = 0.1$, implying $(1-\gamma)^{-1} \approx 1.1$).

A natural question is whether the overall macroeconomic objectives of each country are identical. This is a joint test of whether the distribution of the preferences for macroeconomic stabilization and interest rate smoothing are the same. Rather than averaging the CIC criterion across the preference parameters, we construct a multivariate version of the CIC by generating a three dimensional histogram of joint draws from the posterior. For convenience we set $\gamma = 0$ and compare the volumes generated by integrating over the preference parameters for each country. We use 500,000 draws from the posterior and use a total of 625 bins to characterise the joint distribution.

This joint test returns a high degree of similarity across the distributions. Between Australia and Canada, we find that 90.6 percent of the draws can be characterised by the same distribution; and this figure remains high between Australia and New Zealand (93.4 percent) and between Canada and New Zealand (94.3 percent). Thus our results indicate that the preferences of these three small open economy inflation targeters are, in fact, pretty similar.

# 5 Conclusion

We estimate the macroeconomic policy objectives of the central banks of Australia, Canada and New Zealand within the context of an optimizing DSGE model. Our parameter estimates reveal the objectives of these small open economy inflation targeters. We find key differences in the structural parameters of each economy that imply different behaviour in the setting of monetary policy across countries – even if these countries share identical monetary policy objectives.

We emphasize the similarities rather than the differences in the macroeconomic objectives of the central banks of Australia, Canada and New Zealand. Over the period considered, all three central banks show no concern for mitigating exchange rate volatility as an objective in its own right. However, all three central banks show a substantial concern for interest rate smoothing. The Reserve Bank of Australia shows the most desire to mitigate volatility in the output gap but in all three cases the estimated weight on the output gap is substantially lower than the weight on the deviation of annual inflation from target. Neverthelss, all central banks would be sensibly classified as flexible in their approach to inflation targeting.

Our analysis has important implications for assessing the accountability and transparency of monetary policy. By jointly estimating the parameter estimates conditional on the same DSGE model we can make inferences about objectives conditional on the environment each central bank operates under. Such joint estimates result in very different conclusions relative to uninformed inference based on the unconditional distributions of goal variables such as annual inflation, the output gap, interest rates and the exchange rate. Future work could usefully extend the model to incorporate the potential effects of labour market behavior, credit constraints and policymaking under uncertainty on the estimates of central banks objectives.

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# Appendix

# A Log-linear approximations to firms' optimal pricing rule

## A.1 Domestic goods pricing

Given our specific assumption on period utility of households, re-write the first-order condition in (23), using the $s$-period iterate on the Euler operator (10) to replace $Q_{t,t+s}$, as

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \frac{(C_{t+s}-H_{t+s})^{-\sigma}}{P_{t+s}} Y_{t+s}(i) \times \left[ \widetilde{P}_{H,t} \left( \frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} - \left( \frac{\varepsilon}{\varepsilon-1} \right) P_{H,t+s} MC_{H,t+s} \exp\left(\varepsilon_{H,t+s}\right) \right] = 0$$

Log-linearize this around the deterministic steady state to obtain

$$\begin{aligned} \widetilde{p}_{H,t} - \delta_H p_{H,t-1} &\approx (1-\beta\theta_H) \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \left[ p_{H,t+s} - \delta_H p_{H,t+s-1} + mc_{H,t+s} + \varepsilon_{H,t+s} \right] \\ &= (1-\beta\theta_H) \left[ p_{H,t} - \delta_H p_{H,t-1} + mc_{H,t} + \varepsilon_{H,t} \right] \\ &+ \beta\theta_H (1-\beta\theta_H) \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \left[ p_{H,t+s+1} - \delta_H p_{H,t+s} + mc_{H,t+s+1} + \varepsilon_{H,t+s+1} \right]. \end{aligned}$$

This expression can be written recursively as

$$\widetilde{p}_{H,t} - \delta_H p_{H,t-1} \approx (1-\beta\theta_H) \left[ p_{H,t} - \delta_H p_{H,t-1} + mc_{H,t} + \varepsilon_{H,t} \right] + \beta\theta_H \left[ \mathbb{E}_t \widetilde{p}_{H,t+1} - \delta_H p_{H,t-1} \right]. \quad (50)$$

Log-linearizing (19) yields

$$p_{H,t} = (1-\theta_H) \widetilde{p}_{H,t} + \theta_H p_{H,t-1} + \theta_H \delta_H \pi_{H,t-1}. \quad (51)$$

Substituting (51) into (50) yields the expression (24).

Now, equating firms' labour demand (22) to households labour supply (9):

$$\frac{MC_{H,t}\varepsilon_{a,t}P_{H,t}}{P_t} = (C_t - H_t)^{\sigma} N_t^{\varphi} \quad (52)$$

Log-linearizing this, and using the log-linearized production function $y_t = n_t + \varepsilon_{a,t}$, we have

$$mc_{H,t} = p_t - p_{H,t} + \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \varphi y_t - (1+\varphi)\,\varepsilon_{a,t}. \tag{53}$$

Utilizing the log-linearized CPI index, which implies $p_t - p_{H,t} = -\alpha\,(p_{H,t} - p_{F,t}) = \alpha s_t$, and also (14), in (53) we obtain (25).

## A.2 Imports pricing

Re-write (30) as

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_F)^s \frac{(C_{t+s} - H_{t+s})^{-\sigma}}{P_{t+s}} Y_{F,t+s}(j) \times \left[ \widetilde{P}_{F,t} \left( \frac{P_{F,t+s-1}}{P_{F,t-1}} \right)^{\delta_F} - \left( \frac{\varepsilon}{\varepsilon - 1} \right) \widetilde{e}_{t+s} P^*_{F,t+s}(j) \exp(\varepsilon_{F,t+s}) \right] = 0$$

Log-linearizing, and substituting with $\psi_{F,t+s} + \varepsilon_{F,t+s} = e_{t+s} + p^*_{t+s}$, we obtain

$$\widetilde{p}_{F,t} - \delta_F p_{F,t-1} \approx (1-\beta\theta_F)\,\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_F)^s \left[ p_{F,t+s} + \psi_{F,t+s} + \varepsilon_{F,t+s} - \delta_F p_{F,t+s-1} \right].$$

Log-linearize (27) to get

$$p_{F,t} = (1-\theta_F)\,\widetilde{p}_{F,t} + \theta_F p_{F,t-1} + \theta_F \delta_F \pi_{F,t-1}. \tag{54}$$

Making use of the last two expressions yields (31).

# B Pseudo-code for MCMC procedure

The random walk metropolis algorithm for a linear RE model:

1. Begin with an initial prior $\theta_0 \in \Theta$ and its corresponding prior density $p(\theta_0|M)$ for model $M$.
2. Solve the linear RE model to obtain (46) and construct observation equation (47).
3. For each $n = 0, 1, ...N$, Use (46)-(47), the given data set $y = \{y_t^o\}_{t=0}^T$, and $\theta_n$ to compute the model likelihood, $L(\theta_n|y, M)$ using a Kalman filter. Then calculate the associated posterior density, $p(\theta_n|y, M) = \frac{p(\theta_n|M)L(\theta_n|y,M)}{\int_\Theta p(\theta_n|M)L(\theta_n|y,M)d\mu(\theta_n)}$.
4. Generate a new candidate draw using a random walk model: $\theta_{n+1} = \theta_n + z_{n+1}$, where we assume $z_{n+1} \sim N(0, s\Sigma)$, and $s > 0$ is a scalar factor for scaling the size of the jump in the draws. Compute the associated posterior density, $p(\theta_{n+1}|y, M)$ by repeating Step 3, for $\theta_{n+1}$.
5. Compute the acceptance probability, $\alpha(\theta_n, \theta_{n+1}|y) := \min\left\{\frac{p(\theta_{n+1}|y,M)}{p(\theta_n|y,M)}, 1\right\}$. In words, if $p(\theta_{n+1}|y, M) > p(\theta_n|y, M)$, accept the new candidate $\theta_{n+1}$ with probability 1. Otherwise, accept the new candidate $\theta_{n+1}$ with probability $p(\theta_{n+1}|y, M)/p(\theta_n|y, M)$.
6. Repeat Steps 3-4 for $N$ sufficiently large to ensure that the sequence $\{\theta_n\}_{n=0}^N$ is drawn from an ergodic distribution, $\pi$.
7. Under some sufficient conditions, we can apply the ergodic theorem of an irreducible Markov chain and approximate the posterior expected value of a (bounded) function of interest, $f(\theta)$ using the sample mean of the functions, $N^{-1}\sum_n^N f(\theta_n)$.

# C Prior and posterior parameter estimates

**Table 1**
**Prior parameter densities for all models.**

| | Prior mean | 2.5% | 97.5% | Domain | Density function |
|---|---|---|---|---|---|
| $h$ | 0.60 | 0.19 | 0.93 | $(0,1)$ | Beta |
| $\sigma$ | 1.00 | 0.27 | 2.19 | $\mathbb{R}_+$ | Gamma |
| $\phi$ | 1.50 | 1.01 | 1.99 | $\mathbb{R}_+$ | Gamma |
| $\eta$ | 1.00 | 0.27 | 2.19 | $\mathbb{R}_+$ | Gamma |
| $\delta_H$ | 0.70 | 0.25 | 0.98 | $(0,1)$ | Beta |
| $\delta_F$ | 0.70 | 0.25 | 0.98 | $(0,1)$ | Beta |
| $\theta_H$ | 0.50 | 0.13 | 0.87 | $(0,1)$ | Beta |
| $\theta_F$ | 0.50 | 0.13 | 0.87 | $(0,1)$ | Beta |
| $a_1$ | 0.50 | 0.19 | 0.96 | $(0,1)$ | Beta |
| $b_2$ | 0.50 | 0.19 | 0.96 | $(0,1)$ | Beta |
| $c_3$ | 0.50 | 0.19 | 0.96 | $(0,1)$ | Beta |
| $\rho_a$ | 0.50 | 0.13 | 0.87 | $(0,1)$ | Beta |
| $\rho_q$ | 0.90 | 0.23 | 1.00 | $(0,1)$ | Beta |
| $\rho_s$ | 0.25 | 0.01 | 0.72 | $(0,1)$ | Beta |
| $\mu_q$ | 0.50 | 0.13 | 1.07 | $\mathbb{R}_+$ | Gamma |
| $\mu_y$ | 0.50 | 0.09 | 1.24 | $\mathbb{R}_+$ | Gamma |
| $\mu_r$ | 0.50 | 0.09 | 1.24 | $\mathbb{R}_+$ | Gamma |
| $\sigma_H$ | 2.66 | 0.91 | 7.32 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_F$ | 2.67 | 0.91 | 7.33 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_a$ | 1.19 | 0.52 | 2.66 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_q$ | 0.53 | 0.32 | 0.87 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_s$ | 1.19 | 0.52 | 2.66 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_{\pi*}$ | 1.19 | 0.52 | 2.66 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_{y*}$ | 1.19 | 0.52 | 2.66 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_{r*}$ | 1.19 | 0.52 | 2.66 | $\mathbb{R}_+$ | Inverse Gamma |
| $\sigma_r$ | 1.19 | 0.52 | 2.66 | $\mathbb{R}_+$ | Inverse Gamma |

For $\mu_q = 0$ the prior and posterior distributions will be degenerate at zero.

**Table 2**
**Posterior parameters and convergence diagnostics: Australia ($\mu_q \neq 0$)**

| | Post Mean | Post Std | 2.5% | 97.5% | NSE[a] | p-value[b] | B-G[c] |
|---|---|---|---|---|---|---|---|
| $\beta$ | 0.990 | 0.000 | 0.990 | 0.990 | 0.000 | 1.000 | 1.000 |
| $\alpha$ | 0.450 | 0.000 | 0.450 | 0.450 | 0.000 | 1.000 | 1.000 |
| $h$ | 0.917 | 0.022 | 0.871 | 0.953 | 0.002 | 0.234 | 1.029 |
| $\sigma$ | 0.809 | 0.259 | 0.395 | 1.440 | 0.045 | 0.464 | 1.026 |
| $\phi$ | 1.586 | 0.245 | 1.111 | 2.059 | 0.011 | 0.993 | 1.000 |
| $\eta$ | 0.363 | 0.101 | 0.210 | 0.594 | 0.012 | 0.351 | 1.021 |
| $\delta_H$ | 0.257 | 0.101 | 0.096 | 0.504 | 0.013 | 0.353 | 1.023 |
| $\delta_F$ | 0.046 | 0.027 | 0.010 | 0.109 | 0.001 | 0.651 | 1.001 |
| $\theta_H$ | 0.777 | 0.026 | 0.726 | 0.829 | 0.003 | 0.666 | 1.004 |
| $\theta_F$ | 0.682 | 0.036 | 0.612 | 0.754 | 0.004 | 0.951 | 1.000 |
| $a_1$ | 0.259 | 0.084 | 0.113 | 0.439 | 0.002 | 0.041 | 1.003 |
| $b_2$ | 0.719 | 0.061 | 0.583 | 0.822 | 0.003 | 0.955 | 1.000 |
| $c_3$ | 0.891 | 0.059 | 0.770 | 1.005 | 0.001 | 0.258 | 1.001 |
| $\rho_a$ | 0.809 | 0.035 | 0.735 | 0.870 | 0.002 | 0.657 | 1.001 |
| $\rho_q$ | 0.684 | 0.050 | 0.576 | 0.773 | 0.004 | 0.832 | 1.000 |
| $\rho_s$ | 0.811 | 0.049 | 0.696 | 0.893 | 0.004 | 0.830 | 1.001 |
| $\mu_q$ | 0.005 | 0.003 | 0.001 | 0.012 | 0.000 | 0.729 | 1.001 |
| $\mu_y$ | 0.412 | 0.156 | 0.165 | 0.766 | 0.021 | 0.759 | 1.003 |
| $\mu_r$ | 0.611 | 0.186 | 0.307 | 0.988 | 0.028 | 0.198 | 1.062 |
| $\sigma_H$ | 1.057 | 0.317 | 0.565 | 1.827 | 0.031 | 0.576 | 1.005 |
| $\sigma_F$ | 4.430 | 1.629 | 1.393 | 7.121 | 0.288 | 0.517 | 1.021 |
| $\sigma_a$ | 5.178 | 1.021 | 3.395 | 7.325 | 0.162 | 0.154 | 1.079 |
| $\sigma_q$ | 0.746 | 0.123 | 0.542 | 1.023 | 0.009 | 0.736 | 1.001 |
| $\sigma_s$ | 5.452 | 0.543 | 4.494 | 6.515 | 0.061 | 0.061 | 1.066 |
| $\sigma_{\pi*}$ | 0.418 | 0.043 | 0.341 | 0.509 | 0.001 | 0.435 | 1.000 |
| $\sigma_{y*}$ | 0.547 | 0.071 | 0.421 | 0.701 | 0.002 | 0.888 | 1.000 |
| $\sigma_{r*}$ | 0.220 | 0.021 | 0.182 | 0.265 | 0.000 | 0.954 | 1.000 |
| $\sigma_r$ | 0.363 | 0.051 | 0.273 | 0.471 | 0.002 | 0.111 | 1.004 |

Notes:
a. The numerical standard error (NSE) as given in Geweke (1999).
b. The p-value is computed using $L = 0.08$ in Geweke (1999).
c. The B-G univariate “shrink factor” as in Brooks and Gelman (1998).

**Table 3**
**Posterior parameters and convergence diagnostics: Australia ($\mu_q = 0$)**

| | Post Mean | Post Std | 2.5% | 97.5% | NSE[a] | p-value[b] | B-G[c] |
|---|---|---|---|---|---|---|---|
| $\beta$ | 0.990 | 0.000 | 0.990 | 0.990 | 0.000 | 1.000 | 1.000 |
| $\alpha$ | 0.450 | 0.000 | 0.450 | 0.450 | 0.000 | 1.000 | 1.000 |
| $h$ | 0.925 | 0.022 | 0.876 | 0.963 | 0.003 | 0.899 | 1.000 |
| $\sigma$ | 1.029 | 0.241 | 0.661 | 1.646 | 0.036 | 0.014 | 1.244 |
| $\phi$ | 1.492 | 0.261 | 0.968 | 1.995 | 0.016 | 0.172 | 1.011 |
| $\eta$ | 0.219 | 0.097 | 0.079 | 0.430 | 0.014 | 0.008 | 1.231 |
| $\delta_H$ | 0.399 | 0.162 | 0.142 | 0.717 | 0.021 | 0.000 | 1.382 |
| $\delta_F$ | 0.047 | 0.025 | 0.010 | 0.108 | 0.001 | 0.436 | 1.002 |
| $\theta_H$ | 0.797 | 0.026 | 0.743 | 0.845 | 0.003 | 0.824 | 1.001 |
| $\theta_F$ | 0.720 | 0.035 | 0.649 | 0.785 | 0.005 | 0.701 | 1.004 |
| $a_1$ | 0.257 | 0.084 | 0.110 | 0.433 | 0.002 | 0.984 | 1.000 |
| $b_2$ | 0.750 | 0.063 | 0.617 | 0.861 | 0.005 | 0.144 | 1.022 |
| $c_3$ | 0.891 | 0.060 | 0.772 | 1.007 | 0.001 | 0.621 | 1.000 |
| $\rho_a$ | 0.728 | 0.101 | 0.465 | 0.847 | 0.013 | 0.019 | 1.197 |
| $\rho_q$ | 0.703 | 0.049 | 0.602 | 0.796 | 0.004 | 0.719 | 1.001 |
| $\rho_s$ | 0.852 | 0.048 | 0.737 | 0.927 | 0.006 | 0.870 | 1.001 |
| $\mu_y$ | 0.404 | 0.354 | 0.202 | 1.482 | 0.021 | 0.000 | 1.697 |
| $\mu_r$ | 0.517 | 0.153 | 0.265 | 0.845 | 0.022 | 0.287 | 1.035 |
| $\sigma_H$ | 2.058 | 0.994 | 0.728 | 4.223 | 0.167 | 0.137 | 1.098 |
| $\sigma_F$ | 1.504 | 1.290 | 0.355 | 5.405 | 0.168 | 0.227 | 1.063 |
| $\sigma_a$ | 6.758 | 1.105 | 4.692 | 8.891 | 0.167 | 0.006 | 1.280 |
| $\sigma_q$ | 0.819 | 0.121 | 0.602 | 1.069 | 0.009 | 0.130 | 1.022 |
| $\sigma_s$ | 5.716 | 0.661 | 4.388 | 7.167 | 0.086 | 0.013 | 1.162 |
| $\sigma_{\pi*}$ | 0.419 | 0.043 | 0.341 | 0.509 | 0.001 | 0.651 | 1.000 |
| $\sigma_{y*}$ | 0.533 | 0.071 | 0.410 | 0.686 | 0.002 | 0.411 | 1.001 |
| $\sigma_{r*}$ | 0.219 | 0.021 | 0.182 | 0.265 | 0.000 | 0.866 | 1.000 |
| $\sigma_r$ | 0.342 | 0.042 | 0.267 | 0.430 | 0.001 | 0.986 | 1.000 |

Notes:
a. The numerical standard error (NSE) as given in Geweke (1999).
b. The p-value is computed using $L = 0.08$ in Geweke (1999).
c. The B-G univariate "shrink factor" as in Brooks and Gelman (1998).

**Table 4**
**Posterior parameters and convergence diagnostics: Canada ($\mu_q \neq 0$)**

| | Post Mean | Post Std | 2.5% | 97.5% | NSE[a] | p-value[b] | B-G[c] |
|---|---|---|---|---|---|---|---|
| $\beta$ | 0.990 | 0.000 | 0.990 | 0.990 | 0.000 | 1.000 | 1.000 |
| $\alpha$ | 0.450 | 0.000 | 0.450 | 0.450 | 0.000 | 1.000 | 1.000 |
| $h$ | 0.912 | 0.030 | 0.850 | 0.966 | 0.003 | 0.415 | 1.010 |
| $\sigma$ | 1.241 | 0.354 | 0.557 | 1.875 | 0.061 | 0.409 | 1.032 |
| $\phi$ | 1.477 | 0.248 | 1.009 | 1.976 | 0.006 | 0.057 | 1.004 |
| $\eta$ | 0.416 | 0.119 | 0.206 | 0.668 | 0.010 | 0.443 | 1.009 |
| $\delta_H$ | 0.644 | 0.179 | 0.227 | 0.937 | 0.024 | 0.221 | 1.050 |
| $\delta_F$ | 0.776 | 0.145 | 0.423 | 0.977 | 0.020 | 0.546 | 1.012 |
| $\theta_H$ | 0.933 | 0.017 | 0.901 | 0.966 | 0.002 | 0.099 | 1.060 |
| $\theta_F$ | 0.849 | 0.031 | 0.785 | 0.907 | 0.002 | 0.407 | 1.006 |
| $a_1$ | 0.266 | 0.084 | 0.119 | 0.442 | 0.002 | 0.976 | 1.000 |
| $b_2$ | 0.748 | 0.064 | 0.611 | 0.865 | 0.003 | 0.706 | 1.000 |
| $c_3$ | 0.893 | 0.061 | 0.770 | 1.009 | 0.001 | 0.566 | 1.000 |
| $\rho_a$ | 0.433 | 0.151 | 0.144 | 0.723 | 0.022 | 0.522 | 1.013 |
| $\rho_q$ | 0.704 | 0.048 | 0.607 | 0.795 | 0.004 | 0.852 | 1.000 |
| $\rho_s$ | 0.229 | 0.167 | 0.011 | 0.607 | 0.024 | 0.122 | 1.077 |
| $\mu_q$ | 0.007 | 0.003 | 0.002 | 0.015 | 0.000 | 0.511 | 1.002 |
| $\mu_y$ | 0.157 | 0.094 | 0.033 | 0.409 | 0.012 | 0.891 | 1.000 |
| $\mu_r$ | 0.855 | 0.424 | 0.186 | 1.779 | 0.068 | 0.014 | 1.266 |
| $\sigma_H$ | 20.646 | 1.569 | 18.025 | 24.428 | 0.243 | 0.270 | 1.053 |
| $\sigma_F$ | 0.752 | 0.411 | 0.287 | 1.869 | 0.036 | 0.431 | 1.008 |
| $\sigma_a$ | 2.121 | 1.004 | 0.584 | 4.539 | 0.144 | 0.590 | 1.009 |
| $\sigma_q$ | 0.841 | 0.111 | 0.640 | 1.077 | 0.008 | 0.659 | 1.001 |
| $\sigma_s$ | 2.271 | 0.369 | 1.584 | 3.014 | 0.035 | 0.296 | 1.017 |
| $\sigma_{\pi*}$ | 0.367 | 0.040 | 0.296 | 0.452 | 0.000 | 0.964 | 1.000 |
| $\sigma_{y*}$ | 0.533 | 0.071 | 0.406 | 0.687 | 0.002 | 0.651 | 1.000 |
| $\sigma_{r*}$ | 0.222 | 0.022 | 0.183 | 0.269 | 0.000 | 0.102 | 1.000 |
| $\sigma_r$ | 0.360 | 0.045 | 0.281 | 0.457 | 0.001 | 0.726 | 1.000 |

Notes:
a. The numerical standard error (NSE) as given in Geweke (1999).
b. The p-value is computed using $L = 0.08$ in Geweke (1999).
c. The B-G univariate "shrink factor" as in Brooks and Gelman (1998).

**Table 5**
**Posterior parameters and convergence diagnostics: Canada ($\mu_q = 0$)**

| | Post Mean | Post Std | 2.5% | 97.5% | NSE[a] | p-value[b] | B-G[c] |
|---|---|---|---|---|---|---|---|
| $\beta$ | 0.990 | 0.000 | 0.990 | 0.990 | 0.000 | 1.000 | 1.000 |
| $\alpha$ | 0.450 | 0.000 | 0.450 | 0.450 | 0.000 | 1.000 | 1.000 |
| $h$ | 0.906 | 0.030 | 0.851 | 0.964 | 0.003 | 0.290 | 1.015 |
| $\sigma$ | 1.285 | 0.338 | 0.578 | 1.875 | 0.055 | 0.175 | 1.075 |
| $\phi$ | 1.456 | 0.254 | 0.961 | 1.952 | 0.006 | 0.036 | 1.004 |
| $\eta$ | 0.476 | 0.114 | 0.268 | 0.704 | 0.009 | 0.812 | 1.001 |
| $\delta_H$ | 0.657 | 0.177 | 0.237 | 0.929 | 0.023 | 0.129 | 1.084 |
| $\delta_F$ | 0.873 | 0.082 | 0.684 | 0.989 | 0.007 | 0.146 | 1.026 |
| $\theta_H$ | 0.922 | 0.016 | 0.891 | 0.952 | 0.001 | 0.367 | 1.009 |
| $\theta_F$ | 0.852 | 0.037 | 0.760 | 0.905 | 0.003 | 0.039 | 1.088 |
| $a_1$ | 0.265 | 0.084 | 0.119 | 0.443 | 0.001 | 0.670 | 1.000 |
| $b_2$ | 0.718 | 0.066 | 0.578 | 0.840 | 0.003 | 0.552 | 1.001 |
| $c_3$ | 0.897 | 0.059 | 0.775 | 1.011 | 0.001 | 0.897 | 1.000 |
| $\rho_a$ | 0.366 | 0.137 | 0.120 | 0.639 | 0.016 | 0.530 | 1.009 |
| $\rho_q$ | 0.731 | 0.044 | 0.640 | 0.812 | 0.003 | 0.720 | 1.001 |
| $\rho_s$ | 0.257 | 0.180 | 0.012 | 0.658 | 0.025 | 0.583 | 1.009 |
| $\mu_y$ | 0.147 | 0.069 | 0.049 | 0.313 | 0.006 | 0.941 | 1.000 |
| $\mu_r$ | 0.672 | 0.233 | 0.248 | 1.106 | 0.033 | 0.061 | 1.110 |
| $\sigma_H$ | 20.462 | 3.145 | 13.842 | 24.834 | 0.545 | 0.214 | 1.081 |
| $\sigma_F$ | 0.682 | 0.490 | 0.277 | 2.347 | 0.033 | 0.197 | 1.036 |
| $\sigma_a$ | 2.397 | 1.618 | 0.774 | 6.779 | 0.221 | 0.169 | 1.085 |
| $\sigma_q$ | 0.780 | 0.104 | 0.600 | 1.004 | 0.004 | 0.568 | 1.001 |
| $\sigma_s$ | 2.154 | 0.362 | 1.448 | 2.856 | 0.039 | 0.712 | 1.002 |
| $\sigma_{\pi*}$ | 0.368 | 0.040 | 0.297 | 0.453 | 0.000 | 0.755 | 1.000 |
| $\sigma_{y*}$ | 0.546 | 0.073 | 0.419 | 0.704 | 0.001 | 0.073 | 1.002 |
| $\sigma_{r*}$ | 0.220 | 0.021 | 0.183 | 0.266 | 0.000 | 0.409 | 1.000 |
| $\sigma_r$ | 0.315 | 0.035 | 0.252 | 0.389 | 0.001 | 0.787 | 1.000 |

Notes:
a. The numerical standard error (NSE) as given in Geweke (1999).
b. The p-value is computed using $L = 0.08$ in Geweke (1999).
c. The B-G univariate "shrink factor" as in Brooks and Gelman (1998).

**Table 6**
**Posterior parameters and convergence diagnostics: New Zealand ($\mu_q \neq 0$)**

| | Post Mean | Post Std | 2.5% | 97.5% | NSE[a] | p-value[b] | B-G[c] |
|---|---|---|---|---|---|---|---|
| $\beta$ | 0.990 | 0.000 | 0.990 | 0.990 | 0.000 | 1.000 | 1.000 |
| $\alpha$ | 0.450 | 0.000 | 0.450 | 0.450 | 0.000 | 1.000 | 1.000 |
| $h$ | 0.785 | 0.040 | 0.729 | 0.896 | 0.004 | 0.195 | 1.060 |
| $\sigma$ | 1.568 | 0.396 | 0.822 | 2.257 | 0.070 | 0.186 | 1.085 |
| $\phi$ | 1.550 | 0.247 | 1.053 | 2.028 | 0.013 | 0.654 | 1.001 |
| $\eta$ | 1.011 | 0.329 | 0.421 | 1.683 | 0.041 | 0.805 | 1.001 |
| $\delta_H$ | 0.175 | 0.074 | 0.055 | 0.338 | 0.006 | 0.761 | 1.001 |
| $\delta_F$ | 0.087 | 0.045 | 0.020 | 0.193 | 0.003 | 0.483 | 1.003 |
| $\theta_H$ | 0.775 | 0.029 | 0.711 | 0.825 | 0.004 | 0.786 | 1.002 |
| $\theta_F$ | 0.697 | 0.021 | 0.649 | 0.729 | 0.002 | 0.004 | 1.171 |
| $a_1$ | 0.237 | 0.085 | 0.100 | 0.422 | 0.002 | 0.104 | 1.004 |
| $b_2$ | 0.698 | 0.050 | 0.587 | 0.791 | 0.004 | 0.310 | 1.013 |
| $c_3$ | 0.890 | 0.060 | 0.770 | 1.007 | 0.001 | 0.187 | 1.001 |
| $\rho_a$ | 0.544 | 0.187 | 0.165 | 0.830 | 0.031 | 0.625 | 1.011 |
| $\rho_q$ | 0.695 | 0.039 | 0.596 | 0.760 | 0.004 | 0.576 | 1.006 |
| $\rho_s$ | 0.682 | 0.068 | 0.547 | 0.827 | 0.006 | 0.952 | 1.000 |
| $\mu_q$ | 0.006 | 0.005 | 0.001 | 0.018 | 0.000 | 0.965 | 1.000 |
| $\mu_y$ | 0.273 | 0.138 | 0.100 | 0.623 | 0.020 | 0.574 | 1.010 |
| $\mu_r$ | 0.850 | 0.252 | 0.312 | 1.221 | 0.034 | 0.007 | 1.287 |
| $\sigma_H$ | 2.100 | 1.298 | 0.579 | 4.984 | 0.230 | 0.984 | 1.000 |
| $\sigma_F$ | 0.775 | 0.362 | 0.307 | 1.647 | 0.031 | 0.032 | 1.064 |
| $\sigma_a$ | 20.023 | 1.870 | 15.376 | 22.747 | 0.277 | 0.003 | 1.368 |
| $\sigma_q$ | 0.800 | 0.135 | 0.590 | 1.122 | 0.015 | 0.723 | 1.003 |
| $\sigma_s$ | 2.656 | 0.471 | 1.858 | 3.689 | 0.045 | 0.093 | 1.056 |
| $\sigma_{\pi*}$ | 0.412 | 0.043 | 0.335 | 0.503 | 0.000 | 0.924 | 1.000 |
| $\sigma_{y*}$ | 0.553 | 0.073 | 0.423 | 0.708 | 0.002 | 0.177 | 1.003 |
| $\sigma_{r*}$ | 0.222 | 0.022 | 0.184 | 0.269 | 0.000 | 0.049 | 1.001 |
| $\sigma_r$ | 0.374 | 0.059 | 0.268 | 0.500 | 0.003 | 0.486 | 1.004 |

Notes:
a. The numerical standard error (NSE) as given in Geweke (1999).
b. The p-value is computed using $L = 0.08$ in Geweke (1999).
c. The B-G univariate “shrink factor” as in Brooks and Gelman (1998).

**Table 7**
**Posterior parameters and convergence diagnostics: New Zealand ($\mu_q = 0$)**

| | Post Mean | Post Std | 2.5% | 97.5% | NSE[a] | p-value[b] | B-G[c] |
|---|---|---|---|---|---|---|---|
| $\beta$ | 0.990 | 0.000 | 0.990 | 0.990 | 0.000 | 1.000 | 1.000 |
| $\alpha$ | 0.450 | 0.000 | 0.450 | 0.450 | 0.000 | 1.000 | 1.000 |
| $h$ | 0.812 | 0.036 | 0.752 | 0.891 | 0.005 | 0.999 | 1.000 |
| $\sigma$ | 1.312 | 0.318 | 0.674 | 2.015 | 0.046 | 0.236 | 1.062 |
| $\phi$ | 1.586 | 0.264 | 1.041 | 2.082 | 0.010 | 0.149 | 1.011 |
| $\eta$ | 0.917 | 0.307 | 0.369 | 1.553 | 0.031 | 0.617 | 1.005 |
| $\delta_H$ | 0.173 | 0.074 | 0.043 | 0.324 | 0.007 | 0.275 | 1.019 |
| $\delta_F$ | 0.083 | 0.044 | 0.019 | 0.190 | 0.002 | 0.390 | 1.004 |
| $\theta_H$ | 0.767 | 0.027 | 0.712 | 0.819 | 0.003 | 0.073 | 1.074 |
| $\theta_F$ | 0.683 | 0.020 | 0.645 | 0.725 | 0.002 | 0.017 | 1.095 |
| $a_1$ | 0.240 | 0.079 | 0.103 | 0.408 | 0.002 | 0.501 | 1.000 |
| $b_2$ | 0.722 | 0.048 | 0.612 | 0.809 | 0.004 | 0.414 | 1.007 |
| $c_3$ | 0.891 | 0.060 | 0.769 | 1.006 | 0.001 | 0.274 | 1.000 |
| $\rho_a$ | 0.622 | 0.209 | 0.101 | 0.821 | 0.024 | 0.005 | 1.296 |
| $\rho_q$ | 0.708 | 0.035 | 0.631 | 0.769 | 0.003 | 0.199 | 1.017 |
| $\rho_s$ | 0.717 | 0.057 | 0.606 | 0.830 | 0.006 | 0.695 | 1.003 |
| $\mu_y$ | 0.217 | 0.113 | 0.091 | 0.534 | 0.013 | 0.573 | 1.006 |
| $\mu_r$ | 0.732 | 0.222 | 0.394 | 1.322 | 0.030 | 0.232 | 1.058 |
| $\sigma_H$ | 1.325 | 1.558 | 0.629 | 6.497 | 0.089 | 0.004 | 1.348 |
| $\sigma_F$ | 0.909 | 0.478 | 0.319 | 2.141 | 0.051 | 0.394 | 1.012 |
| $\sigma_a$ | 17.959 | 1.596 | 15.579 | 21.365 | 0.246 | 0.044 | 1.164 |
| $\sigma_q$ | 0.794 | 0.128 | 0.587 | 1.097 | 0.009 | 0.544 | 1.003 |
| $\sigma_s$ | 2.633 | 0.454 | 1.738 | 3.530 | 0.041 | 0.003 | 1.114 |
| $\sigma_{\pi*}$ | 0.412 | 0.042 | 0.336 | 0.501 | 0.000 | 0.492 | 1.000 |
| $\sigma_{y*}$ | 0.546 | 0.072 | 0.417 | 0.702 | 0.002 | 0.368 | 1.001 |
| $\sigma_{r*}$ | 0.222 | 0.022 | 0.184 | 0.269 | 0.000 | 0.967 | 1.000 |
| $\sigma_r$ | 0.338 | 0.047 | 0.255 | 0.441 | 0.003 | 0.080 | 1.018 |

Notes:
a. The numerical standard error (NSE) as given in Geweke (1999).
b. The p-value is computed using $L = 0.08$ in Geweke (1999).
c. The B-G univariate "shrink factor" as in Brooks and Gelman (1998).

**Table 8**
**Posterior odds model comparison**

| **Country (Model, $M_i$)** | $p(y\|M_i)$ | $\ln \frac{p(y\|M_1)}{p(y\|M_2)}$ | **Bayes factor** |
|---|---|---|---|
| Australia ($i = 1$) | -1955.0 | | |
| Australia ($i = 2$) | -1941.7 | -13.3 | $5.97 \times 10^5$ |
| | | | |
| Canada ($i = 1$) | -1815.6 | | |
| Canada ($i = 2$) | -1805.3 | -10.3 | $2.97 \times 10^4$ |
| | | | |
| New Zealand ($i = 1$) | -1994.6 | | |
| New Zealand ($i = 2$) | -1980.5 | -14.1 | $1.33 \times 10^6$ |

Notes:
a. $M_1 : \mu_q > 0$ and $M_2 : \mu_q = 0$.
b. Marginal likelihood for Geweke's $p = 0.1$ are reported, where $p \in (0, 1)$.
c. The Bayes factor is calculated as $\frac{p(y|M_2)}{p(y|M_1)}$.