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**Modelling the long-run real effective
exchange rate of the New Zealand Dollar**

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Modelling the long-run real effective exchange rate of the New Zealand Dollar

Abstract^{1,2}

The usefulness of the concept of an equilibrium exchange rate has been brought into sharp focus by the dramatic depreciation of the euro since its inception in 1999. Does this movement reflect a movement of the actual exchange rate relative to its equilibrium or has the equilibrium shifted relative to the perception of where the euro was in 1999? Similar kinds of questions have been asked about the behaviour of the New Zealand dollar since the latter part of 1999. To answer these kinds of questions it is necessary to have some measure of an equilibrium exchange rate and there are a plethora of alternative approaches available in the literature. In this paper we use the behavioural equilibrium exchange rate (BEER) approach of Clark and MacDonald (1999) to produce long-run equilibrium exchange rates for the effective real exchange rates of the New Zealand dollar. We demonstrate that a well founded measure of the equilibrium value of the dollar may be recovered from a relatively small set of fundamental variables and that this can be used to produce an assessment of the dollar in terms of periods of misalignment.

The remainder of this paper is organised as follows. In the next section we present a brief overview of the BEER based approach to assessing equilibrium exchange rates. In section 2 we present a set of estimates for the *rtwi* measure of the real effective exchange rate. In section 3 some robustness checks are presented, in terms of both the measure of the real effective and sample stability tests. Finally, in section 4, we discuss some assessment and calibration issues.

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1 The behavioural equilibrium exchange rate approach

There are currently a large range of alternative approaches available for assessing if a currency is misaligned (See MacDonald (2000) and Driver and Westaway (2001) for overviews), and these models are distinguished by an exotic array of mnemonics such as BEERs, CHEERs, DEERs, FEERs and PEERs. Although many of these approaches have much in common, the key factor which distinguishes them is the amount of normative structure used to calculate the equilibrium rate. For example, in the Fundamental Equilibrium Exchange Rate (FEER) approach of Williamson (1994), the equilibrium exchange rate – the FEER – is calibrated at a level consistent with both internal and external balance. Although this approach has been fairly widely adopted, there are various advantages, detailed in Clark and MacDonald (1999), from separating the purely normative aspects of exchange rate modelling from the behavioural. The Behavioural Equilibrium Exchange Rate (BEER) approach of Clark and MacDonald attempts to achieve this by adopting a two-step procedure. In the first step a simple behavioural equilibrium exchange rate relationship is estimated and then in a second stage this estimated relationship is used to construct an assessment of whether an exchange rate is overvalued or not. In the latter stage some normative structure may be placed on the calculated equilibrium.

Since a behavioural model is at the heart of the BEER approach, a range of different exchange rate models – from monetary to real – may be deemed consistent with this approach. The vehicle we have chosen in this paper to motivate the BEER approach exploits the familiar uncovered interest parity (UIP) condition:³

$$E_t(\Delta s_{t+k}) = -(i_t - i_t^*), \quad (1)$$

where s_t is the foreign currency price of a unit of home currency, and therefore implies that an increase s_t implies that the currency has

³ Our discussion here draws on Clark and MacDonald (2000).

appreciated, an i_t denotes a nominal interest rate, Δ is the first difference operator, E_t is the conditional expectations operator, $t+k$ defines the maturity horizon of the bonds, and a * denotes a foreign variable. Equation (1) can be converted into a relationship between real variables by subtracting the expected inflation differential, $E_t(\Delta p_{t+k} - \Delta p_{t+k}^*)$, from both sides of the equation. After rearrangement this gives:

$$q_t = E_t(q_{t+k}) + (r_t - r_t^*) + e_t \quad (2)$$

where $r_t = i_t - E_t(\Delta p_{t+k})$ is the *ex ante* real interest rate, $q_t = s_t - E_t(\Delta p_{t+k})$, is the *ex ante* real exchange rate, and e_t is a disturbance term. Expression (2) describes the current equilibrium exchange rate as being determined by two components, the expectation of the real exchange rate in period $t+k$ and the real interest differential with maturity $t+k$.

The use of the uncovered interest parity relationship may appear suspect, as many researchers (see, for example, Meese and Rogoff (1988), Campbell and Clarida (1987), Froot (1990), and Edison and Pauls (1993)) have failed to uncover a systematic, robust relationship between exchange rates and interest rates. However, some recent research provides greater support for UIP. For example, Meredith and Chinn (1998) find that looking at longer time horizons, namely, from three years up to ten years, reveals quite a strong and consistent relationship between the change in the nominal exchange rate and the level of the interest differential across a range of currencies. MacDonald (1997), Edison and Melick (1999) and MacDonald and Nagayasu (1999) confirm that the findings of Meredith and Chinn also hold for the real uncovered interest differential. Thus there is greater support than a few years ago for using UIP as our point of departure in modeling real exchange rates.

We assume that the unobservable expectation of the exchange rate, $E_t(q_{t+k})$, represents the influence of fundamentals exclusive of interest rates on the equilibrium exchange rate and we label this component \bar{q}_t .

The current equilibrium rate is defined as q_t^* to distinguish it from the actual rate q_t .⁴

$$q_t^* = \bar{q}_t + (r_t - r_t^*) \quad (3)$$

In summary, therefore, our approach posits that the current equilibrium exchange rate given by (3) comprises two components: a systematic component, \bar{q}_t , and the real interest differential.

The factors likely to introduce systematic variability into \bar{q}_t have been discussed extensively elsewhere in some detail (see Faruqee, (1995), MacDonald (1997) and Stein (1999)) and are therefore not considered here in any depth. Suffice to say that in our analysis of the New Zealand REER, we have assumed the long-run equilibrium exchange rate is a function of four key variables:

$$\bar{q}_t = f(nfa_t^+, prod_t^+, gapd_t^+, nztot_t^+), \quad (4)$$

where nfa is the ratio of net foreign assets to GDP and from the perspective of a stock-flow consistent exchange rate model (such as the portfolio balance or Obstfeld-Rogoff (1995) models) would be expected to be positively related to the real exchange rate. As in the internal-external balance approach to modeling the real exchange rate, we see nfa as being driven by the determinants of national savings and investment and, in particular, demographics and structural fiscal balances.⁵ $prod_t$ is New Zealand's labour productivity relative to that of her trading partners. With linearly homogeneous production technology this may be interpreted as a Balassa-Samuelson effect and on this basis should be positively related to the real exchange rate; ie if

⁴ It is straightforward to modify (3) to include a risk premium – see Meese and Rogoff (1988). However, in the work of Clark and MacDonald (1999) the risk premium always proved to be insignificant and therefore it is not included in the present analysis.

⁵ For a recent analysis that derives this relationship from internal and external balance considerations, see Alberola, et al (1999).

New Zealand has relatively fast productivity growth in its tradable sector its currency will appreciate.⁶ The variable *gapd* is the output gap in New Zealand relative to the output gap in the trading partner countries. This variable is included for two reasons. First, it may be viewed as an alternative measure of growth, or growth potential, in an economy. Recent hypothecation over the relative strength of the US dollar against the euro has emphasised this kind of variable. The second reason for including this is to allow counterfactuals to be conducted where output can be calibrated at its full employment, or potential, level, as in the FEER approach. Finally, *nztot* represents the New Zealand terms of trade. The definitions of the three variables entering (4) are described in some detail in the data appendix.

The variables included in our equilibrium measure of the real exchange rate likely have different periodic influences on the real exchange rate. The *nfa* and *prod* terms may be thought of as driving the longer run systematic component in the real exchange rate, while the remaining variables have a more cyclical – medium run – relationship with the real exchange rate.

2 Econometric results

We used the methods of Johansen (1995) to determine both the existence of cointegration, or long-run relationships, and to produce estimates of the BEER. These estimates are based on the following vector:

$$x_t = [rtwi_t, nfa_t, longr_t, prod, gapd, nztot_t]' \quad (5)$$

where, of terms not already defined, *rtwi_t* is the log of the real effective exchange rate ($=\ln zdtw - ltwlcpd$) and *longr_t* is the New Zealand long-term real interest rate relative to the trade weighted real foreign long term interest rate.⁷ In summary form, the Johansen approach involves

⁶ The importance of net foreign assets as a determinant of real exchange rates has recently been highlighted in Lane and Milesi-Ferretti (2000).

the following. Assume that the (nx1) vector has an autoregressive representation of the form:

$$x_t = \eta + \sum_{i=1}^p \Pi_i x_t + \varepsilon_t, \quad (6)$$

where η is a (nx1) vector of deterministic terms, p is the lag length and ε is a (nx1) vector of white noise disturbances, with mean zero and covariance matrix Ξ . Expression (6) may be reparameterised into the vector error correction mechanism (VECM) as:

$$\Delta x_t = \eta + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \Pi x_{t-1} + \varepsilon_t \quad (7)$$

where Δ denotes the first difference operator, Φ_i is a (nxn) coefficient matrix (equal to $-\sum_{j=i+1}^p \Pi_j$), Π is a (nxn) matrix (equal to $\sum_{i=1}^p \Pi_i - 1$) whose rank determines the number of cointegrating vectors. If Π is of either full rank, n , or zero rank, $\Pi=0$, there will be no cointegration amongst the elements in the long-run relationship (in these instances it will be appropriate to estimate the model in, respectively, levels or first differences). If, however, Π is of reduced rank, r (where $r < n$), then there will exist (nxr) matrices α and β such that $\Pi = \alpha\beta'$ where β is the matrix whose columns are the linearly independent cointegrating vectors and the α matrix is interpreted as the adjustment matrix, indicating the speed with which the system responds to last period's deviation from the equilibrium level of the exchange rate. Hence the existence of the VECM model, relative to say a VAR in first differences, depends upon the existence of cointegration.⁸

⁷ This is calculated by subtracting a measure of expected inflation from the nominal long term interest differential. The latter is derived from a kernel regression (using the Kernreg.src written for RATS, where the particular kernel chosen is Epanechnikov)

⁸ The so-called Granger representation theorem (see Engle and Granger (1987)) implies that if there exists cointegration amongst a group of variables there must also exist an error correction representation.

We test for the existence of cointegration amongst the variables contained in x_t using the Trace test proposed by Johansen (1995). For the hypothesis that there are at most r distinct cointegrating vectors, this has the form:

$$TR = T \sum_{i=r+1}^N \ln(1 - \hat{\lambda}_i), \quad (8)$$

where $\hat{\lambda}_{r+1}, \dots, \hat{\lambda}_N$ are the $N-r$ smallest squared canonical correlations between x_{t-k} and Δx_t series (where all of the variables entering x_t are assumed $I(1)$), corrected for the effect of the lagged differences of the x_t process.⁹

The sample period is 1985 quarter 4 to 2000, quarter 1 (with the first two observations used to construct the lags in the VAR) and p set equal to 2. The deterministic specification consisted of 3 seasonal dummies and the constant restricted to the cointegrating space. The Trace tests, reported in table 1, indicate that there may be up to two cointegrating vectors. Using the small sample corrected Trace test of Reimers (1992) (which uses $(T-n^*p)$ as a scaling factor rather than T in (8)) there is evidence of one statistically significant cointegrating relationship.

Table 1: Significance of cointegrating vectors

$H_0:r$	Trace	ATrace	Trace95
0	127.86	103	102.14
1	80.56	64	76.07
2	49.17	39	53.12
3	28.59	22	34.91
4	14.55	12	19.96
5	4.78	4	9.24

Notes: The numbers in the column labeled Trace are the estimated values of (8), while those in ATrace are the small sample corrected Trace statistics using the correction of Reimers (1992); the numbers in the Trace95 column are the 95% significance levels from Osterwald-Lenum (1993).

⁹ For details of how to extract the λ 's see Johansen (1988), and Johansen and Juselius (1992).

Using the residuals from the VAR model, we calculated a number of portmanteau statistics. They are: LM(1) and LM(4), which are multivariate Godfrey (1988) LM-type statistics for first and fourth-order autocorrelation; and NM(8), which is a Doornik and Hansen (1994) multivariate normality test. The estimated values of these statistics (p-values in parenthesis) are reported in table 2. The LM(1) test is insignificant, although both the LM(4) and NM(12) statistics are marginally significant.¹⁰

Table 2: Multivariate residual diagnostics

LM (1)	LM(4)	NM(12)
33.47	58.01	24.98
(0.59)	(0.01)	(0.01)

However, implementing a standard set of long-run exclusion and weak exogeneity tests on this system indicates that this is not the final representation. In particular, the exclusion test indicates that both the net foreign asset and relative productivity terms can be excluded from the long-run relationship and the weak exogeneity test indicates that these variables are also weakly exogenous. Additionally, the latter test suggests that all remaining variables with the exception of the real exchange rate are also weakly exogenous. We take the latter finding to be reflective of New Zealand being a small open economy.

Table 3: Exclusion and weak exogeneity tests

	rtwi	prod	longr	Nfa	gapd	lnztot	Chi(1)
Exclude	15.65	0.90	10.62	1.78	9.57	12.22	3.84
Weak Ex	10.93	1.16	1.60	3.45	2.27	0.02	3.84

¹⁰ The significance of the LM(4) statistic seems to be attributable to significant fourth order serial correlation in the productivity and real interest rate equations. However, increasing the lag length of the VAR to 4 does not remove these significant terms and we suggest that they probably represent (multiplicative) seasonal spikes.

Assuming for the time being that only *nfa* and *ltwlpd* are weakly exogenous, we may represent the effects of these tests on our original model in the following way. By partitioning the x vector into two groups, such that $x_t = (y_t, z_t)'$, where $y_t = (rtwi, longr, gapd, nztot)'$ and $z_t = (prod, nfa)'$, the model (7) may be reformulated as:

$$\Delta y_t = \eta + \Phi_0 \Delta z_t + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \Pi y_{t-1} + \varepsilon_t \quad (9)$$

Estimating this model we obtain the Trace statistics reported in table 4 and these indicate evidence of one significant vector on the basis of both the unadjusted and small sample adjusted Trace statistics.

Table 4:
Significance of cointegrating vectors in reduced system

$H_0:r$	Trace	ATrace	Trace95
0	73.68	59	53.12
1	33.09	26	34.91
2	18.68	15	19.96
3	7.98	6	9.24

Note: For definitions see table 1.

The multivariate diagnostics reported in table 5 clearly indicate the benefits of this partial system, in the sense that all of the statistics are now insignificant at standard significance levels.

Table 5: Multivariate residual diagnostics

LM (1)	LM(4)	NM(12)
18.64	19.12	5.78
(0.29)	(0.26)	(0.67)

The univariate statistics reported in table 6 support the multivariate statistics and confirm that the partial model is well specified. One

noteworthy feature of these statistics is that the explanatory power of the real exchange rate equation is relatively high.

Table 6: Univariate residual diagnostics

	SK	KU	ARCH	NORM	R ²
Rtwi	-0.0731	3.18	0.023	1.068	0.533
Longr	0.346	3.817	1.721	4.130	0.161
Gapd	-0.201	2.246	0.117	2.028	0.506
Nztot	-0.220	2.806	7.670	0.581	0.281

Notes: SK denotes skewness, KU denotes Kurtosis, ARCH is a second order autoregressive heteroscedasticity test, NORM is a univariate Jacque-Bera test statistic and R² represents the improvement in explanatory power relative to the random walk with drift.

A visual analysis of the standardised residuals presented in figures 1 to 4 confirms our conclusion that the estimated model is econometrically sound. In figure 5 we report the plots of $\hat{\beta}' z_t$ and $\hat{\beta}' R_{it}$ and since both of these are similar and appear stationary this offers further support for our specification. Normalising the first significant cointegrating vector on the real effective exchange rate we obtain the following relationship, where standard errors are reported in brackets:

$$rtwi_t = 0.058 \underset{(0.01)}{longr_t} + 0.085 \underset{(0.01)}{gapd_t} + 1.848 \underset{(0.34)}{nztot_t} + 14.812 \underset{(2.13)}{.} \quad (10)$$

All of the variables are statistically significant and enter with the correct signs. The coefficient on the real interest differential is consistent with a traditional capital flow interpretation. For example, a 100 basis point increase in the New Zealand real interest rate relative to her trading partners appreciates the real exchange rate by 6 per cent. The coefficient on the terms of trade is correctly signed and highly significant and suggests that a one per cent improvement in New Zealand's terms of trade appreciates the exchange rate by 1.85 per cent. The coefficient on the *gapd* term is also highly significant and indicates that one per cent increase in domestic output relative to trend appreciates the dollar by 8.5 per cent.

The alpha matrix associated with the significant cointegrating relationship, discussed above, is reported in table 7 and indicates that the real exchange rate adjusts significantly negatively to the disequilibrium exchange rate error; significant adjustment to this error also occurs in the gap and real interest rate equations. The implied half-life of exchange rate adjustment is approximately three quarters of one year, which is much faster than the typical half-life reported in simple PPP-based regressions (which typically give half-lives of around 4 years), although it is similar to the half-lives recovered in other BEER estimates (see for example Clark and MacDonald (1999)). We believe that this rapid adjustment speed illustrates the value of conditioning the real exchange rate on ‘real’ fundamentals.

Table 7: Alpha adjustment matrix

Variable	alpha1	t-alpha
$\Delta rtwi$	-0.188	-5.432
$\Delta longr$	1.864	2.024
$\Delta dgap$	4.514	4.442
$\Delta nztot$	0.003	0.091

In table 3 we noted that all variables in the general system, with the exception of the real effective exchange rate, appeared to be weakly exogenous.¹¹ Formal tests for the significance of the alpha1 coefficient on $nztot$ indicated that this was insignificant ($\chi(1)=0.01(0.94)$) as were the joint tests for zero alpha coefficients on the terms of trade and the real interest differential ($\chi(2)=3.33(0.19)$), but not the terms of trade and the $gapd$ term ($\chi(2)=14.14(0.00)$). The normalised cointegrating vector and associated alpha matrix for the system in which the alpha term on both the terms of trade and the real interest differential are zero are reported below:

$$rtwi_t = 0.052_{(0.01)} longr_t + 0.089_{(0.01)} gapd_t + 1.669_{(0.31)} nztot_t + 13.550_{(2.16)} \quad (11)$$

¹¹ The assumption of weak exogeneity assumes no cross correlations between the explanatory variables and this appears to be the case here as evidenced by the multivariate statistics reported in Table 5.

Table 8: Alpha adjustment matrix

Variable	alpha1	t-alpha
$\Delta nztwg$	-0.168	-4.841
$\Delta twreal$	0	0
$\Delta dgap$	4.820	4.788
$\Delta nztot$	0	0

Since these results are very close to those reported above, without the weak exogeneity restrictions imposed, our future discussion focuses on the system with only $prod$ and nfa weakly exogenous.

3 Robustness checks

In this section we present various robustness checks on the basic model. First, we rerun the basic model using various alternative measures of the real effective exchange rate. We then consider the sensitivity of the results discussed above to changes in the sample period by generating Hansen-Johansen stability tests. Finally, we check the sensitivity of the estimates of the equilibrium relationship using a Dynamic OLS (DOLS) estimator.

3.1 Alternative exchange rate measures

The model specification introduced above was re-estimated using four alternative real exchange rates, namely LRTWC, LRTWU, LRTWP, LIMF. The results for the Trace tests are reported in table 9 and indicate that irrespective of the real exchange rate used, there is very clear evidence of one significant cointegration vector in all of the systems. For the system with LRTWU as the real exchange rate measure, there is some evidence to suggest there are two significant cointegrating relationships.

Table 9:
Trace statistics for alternative real exchange rate models

LRTWC	LRTWU	LRTWP	LIMF	95%
74.65	68.20	72.07	62.18	53.12
32.45	40.76	32.05	28.53	34.91
17.79	20.70	17.60	17.47	19.96
6.89	9.23	6.72	6.84	9.24

In table 10 the cointegrating vectors normalised on the different measures of the real exchange rates are reported. We note first that all of the coefficients, irrespective of the specification, are correctly signed (and therefore have the same signs as in the base-line model). Furthermore, the absolute value of the coefficients are also very similar across currency specifications. We do not believe, therefore, that the choice of a different effective exchange rate would affect the assessment of the New Zealand Dollar.

Table 10:
Normalised cointegration relationships for alternative real exchange rates

	<i>longr</i>	<i>gapd</i>	<i>nztot</i>	Constant
LRTWC	0.054 (0.06)	0.069 (0.01)	2.032 (0.24)	-9.779 (1.68)
LRTWU	0.095 (0.02)	0.153 (0.028)	3.361 (0.75)	-19.17 (5.28)
LRTWP	0.073 (0.01)	0.110 (0.01)	2.922 (0.38)	-15.924 (2.69)
	0.094	0.114	3.338	-18.958
LIMF	(0.01)	(0.02)	(0.47)	(3.31)

3.2 Estimation method

In this section we check the sensitivity of our point estimates to the use of the Johansen methodology. In particular, we use a single equation Dynamic OLS (DOLS) estimator. This estimator uses leads and lags of the differences of the right hand side variables to orthogonalise the error term. Being a single equation estimator it has the added advantage in the current context of providing point estimates of the long-run coefficients which reflect an amalgam of all of the underlying cointegrating relationships (in other words if there are two cointegrating vectors in our system the DOLS estimator will combine them in a straightforward way). The basic form of the DOLS estimator is:

$$y_t = \theta_1 + \theta_3 x_t + \sum_{j=-p}^{+n} \theta_{4j} \Delta x_{t+j} + \omega_t, \quad (12)$$

where y_t is a scalar, x_t is a vector with dimension k , θ_3 represents a cointegration vector, p is the maximum lag length, n is the maximum lead length and ω is a Gaussian vector error process. The estimates derived from this method are noted here as equation (13):

$$rtwi_t = -0.852_{(0.40)} prod_t + 0.063_{(0.01)} longr_t - 0.075_{(0.28)} nfa_t + 0.071_{(0.008)} gapd_t + 1.404_{(0.31)} nztot_t - 11.756_{(2.27)} \text{Leads and Lags}, \quad (13)$$

where two leads and lags proved sufficient to orthogonalise the error term. These estimates essentially confirm our findings derived using the Johansen methodology: neither the relative productivity term nor the *nfa* term are significantly different from zero and both are wrongly signed. The remaining terms, are as in our conditional VAR system, are correctly signed and statistically significant and have coefficients which are numerically close to the estimated Johansen values.

3.3 Stability tests

In order to check the stability of our BEER estimates we subjected the base-line system to Hansen-Johansen sub-sample stability tests. The results for the significance of the trace tests and the stability of the β vector are reported in figures 6 and 7, respectively (the sample period noted on the horizontal axis is the longest available for this period). Given the way the Trace statistic is calculated the time paths of the significant statistics should be upward sloping and above unity, and the time paths reported in figure 6 indicate that this is indeed the case. We further note that the β vector is less than unity in all sub-samples, thereby indicating that hypothesis of stability is accepted for all of the investigated sub-samples.

4 Exchange rate misalignment

In figure 8 the estimated BEER derived from equation (10) is plotted against the actual real effective exchange rate. The first thing to note about the estimated BEER is that it is more volatile than the actual real exchange rate series, particularly in the early part of the sample period, and this in large part reflects the volatility of cyclically related determinants of the BEER, such as the real interest differential and the terms of trade. The predominant impression to be gleaned from this figure is that the New Zealand effective rate has been undervalued for much of the floating rate period. The two periods when this is not the case are 1991 to 1993 and in 1997. With the exception of one quarter, the currency has clearly been undervalued in the post-1999 period.

Of course the measures of undervaluation and overvaluation discussed above are contingent on what is effectively a data determined measure of equilibrium. Clark and MacDonald (1999) refer to this as a current measure of misalignment. However, it may be that the variables entering the BEER calculation are not themselves at what is deemed to be equilibrium values. A measure of misalignment in which one or all of the forcing variables are calibrated at some measure of equilibrium is labeled a total misalignment by Clark and MacDonald. One simple way

of calibrating the BEER is to remove the business cycle from the data using a Hodrick-Prescott (HP) filter. An alternative measure is proposed by Clark and MacDonald (2001). Like the HP filter this involves using an atheoretical decomposition of the series into their permanent and transitory components. This may be explained in the following way.

Johansen (1995) has demonstrated that equation (7) has a vector moving average representation of the following form:

$$x_t = C \sum_{i=1}^t \varepsilon_i + C\eta + C(L)(\varepsilon_t + \eta), \quad (13)$$

where

$$C = \beta_{\perp} (\alpha'_{\perp} (I - \sum_{i=1}^{k-1} \Phi_i) \beta_{\perp})^{-1} \alpha'_{\perp}$$

where α_{\perp} and β_{\perp} denote the orthogonal complements to α and β (that is, $\alpha' \alpha_{\perp} = 0$ and

$\beta' \beta_{\perp} = 0$) and α_{\perp} determines the vectors defining the space of the common stochastic trends, and therefore should be informative about the key 'driving' variable(s) in each of the systems. The β_{\perp} vector gives the loadings associated with α_{\perp} , i.e., the series which are driven by the common trends. Thus the C matrix measures the combined effects of these two orthogonal components.

If the vector x is of reduced rank, r , then Granger and Gonzalo (1995) have demonstrated that the elements of x can be explained in terms of a smaller number ($n-r$) of I(1) variables called common factors, f_t , plus some I(0) components, the transitory elements, \tilde{x}_t :

$$x_t = A_t f_t + \tilde{x}_t. \quad (14)$$

The identification of the common factors may be achieved in the following way. If it is assumed that the common factors, f_t , are linear combinations of the variables x_t :

$$f_t = B_1 x_t, \quad (15)$$

and if $A_1 f_t$ and \tilde{x}_t form a permanent-transitory decomposition of x_t , then from the VECM representation (7) the only linear combination of x_t such that \tilde{x}_t has no long-run impact on x_t is:

$$f_t = \alpha'_1 x_t. \quad (16)$$

As Granger and Gonzalo point out, these are the linear combinations of Δx_t which have the ‘common feature’ of not containing the levels of the error correction term in them. This identification of the common factors enables Granger and Gonzalo to obtain the following permanent-transitory decomposition of x_t :

$$x_t = A_1 \alpha'_1 x_t + A_2 \beta' x_t, \quad (17)$$

where, of terms not previously defined, $A_1 = \beta_\perp (\alpha'_\perp \beta_\perp)^{-1}$ and $A_2 = \alpha (\beta' \alpha)^{-1}$. It is straightforward to demonstrate that the common factor, f_t , corresponds to the common trend in the analysis of Stock and Watson (1988).

In figure 9 we present the estimated PEER calculated using (17) alongside the actual REER. Compared to figure 8 we note that the basic difference between the BEER and PEER estimates of equilibrium is that the latter provides a much smoother – less volatile - measure of equilibrium, and this in large measure reflects the origin of the PEER as the ‘permanent’ component of the REER. The closeness between the PEER and REER means that periods of over- and under-valuation do not appear to be so dramatic in this picture. Nevertheless, the period 1990-1992 is still one of overvaluation and the recent experience one of undervaluation.

Of course, we emphasize again that the PEER based measure of equilibrium is essentially an atheoretical (ie statistical) way of constructing an equilibrium exchange rate. An alternative statistical means of calibration involves simply taking the cyclical components out of the data using a Hodrick-Prescott filter and generating a measure of total misalignment based on this. In figure 10 we present this result. We note that on the basis of an HP calibration the dollar was clearly overvalued in 1996 and 1997 and has been persistently undervalued since 1999.

In figure 11 we present a final measure of total misalignment. This has both the terms of trade and productivity terms calibrated using the HP filter, while the real interest differential is assumed to be 100 basis points in favour of New Zealand throughout the sample. This picture is quite similar in many ways to the purely HP adjusted figure 10, although it does bring into sharp relief the dramatic undervaluation of the New Zealand dollar post 1998.

5 Concluding comments

In this paper we have proposed using the BEER approach of Clark and MacDonald (1999) to provide measures of equilibrium for the floating rate period. The approach was implemented for *rtwi* real effective exchange rate. Our starting point consisted of conditioning this variable on home-foreign differentials of productivity, real interest rate, the terms of trade, a gap term and net foreign assets as a proportion of GDP. Using the methods of Johansen, the last two variables proved to be weakly exogenous and could be excluded from the long run relationship. A measure of current equilibrium was therefore obtained in which the real exchange rate was a long run function of the productivity gap, a real interest differential and the terms of trade term. All of these variables entered with the correct sign and were statistically significant. The relationship was shown to be robust with respect to using different measures of the real effective exchange rates, using a different estimator and in terms of stability tests. The coefficient estimates from this relationship were then used to construct a variety of current and total misalignments. All of our estimates

clearly indicated that the New Zealand dollar has been sharply undervalued in the period post-1999.

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Figure 1:
The standardised residuals for rtwi

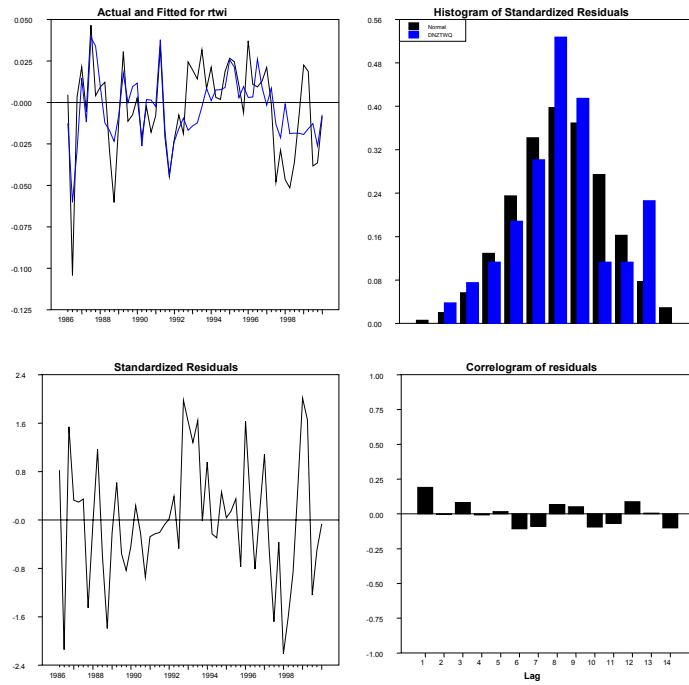


Figure 2:
The standardised residuals for longr

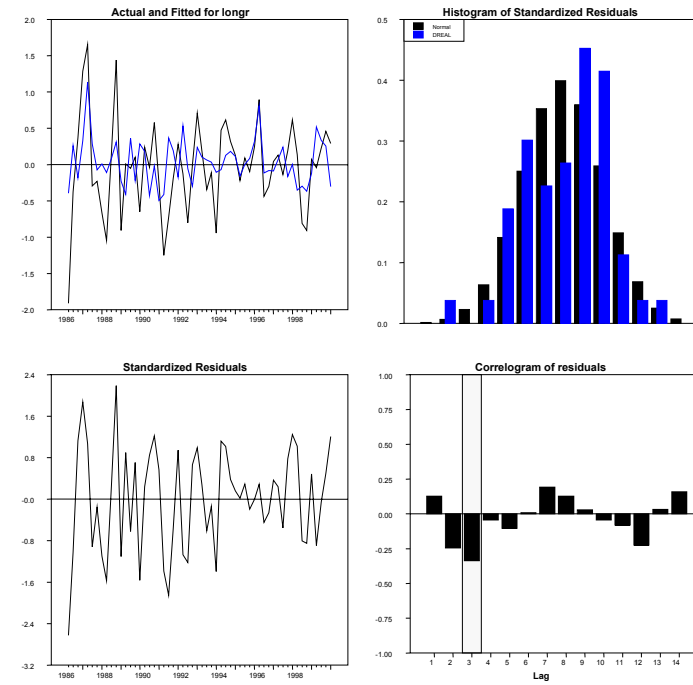


Figure 3:
The standardised residuals for nztot

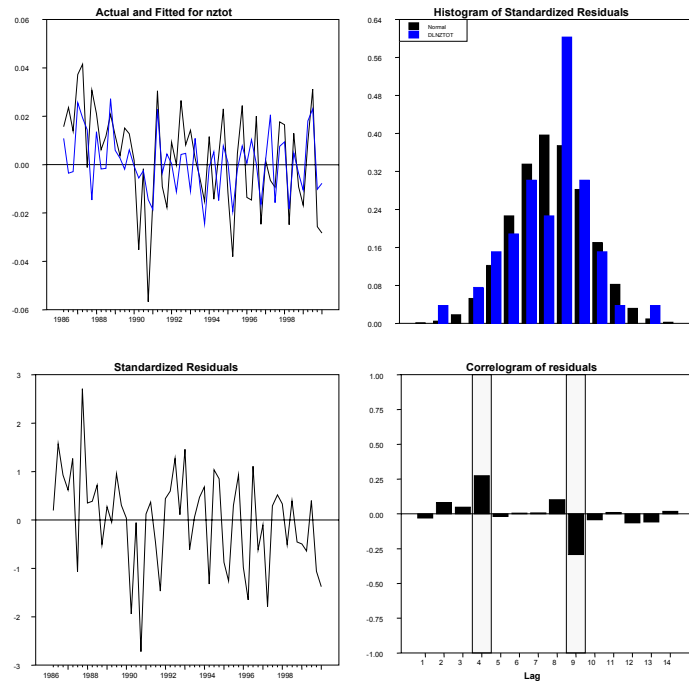


Figure 4:
The standardised residuals for gapd

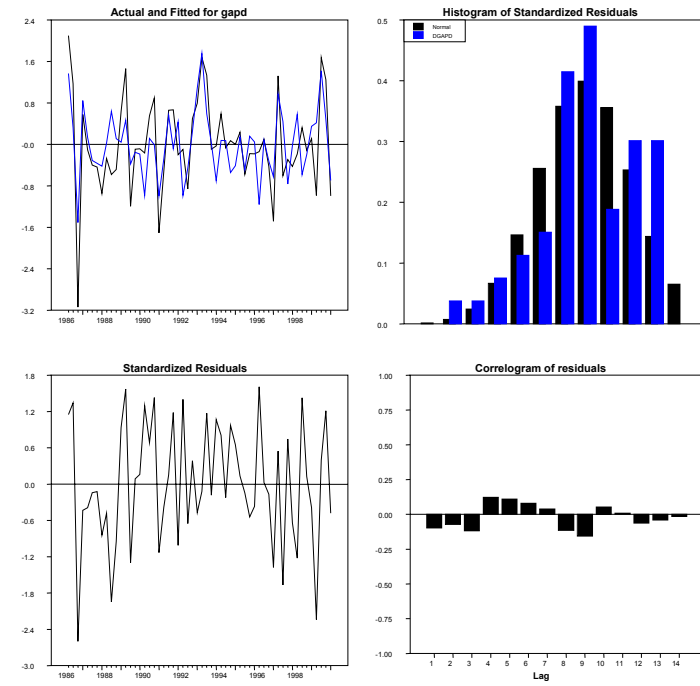


Figure 5:
The first cointegrating vector

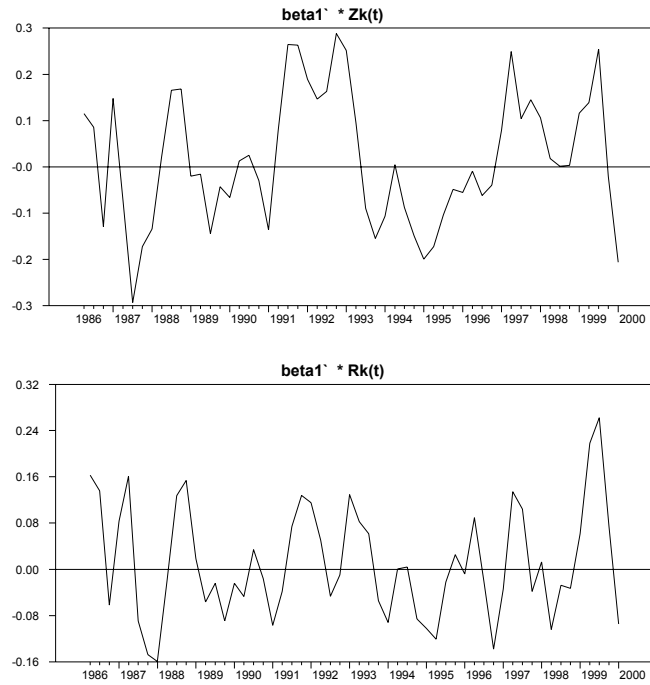


Figure 6:
The Recursive trace tests

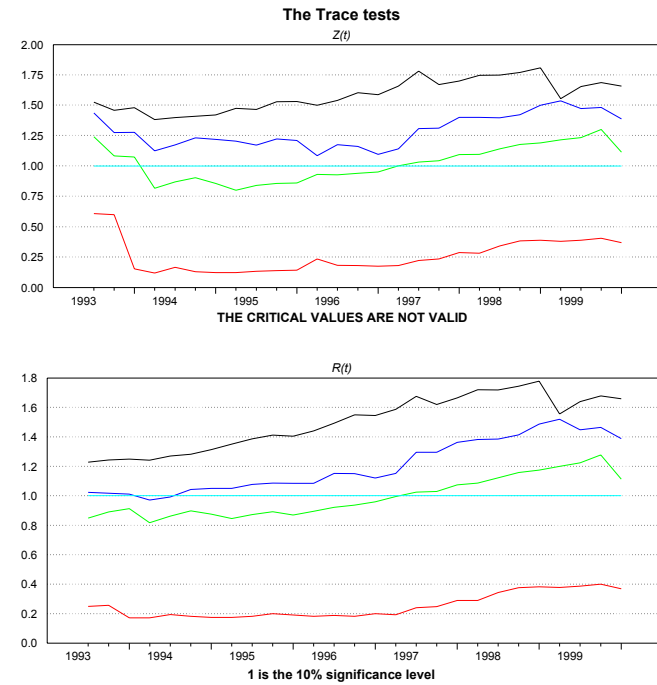


Figure 7:
The recursive estimates of beta

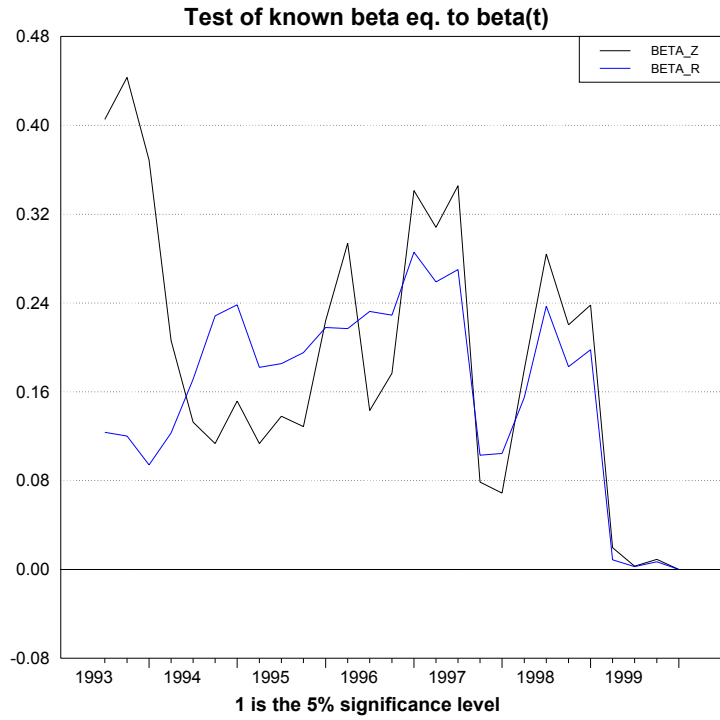


Figure 8. New Zealand BEER Current Misalignment

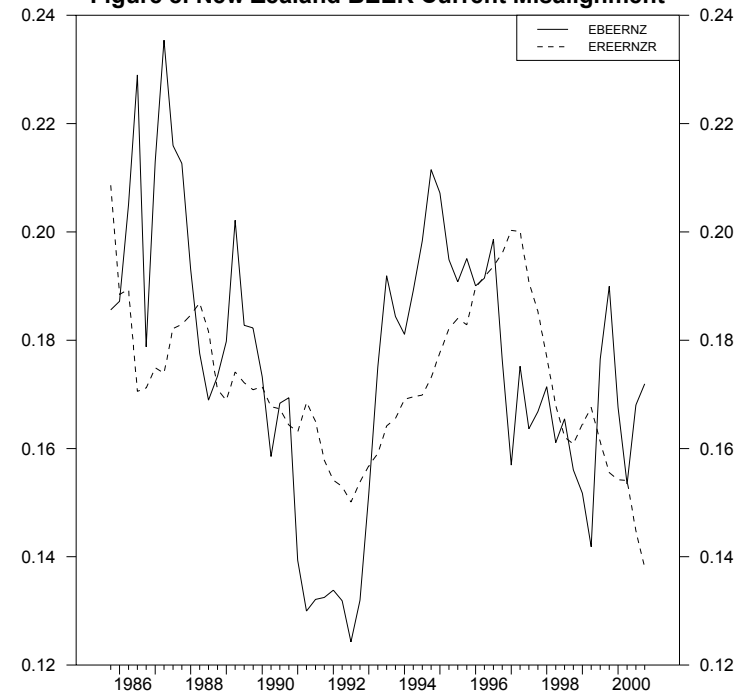


Figure 9. New Zealand Real Effective REER and PEER

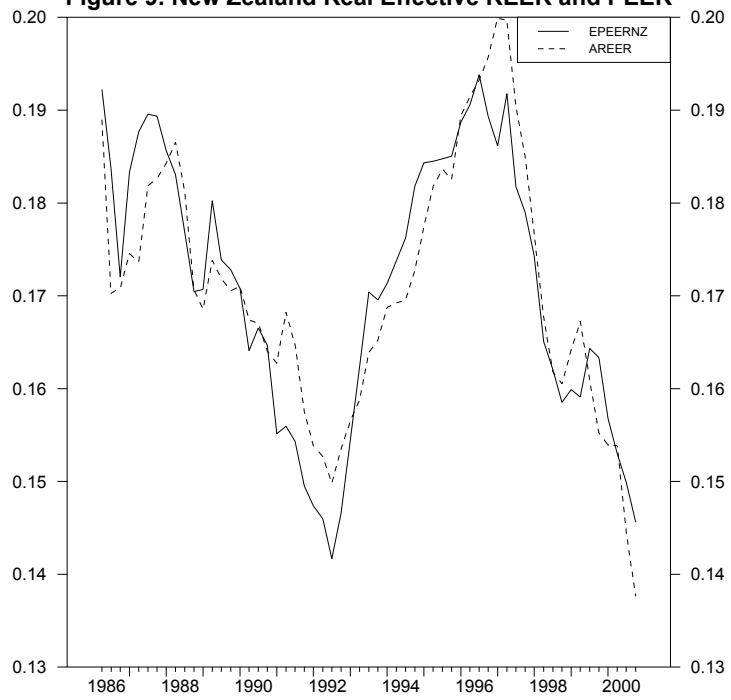
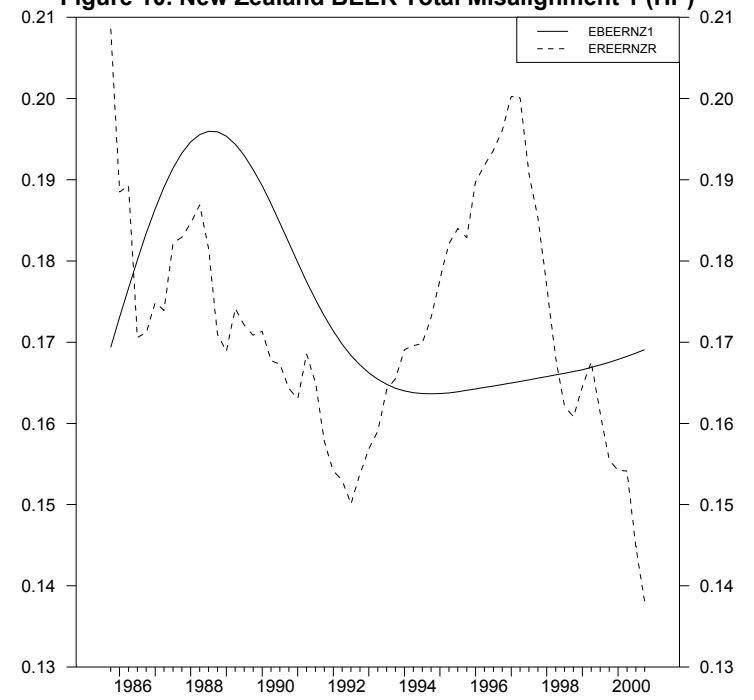
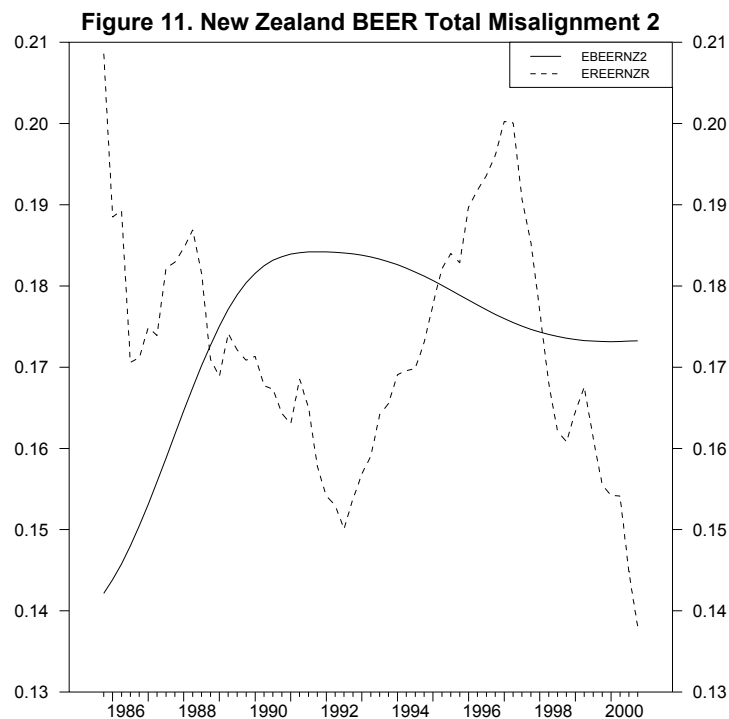


Figure 10. New Zealand BEER Total Misalignment 1 (HP)





Appendix: Data definitions and source

prod: the log of NZ labour productivity less the log of trade-weighted foreign labour productivity. Each quarterly productivity index series was interpolated from the following annual data:

New Zealand: Labour Productivity Index – Business Sector. *Source:* Datastream NZOCFLBP

Australia: Labour Productivity Index – Business Sector. *Source:* Datastream AUOCFLBP

Germany: Labour Productivity Index – Business Sector. *Source:* Datastream BDOCFLBP

Japan: Labour Productivity Index – Business Sector. *Source:* Datastream JPOCFLBP

UK: Labour Productivity Index – Business Sector. *Source:* Datastream UKOCFLBP

US: Labour Productivity Index – Business Sector. *Source:* Datastream USOCFLBP

nfa: ratio of net foreign assets to GDP.

Source: from 1989q1 we used official Statistics NZ net foreign assets data (TNII) interpolated from annual to quarterly frequency. Prior to 1989 we backdated the NFA data using quarterly Statistics New Zealand data for the current account balance (TBC). That is, we assume that $\Delta NFA_t = TBC_{t-1}$, (or $NFA_{t-1} = NFA_t - TBC_{t-1}$). The denominator of the NFA/GDP ratio is the annual total of nominal GDP (NGDPP).

gapd: output gap in NZ less the trade-weighted foreign output gap. *Source:* The New Zealand output gap measure used was the official RBNZ multivariate filter measure from the December 2000 Monetary Policy Statement. The world output gap measure was calculated by

running an HP Filter through a trade-weighted measure of GDP for New Zealand's 14 main trading partners.

nztot: NZ terms of trade. *Source:* Statistics New Zealand.

longr: NZ long-term real interest rate relative to the trade-weighted real foreign long term interest rate. Monthly data were averaged to create quarterly data.

New Zealand: *Source:* Datastream NZI61...

Australia: *Source:* Datastream AUI61...

Germany: *Source:* Datastream BDI61...

Japan: *Source:* Datastream JPI61...

UK: *Source:* Datastream UKI61...

US: *Source:* Datastream USI61...

rtwi: the log of the real effective exchange rate, calculated as $\ln(\text{NZDTW}) - \ln(\text{TWCPD})$, where NZDTW = a trade-weighted effective exchange rate using the following fixed weights over the period: Australia: 0.3, Germany: 0.15, Japan: 0.15, UK: 0.2, US: 0.2. and TWCPD = a trade-weighted foreign CPI series using the above fixed weights.

While this is a rough approximation, section 4.1 shows that the results are robust to superior measures of the real exchange rate.

Bilateral exchange rate data:

$\text{NZD/AU} = \text{AUUS/NZUS}$, where AUUS *Source* = Datastream AUI..RF.

$\text{NZD/BD} = \text{BDUS/NZUS}$, where BDUS *Source* = Datastream BDUI..RF.

$\text{NZD/JP} = \text{JPUS/NZUS}$, where JPUS *Source* = Datastream JPUI..RF.

$\text{NZD/UK} = \text{UKUS/NZUS}$, where UKUS *Source* = Datastream UKUI..RF.

$\text{NZD/US} = \text{USUS/NZUS}$, where USUS = 1.

And NZUS: *Source:* Datastream NZI..RF.

CPI data

NZ: GST and interest exclusive New Zealand CPI (*Source:* RBNZ)

Australia: *Source:* Datastream AUCP....F Subsequently adjusted for GST in 2000Q3¹².

Germany: *Source:* Datastream BDCP....F

Japan: *Source:* Datastream JPI64....F

UK: *Source:* Datastream UKRP....F

US: *Source:* Datastream USCP....F

Alternative real effective exchange rate measures used for robustness checks in 4.1

LRTWC: Relative CPI measure of the real effective exchange rate. *Source:* RBNZ

LRTWU: Relative ULC measure of the real effective exchange rate. *Source:* RBNZ

LRTWP: Relative PPIO measure of the real effective exchange rate. *Source:* RBNZ

LIMF: IMF real effective exchange rate. *Source:* Datastream NZI..RECE

¹² We base our estimate of the GST effect on the Australian Bureau of Statistics estimate of constant GST CPI in 2000Q3. The Constant GST estimate was 2.0 per cent for the quarter, versus 3.7 per cent in the headline CPI. Thus, we adjust headline CPI for a GST effect of 1.7 per cent.