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Analytical Notes

Technical appendix to

Monetary Policy Easing and the Distribution of Wealth in New Zealand.

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This appendix describes the details and calibration of the model presented in the *Analytical Note*.

In order to study the effect of monetary policy on wealth inequality, we utilize a structural macro-economic model in which a nontrivial distribution over wealth and income arises in equilibrium and in which central bank policy responds to inflation. To generate inequality, we follow the approach pioneered by Huggett (1993), Aiyagari (1994) and Krusell and Smith (1998) (among others). Such environments were extended to a continuous time setting by Achdou et al. (2021) and further extended to include nominal rigidities by Kaplan et al. (2018) and Ahn et al. (2017).

In this framework, households are subject to idiosyncratic labour income risk which cannot be fully insured away, resulting in diverse employment histories and resulting wealth holdings. Following the modelling structure found in Ahn et al. (2017), we combine this with nominal rigidities of the form in Rotemberg (1982) to generate a role for monetary policy in setting the price level of consumption.

Households

There is a unit measure of infinitely lived households having preferences over utility flows from future consumption $\{c_t\}_{0<t<\infty}$ and hours worked $\{n_t\}_{0<t<\infty}$ discounted at rate ρ,

$$U(\{c_t\}_{0\leq t<\infty},\{n_t\}_{0\leq t<\infty}) = \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t, n_t)dt.$$

Instantaneous utility takes the form

$$u(c_t) = \begin{cases} \frac{c_t^{1-\sigma}}{1-\sigma} - \psi\frac{n_t^{1+\varphi}}{1+\varphi}, & \sigma > 0,\ \sigma \neq 1 \\ \log(c_t) - \psi\frac{n_t^{1+\varphi}}{1+\varphi}, & \sigma = 1 \end{cases}$$

where σ denotes the coefficient of relative risk aversion, ψ the weight of disutility from labour relative to utility from consumption, and φ the reciprocal of the Frisch elasticity of labour supply.

Households may take positions in bond holdings a_t, by investing or borrowing real government bonds. These holdings evolve at a rate which is given by a household's total income net of consumption. Household income, meanwhile, is composed of earnings, government transfers T_t, its share of firm profits d_t, plus the return on its savings (or minus interest payments on borrowing). Labour is compensated at a wage rate W_t and taxed at a rate τ_t, so that the net labour income of a houshold working n_t hours at efficiency rate z_t is $W_t(1-\tau_t)z_t n_t$. Denoting the real return on savings/interest rate on borrowing by R_t we obtain the evolution equation for assets (the household budget constraint)

$$\frac{da}{dt} = R_t a_t + W_t(1-\tau_t)z_t n_t + d_t + T_t - c_t$$

Individuals also face a borrowing constraint

$a_t \geqslant \underline{a}.$

Individual efficiency endowments can take one of several states, and transitions between states follow an exogenous Poisson process. Given the evolution of the wage rate, interest rate, and the individual efficiency process, households choose a stochastic process for their consumption stream to maximize their expected discounted lifetime utility, subject to their budget constraint.

As in Achdou et al (2021), the solution to the household problem may be characterized by a Hamilton Jacobi Bellman equation, while the associated evolution of the distribution of households over asset holdings and efficiency states may be characterized by a Kolmogorov Forward equation.

Firms

We take the standard New Keynesian approach of introducing a role for monetary policy by invoking differentiated goods produced by monopolistic firms whose profit maximization behaviour is subjected to price adjustment frictions. These frictions take the form of quadratic adjustment costs as in Rotemberg (1982), which allows for a symmetric equilibrium in which all firms set the same price at all times, greatly simplifying the analysis and allowing for the derivation of an explicit nonlinear Phillips curve relating real economic activity and inflation. As is typical, the differentiated goods are intermediates which are assembled into a final consumption good by competitive firms using a constant elasticity of substitution (CES) technology.

The time t final good Y_t is assembled from the intermediate goods y_{jt} by competitive firms using the CES technology

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$ denotes elasticity of substitution between goods. The representative final goods firm takes as given the prices p_{jt} of the intermediate goods and chooses inputs to maximize profits in each period

$$Y_t - \int_0^1 p_{jt} y_{jt} dj$$

subject to the above technological constraint. The solution of this problem gives an optimal demand schedule of the final goods producer as

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\varepsilon} Y_t$$

This demand schedule is taken as given by monopolistic intermediate goods producers, who choose labour demand and prices with the objective of maximizing the expected discounted value of future profits. Given labour input n_{jt} and total factor productivity Z_t, firm output is given by

$$y_{jt} = Z_t n_{jt}^{1-\alpha}$$

With $1-\alpha$ denoting the returns to scaling labour input. The logarithm of total factor productivity follows an exogenous Ornstein Uhlenbeck process,

$$Z_t = Ze^{\zeta_t}$$
$$d\zeta_t = -(1-\rho_Z)\zeta_t \, dt + \sigma_Z \, dW_t^Z$$

Where W_t^Z denotes Brownian motion and Z denotes mean total productivity. Firm j uses its market power to set the price p_{jt} of the intermediate good that it produces, but in so doing it incurs adjustment costs

$$\begin{aligned}\Theta_t\left(\frac{dp_t/dt}{p_{jt}}\right) &= \frac{\theta}{2}\left(\frac{dp_t/dt}{p_{jt}}\right)^2 y_{jt} \\ &= \frac{\theta}{2}\left(\pi_{jt}\right)^2 y_{jt}\end{aligned}$$

where

$$\pi_{jt} = \tfrac{1}{p_{jt}}\tfrac{dp_{jt}}{dt}$$

Real profits at time t are given by revenue net of wages and adjustment costs,

$$\tfrac{p_{jt}}{P_t} y_{jt} - W_t n_{jt} - \frac{\theta}{2}\left(\pi_{jt}\right)^2 y_{jt}$$

so that firm j's objective is to maximize

$$\mathbb{E}_t \int e^{-\rho s}\left(\frac{p_{j,t+s}}{P_{t+s}} y_{j,t+s} - W_{t+s} n_{j,t+s} - \frac{\theta}{2}\left(\pi_{j,t+s}\right)^2 y_{j,t+s}\right) ds$$

over its choice of processes for prices $p_{j,t+s}$ and labour demand $n_{j,t+s}$. The solution of this problem indicates an optimal labour demand schedule in terms of the demand for each firm's good as

$$n_{jt} = \left(\frac{y_{jt}}{Z_t}\right)^{\frac{1}{1-\alpha}}$$

Moreover, optimal price setting leads to a non-linear Phillips curve

$$\left(\rho - \frac{1}{Y_t}\frac{1}{dt}\mathbb{E}_t dY\right)\Pi_t = \frac{\varepsilon}{\theta}\left(\frac{1}{1-\alpha}\frac{W_t}{Z_t^{\frac{1}{1-\alpha}}}Y_t^{\frac{\alpha}{1-\alpha}} - \frac{\varepsilon-1}{\varepsilon}\right) + \frac{1}{dt}\mathbb{E}_t d\Pi$$

in which

$$\Pi_t = \frac{1}{P_t}\frac{dP_t}{dt}$$

is the aggregate inflation rate. For details, see Kaplan et al. (2018).

Government

The government taxes labour income and issues bonds in amount B_t to fund uncertain spending G_t and transfers T_t. The government budget constraint is

$$G_t + R_t B_t + T_t = \tau W_t \int n_t(a,\ell) z(\ell)\, dF_t(a,\ell) + \frac{dB}{dt}$$

Lump sum transfers T_t adjust in response to the level of government debt away from its steady state level B according to the rule

$$T_t - T = -\xi(\ B_t\ -\ B\ ) + \omega_t^F.$$

Here ω_t^F is an exogenous shock which follows a similar specification to that for total factor productivity, as does the logarithm of government expenditures.

Monetary Policy

Monetary policy follows a Taylor rule which sets the nominal interest rate

$$I_t = R + \mu \Pi_t + \omega_t^M$$

where R is the stationary equilibrium real interest rate, μ is the Taylor coefficient which governs feedback of inflation to policy, and ω_t^M is a policy shock which once again follows a specification similar to those for productivity, government expenditures and fiscal policy. A Fisher equation relates the nominal and real interest rates,

$$R_t = I_t - \Pi_t$$

Calibration

Table 1 displays most of the key parameter values in the model, their interpretation, and where applicable their source.

On the household side, the discount rate is chosen to correspond to a quarterly discount factor of 0.986, a standard value. The coefficient of relative risk aversion and reciprocal of the Frisch elasticity are both set to 2, also a standard value. The weight on disutility from labour in the period utility function is 32.4, which is selected to target steady state labour equal to about one third of the households' unit endowment of time. Mean labour efficiency is consequently chosen such that there is on average a unit of labour efficiency.

Table 1: Key Model Parameters

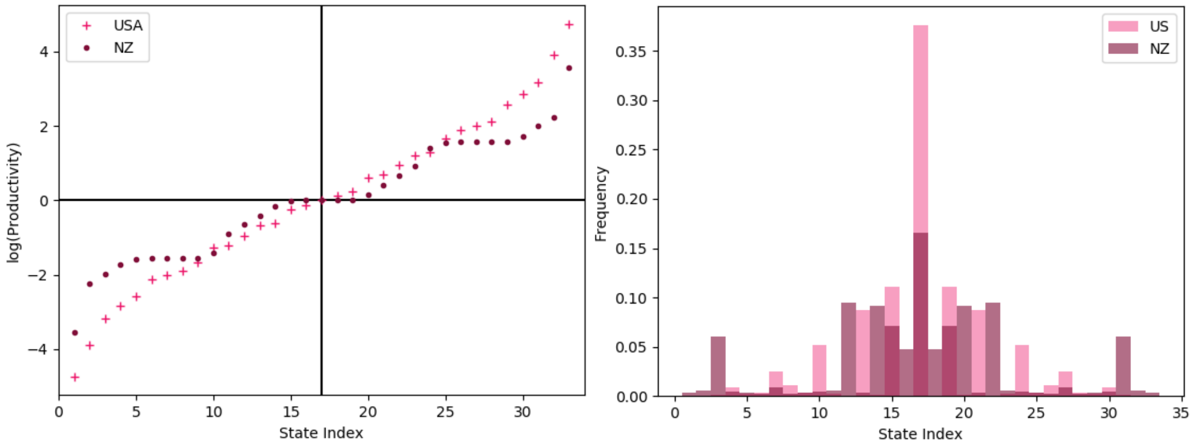
Parameter	Value	Description	Target
Households			
ρ	0.014	Discount rate	Quarterly factor 0.986
σ	2	Risk aversion	Standard Value
ψ	32.4	Disutility weight	Hours ≈ 1/3
φ	2	1/(Frisch Elasticity)	Standard Value
z	3	Labor efficiency	Eff. hours = 1
Intermediate Goods			
α	1	Labor's share	Const. Ret. to Scale
θ	100	Price stickiness	P.C. slope = 0.1
Final Goods			
ϵ	10	Goods elas. of subs.	SS Markup = 0.11
Fiscal Policy			
τ	0.25	Income tax rate	
Monetary Policy			
ρ_{MP}	MP pers.	0.4	
σ_{MP}	MP st. dev.	0.0071	Christiano et al. (1999)

On the firm side, technology is taken to have constant returns to scaling of labour input, and the weight on adjustment costs is taken to be 100, which gives a Phillips curve slope of 0.1. Finally, the elasticity of substitution between intermediate goods is set to 10, giving a steady state mark-up of firm prices above marginal costs 0f 0.11

On the policy side, the income tax rate is set to 25%, the persistence of the monetary policy shock is set to be 0.4, and its standard deviation is set to 0.0071, following Christiano et al. (1999).

As described in the main text, the process for household efficiency is calibrated from the first four moments of changes in the logarithm of earnings at one year and five year horizons between 2014 and 2019. We use the method of simulated moments to estimate a process for household efficiency with two components, one transitory and one persistent. The set of efficiency outcomes is then discretized to give a final continuous time Poisson process which serves as input to the model. Figure 1 displays the final, discretized states for the logarithm of household efficiency, as well the histogram of frequencies at which a typical household will

Figure 1: Income Process



encounter each state in the model, for both the United States calibration[1] and the New Zealand calibration. Further details can be found in Kaplan et al. (2018).

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[1] Taken from Kaplan et al. (2018).