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GDP Plus: An economic activity indicator for New Zealand

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1. Technical Appendix

Our estimation follows the work of Aruoba et al. (2013) and Rees et al. (2014). We treat quarterly GDP growth as an unobserved variable that follows an AR(1) process,

$$\Delta y_t = \mu(1-\rho) + \rho \Delta y_{t-1} + \varepsilon_{G,t},$$

The above equation is a standard representation of GDP growth in the literature, where μ represents the mean growth rate of actual GDP and ρ is the persistence parameter for GDP. The innovation term, $\varepsilon_{G,t}$, is normally distributed.

The key assumption is that the two observed GDP measures, production GDP (GDP_P) and expenditure GDP (GDP_E) are indicators of 'true' underlying GDP. In other words our two observed estimates are equal to true GDP plus some form of measurement error. Therefore in matrix notation,

$$\begin{bmatrix} \Delta y_t^E \\ \Delta y_t^P \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Delta y_t + \begin{bmatrix} \varepsilon_{E,t} \\ \varepsilon_{P,t} \end{bmatrix},$$

where $(\varepsilon_{E,t}, \varepsilon_{P,t}, \varepsilon_{G,t})' \sim iid\ N(0, \Sigma)$. We assume that the measurement errors of the two estimates can be correlated and therefore,

$$\Sigma = \begin{bmatrix} \sigma_{GG}^2 & \sigma_{GE}^2 & \sigma_{GP}^2 \\ \sigma_{EG}^2 & \sigma_{EE}^2 & \sigma_{EP}^2 \\ \sigma_{PG}^2 & \sigma_{PE}^2 & \sigma_{PP}^2 \end{bmatrix}$$

As is shown in Aruoba et al. (2013) this model is only identified when at least one restriction is placed on Σ. If Σ is left unrestricted then there may not be a unique solution to the model. To achieve identification, we estimate a 3-equation model, which includes a third variable whose measurement errors are unrelated to the measurement errors of GDP_E, and GDP_P.

$$\begin{bmatrix} \Delta y_t^E \\ \Delta y_t^P \\ X_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \kappa \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \lambda \end{bmatrix} \Delta y_t + \begin{bmatrix} \varepsilon_{E,t} \\ \varepsilon_{P,t} \\ \varepsilon_{X,t} \end{bmatrix}$$

where $(\varepsilon_{E,t}, \varepsilon_{P,t}, \varepsilon_{G,t}, \varepsilon_{X,t})' \sim iid\ N(0, \Omega)$, and X_t satisfies the above conditions for achieving identification. Therefore,

$$\Omega = \begin{bmatrix} \sigma_{GG}^2 & \sigma_{GE}^2 & \sigma_{GP}^2 & \sigma_{GX}^2 \\ \sigma_{EG}^2 & \sigma_{EE}^2 & \sigma_{EP}^2 & 0 \\ \sigma_{PG}^2 & \sigma_{PE}^2 & \sigma_{PP}^2 & 0 \\ \sigma_{XG}^2 & 0 & 0 & \sigma_{XX}^2 \end{bmatrix}$$

Hence, the three equation model is identified. The 3x3 upper left part of the variance covariance matrix Ω is the un-restricted variance covariance matrix of interest. Adding the X_t equation has allowed us to maintain our assumption about the correlation between the measurement errors of unobserved 'true' GDP and the observed measures.

Valid identification of the three equation model, presented in the technical appendix, relies on our choice of X_t. In practice it is difficult to find a variable whose measurement error is unrelated to the two observed measures. However labour market variables from the *Household Labour Force Survey* (HLFS) potentially satisfy the necessary conditions for the X_t variable as they are generally constructed from consumer side surveys which are not used in the construction of GDP measures (i.e. measurement errors will be unrelated).[1] We also note that the labour market variables from the HLFS are generally quite correlated with other activity measures, which makes them a good candidate for our third variable.[2]

We examine some of the properties of the unemployment rate and employment growth (growth in the number of people employed), from the HLFS to decide which labour market indicator should be used.

Table 1: Correlation matrix

	Production GDP	Expenditure GDP	Unemployment rate	Employment growth
Production GDP	1			
Expenditure GDP	0.70	1		
Unemployment rate	-0.34	-0.20	1	
Employment growth	0.41	0.24	-0.60	1

[1] By contrast, *Quarterly Employment Survey* (QES) labour market data is used in the construction of GDP, so could not be used as the measurement errors are likely to be correlated.
[2] We do however expect a somewhat large one-off revision to HLFS data over 2019 due to the change in population growth data.

Note: Sample period is 1987 Q3 to 2018 Q4.

Table 1 shows the correlation matrix of several variables of interest, it indicates that the measures of GDP are more correlated with employment growth rather than unemployment, therefore it follows that employment growth captures more information that is statistically relevant for creating a new GDP measure.

In order to estimate our three equation system we follow the methodology of Aruoba et al. (2013). We use a Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm to estimate the parameters