

# Short and Long Interest Rate Targets\*

Bernardino Adão  
Banco de Portugal

Isabel Correia  
Banco de Portugal, Universidade Catolica Portuguesa and CEPR

Pedro Teles  
Banco de Portugal, Universidade Catolica Portuguesa and CEPR

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## Abstract

We show that short and long nominal interest rates are independent monetary policy instruments. The peg of both allows to solve the classic problem of multiplicity that arises when only short rates are used as the instrument of policy. A peg of the nominal returns on assets of different maturities is equivalent to a peg of state-contingent interest rates. These are the rates that should be targeted in order to implement unique equilibria. At the zero bound, while it is still possible to target state-contingent interest rates, that is no longer equivalent to the target of the term structure.

*Key words:* Monetary policy; monetary policy instruments; maturities; short rates; multiplicity of equilibria; sticky prices.

*JEL classification:* E31; E40; E52; E58; E62; E63.

## 1. Introduction

When short term nominal interest rates are close to zero, because zero is the lower bound on these rates, the question typically arises of whether policy can be

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conducted with both short and long interest rates. The conventional wisdom seems to be that short and long interest rates are not independent policy instruments (see, for example, Woodford, 2005).<sup>1</sup> We show that this not the case. Short and long rates are, in general, independent instruments, and they should both be used as a means of solving the classic problem of multiplicity of equilibria arising when only short rates are used as the monetary policy instrument. Interestingly enough, the zero bound is an exception to the general result. Precisely at the zero bound, when the economy stays persistently there, short and long rates are no longer independent instruments, and targeting both does not solve the multiplicity problem.

The conventional arguments can be loosely stated as follows. If monetary policy is conducted using an interest rate feedback rule for the short term interest rate, then, under certain conditions, there is a locally determinate equilibrium. This means that the linear system of equations that approximates the equilibrium conditions in the neighborhood of a steady state, has a unique solution in that neighborhood and multiple solutions outside that neighborhood. At the determinate solution, the long rates are obtained by arbitrage from the short rates. There would not seem to be possible to use both short and long rates if the locally determinate equilibrium was indeed the single equilibrium. But it is not.

The locally determinate equilibrium is only one of the possible equilibria, one that has the particularly attractive property of being close to the steady state. All the other solutions of the system that linearly approximate the equilibrium conditions suggest the existence of other equilibria, that have been analyzed by Benhabib, Schmitt-Grohe and Uribe (2001b) among many others.<sup>2</sup> An interest rate peg of both the short and long rates, allows to implement one, and only one, of the equilibria in the set of possible equilibria.

The problem of multiplicity of equilibria when the monetary policy instrument is the short term interest rate was first formally addressed by Sargent and Wallace (1975). In an attempt to side step the multiplicity problem, McCallum (1981) proposes an interest rate feedback such that there is a locally determinate equilibria at the expense of multiple explosive solutions. A large literature on local determinacy followed (see Woodford, 2003 among many others<sup>3</sup>). Attention was

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<sup>1</sup>Woodford (2005) is a comment of McGough, Rudebusch, and Williams (2005).

<sup>2</sup>See also Benhabib, Schmitt-Grohe and Uribe (2001b, 2002, 2003) and Schmitt-Grohe and Uribe (2009). They show that the conditions for local determinacy may in fact be conditions for global indeterminacy.

<sup>3</sup>Clarida, Gali and Gertler (1999, 2000), Carlstrom and Fuerst (2001, 2002), Benhabib,

focused on the local determinate equilibrium, and other solutions were ignored on the basis of arbitrary technical restrictions.

There have been various attempts at solving the problem of multiplicity of equilibria under interest rate policy. The fiscal theory of the price level (see Cochrane (2007) for a critical discussion) assumes that tax rates are also exogenous, so that there are additional equilibrium restrictions that can be used to pin down equilibria. In contrast with this literature, we assume throughout the paper that fiscal policy is determined endogenously, in line with the extensive literature on local and global determinacy.

Atkeson, Chari and Kehoe (2009) consider out of equilibrium escape clauses in the spirit of Obstfeld and Rogoff (1983) and Nicolini (1996), and Christiano and Rostagno (2002). The point of Obstfeld and Rogoff and Nicolini was that if some form of convertibility was to be introduced in case prices deviated from some equilibrium, then there would be a unique equilibrium where convertibility would not be used.

While, in general, interest rate feedback rules give rise to multiplicity there are cases of rules that deliver global uniqueness. This has been shown by Loisel (2008) in a linear model and by Adao, Correia and Teles (2009) in standard monetary models. The rules that Adao, Correia and Teles propose are price level targeting rules where the short-term interest rate responds to forecasts of the future price level and economic activity.

In this paper we first show that while policy on (short term) noncontingent interest rates is unable to pin down a unique equilibrium, that is not the case when the policy instruments are the returns on state-contingent nominal assets. If monetary policy targets state-contingent interest rates, as well as the initial money supply there is a unique equilibrium globally. These results are shown in a simple flexible economy (section 2). We also show that the set of equilibria can be implemented with zero net supply of nominal state-contingent assets.

In a flexible price economy, when policy is conducted with interest rate targets, prices are not pinned down but allocations are. Instead, under sticky prices, setting the path for the nominal interest rates not only does not pin down prices, it also generates multiple equilibria in the allocations. We extend the results to a simple sticky price environment where prices are assumed to be set one period in advance (section 4).

The intuition of this first result is simple: In a deterministic flexible price model, if policy sets exogenously the path of nominal interest rates as well as

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Schmitt-Grohe and Uribe (2001a), among many others.

the initial money supply, there is a unique perfect foresight equilibrium. Under uncertainty, setting the initial money supply and the nominal interest rate in every state of the world is not sufficient to pin down a unique equilibrium. The path of the nominal interest rate in a flexible price model pins down the allocations and the average growth rate of the price level, but not the distribution of prices across states. In order to pin down a unique equilibrium, there have to be additional constraints on the returns of state-contingent nominal assets.

This result is related to the results in Nakajima and Polemarchakis (2003) and Bloise, Dreze and Polemarchakis (2004). These papers analyze the degrees of freedom in conducting monetary policy when the instruments are the money supply and the nominal interest rate. Nakajima and Polemarchakis (2003) impose restrictions on the structure of the economy and on the policies so that the policy results are the same whether the economy has a finite or an infinite horizon.

The second main point in the paper is to show that the targets for the state-contingent interest rates can be achieved with targets for nominal assets of different maturities. This is done in section 3. This result is related to Angeletos (2002) and Buera and Nicolini (2004) that have shown that state-contingent debt may be replicated by debt of multiple maturities. Their result is for quantities, ours for prices, but the mechanisms are similar. We also need as many maturities as the number of states.

In order to be able to use assets of different maturities to target the state contingent interest rates there must be enough variability in noncontingent interest rates. Even if the necessary variability is arbitrarily small, this still means that if the economy was permanently exactly at the zero bound, it would not be possible to use the rates on different maturities to achieve a unique equilibrium. At the zero bound, policy will have to target state-contingent interest rates in order to implement unique equilibria.

## 2. A model with flexible prices

We first consider a simple cash in advance economy with flexible prices. The economy consists of a representative household, a representative firm behaving competitively, and a government. The uncertainty in period  $t \geq 0$  is described by the random variable  $s_t \in S_t$  and the history of its realizations up to period  $t$  (state or node at  $t$ ),  $(s_0, s_1, \dots, s_t)$ , is denoted by  $s^t \in S^t$ . We assume that  $s_t$  has a discrete distribution. The number of states in period  $t \geq 0$  is  $\Phi_t$ .

Production uses labor according to a linear technology. We impose a cash-

in-advance constraint on the households' transactions with the timing structure described in Lucas and Stokey (1983). That is, each period is divided into two subperiods, with the assets market operational in the first subperiod and the goods market in the second.

## 2.1. Competitive equilibria

**Households** The households have preferences over consumption  $C(s^t)$ , and leisure  $L(s^t)$ , described by the expected utility function

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C(s^t), L(s^t)) \right\} \quad (2.1)$$

where  $\beta$  is a discount factor. The households start period  $t$ , in state  $s^t$ , with nominal wealth  $\mathbb{W}(s^t)$ . They decide to hold money,  $M(s^t)$ , and to buy  $B(s^t)$  nominal bonds that pay  $R(s^t) B(s^t)$  one period later.  $R(s^t)$  is the gross nominal interest rate in state  $s^t$ . They also buy  $Z(s^{t+1})$  units of state-contingent nominal securities. Each security pays one unit of money at the beginning of period  $t+1$  in state  $s^{t+1}$ . Let  $Q(s^{t+1}/s^t)$  be the beginning of period  $t$  price of these securities normalized by the probability of the occurrence of the state. The households spend  $E_t Q(s^{t+1}/s^t) Z(s^{t+1})$  in state-contingent nominal securities. Thus, in the assets market at the beginning of period  $t$  they face the constraint

$$M(s^t) + B(s^t) + E_t Q(s^{t+1}/s^t) Z(s^{t+1}) \leq \mathbb{W}(s^t) \quad (2.2)$$

where the initial nominal wealth  $\mathbb{W}(s_0)$  is given.

Consumption must be purchased with money according to the cash in advance constraint

$$P(s^t) C(s^t) \leq M(s^t). \quad (2.3)$$

At the end of the period, the households receive the labor income  $W(s^t) N(s^t)$ , where  $N(s^t) = 1 - L(s^t)$  is labor and  $W(s^t)$  is the nominal wage rate and pay lump sum taxes,  $T(s^t)$ . Thus, the nominal wealth households bring to state  $s^{t+1}$  is

$$\mathbb{W}(s^{t+1}) = M(s^t) + R(s^t) B(s^t) + Z(s^{t+1}) - P(s^t) C(s^t) + W(s^t) N(s^t) - T(s^t) \quad (2.4)$$

The households' problem is to maximize expected utility (2.1) subject to the restrictions (2.2), (2.4), (2.3), together with a no-Ponzi games condition on the holdings of assets.

The following are first order conditions of the households' problem:

$$\frac{u_L(s^t)}{u_C(s^t)} = \frac{W(s^t)}{P(s^t)R(s^t)} \quad (2.5)$$

$$\frac{u_C(s^t)}{P(s^t)} = R(s^t) E_t \left[ \frac{\beta u_C(s^{t+1})}{P(s^{t+1})} \right] \quad (2.6)$$

$$Q(s^{t+1}/s^t) = \beta \frac{u_C(s^{t+1})}{u_C(s^t)} \frac{P(s^t)}{P(s^{t+1})} \quad (2.7)$$

From these conditions we get

$$E_t Q(s^{t+1}/s^t) = \frac{1}{R(s^t)} \quad (2.8)$$

Condition (2.5) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage adjusted for the cost of using money,  $R(s^t)$ . Condition (2.6) is an intertemporal marginal condition necessary for the optimal choice of risk-free nominal bonds. Condition (2.7) determines the price of one unit of money at time  $t + 1$ , for each state  $s^{t+1}$ , normalized by the conditional probability of occurrence of state  $s^{t+1}$ , in units of money at time  $t$ , in state  $s^t$ .

**Firms** The firms are competitive and prices are flexible. The production function of the representative firm is linear

$$Y(s^t) = A(s^t) N(s^t)$$

The equilibrium real wage is

$$\frac{W(s^t)}{P(s^t)} = A(s^t). \quad (2.9)$$

**Government** The policy variables are taxes  $T(s^t)$ , nominal interest rates  $R(s^t)$ , state-contingent nominal prices  $Q(s^{t+1}/s^t)$ , money supplies  $M(s^t)$ , state-noncontingent public debt  $B(s^t)$  and state-contingent debt  $Z(s^{t+1})$ . The government expenditures,  $G_t$ , are exogenous.

The government budget constraints are

$$M(s^t) + B(s^t) + E_t Q(s^{t+1}/s^t) Z(s^{t+1}) = M(s^{t-1}) + R(s^{t-1}) B(s^{t-1}) + Z(s^t) + P(s^{t-1}) G(s^{t-1}) - T(s^{t-1}), t \geq 0$$

together with a no-Ponzi games condition. Let  $Q(s^{t+1}) \equiv Q(s^{t+1}/s_0)$ , with  $Q(s_0) = 1$ . If  $\lim_{T \rightarrow \infty} E_t Q(s^{T+1}) \mathbb{W}(s^{T+1}) = 0$ , the budget constraints can be written as

$$\sum_{s=0}^{\infty} E_t Q(s^{t+s+1}/s^t) [M(s^{t+s}) (R(s^{t+s}) - 1) + T(s^{t+s}) - P(s^{t+s}) G(s^{t+s})] = \mathbb{W}(s^t) \quad (2.10)$$

**Market clearing** The goods and labor market clearing conditions are

$$C(s^t) + G(s^t) = A(s^t) N(s^t)$$

and

$$1 - L(s^t) = N(s^t).$$

We have already imposed market clearing in the money and asset markets.

**Equilibria** A competitive equilibrium is a sequence of policy variables, quantities and prices such that the private agents solve their problems given the sequences of policy variables and prices, the budget constraint of the government is satisfied, and markets clear.

The competitive equilibrium conditions for the variables  $\{C(s^t), L(s^t)\}$ , and  $\{R(s^t), Q(s^{t+1}/s^t), M(s^t), B(s^t), Z(s^{t+1}), T(s^t)\}$  are the resource constraints

$$C_t + G_t = A_t(1 - L_t), \quad (2.11)$$

the intratemporal conditions

$$\frac{u_C(s^t)}{u_L(s^t)} = \frac{R(s^t)}{A(s^t)}, \quad (2.12)$$

that are obtained from the households intratemporal conditions (2.12) and the firms optimal condition (2.9), as well as the cash in advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), the budget constraint (2.10).

The equations identified above determine a set of equilibrium allocations, prices and policy variables. In order for a particular equilibrium in this set to be implemented, it is necessary to determine exogenous policy rules for a subset of the policy variables. A policy rule for a particular policy variable can be a function of only the state or of other variables. We will primarily consider the case where policy rules are only functions of the state. An exogenous policy rule is one that is not implied by the other equilibrium conditions.

In the next section we will show that if interest rates are set exogenously in every date and state, as well as the initial money supply, there is multiplicity under uncertainty. The appropriate policy instruments, that allow to implement a unique equilibrium, are the returns on state-contingent nominal assets.

## 2.2. Multiplicity of equilibria with interest rate rules

In this section, we show that when policy is conducted with constant functions for the monetary policy instruments, and fiscal policy is endogenous, if policy sets the nominal interest rates and the initial money supply, it is unable, under uncertainty, to implement a unique equilibrium.

Without imposing restrictions on the policy variables, the equilibrium conditions are the resources constraint, (2.11), the intratemporal condition (2.12), the cash in advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), as well as the budget constraints (2.10). These conditions define a set of equilibrium allocations, prices and policy variables. The set of allocations is the set of implementable allocations. In order for the set of equilibrium conditions to have a unique solution a subset of the policy variables must be set exogenously. We will call this set, the set of instruments of policy.

We will start by showing that if the policy instruments are the nominal interest rate, as well as the initial money supply, it is possible to implement a unique equilibrium in the deterministic economy, but not under uncertainty. We first consider the case in which the policy are sequences of constant functions for the



interest rates. The other policy variables are not exogenous. From the resources constraints, (2.11) and the intratemporal conditions (2.12), we obtain the functions  $C(R(s^t))$  and  $L(R(s^t))$ . The system of equations can then be summarized by the following dynamic equations:

$$\frac{u_C(C(R(s^t)), L(R(s^t)))}{P(s^t)} = \beta R(s^t) E_t \left[ \frac{u_C(C(R(s^{t+1})), L(R(s^{t+1})))}{P(s^{t+1})} \right], t \geq 0 \quad (2.13)$$

together with the cash in advance constraints, (2.3), and the budget constraints, (2.10).

Suppose the nominal interest rate is set exogenously in every date and state. The allocation is pinned down uniquely. The issue is how can a unique sequence of prices levels be pinned down. The proposition clarifies it.

**Proposition 2.1.** *Suppose policy are sequences of numbers for the nominal interest rates. Let the interest rate be determined exogenously in every date and state, as well as the initial money supply. For this policy the allocation and prices are determined uniquely in the deterministic case. Under uncertainty, there is a single solution for the consumption and labor allocations, but not for the price levels.*

Proof: Given the interest rate for every date and state, the allocation is obtained from the functions  $C(R(s^t))$  and  $L(R(s^t))$ .

At any period  $t \geq 1$ , given  $P(s^{t-1})$ , there are  $\Phi_{t-1}$  equations to determine  $\Phi_t$  variables,  $P(s^t)$ . More specifically, for each state  $s^{t-1}$ , there is one equation to determine  $\#S_t$  variables. Except for the deterministic case, there are multiple solutions for the price level, and consequently for the money supply. ■

The initial price level indeterminacy in the deterministic economy under an interest rate peg, is replaced by a terminal price indeterminacy, in every terminal state, under uncertainty. As we will see below this explosion in degrees of multiplicity under uncertainty results from pegging the state-noncontingent nominal interest rates instead of the state-contingent nominal returns. If, instead, these were pegged, there would be a single degree of multiplicity as in the deterministic case. A single equilibrium could be implemented by setting exogenously the money supply in the initial period.

### 2.3. Policy with state-contingent interest rates

In this section we show that a policy that pegs the state-contingent nominal returns, as well as the money supply in the initial period, implements a unique equilibrium.

The equilibrium conditions are the resources constraint, (2.11), the intratemporal condition (2.12), the cash in advance constraints (2.3), the intertemporal conditions (2.6) and (2.7), as well as the budget constraints (2.10).

As before, from the resources constraints (2.11) and the intratemporal condition (2.12) we obtain the functions  $C(R(s^t))$  and  $L(R(s^t))$ .

The system of equilibrium conditions can be summarized by the following dynamic equations:

$$Q(s^t/s^{t-1}) = \beta \frac{u_C(R(s^t))}{u_C(R(s^{t-1}))} \frac{P(s^{t-1})}{P(s^t)}, t \geq 1 \quad (2.14)$$

together with (2.8), the cash in advance condition, (2.3), and the budget constraint that determines, not uniquely, the endogenous taxes and debt levels, (2.10).

Clearly if policy is conducted by setting exogenously the state-contingent nominal interest rates, given the initial price level, the price levels are all determined. In order to have a single equilibrium it would still be necessary to set exogenously the initial money supply. The proposition follows:

**Proposition 2.2.** *If the state-contingent interest rates are set exogenously for every date and state, there is a unique equilibrium for the allocations and prices if the money supply is set exogenously in the initial period.*

Proof:

Let  $P(s_0)$  be given. Given the values for  $\{Q(s^t/s^{t-1}), t \geq 1\}$ ,  $\{R(s^t), t \geq 0\}$  are determined uniquely, and given  $P(s^{t-1})$ ,  $P(s^t)$  is obtained from the dynamic condition (2.14) for  $t \geq 1$ . The condition only holds for  $t \geq 1$ . Cannot use the condition at  $t = 0$ , to determine  $P(s_0)$ .  $M(s_0)$  pins down the initial price from the cash-in-advance constraint that, if  $R(s_0) > 1$ , holds with equality. ■

In these economies there is a unique equilibrium if the policy is to peg the nominal returns on the state-contingent nominal assets and in addition money supply is set exogenously in the initial period. A timeless perspective (see Woodford, 2003) abstracts from this initial period by concentrating on the asymptotic behavior of the economy, as if the initial period had happened at an arbitrarily

early date.<sup>4</sup> According to the timeless perspective all the government is required to do is to set exogenously the state-contingent interest rates.

### 2.3.1. State-contingent debt in zero net supply

Even if the government stands ready to supply and demand any quantity of state-contingent bonds at given state-contingent prices, these assets can be in zero net supply in every equilibrium. To see this notice that, when the supply of state-contingent assets is zero,  $Z(s^{t+1}) = 0$ , the budget constraints of the government are

$$\begin{aligned} & \sum_{s=0}^{\infty} E_t Q(s^{t+s+1}/s^t) [M(s^{t+s})(R(s^{t+s}) - 1) + T(s^{t+s}) - P(s^{t+s})G(s^{t+s})] \\ & = \mathbb{W}(s^t) \end{aligned}$$

where, with  $Z(s^t) = 0$ ,  $\mathbb{W}(s^t) = M(s^{t-1}) + R(s^{t-1})B(s^{t-1}) + P(s^{t-1})G(s^{t-1}) - T(s^{t-1})$  is noncontingent. This must be satisfied for any allocation and prices, at any period and state. Even with  $Z(s^t) = 0$ , there are still multiple solutions of these equations for the endogenous nominal noncontingent debt and the lump sum taxes.

## 3. Term structure

In this section we show the second main result of the paper, that there is an equivalence between pegging state-contingent prices and noncontingent interest rates of different maturities.

We assume there is noncontingent nominal debt of maturity  $j = 1, \dots, n$ , with compound gross interest rate  $R_j(s^t)$ . For simplicity let  $n = 2$ . Then, the following arbitrage conditions must hold

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta^2 R_2(s^t) E_t \left[ \frac{u_C(R_1(s^{t+2}))}{P(s^{t+2})} \right]$$

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<sup>4</sup>One way to interpret the timeless perspective, is that it is possible to use conditions  $Q(s^t/s^{t-1}) = \beta \frac{u_C(R(s^t))}{u_C(R(s^{t-1}))} \frac{P(s^{t-1})}{P(s^t)}$  for  $t = 0$ , instead of only from  $t = 1$  on, as well as the assumption that  $P_{-1}$  is exogenous. These are additional conditions that allow to determine the price level at  $t = 0$ .

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_1(s^t) E_t \left[ \frac{u_C(R_1(s^{t+1}))}{P(s^{t+1})} \right]$$

$$\frac{u_C(R_1(s^{t+1}))}{P(s^{t+1})} = \beta R_1(s^{t+1}) E_{t+1} \left[ \frac{u_C(R_1(s^{t+2}))}{P(s^{t+2})} \right]$$

which imply

$$R_2(s^t) E_t \left[ \frac{u_C(R_1(s^{t+2}))}{P(s^{t+2})} \right] = R_1(s^t) E_t \left[ R_1(s^{t+1}) E_{t+1} \left[ \frac{u_C(R_1(s^{t+2}))}{P(s^{t+2})} \right] \right]$$

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$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_2(s^t) E_t \left[ \frac{u_C(R_1(s^{t+1}))}{R_1(s^{t+1}) P(s^{t+1})} \right]$$

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_1(s^t) E_t \left[ \frac{u_C(R_1(s^{t+1}))}{P(s^{t+1})} \right]$$

If  $cov = 0$ ,

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_2(s^t) E_t \left[ \frac{1}{R_1(s^{t+1})} \right] E_t \left[ \frac{u_C(R_1(s^{t+1}))}{P(s^{t+1})} \right]$$

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_1(s^t) E_t \left[ \frac{u_C(R_1(s^{t+1}))}{P(s^{t+1})} \right]$$

This implies  $R_2(s^t) E_t \left[ \frac{1}{R_1(s^{t+1})} \right] = R_1(s^t)$  or  $\frac{1}{R_2(s^t)} = \frac{1}{R_1(s^t)} E_t \left[ \frac{1}{R_1(s^{t+1})} \right]$

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If the conditional covariance to state  $s^t$  between  $R_1(s^{t+1})$  and  $E_{t+1} \left[ \frac{u_C(R_1(s^{t+2}))}{P(s^{t+2})} \right]$

was equal to zero, then it would follow that

$$R_2(s^t) = R_1(s^t) E_t [R_1(s^{t+1})].$$

The long rate is given by the sequence of short rates. In this case there would be no degrees of freedom in targeting the long rates, once the short rates are set. This may be the basis for the common intuition that short and long rates are not independent policy instruments. But this case of zero covariance is by no means the general case. In general the covariance is not zero, long rates may be additional policy instruments that can be used to uniquely implement equilibria.

In order to illustrate the results that we will derive more generally below, we consider that in each period there are two possible occurrences of uncertainty,  $s_t \in \{h, l\}$ ,  $t \geq 1$ , and suppose as above that there are one and two period noncontingent bonds. Then the following conditions must be satisfied

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_1(s^t) \left[ \pi(h/s^t) \frac{u_C(R_1(s^t, h))}{P(s^t, h)} + \pi(l/s^t) \frac{u_C(R_1(s^t, l))}{P(s^t, l)} \right] \quad (3.1)$$

$$\begin{aligned} & \frac{u_C(R_1(s^t))}{P(s^t)} \\ &= \beta^2 R_2(s^t) \left[ \begin{aligned} & \pi((h, h)/s^t) \frac{u_C(R_1(s^t, h, h))}{P(s^t, h, h)} + \pi((h, l)/s^t) \frac{u_C(R_1(s^t, h, l))}{P(s^t, h, l)} \\ & + \pi((l, h)/s^t) \frac{u_C(R_1(s^t, l, h))}{P(s^t, l, h)} + \pi((l, l)/s^t) \frac{u_C(R_1(s^t, l, l))}{P(s^t, l, l)} \end{aligned} \right] \end{aligned} \quad (3.2)$$

$$\begin{aligned} & \frac{u_C(R_1(s^t, h))}{P(s^t, h)} \\ &= \beta R_1(s^t, h) \left[ \pi(h/(s^t, h)) \frac{u_C(R_1(s^t, h, h))}{P(s^t, h, h)} + \pi(l/(s^t, h)) \frac{u_C(R_1(s^t, h, l))}{P(s^t, h, l)} \right] \end{aligned} \quad (3.3)$$

$$\begin{aligned} & \frac{u_C(R_1(s^t, l))}{P(s^t, l)} \\ &= \beta R_1(s^t, l) \left[ \pi(h/(s^t, l)) \frac{u_C(R_1(s^t, l, h))}{P(s^t, l, h)} + \pi(l/(s^t, l)) \frac{u_C(R_1(s^t, l, l))}{P(s^t, l, l)} \right] \end{aligned} \quad (3.4)$$

The last three conditions, (3.2), (3.3), and (3.4), can be used to obtain the arbitrage condition

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_2(s^t) \left[ \pi(h/s^t) \frac{u_C(R_1(s^t, h))}{R_1(s^t, h) P(s^t, h)} + \pi(l/s^t) \frac{u_C(R_1(s^t, l))}{R_1(s^t, l) P(s^t, l)} \right]$$

Given  $P(s^t)$ , conditions (3.1) and (??) determine  $P(s^t, h)$  and  $P(s^t, l)$ , provided  $R_1(s^t, h) \neq R_1(s^t, l)$ . It follows that if  $R_1(s^t)$  and  $R_2(s^t)$  are set exogenously, and  $R_1(s^t, h) \neq R_1(s^t, l)$ , for all  $s^t$ , for a given initial price level  $P(s_0)$ , there is a unique solution for the allocations and prices.

The proposition follows.

**Proposition 3.1.** *Let  $S_t = \{s_1, s_2, \dots, s_n\}$  and suppose there are nominal non-contingent assets of maturity  $j = 1, \dots, n$ . If the returns on these assets are set exogenously, then, in general, there is a unique equilibrium for the allocations and prices (if the money supply is set exogenously in the initial period).*

Proof: See Appendix 1. ■

### 3.1. The zero bound

While the question of using long rates as an instrument of policy is typically raised when short rates are very close to the zero lower bound, it turns out that at the zero bound the conditions for the result in proposition 3.1 may not be verified.

Indeed, suppose for simplicity that the economy was permanently at the zero bound. Then  $R_1(s^t, h) = R_1(s^t, l) = 1$ , and it follows that  $R_2(s^t) = 1$ . The condition of enough variability in the interest rates is not fulfilled and there are multiple equilibria.

It is possible for the nominal interest rate to be temporarily at the zero bound and there be enough variability in future rates, so that the proposition still holds. Furthermore, the variability in interest rates that is necessary can be arbitrarily small, so in that sense the results are quite robust.

## 4. Price setting restrictions

In this section we show that the results derived above extend to an environment with prices set in advance. We modify the environment to consider price setting restrictions. There is a continuum of goods, indexed by  $i \in [0, 1]$ . Each good  $i$  is produced by a different firm. The firms are monopolistic competitive and set prices in advance with different lags.

The households have preferences described by (2.5) where  $C(s^t)$  is now the composite consumption

$$C(s^t) = \left[ \int_0^1 c_i(s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1.$$

Households have a demand function for each good given by

$$c_i(s^t) = \left( \frac{p_i(s^t)}{P(s^t)} \right)^{-\theta} C(s^t),$$

where  $P(s^t)$  is the price level,

$$P(s^t) = \left[ \int p_i^{1-\theta}(s^t) di \right]^{\frac{1}{1-\theta}}. \quad (4.1)$$

The households' intertemporal and intratemporal conditions are as before, (2.5), (2.6) and (2.7).

The government must finance an exogenous path of government purchases  $\{G(s^t)\}_{t=0}^{\infty}$ , such that

$$G(s^t) = \left[ \int_0^1 g_i^{\frac{\theta-1}{\theta}}(s^t) di \right]^{\frac{\theta}{\theta-1}} \quad (4.2)$$

Given the prices on each good  $i$  in units of money,  $p_i(s^t)$ , the government minimizes expenditure on government purchases by deciding according to

$$\frac{g_i(s^t)}{G(s^t)} = \left( \frac{p_i(s^t)}{P(s^t)} \right)^{-\theta} \quad (4.3)$$

Aggregate resource constraints can be written as

$$(C(s^t) + G(s^t)) \int_0^1 \left( \frac{p_i(s^t)}{P(s^t)} \right)^{-\theta} di = A(s^t) N(s^t). \quad (4.4)$$

A fraction  $\alpha_j$  firms set prices  $j$  periods in advance with  $j = 0, \dots, J-1$ . Firms decide the price for period  $t$  with the information up to period  $t-j$  to maximize:

$$E_{t-j} [Q(s^{t+1}/s^{t-j}) (p_i(s^t) y_i(s^t) - W(s^t) n_i(s^t))]$$

subject to the production function

$$y_i(s^t) \leq A_t n_i(s^t)$$

and the demand function

$$y_i(s^t) = \left( \frac{p_i(s^t)}{P(s^t)} \right)^{-\theta} Y(s^t) \quad (4.5)$$

where  $y_i(s^t) = c_i(s^t) + g_i(s^t)$

The optimal price is

$$p_i(s^t) = p_j(s^t) = \frac{\theta}{(\theta - 1)} E_{t-j} \left[ \eta_j(s^{t+1}) \frac{W(s^t)}{A(s^t)} \right] \quad (4.6)$$

where

$$\eta_j(s^{t+1}) = \frac{Q(s^{t+1}/s^{t-j}) P(s^t)^\theta Y(s^t)}{E_{t-j} \left[ Q(s^{t+1}/s^{t-j}) P(s^t)^\theta Y(s^t) \right]}.$$

Compared with the equilibrium conditions under flexible prices, the set of equilibrium conditions when prices are set in advance includes more variables, the prices of the different firms, but it also includes more restrictions, the price setting restrictions. The number of additional variables and restrictions is the same, and the degrees of freedom are the same as under flexible prices. The degree of indeterminacy is the same as under flexible prices and therefore the statement in proposition 2.2, that a peg of state-contingent interest rates delivers a unique equilibrium, still holds when prices are set in advance.

Substituting the state-contingent prices  $Q(s^{t+1}/s^{t-j})$  in the price setting conditions (4.6), and using the intertemporal condition (2.6) as well as the households' intratemporal condition (2.5), we obtain the intratemporal conditions<sup>5</sup>

$$E_{t-j} \left[ \begin{array}{l} \frac{u_C(s^t)}{R(s^t)} P(s^t)^{\theta-1} A(s^t) (1 - L(s^t)) - \\ \frac{\theta}{(\theta-1)} u_L(s^t) P(s^t)^{\theta-1} (1 - L(s^t)) \frac{P(s^t)}{p_j(s^t)} \end{array} \right] = 0, j = 0, \dots, J-1. \quad (4.8)$$

The equilibrium conditions can then be summarized by the intertemporal conditions (2.7) and (2.8), the intratemporal conditions (4.8), as well as the conditions for the price level at date  $t$ , in state  $(s^t)$  that can be written as

$$P(s^t) = \left[ \sum_{j=0}^{J-1} \alpha_j (p_j(s^t))^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (4.9)$$

---

<sup>5</sup>Notice that, if  $J = 1$ , meaning that there are only flexible price firms,  $p_0(s^t) = P(s^t)$  and we would get the intratemporal condition obtained under flexible prices,

$$\frac{u_C(s^t)}{u_L(s^t)} = \frac{\theta R(s^t)}{(\theta - 1) A(s^t)}, \quad (4.7)$$

corresponding to (2.12), for the case where  $\theta \rightarrow \infty$ .



and the resource constraints, written as

$$[C(s^t) + G(s^t)] \sum_{j=0}^{J-1} \alpha_j \left( \frac{p_j(s^t)}{P(s^t)} \right)^{-\theta} = A(s^t) N(s^t). \quad (4.10)$$

The proposition follows:

**Proposition 4.1.** *(Prices are set in advance) If the state-contingent interest rates are set exogenously for every date and state, and the money supply is set exogenously in the initial period, there is a unique equilibrium for the allocations and prices.*

Proof: Let  $\{Q(s^{t+1}/s^t), t \geq 0\}$  be set exogenously. Then  $\{R(s^t), t \geq 0\}$  are determined uniquely.

At any  $t \geq J$ , given  $P(s^{t-1})$ ,  $C(s^{t-1})$  and  $L(s^{t-1})$  there are  $\Phi_t$  intertemporal conditions,  $\Phi_t$  resource constraints,  $\Phi_t$  price level conditions,  $\Phi_{t-j}$  price setting conditions,  $j = 0, \dots, J-1$ . The variables are  $\Phi_t$  consumption levels  $C(s^t)$ ,  $\Phi_t$  levels of leisure  $L(s^t)$ ,  $\Phi_t$  price levels and  $\Phi_{t-j}$  prices for the different firms,  $j = 0, \dots, J-1$ .

For  $t = 0$ , there is one price level condition, one resource constraint, one price setting condition. The variables are  $C(s_0)$ ,  $L(s_0)$ ,  $P(s_0)$  and one price for the flexible firms in period 0. The other prices are historical. Can use the cash in advance constraint with exogenous  $M_0$  to determine all the variables in period 0.

For  $t = 1$ , given  $P(s_0)$ ,  $C(s_0)$  and  $L(s_0)$ , there are  $\Phi_1$  price level conditions,  $\Phi_1$  resources constraints,  $\Phi_1 + \Phi_0$  price setting conditions,  $\Phi_0$  intertemporal conditions to determine the same number of variables. The variables are  $\Phi_1$  consumptions  $C(s^1)$ ,  $\Phi_1$  levels of leisure  $L(s^1)$ ,  $\Phi_1$  price levels,  $\Phi_1$  prices for the flexible firms in period 1 and  $\Phi_0$  prices for the firms setting the price in period 0 for period 1. Similarly for any period  $1 \leq t \leq J-1$ . ■

Similarly, if, instead of the state contingent returns, there was a peg of the noncontingent maturities, under the conditions of Proposition 3.1, then there would be a unique equilibrium for quantities and prices.

## 5. Concluding Remarks

We have shown that a monetary policy that targets the state-contingent nominal returns, as opposed to the state-noncontingent nominal interest rates, is able to eliminate the indeterminacy associated with uncertainty. In economies with

flexible prices and prices set in advance, that policy, for a given initial money supply, implements a unique equilibrium.

A policy that pegs the returns on state-contingent nominal assets may seem a difficult task for the monetary authority. This is less so, because setting state contingent interest rates is equivalent to setting the noncontingent interest rates on nominal assets of different maturities.

It follows that, in contrast to conventional wisdom, long rates can be used as additional instruments of policy, provided that there is enough variability in interest rates. This condition of mostly theoretical interest, means that at the zero bound, if the economy was to stay persistently there, targeting different maturities would not solve the multiplicity problem. A target of the state contingent interest rates would have to be used to implement a unique equilibrium at the, consistently optimal, Friedman rule of zero nominal interest rates.

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## A. Appendix

### A.1. Proposition 3.1

Let  $S_t = \{s_1, s_2, \dots, s_n\}$ . Then it must be the case that

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_1(s^t) \left[ \begin{array}{c} \pi(s_1|s^t) \frac{u_C(R_1(s^t, s_1))}{P(s^t, s_1)} + \pi(s_2|s^t) \frac{u_C(R_1(s^t, s_2))}{P(s^t, s_2)} + \\ \dots + \pi(s_n|s^t) \frac{u_C(R_1(s^t, s_n))}{P(s^t, s_n)} \end{array} \right] \quad (\text{A.1})$$

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_2(s^t) \left[ \begin{array}{c} \pi(s_1|s^t) \frac{u_C(R_1(s^t, s_1))}{R_1(s^t, s_1)P(s^t, s_1)} + \pi(s_2|s^t) \frac{u_C(R_1(s^t, s_2))}{R_1(s^t, s_2)P(s^t, s_2)} + \\ \dots + \pi(s_n|s^t) \frac{u_C(R_1(s^t, s_n))}{R_1(s^t, s_n)P(s^t, s_n)} \end{array} \right] \quad (\text{A.2})$$

...

$$\frac{u_C(R_1(s^t))}{P(s^t)} = \beta R_n(s^t) \left[ \begin{array}{c} \pi(s_1|s^t) \frac{u_C(R_1(s^t, s_1))}{R_{n-1}(s^t, s_1)P(s^t, s_1)} + \pi(s_2|s^t) \frac{u_C(R_1(s^t, s_2))}{R_{n-1}(s^t, s_2)P(s^t, s_2)} + \\ \dots + \pi(s_n|s^t) \frac{u_C(R_1(s^t, s_n))}{R_{n-1}(s^t, s_n)P(s^t, s_n)} \end{array} \right] \quad (\text{A.3})$$

Let  $R_j(s^t)$ ,  $j = 1, \dots, n$  be set exogenously, and let there be enough variability in  $R_j(s^t, s_l)$ ,  $j = 1, \dots, n$ ,  $l = 1, \dots, n$ , so that the matrix with row vectors  $\left[ \pi(s_1|s^t) \frac{u_C(R_1(s^t, s_1))}{R_{j-1}(s^t, s_1)} \quad \pi(s_2|s^t) \frac{u_C(R_1(s^t, s_2))}{R_{j-1}(s^t, s_2)} \quad \dots \quad \pi(s_n|s^t) \frac{u_C(R_1(s^t, s_n))}{R_{j-1}(s^t, s_n)} \right]$ ,  $j = 1, \dots, n$ , with  $R_0(s^t, s_l) = 1$  is invertible. Then, for a given initial price level  $P(s_0)$ , there is a unique solution for the allocations and prices.

If the initial money supply is also set, then there is a unique equilibrium for  $P(s_0)$  as well.