

On a Tractable Small Open Economy Model with Endogenous Monetary-policy Trade-offs★

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ABSTRACT

We propose a new highly tractable small open economy model with incomplete financial markets. The key aspect of the model is that it is not isomorphic to a closed economy, as has been shown in similarly tractable small open economy models. Under incomplete markets the real exchange rate is a fundamental variable in determining the equilibrium. This exchange rate channel introduces an endogenous trade-off for monetary policy. We also show that the welfare consistent monetary-policy loss function includes the real exchange rate as an argument, reflecting the endogenous monetary-policy trade-off.

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1 Introduction

In this paper, we revisit the monetary policy design question in the context of a small open economy business cycle model with “new Keynesian” frictions. Our contribution is twofold. First, we provide a highly tractable model whose approximate equilibrium characterization admits an *explicit* and non-trivial real exchange rate channel. This arguably realistic feature is missing from the existing literature on tractable small open economy models. Second, given this new channel, we show that there is an endogenous trade-off between stabilizing domestic inflation and output gap. This trade-off will also be reflected in a second-order accurate approximation of the utility-based social loss function.

In existing works based on the tractable model in Galí and Monacelli [2005] or Clarida, Galí, and Gertler [2001], the open economy dimension merely alters the equilibrium conditions that are familiar to a closed economy model in terms of the slopes of an IS curve and a Phillips curve. Quite starkly, Clarida, Galí, and Gertler [2001] thus concluded that the monetary policy problem for the small open economy is isomorphic to its closed economy cousin. Also, in a recent application of the Galí and Monacelli [2005] or Clarida, Galí, and Gertler [2001] model, Llosa and Tuesta [2008] show that the open economy dimension further restricts the region of the policy-rule parameter space that would ensure a notion of “E-stability” of a rational expectations equilibrium.¹

In our model, since the real exchange rate will appear as an endogenous and explicit channel of monetary policy trade-off, it will no longer be the case that domestic (producer price) inflation targeting will be sufficient in monetary policy design. In contrast to Llosa and Tuesta [2008] who analyze the stabilizing properties of various policy rules with arbitrary inflation measures, our framework affords an explicit deduction of what the central bank objective, and hence, policy rule should look like in a small open economy.

More precisely, we provide a theoretical foundation for the hypothesis that monetary policy in this economy cannot just respond to the usual suspects of domestic output gap and producer price inflation index. Movements in the international real exchange rate must also be taken into account explicitly in the small open economy monetary policy problem. Arguably, such a conclusion is indeed what practitioners in small open economy central banks have already been doing. For example, clause 4(b) of New Zealand’s 2002 *Policy Targets Agreement* states that:

“In pursuing its price stability objective, the Bank shall seek to avoid unnecessary instability in output, interest rates and the exchange rate”.

¹The degree of international trade openness is measured as the share of home-produced goods in the aggregate final consumption good basket. In our model, we not only index international trade openness by the share of home-produced goods in the final consumption good, but also measure trade openness by the share of an imported intermediate good in the production side of the economy. For example, one may interpret this good as energy.

We also show that how this endogenous monetary policy trade-off channel works will depend greatly on two competing notions of trade openness in the model. One is the degree of home bias γ in the final consumption good basket. The other is the degree of dependence of the small open economy on imported intermediate inputs into production δ .

There are, of course, departures from the “isomorphism” result outlined above. Examples are [Monacelli \[2005\]](#) and other medium-scale versions of similar models used in central banks around the world. However, these departures sacrifice tractability and resort usually to numerical simulations and examples. Thus, it is our aim in this paper to complete this gap in the literature by providing a tractable version of a model whose equilibrium characterization, under alternative monetary policy decision rules, can still be analytically studied.

2 An alternative model

Our model is basically built on the small open-economy model with imperfect competition and nominal price rigidities introduced by [Galí and Monacelli \[2005\]](#). We consider a world economy composed of two open economies: (i) the domestic economy that is small in the sense that their decisions do not have any impact in the rest of the world; and the foreign economy (or rest of the world) that we treat as large. We will use variables with an asterisk superscript to refer to the foreign country and variables without an asterisk to denote the small domestic economy. Subscripts “ H ” (for Home) and “ F ” (for Foreign) on certain variables will denote the production origin of the goods.

There are two key differences with the [Galí and Monacelli \[2005\]](#) model. First, we allow for a friction in the global financial markets by assuming that these markets are incomplete. Second, we generalize the notion of trade openness to also allow for imports of intermediate inputs into producing a final good, along the lines of [McCallum and Nelson \[1999\]](#) and [Chari, Kehoe, and McGrattan \[2002\]](#).

2.1 Primitive stochastic processes

Denote the state space of primitive exogenous shocks as $Z := \times_{m=1}^M [z_m, \bar{z}_m]$, $-\infty < z_m < \bar{z}_m < +\infty$ for each $m \in \{1, \dots, M\}$, and, let $t \in \mathbb{N} := \{0, 1, \dots\}$.

Assume that at the beginning of each $t \in \mathbb{N}$, a vector of domestic and foreign (i.e. global) exogenous shocks $z_t \in Z$ is publicly known. In this model, $M = 2$ and Z is the set of all possible total factor productivity shocks (z, z^*) , respectively, in the small open economy and in the rest of the world. Assume $Z^\infty := \times_{t \in \mathbb{N}} Z$ has the topology induced by the usual product norm $\|\{z_t\}_{t \in \mathbb{N}}\| := \sum_{t=0}^{\infty} z_t / 2^t$. Then, $\mathcal{Z} := \mathcal{B}(Z)$ denotes the

Borel σ -algebra generated by compact cubes $Z \subset \mathbb{R}^M$, and, $\mathcal{P}(Z^\infty)$ denotes the space of countably-additive probability measures defined on the measurable space $(Z^\infty, \mathcal{Z}^\infty)$.

Denote $h^t := \{z_0, z_1, \dots, z_t\}$ as a t -history of global shocks. Denote $\mu_t \in \mathcal{P}(Z^{t+1})$ as a time- t probability measure on the product Borel σ -algebra, $\times_{s=0}^t \mathcal{Z} := \mathcal{Z}^{t+1}$. Let $\mu_t(dh^t) := \mu_t(H)$ where $H \subset \mathcal{Z}^{t+1}$. Assume without loss of generality, $\mu_0(h^0) = \mu_0(z_0) = 1$.

2.2 Households

As in [McCallum and Nelson \[1999\]](#) or [Benigno and Thoenissen \[2008\]](#), individuals in our small open-economy have access only to a pair of domestic and foreign nominal un-contingent bonds denominated in their own currencies, respectively, B_t and B_t^* . More precisely, $B_{t+1}(h^t)$ or $B_{t+1}^*(h^t)$ denotes a claim on one unit of currency following, and irrespective of, any history h^{t+1} that may occur at $t+1$. Let $S_t(h^t)$ denote the nominal exchange rate, defined as the domestic currency price of a unit of foreign currency. In domestic currency terms, the prices of one unit of the nominal bonds $B_{t+1}(h^t)$ and $B_{t+1}^*(h^t)$ are, respectively, $1/[1+r_t(h^t)]$ and $S_t(h^t)/[1+r_t^*(h^t)]$.

The representative consumer in the domestic country faces the following sequential budget constraint, for each $t \in \mathbb{N}$, and each $h^t \in \mathcal{Z}^{t+1}$,

$$\begin{aligned} P_t(h^t)C_t(h^t) + \frac{B_{t+1}(h^t)}{1+i_t(h^t)} + \frac{S_t(h^t)B_{t+1}^*(h^t)}{1+i_t^*(h^t)} \\ \leq W_t(h^t)N_t(h^t) + B_t(h^{t-1}) + S_t(h^t)B_t^*(h^{t-1}) + \Pi_t(h^t), \end{aligned} \quad (1)$$

where P_t is the domestic consumer price indexes, C_t is a composite consumption index, i_t and i_t^* are the domestic and the foreign nominal interest rates, respectively; W_t is the nominal wage rate, N_t denotes the hours of labor supplied; and Π_t are the total nominal dividends received by the consumer from holding equal shares of the domestic firms.

A minor difference of our model to [Galí and Monacelli \[2005\]](#) and [McCallum and Nelson \[1999\]](#) is that consumers exhibit an endogenous discount factor that we denote by ρ_t . This assumption is introduced in order to ensure a unique nonstochastic steady-state consumption level, following [Schmitt-Grohe and Uribe \[2003\]](#).² However, this is not a fundamental assumption for our conclusions with respect to the endogenous monetary-policy trade off arising from the real-exchange-rate channel.³

²[Galí and Monacelli \[2005\]](#) assume the existence of an international market for complete state-contingent claims. In doing so, they thus avoid the problem of steady-state allocations being dependent on initial conditions. [McCallum and Nelson \[1999\]](#) assume incomplete markets which would mean the opposite for steady state consumption; but this issue is not discussed by the authors.

³Other ways of closing open-economy models are also discussed in [Schmitt-Grohe and Uribe \[2003\]](#). In our framework the most natural alternative could be to assume endogenous transaction cost in taking position in foreign bonds (see, e.g., [Benigno and Thoenissen \[2008\]](#)). The model with this alternative assumption would be analytically less tractable, and the equilibrium dynamics requires a specific law of motion for

The consumers' preferences are given by the following present-value total expected utility function:

$$\begin{aligned} \sum_{t=0}^{\infty} \rho_t \int_{Z^{t+1}} \{U [C_t (h^t)] - V [N_t (h^t)]\} d\mu_t(h^t) \\ := \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \rho_t \{U [C_t (h^t)] - V [N_t (h^t)]\} \right\}, \end{aligned} \quad (2)$$

where

$$\rho_t = \begin{cases} \beta (C_{t-1}^a (h^{t-1})) \rho_{t-1} & \text{for } t \in \mathbb{N} \setminus \{0\} \\ 1 & \text{for } t = 0 \end{cases}, \quad (3)$$

and C_t^a denotes the cross-economy average level of consumption.⁴

For concreteness, we will consider the following parametric form for the function $\beta : \mathbb{R}_+ \rightarrow (0, 1)$, following [Ferrero, Gertler, and Svensson \[2007\]](#):

$$\beta(C_t^a) = \frac{1}{1 + \phi (\ln C_t^a - \vartheta)}. \quad (4)$$

We do not impose a priori any condition on the sign of the dependence of the discount factor on average consumption, i.e., we only assume that $\beta'(C_t^a) \neq 0$.⁵

We also assume that per-period utility of consumption and labor have the respective forms,

$$U [C_t (h^t)] = \frac{C_t (h^t)^{1-\sigma}}{1-\sigma}, \quad V [N_t (h^t)] = \psi \left(\frac{N_t (h^t)^{1+\varphi}}{1+\varphi} \right),$$

where $\sigma > 0$, $\varphi > 0$, and $\psi > 0$.

The household chooses an optimal plan $\{C_t(h^t), N_t(h^t), B_{t+1}(h^t), B_{t+1}^*(h^t)\}_{t \in \mathbb{N}}$ to maximize (2) subject to (1). Unilaterally, the household will take the aggregate outcome $C_t^a(h^t)$, nominal prices $\{W_t(h^t), P_t(h^t), S_t(h^t)\}_{t \in \mathbb{N}}$ and policy $\{i_t(h^t), i_t^*(h^t)\}_{t \in \mathbb{N}}$ as fixed for each measurable h^t , $B_0(h^0)$ and $B_0^*(h^0)$ are known.

bonds. Our assumption will make clear that what is crucial for the policy trade off is just the incompleteness of financial markets, and not the random walk property of the asset/consumption dynamics implied by this incompleteness (in the absence of the endogenous discounting assumption).

⁴As in [Schmitt-Grohe and Uribe \[2003\]](#) we may also include average labor supply N_t^a as an argument of the function $\beta : \mathbb{R}_+ \rightarrow (0, 1)$. However, this would not affect the qualitative conclusions of the paper.

⁵There is a considerable disagreement over whether the instantaneous discount rate is an increasing or a decreasing function of consumption. On the one hand, authors like, for instance, [Blanchard and Fischer \[1989\]](#), find counterintuitive that people would be more impatient as the levels of consumption rise. On the other hand, [Epstein \[1987\]](#) argues that the proper interpretation of a discount rate as considered in this paper is that individuals who know that will have a large level of consumption in the future evaluate current consumption more highly. In any case, this assumption is not relevant for our results, so that we do not care about this debate.

Denote a measurable selection as $X_t(h^t) =: X_t$. Define the real exchange rate as $Q_t := S_t P_t^* / P_t$. Given the functional forms, the respective first order conditions of the household's problem, for each $h^t \in \mathcal{Z}^{t+1}$ and $t \in \mathbb{N}$, are:

$$-\frac{U_C(C_t)}{V_N(N_t)} = \frac{W_t}{P_t} \quad \Rightarrow \quad \psi N_t^\varphi C_t^\sigma = \frac{W_t}{P_t}, \quad (5)$$

$$1 = (1 + i_t) \mathbb{E}_t \left\{ \beta (C_{t+1}^a) \left(\frac{P_t}{P_{t+1}} \right) \frac{U_C(C_{t+1})}{U_C(C_t)} \right\} \\ \Rightarrow C_t^{-\sigma} = (1 + i_t) \mathbb{E}_t \left\{ \beta (C_{t+1}^a) \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma} \right\}, \quad (6)$$

$$1 = (1 + i_t^*) \mathbb{E}_t \left\{ \beta (C_{t+1}^a) \left(\frac{P_t^* Q_{t+1}}{P_{t+1}^* Q_t} \right) \frac{U_C(C_{t+1})}{U_C(C_t)} \right\} \\ \Rightarrow C_t^{-\sigma} = (1 + i_t^*) \mathbb{E}_t \left\{ \beta (C_{t+1}^a) \left(\frac{P_t^* Q_{t+1}}{P_{t+1}^* Q_t} \right) C_{t+1}^{-\sigma} \right\}. \quad (7)$$

Each optimally chosen C_t will be consistent with the household's intra-period choice of a home-produced final consumption good, $C_{H,t}$ and an imported final good $C_{F,t}$, where C_t is defined by a CES aggregator

$$C_t = \left[(1 - \gamma)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}; \quad \gamma \in (0, 1), \eta > 0. \quad (8)$$

Furthermore, each type of final good, $C_{H,t}$ and $C_{F,t}$, are aggregates of a variety of differentiated goods indexed by $i, j \in [0, 1]$. Respectively, these aggregates are

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{F,t} = \left[\int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}; \quad \varepsilon > 0 \quad (9)$$

As is well known from [Galí and Monacelli \[2005\]](#), optimal allocation of the household expenditure across each good type gives rise to the demand functions:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}, \quad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \quad (10)$$

for all $i, j \in [0, 1]$, where the aggregate price levels are solved as

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, \quad P_{F,t} = \left(\int_0^1 P_{F,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \quad (11)$$

Likewise, optimal consumption demand of final home and foreign goods can be derived, respectively, as

$$C_{H,t} = (1 - \gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t.$$

Substitution of these demand functions into (8) yields the consumer price index as

$$P_t = \left[(1 - \gamma) P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \quad (12)$$

3 Differentiated goods technology and pricing

In this section, we make our first departure from the Galí and Monacelli [2005] setup. Each domestic firm $i \in [0, 1]$ produces a differentiated good and faces a demand curve as shown in (10). Following McCallum and Nelson [1999], we consider that production uses the CES technology

$$Y_t(i, h^t) = \left\{ \alpha \left[A_t N_t^d(i, h^t) \right]^\nu + (1 - \alpha) \left[IM_t(i, h^t) \right]^\nu \right\}^{\frac{1}{\nu}}; \quad \alpha \in (0, 1], -\infty \leq \nu \leq 1, \quad (13)$$

where $N_t(i)$ is labor hired by the firm and $IM_t(i)$ is an index of imported intermediate goods.⁶ The random variable $A_t := \exp\{a_t\}$ is an exogenous embodied labor productivity; $\alpha \in (0, 1]$ measures the productive dependence of the domestic economy; and $1/(1 - \nu)$ is the elasticity of substitution between labor and the imported intermediate good. Note that in the limit as $\nu \rightarrow 0$, we have a Cobb-Douglas function. The law of one price and the limiting closed-economy assumption for the rest of the world ensures that the price of IM_t is equal to $S_t P_t^*$.

The cost-minimization problem for the firm's production is

$$\min P_t(h^t) \left\{ Q_t(h^t) IM_t(i, h^t) + \frac{W_t(h^t)}{P_t(h^t)} N_t^d(i, h^t) \right\}$$

subject to (13).

The first-order conditions with respect to optimal labor and intermediate-imports are

$$\frac{W_t(h^t)}{P_t(h^t)} = \alpha \frac{MC_t^n(h^t)}{P_t(h^t)} A_t^\nu \left(\frac{Y_t(h^t)(i, h^t)}{N_t^d(i, h^t)} \right)^{1-\nu}$$

and

$$Q_t(h^t) = (1 - \alpha) \frac{MC_t^n(h^t)}{P_t(h^t)} \left(\frac{Y_t(i, h^t)}{IM_t(i, h^t)} \right)^{1-\nu},$$

respectively, where MC_t^n is nominal marginal cost. With a homogeneous of degree one

⁶This intermediate good can be interpreted as two equivalent forms. On the one hand, we can assume that the imported goods can be either devoted to consumption $C_{F,t}$ or used as a production input IM_t . On the other hand, we can assume that the domestic economy imports two differentiated goods: consumption goods $C_{F,t}$ and intermediated goods IM_t .

production function, the same first-order conditions also hold in the aggregate:

$$\frac{W_t(h^t)}{P_t(h^t)} = \alpha \frac{MC_t^n(h^t)}{P_t(h^t)} A_t^\nu \left(\frac{Y_t(h^t)}{N_t^d(h^t)} \right)^{1-\nu} \quad (14)$$

and

$$Q_t(h^t) = (1 - \alpha) \frac{MC_t^n(h^t)}{P_t(h^t)} \left(\frac{Y_t(h^t)}{IM_t(h^t)} \right)^{1-\nu}, \quad (15)$$

respectively.

Since the firm $i \in [0, 1]$ is assumed to be imperfectly competitive, it gets to set an optimal price $P_{H,t}(i, h^t)$ given a Calvo-style random time-independent signal to do so. With a per-period probability $(1 - \theta)$ the firm gets to reset price.

As this is quite a standard model in the literature, we relegate the derivation to Appendix A which shows that the firm will set a dynamic and stochastic price markup over marginal cost. This is characterized by the first-order condition:

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \left(\prod_{\tau=t}^{t+k-1} \beta(C_\tau^a) \right) \frac{\lambda_{t+k}}{\lambda_t} Y_{t+k}(i) \left[\tilde{P}_{H,t}(i) - \left(\frac{\varepsilon}{\varepsilon - 1} \right) MC_{t+k}^n \right] \right\} = 0, \quad (16)$$

where $\lambda_t := U_C[C_t(h^t)]$, and the demand faced by the firm at some time $t + k$ (and following history h^{t+k}), conditional on the firm maintaining a sale price of $\tilde{P}_{H,t}(i)$ is

$$Y_{t+k}(i) = \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+k}} \right)^{-\varepsilon} [C_{H,t+k} + C_{H,t+k}^*]. \quad (17)$$

In a symmetric pricing equilibrium, where $\tilde{P}_{H,t} := \tilde{P}_{H,t}(h^t) = \tilde{P}_{H,t}(i, h^t)$, the law of motion for the aggregate price is

$$P_{H,t} = \left[\theta P_{H,t-1}^{1-\varepsilon} + (1 - \theta) \tilde{P}_{H,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

4 Competitive equilibrium

In a competitive equilibrium we require that given monetary policy and exogenous processes, the decisions of households and firms are optimal, as characterized earlier, and that markets clear. First, the labor market must clear, so that (5) equals (36) for all states and dates. Second, the final Home-produced goods market for each variety $i \in [0, 1]$ clears so that:

$$Y_t(i, h^t) = C_{H,t}(i, h^t) + C_{F,t}(i, h^t). \quad (18)$$

Third, the no-arbitrage condition for international bonds will be given by the equality of (6) and (7). In the rest of the world, assumed to be the limiting case of a closed economy, we have market clearing as $Y_t^* = C_t^*$.

5 First-order equilibrium dynamics

In this section we characterize the linearized equilibrium dynamics of our small open-economy conditional on the process of the nominal interest rate i_t , and the exogenous processes $\{y_t^*, \pi_t^*, a_t\}$. To this end, consider the gaps of the aggregate variables with respect to their potential level in an equilibrium with fully flexible domestic prices (*i.e.* when the marginal cost mc_t is zero at any time t and in any state).

Denote \bar{y}_t and \bar{q}_t as the levels of output and real exchange rate, respectively, at the flexible-price equilibrium. Let \tilde{x}_t and \tilde{q}_t denote the domestic output gap and the real exchange rate gap (in percentage deviation), respectively, where $\tilde{x}_t = y_t - \bar{y}_t$ and $\tilde{q}_t = q_t - \bar{q}_t$. In Appendix B we prove that the levels of both \bar{y}_t and \bar{q}_t only depend on exogenous variables. We will show that the equilibrium dynamics can be fully approximated to first-order accuracy as a system of stochastic dynamic equations on \tilde{x}_t , $\pi_{H,t}$ and \tilde{q}_t .

5.1 Approximating demand side and UIP

By log linearizing the previous conditions (5), (6) and (7), and by imposing the equilibrium condition $C_t = C_t^a$, we obtain in Appendix B that the optimal consumer's plan approximately satisfies:⁷

$$\sigma c_t = w_t - p_t - \phi n_t, \quad (19)$$

and

$$-(\sigma - \phi) c_t = i_t - \mathbb{E}_t \{\pi_{t+1}\} - \sigma \mathbb{E}_t \{c_{t+1}\} + \phi \vartheta, \quad (20)$$

and

$$-(\sigma - \phi) c_t = r_t^* - q_t + \mathbb{E}_t \{q_{t+1}\} - \sigma \mathbb{E}_t \{c_{t+1}\} + \phi \vartheta, \quad (21)$$

where q_t denotes the logarithm of the real exchange rate Q_t ; π_{t+1} is the rate of change in the domestic consumer price index from t to $t + 1$, *i.e.*, $\pi_{t+1} = p_{t+1} - p_t$; and r_t^* is the foreign interest rate, *i.e.*, $r_t^* = i_t^* - \mathbb{E}_t \{\pi_{t+1}^*\}$, with π_{t+1}^* denoting the inflation rate of the foreign consumer price index. In order to maintain the same sign of slope of the Euler equation as in Galí and Monacelli [2005] we will assume that $\sigma - \phi > 0$. The latter

⁷From now on, a lower-case letters will denote the logarithms of the corresponding upper-case variables.

condition ensures that the our assumption on the subjective discount factor is only a technical issue without any qualitative consequences on the monetary policy problem.

The log-linear form for the consumption index (8) is given by:

$$c_t = (1 - \gamma) c_{H,t} + \gamma c_{F,t}, \quad (22)$$

where $c_{H,t}$ is also an index of consumption of domestically produced goods; $c_{F,t}$ is an index of imported goods; and $\gamma \in [0, 1]$ measures the degree of openness. The domestic consumer price index (12) is log-linearized as

$$p_t = (1 - \gamma) p_{H,t} + \gamma p_{F,t}, \quad (23)$$

where $p_{H,t}$ is the price index for domestically produced goods; and $p_{F,t}$ is the price index for imported goods. Similarly, domestic output in log-linear form is given in equilibrium by

$$y_t = (1 - \gamma) c_{H,t} + \gamma c_{H,t}^*, \quad (24)$$

where $c_{H,t}^*$ is the export of domestically produced goods. The optimal allocation of expenditure within each category of goods yields the following demands:

$$c_{H,t} = \eta (p_t - p_{H,t}) + c_t, \quad (25)$$

$$c_{F,t} = \eta (p_t - p_{F,t}) + c_t, \quad (26)$$

$$c_{H,t}^* = \eta (s_t + p_t^* - p_{H,t}) + c_t^*. \quad (27)$$

We assumed that the law of one price holds at all periods, so that $p_{F,t} = s_t + p_t^*$. Combining this expression with the consumption price index (23) and the definition of the real exchange rate, we obtain that

$$p_t - p_{H,t} = \left(\frac{\gamma}{1 - \gamma} \right) q_t. \quad (28)$$

From this equation we directly follow that the rate of change in the price index of the domestically produced goods (or domestic inflation) differs from the inflation rate of the domestic consumer price index. In particular we obtain that

$$\pi_t - \pi_{H,t} = \left(\frac{\gamma}{1 - \gamma} \right) (q_t - q_{t-1}), \quad (29)$$

where $\pi_{H,t}$ is the domestic inflation rate. Observe that the gap between the two inflation rates is proportional to the variation in the real exchange rate, with the degree of proportionality determined by the degree of openness γ .

By combining (28) with (24), (25) and (27), we obtain that domestic output is equal to

$$y_t = (1 - \gamma) c_t + \left[\frac{\eta\gamma(2 - \gamma)}{1 - \gamma} \right] q_t + \gamma y_t^*, \quad (30)$$

where we have used the fact that the foreign economy is large, which implies that the weight in the foreign composite consumption C_t^* of the domestic goods is negligible, so that $y_t^* = c_t^*$.

From the optimality conditions for the consumers' problem we can derive the uncovered interest parity (UIP) condition. More specifically, from the conditions (20) and (21) we obtain the following log-linear approximation of the UIP condition:

$$i_t = r_t^* + \mathbb{E}_t \{ \pi_{t+1} \} - q_t + \mathbb{E}_t \{ q_{t+1} \}. \quad (31)$$

Under complete markets the Euler equations of the domestic and the foreign countries gives a tight state-by-state and date-by-date relation linking domestic consumption with foreign consumption and real exchange rate[see. e.g. Galí and Monacelli, 2005]. In particular, under our utility function (2) this relationship is in log-linear terms given by: $c_t = c_t^* + (\frac{1}{\sigma}) q_t$. This implies that the dynamics of output and the real exchange rate are perfectly correlated. However, this is no longer true in the case of incomplete asset markets. As Chari, Kehoe, and McGrattan [2002] show, in this case the relation between real exchange rate and consumption only holds in conditional expectation terms. In particular, combining condition (21) with the Euler equation of the foreign economy yields

$$\mathbb{E}_t \{ q_{t+1} \} - q_t = [\sigma \mathbb{E}_t \{ y_{t+1}^* \} - (\sigma - \phi) y_t^*] - [\sigma \mathbb{E}_t \{ c_{t+1} \} - (\sigma - \phi) c_t], \quad (32)$$

where we have also used the condition $y_t^* = c_t^*$. Therefore, output and real exchange are not perfect correlated, so that we need equation (31) to characterize the full equilibrium dynamics of our model as we will show below [see also Benigno, 2008; Benigno and Thoenissen, 2008].

Using (31) and (29), we obtain the following dynamic equation for the gap in the real exchange rate:

$$\tilde{q}_t = \mathbb{E}_t \{ \tilde{q}_{t+1} \} - (1 - \gamma) [i_t - \mathbb{E}_t \{ \pi_{H,t+1} \}] + u_t, \quad (33)$$

where $u_t = (1 - \gamma) r_t^* + \mathbb{E}_t \{ \bar{q}_{t+1} \} - \bar{q}_t$ is shown to be an exogenous variable that captures variations in the foreign real interest rate and the underlying dynamics of the real exchange rate in the flexible-price version of the economy.

Combining (20) with (29), (30) and (33), and after some tedious algebra, we obtain in

Appendix B that

$$\tilde{x}_t = \omega \mathbb{E}_t \{ \tilde{x}_{t+1} \} - \mu [i_t - \mathbb{E}_t \{ \pi_{H,t+1} \}] + \chi \mathbb{E}_t \{ \tilde{q}_{t+1} \} + \epsilon_t, \quad (34)$$

where

$$\begin{aligned} \omega &= \frac{\sigma}{\sigma - \phi}, \\ \mu &= \left[\frac{1 - \gamma}{\sigma - \phi} \right] \left[1 - \gamma + \frac{\eta \gamma (2 - \gamma) (\sigma - \phi)}{1 - \gamma} \right], \\ \chi &= \frac{\eta \gamma \phi (2 - \gamma)}{(1 - \gamma) (\sigma - \phi)}, \end{aligned}$$

and ϵ_t is a stochastic, exogenous variable that is just a linear function of shocks in the the rest of the world and the domestic productivity. The previous equation is a forward looking IS-type equation that represents the sensitivity of the output gap to changes in the real interest rate.

However, in contrast to the canonical model in Galí and Monacelli [2005], movements in the real exchange rate in our model also affects the output gap directly. In our small open-economy the real exchange rate indirectly affects the output gap by generating changes in the real interest rate faced by domestic consumers. The depreciation in the real exchange rate raises the consumer price index because: (i) it increases the price of the imported consumption goods; and (ii) increases export demand, which results into a rise in the price of domestically produced goods. This increase in the consumer price index reduces the expected rate of inflation for a given expected future price level. This leads to an increase in the real interest rate facing consumers and, therefore, a decrease in consumption and output for a given the expected future exchange rate.

Furthermore, observe that movements in the expected real exchange rate in our model also affect the output gap because those by directly modifying the marginal rate of substitution of consumption between different periods and across states. This effect comes from the endogenous discount factor, $\beta(C_t^a)$, so that changes in the aggregate demand also affects the discounting path. This direct effect does not arise when we assume $\phi = 0$ because in this case $\chi = 0$. Hence, this is the main qualitative consequence of our assumption on the discount rate. In any case, the effects of real exchange rate on output gap depends on the degree of openness γ . However, the productive dependence given by δ does not determines the sensitivity of output gap to changes in real exchange rate because this parameter only determines the supply-side channel.

Finally, we can see that the elasticity of the discounting rate given by ϕ affects to size of the impacts from the expectation on future output gap and from the changes in the net domestic interest rate. However, under the assumptions in our model this parameter

does not alter the sign of these two effects.

5.2 Approximating the supply side

Log-linearizing (13) around the zero-inflation steady state, we obtain that the aggregate output in logarithmic terms is given by

$$y_t = (1 - \delta)(a_t + n_t) + \delta im_t, \quad (35)$$

with $\delta = (1 - \alpha)(IM_{ss}/Y_{ss})^\nu$, where IM_{ss} and Y_{ss} are the stationary values of imported intermediate good and aggregate output, respectively. Moreover, from cost minimization we obtain that the demand of both inputs are given in logarithmic terms as⁸

$$n_t = y_t + \left(\frac{\nu}{1 - \nu}\right) a_t + (1 - \nu)^{-1} [mc_t - (w_t - p_t) + (p_{H,t} - p_t)], \quad (36)$$

$$im_t = y_t + (1 - \nu)^{-1} [mc_t - q_t + (p_{H,t} - p_t)], \quad (37)$$

where $mc_t := \ln(MC_t^n/P_{H,t}) - \ln[(1 - \varepsilon)/\varepsilon]$ is the percentage deviation in real marginal cost.

In Appendix A, we solve the firms' problem consisting on maximizing the expected discounted value of profits, and we obtain from this solution that the dynamics of the domestic inflation around the zero-inflation steady-state can be approximated by the log-linear dynamic equation:

$$\pi_{H,t} = \widehat{\beta} \mathbb{E}_t \{ \pi_{H,t+1} \} + \left[\frac{(1 - \theta)(1 - \theta \widehat{\beta})}{\theta} \right] mc_t, \quad (38)$$

with $\widehat{\beta} = \beta(C_{ss})$, where C_{ss} is the level of domestic consumption at the zero-inflation steady state. Combining (38) with (19), (30), (35), (36) and (37), we obtain in Appendix B.2 that

$$\pi_{H,t} = \widehat{\beta} \mathbb{E}_t \{ \pi_{H,t+1} \} + \lambda (\kappa_1 \tilde{x}_t + \kappa_2 \tilde{q}_t), \quad (39)$$

where

$$\lambda = \left[\frac{(1 - \theta)(1 - \theta \widehat{\beta})}{\theta} \right] \left[\frac{(1 - \nu)(1 - \delta)}{(1 - \delta)(1 - \nu) + \delta(1 - \nu + \delta\varphi)} \right],$$

⁸I am not sure whether or not we must give present the minimization problem and the conditions before the linearization. However, the solution of the problem are in McCallum, although without labor.

$$\kappa_1 = \left[\frac{\sigma + \varphi(1 - \gamma)}{1 - \gamma} \right],$$

and

$$\kappa_2 = \left[\frac{\gamma(1 - \delta)^2(1 - \nu) + (1 - \gamma)\delta(1 - \nu + \varphi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} + \frac{\sigma\eta\gamma^2}{(1 - \gamma)^2} - \frac{2\sigma\eta\gamma}{(1 - \gamma)^2} \right].$$

Equation (39) is an augmented New Keynesian Phillips curve representing the dynamics of the short-run aggregate supply. This curve relates the domestic inflation to the output gap and the real exchange rate gap. Observe that, in contrast with the standard open-economy model, in our model it is not necessary to introduce an ad-hoc "cost-push shock" in order to make the monetary policy trade-offs non-trivial.⁹ In our model the marginal cost is not fully determined by the output gap because that cost also depends on the real exchange rate as is shown in Appendix B. Moreover, as (33) shows, the dynamics of the real exchange rate depends on the exogenous stochastic variable u_t for any given path of the nominal interest rate i_t . Therefore, the marginal cost also moves with the foreign real interest rate and the domestic and foreign productivity shocks, which generates the necessary trade-offs between \tilde{x}_t and $\pi_{H,t}$.

This feature of our model derives directly from the incompleteness of the financial markets and does not rely on additional price stickiness in the additional imported goods sector as in Monacelli [2005]. As was shown above, under complete asset markets the dynamics of the real exchange rate is fully given by the evolution of output. Hence, in this case the New Keynesian Phillips curve relates the inflation rate only with the output gap. This is the reason why some ad-hoc cost-push shock must be introduced in the model with complete markets to generate short-run trade-offs between \tilde{x}_t and $\pi_{H,t}$.

In our model, the direct link between real exchange rate movements and the real marginal cost can be dissected into three effects. These correspond to the three terms in the composite parameter κ_2 . First, an increase in the real exchange rate (or an exchange rate depreciation) increases the prices of the imported consumption goods faced by domestic consumers. This first effect has a substitution and an income effect on the marginal cost and the domestic inflation.

On the one hand, this increase in the price of imported consumption goods leads consumers to reduce the demand of these goods and therefore to reduce aggregate consumption and to increase leisure. This translates into an increase in the marginal product of labor that drives the marginal cost up. This negative effect of the depreciation in the real exchange rate is given by the second term in κ_2 . On the other hand, this increase in the price of imported consumption goods reduces the real wage rate facing by consumers, who react by increasing labor supply to compensate the reduction in the purchasing power of

⁹See Clarida et al. (1999) for a detailed discussion on this ad-hoc cost-push term.

their given income. This leads to a reduction in marginal product of labor that pushes the marginal cost down. This positive effect of the depreciation in the real exchange rate is given by the third term in κ_2 . Observe that the substitution effect dominates, so that the net effect of the increase in the price of the imported consumption goods on marginal cost is always negative.

Furthermore, exchange rate depreciation in our model increases the unit price of the imported intermediate good, which obviously drives the marginal cost inflation up. This other effect of real exchange rate on the domestic inflation rate is given by the first term in the value of κ_2 .

Therefore, the overall impact of the real exchange rate on the domestic inflation (i.e., the sign of κ_2) is then ambiguous, and it depends on the degree of openness γ and the productive dependence α (through their effect on δ).

As in the closed economy a change in the domestic output has a positive effect on the domestic inflation rate. In our model the size of this effect depends on the degree of openness γ and on the productive dependence α (or, equivalently, δ). An increase in the domestic output has an effect on the marginal cost through: (i) an increase in the equilibrium level of employment, that is captured by the first term φ in κ_1 ; (ii) a rise in the terms of trade given by the second term in κ_1 , which is negatively related with the degree of openness γ ; and (iii) an increase in the equilibrium level of the imported intermediate good given by the second term in λ , which is positively related with the productive dependence α (or, equivalently, δ).

5.3 Discussion

Given a process for the domestic nominal interest rate i_t , the set of paths $\{\tilde{x}_t, \pi_{H,t}, \tilde{q}_t\}$ solving the forward-looking dynamic system (33), (39) and (34), gives a first-order approximation of the equilibrium dynamics of the small open-economy.

To reiterate, in our model the dynamics of the real exchange rate is a fundamental variable is describing the equilibrium process of the small open economy. Movements in the real exchange rate affects output gap by altering the real interest rate as is the case of the canonical open-economy model. However, in our model these movements in the exchange rate also affect directly the output gap (as an artefact of endogenous discounting) and domestic inflation rate (via the two trade channels and market incompleteness).

5.4 The canonical new-Keynesian model as special case

Observe that these direct effects of real exchange rate in the dynamics of the domestic inflation (39) through marginal cost disappear either when $\delta = \gamma = 0$ or, as was explained before, when the financial markets are complete. Furthermore, if $\phi = 0$, then there is no

direct real exchange rate channel in the IS relation (34) as well. We summarize this in the following observation.

Proposition 1 *If the economy: (i) does not rely on imported intermediate inputs and final consumption goods, $\delta = \gamma = 0$, and (ii) thus endogenous discounting is an irrelevant assumption, so that without loss of generality, $\phi = 0$, then, the model is equivalent to the canonical new-Keynesian two equation model.*

6 Monetary policy implications

We begin with a simple observation that the model no longer inherits an isomorphic monetary policy design problem as in its closed economy counterpart. The competitive equilibrium now is characterized by the approximate system:

$$\pi_{H,t} = \widehat{\beta} \mathbb{E}_t \{ \pi_{H,t+1} \} + \lambda (\kappa_1 \tilde{x}_t + \kappa_2 \tilde{q}_t), \quad (40)$$

$$\tilde{x}_t = \omega \mathbb{E}_t \{ \tilde{x}_{t+1} \} - \mu [i_t - \mathbb{E}_t \{ \pi_{H,t+1} \}] + \chi \mathbb{E}_t \{ \tilde{q}_{t+1} \} + \epsilon_t, \quad (41)$$

$$\tilde{q}_t = \mathbb{E}_t \{ \tilde{q}_{t+1} \} - (1 - \gamma) [i_t - \mathbb{E}_t \{ \pi_{H,t+1} \}] + u_t \quad (42)$$

where ϵ_t and u_t are exogenous processes.

6.1 Endogenous monetary policy trade-off

In the canonical model of Galí and Monacelli [2005], an equivalent exogenous variation in ϵ_t can be fully offset by the monetary policy instrument, i_t , thus removing any impact on output gap \tilde{x}_t . Hence the non-existence of any policy trade-off between domestic producer price inflation $\pi_{H,t}$ and output gap \tilde{x}_t . This is also the case, qualitatively, in the closed economy canonical model [see e.g. Woodford, 2003] and in our nested cases when $\delta = \gamma = 0$ and $\phi = 0$. Typically, to ensure that there is a relevant monetary policy trade-off, the literature using the canonical model would introduce exogenous “cost-push” shocks [see e.g. Woodford, 2003] to the special case of the Phillips equation (40).

In this model, bond market incompleteness results in the equilibrium relation (42) and also the explicit linkage between the real exchange rate and real marginal cost, and therefore, the domestic producer price inflation in (40). The resulting monetary policy implication is that any exogenous variation encapsulated in ϵ_t (i.e. foreign output, foreign real interest rate, or domestic labor productivity) cannot, now, be fully offset by changing i_t , since this will also affect the real exchange rate via (42) and domestic inflation via (40).

7 Concluding remarks

In this paper, we have developed a tractable version of a small open economy whose monetary policy implications are no longer similar to its closed-economy counterpart. In ongoing work, we aim to show that this model results in a consumer welfare-consistent approximation for the monetary policy loss function that requires the real exchange rate to be an argument, in addition to domestic producer price inflation and the output gap arguments. In a related paper [Alonso-Carrera and Kam, 2009], we also study the stability properties for optimal monetary policy when both the monetary authority and the private agents have to statistically learn about the equilibrium probabilistic laws of the economy.

Appendix

A Optimal price-setting

We assumed in the paper a function $\mathbb{R}_+ \ni C_t^a \mapsto \beta(C_t^a) \in (0, 1)$ that represents endogenous (aggregate consumption dependent) one-period discounting. Note that for a fixed sequence of aggregate consumption $\{C_\tau^a\}_{\tau=0}^{k-1}$ the k -period ahead endogenous discount factor is

$$\prod_{\tau=0}^{k-1} \hat{\beta}_{\tau+1} = \prod_{\tau=0}^{k-1} \beta(C_\tau^a),$$

where $\hat{\beta}_{t+1} = \beta(C_t^a)$ and $\beta_0 = 1$.

A.1 The Calvo-style pricing problem

The monopolistically competitive firm $i \in [0, 1]$ chooses pricing strategy $\{P_{H,t}(i, h^t)\}$. Given Markovian public histories of states, $h^t := \{z_0, \dots, z_t\}$, we further restrict the optimal pricing strategy to belong to a class induced by stationary pricing decision functions $\tilde{P}_{H,t}$ such that $\tilde{P}_{H,t}(i, z_t[h^t]) = \tilde{P}_H(i, z_t)$.

Let the aggregate price in the home goods industry H , in time $t + k$ and state z_{t+k} , be $P_{H,t+k}(z_{t+k}) := P_{H,t+k}$. The aggregate domestic and foreign consumption demands are, respectively, $C_{H,t+k}(z_{t+k}) := C_{H,t+k}$ and $C_{H,t+k}^*(z_{t+k}) := C_{H,t+k}^*$. The output of firm i in that state and date is then $Y_{t+k}(i, z_{t+k}) := Y_{t+k}(i)$. The demand faced by firm i in state z_{t+k} , if the price $\tilde{P}_{H,t}(i, z_t) := \tilde{P}_H(i)$ still prevails (with probability θ^k) in period $t + k$ is

given by

$$Y_{t+k}(i) = \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+k}} \right)^{-\varepsilon} [C_{H,t+k} + C_{H,t+k}^*], \quad (43)$$

where we have suppressed the explicit state-dependence notation.

Let $\lambda_t := U_C[C(z_t)]$. The firm i solves the following expected total discounted nominal profit maximization problem:

$$\max_{\tilde{P}_{H,t}(i)} \sum_{k=0}^{\infty} \mathbb{E}_t \left\{ \left(\prod_{\tau=t}^{t+k-1} \beta(C_{\tau}^a) \right) \frac{\lambda_{t+1}}{\lambda_t} [P_{H,t}(i) - MC_{t+k}^n] \right\}, \quad (44)$$

subject to (43).

A.2 Optimal pricing plan in a symmetric pricing equilibrium

The first order condition with respect to $\tilde{P}_{H,t}(i)$ is

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \left(\prod_{\tau=t}^{t+k-1} \beta(C_{\tau}^a) \right) \frac{\lambda_{t+k}}{\lambda_t} Y_{t+k}(i) \left[\tilde{P}_{H,t}(i) - \left(\frac{\varepsilon}{\varepsilon - 1} \right) MC_{t+k}^n \right] \right\} = 0. \quad (45)$$

We re-write the problem in real terms, by dividing both sides of (45) with the aggregate home-goods price level, $P_{H,t}$:¹⁰

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \left(\prod_{\tau=t}^{t+k-1} \beta(C_{\tau}^a) \right) \frac{\lambda_{t+k}}{\lambda_t} Y_{t+k}(i) \left[\frac{\tilde{P}_{H,t}(i)}{P_{H,t}} - \left(\frac{\varepsilon}{\varepsilon - 1} \right) \left(\frac{P_{H,t+k}}{P_{H,t}} \right) MC_{t+k} \right] \right\} = 0. \quad (46)$$

where real marginal cost in time $t + k$ and state z_{t+k} is

$$MC_{t+k} =: \frac{MC_{t+k}^n}{P_{H,t+k}}.$$

We restrict attention to a symmetric pricing equilibrium.

Definition 1 A symmetric pricing equilibrium is a stationary Markovian strategy $\{\tilde{P}_{H,t}(i, z_t)\}_{t=0}^{\infty} = \{\tilde{P}_{H,t}(z_t)\}_{t=0}^{\infty}$ for all firms $i \in [0, 1]$ that re-set prices at time t , and $\{\tilde{P}_{H,t}(z_t)\}_{t=0}^{\infty}$ satisfies (46).

A.3 Log-linear approximation of optimal pricing policy function

For small perturbations around a zero-inflation steady state, (46) can be approximated to first-order accuracy as:

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} [\beta(C_{ss}^a) \theta]^k \lambda_{ss} Y_{ss}(i) \left[\tilde{p}_{H,t} - \ln \left(\frac{P_{H,t+k}}{P_{H,t}} \right) - mc_{t+k} \right] \right\}, \quad (47)$$

¹⁰This also renders the characterization of optimal pricing in terms of stationary variables.

where

$$\begin{aligned}\tilde{p}_{H,t} &:= \ln\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right), \\ mc_t &:= \ln\left(\frac{MC_t^n}{P_{H,t}}\right) - \ln\left(\frac{\varepsilon - 1}{\varepsilon}\right).\end{aligned}$$

Define $\hat{P}_{H,t} = \ln(P_{H,t}/P_{H,ss})$. So then (47) can be re-written as

$$\begin{aligned}\left(\frac{1}{1 - \beta(C_{ss}^a)\theta}\right)\tilde{p}_{H,t} &= \mathbb{E}_t \sum_{k=0}^{\infty} [\beta(C_{ss}^a)\theta]^k [\hat{P}_{H,t+k} + mc_{t+k}] - \left(\frac{1}{1 - \beta(C_{ss}^a)\theta}\right)\hat{P}_{H,t} \\ \Rightarrow \tilde{p}_{H,t} + \hat{P}_{H,t} &= (1 - \beta(C_{ss}^a)\theta) \mathbb{E}_t \sum_{k=0}^{\infty} [\beta(C_{ss}^a)\theta]^k [\hat{P}_{H,t+k} + mc_{t+k}].\end{aligned}$$

By the law of iterated expectations (with respect to the σ -algebra generated by the approximately linear model), this can be written recursively as

$$\begin{aligned}\tilde{p}_{H,t} + \hat{P}_{H,t} &= [1 - \beta(C_{ss}^a)](\hat{P}_{H,t} + mc_t) + [\beta(C_{ss}^a)\theta] \mathbb{E}_t(\tilde{p}_{H,t+1} + \hat{P}_{H,t+1}) \\ \Rightarrow \tilde{p}_{H,t} &= [1 - \beta(C_{ss}^a)\theta] mc_t + [\beta(C_{ss}^a)\theta] \mathbb{E}_t(\tilde{p}_{H,t+1} + \pi_{H,t+1}).\end{aligned}\quad (48)$$

Recall that in the [Calvo \[1983\]](#)-style model, by the law of large numbers, the evolution of the aggregate home goods price level is such that

$$P_{H,t}^{1-\varepsilon} = (1 - \theta)\tilde{P}_{H,t}^{1-\varepsilon} + \theta P_{H,t-1}^{1-\varepsilon}.$$

A log-linear approximation of this yields a linear relationship between home goods price inflation, $\pi_{H,t} := \ln(P_{H,t}/P_{H,t-1})$ and the percentage deviation of the optimal price from the aggregate price level:

$$\pi_{H,t} = \frac{1 - \theta}{\theta} \tilde{p}_{H,t}.$$

Using this identity, and the one-period ahead forecast of the same identity, in (48), we obtain the characterization of the approximately log-linear optimal pricing function $\tilde{p}_{H,t}$ in terms of a forward-looking Phillips curve:

$$\pi_{H,t} = \beta(C_{ss}^a) \mathbb{E}_t \pi_{H,t+1} + \frac{[1 - \beta(C_{ss}^a)\theta](1 - \theta)}{\theta} mc_t.\quad (49)$$

Equation (49) is almost identical, to first-order accuracy, to the Phillips curve derived in models such as [Galí and Monacelli \[2005\]](#) and [Clarida, Galí, and Gertler \[2001\]](#). What

is different is that the one-period discount factor at the deterministic steady state is now a function of fundamental outcomes in the steady-state competitive equilibrium. In this case, it will be a function of steady-state aggregate consumption, C_{ss} . More generally, if we do not focus on local linearized dynamics around the deterministic steady state, the global dynamics of inflation will also depend non-linearly on the endogenous discount factor function $\beta : \mathbb{R}_+ \rightarrow (0, 1)$.

B Deriving the log-linear three-equation equilibrium system

In this appendix we provide the details of the log-linearization of the three key conditions that locally approximate the equilibrium characterizations of recursive equilibrium in the model.

B.1 Equilibrium with flexible prices

We first characterize the equilibrium when prices are fully flexible, i.e., $mc_t = 0$. From (19), (35), (36) and (37), we obtain that the equilibrium with flexible prices

$$\sigma \bar{c}_t + \varphi \bar{y}_t + \left[\frac{\gamma(1-\delta)^2(1-\nu) + (1-\gamma)\delta(1-\nu+\varphi)}{(1-\nu)(1-\delta)^2(1-\gamma)} \right] \bar{q}_t - (1+\varphi) a_t = 0. \quad (50)$$

Moreover, combining the last equation with (30), we also obtain

$$\bar{c}_t = \Omega_1 a_t - \Omega_2 \bar{q}_t - \Omega_3 y_t^*, \quad (51)$$

where

$$\Omega_1 = \frac{1+\varphi}{\sigma + \varphi(1-\gamma)},$$

$$\Omega_2 = \left[\frac{1}{\sigma + \varphi(1-\gamma)} \right] \left[\frac{\eta\gamma\varphi(2-\gamma)}{1-\gamma} + \frac{\gamma(1-\delta)^2(1-\nu) + (1-\gamma)\delta(1-\nu+\varphi)}{(1-\nu)(1-\delta)^2(1-\gamma)} \right],$$

and

$$\Omega_3 = \frac{\gamma\varphi}{\sigma + \varphi(1-\gamma)}.$$

Using (51) and (32), we derive that

$$\begin{aligned}\bar{q}_t &= \left[\frac{(1 - \sigma\Omega_2)}{1 - (\sigma - \phi)\Omega_2} \right] \mathbb{E}_t \{ \bar{q}_{t+1} \} \\ &+ \left[\frac{\Omega_1}{1 - (\sigma - \phi)\Omega_2} \right] [\sigma \mathbb{E}_t \{ a_{t+1} \} - (\sigma - \phi) a_t] \\ &- \left[\frac{1 + \Omega_3}{1 - (\sigma - \phi)\Omega_2} \right] [\sigma \mathbb{E}_t \{ y_{t+1}^* \} - (\sigma - \phi) y_t^*].\end{aligned}$$

Hence, by solving forward the previous dynamic equation, we can see that the level of \bar{q}_t at the frictionless equilibrium only depends on the exogenous stochastic variables a_t and y_t^* . From (50) and (51) we check that the previous conclusion is also true for the case of the levels of \bar{y}_t and \bar{c}_t .

B.2 Supply side of the equilibrium with sticky-prices

By combining (36) with (19), (30), (35) and (37) we obtain

$$\begin{aligned}mc_t &= \left[\frac{(1 - \nu)(1 - \delta)}{(1 - \delta)(1 - \nu) + \delta(1 - \nu + \delta\phi)} \right] \left\{ \left[\frac{\sigma + \phi(1 - \gamma)}{1 - \gamma} \right] y_t \right. \\ &+ \left[\frac{\gamma(1 - \delta)^2(1 - \nu) + (1 - \gamma)\delta(1 - \nu + \phi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} - \frac{\sigma\eta\gamma(2 - \gamma)}{(1 - \gamma)^2} \right] q_t \\ &\left. - (1 + \phi) a_t - \left(\frac{\gamma(1 + \phi - \nu)}{1 - \gamma} \right) y_t^* \right\}.\end{aligned}\quad (52)$$

In the flexible price equilibrium, we obtain that

$$\begin{aligned}0 &= \left[\frac{(1 - \nu)(1 - \delta)}{(1 - \delta)(1 - \nu) + \delta(1 - \nu + \delta\phi)} \right] \left\{ \left[\frac{\sigma + \phi(1 - \gamma)}{1 - \gamma} \right] \bar{y}_t \right. \\ &+ \left[\frac{\gamma(1 - \delta)^2(1 - \nu) + (1 - \gamma)\delta(1 - \nu + \phi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} - \frac{\sigma\eta\gamma(2 - \gamma)}{(1 - \gamma)^2} \right] \bar{q}_t \\ &\left. - (1 + \phi) a_t - \left(\frac{\gamma(1 + \phi - \nu)}{1 - \gamma} \right) y_t^* \right\}.\end{aligned}\quad (53)$$

Subtracting (53) to (52) we obtain

$$\begin{aligned}mc_t &= \left[\frac{(1 - \nu)(1 - \delta)}{(1 - \delta)(1 - \nu) + \delta(1 - \nu + \delta\phi)} \right] \left\{ \left[\frac{\sigma + \phi(1 - \gamma)}{1 - \gamma} \right] \tilde{x}_t \right. \\ &+ \left[\frac{\gamma(1 - \delta)^2(1 - \nu) + (1 - \gamma)\delta(1 - \nu + \phi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} - \frac{\sigma\eta\gamma(2 - \gamma)}{(1 - \gamma)^2} \right] \tilde{q}_t \left. \right\}.\end{aligned}\quad (54)$$

Introducing (54) in (38) we directly obtain equation (39).

B.3 Demand side of the equilibrium with sticky-prices

By combining (20) with (29) and (30), we obtain after some simple algebra that

$$\begin{aligned}
y_t &= \left(\frac{\sigma}{\sigma - \phi} \right) \mathbb{E}_t \{y_{t+1}\} - \left(\frac{1 - \gamma}{\sigma - \phi} \right) [i_t - \mathbb{E}_t \{\pi_{H,t+1}\}] \\
&+ \left[\frac{\gamma}{\sigma - \phi} - \frac{\sigma \eta \gamma (2 - \gamma)}{(1 - \gamma)(\sigma - \phi)} \right] \mathbb{E}_t \{q_{t+1}\} \\
&- \left[\frac{\gamma}{\sigma - \phi} - \frac{\eta \gamma (2 - \gamma)}{1 - \gamma} \right] q_t - \left(\frac{\sigma \gamma}{\sigma - \phi} \right) \mathbb{E}_t \{y_{t+1}^*\} + \gamma y_t^* - \frac{\phi \vartheta (1 - \gamma)}{\sigma - \phi} \quad (55)
\end{aligned}$$

From (53) we can derive the following condition on the equilibrium with flexible prices:

$$\begin{aligned}
\bar{y}_t &= \left(\frac{\sigma}{\sigma - \phi} \right) E_t \{\bar{y}_{t+1}\} \\
&- \left[\frac{1 - \gamma}{\sigma + \phi (1 - \gamma)} \right] \left[\frac{\gamma (1 - \delta)^2 (1 - \nu) + (1 - \gamma) \delta (1 - \nu + \phi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} - \frac{\sigma \eta \gamma (2 - \gamma)}{(1 - \gamma)^2} \right] \bar{q}_t \\
&+ \left[\frac{\sigma (1 - \gamma)}{(\sigma - \phi) [\sigma + \phi (1 - \gamma)]} \right] \\
&\quad \times \left[\frac{\gamma (1 - \delta)^2 (1 - \nu) + (1 - \gamma) \delta (1 - \nu + \phi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} - \frac{\sigma \eta \gamma (2 - \gamma)}{(1 - \gamma)^2} \right] E_t \{\bar{q}_{t+1}\} \\
&+ \left[\frac{(1 - \gamma)(1 + \phi)}{\sigma + \phi (1 - \gamma)} \right] a_t - \left[\frac{\sigma (1 - \gamma)(1 + \phi)}{(\sigma - \phi) [\sigma + \phi (1 - \gamma)]} \right] E_t \{a_{t+1}\} \\
&+ \left[\frac{\sigma \gamma}{\sigma + \phi (1 - \gamma)} \right] y_t^* - \left[\frac{\gamma \sigma^2}{(\sigma - \phi) [\sigma + \phi (1 - \gamma)]} \right] E_t \{y_{t+1}^*\}. \quad (56)
\end{aligned}$$

Subtracting (56) from (55), we obtain

$$\begin{aligned}
\tilde{x}_t &= \left(\frac{\sigma}{\sigma - \phi} \right) E_t \{\tilde{x}_{t+1}\} - \left[\frac{\gamma}{\sigma - \phi} - \frac{\eta \gamma (2 - \gamma)}{1 - \gamma} \right] \tilde{q}_t \\
&+ \left[\frac{\gamma}{\sigma - \phi} - \frac{\sigma \eta \gamma (2 - \gamma)}{(1 - \gamma)(\sigma - \phi)} \right] E_t \{\tilde{q}_{t+1}\} - \left(\frac{1 - \gamma}{\sigma - \phi} \right) [i_t - E_t \{\pi_{H,t+1}\}] + \xi_t, \quad (57)
\end{aligned}$$

with

$$\begin{aligned}
\xi_t &= \left[\frac{\eta \gamma \phi (2 - \gamma)}{\sigma + \phi (1 - \gamma)} + \left(\frac{1 - \gamma}{\sigma + \phi (1 - \gamma)} \right) \frac{\gamma (1 - \delta)^2 (1 - \nu) + (1 - \gamma) \delta (1 - \nu + \phi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} - \frac{\gamma}{\sigma - \phi} \right] \bar{q}_t \\
&+ \left[\frac{\gamma}{\sigma - \phi} - \frac{\sigma \eta \gamma \phi (2 - \gamma)}{(\sigma - \phi) [\sigma + \phi (1 - \gamma)]} \right] \\
&\quad - \left(\frac{\sigma}{\sigma - \phi} \right) \left(\frac{1 - \gamma}{\sigma + \phi (1 - \gamma)} \right) \frac{\gamma (1 - \delta)^2 (1 - \nu) + (1 - \gamma) \delta (1 - \nu + \phi)}{(1 - \nu)(1 - \gamma)(1 - \delta)^2} \right] E_t \{\bar{q}_{t+1}\} \\
&- \left[\frac{(1 - \gamma)(1 + \phi)}{\sigma + \phi (1 - \gamma)} \right] a_t + \left[\frac{\sigma (1 - \gamma)(1 + \phi)}{(\sigma - \phi) [\sigma + \phi (1 - \gamma)]} \right] E_t \{a_{t+1}\} \\
&- \left[\frac{\gamma \phi (1 - \gamma)}{\sigma + \phi (1 - \gamma)} \right] y_t^* + \left[\frac{\gamma \sigma \phi (1 - \gamma)}{(\sigma - \phi) [\sigma + \phi (1 - \gamma)]} \right] E_t \{y_{t+1}^*\} - \frac{\phi \vartheta (1 - \gamma)}{\sigma - \phi},
\end{aligned}$$

where ξ_t is a stochastic, exogenous variable as can be followed from the previous characterization of the frictionless equilibrium. Finally, by introducing (33) into (57), we directly

obtain equation (34) in the main text, where

$$\epsilon_t = \xi_t - \left[\frac{\gamma}{\sigma - \phi} - \frac{\eta\gamma(2 - \gamma)}{1 - \gamma} \right] u_t.$$

C Loss function approximation of welfare criterion

Here we derive a second-order approximation of the representative consumer's welfare function (2) taking into account the approximate characterization of equilibrium behavior of all agents in (33), (39) and (34). We use the method proposed by Benigno and Woodford [2006].

[To be completed]

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