

TAX, CREDIT CONSTRAINTS, AND THE BIG COSTS OF SMALL INFLATION.

Andrew Coleman*
University of Otago

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*Department of Economics, University of Otago, P.O. Box 56, Dunedin, New Zealand. 644-4797494.
andrew.coleman@otago.ac.nz

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Abstract

This paper develops a heterogeneous-agent overlapping generations model that examines how the neutrality of the tax system with respect to inflation depends on the price elasticity of the housing supply. The model, which endogenises house prices and rents, and which incorporates detailed tax regulations and bank-imposed credit constraints, shows (i) inflation has large effects on the tenure arrangements of young households irrespective of the housing supply elasticity; and (ii) inflation can improve the welfare of some low income young households if the supply is sufficiently elastic. The welfare costs of inflation are reduced by taxing real rather than nominal interest.

JEL classification: E40, E58

I. INTRODUCTION

This paper analyses how the interaction of inflation, the tax system, and bank-imposed capital constraints affects housing markets. It has been sparked by concern that even in New Zealand's post-1990 low inflation environment the home-ownership rates of young low-income households are declining because of tax laws that favour investment in rental property rather than interest earning assets. By itself, the interaction of inflation with the tax system is not sufficient to reduce home ownership rates, for property acts as a tax shelter for owner-occupiers. But the credit constraints borrowers face are also exacerbated by inflation, as banks typically do not alter their loan conditions when inflation and interest rates change. If banks do not increase the amount credit-constrained households can borrow when debt servicing payments increase, it becomes more difficult for them to purchase houses (Modigliani (1976); Kearn (1979)).

The paper uses a version of the Modigliani-Brumberg overlapping generations model developed by Ortalo-Magné and Rady (1998, 2006) to explore the issue. The model has a plethora of households who differ by age, income, marginal tax rate, and the extent they can borrow. These households choose their consumption levels, save for retirement, borrow and lend subject to realistic borrowing constraints, buy or rent different sized houses for their own use, and decide whether or not to invest in housing. House prices are determined endogenously by matching the supply of houses with the collective demand for housing by owner-occupiers and landlords.

The model suggests small increases in the inflation rate *can* have large effects on home ownership rates even when inflation is perfectly anticipated. If the housing supply is inelastic, landlords – attracted by tax-free capital gains – bid up property prices and young households are squeezed out of the market because mortgage repayments are more onerous. If the housing supply is

elastic, landlords bid down rents, and credit constrained households are induced to rent instead of purchasing a house. The increase in the number of households renting in response to an increase in the inflation rate is a decreasing function of real interest rates. When real interest rates are above 6 percent (as they were in New Zealand between 1985 and 1998) moderate inflation has relatively little effect on the fraction of households choosing to rent. But when real mortgage rates are 5% or lower, a one percentage point increase in inflation leads to an 8 - 11 percentage point decline in home ownership rates among young households.

The effect of inflation on housing markets depends on the interaction of taxes and credit constraints, and largely disappears if there is only one distortion. Without borrowing restrictions, variation in the inflation rate has little effect on homeownership rates as households can alter their repayment schedules to maintain real consumption patterns. If banks impose credit constraints but the inflation component of interest earnings is exempted from tax, a one percentage point rise in inflation reduces homeownership rates among young households by 2 - 3 rather than 8-11 percentage points. This reinforces the traditional view of economists that taxing the inflation component of interest earnings is highly distortionary, even at low inflation rates (Viner (1923); Nowotny (1980); Fischer and Summers (1989); Feldstein (1996, 1997)).

The model is used to calculate how inflation affects welfare. When nominal interest income is taxed, agents have log-linear utility functions, and the supply of housing is inelastic, an increase in the inflation rate from 0 to 3 percent reduces lifetime welfare for almost all agents by 2 - 4 percent. This loss occurs for two reasons. First, nominal interest rates and real house prices increase, forcing young people to rent rather than buy or to buy a small rather than a large house. Secondly, real after-tax interest rates decrease, reducing the incentive to accumulate capital for

retirement. However, inflation is not necessarily bad when the supply of housing is elastic, as rents fall. The welfare of some low income households increases as lower rents allow them to increase consumption when they are young and credit constrained. Inflation still causes welfare losses for middle and high income households, however, as it makes house purchase more difficult while young, and reduces the return to saving as after tax-real interest rates decline.

The implication that young people can benefit from inflation is not prominent in the literature (for an exception, see Pozdena 1988). While this result may seem perverse, it occurs because of the existence of two distortions: the restrictions imposed on young people that prevent them borrowing against future income; and the excessive taxation of real interest earnings when inflation occurs. In a world of second best, inflation can improve the welfare of credit constrained young people by inducing unconstrained landlords to offer them low rents.

The model highlights two issues relevant to current policy in New Zealand. First, the deterioration in housing affordability that occurred after 2001 may have been partly caused by the increase in the average inflation rate that occurred after the inflation target was raised in 2001. This paper suggests it matters whether inflation is in the top or bottom end of the 1 – 3 percent inflation target range. Secondly, the adverse effects of inflation can be ameliorated through tax reform if it is too difficult for the central bank to achieve very low rates of inflation.

The model is related to several earlier papers. It is conceptually similar to Feldstein (1997) but clarifies his insights by incorporating a rental housing sector and different after-tax interest rates for borrowers and lenders. It extends the models used by Slemrod (1982) and Hayashi, Ito, and Slemrod (1988) to study individual lifecycle optimisation by endogenising house prices. It adopts

the Ortalo-Magné and Rady (2006) framework (rather than various alternatives) so house prices can be determined endogenously, but extends it by incorporating taxes and by allowing an elastic housing supply. Lastly, it complements Coleman (2010) which uses a similar model to examine the effects of capital gains taxes and rather than the distortionary effects of inflation.

This paper complements several other papers that investigate how mortgage finance affects housing markets (for a review see Kursten (2005)). Using a version of the life-cycle model developed by Aiyagari (1994), Li and Yao (2007) investigate the determinants of household decisions to rent or buy when house prices are determined exogenously. They show the design features of long term mortgages have important effects on the housing choices of young households when the inflation rate is non-zero. Chambers, Garriga, and Schlagenhaut (2009) also explore how mortgage contracts affect housing demand in an inflationary environment, arguing that contract designs that adjust for inflation increase the welfare of young credit-constrained households. They leave housing supply issues to further research. Kiyotaki et al (2011) use a lifecycle model with housing demand and credit constraints to examine long term distribution issues, but do not use it to explore the interaction of inflation with the tax system.

The details of the model are presented in section 2. In section 3, key results of the model are shown for four different scenarios: when the housing supply is elastic or inelastic; and when the inflation component of interest is taxed or tax exempt. This is followed by an analysis of the dependence of the results on key parameters. Finally, conclusions are offered in section 5.

II. AN INTERGENERATIONAL MODEL OF HOUSING DEMAND

a) The basic framework

The model is based on the overlapping generations housing model of Ortalo-Magné and Rady (2006). There are four overlapping cohorts, each living for four equal periods $i = \{0,1,2,3\}$ that represent three working and one retired life stages. Agents differ by income. In period t agent j born in period $t-i$ supplies labour inelastically and has real labour income

$$Y_t^{i,j} = \omega_j g_i Y_{t-i}^0 \quad (1)$$

where ω_j = an idiosyncratic factor affecting agent j relative to average cohort earnings; g_i = a factor reflecting the life-cycle earnings of the cohort in its i^{th} period; and Y_{t-i}^0 = the average income of the cohort at time of birth. Agents have a constant place in the within-cohort income distribution (agent 1 has the lowest income). Nominal income is $P_t Y_t^{i,j}$, where P_t is the pre-tax price of the good. An indirect tax τ^g is applied to goods other than housing, so the post tax prices of goods are $(1 + \tau^g)P_t$. Incomes and goods prices increase at the constant inflation rate π .

Agents obtain utility from the consumption of goods and housing. In each period an agent chooses real consumption $c_t^{i,j}$, and one of three housing options: they can rent a small house or flat (R), purchase a flat (F), or purchase a larger house (H). Age zero agents can also live with their parents at no cost, although they gain no utility from doing so.¹ In period t agents get utility

$$u(c_t^{i,j}, \mathbf{I}_t^{i,j,h}) = \ln(c_t^{i,j}) + \sum_h v^h I_t^{i,j,h} \quad (2)$$

where $\mathbf{I}_t^{i,j,h} = \{I_t^{i,j,R}, I_t^{i,j,F}, I_t^{i,j,H}\}$ is a vector of indicator variables equal to one if the agent has housing tenure h in period i of his or her life at time t . $v^H > v^F$ as houses are bigger than flats, and

$v^F > v^R$, as agents can modify an owned flat to their own taste. Agents born at time t choose consumption and housing paths to maximise discounted lifetime utility:

$$U = \sum_{i=0}^3 \beta^i u(c_{t+i}^{i,j}, \mathbf{I}_{t+i}^{i,j,h}) \quad (3)$$

At the start of each period households receive income and consume; borrow or lend; and rent, buy, or sell property. In the last period, they sell all assets except their last house, repay any debts, and consume their remaining wealth. They die at the end of period 3, bequeathing their house to a younger agent. At time t a fraction κ_i is left to members of the cohort born at $t-i$; it is either left to the j th agent, preserving the intergenerational income ranking, or equally distributed across all agents of the cohort.² In this paper, $\kappa_2 = 1$ so agents do not inherit until late in life: if agents inherit early, they have less need to save for housing.

b) Taxes and the housing market

Five features of the New Zealand tax system are modelled. (1) There are two marginal tax rates, τ_1 and $\tau_2 \geq \tau_1$ for agents with income greater than a constant real threshold τ^* . These tax rates are applied to rental and interest income. (2) Imputed rent is tax exempt. (3) There is no capital gains tax. (4) Landlords but not home-owners can deduct mortgage interest payments from their taxable income. (5) There is an indirect tax τ^s on consumption expenditure but not rent or housing. τ^s is determined endogenously so that sum of indirect taxes and capital income taxes equal a fraction $\tau^{s*} = 10$ percent of labour income. This ensures that changes in the tax system do not alter the government's tax revenue. Government expenditure does not generate utility.

Flats and houses cost P_t^F and P_t^H to purchase. Flats can also be leased, at price P_t^R that is paid in advance at the beginning of the lease. The rent is paid to a landlord, who, for convenience, is restricted to be an agent in period 2 of their lives. The number of landlords is endogenous; an indicator variable $I_t^{i,j,R*}$ indicates whether or not the j^{th} agent owns a rental property.³ Because there is no uncertainty, the after-tax return from purchasing a flat in period t , leasing it, and selling it in period $t+1$ is equal to the after-tax return from lending money. As such, the relationship between rent, tax rates, flat prices, and interest rates is

$$P_t^R (1 - \tau_2)(1 + r_t(1 - \tau_2))^T + P_{t+1}^F = P_t^F (1 + r_t(1 - \tau_2))^T \quad (4)$$

or

$$P_t^R = P_t^F \left(\frac{(1 + r_t(1 - \tau_2))^T - (1 + \pi_t^F)}{(1 - \tau_2)(1 + r_t(1 - \tau_2))^T} \right) \quad (5)$$

where π_t^F is the rate of price appreciation for flats. The right hand side of equation 4 is the after-tax return in period $t+1$ from investing P_t^F in interest earning bonds. The left hand side is the after-tax return at $t+1$ from using the same sum to purchase a rental flat at time t . It comprises the after-tax rent paid at time t and reinvested at interest, plus the untaxed proceeds from selling the rental unit at time $t+1$. Since interest payments by landlords are fully tax deductible, the return to a landlord is independent of their level of gearing. Landlords are high income agents in period 2 of their lives, so after-tax returns are calculated using the top marginal tax rate τ_2 .

In each period, agents choose one of the three housing options, or live with their parents, so there are 256 possible different housing patterns through an agent's lifetime. Rather than calculate the utility of each of these patterns, I restrict agents to a smaller set of patterns, \mathbf{H} , by imposing three rules. The three restrictions are: (i) only 0 period agents may choose no housing;

(ii) only period 0 and period 1 agents may choose to rent; and (iii) except in the last period, agents' housing choices must not worsen through time⁴. By this means, \mathbf{H} has 23 options, $\mathbf{H} = \{ORFF, ORHF, ORHH, OFFF, OFHF, OFHH, OHHF, OHHH, RFFF, RRHF, RRHH, RFFF, RFHF, RFHH, RHHF, RHHH, FFFF, FFHF, FFHH, FHHF, FHHH, HHHF, HHHH\}$. An agent's optimal discounted utility is calculated for each of these patterns, and the agent is assumed to choose the pattern that provides the greatest discounted utility.

The model is solved for two different housing supply assumptions. First, the number of houses, n^H , and flats, n^F , is determined exogenously. To ensure property prices are positive in equilibrium, $n^H + n^F < 4N$. Secondly, there is an elastic supply of flats and houses, and the quantity of each is determined in equilibrium along with rents and prices. The supply functions are:

$$\begin{aligned} P_t^F &= \alpha_0^F + \alpha_1^F (Q_t^F + Q_t^H) \\ P_t^H &= P_t^F + \alpha_0^H + \alpha_1^H Q_t^H \end{aligned} \tag{6}$$

The price of flats is an increasing function of the total number of properties (to reflect the possible scarcity of land), while house price exceed the price of flats by a premium that reflects additional building costs. When the housing supply is elastic, the price elasticity of flats is approximately 1.

c) The lending market

Agents can lend through a zero profit bank at an interest rate r . There are no restrictions on deposit contracts, and deposits made at t earn interest at $t+1$. Agents pay tax on this interest at their marginal tax rate, but do not get a tax deduction for interest paid on borrowed funds unless they borrow to fund a rental property.⁵ An agent's positive funds are labelled $B_t^{i,j}$. Agents do not lend or borrow in period 3 as they consume all wealth except their final house.

Agents can also borrow with a mortgage contract that is subject to three restrictions.⁶

i) *The loan to value restriction.* All lending is collateralised against property. The gross amount borrowed, $D_t^{i,j^-} \geq 0$, cannot exceed the value of property multiplied by the loan to value ratio θ :

$$D_t^{i,j^-} \leq \sum_{h \in F, H} \theta P_t^h I_t^{i,j,h} \quad (7)$$

ii) *The regular cash payment restriction.* Banks issue η -year table mortgages, and require customers to make regular cash repayments CP throughout the life of the mortgage rather than a large repayment at its terminal date. The payment size CP ensures the mortgage is retired at the end of the term: if D^0 is initially borrowed, the equal-sized annual payments are

$$CP = D^{0-} r \left[\frac{(1+r)^\eta}{(1+r)^\eta - 1} \right] \quad (8)$$

$\eta = 25$ years. To model this feature of a mortgage, a household with gross debt of D_t^{i,j^-} has a separate account with the bank and make a deposit earning untaxed interest at rate r_t of size

$$D_t^{i,j^*} = D_t^{i,j^-} \frac{r_t}{1+r_t} \left[\frac{(1+r_t)^{\eta/T}}{(1+r_t)^{\eta/T} - 1} \right] \quad (9)$$

This means the net borrowing position of a borrowing agent, $D_t^{i,j} = D_t^{i,j^-} - D_t^{i,j^*}$, is less than the gross borrowing position. Without this “cash payment” feature, many agents would prefer to purchase rather than rent because the interest payment occurs a period later than the rental payment. When the “cash payment” requirement is imposed, purchasing a house requires a larger payment to the bank in period t than the cost of renting a house.

iii) *The mortgage-repayment-to-income restriction.* The amount an agent can borrow is restricted to ensure the mortgage repayment given by equation 9 is smaller than a fraction δ of income:

$$D_t^{i,j-} \frac{r_t}{1+r_t} \left[\frac{(1+r_t)^{\eta/T}}{(1+r_t)^{\eta/T} - 1} \right] \leq \delta P_t Y_t^{ij} \quad (10)$$

The three mortgage conditions are only imposed on period 0 and period 1 agents to simplify the solution algorithm. In period 2 agents can borrow unrestricted amounts. The absence of a restriction in period 2 has little effect because agents are in their peak earning years, receive their inheritance at this time, and are actively saving to finance their retirement.

d) Utility maximisation

An agent born at t solves the following maximisation problem (the j th superscript is omitted):

$$\begin{aligned} \text{Max}_{\{c_{t+i}, \mathbf{I}_{t+i}^{i,h}\}} U = & \sum_{i=0}^3 \beta^i u(c_{t+i}^i, \mathbf{I}_{t+i}^{i,h}) \quad (11) \\ & - \lambda_0 \left(P_t Y_t^0 - B_t^0 + D_0^t - (1 + \tau^g) P_t c_t^0 - \sum_h P_t^h I_t^{0,h} \right) \\ & - \sum_{i=1}^3 \lambda_i \left(\begin{aligned} & (1 + \pi)^i P_t Y_{t+i}^i + B_{t+i-1}^i (1 + r_{t+i-1} (1 - \tau^i)) - D_{t+i-1}^i (1 + r_{t+i-1}) - B_{t+i}^i + D_{t+i}^i \\ & - (1 + \pi)^i (1 + \tau^g) P_t c_t^i - \sum_h P_{t+i}^h I_{t+i}^{i,h} + \sum_{h=F,H} P_{t+i}^h I_{t+i-1}^{i-1,h} + \kappa_i \text{Inherit}_{t+i} \\ & + \left[P_{t+i}^R (1 - \tau^i) I_{t+i}^{i,R*} + (P_{t+i}^F - P_{t+i-1}^F (1 + r_{t+i-1} (1 - \tau^i))) I_{t+i-1}^{i-1,R*} \right] \end{aligned} \right) \\ & - \sum_{i=0}^1 \chi_i \left(D_{t+i}^{i,-} - \sum_h \theta P_{t+i}^h I_{t+i}^{i,h} \right) - \sum_{i=0}^1 \phi_i \left(D_{t+i}^{i,-} \frac{r_{t+i}}{1+r_{t+i}} \left[\frac{(1+r_{t+i})^{\tau/T}}{(1+r_{t+i})^{\tau/T} - 1} \right] - \delta Y_{t+i}^i \right) \\ & - \sum_{i=0}^3 \varsigma_i (B_{t+i}^i) - \sum_{i=0}^3 \nu_i (D_{t+i}^i) \end{aligned}$$

Lines 2 and 3 of equation (11) are the budget constraints facing the agent in the four periods. There are terms reflecting rental income and inheritance, and lending and borrowing are entered separately. τ^i is the marginal tax rate applying in period i of the agent's life. The Kuhn-Tucker conditions χ_i and ϕ_i impose the loan-to-value and the mortgage-repayment-to-income

constraints, while those in the last line mean non-negative amount are lent and borrowed. The agent calculates the maximum utility for each housing pattern in the set H , and then selects the housing pattern with the highest utility. (Analytical solutions for the optimal consumption path given a particular housing pattern are found.) Each solution has 48 parts corresponding to the 48 possible combinations of Kuhn-Tucker conditions.

e) Equilibrium conditions

The economy can either be open or closed. The solution presented below is for an open economy where real interest rates are determined exogenously and the net foreign asset position can be non-zero. The following price relationships hold along the dynamic steady state path:

$$(1 + r_t) / (1 + \pi_t) = 1 + r^* \quad (12a)$$

$$P_{t+1}^F / P_t^F = 1 + \pi^F \quad (12b)$$

$$P_t^H / P_t^F = \rho^H \quad (12c)$$

$$\frac{P_t^R}{P_t^F} = \left(\frac{(1 + r_t(1 - \tau_2))^T - (1 + \pi_t^F)}{(1 - \tau_2)(1 + r_t(1 - \tau_2))^T} \right) = \rho^R \quad (12d)$$

Equation (12a) states that real interest rates are constant and equal to the foreign real interest rate r^* . Equation (12b) states that flat prices appreciate at a constant rate.⁷ Equation (12c) states that the ratio of house prices to flat prices is constant. Equation (12d) is a restatement of equation 5, linking rents to interest rates and the flat price appreciation rate.

For a set of parameters $\{N, T, Y_t^0, \omega_j, g_i, \pi, r^*, \beta, v_h, \kappa_i, H, n^H, n^F, \eta, \theta, \delta, \tau^{g^*}, \tau_1, \tau_2, \tau^*\}$ and housing parameters either $\{n^F, n^H\}$ or $\{\alpha_0^F, \alpha_1^F, \alpha_0^H, \alpha_1^H\}$ the steady state equilibrium is described by a set of prices $\{r, \pi^F, \rho^H, \rho^R\}$, an indirect tax rate τ^g , a set of housing and

consumption demands $\{c_{t-i+s}^{s,j}, \mathbf{I}_{t-i+s}^{s,j,h}\}_{s=0,\dots,3}$ for each agent j in each cohort born in period $t-i$, and a net foreign asset position B_t^{net} such that all agents have maximal utility and

$$\sum_{i=0}^3 \sum_{j=1}^N c_t^{i,j} = (1 - \tau^{g*}) \sum_{i=0}^3 \sum_{j=1}^N y_t^{i,j} - \left(\frac{r - \pi}{1 + \pi} \right) B_t^{net} \quad (13a)$$

$$\sum_{i=0}^3 \sum_{j=1}^N (B_t^{i,j} - D_t^{i,j}) - P_t^F \sum_{j=1}^N I_t^{2,j,R*} = B_t^{net} \quad (13b)$$

$$\tau^{g*} \sum_{i=0}^3 \sum_{j=1}^N y_t^{i,j} = \tau^g \sum_{i=0}^3 \sum_{j=1}^N c_t^{i,j} + \sum_{i=0}^3 \sum_{j=1}^N B_{t-1}^{i,j} r \tau_t^{ij} + \sum_{j=1}^N P_t^R \tau_t^{2,j} I_t^{2,j,R*} - \sum_{j=1}^N P_{t-1}^F r \tau_t^{3,j} I_{t-1}^{2,j,R*} \quad (13c)$$

and either

$$\sum_{i=0}^3 \sum_{j=1}^N (I_t^{i,j,R} + I_t^{i,j,F}) = n^F \quad (13d')$$

$$\sum_{i=0}^3 \sum_{j=1}^N I_t^{i,j,H} = n^H \quad (13e')$$

or

$$\sum_{i=0}^3 \sum_{j=1}^N (I_t^{i,j,R} + I_t^{i,j,F}) = Q^F = \frac{P^F - \alpha_0^F}{\alpha_1^F} - Q^H \quad (13d'')$$

$$\sum_{i=0}^3 \sum_{j=1}^N I_t^{i,j,H} = Q^H = \frac{P^H - P^F - \alpha_0^H}{\alpha_1^H} \quad (13e'')$$

Equation (13a) requires that total consumption plus tax plus real earnings on the net bond position in each period equals total production. Equation (13b) is the net supply of foreign bonds, given that landlords are assumed to borrow 100 percent of the price of a flat. This will change through time if there is economic growth or inflation. Equation 13c says that the total tax take is equal to total indirect tax revenue plus the tax on interest and rent minus the tax deduction for landlords. Equations (13d') and (13e') require that the total demand for flats equals the supply of flats, and that the total demand for houses equals the supply of houses,

when the supply of properties is determined exogenously. Equations (13d'') and (13e'') are the equivalent equations for the case when the supply of property is elastic.

The solution is found numerically. The algorithm searches for prices $\{\tau^g, P_t^R, P_t^F, P_t^H\}_{t=-3, \dots, 3}$ so that when each agent j born in period $t-i, i=0, \dots, 3$ consumes a sequence of goods and tenure options $\{C_{t-i+s}^{s,j}, \mathbf{I}_{t-i+s}^{s,j,h}\}_{s=0, \dots, 3}$ that solves their constrained utility problem given by equation (11), the aggregation conditions 13a – 13e at time t are satisfied.

f) Parameterisation

The set of baseline parameters $\{N, T, Y_t^0, \omega_j, g_i, \pi, r^*, \beta, v_h, \kappa_i, H, n^H, n^F, \eta, \theta, \delta, \tau^{g*}, \tau_1, \tau_2, \tau^*\}$ and housing parameters $\{n^F, n^H\}$ are listed in table 1 and were chosen to approximate features of the New Zealand economy in the middle of the post 1990 low inflation era. The income parameters approximately match the lifecycle and cohort income patterns of New Zealanders reported in census documents, 1966-2001, under the assumption that the basic agent is a household comprised of a male and female of the same age. For simplicity, income is assumed to be uniformly distributed over the range \$20000 to \$80000.

The tax rates reflect New Zealand tax settings at the start of 2000. In the baseline model, $(\tau_1, \tau_2) = (20\%, 33\%)$ with a threshold $\tau^* = \$50000$. The model was also solved under four alternative tax regimes to show the effect of inflation: $(\tau_1, \tau_2) = (0\%, 0\%), (10\%, 10\%), (20\%, 20\%),$ and $(20\%, 39\%)$. (The top tax rate was increased to 39% in 2000 for a decade.) The model is also solved with tax rules that exempt the inflation component of interest income from tax, and only allow landlords to deduct real interest payments from their taxable income.⁸ The GST rate was chosen to ensure that capital income taxes and consumption taxes total to 10 percent of labour income.

In the baseline model, the intertemporal discount factor is 3 percent, and the real mortgage rate is 5 percent. Households can only borrow up to 90 percent of the value of a property and pay no more than 30 percent of their income in debt servicing. The borrowing restrictions reflect the conditions facing New Zealand borrowers between 2000 and 2013. In 2013, the Reserve Bank of New Zealand introduced regulations tightening the loan-to-value restrictions that banks imposed. These tighter restrictions have been altered in some simulations, but do not qualitatively affect the key results on the way the interaction of the borrowing restrictions and the inflation rate affect the housing market.

The model was solved for inflation rates ranging from 0 to 3 percent, reflecting the legal requirement that the Reserve Bank of New Zealand achieve stability in the general level of prices. This requirement currently requires the Bank to keep inflation between 1 and 3 percent. The effects of higher inflation rates were not investigated as the purpose of the paper was to ascertain whether low rates of inflation have an appreciable effect on economic welfare.

When the housing supply is inelastic, 57 percent of the properties are large houses, and 42 percent of the properties are small houses. The total number of properties is 1 percent less than the number of agents, to ensure rents are positive in equilibrium. The parameters $(\nu^R, \nu^F, \nu^H) = (0.33, 0.35, 0.45)$ mean (approximately) that (i) at the margin a household would be prepared to spend a third of their income on rent rather than have no accommodation; (ii) the additional benefit from living in an owner-occupied flat rather than a rented flat is 2%; and, (iii) the additional benefit from living in a large house is a further 10 percent. Housing supply parameters

were chosen so that that the price elasticity of flats was approximately one percent, but that the number of houses and flats would be approximately the same in the elastic and inelastic cases.

III. RESULTS

The key results are in table 2 (inelastic housing supply) and table 3 (elastic housing supply) and show the equilibrium prices as the inflation rate is varied from 0 to 3 percent.. The tables show how rents, flat prices, house prices, the number of flats and houses, and the number of people renting vary with the inflation rate for different tax combinations. The top half of each table presents outcomes when all interest earnings are taxed while the bottom half shows outcomes when the inflation component of interest earning are tax exempt. Results are presented for marginal tax rate combinations $(\tau_1, \tau_2) = \{(0\%, 0\%), (20\%, 20\%), (20\%, 33\%)\}$.

a) Inelastic housing supply, all interest earnings taxed.

Table 2 and figures 1 and 2 show how inflation and tax affect the housing market when the supply of housing is perfectly inelastic. If capital income is not taxed, inflation has almost no effect on real house prices and rents, although the fraction of the youngest cohort renting increases substantially as the inflation rate increases. Neither rents nor prices change much because the price of a flat is determined by its value to the marginal resident, who is a low income young person choosing whether to rent or live in the parental home, and the benefit of living independently varies only slightly with the inflation rate.⁹ The housing tenure choice is affected by the inflation rate, however, because an increasing fraction of low income households find it preferable to rent rather than purchase as nominal interest rates increase, as they cannot afford the mortgage payments without deep cuts in consumption.

When the tax rates are positive, there are three competing tendencies. If the inflation rate is zero, the fraction of the young cohort renting is decreasing in the tax rate as imputed rent is tax exempt. This incentive to own a home rather than to rent and accumulate financial assets is sufficiently large that the fraction renting falls to zero when $(\tau^1, \tau^2) = (20\%, 33\%)$. As the inflation rate increases, however, there are two other effects. First, low income, credit constrained households find it increasingly difficult to make mortgage payments as inflation and nominal interest rates increase. Secondly, landlords are attracted to the housing market to take advantage of tax free capital gains. As they have higher marginal tax rates, they value the tax concession more highly than low income tenants and the amount they are prepared to pay for rental property is a steeply increasing function of the inflation rate (Litzenberg and Sosin (1979)). The result is a squeeze that sees landlords replace young households as the owners of flats.

Figure 3 shows the fraction of the young cohort that rents as a function of tax rates and inflation rates. When the inflation rate is less than 1 percent, the fraction of the young cohort that rents declines as tax rates increase, due to the rising value of the imputed rent tax concession. When the inflation rate is 2 or 3 percent, the effect of higher mortgage rates and the increasing attractiveness of property to high marginal rate investors dominates the imputed rent tax concession and the fraction who rent is an increasing function of the top marginal tax rate.

b) Elastic housing supply, all interest earnings taxed.

Table 3 and figure 4 show the results when the supply of housing is elastic. As before, inflation has little effect on real flat prices, house prices, rents, or the total number of houses when capital income is not taxed, but the fraction of the youngest cohort renting increases as the inflation rate increases. The composition of the housing stock also changes as the inflation rate increases,

with fewer large houses. This is because fewer young households can afford the higher nominal mortgage payment on large houses so they purchase a small house as a temporary measure.

When capital income is taxed, the fraction of young cohorts renting increases with inflation by a similar amount as when the housing supply is inelastic. The mechanism is different, however. In the inelastic case, flat prices increase as inflation increases because landlords bid up prices to take advantage of the tax concession. In the elastic case, prices of houses do not increase by very much, but landlords bid rents down. The combination of slightly higher flat prices and significantly lower rents induces a large number of low income households to rent. It also increases the number of flats and reduces the number of households living with their parents.

c) Exempting the inflation component of interest earnings from tax.

Many economists have argued that the distortions caused by the interaction of inflation and the tax system would not occur if the inflation component of interest earnings were exempt from income tax, and borrowers could only deduct the real interest component of interest payments from taxable income¹⁰ (Aaron 1976; Fischer and Summers 1989; Feldstein 1996). The effects of these two changes in the tax code on the housing market are shown in the bottom half of tables 2 and 3, and in figures 1 – 2 and 3 – 4.¹¹ (The indirect tax rate is adjusted so keep total nominal taxes the same). The results show inflation has only small effects on house prices and home ownership rates when real rather than nominal interest income is taxed; indeed, the effects are very similar to the case when capital income taxes are zero. The difference occurs because wealthy households have less incentive to become landlords as they no longer pay tax on the inflation component of interest earnings. When the supply of housing is inelastic, the price of flats is not bid up by wealthy households competing to become landlords as inflation increases;

alternatively, when the supply of housing is elastic, rents are not bid down. In both cases, home-ownership rates amongst the young scarcely change as inflation changes.

It is worth noting that taxing the inflation component of interest earnings raises little revenue in the model. When the inflation rate is 3 percent, the indirect tax rate needs to be increased by less than 0.2% to raise the same revenue, as high income households rearrange their finances to avoid the tax, by becoming landlords. The additional tax revenue on rental income is offset by a decrease in tax paid on interest as the mortgage payments made by landlords are tax deductible.

d) Utility calculations

The lifetime utility levels of each agent can be used to analyse the welfare consequences of inflation. Figure 5 shows how utility levels change when the inflation rate increases from 0 to 3 percent, the supply of houses is inelastic and the tax rates are $(\tau_1, \tau_2) = (20\%, 33\%)$. Inflation reduces welfare for all households by between 2 and 4 percent when nominal interest earnings are taxed. The effect is largest on low and middle income households, as they have the largest change in their housing patterns, choosing to rent or to delay the purchase of large houses as inflation and nominal interest rates increase. High income households, whose housing is less affected by the inflation rate, experience a welfare reduction because they change the timing of their consumption in response to the higher real capital income taxes.

Figure 5 also shows the inflation has much smaller welfare effects when real interest earnings are taxed. Low income agents experience small welfare reductions, as they rent rather than own their homes when nominal interest rates increase; middle income agents have small welfare gains

because indirect tax rates drop when inflation rises (as the Government obtains more tax revenue from rent); but for both groups the welfare changes are typically less than 0.5 percent.

Figure 6 shows the change in utility that occurs when the housing supply is elastic. In this case, inflation improves the welfare of low income households, by up to 4 percent. There are two reasons for this improvement. First, rents fall, more than offsetting the decline in welfare that comes from being a tenant rather than an owner-occupier. Secondly, more flats are built, and the lowest income households gain from the opportunity to live independently rather than in the parental home. (The peak welfare improvement accrues to the highest income person that lived at home when the inflation rate was zero.) These gains are offset by welfare losses of middle income and high income households, the latter of approximately 2 percent, as these households pay higher taxes on their interest income. For the economy as a whole, total welfare is reduced, although this result depends on the distribution of income.

This result warrants further comment. Low income agents who rent when they are young have very low consumption because they cannot borrow against their higher future incomes. Inflation improves their welfare because it leads to a reduction in rent that raises their consumption, as landlords are prepared to accept lower rents when the inflation rate increases. In essence, the effect of one economic distortion (credit constraints preventing young people from borrowing against future income) is reduced by a second economic distortion (the asymmetric taxes on the inflation component of interest earnings and capital gains) when there is inflation. If young people were able to borrow against future income for consumption purposes, inflation would still reduce rents but it would have minimal effects on welfare as it would primarily affect the amount young people borrowed, not the amount they consumed.

When the inflation component of interest is not taxed, the effects of inflation on welfare depend little on the housing supply elasticity. This is because neither rent nor the total quantity of housing change by much in either the elastic or inelastic cases. This means that if the inflation component of interest earnings were tax exempt and the housing supply was sufficiently elastic, welfare would decline for some low income households, as high income households would no longer compete with each other to provide low rent accommodation to low income households.

IV. PARAMETER VARIATIONS

Table 4 shows the effects of changing some of the model parameters when the housing supply is inelastic. Each row shows the rent, flat price, and house price when $\pi=2\%$, and the fraction of the young cohort that rents when $\pi=0$ and $\pi=2\%$. These numbers are used to calculate the “slope”: the additional number of young households renting when inflation increases by 1 percentage point. Table 5 presents the same information for the elastic housing supply case.

Table 4 shows that the extent that inflation affects the home-ownership rate of young households depends little on the tightness of credit constraints, the mix of housing types, or the relative utility of different tenure options. However, it is sensitive to the real interest rate and the discount rate. When real interest rates are 6% or more, inflation has little effect on home-ownership rates, as property prices are sufficiently low that almost all households purchase flats or houses rather than rent. In contrast, when real interest rates fall to 4%, each percentage point increase in the inflation rate is associated with a 24% increase in the fraction of young households that rent. At even lower real interest rates, almost all young households rent rather than purchase, as property prices become very high.

Homeownership rates are sensitive to real interest rates for two reasons. First, house prices are sensitive to real interest rates, declining as real interest rates increase. By itself, this would not change homeownership rates, as the real financing cost (the real interest rate multiplied by the flat price) changes little as interest rates change. However, the required loan repayment increases as real interest rates fall (equation 8). Consequently, credit constrained households that rent can consume more than credit constrained households that purchase. When interest rates are low this option appears sufficiently attractive that a large number of young households reduce their lifetime consumption to consume more at a young age.

The results are similar when the housing supply is elastic. The effect of inflation on homeownership rates is not particularly sensitive to the tightness of credit constraints, building costs, or the utility value of different tenure options, but is sensitive to the discount rate and real interest rates. However, the mechanism is different. There is little effect on property prices as parameters change, but rents decline, and the home-ownership rates of young households decline as credit-constrained households choose to increase consumption by renting.

The results are also sensitive to the intertemporal discount rate. When households are more patient (modelled by raising the intertemporal discount rate β from 0.97 to 0.98), inflation causes a smaller reduction in the home-ownership rate, as fewer young households choose to increase current consumption by renting. When the model was reconfigured so half the households have $\beta=0.98$ and half have $\beta=0.97$, the effect of inflation on home-ownership rates was midway between the cases when all households have $\beta = 0.97$ and $\beta=0.98$. This suggests the effect of

inflation on the housing market depends on the on the distribution of discount rates as well as the distribution of income

5. Conclusion

This paper extends the literature that examines how the interaction of inflation with the tax system affects the economy even when inflation is constant and perfectly anticipated. It focuses on two nominal rigidities: (i) government imposed taxes on the inflation component of interest earnings; and (ii) financial sector imposed restrictions on the amount households can borrow, restrictions that are expressed in nominal not real terms. The paper's key contribution is to endogenise property prices in a model that incorporates owner-occupied and rental sectors.

Four results of the model stand out. First, small increases in inflation can cause large reductions in the home-ownership rates of young cohorts when nominal interest is taxed. These cohorts find it difficult to maintain consumption when nominal mortgage payments increase in response to higher inflation, as they cannot increase their borrowing. When the model is parameterised to mimic New Zealand tax rates, and the real interest rate is 5%, a one percent increase in the inflation rate is associated with an 8 - 10% decrease in the home-ownership rates of young households, irrespective of the elasticity of the housing supply. This suggests it matters whether inflation is in the upper half rather than the lower half of the inflation target band.

Secondly, the welfare effects of inflation depend on the elasticity of the housing supply. When the supply of housing is inelastic, inflation reduces the welfare of all households (except the first generation of owners) because landlords bid up house prices to avoid the tax on the inflation component of interest income. Inflation reduces the welfare of low income households because

they rent rather than purchase a home to maintain consumption levels early in life; it reduces the welfare of middle and high income because they delay purchasing large houses and earn lower real after tax returns from their investments. When the supply of housing is elastic, competition between landlords leads to lower rents and the construction of additional houses. In this case inflation raises the welfare of the lowest income groups, for even though a large fraction choose to rent rather than purchase a home, the lower rents more than offset the welfare losses stemming from being a tenant rather than an owner-occupier. Middle and high income households lose from inflation, however, as they delay purchasing large houses. The overall welfare consequences of inflation will depend on the relative numbers of high and low income households. In the main parameterisations studied in this paper, the losses to middle and high income agents are larger than the gains to low income households, but this need not be the case.

Thirdly, the effects of inflation on home-ownership rates are decreasing in the real interest rate. The ratio of mortgage repayments (including capital repayments) to rents reduces as real interest rates increase, reducing the disincentive for credit constrained households to purchase a house. In the model simulations, moderate inflation had little effect on home-ownership rates when real interest rates were 6 percent or more. This feature of the model suggests that the effect of low inflation on the housing market may have been minimal in New Zealand prior to 2000, for real interest rates between 1985 and 2000 typically exceeded six percent.

Fourthly, the effects of inflation on the housing market would largely be avoided if the inflation component of nominal interest earnings were exempt from income. The main effect of inflation on home ownership patterns stem from the incentives that induce high income households to

become landlords when the inflation rate increases, to avoid the extremely high taxes on real interest income and to take advantage of tax free capital gains from property appreciation.

These findings are consistent with Feldstein's (1997) argument that low inflation can cause significant welfare losses because of its effect on housing markets. The paper adds sophistication to his analysis, however, by simultaneously considering the effects of inflation on inter-temporal housing and consumption patterns, by allowing for a rental market as well as home-ownership, and by endogenising house prices. With one exception, none of these factors overturn his essential insights into the harmful consequences of low inflation. The exception concerns the way inflation can improve the welfare of young low income agents who rent, by lowering their rents and raising their consumption while young.

This paper also contributes to the literature that suggests welfare could be improved if the inflation component of interest income were not taxed. This paper suggests that the extent of the welfare gains depends on the elasticity of housing supply, but the gains could be substantial – not surprising, perhaps, since the inflation component of interest earnings is not income.

Table 1. Parameterisation of the model.

Parameter	Description	Value	Source/Rationale
Y_t^0	Average income of 25-35 cohort	50000	NZ Census 2001: average male and female earnings, 25-35 year olds, are \$32800 and \$23300 respectively
ω_j	Income distribution	Uniform on [20000,80000]	
g_i	Lifecycle income pattern	{1, 1.5, 1.5, 0.75+20000}	NZ Census, 1966- 2001. Based on real lifecycle earnings of cohort turning 20 in 1946, 1961.
B	Discount factor	0.97 annualised	Arbitrary
$\{v^R, v^F, v^H\}$	Utility from housing	{0.33, 0.35, 0.45}	Arbitrary
κ_i	Inheritance timing	{0,0,1,0}	Arbitrary
n^H/N	Fraction of houses	0.57	Arbitrary
n^F/N	Fraction of flats	0.42	Arbitrary
H	Mortgage term	25 years	Standard mortgage term in 1990s
δ	Maximum debt service-income ratio	30%	Reflects NZ banking conditions
Θ	Maximum loan to value ratio	90%	Reflects NZ banking conditions
τ^{g^*}	GST rate	0.10	Tax take equals 10% of labour income; arbitrary, but close to NZ rate.
τ_1, τ_2, τ^*	Income tax rates and threshold	20%, 33% \$50000	Reflects NZ rates in 2000.
α_0^F, α_1^F α_0^H, α_1^H	Housing supply parameters	10, -1250 1, 9300	Arbitrary, generates approximately 1% price elasticity for flats.

Table 2: Output of Model with inelastic supply.

Tax rates	Inflation rate			
	0	1	2	3
Normal taxes – all nominal interest income taxed				
	Flat prices			
(0,0)	\$166,286	\$166,397	\$165,788	\$165,034
(20,20)	\$154,558	\$158,922	\$162,615	\$166,516
(20,33)	\$147,207	\$154,534	\$163,247	\$171,990
	House prices			
(0,0)	\$269,364	\$268,470	\$267,434	\$266,331
(20,20)	\$256,960	\$260,735	\$262,004	\$264,058
(20,33)	\$249,712	\$255,027	\$260,838	\$266,629
	Rents			
(0,0)	\$6,425	\$6,425	\$6,400	\$6,375
(20,20)	\$6,469	\$6,475	\$6,475	\$6,475
(20,33)	\$6,521	\$6,467	\$6,482	\$6,468
	Fraction of young cohort renting			
(0,0)	21.5%	24.5%	28.3%	31.5%
(20,20)	5.8%	16.0%	20.8%	32.3%
(20,33)	0.0%	10.0%	21.8%	42.3%
	Number of flats and houses			
(0,0)	580 + 900	580 + 900	580 + 900	580 + 900
(20,20)	580 + 900	580 + 900	580 + 900	580 + 900
(20,33)	580 + 900	580 + 900	580 + 900	580 + 900
The inflation component of interest is tax exempt.				
	Flat prices			
(0,0)	\$166,286	\$166,397	\$165,788	\$165,034
(20,20)	\$154,419	\$154,638	\$154,613	\$154,811
(20,33)	\$146,588	\$146,507	\$146,270	\$146,917
	House Prices			
(0,0)	\$269,364	\$268,470	\$267,434	\$266,331
(20,20)	\$256,835	\$256,847	\$256,684	\$256,404
(20,33)	\$248,971	\$247,772	\$247,330	\$248,066
	Rents			
(0,0)	\$6,425	\$6,425	\$6,400	\$6,375
(20,20)	\$6,467	\$6,475	\$6,475	\$6,475
(20,33)	\$6,485	\$6,489	\$6,476	\$6,502
	Fraction of young cohort renting			
(0,0)	21.5%	24.5%	28.3%	31.5%
(20,20)	5.5%	10.0%	14.0%	17.3%
(20,33)	0.0%	2.0%	4.8%	7.5%
	Number of flats and houses			
(0,0)	580 + 900	580 + 900	580 + 900	580 + 900
(20,20)	580 + 900	580 + 900	580 + 900	580 + 900
(20,33)	580 + 900	580 + 900	580 + 900	580 + 900

Table 3: Output of Model with elastic supply.

Tax rates	Inflation rate			
	0	1	2	3
Normal taxes – all nominal interest income taxed				
	Flat prices			
(0,0)	\$147,266	\$147,279	\$147,242	\$147,263
(20,20)	\$146,388	\$146,716	\$147,043	\$147,275
(20,33)	\$145,634	\$146,393	\$147,059	\$147,535
	House Prices			
(0,0)	\$249,551	\$249,482	\$249,343	\$249,297
(20,20)	\$248,461	\$248,747	\$248,922	\$248,979
(20,33)	\$247,695	\$248,238	\$248,763	\$249,028
	Rents			
(0,0)	\$5,681	\$5,683	\$5,681	\$5,682
(20,20)	\$6,123	\$5,983	\$5,857	\$5,735
(20,33)	\$6,442	\$6,130	\$5,842	\$5,551
	Fraction of young cohort renting			
(0,0)	23.0%	24.5%	25.5%	28.5%
(20,20)	6.3%	15.3%	22.5%	27.8%
(20,33)	0.0%	9.5%	22.5%	35.3%
	Number of flats and houses			
(0,0)	669 + 929	678 + 920	687 + 910	694 + 903
(20,20)	682 + 907	689 + 903	708 + 888	727 + 870
(20,33)	675 + 906	704 + 884	725 + 870	751 + 849
The inflation component of interest is tax exempt.				
	Flat prices			
(0,0)	\$147,266	\$147,279	\$147,242	\$147,263
(20,20)	\$146,388	\$146,377	\$146,404	\$146,295
(20,33)	\$145,585	\$145,637	\$145,571	\$145,645
	House Prices			
(0,0)	\$249,551	\$249,482	\$249,343	\$249,297
(20,20)	\$248,461	\$248,430	\$248,448	\$248,327
(20,33)	\$247,639	\$247,553	\$247,461	\$247,548
	Rents			
(0,0)	\$5,681	\$5,683	\$5,681	\$5,682
(20,20)	\$6,123	\$6,123	\$6,124	\$6,121
(20,33)	\$6,438	\$6,439	\$6,436	\$6,443
	Fraction of young cohort renting			
(0,0)	23.0%	24.5%	25.5%	28.5%
(20,20)	6.3%	10.3%	13.8%	17.5%
(20,33)	0.0%	2.3%	4.8%	7.3%
	Number of flats and houses			
(0,0)	669 + 929	678 + 920	687 + 910	694 + 903
(20,20)	682 + 907	684 + 905	685 + 904	685 + 903
(20,33)	675 + 905	690 + 892	692 + 889	691 + 890

Table 4: Parameter variations with inelastic supply.

Parameter change	Rent	Flat price	House price	% young renting $\pi = 0\%$	% young renting $\pi = 2\%$	Slope
Base Parameters	6480	163200	260800	0%	22%	11
Changing credit constraints (loan to value ratio)						
$\theta = 0.8$	6470	163100	259300	2	29	14
$\theta = 0.9$	6480	163200	260800	0%	22%	11
$\theta = 1.0$	6500	163500	262700	0	18	9
Changing number of flats and houses						
$N_h=600, N_f=980$	6450	162700	298300	0%	29%	15
$N_h=900, N_f=680$	6480	163200	260800	0%	22%	11
$N_h=900, N_f=660$	7357	185600	281600	0%	26%	13
Changing the valuation of houses						
value= 32 35 45	6300	158800	258000	0%	17%	9
value= 33 35 45	6480	163200	260800	0%	22%	11
value= 33 35 46	6500	163500	270600	0%	20%	10
Changing real interest rates, discount $\beta = 0.97$						
$\beta = 0.97$ $r = 4$	6440	207600	298400	15%	63%	24
$\beta = 0.97$ $r = 4.5$	6490	183400	278400	7%	51%	22
$\beta = 0.97$ $r = 5$	6480	163200	260800	0%	22%	11
$\beta = 0.97$ $r = 5.5$	6510	148500	247200	0%	9%	5
$\beta = 0.97$ $r = 6$	6850	133300	234300	0%	0%	0
Changing real interest rates, discount $\beta = 0.98$						
$\beta = 0.98$ $r = 4$	6870	220900	324100	6%	50%	22
$\beta = 0.98$ $r = 4.5$	6410	180900	284400	1%	21%	10
$\beta = 0.98$ $r = 5$	6390	161800	267300	0%	10%	5
$\beta = 0.98$ $r = 5.5$	6320	145600	252300	0%	2%	1
$\beta = 0.98$ $r = 6$	6990	135600	242300	0%	0%	0

Prices worked out when the inflation rate $\pi = 2\%$.

“Slope” is the change in the fraction of young cohort renting when there is a 1 percentage point increase in the inflation rate.

Table 5: Parameter variations with elastic supply.

Parameter change	Rent	Flat price	House price	% young renting $\pi = 0\%$	% young renting $\pi = 2\%$	Slope	N_h	N_f	Total
Base Parameters	5840	147100	248800	0%	23%	11	870	725	1595
Changing credit constraints (loan to value ratio)									
$\theta = 0.8$	5830	147000	248500	3%	25%	11	853	741	1594
$\theta = 0.9$	5840	147100	248800	0%	23%	11	870	725	1595
$\theta = 1.0$	5840	147000	248800	0%	19%	10	880	715	1595
Changing additional cost of constructing a large house									
$a_{0h} = 93000$	5840	147100	248800	0%	23%	11	870	725	1595
$a_{0h} = 143000$	6440	145600	293800	0	5%	3	528	1053	1581
Changing the valuation of houses									
$vv = 32\ 35\ 45$	5820	146600	248400	0%	16%	8	877	715	1592
$vv = 33\ 35\ 45$	5840	147100	248800	0%	23%	11	870	725	1595
$vv = 33\ 35\ 46$	5840	147100	249700	0%	20%	10	964	632	1596
Changing real interest rates, discount rate $\beta = 0.97$									
$\beta = 0.97\ r = 4$	4580	147500	249200	15%	59%	22	876	724	1600
$\beta = 0.97\ r = 4.5$	5230	147500	249200	7%	39%	16	866	734	1600
$\beta = 0.97\ r = 5$	5840	147100	248800	0%	23%	11	870	725	1595
$\beta = 0.97\ r = 5.5$	6380	145700	247400	0%	9%	4	863	719	1582
$\beta = 0.97\ r = 6$	6850	144500	246200	0%	2%	1	866	705	1571
Changing real interest rates, discount rate $\beta = 0.98$									
$\beta = 0.98\ r = 4$	4580	147500	250000	7%	31%	11	945	655	1600
$\beta = 0.98\ r = 4.5$	5220	147500	250000	2%	21%	10	951	649	1600
$\beta = 0.98\ r = 5$	5830	146900	249400	0%	10%	5	948	646	1594
$\beta = 0.98\ r = 5.5$	6330	145500	247900	0%	2%	1	942	638	1580
$\beta = 0.98\ r = 6$	6890	144600	247000	0%	0%	0	942	629	1571

Prices worked out when the inflation rate $\pi = 2\%$.

“Slope” is the change in the fraction of young cohort renting when there is a 1 percentage point increase in the inflation rate.

N_h, N_f are the equilibrium number of houses and flats.

Figure 1: Price of small houses as inflation increases.
 Inelastic supply, $r=5\%$.

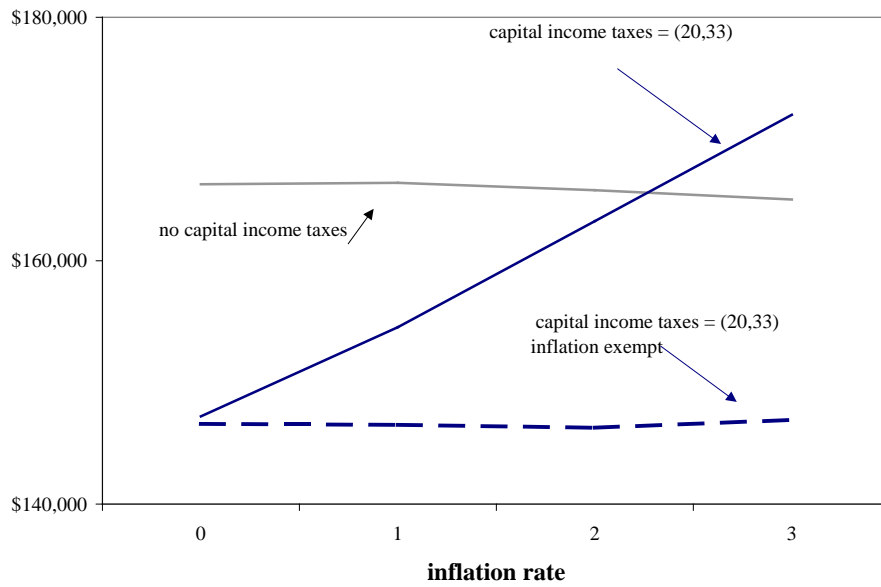


Figure 2: Fraction of young cohort renting as inflation increases.
 Inelastic supply, $r=5\%$.

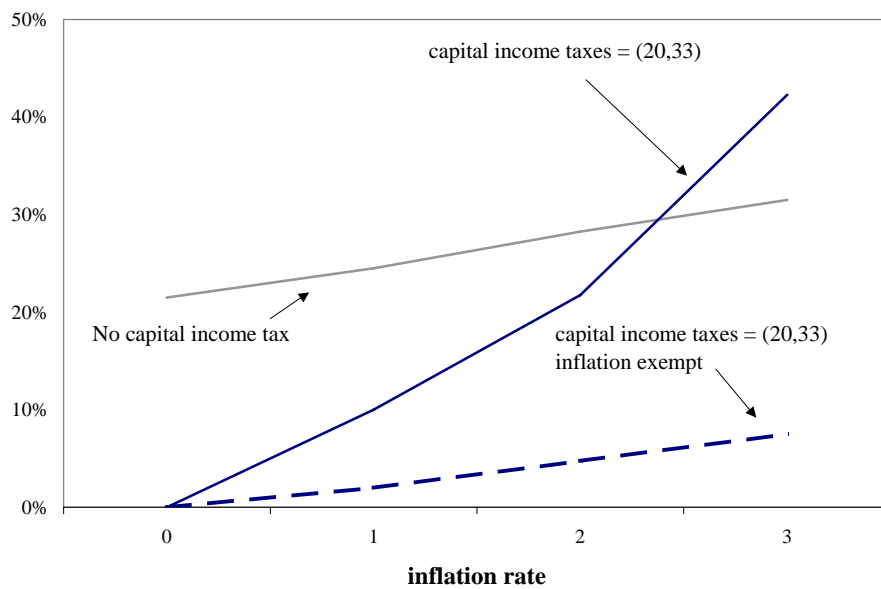


Figure 3: Fraction of young cohort renting as a function of tax and the inflation rate
 Inelastic supply, $r = 5\%$.

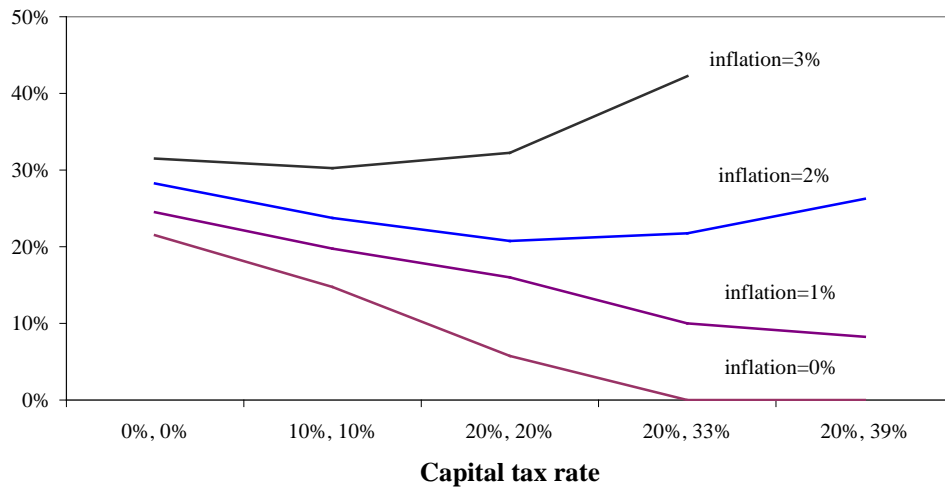


Figure 4: Rent as inflation increases.
 Elastic supply, $r = 5\%$.

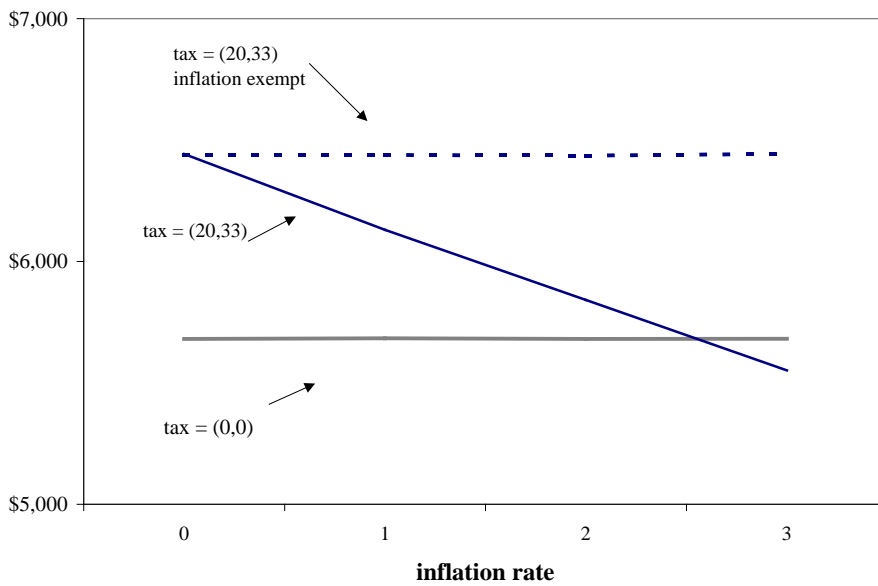


Figure 5: Welfare change when inflation increases from 0% to 3%
 Inelastic supply, tax = (20%, 33%) r= 5%

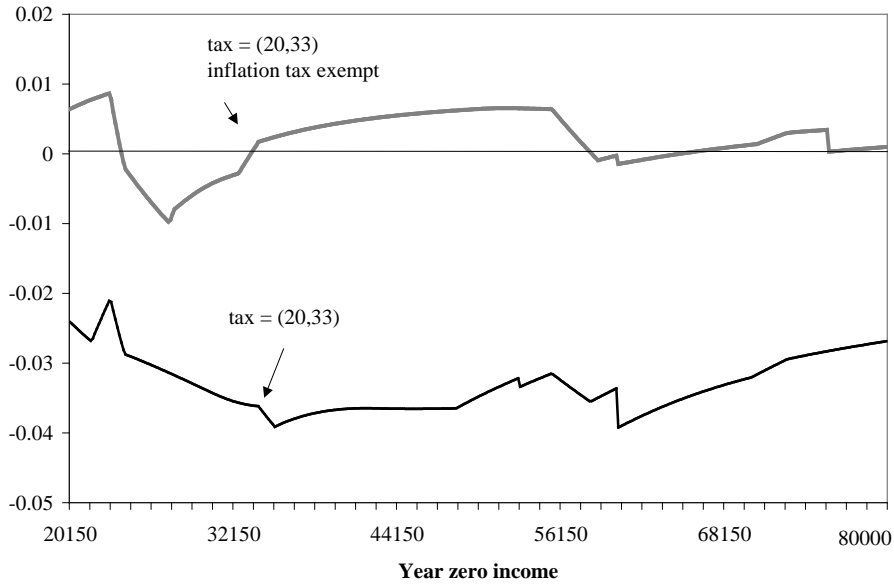
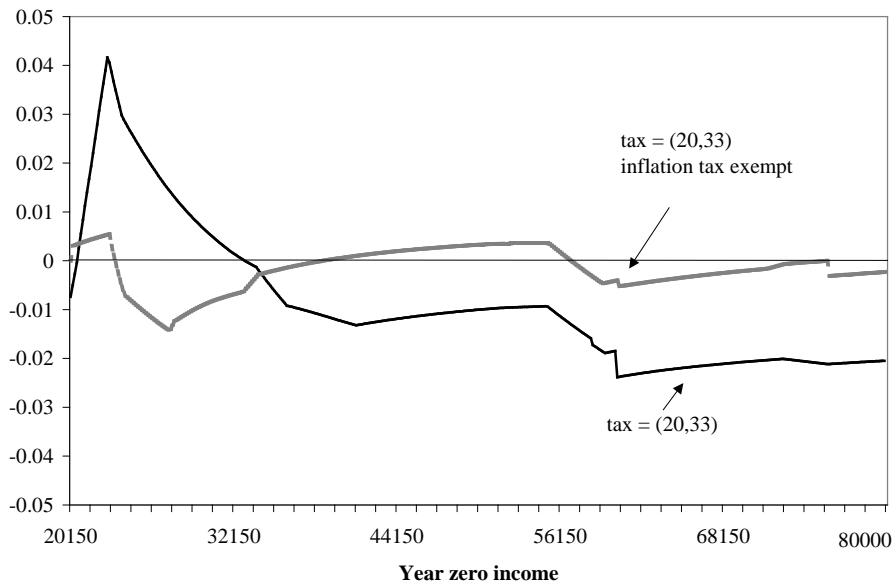


Figure 6: Welfare change when inflation increases from 0% to 3%
 Elastic and inelastic supply, tax = (20%, 33%) r= 5%.



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¹ The model has also been solved under the assumption that young households can share and rent half a house instead of living with their parents. The results are similar to those presented here.

² In a third option, odd numbered agents received no inheritance, while even numbered agents inherited two properties. The overall pattern of results was similar, showing the results were robust to the inheritance assumption.

³ If there is demand for f flats, the f highest income individuals are assumed to own one flat each.

⁴ Coleman (2009) relaxes these restrictions further, allowing agents rent in their third and fourth periods if they rent throughout their lives. The results are qualitatively similar, although the fraction of the population renting increases.

⁵ To reduce computational complexity, the marginal tax rate is based on labour income, not total income.

⁶ Note that banks impose these restrictions even though there is no uncertainty in the model

⁷ If the number of flats and houses is determined exogenously, an equilibrium can be found in which incomes in the economy grow at a constant rate, and in this case the steady state equilibrium will have property prices growing at a faster rate than the rate of inflation. If the number of properties is determined endogenously and the income growth rate is positive, the only possible steady states occur when all people live in large houses, or when the quality of flats and houses steadily improves. This paper does not analyse these cases although the model is set up to do so.

⁸ In this case the constraints in equation 11 and the aggregation condition (13c) are modified accordingly.

⁹ If a low income young person spends all his or her income, the amount he or she is willing to spend on rent is 33 percent of their income when $v^R = 0.33$.

¹⁰ Depreciation allowances should also be indexed and the inflation component of capital gains exempted from tax.

¹¹ Note that the indirect tax rate is adjusted to keep total nominal taxes the same in all cases.