

The Yield Curve in a Small Open Economy

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Abstract

Long-term nominal interest rates in a number of inflation-targeting small open economies have tended to be highly correlated with those of the United States. This observation has recently led support to the view that in these economies the long-end of the yield curve has decoupled from its short-end. We set up and estimate a micro-founded two-block small open economy model to study the co-movement of long-term nominal interest rates of different currencies. The expectations hypothesis together with uncovered interest rate parity, which both hold in our model, can account for the co-movement of interest rates observed in the data.

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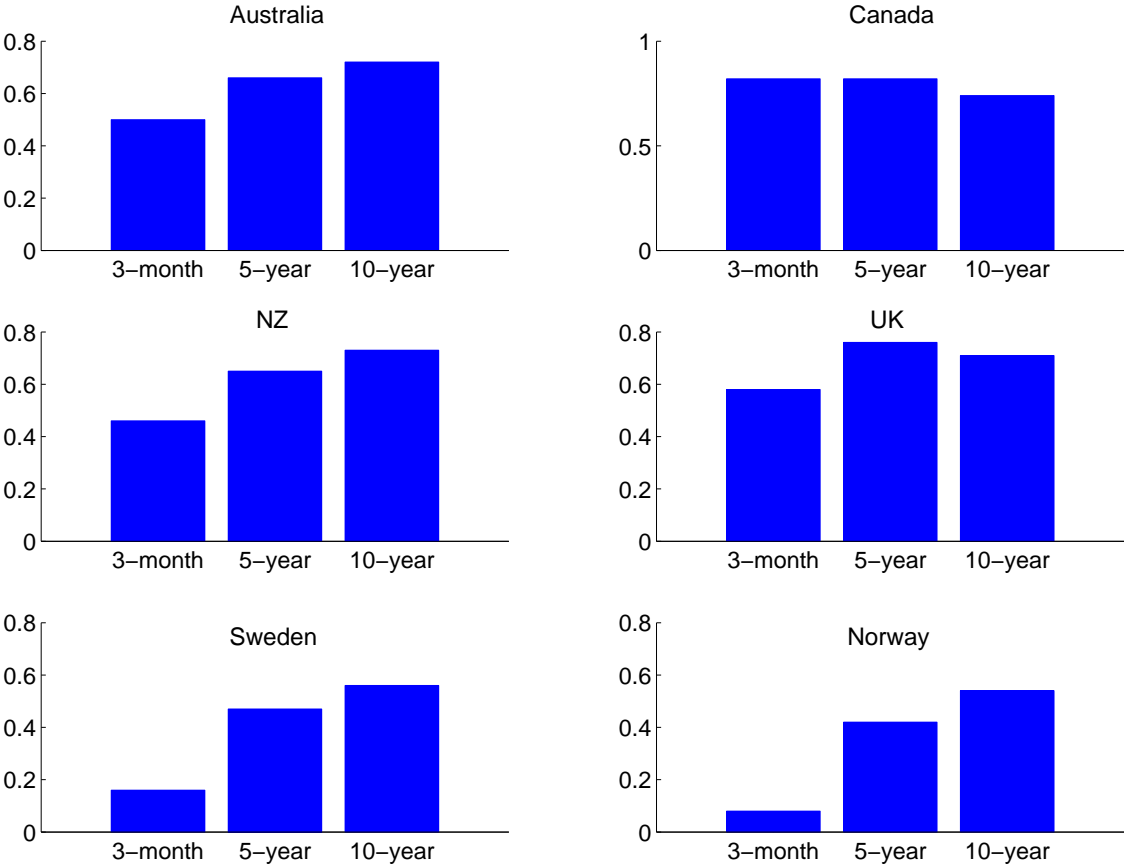
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1 Introduction

Long-term nominal interest rates in a number of inflation-targeting small-open economies, such as Australia, Canada, New Zealand, Norway, Sweden and the United Kingdom, have moved very closely with those of the United States over the past fifteen years or so. Figure 1 shows the pattern of interest rate correlations at different points on the yield curve for each country with the US.¹ The pattern is stark: long-term nominal rates are highly correlated with their US counterparts and generally more so than rates at shorter maturities. This pattern has led to the view that long-term nominal interest rates in these small economies have somehow decoupled from short-term nominal interest rates.²

Figure 1: Cross Correlations with US Interest Rates
HP-filtered quarterly data



Note: Australia, Norway, NZ, Sweden and UK cover 1993–2009; Canada covers 1997–2009.

The view that the long-end decoupled from its short-end was probably first advanced

¹The pattern is robust to the filtering of the data. For example, for the unfiltered data, in the case of Australia, these correlations are 0.56, 0.76 and 0.85 for 3 month, 5 year and 10 year maturities, respectively.

²See for example, NorgesBank [2005], Bollard [2007], and RBA [2007]

by Greenspan [2005] when referring to the behaviour of bond markets as a conundrum. In his February 2005 testimony to Congress Greenspan stated that: *“For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum. Bond price movements may be a short-term aberration, but it will be some time before we are able to better judge the forces underlying recent experience.”*

Traditionally, the transmission mechanism of monetary policy is understood as going from a short-term nominal interest rate to a long-term real interest rate, which in turn influences aggregate demand. Of course, for a short-term *nominal* rate to influence a long-term *real* rate, the short-term *nominal* rate must influence long-term *nominal* rates.

The view of a decoupled yield curve is therefore problematic. It is not a theory with refutable propositions and is inconsistent with optimizing models. This is because the theory of the yield curve that is compatible with optimizing behaviour is the expectations hypothesis, according to which the yield curve never decouples. In standard monetary models, monetary policy matters precisely because long-term nominal interest rates are never fully decoupled from the expected path of the policy rate.³

Are the reduced-form patterns of correlations of Figure 1 consistent with theory? What are the underlying forces at work in the determination of the domestic yield curve? We address these questions and find that the expectations hypothesis together with uncovered interest rate parity can account for the observed co-movement in interest rates of different currencies. Indeed, the main contribution of our paper is to uncover a mechanism that can give rise to the observed reduced-form correlations in optimizing general equilibrium; as we show below, differences in the persistence of domestic and foreign disturbances can give rise to the pattern of correlations of Figure 1. The paper offers a structural explanation—for which we find support in the data—for the reduced-form correlations of Figure 1.

Other papers have tackled the related question of how much domestic and foreign factors influence domestic interest rates. For example, Campbell and Lewis [1998] use an event study to examine how Australian bond yields respond to new information and find that US economic news has a larger effect than domestic news on Australian yields. Tarditi [1996] estimates a reduced-form model of the Australian 10-year bond yield and finds that a one percentage point increase in the US 10-year bond yield is associated with around half a percentage point increase in Australian long-term yields.

There is also a large literature that analyses the yield curve with affine term structure models. These studies typically assume that bond yields are affine functions of unobservable factors and incorporate cross-equation restrictions that eliminate arbitrage opportunities (see Backus et al. [2001], Backus and Wright [2007], Knez et al. [1994], Duffie and Kan [1996], and Dai and Singleton [2000]). But while factor models have been relatively successful in matching key statistical properties of the yield curve, factor models are not structural. Recent work addresses this issue by fitting the term structure to macroeconomic factors, either by combining them within unobserved factors, as in Ang and Piazzesi [2003] and Bernanke et al. [2004], or by incorporating a no-arbitrage model of the term structure

³This point is convincingly made by Rotemberg and Woodford [1997].

within a macroeconomic model as in Rudebusch and Wu [2004], Bekaert et al. [2006] and Hordahl et al. [2006].

Our approach here is different. We set up a micro-founded two-block model consisting of a small open economy and a large (closed) economy and extend the set of equilibrium conditions in both the large and small economies to allow for an explicit consideration of the co-movement of foreign and domestic interest rates. In our model, the expectations hypothesis links interest rates of different maturities and uncovered interest rate parity links interest rates of different currencies. Short-term nominal rates are set by the monetary authorities on the basis of the fundamentals of their economies. In this respect our analysis resembles that of Evans and Marshall [1998], but unlike them, we study the behaviour of a *small open* economy's yield curve and pay particular attention to its relation to the *large* economy's yield curve. We then estimate the model's parameters and examine its ability to match the co-movement of interest rates of different currencies. As we show below, the estimated model captures the pattern of Figure 1 well.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 examines the dynamics of the yield curve in this model. Section 4 discusses the estimation of the model and presents some additional independent evidence for differences in exogenous persistence. Section 5 then contrasts the model's moments with their empirical counterparts. Section 6 concludes.

2 The Model

We extend the Galí and Monacelli [2005] small open economy model in two ways. First, we increase the set of equilibrium conditions in both the large and small economies to incorporate interest rates of longer maturities. Second, we add foreign and domestic demand shocks. Instead of working through the details of the derivation, which are in Galí and Monacelli [2005], we discuss the log-linear aggregate equations and the role of the yield curve in the model.

2.1 The Large Economy

Variables with a *star superscript* correspond to the large economy, which obeys a standard set of New Keynesian closed economy equations.⁴ All variables are expressed in percentage deviations from their steady states.

The aggregate demand schedule links the current level of foreign output, y_t^* , to its expected future level, the *ex-ante* short-term real interest rate, and a foreign aggregate demand

⁴Ireland [2004] and Woodford [2003] contain detailed discussions of the New Keynesian closed economy model.

disturbance, g_t^* , as follows

$$y_t^* = E_t y_{t+1}^* - \sigma^{-1} (R_{1,t}^* - E_t \pi_{t+1}^*) + \sigma^{-1} (1 - \rho_g^*) g_t^* \quad (1)$$

where: $R_{1,t}^*$ is the foreign short-term nominal interest rate; π_t^* is the foreign inflation rate; $\sigma > 0$ governs intertemporal substitution; ρ_g^* is the persistence of g_t^* .

It can be shown that in this model the theory of the term structure implied by optimizing behaviour is the expectations hypothesis. Thus, the nominal interest rate at period t associated with a bond that promises to pay one unit of foreign currency at the end of period $t + m - 1$ is determined by

$$R_{m,t}^* = \frac{1}{m} E_t \sum_{j=1}^m R_{1,t+j-1}^* \quad m = 2, 3, 4, \dots \quad (2)$$

Firms operate in a monopolistically competitive goods market and are subject to Calvo-price stickiness. Factor markets are competitive and goods are produced with a constant returns to scale technology. These assumptions yield the New Phillips curve:

$$\pi_t^* = \lambda (\sigma + \varphi) y_t^* + \beta E_t \pi_{t+1}^* - \lambda (1 + \varphi) a_t^* \quad (3)$$

where: $\lambda \equiv (1 - \theta)(1 - \beta\theta)/\theta$; θ governs the degree of price stickiness; $\varphi > 0$ captures the elasticity of labour supply, $\beta \in (0, 1)$ is the households' discount factor and a_t^* is foreign total factor productivity shock.

The foreign monetary authority is assumed to follow a Taylor-type rule of the form

$$R_{1,t}^* = \rho_r^* R_{1,t-1}^* + \alpha_\pi^* \pi_t^* + \alpha_y^* y_t^* + \varepsilon_{r,t}^* \quad (4)$$

where $\varepsilon_{r,t}^*$ is an independent and identically distributed (*iid*) foreign monetary disturbance with zero mean and standard deviation $\sigma_{\varepsilon_r^*}$.

The technology shock, a_t^* , and the demand shock, g_t^* , follow autoregressive processes of the form

$$a_t^* = \rho_a^* a_{t-1}^* + \varepsilon_{a,t}^* \quad (5)$$

$$g_t^* = \rho_g^* g_{t-1}^* + \varepsilon_{g,t}^* \quad (6)$$

where: the persistence parameters, ρ_a^* and ρ_g^* , are less than unity in absolute value; and the shocks $\varepsilon_{a,t}^*$ and $\varepsilon_{g,t}^*$ are zero mean *iid* disturbances with standard deviations $\sigma_{\varepsilon_a^*}$ and $\sigma_{\varepsilon_g^*}$, respectively.

2.2 The Small Open Economy

The small economy's IS-curve links consumption, c_t , to its expected future value, the *ex-ante* short-term real interest rate, $R_{1,t} - E_t \pi_{t+1}$, and a domestic aggregate demand distur-

bance, g_t . Following Galí and Monacelli [2005] the small open economy's IS-curve can be shown to take the form

$$c_t = E_t c_{t+1} - \sigma^{-1} (R_{1,t} - E_t \pi_{t+1}) + \sigma^{-1} (1 - \rho_g) g_t \quad (7)$$

where ρ_g governs the persistence of g_t . Equilibrium in the goods market implies that domestic output, y_t , satisfies,

$$y_t = c_t + \frac{\alpha \omega}{\sigma} s_t \quad (8)$$

where $\omega \equiv \sigma \tau + (1 - \alpha)(\sigma \iota - 1)$; $\alpha \in [0, 1]$ is the share of foreign goods in the consumption basket, and therefore serves as a measure of openness; τ is the intratemporal elasticity of substitution between foreign and domestically produced goods, while ι is the elasticity of substitution across varieties of foreign goods. It is worth noting that for $\alpha = 0$, the small economy's equations reduce to the standard set of closed economy equations discussed above. Thus, the small economy has all of the structural features of the large economy, overlaid, of course, by openness. Indeed, as discussed in Galí and Monacelli [2005], the linearized equations hold around a symmetric steady state.

Finally, in Equation (8), s_t stands for the terms of trade defined as the price of foreign goods, $p_t^* + e_t$, relative to the price of domestic goods, $p_{h,t}$. In turn, the nominal exchange rate, e_t , is defined as the price of foreign currency in terms of the domestic currency. That is, $s_t = p_t^* + e_t - p_{h,t}$. Around a symmetric steady state, the consumer price index of the small economy is a weighted average of the form $p_t = (1 - \alpha)p_{h,t} + \alpha(p_t^* + e_t)$. It is straightforward to show that $p_t = p_{h,t} + \alpha s_t$, which implies that consumer price inflation and domestically-produced goods inflation are linked by the expression below.

$$\pi_t = \pi_{h,t} + \alpha \Delta s_t \quad (9)$$

In equilibrium, the nominal interest rate at t associated with a bond that promises to pay one unit of *domestic* currency at the end of period $t + m - 1$ is determined by

$$R_{m,t} = \frac{1}{m} E_t \sum_{j=1}^m R_{1,t+j-1} \quad m = 2, 3, 4, \dots \quad (10)$$

The dynamics of *domestically-produced* goods price inflation, $\pi_{h,t}$, are governed by an analogous New Phillips curve

$$\pi_{h,t} = \lambda [\varphi y_t + \sigma c_t + \alpha s_t] + \beta E_t \pi_{h,t+1} - \lambda(1 + \varphi) a_t \quad (11)$$

Monetary policy in the small economy is also assumed to follow a Taylor-type rule of the form

$$R_{1,t} = \rho_r R_{1,t-1} + \alpha_\pi \pi_t + \alpha_y y_t + \varepsilon_{r,t} \quad (12)$$

where $\varepsilon_{r,t}$ is an *iid* monetary policy shock with zero mean and standard deviation σ_{ε_r} .

The real exchange rate, q_t , is defined as $q_t \equiv e_t + p_t^* - p_t$, which implies that changes in the nominal exchange rate are equal to changes in the real exchange rate and the differential in consumer price inflation.

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^* \quad (13)$$

Combining these expressions, it is easy to show that changes in the real exchange rate are proportional to changes in the terms of trade as follows

$$\Delta q_t = (1 - \alpha) \Delta s_t \quad (14)$$

Complete international securities markets together with market clearing, imply that the international risk-sharing condition takes the form⁵

$$c_t = y_t^* + \frac{1}{\sigma} (g_t - g_t^*) + \frac{(1 - \alpha)}{\sigma} s_t \quad (15)$$

Finally, exogenous domestic processes evolve according to

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \quad (16)$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \quad (17)$$

where the shocks $\varepsilon_{a,t}$, and $\varepsilon_{g,t}$ are *iid* with zero-mean and standard deviations σ_{ε_a} and σ_{ε_g} , respectively. The persistence parameters, ρ_a and ρ_g , are as before, less than unity in absolute value.

2.3 The Foreign and Domestic Yield Curves

The linearized dynamics of the model, as we mentioned above, are valid around a symmetric steady state in which the condition of uncovered interest rate parity holds:

$$R_{1,t} = R_{1,t}^* + E_t \Delta e_{t+1} \quad (18)$$

Equation (18) is not an independent equilibrium condition because it can be recovered from the Euler equations for consumption and the international risk-sharing condition. The expectations hypothesis, Equations (2) and (10), can be combined with the uncovered interest parity condition, Equation (18), to relate foreign and domestic interest rates of equivalent

⁵Demand shocks, g_t , enter the household's lifetime expected utility as follows: $E_0 \sum_{t=0}^{\infty} \beta^t e^{g_t} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$. It follows that demand shocks enter the international risk-sharing condition as in Equation (15). In the absence of these demand shocks Equation (15) is similar to Galí and Monacelli [2005]'s one.

maturities as follows,

$$R_{m,t} = R_{m,t}^* + \frac{1}{m} \sum_{j=1}^m E_t \Delta e_{t+j} \quad (19)$$

Equations (19) highlights the fact that the expected path of the nominal exchange rate governs the degree to which movements in foreign rates are reflected into movements in domestic rates. For example, in the extreme case in which the small economy is closed, movements in foreign rates would be fully absorbed by expected movements in the exchange rate and not translate into movements in domestic rates.⁶

In this model, long-term nominal interest rates are central in the transmission of monetary policy for both economies. As emphasized by Rotemberg and Woodford [1997], in sticky-price models it is the *ex-ante* long-term real interest rate that matters for aggregate demand. The same is true in the small open economy version of the model. To see this, take the IS curve for the small economy, Equation (7), set all disturbances to zero for simplicity, and assume that the large economy is in steady state. This implies that

$$c_t = E_t c_{t+1} - \sigma^{-1} (R_{1,t} - E_t \pi_{t+1})$$

Advance the equation one period and substitute the resulting expression to obtain,

$$c_t = -\sigma^{-1} (R_{1,t} - E_t \pi_{t+1}) - \sigma^{-1} (E_t R_{1,t+1} - E_t \pi_{t+2}) + E_t c_{t+2}$$

Repeating this operation many times and using Equation (10) we may write

$$c_t = -\sigma^{-1} m \left(R_{m,t} - \frac{1}{m} E_t \sum_{j=1}^m \pi_{t+j} \right) \quad (20)$$

since in a stationary equilibrium $E_t c_{t+m}$ is approximately zero for large m . Equation (20) reveals that the current level of consumption depends on an *ex-ante* long-term real interest rate whose impact is magnified by maturity and scaled by the economy's degree of intertemporal substitution. If the small economy were closed, Equation (20) would hold also for output, because if $\alpha = 0$, then $c_t = y_t$.

Thus, the sticky-price small open economy model puts long-term nominal interest rates at the very heart of the transmission mechanism in much the same way as the closed economy sticky-price model does: the expectations hypothesis gives monetary policy leverage over long-term nominal interest rates and nominal rigidities translate this leverage to long-term real interest rates which in turn affect aggregate demand.

It is worth noting that the theory of monetary transmission embedded in the model stands in contrast to any hypothesis of a decoupled yield curve; in this model, the yield curve is always and everywhere linked to the expected future path of its policy rate. In the

⁶This would be the case in our model if $\alpha = 0$. It would also be the case in the infinitely many equilibria consistent with a constant interest rate rule of the form $R_{1,t} = R_{1,t-1}$.

next section we show how this model can nevertheless bring about the co-movement between the yield curves that we observed in the data.

3 Dynamics and Persistence

The model, given by Equations (1) to (17), can be viewed as a theory about the joint determination of the large and small economy's yield curves. According to the model, any co-movement between foreign and domestic variables, which does not happen by chance, has to be driven by foreign shocks. These are the shocks that generate co-movement. Domestic shocks, on the other hand, reduce co-movement because they influence domestic variables alone.⁷ So, the relative size of foreign shocks—relative to that of domestic shocks—is an important determinant of the overall level of correlation between foreign and domestic interest rates.

The model predicts that a small economy which is subject to relatively large idiosyncratic shocks should find little correlation between its yield curve and that of foreign economies.⁸ This explains why, in the empirical analysis that follows which uses Australian and US data, we restrict our attention to the Australian inflation-targeting period. If one considers the inflation target—like Ireland [2007]—as an additional shock to which an economy may be exposed to, then an inflation-targeting framework can be thought to remove variations that originate from perceptions of a changing inflation target. Such variations are particularly important for the correlations of Figure 1 because they affect inflation expectations, which are important determinants of long-term nominal interest rates. Thus, relatively stable inflation expectations in the small economy is a necessary—though not sufficient—condition for the high correlation that we observed in Figure 1.

The discussion so far explains how the model determines the *level* of the correlations of Figure 1. Next we discuss what can explain the *slope* of the pattern of correlations of Figure 1. To develop the intuition it is useful to parameterize the model symmetrically. Below we will estimate these parameters and ask the data for its preferred values, but for the time being, they are set as shown in Table 1. As the unconditional second moments are highly non-linear functions of the structural parameters, we can gain insights by analyzing the impulse responses of the yield curve.

Figure 2 shows two sets of impulse responses to a domestic demand shock. One for which the persistence of g_t is assumed to be low and one for which the persistence of g_t is assumed to be high. The less persistent the shock, the smaller the impact on longer-term rates. The intuition for this is straightforward: less persistent shocks induce less persistent

⁷This is true under uniqueness of the rational expectations equilibrium. See Jääskelä and Kulish [2007] for an analysis of indeterminacy of the equilibrium in which this is not the case.

⁸As an example, take the case of Argentina for the period 1997Q3-2008Q4 for which the correlation between the Argentinean and the US short-term rates is -0.22. During this period large idiosyncratic shocks buffeted Argentina. In the last quarter of 2001 the Argentinean government defaulted on its debt and short-term rates spiked to 117 percent per annum. At the same time in the US, the Federal Reserve, concerned with deflation, cut the Federal Funds rate.

Table 1: **Symmetric Parametrization**

σ^{-1}	1.0	β	0.99	λ	0.3	α	0.4
φ	0.9	ω	1.0	ρ_r^*	0.8	ρ_r	0.8
α_π^*	0.5	α_π	0.5	α_y^*	.01	α_y	.01
ρ_g^*	0.9	ρ_g	0.9	ρ_a^*	0.9	ρ_a	0.9

expected movements of the short-term rate. Long-term rates depend on the expected path of the short rate, and so, they too respond less strongly.

Perhaps less obvious is that a less persistent shock moves shorter-rates on impact by more than a more persistent shock. This is because when the demand process is less persistent output and inflation respond on impact by more. To understand why recall that g_t enters the household's lifetime expected utility as follows $E_0 \sum_{t=0}^{\infty} \beta^t e^{g_t} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$. So, the difference between the marginal utilities in the impact period and that in subsequent periods is greater when the demand shock process is not persistent.

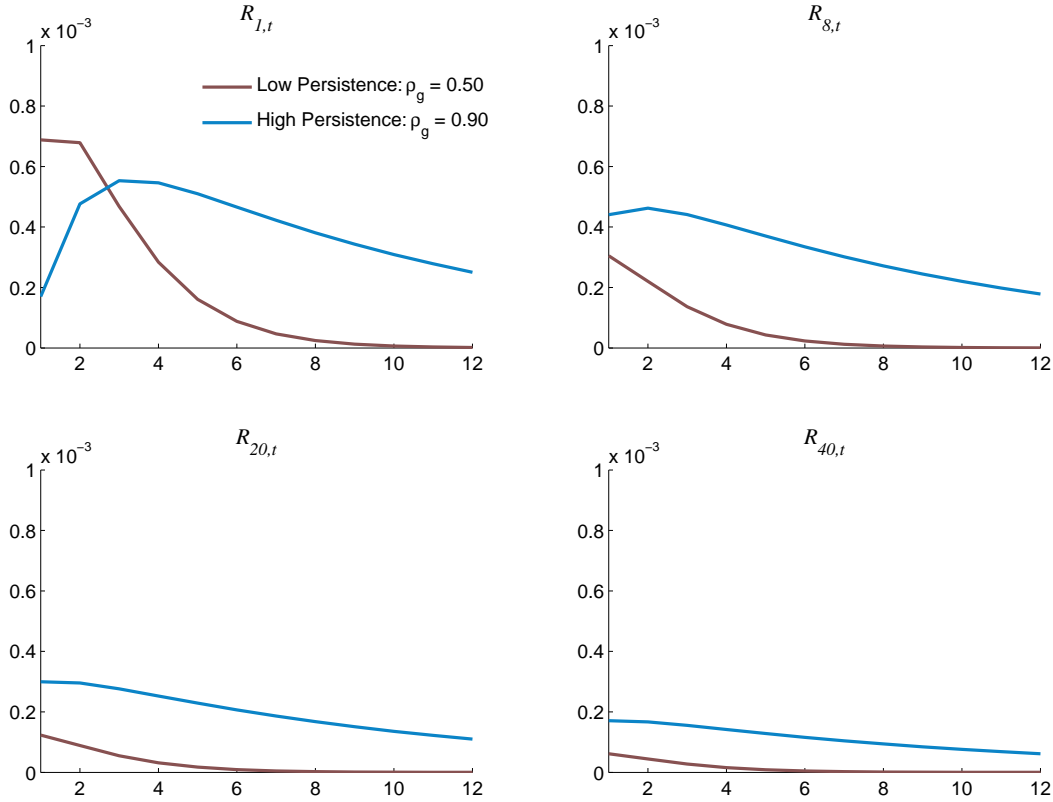
Take the case in which the g_t process exhibits no persistence, $\rho_g = 0$; the difference that a shock to g_t causes between the marginal utilities of period t and that of future periods is as large as possible. Forward looking agents respond to this by bringing forward future consumption and cutting the contemporaneous supply of hours by relatively more, which results in relatively more upward pressure on inflation. Monetary policy, in turn, responds by increasing the short-term rate by more than it would otherwise. In contrast, forward looking agents would respond contemporaneously little when the shock to g_t is known to be long lasting. Agents with rational expectations know that what has magnified their utility today, g_t , will also do so tomorrow, $\rho_g g_t$. In other words, the difference between g_t and $\rho_g g_t$ decreases as the persistence of the shock increases.⁹

As Figure 3 illustrates the response of the yield curve is similar in the case of technology shocks: more persistent shocks move long-term rates by more than less persistent shocks would; and less persistent shocks move short-term rates by a greater degree than more persistent shocks would.

The mechanism in the case of technology shocks, however, is different. Firms' demand for labour increases after a positive shock to technology. When the shock is known to be highly persistent, agents expect real wages to remain high for a long time. As consumption depends on permanent income, consumption demand increases by more than it would otherwise. Firms, in turn, face a higher demand than they would otherwise, which allows them to hire relatively more labour. The production function, $y_t = a_t + n_t$, and the fact that the value of a_t on the impact period is the same regardless of the persistence of the process, implies that output would expand by more than it would if the shock were less persistent.

⁹In the extreme case in which g_t followed a unit root, demand shocks would operate merely by re-scaling the intertemporal utility function. Because preferences are ordinal (as opposed to cardinal) defined, these shocks would then have no impact on behaviour.

Figure 2: Impulse Responses to a Domestic Demand Shock



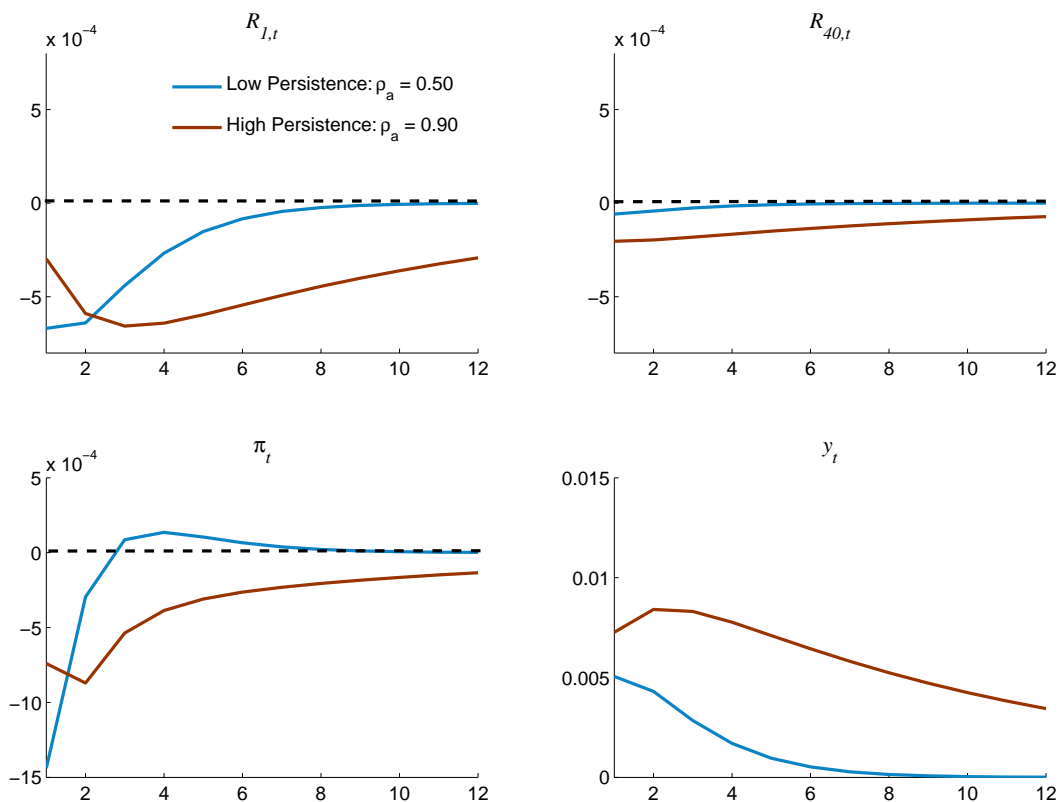
Note: The size of the shock is one standard deviation set at 0.01.

When the shock is known to have low persistence, the expected income stream from higher real wages in the future would be lower and consumption would increase by less than it would otherwise. For markets to clear, prices would have to fall by more than they would otherwise to induce households to purchase the increased output. But at the same time, firms can lower prices by more as the real marginal cost of production falls relatively more after a less persistent positive shock to technology. Monetary policy would therefore react more strongly initially because the fall in inflation is more pronounced the less persistent the shock.¹⁰

Figure 4 shows impulse responses of both domestic and foreign yield curves for two different parameterizations of the persistence of the foreign demand process: $\rho_g^* = 0.9$ and

¹⁰There is one caveat. The intuition changes, however, if monetary policy reacts strongly to output. It would then be possible for the short-rate to move by a greater amount in the more persistent case. As output expands by more in the highly persistent case, a sufficiently strong response to output could move the short-rate contemporaneously to a greater degree. This is because output and inflation move in opposite directions, which implies that the overall direction of the response of the short-rate is ambiguous. The response would then depend on the particular settings of the rule. If the policy rule were to be specified in terms of the output gap, however, then the response of the short-rate would be unambiguous and less persistent shocks would unambiguously move the short-rate more strongly.

Figure 3: Impulse Responses to a Domestic Technology Shock

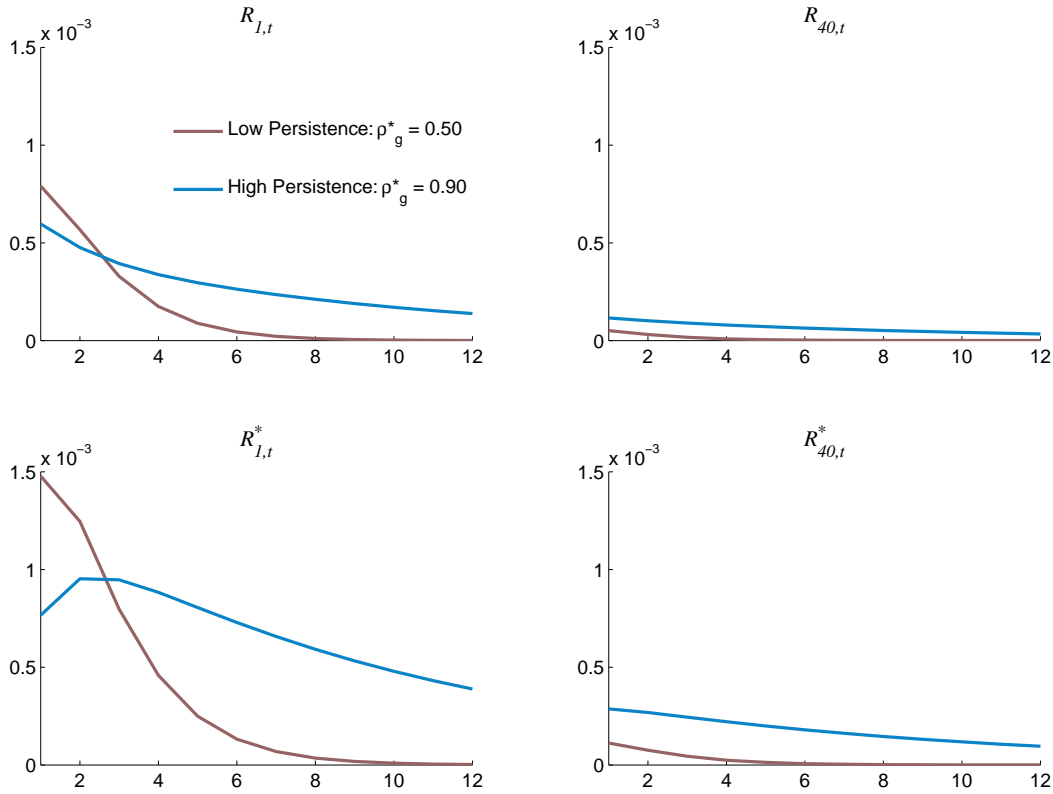


Note: The size of the shock is one standard deviation set at 0.01.

$\rho_g^* = 0.5$. As before, the less persistent the shock, the smaller is the impact on both foreign and domestic longer-term rates and the larger is the impact on both foreign and domestic short-term rates. Clearly, foreign shocks, unlike domestic ones, constitute a source of variation for both yield curves and therefore drive up the overall level of co-movement between interest rates of all maturities and currencies.

The key to understanding the model's ability to reproduce the pattern of correlations of Figure 1 is that less persistent domestic shocks produce a source of variation in the yield curve that is relatively stronger at the short-end of the yield curve than at the long-end of the yield curve. Other things equal, if the persistence of a domestic shock decreases, the correlation between the short-term rates of the two economies would decrease, while the correlation between their longer-term rates would increase. The correlation at the long-end increases because, if the persistence of domestic shocks decreases, foreign shocks – the cause of variability of foreign rates – become a relatively more important source of variation for domestic long-term rates. Recall that domestic shocks, regardless of their persistence, are a source of variation for the domestic yield curve but not one for the foreign yield curve. For the model to be able to produce the upward sloping pattern of correlations of Figure 1 foreign shocks have to be relatively more persistent than domestic shocks. In the next sec-

Figure 4: Impulse Responses to a Foreign Demand Shock



Note: The size of the shock is one standard deviation set at 0.01.

tion, we will estimate the model and ask the data if this is indeed the case.

4 Estimation

4.1 2-Step Estimation

For estimation purposes, the discount factor, β , is set at 0.99, which at a quarterly frequency corresponds to a steady-state real rate of interest of 4.1 per cent. The degree of openness, α , is set at 0.4, consistent with the value of the share of foreign goods in the Australian consumption basket.¹¹

The rest of the model's parameters are estimated with Bayesian techniques, as discussed in An and Schorfheide [2006], Lubik and Schorfheide [2005] and Griffoli [2007].¹² We do so

¹¹In preliminary attempts to estimate the model, we found that α would invariably tend towards zero for a range of prior distributions.

¹²We used the MATLAB package Dynare for the estimation of the model. The code that reproduces our

Table 2: **Large Economy**

Parameters	Prior	Posterior	Posterior			Prior	Prior Std.
	Mean	Mode	Mean	90 percent C.I.		Density	
σ^{-1}	1.500	1.485	1.786	[0.926	2.597]	Gamma	0.500
λ	0.300	1.073	1.103	[0.822	1.381]	Normal	0.250
φ	0.900	1.236	1.318	[0.913	1.725]	Gamma	0.200
ρ_r^*	0.900	0.906	0.903	[0.838	0.970]	Beta	0.050
α_π^*	0.500	0.463	0.521	[0.382	0.659]	Normal	0.200
α_y^*	0.250	-0.036	-0.041	[-0.069	-0.012]	Normal	0.100
ρ_g^*	0.500	0.858	0.862	[0.795	0.927]	Beta	0.150
ρ_a^*	0.500	0.909	0.891	[0.832	0.956]	Beta	0.150
Standard Deviations							
$\sigma_{\varepsilon_a^*}$	0.015	0.004	0.004	[0.003	0.005]	Uniform	.009
$\sigma_{\varepsilon_g^*}$	0.020	0.010	0.011	[0.007	0.015]	Uniform	.011
$\sigma_{\varepsilon_r^*}$	0.015	0.002	0.002	[0.0017	0.003]	Uniform	.009

in two steps: in the first, we estimate the large economy's parameters; and in the second, we estimate the remaining small economy's parameters, taking the posterior mode values of the common parameters as given from the first step.

Since our focus is on the cross correlations of domestic interest rates with their US counterparts, we take the US to be the large economy. We use quarterly HP-filtered data on real US GDP per capita, US CPI-inflation and a US 3-month nominal interest rate for the sample period 1983Q1-2008Q4.¹³

Table 2 summarizes results for this first step of the estimation. These are in line with those of previous studies. Like Ireland [2004], we find exogenous processes to be highly persistent and demand disturbances to be the most important source of aggregate fluctuations. And also, just as Ireland [2004] and Lubik and Schorfheide [2004], we find that monetary policy responds much more strongly to inflation than it does to output.

Since the large economy is exogenous to the small economy, we take the posterior mode parameter values as given from the first step of estimation and estimate the remaining small open economy parameters on Australian and US data. For the small economy we use quarterly HP-filtered data on real GDP per capita, CPI-inflation and a 3-month nominal interest rate for the sample period 1993Q1-2008Q4. We take the same foreign variables as in the first step, y_t^* , π_t^* and $R_{1,t}^*$, for the sample period 1993Q1-2008Q4. Table 3 below summarizes results for this second step of the estimation.

Our estimation results from this second step are also in line with those found in other

results is available upon request.

¹³Appendix A contains a description of the data sources.

Table 3: **Small Economy**

Parameters	Prior	Posterior	Posterior			Prior	Prior Std.
	Mean	Mode	Mean	90 percent C.I.		Density	
ω	2.000	1.141	1.179	[0.855	1.496]	Gamma	0.500
ρ_r	0.900	0.929	0.915	[0.855	0.977]	Beta	0.050
α_π	0.500	0.565	0.622	[0.441	0.796]	Normal	0.200
α_y	0.250	0.080	0.077	[0.014	0.139]	Normal	0.100
ρ_a	0.500	0.570	0.574	[0.426	0.726]	Beta	0.150
ρ_g	0.500	0.615	0.615	[0.467	0.767]	Beta	0.150
Standard Deviations							
σ_{ε_a}	0.015	0.004	0.005	[0.004	0.005]	Uniform	0.009
σ_{ε_g}	0.015	0.011	0.012	[0.009	0.015]	Uniform	0.009
σ_{ε_r}	0.005	0.002	0.002	[0.002	0.003]	Uniform	0.003

studies. The persistence parameters, as we emphasized above, are of particular importance for the model to be able to match the pattern of Figure 1; our estimates for these parameters are similar to those of other studies on estimated small open economy models.¹⁴

Figure 5 shows the prior and posterior densities for the AR(1) parameters that govern exogenous persistence. Our choice of prior distributions is the same for foreign and domestic exogenous processes. In this way, our priors are silent about differences in exogenous persistence. Notice, however, that the data shift the prior distributions of the *foreign* AR(1)s towards more persistent processes; as shown, the posterior distributions are fairly tight around highly persistent values. And although for the small economy, the data too prefer more persistent processes relative to the priors, the posterior distributions are centered around less persistent processes than those of the foreign economy.

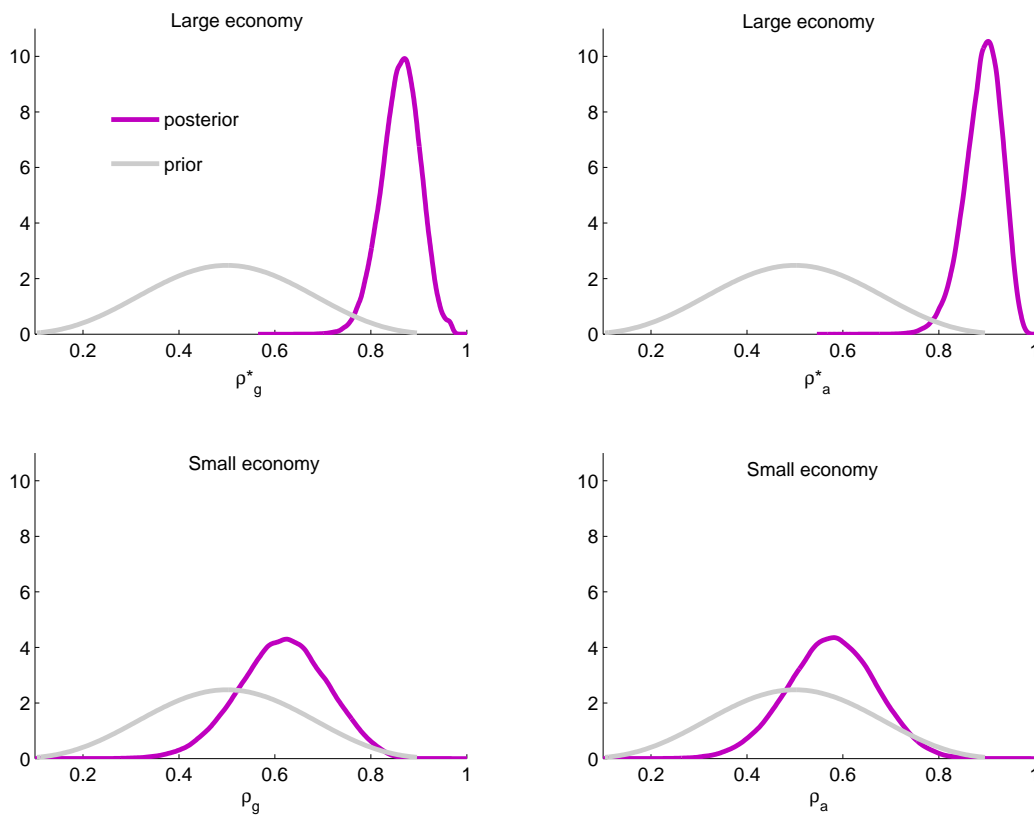
4.2 Independent Evidence of Differences in Persistence

The estimates from Tables 2 and 3 suggest that US exogenous shock processes are considerably more persistent than their Australian counterparts. This result plays an important role, as we discussed above, in determining the model's ability to match the reduced form cross-country interest rate correlations.

As additional evidence to support the plausibility of this result, we estimate the shock processes of Equations (5) (6), (16), and (17), directly from the data. We focus first on technology shocks. Using the fact that the the model's production function implies that output per capita is the product of hours worked per capita and technology, we construct a technology series for the US and Australia. We then regress the HP-filtered technology series

¹⁴See for example, Nimark [2009] and Kam et al. [2009], studies that also use Australian data, who find values for the persistence parameters which are in line with our findings.

Figure 5: Prior and Posterior Distributions



for each country on a single lag of itself. The coefficient on the lagged technology variable represents the estimate of the persistence of the technology disturbances in each country. Table 4 shows the results of this exercise. Columns (1) and (4), which correspond to the samples used in the estimation of our model, support the conclusion that US technology shocks are more persistent than their Australian counterparts. Columns (2) and (3) illustrate that this result is robust to alternative sample periods.

Although there is no obvious mapping from the model's demand shocks into an observed data series, variations in government final consumption expenditure could represent a plausible exogenous source of demand disturbances. Table 4 shows the results of regressions of HP-filtered government final consumption expenditure on a single lag of itself for both the US and Australia. Interpreting the coefficients from these regressions as indicators of the persistence of demand shocks in the two countries, we once again find that US shocks are considerably more persistent than their Australian counterparts over the samples used in our model-based estimation (columns (1) and (4)). These results are also robust to alternative choices of samples.¹⁵

¹⁵The results are also broadly similar if we use the sum of government final consumption expenditure and public gross fixed capital formation for each country. However, we prefer the former variable as the Australian public gross fixed capital expenditure series is distorted by the privatization of several large public business enterprises during our sample.

Table 4: **Independent evidence**

	United States		Australia	
	1983Q1 - 2008Q4	1993Q1 - 2008Q4	1983Q1 - 2008Q4	1993Q1 - 2008Q4
	(1)	(2)	(3)	(4)
Technology				
a_{t-1}^*	0.83 (15.69)	0.81 (11.00)	a_{t-1} 0.44 (5.16)	0.69 (7.81)
R^2	0.71	0.66	0.21	0.50
Demand				
g_{t-1}^*	0.71 (10.19)	0.69 (7.35)	g_{t-1} 0.21 (0.10)	0.36 (3.02)
R^2	0.50	0.46	0.04	0.13

Note: t-statistics in brackets

5 Unconditional Moments

Table 5 compares the theoretical standard deviations of output, inflation, the nominal exchange rate and nominal interest rates, all computed at the posterior mode values of the parameters with their empirical counterparts. The model matches accurately the volatility of output, the volatility of the short-term interest rate and the volatility of inflation, but under-estimates that of the change in the nominal exchange rate and long-term interest rates. These latter results are not surprising, as it is well-known since Shiller [1979]’s and Meese and Rogoff [1983]’s findings of ‘excess volatility’, that models such as this fail to capture either the variability of long-term interest rates or the variability of the nominal exchange rate.

Table 6 shows the correlations as computed in the data for foreign interest rates, domestic interest rates, output, the change in the nominal exchange rate and inflation with their theoretical counterparts. The correlations between foreign and domestic interest rates are well-captured by the model. As Figure 6 shows, the model, at the posterior mode, generates the upward sloping pattern of correlations between interest rates of equivalent maturities that we observe in Figure 1.

The model is successful in matching these point estimates. And although, it seems to do worse in matching other moments, like those between output and the yield curve, this is not really the case. Point comparisons can be misleading. As it turns out, the correlations between the yield curve and output are generally not statistically significant and the same is true for the correlations that involve the nominal exchange rate. Table 7 illustrates this by removing from Table 6 those correlations which are not significant at the 5 per cent level in the data and according to the model’s posterior distribution. The estimated model does quite well as it tends to find zero or low correlations between variables which have no

Table 5: **Standard Deviations**

	Model (posterior mode)	90 per cent Confidence Interval		Data (1993:1 - 2008:4)
y_t	0.0064	[0.0053	0.0087]	0.0068
π_t	0.0042	[0.0030	0.0052]	0.0036
Δe_t	0.0087	[0.0077	0.0097]	0.0465
$R_{1,t}$	0.0023	[0.0021	0.0026]	0.0019
$R_{20,t}$	0.0005	[0.0004	0.0006]	0.0020
$R_{40,t}$	0.0002	[0.0002	0.0003]	0.0018

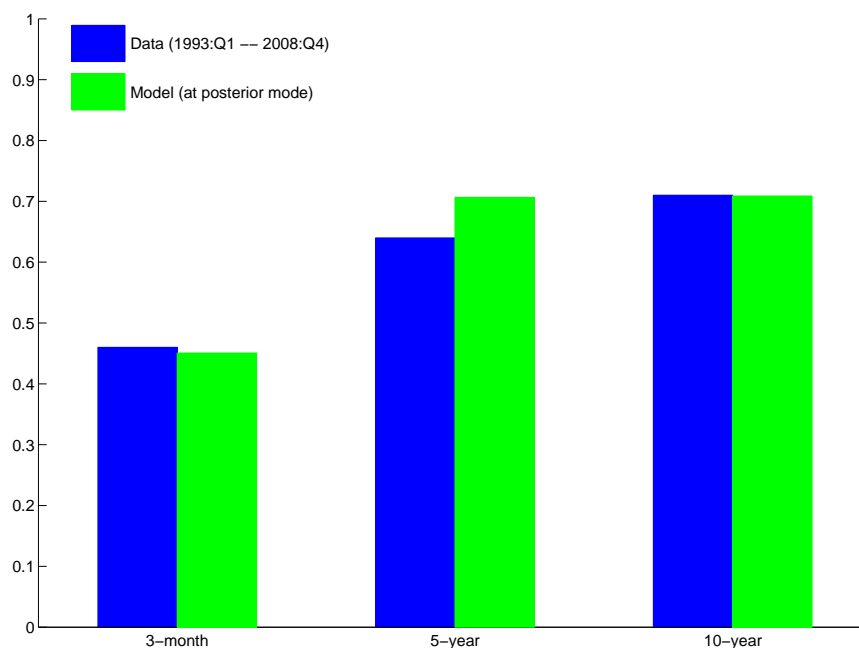
Table 6: **Contemporaneous Correlations**

Data (1993:Q1–2008:Q4)									
	y_t	π_t	Δe_t	$R_{1,t}$	$R_{20,t}$	$R_{40,t}$	$R_{1,t}^*$	$R_{20,t}^*$	$R_{40,t}^*$
y_t	1.00								
π_t	0.11	1.00							
Δe_t	-0.06	0.16	1.00						
$R_{1,t}$	0.23	0.41	0.09	1.00					
$R_{20,t}$	0.29	0.33	-0.05	0.78	1.00				
$R_{40,t}$	0.23	0.34	-0.04	0.72	0.98	1.00			
$R_{1,t}^*$	0.13	0.11	0.02	0.46	0.37	0.30	1.00		
$R_{20,t}^*$	0.21	0.15	0.16	0.48	0.64	0.60	0.81	1.00	
$R_{40,t}^*$	0.18	0.17	0.16	0.50	0.73	0.71	0.64	0.96	1.00
Model (posterior mode)									
	y_t	π_t	Δe_t	$R_{1,t}$	$R_{20,t}$	$R_{40,t}$	$R_{1,t}^*$	$R_{20,t}^*$	$R_{40,t}^*$
y_t	1.00								
π_t	0.02	1.00							
Δe_t	-0.29	0.29	1.00						
$R_{1,t}$	-0.12	0.26	0.00	1.00					
$R_{20,t}$	-0.03	0.27	0.02	0.92	1.00				
$R_{40,t}$	-0.03	0.26	0.03	0.90	1.00	1.00			
$R_{1,t}^*$	0.00	0.08	0.11	0.45	0.71	0.71	1.00		
$R_{20,t}^*$	-0.01	0.09	0.08	0.45	0.71	0.71	0.99	1.00	
$R_{40,t}^*$	-0.01	0.09	0.08	0.44	0.70	0.71	0.99	1.00	1.00

significant correlations in actual data.

It is informative to take into account the distributions of these moments as opposed to point estimates. We construct the prior and posterior densities of the second moments

Figure 6: Cross Correlations with US Interest Rates



by taking 10,000 draws of the parameters from the prior density and another 10,000 draws from the posterior density. For each draw we calculate the correlations of interest. These are plotted in Figure 7 which shows the distributions of the correlation coefficients for interest rates, output and inflation. The correlations of the data along with their 95 per cent confidence intervals are shown as dashed lines.

Consider, for example, the top left panel of Figure 7. This shows the joint posterior distributions of the correlation of the Australian and US 3-month interest rate with the correlation of the Australian and US 5-year interest rate. These empirical correlations lie within the model's posterior. This implies that, not only can the model match independently the correlation of Australian and US 3-month and 5-year bond rates, but also that it matches these moments jointly. In other words, the model does not need to sacrifice fit at the short end of the yield curve to match the co-movement at the long end of the yield curve. This is also the case in the remaining two panels on the left side of Figure 7, which show the joint correlations between the 3-month and 10-year, and between the 5-year and 10-year interest rates. The model does also reasonably well on the other correlations, where the mass of the posterior distributions gets close to the data. Take for example the top right panel of Figure 7, which shows the joint correlations of the Australian 3-month interest rate with inflation and output. Taking into account that the correlation between the short-term interest rate and output is not significant, it is not surprising to find that the mass of the posterior distribution centers close to zero.

But apart from the performance on particular correlations, it is worth stressing that in all cases, the posterior densities are tighter and closer to the data than the prior densities. The data help shift the estimated distributions away from the prior densities and towards

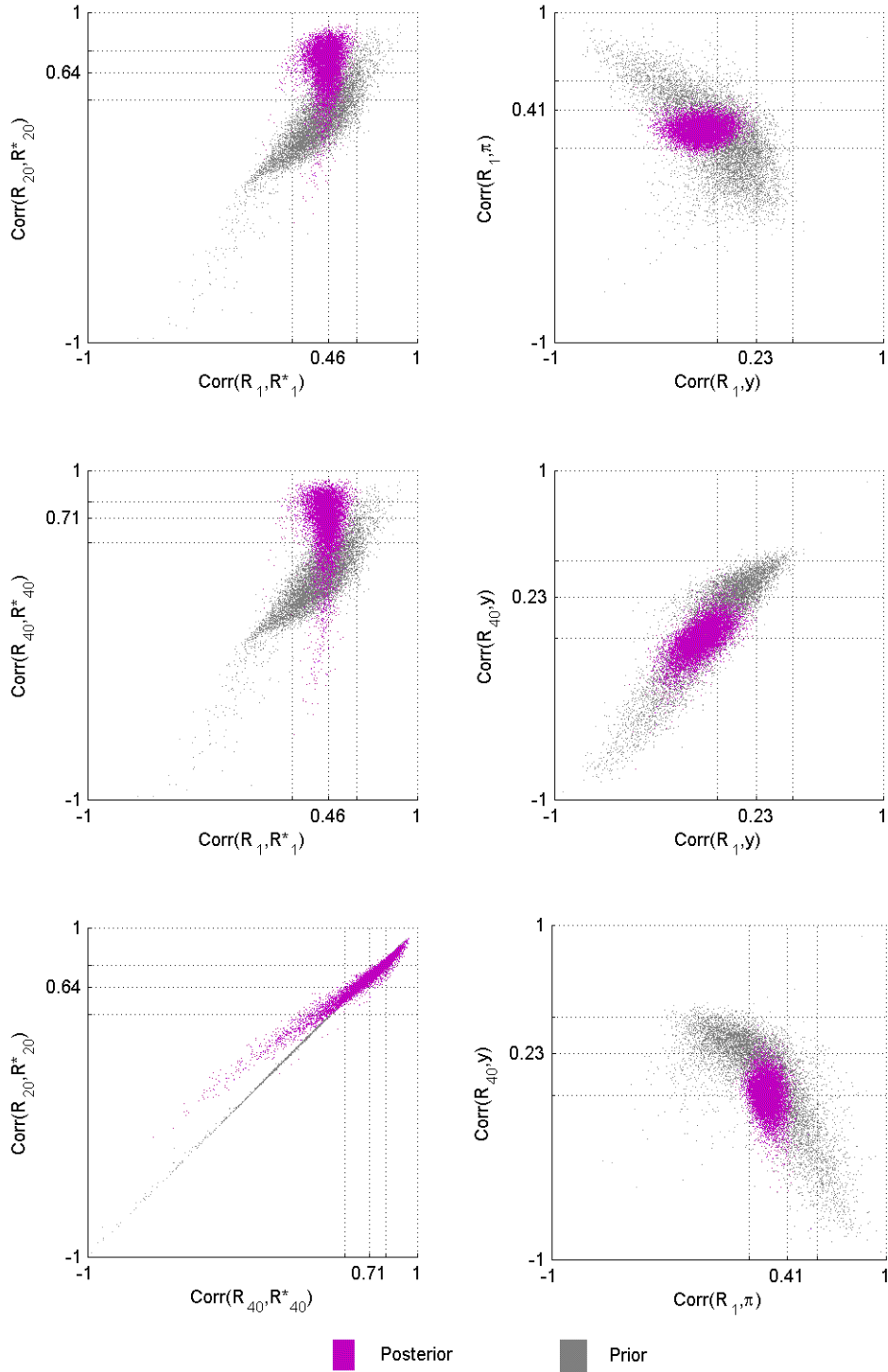
Table 7: **Statistically Significant Correlations**

Data (1993:Q1–2008:Q4)									
	y_t	π_t	Δe_t	$R_{1,t}$	$R_{20,t}$	$R_{40,t}$	$R_{1,t}^*$	$R_{20,t}^*$	$R_{40,t}^*$
y_t	1.00								
π_t	–	1.00							
Δe_t	–	–	1.00						
$R_{1,t}$	–	0.41	–	1.00					
$R_{20,t}$	0.29	0.33	–	0.78	1.00				
$R_{40,t}$	–	0.34	–	0.72	0.98	1.00			
$R_{1,t}^*$	–	–	–	0.46	0.37	0.30	1.00		
$R_{20,t}^*$	–	–	–	0.48	0.64	0.60	0.81	1.00	
$R_{40,t}^*$	–	–	–	0.50	0.73	0.71	0.64	0.96	1.00
Model (posterior mode)									
	y_t	π_t	Δe_t	$R_{1,t}$	$R_{20,t}$	$R_{40,t}$	$R_{1,t}^*$	$R_{20,t}^*$	$R_{40,t}^*$
y_t	1.00								
π_t	–	1.00							
Δe_t	-0.29	0.29	1.00						
$R_{1,t}$	–	0.26	–	1.00					
$R_{20,t}$	–	0.27	–	0.92	1.00				
$R_{40,t}$	–	0.26	–	0.90	1.00	1.00			
$R_{1,t}^*$	–	0.08	–	0.45	0.71	0.71	1.00		
$R_{20,t}^*$	–	0.09	–	0.45	0.71	0.71	0.99	1.00	
$R_{40,t}^*$	–	0.09	–	0.44	0.70	0.71	0.99	1.00	1.00

the location of the empirical correlations. In fact, the posteriors shrink considerably the uncertainty relative to the priors. Although this might not be necessarily surprising in the case of series which are observables in estimation—as one would expect the data to be informative about these moments—it is surprising in the case of those series which are unobserved in estimation, such as the yield curve.

The analysis of Section 3 and the evidence about differences in persistence of Section 4 foreshadowed that the model’s posterior distribution would have a decent chance to match the yield curve moments of Figure 1. Nonetheless, Figure 7 emphasizes not only that the priors alone, like those of Figure 5, do not match the yield curve moments of Figure (1), but also that the model being estimated on inflation, output and short-term interest rate data alone generates the differences in persistence needed to explain the pattern of Figure 1.

Figure 7: Prior and Posterior Joint Distributions of the Correlations Between Interest Rates, Inflation and Output



6 Conclusion

Recently, long-term nominal interest rates in inflation-targeting small open economies, like Australia, Canada, New Zealand, Norway, Sweden and the United Kingdom, have moved very closely with those of the United States. This observation has given support to the view that the long-end of the domestic yield curve is decoupled from its domestic policy rate, and with it, to a concern that monetary policy in these small open inflation-targeting economies may be weaker than it otherwise might be.

In this paper we have set up and estimated a fully micro-founded two-country small open economy model and incorporated long-term nominal interest rates to study the co-movement of interest rates of different currencies. We have shown that the reduced-form correlations at the short- and long-end of the domestic and foreign yield curves can be explained by the model, in which the expectations hypothesis and uncovered interest rate parity hold, remarkably well. In particular, if foreign shocks are more persistent than domestic shocks, a hypothesis for which we find support in recent data, then it makes sense for long-term nominal interest rates in the small and large economies to become highly correlated. These correlations are not evidence that long-term interest rates have decoupled in any way, nor are they evidence of weaker monetary policy.

A Data Descriptions and Sources

Australian gross domestic product *per capita*: Seasonally adjusted quarterly real Australian GDP *per capita* (ABS Cat. 5206.0).

Australian CPI inflation: Percentage change in the seasonally adjusted Australian consumer price index excluding interest and taxes.(ABS Cat. 6401.0).

Australian interest rates: Quarterly average of monthly constant maturity rates (Reserve Bank of Australia).

US gross domestic product *per capita*: Seasonally adjusted quarterly real US GDP *per capita* (Bureau of Economic Analysis National Income and Product Accounts).

US CPI inflation: Percentage change in the seasonally adjusted quarterly average US consumer price index (Datastream code: USCP..F).

US interest rates: Quarterly average of monthly constant maturity rates (Federal Reserve Economic Data).

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