

Policy Trade-offs and International Spillover Effects at the Zero Bound*

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Abstract

This paper examines how the policy trade-off facing the central bank of a small open economy is affected by the presence of the zero lower bound on nominal interest rates both at home and abroad. We demonstrate that the inability of the central bank abroad to stabilise inflation and the output gap generates a spillover for the small open economy that gives rise to a trade-off between inflation and output gap stabilisation. Depending on the trade structure between the small open economy and the rest of the world, these spillovers may improve or worsen the trade-off in the small open economy. This in turn will have an effect on the length of the stay at the zero bound and also the character of policy after nominal interest rates have been raised from zero.

Keywords: Small open economy, Policy trade-offs, Trade structure

JEL codes: E58, F41, F42

*The views in the paper are those of the authors and do not necessarily reflect those of the Bank of England.

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1 Introduction

How does the zero lower bound on nominal interest rates (ZLB) affect optimal monetary policy in a small open economy in response to global shocks? The financial crisis of 2007-2008 and the subsequent “Great Recession” led central banks around the world to lower nominal interest rates to close to zero. Given the international nature of the crisis this question concerning the role of openness and international spillovers for monetary policy at the zero bound is particularly pertinent.

In this paper, we analyse how the policy trade-off faced by the central bank of a small open economy (referred to as ‘home’) is affected by the presence of the ZLB both at home and abroad (in a large economy, referred to as ‘foreign’) in a new Keynesian model along the lines of Gali and Monacelli (2005) and De Paoli (2009). In our model central banks in both countries choose their policies optimally with respect to the loss function that consists of the weighted average of inflation and output gap. We examine a global shock to the natural rate of interest which in normal times central banks can stabilise perfectly by setting their policy rates equal to the natural rate. Our focus is however on the global shock which size is big enough that it causes the natural rate of interest to fall to the extent that nominal interest rates are cut to zero.¹

We find that, in response to a global shock, the presence of the ZLB at home and abroad creates a policy trade-off for the home central bank that has two components. First, the ZLB at home constrains the home central bank’s ability to offset the impact of the global shock on the home natural rate (which depends on foreign shocks as well as home shocks because home households consume foreign goods). Second, the ZLB abroad prevents the foreign central bank from stabilising the foreign economy perfectly. As a result, the foreign output gap is non-zero and thus creates inefficient fluctuations in home output gap and inflation. The sign of the foreign output gap spillover depends on the degree of substitutability of home and foreign goods (see Corsetti and Pesenti (2001)). In our benchmark calibration we assume that home and foreign goods are substitutes based on the micro evidence on trade elasticities (see Obstfeld and Rogoff (2000)). Thus a fall in foreign output gap increases home output gap and decreases home inflation.

The size of the spillover from the foreign policy depends on the choice of both foreign and home central bank to commit. We show that developments in foreign policy can affect not only the policy trade-off faced by the home central bank but also its length of the stay at the ZLB and the subsequent policy of raising rates. In particular, we find that when both central banks act under discretion the length of the stay at the ZLB in home economy is unaffected by foreign policy. This is because the shock is symmetric and the policymaker under discretion is unable to stimulate the economy. As a result, the length of the stay at the ZLB is determined by the duration of the shock itself (see Jung et al. (2005)). Despite this, the fact that foreign economy experiences a big fall in the output gap under discretion improves the policy trade-off faced by the home central bank. This is reflected in both a smaller home output gap and smaller home deflation.

When the foreign and home central banks follow commitment policies, the foreign ZLB lengthens the stay of the home central bank at the ZLB and also induces a more gradual tightening of home policy after nominal

¹This approach is similar to Jung et al. (2005).

rates have been raised from zero. The longer stay at the ZLB results from a contractionary effect of foreign commitment policy on the home economy. The foreign central bank, by committing to a zero interest rate policy for longer, engineers higher inflation expectations which help to stimulate foreign output gap. This however worsens the policy trade-off faced by the home central bank as it produces a decrease in home output gap and an increase in home inflation. Home central bank thus finds it optimal to be accommodative. As a result, under our benchmark parameterisation it lengthens the stay at the ZLB by 1 quarter (in comparison with the situation in which there is no spillover from foreign policy). Moreover, it exits from the ZLB in a gradual manner, i.e. by keeping interest rate lower than the natural rate for additional 2 quarters.

More generally, the commitment policy of the foreign central bank produces a beggar-thy-neighbour effect on home economy. This induces a more accommodative monetary policy by the home central bank, irrespective of whether the home central bank acts under discretion or commitment. Under our benchmark parameterisation, we find that the length of the stay at the ZLB in the home economy is increased by 2 quarters in result of the foreign commitment.

The literature on monetary policy at the ZLB has concentrated mainly on closed economies.² The main finding of these papers is that, while discretionary policy is very costly, optimal commitment, which involves keeping interest rates at the ZLB for longer (than the duration of the shock), can improve macroeconomic stability. A recent paper by Levin et al. (2010) argues that the costs under commitment policy can also be big for a sizeable shock, such as the one associated with the recent crisis and "Great Recession". This means that international spillovers coming from the inability of monetary policy to stabilise the economy will be big, even under commitment. In our paper, we exploit this issue by adopting the size of the shock studied in Levin et al. (2010).

As far as policy in open economies is concerned, Svensson (2001), Svensson (2003a), Coenen and Wieland (2003), and Nakajima (2008) study the zero interest rate policy in an environment where a single country hits the ZLB. Fujiwara et al. (2010) analyses the optimal coordination policy in a two-country world faced with the global shock that leads both countries to the ZLB. The authors find that the nature of coordination policy depends on substitutability of traded goods, since this parameter determines the size of international spillovers. Bodenstein et al. (2009) and Erceg and Linde (2010) study the effects of foreign shocks in an open economy when it is at the ZLB. Both papers find, that in this situation, the effects of foreign shocks are usually amplified. This is because, at the ZLB, monetary policy is constrained and cannot provide the necessary stimulus to its economy. Interestingly, Bodenstein et al. (2009) also show that the spillover effects of foreign shocks do not seem to be much affected by foreign monetary policy. They argue that, although the ZLB makes foreign output fall by more in response to a negative shock, it also reduces the associated home appreciation. Thus the ultimate effect on home output is little changed when compared to the case of no ZLB. This result, as Bodenstein et al. (2009) acknowledges, however, depends on the assumed trade price elasticity. In our framework, spillovers to home economy at the ZLB are summarised by the natural rate and foreign output gap dynamics. The size and

²See Adam and Bili (2006), Adam and Bili (2007), Eggertsson and Woodford (2003), Jung et al. (2005), Nakov (2006).

the sign of foreign output gap channel is determined by, among others, the degree of substitutability of home and foreign goods, which in turn affects the trade price elasticity. The ZLB in the foreign economy induces a decline in the foreign output gap (since foreign monetary policy is not able to stabilise its economy). This spillover can affect the length of the stay at the zero bound and also the way home monetary policy should stabilise its economy once it abandons the ZLB policy.

Finally, our paper is also related to Lipinska et al. (2009), which studies international policy spillovers in case of global cost-push shocks. Lipinska et al. (2009) shows that, in this case, policy trade-offs in a small open economy depend on foreign policy actions precisely because the cost-push shock introduces trade-offs for policymakers. Our paper shows more generally that foreign policy spillovers emerge in situations when foreign monetary policy is not able to stabilise its economy, i.e. faces a policy trade-off between stabilising inflation and the output gap or is constrained by the ZLB because of the size of the shock.

The paper is organised as follows. In section 2 we outline the model we use to conduct the analysis; in section 3 we explain the nature of international spillovers; in section 4 we derive optimal policy under discretion and commitment in a small open economy under the ZLB; in section 5 we analyse the results for our benchmark case of home and foreign goods that are substitutes under assumption that both policies follow discretionary and then commitment policies; in section 6 we provide a discussion on the role of monetary policy design in international spillovers at the ZLB; in section 7 we discuss alternative modelling assumptions that could have an impact on the nature of international spillovers; section 8 concludes.

2 Model

In this section, we describe the model we use to conduct the analysis and its calibration.

2.1 Small open economy model

The analysis is conducted in a standard small open economy new Keynesian model along the lines of Gali and Monacelli (2005) and De Paoli (2009). The relative simplicity of the model is an advantage in that it means that the spillover effects due to the presence of the ZLB will be more transparent. In the model, there are two countries: ‘home’ (indexed by H) and ‘foreign’ (indexed by F). Representative households in each country supply labour to monopolistically competitive firms producing differentiated goods, and consume goods produced in both the home and foreign economies. Wages are assumed to be fully flexible, but prices are assumed to be sticky as in Calvo (1983). We adopt the approach of De Paoli (2009), which first solves for the equilibrium of the two-country model, and then takes the limit of the size of the home economy to zero. As a result, the home economy becomes a small open economy, whereas the foreign economy behaves like a closed economy: although developments in foreign variables affect the home economy, the opposite is not true. This is because the share of home goods in consumption basket of foreign households is infinitesimal. The model is in the class of cashless-limit economies, see e.g. Woodford (2003).

2.1.1 Foreign economy

The foreign economy we consider is the same as the one used to analyse optimal policy at the ZLB in a closed economy setting (e.g. Jung et al. (2005), and Levin et al. (2010)). The non-policy block of the model is represented by two equations: an *IS* curve and a new Keynesian Phillips curve (*NKPC*), which are both derived from the optimising behaviour of households and firms:

$$\widehat{x}_{F,t} = \widehat{x}_{F,t+1} - \frac{1}{\rho}(i_{F,t} - \widehat{\pi}_{F,t+1} - r_{F,t}^n), \quad (1)$$

$$\widehat{\pi}_{F,t} = k(\rho + \eta)\widehat{x}_{F,t} + \beta\widehat{\pi}_{F,t+1}, \quad (2)$$

where $\widehat{x}_{F,t}$ is the foreign output gap, $\widehat{\pi}_{F,t}$ is the foreign inflation rate, $i_{F,t}$ is the foreign short-term nominal interest rate, $r_{F,t}^n$ is the foreign natural interest rate³, $\frac{1}{\rho}$ is the interest rate elasticity of real aggregate demand, k is the slope of the *NKPC* and η is the inverse of elasticity of labour supply.

It can be shown that the foreign natural interest rate depends on demand shocks and productivity shocks:

$$r_{F,t}^n = \frac{\rho\eta}{\rho + \eta}(\Delta\widehat{A}_{F,t+1} - \Delta\widehat{B}_{F,t+1}) + \frac{1 - \beta}{\beta}, \quad (3)$$

where β is the discount factor and $\frac{1-\beta}{\beta}$ is the real interest in the steady state.

2.1.2 Home economy

The home economy can also be represented by an *IS* curve and an *NKPC*:

$$\widehat{x}_{H,t} = \widehat{x}_{H,t+1} - \frac{1 - \lambda}{\rho\lambda} \left(i_{H,t} - \widehat{\pi}_{H,t+1} - r_{H,t}^{n,PPI} \right) + \left(\frac{\rho(1 - \lambda) - \rho\lambda}{\rho(1 - \lambda)} \right) \Delta\widehat{x}_{F,t+1}, \quad (4)$$

$$\widehat{\pi}_{H,t} = k \left(\frac{\eta(1 - \lambda) + \rho\lambda}{1 - \lambda} \widehat{x}_{H,t} + \frac{\rho(1 - \lambda) - \rho\lambda}{1 - \lambda} \widehat{x}_{F,t} \right) + \beta\widehat{\pi}_{H,t+1}, \quad (5)$$

where λ is the degree of openness of home economy⁴, θ is the intratemporal elasticity of substitution between home and foreign goods, $\widehat{x}_{H,t}$ is the home output gap, $\widehat{\pi}_{H,t}$ is the (PPI) inflation rate, $i_{H,t}$ is the home short-term nominal interest rate, $r_{H,t}^{n,PPI}$ is the home natural real interest rate, defined in terms of PPI.⁵ In the home economy, inflation and the output gap additionally depend on developments in the foreign output gap. This dependence is governed by parameter $\rho\lambda$ which is equal to $\frac{\rho(1-\lambda)}{(\rho\theta-1)\lambda(2-\lambda)+1}$. When $\rho\theta = 1$ or if $\lambda = 0$, $\rho\lambda = \rho(1-\lambda)$ and the equations (4, 5) collapse to that for the closed economy.

³The natural rate is defined as the real rate in the flexible price equilibrium or equivalently the real rate consistent with zero inflation.

⁴Similarly to De Paoli (2009) this parameter also governs the degree of home bias in Home economy, i.e. the level of home bias is given by $(1 - \lambda)$.

⁵The natural real interest rate in terms of CPI inflation is given by $r_{H,t}^n = i_t - \widehat{\pi}_{t+1}$, where CPI inflation is defined as

$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{\lambda}{1-\lambda} \Delta\widehat{R}\widehat{S}_t$. Therefore, the natural real rate in terms of PPI inflation is $r_{H,t}^{n,PPI} = i_t - \widehat{\pi}_{t+1} + \frac{\lambda}{1-\lambda} \Delta\widehat{R}\widehat{S}_{t+1}$.

Table 1: Model parameters

Intertemporal elasticity of substitution (ρ^{-1})	1
Intratemporal elasticity of substitution (θ)	3
Frisch elasticity of labour supply (η^{-1})	0.47^{-1}
Degree of openness (λ)	0.5
Subjective discount factor (β)	0.99
Elasticity of substitution across the differentiated products (σ)	10
Probability of not being able to reset price (α)	0.66
$k = (1 - \alpha\beta)(1 - \alpha) / \alpha(1 + \sigma\eta)$	
$k^* = (1 - \alpha^*\beta)(1 - \alpha^*) / \alpha^*(1 + \sigma\eta)$	

The home natural interest rate ($r_{H,t}^{n,PPI}$) depends on both home shocks and foreign shocks; it can be shown to depend on the foreign natural interest rate and differences between home and foreign demand and supply shocks (this captures movements of the real exchange rate in the flexible price equilibrium):

$$r_{H,t}^{n,PPI} = r_{F,t}^n + \frac{\rho_\lambda \eta}{(\rho_\lambda + \eta(1 - \lambda))} \left(\Delta \left(\widehat{A}_{H,t+1} - \widehat{A}_{F,t+1} \right) - (1 - \lambda) \Delta \left(\widehat{B}_{H,t+1} - \widehat{B}_{F,t+1} \right) \right). \quad (6)$$

Furthermore, if home and foreign shocks are perfectly correlated then domestic and foreign natural interest rate are equalised and $r_{H,t}^{n,PPI} = r_{F,t}^n$.⁶

The values of model parameters are presented in Table 1.

3 Nature of spillovers

3.1 How do foreign developments affect the home economy?

In this paper we consider the effects of a global shock, i.e. a negative shock that hits both the home and foreign economies in the same way: $r_{H,t}^{n,PPI} = r_{F,t}^n$. In what follows, we describe how the home economy is affected by the foreign economy's response to the shock. In particular, developments in the foreign economy can affect the home economy through two channels.

First, the home economy is affected by a real spillover. This spillover comes from the fact that the natural real interest rate in the home economy will be affected by shocks in the foreign economy, reflecting the assumption that a proportion of total demand for home goods and services is from foreign consumers. In response to the effects from this spillover, monetary policy would optimally adjust nominal interest rates in line with the change in the natural real rate, thereby stabilising the output gap and inflation.

Second, the home output gap and inflation are influenced by the foreign output gap, which is apparent from the home economy's *IS* curve and *NKPC*. Given that foreign monetary policy determines the size and time

⁶When home and foreign shocks are perfectly correlated the real exchange rate does not change in the flexible price equilibrium.

path of the foreign output gap, the nature of this spillover channel will depend on the foreign central bank's behaviour. Furthermore, this spillover is akin to a mark-up shock from the viewpoint of the home economy: it pushes the output gap and inflation in opposite directions, given rise to a trade-off between output gap and inflation stabilisation for the home policymaker. In response to shocks small enough not to drive nominal rates to the zero bound, foreign monetary policy would be able to stabilise the foreign output gap perfectly and this channel would not be present.

Overall, if the foreign policymaker stabilises the foreign output gap, spillovers from efficient foreign shocks would not create a trade-off between stabilising inflation and the output gap for home monetary policy.⁷ When there are efficient shocks in the foreign economy that are large enough to drive foreign nominal rates to the ZLB, the foreign policymaker is unable to stabilise the output gap and inflation, and this gives rise to a spillover for the home economy.

3.2 What is the nature of the spillover?

The sign of the impact of the foreign output gap on the home output gap and inflation depends on the substitutability of home and foreign goods for home households (i.e., if $\rho\theta > 1$ (< 1), home and foreign goods are substitutes (complements) in the utility and $\rho(1 - \lambda) > \rho_\lambda$ ($\rho(1 - \lambda) < \rho_\lambda$)).

When goods are substitutes (complements), home inflation is increasing (decreasing) in the foreign output gap. These differences arise because foreign variables affect home real marginal costs in two opposing ways. Real marginal cost in the home economy will depend on the real wage demanded by households in units of home production and home productivity. The influence of foreign variables on home real marginal cost therefore results from their impact on the real wage, given that home productivity is determined by home technology.

Considering the effects of a foreign monetary contraction sheds light on the opposing effects. In response to the contraction, foreign output falls, leading to a negative foreign output gap, and the home real exchange rate depreciates. On the one hand, the fall in foreign output, for a given level of home output (and hence consumption of home goods), reduces home consumption of foreign goods. This reduces home consumption overall, thereby raising the marginal utility of consumption. Given labour demand, in order to restore the ratio of the marginal utilities of consumption and leisure, households reduce the amount of time spent as leisure – that is, they increase their labour supply. This pushes down on real wages and hence marginal costs. On the other hand, the real depreciation reduces the value of home production in units of home consumption, leading households to supply less labour. This pushes up on real marginal costs. For substitutes (complements), the real exchange rate adjustment is smaller (bigger) so the negative foreign output gap has a negative (positive) effect on home real marginal costs and hence inflation.

The home output gap is decreasing (increasing) in the foreign output gap for substitutes (complements). Again, the intuition can be understood by considering the effects of a foreign monetary contraction. The decrease

⁷Lipinska et al. (2009) shows that in case of global inefficient cost-push shock the foreign policymaker is not able to stabilise the foreign output gap which induces a trade-off for home policymaker.

in foreign demand directly decreases demand for home output – referred to as the aggregate demand effect by Corsetti and Pesenti (2001). But the real depreciation induces expenditure switching that raises demand for home output – the expenditure switching effect. For substitutes (complements), the expenditure switching effect dominates (is dominated by) the aggregate demand effect, and overall demand for home output rises (falls), leading to a positive (negative) home output gap.

4 Optimal monetary policy in a small open economy

In this section, we outline the problem facing the monetary policymakers in the home and foreign economies, and characterise the solutions for optimal policy under discretion and commitment in the small open economy.

4.1 Objective of monetary policy

The objective of policymakers in the home and foreign economies is to stabilise a weighted average of squared deviations of inflation and the output gap from their steady states in their respective economies.⁸ Therefore, the central banks in the home and foreign economies choose the path of the short-term nominal interest rate to minimise:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\widehat{\pi}_{j,t}^2 + \omega_j \widehat{x}_{j,t}^2 \right), \quad (7)$$

where β is the discount factor, $j = H, F$ and ω_j denotes the weight each central bank assigns to the output gap. Nominal interest rates cannot be negative, i.e. there is a ZLB:

$$i_t^j \geq 0. \quad (8)$$

In our set-up, the central banks are assumed to be able to adopt perfectly credible policies. Furthermore, they are assumed not to have access to quantitative measures such as asset purchases when nominal interest rates are zero. The latter assumption captures a view that asset purchases and other such measures may not necessarily represent a perfect substitute for further cuts to nominal interest rates.

In what follows, we will show that the path of inflation and output gap determined by the optimal policy in a small open economy differs in two aspects from the path of these variables in the closed economy. First, the path of inflation and output gap depends on the degree of openness and substitutability of home and foreign goods. Second, it also depends on the path of the foreign output gap.

4.2 Optimal policy under discretion

4.2.1 Optimisation

The central bank in the home economy minimises (7) with respect to the economy’s structural equations ((4) and (5)) and the non-negativity constraint on nominal interest rates (8). Under discretion, the central bank

⁸As argued by Svensson (2003b) this policy objective coincides with the objectives of inflation-targeting central banks.

re-optimises each period. The optimisation can be represented by the following Lagrangian:

$$\begin{aligned}
L = & \sum \beta^t \{ \widehat{\pi}_{H,t}^2 + \omega_H \widehat{x}_{H,t}^2 \} + \\
& + 2\gamma_{1,t} \left(- \left(\frac{\rho_\lambda}{\rho(1-\lambda)} \right) \Delta \widehat{x}_{H,t+1} - \left(1 - \frac{\rho_\lambda}{\rho(1-\lambda)} \right) \Delta \widehat{x}_{F,t+1} + \frac{1}{\rho} \left(i_t - \widehat{\pi}_{H,t+1} - r_{H,t}^{n,PPI} \right) \right) + \\
& + 2\gamma_{2,t} \left(\widehat{\pi}_{H,t} - k \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \widehat{x}_{H,t} + \frac{\rho(1-\lambda) - \rho_\lambda}{1-\lambda} \widehat{x}_{F,t} \right) - \beta \widehat{\pi}_{H,t+1} \right).
\end{aligned} \tag{9}$$

The first order conditions with respect to $\widehat{\pi}_{H,t}$, $\widehat{x}_{H,t}$ and $i_{H,t}$ are as follows:

$$\widehat{\pi}_{H,t} + \gamma_{2,t} = 0, \tag{10}$$

$$\omega_H \widehat{x}_{H,t} + \left(\frac{\rho_\lambda}{\rho(1-\lambda)} \right) \gamma_{1,t} - k \left(\frac{\eta(1-\lambda) + \rho_\lambda}{(1-\lambda)} \right) \gamma_{2,t} = 0, \tag{11}$$

$$i_t \gamma_{1,t} = 0, \tag{12}$$

$$i_t \geq 0, \tag{13}$$

$$\gamma_{1,t} \geq 0, \tag{14}$$

where $\gamma_{1,t}$ and $\gamma_{2,t}$ are the Lagrange multipliers on the constraints. The form of the first order conditions (FOCs) is the same as for a closed economy. The FOCs for the small open economy differ insofar as the slope of the Phillips curve depends on its degree of openness. Equation 12 and inequalities 13 and 14 are the Kuhn-Tucker conditions for the non-negativity constraint on the nominal interest rate. When the nominal interest rate is zero, from 12 and 14, it must be the case that $\gamma_{1,t}$ is strictly positive (that is, 8 is binding). Similarly, when the nominal interest rate is positive, $\gamma_{1,t}$ will be equal to zero.

4.2.2 Dynamic Path

The dynamic path for the endogenous variables in the home economy is characterised by two phases.⁹ In the first phase, the nominal interest rate is equal to zero (the non-negativity constraint binds, $\gamma_{1,t} > 0$). Since the shock to the natural rate is assumed to dissipate over time, it will gradually converge to its steady state value. As a result, the endogenous variables also converge to their (interior solution) steady state values.¹⁰ In the case of the nominal interest rate, this is strictly greater than zero; therefore, at some point the non-negativity constraint on the nominal interest rate will cease to bind and it will become positive. The final period in the first phase we denote by T^d (i.e. $i_{H,T^d} = 0$ and $i_{H,T^d+1} > 0$). From the IS curve and NKPC, the dynamics of the output gap and inflation in the first phase are governed by the following difference equation:

$$\mathbf{z}_{H,t+1} = \mathbf{A} \mathbf{z}_{H,t} - \mathbf{a} r_{H,t}^{n,PPI} - \mathbf{B}_1 \widehat{x}_{F,t+1} - \mathbf{B}_2 \widehat{x}_{F,t},$$

where $\mathbf{z}_{H,t} = \left[\widehat{\pi}_{H,t} \quad \widehat{x}_{H,t} \right]'$. This can be solved forward to give a unique bounded solution for the home output gap and inflation:

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} \mathbf{a} r_{H,k}^{n,PPI} + \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_1 \widehat{x}_{F,k+1} - \mathbf{B}_2 \widehat{x}_{F,k}) + \mathbf{A}^{-(T^d-t+1)} \mathbf{z}_{H,T^d+1}.$$

⁹Details are given in the Appendix.

¹⁰The steady state under discretion is described in the Appendix.

This differs from the solution for the closed economy in several ways. First, the coefficient matrices \mathbf{A} and \mathbf{a} will be different to their closed economy counterparts insofar as the parameters of the home economy depend on the degree of openness and substitutability of home and foreign goods. Second, the paths of the home variables depend on current and future values of the foreign output gap (\widehat{x}_F). Third, it will not necessarily be the case that $\mathbf{z}_{H,T^d+1} = 0$, in contrast to the equivalent case for the closed economy. This reflects the mark-up shock-like nature of the foreign output gap for the home economy. In case of asymmetric shocks, even once the natural rate has become positive, it will not necessarily be optimal to set $i_{H,t} = r_{H,t}^{n,PPI}$.

In the second phase, $t = T^d + 1, \dots$, the nominal interest rate is positive and the Lagrange multiplier on the non-negativity constraint is zero $\gamma_{1,T^d+1} = \gamma_{1,T^d+2} = \dots = 0$. Using this fact, first order conditions 10 and 11 and the economy's structural equations (4) and (5), it is possible to obtain a unique bounded solution for the home output and inflation, which is given by:

$$\mathbf{z}_{H,t} = \begin{bmatrix} 1 \\ -\frac{k}{\omega_H} \left(\frac{\eta(1-\lambda) + \rho\lambda}{1-\lambda} \right) \end{bmatrix} \Psi_2 \sum_{k=t}^{\infty} \Psi_1^{-(k-t)} \widehat{x}_{F,k},$$

where $\Psi_1 = \frac{\omega_H \beta}{\omega_H + k^2 \left(\frac{\eta(1-\lambda) + \rho\lambda}{1-\lambda} \right)^2}$ and $\Psi_2 = \frac{\omega_H k \left(\frac{\rho(1-\lambda) - \rho\lambda}{1-\lambda} \right)}{\omega_H + k^2 \left(\frac{\eta(1-\lambda) + \rho\lambda}{1-\lambda} \right)^2}$. The form of this equation is the same as the reduced form expressions for the output gap and inflation under optimal policy with discretion in response to cost-push shocks in Clarida et al. (2002), highlighting the mark-up shock-like nature of the foreign output gap for the home economy. To the extent that the foreign output gap is non-zero for $t \geq T^d + 1$, the home output gap and inflation will also be non-zero.

4.3 Optimal policy under commitment

4.3.1 Optimisation

Under commitment, the home central bank's optimisation problem can be represented by the same Lagrangian (9) as for discretion. But in contrast to the case of discretionary policy, under commitment the policymaker is assumed to be able to choose the entire paths of inflation and the output gap to minimise its loss. The first order conditions to the policymaker's problem with respect to $\widehat{\pi}_{H,t}$, $\widehat{x}_{H,t}$ and $i_{H,t}$ are as follows:

$$\widehat{\pi}_{H,t} + \frac{1}{\beta\rho} \gamma_{1,t} + \gamma_{2,t} - \gamma_{2,t-1} = 0, \quad (15)$$

$$\omega_H \widehat{x}_{H,t} - \gamma_{1,t} + \frac{1}{\beta} \gamma_{1,t-1} - k \frac{\eta(1-\lambda) + \rho\lambda}{1-\lambda} \gamma_{2,t} = 0, \quad (16)$$

$$\gamma_{1,t} i_{H,t} = 0, \quad (17)$$

$$\gamma_{1,t} \geq 0, \quad (18)$$

$$i_{H,t} \geq 0, \quad (19)$$

where $\gamma_{1,t}$ and $\gamma_{2,t}$ are the Lagrange multipliers on the constraints. As with discretion, the form of the first order conditions (FOCs) is the same as that for a closed economy, differing only to the extent that the slope of the Phillips curve depends on its degree of openness and substitutability of home and foreign goods. The

implications of the Kuhn-Tucker conditions (17), (18) and (19) for whether the nominal interest rate is positive or not are similar to those for discretionary policy.

4.3.2 Dynamic path

As in the closed economy case studied by Jung et al. (2005), the dynamic path for the home economy is characterised by three distinct phases.¹¹

In the first phase, the nominal interest rate is zero. Given that the system converges back to its steady state¹² as the effects of the shock dissipate, the nominal interest rate will eventually be increased from zero. The final period in the first phase is denoted by T^c . After substituting for $i_{H,t} = 0$, the *IS* curve (4) and *NKPC* (5) give rise to a difference equation for $t = 1, \dots, T^c$ of the form:

$$\mathbf{z}_{H,t+1} = \mathbf{A}\mathbf{z}_{H,t} - \mathbf{a}r_{H,t}^{n,PPI} - \mathbf{B}_1\hat{x}_{F,k+1} - \mathbf{B}_2\hat{x}_{F,k}. \quad (20)$$

By solving this forward, and using the FOCs (16) and (15), we obtain the following equations for the path of the endogenous variables in the home economy and Lagrange multipliers $\boldsymbol{\gamma}_t = \begin{bmatrix} \gamma_{1,t} & \gamma_{2,t} \end{bmatrix}'$ up to, and including, period T^c :

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} \mathbf{a}r_{H,k}^{n,PPI} + \mathbf{A}^{-(T^c-t+1)} \mathbf{z}_{H,T^c+1} + \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_1\hat{x}_{F,k+1} + \mathbf{B}_2\hat{x}_{F,k}), \quad (21)$$

$$\boldsymbol{\gamma}_t = \mathbf{C}\boldsymbol{\gamma}_{t-1} - \mathbf{D}\mathbf{z}_{H,t}. \quad (22)$$

The form of these equations differs from the closed economy case insofar as the paths for the home output gap and inflation also depend on the path of the foreign output gap. In addition, the elements of the coefficient matrices will differ from their closed economy counterparts since the parameters of the home economy depend on the home economy's degree of openness and substitutability of home and foreign goods.

The equations show that during the phase up to, and including, period T^c , the endogenous variables in the home economy depend on current and future values of the home natural rate, a terminal condition for the zero interest rate policy phase, \mathbf{z}_{H,T^c+1} (referred to by Levin et al. (2010) as the "forward guidance vector"), and current and future values of the foreign output gap. As discussed by Levin et al. (2010), the forward guidance vector pins down the rational expectations equilibrium for the economy in the first phase.

The second phase occurs at period $T^c + 1$ and is distinguished from the first and third phases since in (15) $\gamma_{1,t} = \gamma_{1,T^c+1} = 0$, but $\gamma_{1,t-1} = \gamma_{1,T^c} > 0$. This phase acts as a bridge between the other two phases: the first phase depends on the outcome in the second phase since this is when the value of forward guidance vector is determined. In turn, the forward guidance vector and γ_{2,T^c+1} depend on the values of the endogenous variables in period $T^c + 2$, which is the initial period of the final phase:

$$\begin{bmatrix} \mathbf{z}_{H,T^c+1} \\ \gamma_{2,T^c+1} \end{bmatrix} = \mathbf{F}^{-1}\mathbf{B}\mathbf{z}_{H,T^c+2} + \mathbf{F}^{-1}\mathbf{H}\boldsymbol{\gamma}_{T^c} + \mathbf{F}^{-1}\mathbf{K}\hat{x}_{F,T^c+1}. \quad (23)$$

¹¹ More details on the solution are given in the Appendix.

¹² The steady state under commitment is described in Appendix.

These equations for phase 2 (obtained by substituting $\gamma_{1,T^c+1} = 0$ into the first order conditions to the policy problem, (16) and (15), and using (5)) also differ from those for the closed economy only to the extent that they include the foreign output gap and the parameters depend on the home economy's degree of openness and substitutability of home and foreign goods.

In the final phase ($t = T^C + 2, \dots$), it will be the case that $\gamma_{1,T^C+1} = \gamma_{1,T^C+2} = \dots = 0$. Using this fact, (16) and (15), and the structural equations of the economy, (5) and (4), we obtain a unique bounded solution given by:

$$\begin{bmatrix} \hat{\pi}_{H,t} \\ \hat{x}_{H,t} \\ \gamma_{2,t} \end{bmatrix} = \begin{bmatrix} (1 - \lambda_2) \\ \frac{k}{\omega_H} \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right) \lambda_2 \\ \lambda_2 \end{bmatrix} \gamma_{2,t-1} + \begin{bmatrix} c_{1,t} \\ c_{2,t} \\ c_{3,t} \end{bmatrix}, \quad (24)$$

where λ_2 is a real eigenvalue of an associated matrix and $\mathbf{c}_t = \begin{bmatrix} c_{1,t} & c_{2,t} & c_{3,t} \end{bmatrix}'$ is a function of current and future values of the foreign output gap. As before, this differs from the closed economy solution because the home output gap and inflation depend on developments in the foreign economy, the degree of openness and substitutability of home and foreign goods.

4.4 Model solution

To solve for the optimal path of the endogenous variables under discretion (commitment) it is necessary to determine the value of T^d (T^c). Following Jung et al. (2005), we apply an algorithm that computes the path for $\gamma_{1,t}$ for an initially high value for T^d (T^c), and then reduces T^d (T^c) by one and re-computes the path for $\gamma_{1,t}$ until $\gamma_{1,T^d} > 0$ and $\gamma_{1,T^d+1} = 0$ ($\gamma_{1,T^c} > 0$ and $\gamma_{1,T^c+1} = 0$).

5 Results

In this section, we present the simulation results for discretionary and commitment policies in the home and foreign economy in response to a large adverse shock to the natural real rate of interest.

5.1 Benchmark case: home and foreign goods are substitutes; "Great Recession" shock

Our benchmark case is characterised by home and foreign goods that are substitutes. This follows work by Obstfeld and Rogoff (2000), who survey the literature and find that the intratemporal elasticity of substitution between home and foreign goods is quite high (in the neighbourhood of 5 or 6), consistent with them being substitutes.

In our benchmark case, we consider the case of a large negative demand shock at period 0 that causes the natural rate to become negative in that period, following Jung et al. (2005).¹³ We focus on the effects of a

¹³Note that our results would not change if we considered a productivity shock.

simultaneous negative demand shock in both the home and foreign economies (that is, a global shock). The shock is calibrated to match the “Great Recession” shock considered by Levin et al. (2010) and involves an 8 percentage point p.a. fall in the home and foreign natural real rates relative to steady state on impact. The effect of the shock gradually dissipates from period 1 onwards deterministically, slowly returning the natural rate to its steady state level with the persistence parameter equal to 0.85. As in Jung et al. (2005), this assumption allows us to focus on the optimal path for nominal interest rates in response to the shock in a perfect foresight environment. In this setting, after the shock has occurred agents know that no further shocks will hit. They are therefore able to foresee perfectly the future paths of the natural rate and, consequently, all endogenous variables.

5.1.1 Policies under discretion

We consider optimal policy under discretion for the home central bank in response to the global shock when the foreign central bank is also assumed to be following optimal policy with discretion. We find that the responses in the home and foreign economies are the same: nominal rates are held at zero until the natural real rate is positive; thereafter, nominal rates are set equal to the natural real rate. The finding that the responses are identical reflects the symmetry of the shock and the fact that policy is set under discretion in both economies.

Figure 1 shows the responses the home and foreign economies to the shock. It shows nominal and natural real interest rates (first panel), inflation (second panel), the output gap (third panel) and short-term and natural real interest rates (fourth panel).

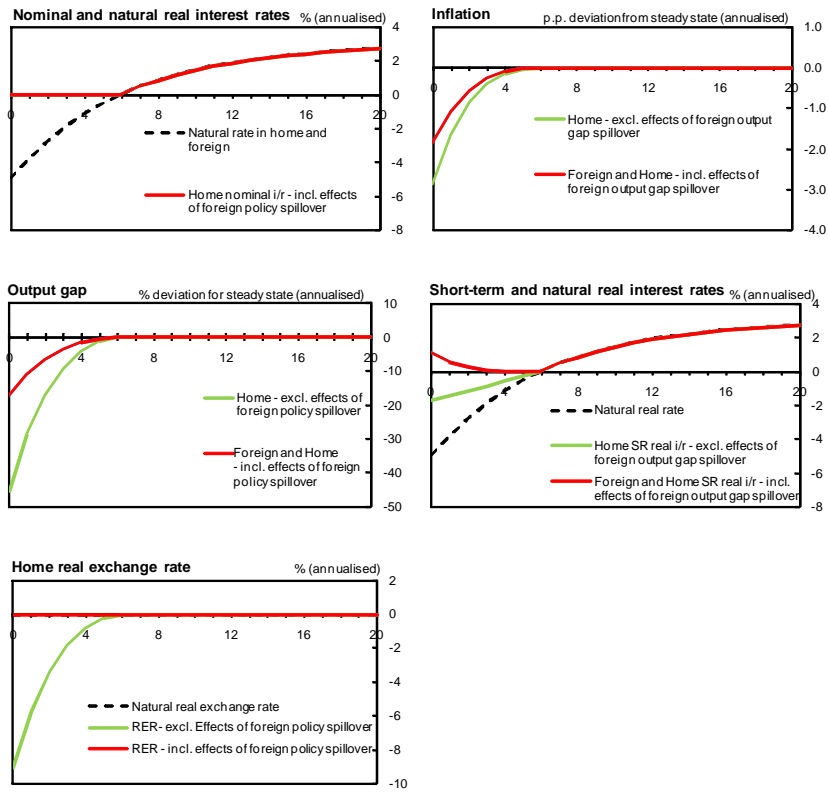
The responses of the home and foreign economies are the same qualitatively as those in Jung et al. (2005). Quantitatively, the responses in Figure 1 are much larger compared to those presented in Jung et al. (2005), reflecting the much bigger shock we consider. Nominal interest rates are cut to zero and held at that level until the natural real rate becomes positive. Thereafter, it is optimal each period for the central bank to set the nominal rate equal to the natural real rate, stabilising the output gap and inflation. However, this policy fails to ward off deflation, and is consistent with a large negative output gap.

In the home economy, although it is affected by a spillover from the negative foreign output gap, the symmetric nature of the shock and the fact that monetary policy in both economies is conducted under discretion imply that the real exchange rate remains unchanged in response to the shock. As a result, the dynamics of the home output gap and inflation – and hence the path of the nominal interest rate – are the same as in the foreign economy.

5.1.2 Policies under commitment

We consider optimal policy under commitment for the home central bank in response to the global shock when the foreign central bank also follows commitment policy. In this case, nominal rates in both economies are held at the zero bound for several periods after the natural real rate has become positive (as in Jung et al. (2005)). The spillover from foreign policy lengthens home economy’s stay at the ZLB compared to what it

Figure 1: Home and foreign responses to a global shock for optimal policy under discretion



would be if the foreign policy was not constrained by the ZLB. Nevertheless, in the face of a global shock, the home economy still exits the ZLB earlier than the foreign. This reflects the fact that policy inertia in the home economy induces a real depreciation that provides additional stimulus, reducing the extent of policy inertia that is required. After leaving the ZLB, the subsequent tightening of home policy is more gradual.

Figure 2 plots the responses to the shock of macroeconomic variables in the home and foreign economies under commitment policy.

As with discretionary policy, the responses of the foreign economy are the same qualitatively as those in Jung et al. (2005). The foreign economy's responses under commitment are also much larger compared to those presented in Jung et al. (2005), consistent with the larger shock. In contrast to policy under discretion, nominal interest rates are held at zero until a number of periods after the natural rate has turned positive – there is policy inertia. The policymaker avoids deflation today effectively by “borrowing” stimulus from the future. By creating positive inflationary expectations, the central bank reduces the size of the negative output gap relative to discretionary policy.

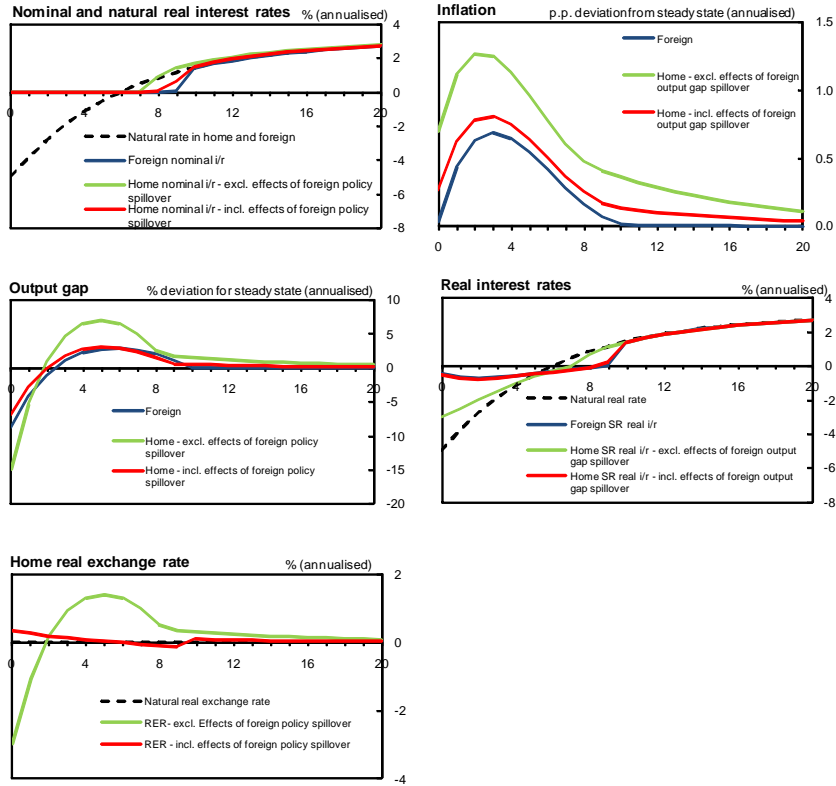
Optimal monetary policy in response to the global shock in the home economy has some key differences with the foreign central bank's optimal policy. In the home economy, nominal rates are raised from zero earlier than in the foreign. Once rates are increased in the home economy, they are brought back in line with the natural real rate more gradually. These differences reflect the fact that the home economy is affected by the spillover from foreign policy through the foreign output gap's influence on the home output gap and inflation. They also reflect the propagation of the demand shock in the home economy that is different compared to the foreign economy on account of the home economy's openness. We now consider these effects in turn.

The optimal response of home monetary policy to the effects of the demand shock (excluding the effects of the foreign output gap spillover) would be similar to that in the foreign economy: cut nominal rates to zero and hold them there for several periods after the natural rate has become positive (green line in Figure 2). But compared to in the foreign economy, home nominal rates would be raised from the ZLB earlier – there would be less policy inertia (green line compared to blue in Figure 2). This reflects the fact that, in the home economy, the policy inertia leads to a depreciation of the real exchange rate, which provides additional stimulus to demand and raises inflation compared to the foreign economy. As a result, the home central bank can generate expectations of future inflation with less inertial policy, and hence raises the nominal rate earlier than in the foreign economy.

The spillover from the foreign output gap on the home economy behaves akin to a mark-up shock that changes over time. Under the assumption that goods are substitutes for home consumers, the foreign output gap boosts and then weighs on the home output gap in a way that broadly offsets the underlying influence of the natural rate shock on the home economy when it is constrained by the ZLB. Home inflation is initially depressed by the negative foreign output gap. Further out, once the foreign output gap has turned positive, this creates upward pressure on home inflation.

Overall, monetary policy in the home economy aims optimally to contain the combined effects of the demand

Figure 2: Home and foreign responses to a global shock for optimal policy under commitment



shock and the mark-up shock-like spillover from foreign policy on the output gap and inflation. When the home economy is affected by both the spillover from the foreign output gap and the negative demand shock it remains at the ZLB for longer than it would if it were only affected by the demand shock (red line compared to green in Figure 2). This is because the foreign central bank’s policy of forward guidance stimulates a positive foreign output gap. This weighs on the home output gap, though increases home inflation, like a positive mark-up shock. Monetary policy in the home economy, when faced with the trade-off between inflation and the output gap resulting from the positive foreign output gap, is optimally looser relative to policy in response to the effects of the demand shock alone. Home monetary policy therefore accommodates some of the inflationary pressure from the spillover from foreign policy in order to reduce the costs it generates in terms of the downward pressure on the home output gap. After exiting the ZLB, home policy tightens gradually, reflecting the continued downward pressure on the home output gap from the positive foreign output gap. The gradualism of the policy tightening smoothes the costs imposed by the spillover from the foreign output gap over a longer time period, reducing their absolute size, which is consistent with the canonical optimal response to a mark-up shock.

6 Foreign policy design

Optimal policy in the home economy depends on foreign policy through its effect on the foreign output gap. In this section, we examine how differences in the foreign output gap, resulting from alternative assumptions about foreign policy, affect the home economy for given home policies. In particular, we consider two dimensions in which foreign policy could differ from home policy. These are specific preferences of the foreign policymaker and the choice of foreign policy to whether commit or act under discretion.

Preferences of foreign central bank affect the size of the foreign output gap fluctuations and thus also the size of the foreign spillover to home economy. Intuitively, under a dovish foreign central bank the size of the foreign output gap fluctuations is smaller and as a result the size of the spillover is smaller.¹⁴

We study also situations in which foreign policy differs from the home policy with respect to its choice whether to commit or not. When home policy is set under discretion, it remains looser if foreign policy is set under commitment relative to if foreign policy is discretionary (see Figure 3). Intuitively, the home central bank keeps nominal interest rates lower when foreign policy is set under commitment because the foreign central bank's policy inertia induces an appreciation of the home real exchange rate. This gives rise to an expenditure switching effect that reduces home demand, which is absent when foreign policy is also discretionary. Under assumption of home and foreign goods being substitutes, this effect dominates the aggregate demand effect. As a result, overall demand for home output declines, giving rise to a wider home output gap. Furthermore, a boost in foreign output gap coming from the foreign commitment puts upward pressure on home inflation. In sum, foreign commitment policy raises home inflation and widens the home output gap. Faced with this trade-off, the home central bank optimally allows inflation to overshoot its steady state level in order to reduce the costs in terms of real activity from the negative output gap.

When home policy is set under commitment, there is less policy inertia when the foreign central bank sets discretionary policy compared to commitment (see Figure 4). In the case of foreign discretionary policy, the home commitment policy induces a depreciation of the home real exchange rate, boosting home demand. Although the larger fall in foreign demand when foreign policy is discretionary gives rise to a larger aggregate demand effect, this is offset by the expenditure switching effect. As a result of the boost to home demand from the real depreciation, there is less need for stimulus through policy inertia by the home central bank. Nevertheless, nominal rates remain at the ZLB until after the natural rate has become positive, reflecting the deflationary impact of the negative demand shock and the real depreciation. In the case of foreign commitment policy, the foreign central bank's policy inertia reduces the extent of the real depreciation of the home exchange rate. Therefore, home policy needs to generate greater stimulus through inertia when foreign policy is set under commitment compared to the case of discretionary foreign policy.

¹⁴Lipinska et al. (2009) study this issue in more detail for the case of cost-push shocks.

Figure 3: Home responses to a global shock for optimal policy under discretion

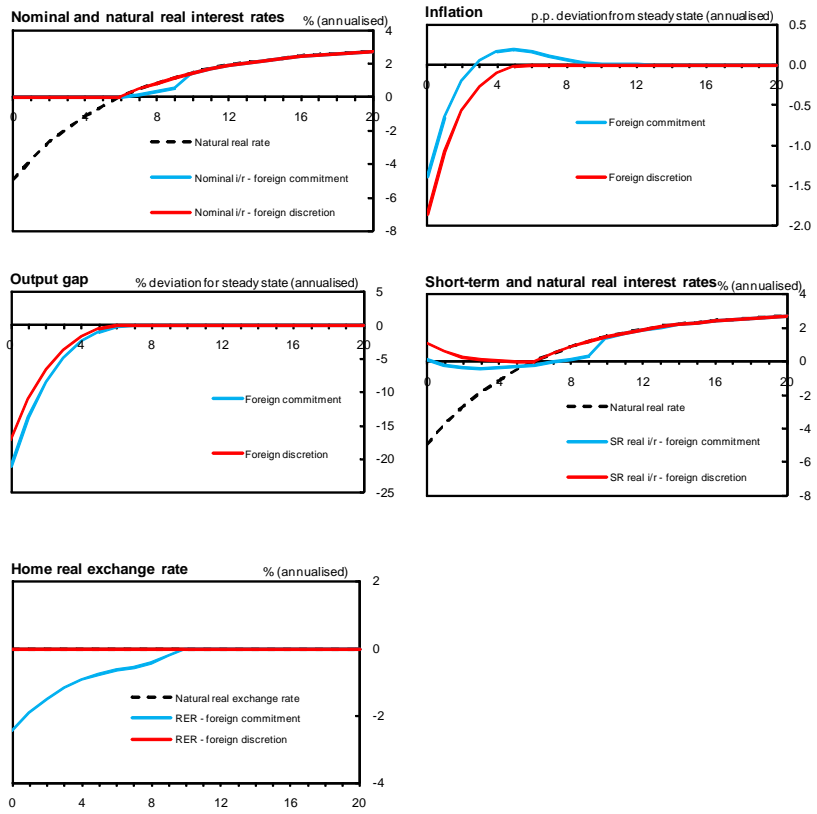
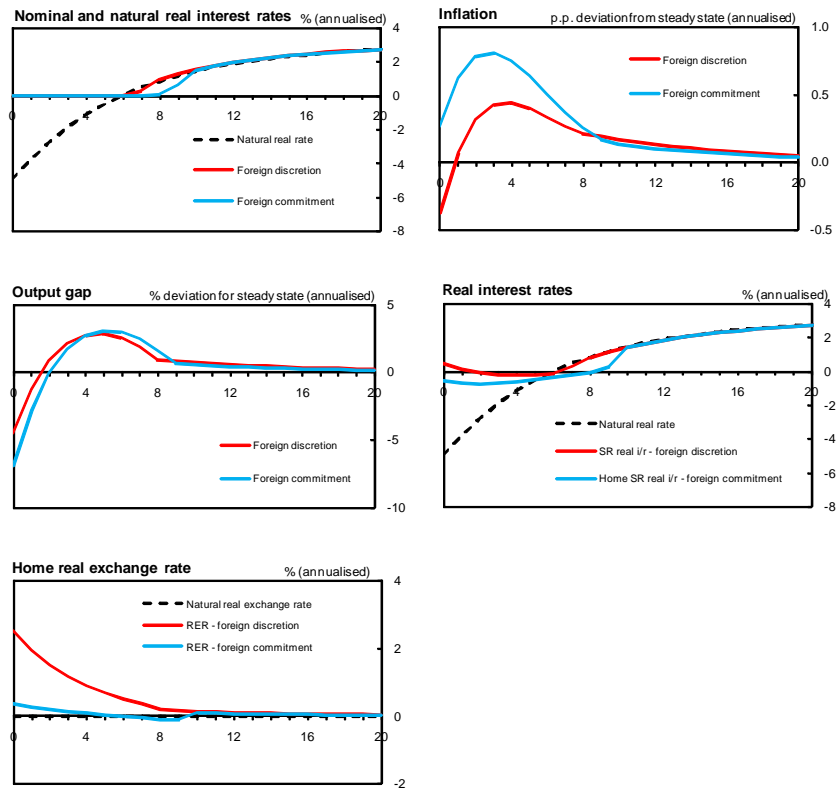


Figure 4: Home responses to a global shock for optimal policy under commitment



7 Discussion

For the purposes of clarity of our analysis we chose a highly stylised model. There are several important alternative modelling assumptions that can have an effect on the nature of international spillovers. These are: imperfect pass-through, limited risk sharing among countries and domestic frictions in the labour market, such as e.g. sticky wages. In the section we will shortly explain the intuition how these alternative features of the model could affect our results.

First, our model is built under assumption that there is a full pass-through from exchange rate movements. This implies that when home and foreign goods are substitutes that the expenditure switching effect resulting from the foreign output gap dynamics is strong and outweighs the aggregate demand effect. However if one takes into account that the pass-through from exchange rate movements is rather limited then the expenditure switching effect is reduced. This implies that the sign of international spillovers can be reversed.

Second, our model assumes that there is a full risk sharing among countries. This implies that countries can run trade imbalances and finance an increase in their consumption by borrowing from abroad. However if one assumes that the degree of financial integration between countries is smaller then the trade balances will be kept closer to zero via the adjustment of quantities produced. As a result, the size of international spillovers will be reduced.

Finally, our model assumes a frictionless labour market. This assumption is important in driving the effects of international spillovers on home inflation. Note that changes in the foreign output gap affect home inflation via a change in home labour supply which in turn affects the real marginal cost. However if one assumed instead that wages were sticky then the effect of changes in home labour supply on the real marginal cost would be much more limited. And thus home inflation would be little changed. Moreover, an additional friction in the labour market will make it harder for the central bank under commitment to engineer inflation expectations in order to boost the economy.

8 Conclusion

In this paper we study how the presence of the ZLB both at home and abroad affects the optimal policy response to a global shock in a standard new Keynesian model of a small open economy.

We show that the inability of the central bank abroad to stabilise both inflation and the output gap generates a spillover for the small open economy. This spillover manifests itself via fluctuations in the foreign output gap, which affect the home economy like a mark-up shock. As a result, they produce a trade-off for home monetary policy. Importantly, this spillover is independent from the type of the shock that hit both economies. The size and sign of the spillover is determined by the degree of substitutability of home and foreign goods and also the foreign central bank's choice of policy design, i.e. discretion versus commitment. We find that, depending on the exact nature of the spillover, the central bank in the small open economy may change the length of the stay at the zero lower bound and also its subsequent policy once it starts raising rates.

To sum up, we show that international spillovers at the ZLB have an inefficient component (foreign output gap channel), which was not explored previously in the literature. This indicates that, as in the case of inefficient shocks such as markup shocks (see Benigno and Benigno (2006)), gains from coordination at the ZLB could be high. Thus it would be interesting to investigate this issue further by comparing fully optimal coordinated and uncoordinated policies at the ZLB.

References

- Adam, K. and M. Bili (2006). Optimal monetary policy under commitment with a zero bound on nominal interest rates. *Journal of Money, Credit and Banking* 7.
- Adam, K. and M. Bili (2007). Discretionary monetary policy under commitment and the zero lower bound on nominal interest rates. *Journal of Monetary Economics* 54(3).
- Benigno, G. and P. Benigno (2006). Designing targeting rules for international monetary policy cooperation. *Journal of Monetary Economics* 53, 473–506.
- Bodenstein, M., C. Erceg, and L. Guerrieri (2009). The effects of foreign shocks when interest rates are at zero. *International Finance Discussion Papers* (983).
- Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12, 383–98.
- Clarida, R., J. Gali, and M. Gertler (2002). A simple model for international monetary policy analysis. *Journal of Monetary Economics* 49, 879–904.
- Coenen, G. and V. Wieland (2003). The zero interest bound and the role of exchange rate for monetary policy in japan. *Journal of Monetary Economics* 50(5).
- Corsetti, G. and P. Pesenti (2001). Welfare and macroeconomic interdependence. *Quarterly Journal of Economics* 116, 421–45.
- De Paoli, B. (2009). Monetary policy and welfare in a small open economy. *Journal of International Economics* 77, 11–22.
- Eggertsson, G. and M. Woodford (2003). The zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity* (1).
- Erceg, C. and J. Linde (2010). Asymmetric shocks in a currency union with monetary and fiscal handcuffs. mimeo.
- Fujiwara, I., N. Sudo, and Y. Teranishi (2010). Global liquidity trap. *International Journal of Central Banking* 6.

- Gali, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies* 72, 707–34.
- Jung, T., Y. Teranishi, and T. Watanabe (2005). Optimal monetary policy at the zero interest rate bound. *Journal of Money, Credit and Banking* 37(5).
- Levin, A., D. Lopez-Salido, E. Nelson, and T. Yun (2010). Limitations on the effectiveness of forward guidance at the zero lower bound. *International Journal of Central Banking* 6, 143–91.
- Lipinska, A., M. Spange, and M. Tanaka (2009). International spillover effects and monetary policy activism. *Bank of England Working Paper* (377).
- Nakajima, T. (2008). Liquidity trap and optimal monetary policy in open economies. *Journal of Japanese and International Economies* 22(1).
- Nakov, A. (2006). Optimal and simple monetary policy rules with zero floor on the nominal interest rate. *International Journal of Central Banking* 6.
- Obstfeld, M. and K. Rogoff (2000). The six major puzzles in international macroeconomics: is there a common cause? *NBER Macroeconomics Annual 2000*, 339–90.
- Svensson, L. (2001). The zero bound in an open economy: A foolproof of escaping from a liquidity trap. *Monetary and Economic Studies* 19(S1).
- Svensson, L. (2003a). Escaping from a liquidity trap and deflation: The foolproof way and others. *Journal of Economic Perspectives* 17(4).
- Svensson, L. E. O. (2003b). The inflation target and the loss function. *Central Banking, Monetary Theory and Practice: Essays in Honour of Charles Goodhart* 1.
- Woodford, M. (2003). *Interest and prices*. Princeton University Press.

9 Appendix

In this section, we provide greater detail on the solutions for optimal policy under discretion and commitment in the small open economy.

9.1 Discretion

9.1.1 Steady state

In the home economy’s steady state, inflation, the output gap, the nominal interest rate, the natural real rate and the Lagrange multipliers are equal to their long run values (indexed by a subscript ∞): $\hat{\pi}_{H,t} = \hat{\pi}_{H,t+1} = \hat{\pi}_{H,\infty}$;

$\widehat{x}_{H,t} = \widehat{x}_{H,t+1} = \widehat{x}_{H,\infty}$; $i_{H,t} = i_{H,\infty}$; $r_{H,t}^n = r_{H,\infty}^{n,PPI}$; $\gamma_{1,t} = \gamma_{1,\infty}$; $\gamma_{2,t} = \gamma_{2,\infty}$. Jung et al. (2005) show that for the closed economy the interior solution for the steady state is first best for optimal policy under discretion and unique for optimal policy under commitment and discretion; this implies that $\widehat{x}_{F,t} = \widehat{x}_{F,t+1} = \widehat{x}_{F,\infty} = 0$. Therefore, using this fact, it is possible to derive the steady state for the small open economy, for which there are also interior and corner solutions. The interior solution is for the case when the nominal interest rate is positive and is given by:

$$\widehat{\pi}_{H,\infty} = 0; \widehat{x}_{H,\infty} = 0; i_{H,\infty} = r_{H,\infty}^{n,PPI}; \gamma_{1,\infty} = 0; \gamma_{2,\infty} = 0. \quad (25)$$

The corner solution occurs when nominal interest rates are equal to zero and is given by:

$$\widehat{x}_{H,\infty} = -\frac{(1-\beta)(1-\lambda)}{k(\eta(1-\lambda) + \rho_\lambda)} r_{H,\infty}^{n,PPI},$$

$$\widehat{\pi}_{H,\infty} = -r_{H,\infty}^{n,PPI} < 0, \quad (26)$$

$$\gamma_{2,\infty} = r_{H,\infty}^{n,PPI},$$

$$\gamma_{1,\infty} = \frac{\rho(1-\lambda)}{\rho_\lambda} \left(\omega_H \frac{(1-\beta)(1-\lambda)}{k(\eta(1-\lambda) + \rho_\lambda)} + k \left(\frac{\eta(1-\lambda) + \rho_\lambda}{(1-\lambda)} \right) \right) r_{H,\infty}^{n,PPI}.$$

Following similar arguments to those in Jung et al. (2005), the interior solution is the first best outcome - it is superior to the corner solution in terms of the central bank's preferences.

9.1.2 Dynamic path

For the phase when the nominal interest rate is zero, $t = 1, \dots, T^d$, the *NKPC* (5), the *IS* curve (4) (after substituting for $i_{H,t} = 0$) and the first order conditions from the policymaker's optimisation ((10) and (11)) characterise the optimal path for the endogenous variables. This is given by:

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} \mathbf{a} r_{H,k}^{n,PPI} + \sum_{k=t}^{T^d} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_1 \widehat{x}_{F,k+1} - \mathbf{B}_2 \widehat{x}_{F,k}) + \mathbf{A}^{-(T^d-t+1)} \mathbf{z}_{H,T^d+1},$$

$$\text{where } \mathbf{A} \equiv \begin{bmatrix} \beta^{-1} & -\beta^{-1} k \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right) \\ -\beta^{-1} \rho_\lambda^{-1} (1-\lambda) & 1 + \beta^{-1} \rho_\lambda^{-1} k (\eta(1-\lambda) + \rho_\lambda) \end{bmatrix}, \mathbf{a} \equiv \begin{bmatrix} 0 \\ \frac{1-\lambda}{\rho_\lambda} \end{bmatrix}, \mathbf{B}_1 \equiv \begin{bmatrix} 0 \\ -\left(\frac{\rho(1-\lambda)}{\rho_\lambda} - 1 \right) \end{bmatrix},$$

$$\mathbf{B}_2 \equiv \begin{bmatrix} -\beta^{-1} k \left(\frac{\rho(1-\lambda) - \rho_\lambda}{1-\lambda} \right) \\ \beta^{-1} \rho_\lambda^{-1} k (\rho(1-\lambda) - \rho_\lambda) + \left(\frac{\rho(1-\lambda)}{\rho_\lambda} - 1 \right) \end{bmatrix}.$$

For $t = T^d + 1, \dots$ the solution is given by:

$$\mathbf{z}_{H,t} = \begin{bmatrix} 1 \\ -\frac{k}{\omega_H} \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right) \end{bmatrix} \Psi_2 \sum_{k=t}^{\infty} \Psi_1^{-(k-t)} \widehat{x}_{F,k},$$

$$\text{where } \Psi_1 = \frac{\omega_H \beta}{\omega_H + k^2 \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right)^2} \text{ and } \Psi_2 = \frac{\omega_H k \left(\frac{\rho(1-\lambda) - \rho_\lambda}{1-\lambda} \right)}{\omega_H + k^2 \left(\frac{\eta(1-\lambda) + \rho_\lambda}{1-\lambda} \right)^2}.$$

9.2 Commitment

9.2.1 Steady state

There is a unique steady state for policy with commitment, given by the interior solution:

$$\widehat{\pi}_{H,\infty} = 0; \widehat{x}_{H,\infty} = 0; i_{H,\infty} = r_{H,\infty}^{n,PPI}; \gamma_{1,\infty} = 0; \gamma_{2,\infty} = 0. \quad (27)$$

In the corner solution, which occurs when the economy is at the zero lower bound, it will be the case that:

$$\gamma_{1,\infty} = -\beta \rho r_{H,\infty}^{n,PPI} < 0.$$

Since this violates the Kuhn-Tucker condition (18), there cannot be a corner steady state solution. This is also the case for a closed economy.

9.2.2 Dynamic path

In the first phase, $t = 1, \dots, T^c$, the *NKPC*, the *IS* curve (after substituting for $i_{H,t} = 0$) and the first order conditions from the policymaker's optimisation characterise the optimal path for the endogenous variables, which is given by:

$$\mathbf{z}_{H,t} = \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} \mathbf{a} r_{H,k}^{n,PPI} + \mathbf{A}^{-(T^c-t+1)} \mathbf{z}_{H,T^c+1} + \sum_{k=t}^{T^c} \mathbf{A}^{-(k-t+1)} (\mathbf{B}_1 \widehat{x}_{F,k+1} + \mathbf{B}_2 \widehat{x}_{F,k}), \quad (28)$$

$$\gamma_t = \mathbf{C} \gamma_{t-1} - \mathbf{D} \mathbf{z}_{H,t},$$

$$\text{where } \mathbf{C} \equiv \begin{bmatrix} \frac{1}{\beta} + \frac{k}{\beta \rho} \frac{\rho}{\rho_\lambda} (\eta(1-\lambda) + \rho_\lambda) & k \frac{\rho}{\rho_\lambda} (\eta(1-\lambda) + \rho_\lambda) \\ \frac{1}{\beta \rho} & 1 \end{bmatrix}, \mathbf{D} \equiv \begin{bmatrix} k \frac{\rho}{\rho_\lambda} (\eta(1-\lambda) + \rho_\lambda) & \omega_H \frac{\rho(1-\lambda)}{\rho_\lambda} \\ 1 & 0 \end{bmatrix}.$$

The path for the variables up to and including period T^c depends on the value of \mathbf{z}_{H,T^c+1} . To solve for this we use the fact that in period $T^c + 1$ the non-negativity constraint on nominal interest rates no longer binds: $\gamma_{1,t} = 0$. Substituting this into the first order conditions for the policy problem and using the *NKPC* gives the following equation for \mathbf{z}_{H,T^c+1} and γ_{2,T^c+1} :

$$\begin{bmatrix} \mathbf{z}_{H,T^c+1} \\ \gamma_{2,T^c+1} \end{bmatrix} = \mathbf{F}^{-1} \mathbf{G} \mathbf{z}_{H,T^c+2} + \mathbf{F}^{-1} \mathbf{H} \gamma_{T^c} + \mathbf{F}^{-1} \mathbf{K} \widehat{\mathbf{x}}_{F,T^c+1}, \quad (29)$$

$$\text{where } \mathbf{F} \equiv \begin{bmatrix} 1 & -\frac{k(\eta(1-\lambda)+\rho_\lambda)}{1-\lambda} & 0 \\ 1 & 0 & 1 \\ 0 & \omega_H & -k \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \end{bmatrix}, \quad \mathbf{G} \equiv \begin{bmatrix} \beta & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{H} \equiv \begin{bmatrix} 0 & 0 \\ \frac{1}{\beta\rho} & 1 \\ \left(\frac{\rho_\lambda}{\beta\rho(1-\lambda)} \right) & 0 \end{bmatrix},$$

$$\mathbf{K} \equiv \begin{bmatrix} \frac{k(\rho(1-\lambda)-\rho_\lambda)}{1-\lambda} \\ 0 \\ 0 \end{bmatrix}.$$

To solve this we draw on the solution for the endogenous variables from $T^C + 2$ onwards. Substituting $\gamma_{1,T^c+1} = \gamma_{1,T^c+2} = \dots = 0$. into the first order conditions for the policy problem and using the *NKPC* gives a system of difference equations governing the behaviour of $\widehat{\pi}_{H,t+1}$ and $\phi_{2,t}$ for $t = T^C + 2$ of the form:

$$\begin{bmatrix} \widehat{\pi}_{H,t+1} \\ \phi_{2,t} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{\pi}_{H,t} \\ \phi_{2,t-1} \end{bmatrix} + \mathbf{n} \widehat{\mathbf{x}}_{F,t},$$

$$\text{where } \mathbf{M} \equiv \begin{bmatrix} \frac{1}{\beta} + \frac{1}{\beta\omega_H} \left(\frac{k(\eta(1-\lambda)+\rho_\lambda)}{1-\lambda} \right)^2 & -\frac{1}{\beta\omega_H} \left(\frac{k(\eta(1-\lambda)+\rho_\lambda)}{1-\lambda} \right)^2 \\ -1 & 1 \end{bmatrix} \text{ and } \mathbf{n} \equiv \begin{bmatrix} -k \left(\frac{\rho(1-\lambda)-\rho_\lambda}{1-\lambda} \right) \\ 0 \end{bmatrix}.$$

The unique bounded solution to this difference equation is given by:

$$\begin{bmatrix} \widehat{\pi}_{H,t} \\ \widehat{\mathbf{x}}_{H,t} \\ \gamma_{2,t} \end{bmatrix} = \begin{bmatrix} (1-\lambda_2) \\ \frac{k}{\omega_H} \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \lambda_2 \\ \lambda_2 \end{bmatrix} \gamma_{2,t-1} + \begin{bmatrix} c_{1,t} \\ c_{2,t} \\ c_{3,t} \end{bmatrix}, \quad (30)$$

$$\text{where } c_{1,t} = k \left(\frac{\rho(1-\lambda)-\rho_\lambda}{\beta(1-\lambda)} \right) \sum_{k=t}^{\infty} \lambda_1^{-(k+1-t)} \widehat{\mathbf{x}}_{F,k},$$

$$c_{2,t} = -\frac{\phi_{21}}{\omega_H} k^2 \left(\frac{\eta(1-\lambda)+\rho_\lambda}{1-\lambda} \right) \left(\frac{\rho(1-\lambda)-\rho_\lambda}{\beta(1-\lambda)} \right) \left(\phi_{22} - \frac{\phi_{21}\phi_{12}}{\phi_{11}} \right)^{-1} \left(\widehat{\mathbf{Y}}_{gap,t}^* + \sum_{k=t+1}^{\infty} \lambda_1^{-(k-t)} \widehat{\mathbf{x}}_{F,k} - \lambda_2 \sum_{k=t}^{\infty} \lambda_1^{-(k+1-t)} \widehat{\mathbf{x}}_{F,k} \right),$$

$$c_{3,t} = -\phi_{21} k \left(\frac{\rho(1-\lambda)-\rho_\lambda}{\beta(1-\lambda)} \right) \left(\phi_{22} - \frac{\phi_{21}\phi_{12}}{\phi_{11}} \right)^{-1} \left(\widehat{\mathbf{Y}}_{gap,t}^* + \sum_{k=t+1}^{\infty} \lambda_1^{-(k-t)} \widehat{\mathbf{x}}_{F,k} - \lambda_2 \sum_{k=t}^{\infty} \lambda_1^{-(k+1-t)} \widehat{\mathbf{x}}_{F,k} \right),$$

$$\text{and } \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \mathbf{\Phi} \mathbf{M} \mathbf{\Phi}^{-1}, \quad \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \mathbf{\Phi}.$$