Incorporating Conjunctural Analysis in Structural Models

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Abstract

This paper develops a methodology to incorporate monthly timely information in estimated quarterly structural DSGE models. We derive the monthly dynamics consistent with the model and augment its state space with auxiliary monthly variables typically used in conjunctural analysis. In this way, we can consider quarterly DSGE based estimates of the parameters as given while obtaining increasingly accurate early estimates of the quarterly variables each time data are released throughout the quarter. We illustrate our method on the basis of a prototypical new-keynesian model augmented with a panel of twenty three monthly indicators for the US economy.

JEL Classification:

Keywords:

1 Introduction

Structural macroeconomic models are important tools for policy analysis. In particular, dynamic stochastic general equilibrium (DSGE) models are becoming increasingly popular as policy analysis tools in many institutions. By explicitly taking into account forward behavior on the part of the agents, they provide a useful framework to analyze the effects of alternative policies. These models are typically estimated on quarterly data and, in recent years, knowledge has been build up on reliability of the estimates, forecasting performance, reasonable values for calibrated parameters and setting of the priors.

However, these tools are not suitable for policy analysis in real time since they do not take advantage of information contained in monthly timely data releases in order to now-cast and forecast the key variables in the model. For the practical use of these models in policy institutions this is an important problem given the publication lag of many key quarterly variables. For example, in the US, the first official figure for GDP is the preliminary estimate published at the end of the month following the end of the reference quarter. In the Euro Area, the first official figure is the flash estimate which is published about six weeks after the end of the reference quarter.

Monthly timely indicators such as surveys, car registrations and other variables used in conjunctural analysis, are typically not the focus of structural analysis. The analysis of this information is performed by short-term forecasters and it is based on either judgement or reduced form models. Usual practice for the structural modeler is to take the early estimates produced by the forecasters and use them as if they were observations as an input in the model. However, treating forecasts as observations, albeit noisy, assigns to the conjunctural analysts a knowledge of the future that is totally unrealistic. It would be like saying that they *see* into the future with some noise, rather than forecasting future economic conditions with their current information set. An alternative has been proposed by Monti (2007) who suggests treating judgemental/conjunctural analysts as forecasters with a possibly larger information set. These forecasts can then be filtered and combined with a structural model to generate forecasts that are model-based, but that incorporate the extra-information available to the judgmental forecasters. The approach proposed in Monti (2007) is similar in spirit to the one used by Coenen, Levin and Wieland (2005) to deal with revisions, but with a key difference: she considers the judgmental forecasts as optimal forecasts made with a different information set, not a noisy signal of the actual variables.

The alternative explored in this paper is to combine a statistical model for bridging staggered monthly releases with quarterly GDP, developed by Giannone, Reichlin and Small (2008) and widely used in central banks, with a DSGE model. Our framework allows us to update the forecast at the time of each data release and monitor in real time key quantities, both observable variables like GDP and unobservable, model-based variables, such as total factor productivity (TFP) or the output gap.

In our approach, we will take the estimated quarterly DSGE model at face value and keep the estimated parameters as they are produced by the quarterly DSGE modellers. To exploit monthly information in a model consistent way, we derive the monthly state space representation that corresponds to the quarterly model and augment it with additional series which are believed to provide early information on the key quarterly variables of the model which are published late. On the basis of this framework, we can update the now-cast and forecast of the key quarterly variables taking into account the real time data flow. That is, we can update the estimates each time new data are released throughout the quarter. This allows us to interpret the early releases with the lenses of the model. By combining structural analysis with conjunctural analysis we can update our "stories", in principle, each day of the month. An additional interesting feature of the model is that we can assess the marginal impact of particular news on the variables of interest.

A key feature of our methodology is that the extra information provided by the monthly panel is valuable only because it is more timely. At the end of the quarter, the DSGE combined with the statistical model for monthly variables and the quarterly DSGE model with no extra information produce the same results.

The method of this paper is related to other approaches that combine reduced form and structural analysis, but differs both in techniques and objectives. Del Negro et al. (2005) have proposed a framework which combines VAR and DSGE analysis to provide the modeller with a tool for attributing the desired weight to the structural model with respect to the VAR via a prior. Boivin and Giannoni (2006), on the other hand, use information in large panels of data to obtain better estimates of the states of the structural model in a framework in which the variables of interest are observed with an error. While their aim is to improve the quarterly estimates of the DSGE modeller we "do not interfere" with her. At the end of the quarter, our augmented model is the same as the structural quarterly DSGE.

The paper is organized in four sections. In the first section we explain the methodology. In the second we illustrate the design of the empirical application based on the new-Keynesian model of Del Negro and Schorfheide (2004) and a panel of twenty three monthly series for the US economy. In the third we describe the design of the forecast exercise and in the fourth we discuss results.

2 The methodology

We consider structural quarterly models whose log-linearized solution have the form:

$$\begin{aligned} S_{t_q} &= \mathcal{T}_\theta S_{t_q-1} + \mathcal{B}_\theta \varepsilon_{t_q} \\ Y_{t_q} &= \mathcal{M}_\theta(L) S_{t_q-1} \end{aligned} \quad (1)$$

where t_q is time in quarters, $Y_{t_q} = (y_{1,t_q}, ..., y_{k,t_q})'$ is a set of observable variables which are transformed to be stationary, s_t are the states of the model and ε_t are structural orthonormal shocks. The filter $\mathcal{M}_\theta(L) = \mathcal{M}_{0,\theta} + \mathcal{M}_{1,\theta}L + ... + \mathcal{M}_{p,\theta}L^p$, the autoregressive matrix $\mathcal{T}_\theta$ and the coefficients $\mathcal{B}_\theta$ are function of the deep, behavioral, parameters which are collected in the vector θ. We will consider the model and the parameters as given by the structural modeler who obtained them by estimation or calibration.

As it is standard, we consider the situation in which the model is estimated at the quarterly frequency and the variables are key quarterly series, such as GDP and national account data or variables that are available at higher frequencies, like financial or price data, but enter the model as quarterly, either as averages over the quarter or as end of the quarter values.

In this standard case, the model can be updated only when the quarterly observations become available: therefore one must wait for the end of the quarter or even later, when the variables that are published the latest are finally released. Notice that in a quarterly model also variables with monthly or higher native frequency are incorporated with a delay when they enter the model.

Our objective is to define a framework in which the statistical model used to exploit monthly data releases, either referring to variables included in the model as quarterly or to variables that can provide early information about GDP or other key quantities, can be linked in a consistent way to the structural model so as to obtain early estimates of the variables considered by the model.

The statistical model we will use is that developed by Giannone, Reichlin and Small (2008) . This is a model that aims at bridging the monthly information as it becomes available throughout the quarter with quarterly quantities. The interesting features of the model is that it can incorporates information in many monthly data and it provide consistent estimates from panels of data with jagged edge, that is data that, due to publication lags, have missing information at the end of the sample. In what follows we will show how to link this statistical framework with the structural model.

Let us define by t_m the time in months and denote by $Y_{t_m} = (y_{1,t_m}, ..., y_{k,t_m})'$ the vector of possible latent monthly counterparts of the variables that enter the quarterly model that are transformed so as to correspond to a quarterly quantity when observed at the last month of each quarter, i.e. when t_m corresponds to March, June, September or December.

For example let y_{i,t_q} be the CPI inflation $(\pi_{t_q} = (\log P_{t_q} - \log P_{t_q-1}) \times 100)$ and suppose that it enters the models as average over the quarter, then:

$$\begin{aligned} y_{i,t_m} &= [(\log P_{t_m} + \log P_{t_m-1} + \log P_{t_m-2}) - (\log P_{t_m-3} + \log P_{t_m-4} + \log P_{t_m-5})] \times 100 \\ &\approx [\log (P_{t_m} + P_{t_m-1} + P_{t_m-2}) - \log (P_{t_m-3} - \log P_{t_m-4} - \log P_{t_m-5})] \times 100 \end{aligned}$$

Let us further consider additional monthly variables that carry out information on current economic conditions. We define by $X_{t_m} = (x_{1,t}, ..., x_{n,t})'$ the vector of these auxiliary stationary monthly variables transformed as above so as to correspond to quarterly quantities at the end of each quarter.

For example let us consider the index of capacity utilization CU_{t_m} and suppose that, to make it stationary, we have to take first differences. Then, assuming CU_{t_m} is in the $j-$th position of the vector of auxiliary variables, we have:

$$x_{j,t_m} = \frac{1}{3}\left[(CU_{t_m} + CU_{t_m-1} + CU_{t_m-2}) - (CU_{t_m-3} + CU_{t_m-4} + CU_{t_m-5})\right]$$

which, when observed at the last month of a quarter, corresponds to the quarterly change of the average capacity utilization over that quarter.[1]

Let us first consider for simplicity how to incorporate the monthly information contained in Y_{t_m}. We cannot use directly the model since the latter specifies the dynamics of the data at a quarterly frequency, hence we need to define a monthly dynamics that is compatible with the model.

In accordance with our definition of the monthly variables, we can define the vector of monthly states s_{t_m} as a set of latent variables which corresponds to its quarterly model-based concept when observed at the last month of each quarter. Hence, it follow that our original state equation

$$s_{t_q} = \mathcal{T}_\theta \ s_{t_q-1} + B_\theta \varepsilon_{t_q}$$

[1]If capacity utilization is instead already stationary in the level then

$$x_{j,t_m} = \frac{1}{3}(CU_{t_m} + CU_{t_m-1} + CU_{t_m-2}))$$

which corresponds to the average capacity utilization over the quarter.

can be rewritten in terms of the monthly latent states as

$$s_{t_m} = \mathcal{T}_\theta \ s_{t_m-3} + B_\theta \varepsilon_{t_m}$$

when t_m corresponds to the last month of a quarter.

There are many monthly VARMA processes, but a unique VAR(1), that can deliver the above relation between s_{t_m} and s_{t_m-3}. We will assume that the monthly states follow a VAR(1), in order to maintain the dynamics of the monthly model as similar as possible to the quarterly one. Hence

$$s_{t_m} = \mathcal{T}_m \ s_{t_m-1} + \mathcal{B}_m \varepsilon_{m,t_m} \tag{2}$$

where ε_{m,t_m} are orthonormal shocks. This implies:

$$s_{t_m} = \mathcal{T}_m^3 \ s_{t_m-3} + [\mathcal{B}_m \varepsilon_{m,t_m} + \mathcal{B}_m \mathcal{T}_m \varepsilon_{m,t_m-1} + \mathcal{B}_m \mathcal{T}_m^2 \varepsilon_{m,t_m-2}].$$

This last equation gives us a unique mapping from the coefficients of the quarterly model to the coefficients of the monthly model, which can be recovered from the following equations.

$$\begin{aligned} \mathcal{T}_m &= \mathcal{T}_\theta^{\frac{1}{3}} \\ vec(\mathcal{B}_m \mathcal{B}_m') &= (I + \mathcal{T}_m \otimes \mathcal{T}_m + \mathcal{T}_m^2 \otimes \mathcal{T}_m^2)^{-1} vec(\mathcal{B}_\theta \mathcal{B}_\theta'). \end{aligned}$$

Let us now turn to the monthly version of the observation equation. We will start by analyzing the (not very realistic) case in which all variables are observable at monthly frequency. The monthly observation equation would then be:

$$Y_{t_m} = \mathcal{M}_m(L) S_{t_m} \tag{3}$$

where

$$\mathcal{M}_m(L) = \left(\mathcal{M}_{0,\theta} + 0 \cdot L + 0 \cdot L^2 + \mathcal{M}_{1,\theta} L^3 + ... + \mathcal{M}_{p,\theta} L^{3p}\right)$$

The equations (2) and (3) therefore describe the dynamics that is compatible with the quarterly model. If all the observables of the model were available at a monthly frequency, we could now simply use the monthly model defined by equations (2) and (3) to immediately incorporate this higher frequency information. However, some variables - think of GDP, for example - are not available at monthly frequency. So let us assume, that the variable in the i-th position of the vector of observables Y_{t_m}, i.e. y_{i,t_m}, is not available at a monthly frequency, but only at the quarterly frequency. This means that y_{i,t_m} is a latent variable when t_m does not correspond to the end of a quarter. Moreover, due to the unsynchronized data releases schedule data are not available on the same span (the dataset has jagged edges). The unavailability of some data does not prevent us from still taking advantage of the monthly information that is available using a Kalman filter. To do so, we follow Giannone Reichlin and Small (2008) and define the following state space model

$$\begin{aligned} s_{t_m} &= \mathcal{T}_m \ s_{t_m-1} + \mathcal{B}_m \varepsilon_{m,t_m} \\ Y_{t_m} &= \mathcal{M}_m(L) s_{t_m} + V_{t_m} \end{aligned}$$

where $V_{t_m} = (v_{1,t_m}, ..., v_{k,t_m})$ is such that $\text{var}(v_{i,t_m}) = 0$ if y_{i,t_m} is available and $\text{var}(v_{i,t_m}) = \infty$ otherwise.

Let us now turn to how we incorporate the auxiliary monthly variables. As a starting point we the define the relation between the auxiliary variables X_{t_q} and the model's observable variables at a quarterly frequency:

$$X_{t_q} = \mu + \Lambda Y_{t_q} + e_{t_q} \tag{4}$$

where e_{t_q} is orthogonal to the quarterly variables entering the model. Given that some of the observables are available only at a quarterly frequency, we will use this equation to estimate the coefficients Λ and the variance-covariance matrix of the shocks $E(e_{t_q} e'_{t_q}) = R$. Let us now focus on incorporating the auxiliary variables in their monthly form. As stressed above, $X_{t_m} = (x_{1,t}, ..., x_{n,t})'$ is the vector of these auxiliary stationary monthly variables transformed so as to correspond to quarterly quantities at the end of each quarter. We can related X_{t_m} to the monthly observables Y_{t_m} using the equivalent of equation (4) for the monthly frequency (the bridge model):

$$X_{t_m} = \mu + \Lambda Y_{t_m} + e_{t_m} \tag{5}$$

where $e_{t_m} = (e_{1,t_m}, ..., e_{k,t_m})$ is such that $\text{var}(e_{i,t_m}) = [R]_{i,i}$ if X_{i,t_m} is available and $\text{var}(e_{i,t_m}) = \infty$ otherwise. This way we take care of the problem of the jagged edge at the end of the dataset, due to the fact that the data is released in an unsynchronized fashion and that the variables have different publishing lags (e.g. Capacity utilization releases refer to the *previous* month's total capacity utilization, while the release of the Philadelphia Business Outlook Survey refers to the *current* month). We will use equation (5) to expand the original state-space:

$$\begin{array}{rcl} s_{t_m} & = & \mathcal{T}_m \ s_{t_m-1} + \mathcal{B}_m \varepsilon_{m,t_m} \\ Y_{t_m} & = & \mathcal{M}_m(L) s_{t_m} + V_{t_m} \\ X_{t_m} - \mu & = & \Lambda \mathcal{M}_m(L) s_{t_m} + e_{t_m} \end{array} \tag{6}$$

where V_{t_m} and e_{t_m} are defined above. The state-space form (6) allows us to account for and incorporate all the information about the missing observables contained in the auxiliary variables.

The choice of modeling X_{t_m} as solely dependent on the observables, rather than depending in a more general way from the states, is motivated by the fact that we want the auxiliary variables to be relevant *only* when the quarterly data is not available. Indeed, with this modeling approach, the monthly auxiliary information becomes redundant, when the quarterly data is available. This would not have been the case, if we had bridged the monthly variables directly with the states, because in this case we could have exploited the auxiliary variables to get better estimates of the latent state variables, even when the data for the observables became fully available.

Moreover, the choice of modeling the bridge model as a function of the observables only is not model dependent. The estimation of the coefficient Λ depends exclusively on the data and the model enters the bridge equation solely by imposing the transitional dynamics. This feature is nice, because it yields more robustness.

In the following section we present an application of the methodology described above.

3 Design of the Forecasting Exercise

We use a simple new-keynesian dynamic stochastic general equilibrium model, as the one used in Del Negro and Schorfheide (2004). The only source of nominal rigidities in this model is the presence of adjustment costs that firms incur in when changing their prices. A detailed description of the model is reported in Appendix, while here we present only the log-linearized model.

The log-linearized system can be reduced to three equations in output inflation and the interest rate:

$$
\begin{aligned}
\hat{y}_t - \hat{g}_t &= E_t\left(\hat{y}_{t+1} - \hat{g}_{t+1}\right) - \frac{1}{\tau}\left(\hat{r}_t - E_t\hat{\pi}_{t+1} - \rho_z\hat{z}_t\right) \\
\hat{\pi}_t &= \beta E_t\hat{\pi}_{t+1} + \kappa(\hat{y}_t - \hat{g}_t) \\
\hat{r}_t &= \psi_1(1-\rho_r)\hat{\pi}_t + \psi_2(1-\rho_r)\hat{y}_t + \rho_r\hat{r}_{t-1} + \varepsilon_{r,t}
\end{aligned}
\tag{7}
$$

The first equation relates current output gap ($\hat{y}_t - \hat{g}_t$ because we are in presence of government spending) with the expected future output gap, the real interest rate ($\hat{r}_t - E_t\hat{\pi}_{t+1}$)and the shocks to the technology process $\hat{z}_t$, which is assumed to evolve following the process:

$$\hat{z}_{t+1} = \rho_z\hat{z}_t + \varepsilon_t^z.$$

Also the government spending shock follows an AR(1) process:

$$\hat{g}_{t+1} = \rho_g\hat{g}_t + \varepsilon_t^g.$$

The second equation is the familiar new Phillips curve and the last equation is a standard Taylor rule.

The relation between log-deviations from steady state and observable output growth, CPI inflation and the annualized nominal interest rate is given by the following measurement equation.

$$
\begin{aligned}
INFL_t &= \pi^* + 4\pi_t \\
RA_t &= \pi^* + r^* + r_t \\
\Delta\ln GDP_t &= \ln\gamma + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t
\end{aligned}
\tag{8}
$$

The model given by equations (7) can then solved with standard techniques, such as those proposed by Blanchard and Kahn (1980), Uhlig (1999), Klein (2000), Sims (2002), among others. More specifically, the model has a solution in terms:

$$s_t = \begin{bmatrix} \hat{r}_t \\ \hat{g}_t \\ \hat{z}_t \end{bmatrix} = A_\theta s_{t-1} + B_\theta\varepsilon_t \tag{9}$$

$$Y_t = \begin{bmatrix} INFL_t \\ RA_t \\ \Delta\ln GDP_t \end{bmatrix} = C_\theta(L)s_t$$

	Prior Distribution			Posterior Distribution		
	Distribution	mean	st.dev	mode	mean	st.dev
γ	Normal	0.5	0.5	0.7023	0.7133	0.1210
π^*	Gamma	5	2	4.4461	4.9926	1.7046
r^*	Gamma	2	1	2.6212	2.5298	0.5104
τ	Gamma	2	0.5	2.7139	2.8401	0.4815
κ	Gamma	0.3	0.1	0.1498	0.1722	0.0613
ψ_1	Gamma	1.5	0.5	1.4112	1.6038	0.3199
ψ_2	Gamma	0.5	0.25	0.4170	0.5594	0.1916
ρ_g	Beta	0.9	0.05	0.9633	0.9595	0.0179
ρ_z	Beta	0.2	0.1	0.3901	0.3740	0.1127
ρ_r	Beta	0.7	0.15	0.8753	0.8820	0.0294
σ_g	InvGamma	1.25	0.65	0.5038	0.5185	0.0747
σ_z	InvGamma	1.25	0.65	0.5685	0.6158	0.0704
σ_z	InvGamma	0.63	0.33	0.6320	0.6523	0.0669

Table 1: Prior and posterior distribution of the parameters of the model estimated over the period 1982Q1 to 1996Q4.

For simplicity, we perform the estimation of the underlying parameters θ only once at the beginning of the evaluation sample, *i.e.* in 1997Q1, using data for the period period 1982Q1 to 1996Q4.

We want to show how to incorporate a set of monthly variables into the prototypical new-keynesian model defined above, obtaining forecasts that are more accurate that the ones based solely on the model and, what is more important, real-time estimates of model-based concepts such as TFP growth and the natural rate of interest.

Clearly the model (9) has the form of the state space form (1) and hence it is possible to determine its monthly dynamics as described in the previous section. We perform the forecasting exercise over the evaluation sample 1997Q1-2007Q4 using quarter-on-quarter GDP growth, CPI annualized quarterly inflation, the annualized Fed Funds rate and a panel of series that are deemed to be informative on the state on the economy, e.g. the ones NBER business cycle dating committee looks at or the ones that Bloomberg reports. More specifically, the series are: Purchasing Managers Index (PMI), Total Construction put in place: Total (CONSTR.), Total Employment on nonag payrolls and Average hourly earnings (EMPL, to identify the series on the employment situation), Total Industrial Production (IP), Total Capacity Utilization (CU), Business Outlook Survey of the Philadelphia Fed: General activity (BOS: Phil Fed), PPI of finished goods (1982=100 for all PPI data), PPI of crude materials, CPI of all items (urban), Total SALES: Manufacturing and Trade, Total INVENTORIES: Manufacturing and Trade, Real disposable personal income (RDPI), PCE: Total.

Table 2 describes a stylized calendar of data releases where variables have been grouped in twenty three clusters according to their timeliness. The stylization consists in associating a date with a group of variables with similar economic content (soft, quantities, prices and so on). This is a quite realistic representation of the calendar and will allow us to evaluate the changes in the forecast with variables with a given economic content.

In the first column we indicate the data release, in the second the series and in the third the date the release refers to which gives us the information on the publication lag. We can see, for example, that the Philadelphia Fed Survey is the first release referring to the current month m and it is published the last day of the first month of the quarter. Hard data arrive later. For example, industrial production is published in the middle of the second month of the quarter and refers to the previous month. GDP, released the last week of the last month of the quarter refers to the previous quarter.

	timing	release	publication lag
1	1^{st} day of the 1^{st} month of the quarter	-	-
2	1^{st} business day of the 1^{st} month of the quarter	PMI and construction	m-1
3	1^{st} Friday of the 1^{st} month of the quarter	Employment situation	m-1
4	15^{th} to 17^{th} of the 1^{st} month of the quarter	Industrial Production and Capacity Utilization	m-1
5	3^{rd} Thursday of the quarter	Business Outlook Survey: Philadelphia Fed	m
6	Middle of the 1^{st} month of the quarter	CPI and PPI	m-1
7	Last week of 1^{st} month of the quarter	GDP release	q-1
8	Day after GDP release	Inventories, Sales, PCE, RDPI	m-2 (INV and sales), m-1 (PCE, RDPI)
9	Last day of the 1^{st} month of the quarter	Fed Funds rate	m
10	1^{st} business day of the 2^{nd} month of the quarter	PMI and construction	m-1
11	1^{st} Friday of the 2^{nd} month of the quarter	Employment situation	m-1
12	15^{th} to 17^{th} of the 2^{nd} month of the quarter	Industrial Production and Capacity Utilization	m-1
13	3^{rd} Thursday of the 2^{nd} month of the quarter	Business Outlook Survey: Philadelphia Fed	m
14	Middle of the 2^{nd} month of the quarter	CPI and PPI	m-1
15	Last week of the 2^{nd} month of the quarter	Inventories, Sales, PCE, RDPI	m-2 (INV and sales), m-1 (PCE, RDPI)
16	Last day of the 2^{nd} month of the quarter	Fed Funds rate	m
17	1^{st} business day of the 3^{rd} month of the quarter	PMI and construction	m-1
18	1^{st} Friday of the 3^{rd} month of the quarter	Employment situation	m-1
19	15^{th} to 17^{th} of the 3^{rd} month of the quarter	Industrial Production and Capacity Utilization	m-1
20	3^{rd} Thursday of the 3^{rd} month of the quarter	Business Outlook Survey: Philadelphia Fed	m-1
21	Middle of the 3^{rd} month of the quarter	CPI and PPI	m-1
22	Last week of the 3^{rd} month of the quarter	Inventories, Sales, PCE, RDPI	m-2 (INV and sales), m-1 (PCE, RDPI)
23	Last day of the 3^{rd} month of the quarter	Fed Funds rate	m

Table 2: Data releases are indicated in rows. Column 1 indicates the progressive number associated to each "vintage". Column 2 indicates the official dates of the publication. Column 3 indicates the releases. Column 4 indicates the publishing lag: e.g. IP is release with 1-month delay (m-1).

The forecast will be updated twenty three times throughout the quarter, corresponding to the stylized calendar 2. In this way we can associate to each update a date and a set of variables. The horizontal axis of the Figures below, reporting the results, indicate the grouping of releases corresponding to the calendar.

4 Empirical results

4.1 Forecast Accuracy

Figure (1), shows how the mean square forecast errors (MSFE) of the nowcasts of GDP growth produced with the quarterly DSGE model (Q) and with the monthly DSGE model that also exploits the information contained in the panel (M+panel) change with the arrival of new information within the quarter. Also a naive benchmark is shown for comparison.[2] In the case of GDP growth the naive benchmark is a constant growth model (random walk in levels) which is estimated as the mean of the last 10 years GDP growths. Notice that it changes in correspondence with panel 7, that is when we can incorporate the data of GDP growth in the last quarter, which was not available in the previous 5 panels.

We also compare the performance of Q and M+panel with the performance of the SPF's nowcast of GDP growth. Since the SPF forecasts are released approximately around the middle of the second month of the quarter, we choose to match it with forecasts produced with approximately the same information, *i.e.* in the second month of the quarter, between the employment release and the Philadelphia Fed Business Outlook Survey release.

As mentioned above, the parameters of the DSGE model are estimated once, at the beginning of the evaluation sample, and are kept fix hence forth. The coefficients that load the observables of the DSGE into the rest of the panel are instead re-estimated at every step.

Results show that, in the first month, the information flow has very little impact on the MSFE of GDP nowcasts made with the monthly model that exploits the panel. This is because, during the first month, until the arrival of the data for that month's Fed funds rate at the very end of the month, all the information being released involves the previous quarter. As soon as information on the current quarter starts to arrive (with the Fed funds rate, PMI, total construction and the employment situation of the first month of the quarter, which all arrive at the beginning of the second month), we start seeing the positive impact of the new information on the accuracy of the predictions. Moreover, comparing the nowcast of GDP produced with the monthly model that exploits the information available at the middle of the second month with the SPF nowcasts, one can see that the M+panel model does as good as, if not better, than the SPF. The smooth decline in the MSFE of GDP nowcasts of the monthly model implies that each of the new releases carries some information that is relevant for predicting today's GDP growth accurately.

Figures (2) and (3) report the mean square forecast errors (MSFE) of the

[2]We chose to present the results like in absolute rather than in relative terms, in order to better highlight the arrival of the information and the impact it has on the various models, including the benchmark.

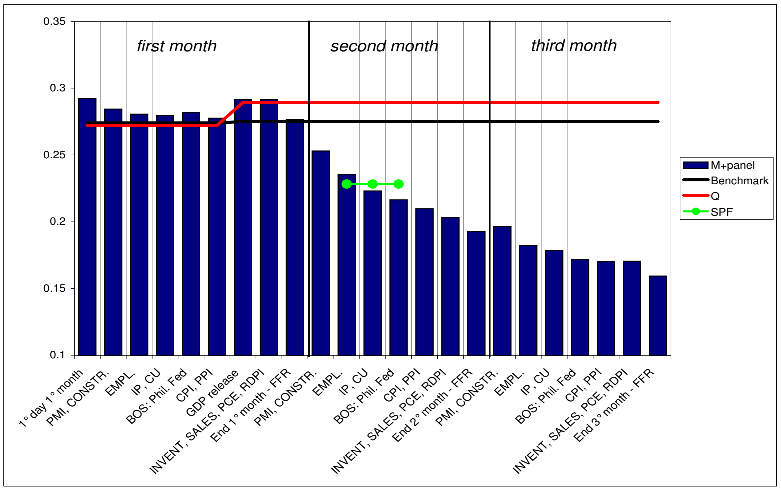

Figure 1: **NOWCAST GDP growth: MSFE across vintages throughout the month**

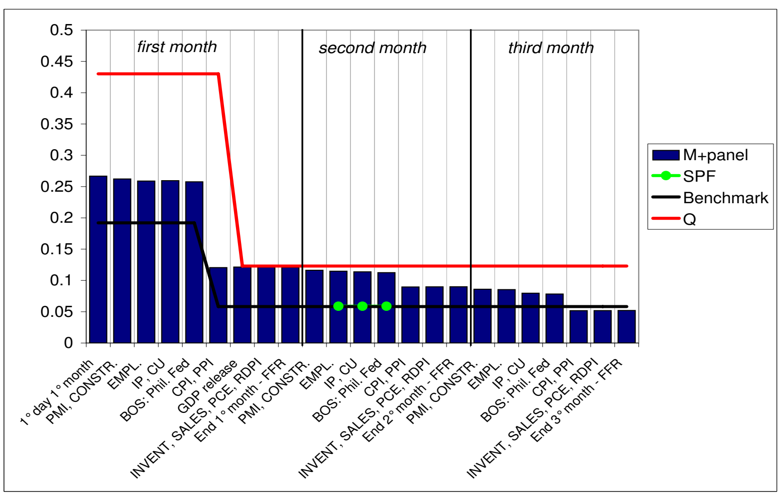

Figure 2: **NOWCAST CPI inflation: MSFE across vintages throughout the month**

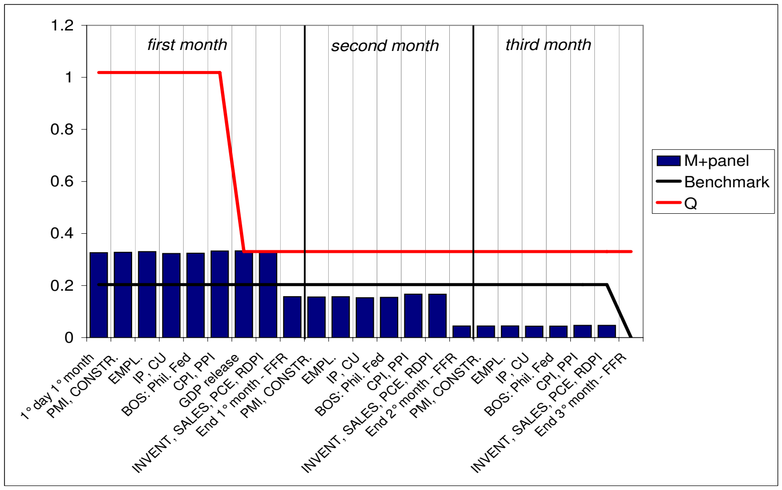

Figure 3: **NOWCAST Fed funds rate: MSFE across vintages throughout the month**

nowcasts for CPI year-on-year inflation and the annualized Fed Funds rate, respectively, produced with the quarterly DSGE model (Q) and with the monthly DSGE model that also exploits the information contained in the panel (M+panel), and compares them to a naive benchmark. We construct nowcasts of CPI year-on-year inflation as the mean of the last three available data points for annualized quarterly CPI inflation and the nowcast of annualized quarterly CPI inflation produced with each of the models we compare. Similarly, the one-step-ahead forecast of CPI year-on-year inflation is the mean of the last two available data points for annualized quarterly CPI inflation and the nowcast and one-step-ahead forecasts of (annualized) quarterly GDP generated by the models under consideration. The naive model for CPI year-on-year inflation is the last available year-on-year inflation: it is constructed, using quarterly data, as the mean of the last 4 available data points. This means that, as is obvious from Figure (3), the benchmark will change when the data for CPI inflation of the last quarter becomes available, i.e. at panel 6. The naive model for the Fed Funds rate is a random walk, that is we assume that the Fed Funds rate today is equal to the Fed Funds rate of the previous quarter. Hence, when the information on the Fed funds rate for this quarter becomes available, i.e. in panel 23, the errors go to zero.

As for GDP growth, also the accuracy of the predictions of CPI y-on-y inflation and the Fed funds rate improves as more information is released. However, the step shape of Figures (2) and (3) indicates that the variables of the panel are not very relevant in improving the accuracy of the forecasts, it is just the arrival of more data points for CPI inflation and the Fed Funds rate, respectively, that

Table 3: Mean square forecast errors of quarter-on-quarter GDP growth forecasts with horizons 0 to 4 for the different model relative to the naive benchmark.

	NB	SPF	Q	M	M+panel
Q0	0.275	0.226	0.289	0.275	0.223
Q1	0.281	0.272	0.278	0.281	0.278
Q2	0.274	0.287	0.269	0.272	0.270
Q3	0.269	0.272	0.261	0.263	0.262
Q4	0.276	0.280	0.264	0.266	0.265

Table 4: Mean square forecast errors of CPI year-on-year inflation forecasts with horizons 0 to 4 for the different model relative to the naive benchmark.

	NB	SPF	Q	M	M+panel
Q0	0.058	0.059	0.123	0.123	0.114
Q1	0.196	0.181	0.440	0.440	0.423
Q2	0.306	0.367	0.494	0.519	0.473
Q3	0.447	0.572	0.713	0.740	0.672
Q4	0.594	0.713	1.092	1.132	1.059

Table 5: Mean square forecast errors of forecasts for the Fed funds rate with horizons 0 to 4 for the different model relative to the naive benchmark.

	NB	Q	M	M+panel
Q0	0.2036	0.3306	0.1586	0.1530
Q1	0.7291	1.0436	0.7705	0.7440
Q2	1.5083	1.8856	1.5822	1.5285
Q3	2.4527	2.7757	2.4775	2.4025
Q4	3.4712	3.6764	3.3953	3.3136

helps.

Tables (3)-(5) report the MSFE of nowcasts and forecasts up to 4 quarters ahead, for GDP growth, year-on-year CPI inflation and the Fed Funds rate, respectively. We compare the the naive model, the quarterly DSGE model (Q), the monthly DSGE model (M) and the monthly DSGE model that also exploits the information contained in the panel (M+panel) with the forecasts produced by SPF. Hence, in order to match the information available to them at the time of the forecast, we generate the forecasts of tables (3)-(5) with "panel 12", *i.e.* corresponding to the release of Industrial Production and Capacity utilization data in the second month of each quarter.

Looking at Tables (3)-(5), two results are worth mentioning. First, the performance of the M model (*i.e.* the monthly model that does not exploit the panel) when nowcasting, is quite close to the performance of the quarterly model. This implies, that the improvement obtained with the M+panel model really derives from the extra-information and not from the monthly dynamics. Second, notice how, as the forecasting horizon increases, the performance of the Q, M and M+panel models becomes more and more similar. That is, the information that can be extracted for the panel of variables is relevant only when nowcasting.

Figures (4)-(6) depict, respectively for GDP growth, year-on year CPI inflation and the Fed Funds rate, the nowcasts produced with by the naive model,

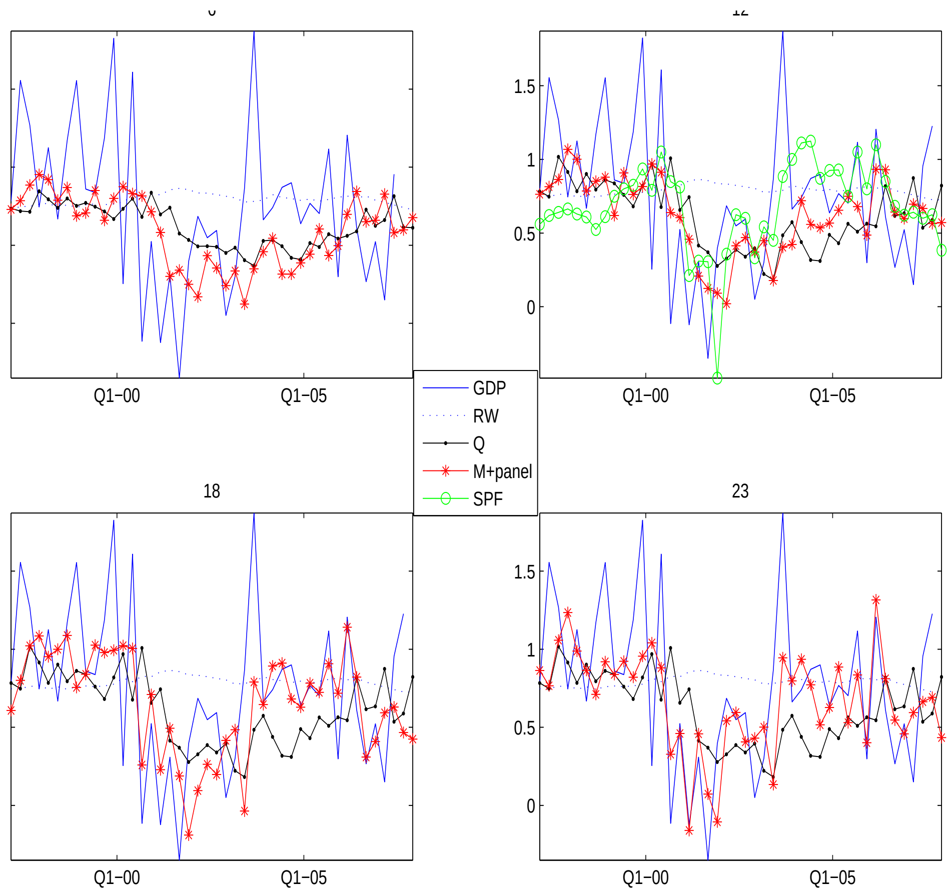

Figure 4: **NOWCAST of GDP for 4 representative vintages**

the Q model and M+panel model, at different dates in the quarter, and hence with different information available to forecast.[3] The top-left panel of each graph report the nowcasts generated with panel 6, *i.e.* in the first month of the quarter, right after the release of the data for prices. The top-right panel graphs the nowcasts produced with "panel 12", the day of the release of Industrial Production and Capacity utilization data in the second month of each quarter. Since the information, in this case, approximately matches the one available to the SPF when the produce their forecasts, we include the latter in the graph. The bottom-left panel of figures (4)-(6) plots the nowcasts generated by the various models the day of the release of employment data in the third month of the quarter (panel 18). Finally, the bottom-right panel reports the nowcasts produced at the end of the third month of each quarter, once the information on that quarter's Fed Funds rate becomes available.

It is evident from Figure (4)that the nowcast produced in real-time with the

[3]The different information sets are identified with the progressive number of table 2. So, for example, panel 12 is the panel obtained as a snapshot of the the information available right after the release, in the second month of the quarter, of the IP and CU for the first month of the quarter.

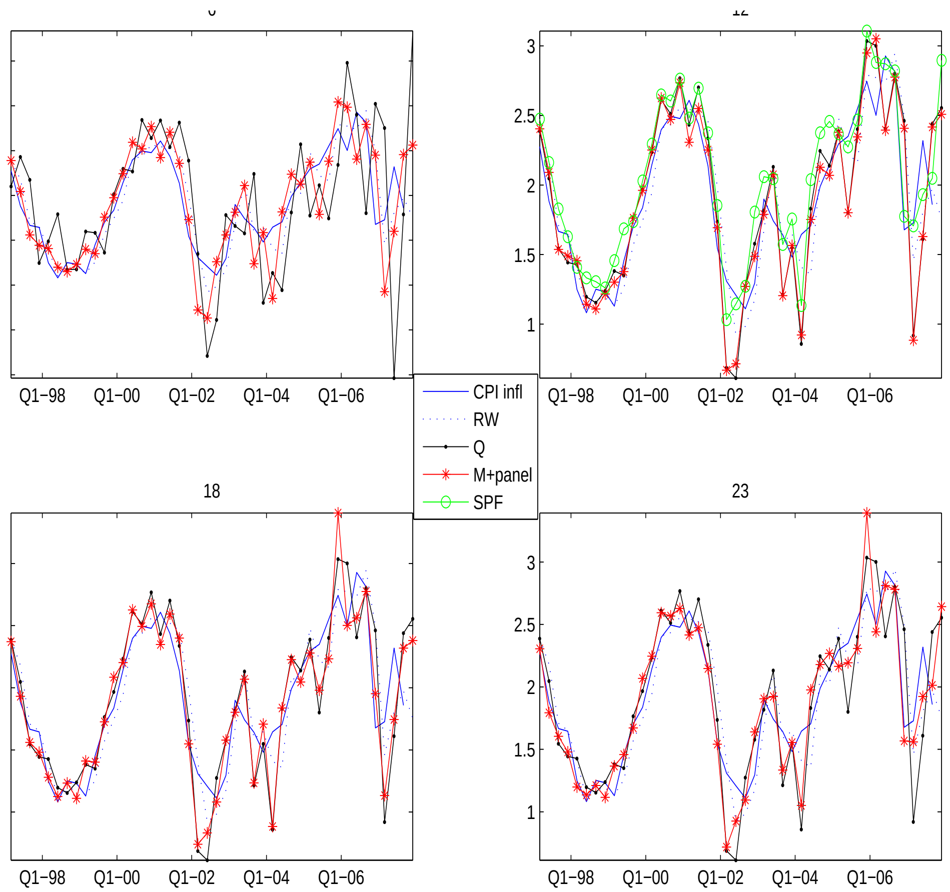

Figure 5: **NOWCAST of CPI for 4 representative vintages**

M+panel model is effective at tracking GDP growth and that it compares well with the SPF. Moreover, a general look to both Figures (1)-(3) and Figures (4)-(6) allows us to make the following conclusions. First, while the performance of the M+panel model in forecasting GDP is exceptional, the M+panel model is not as effective in estimating CPI inflation, where the benchmark beats all models. Its performance with respect to the Fed funds is instead quite good. Figures(1)-(3) highlight the marginal impacts of data releases on the accuracy of the nowcasts of the variables of interest. While the smooth decline in the MSFE of the GDP nowcasts indicates that, from the second month on, all new releases improve the accuracy of the forecast, the step-shape of Figures (2) and (3) implies that the only relevant information for these two variables are their own releases.

4.2 Structural analysis

The second set of results involves the structural features of the models. Given that we have a fully-fledged structural model, we can use it to forecast and analyze quantities that are unobserved and intrinsically meaningful only within

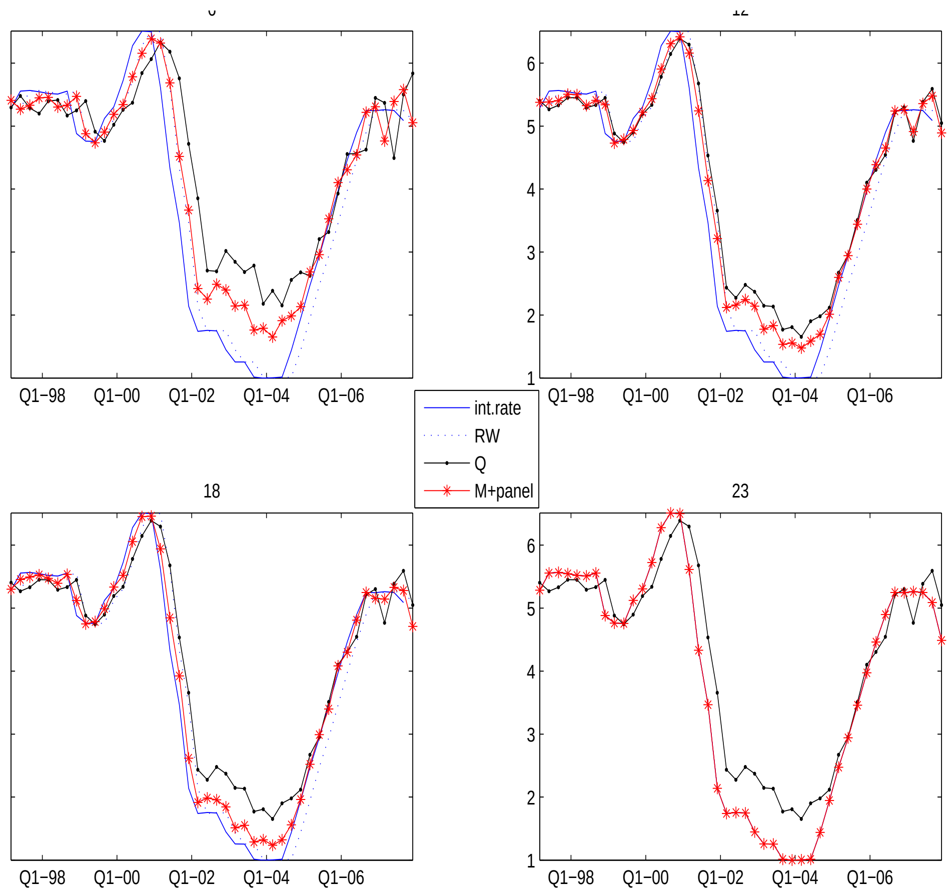

Figure 6: **NOWCAST of RA for 4 representative vintages**

the context of a structural model, such as the TFP.

Since the variable "TFP" is unobserved, we take it's ex-post estimate - *i.e.* the estimate produced by the quarterly DSGE model using all available data up to 2007Q4 - to be the "true" one. Then we try to match, in real-time, this ex-post estimate of the TFP using the Q model (the quarterly model that uses only the "quarterly" observable variables) and the M+panel model (the monthly model that exploits the variables of the panel). We also construct a series of "TFP" estimates intrinsic in the SPF forecasts; we obtain these by taking the SPF nowcasts and forecasts for GDP and CPI year-on-year inflation as if they were "actual" data and that feeds into the quarterly DSGE (Q) model. The filter will return a series for the TFP which now accounts for the SPF nowcasts and forecasts.

Figure (7) shows how the mean square forecast errors (MSFE) of the nowcasts of TFP growth produced with the quarterly DSGE model (Q) and with the monthly DSGE model that also exploits the information contained in the panel (M+panel) change with the arrival of new information within the quarter. We report also the MSFE of a naive benchmark, a constant growth model (random

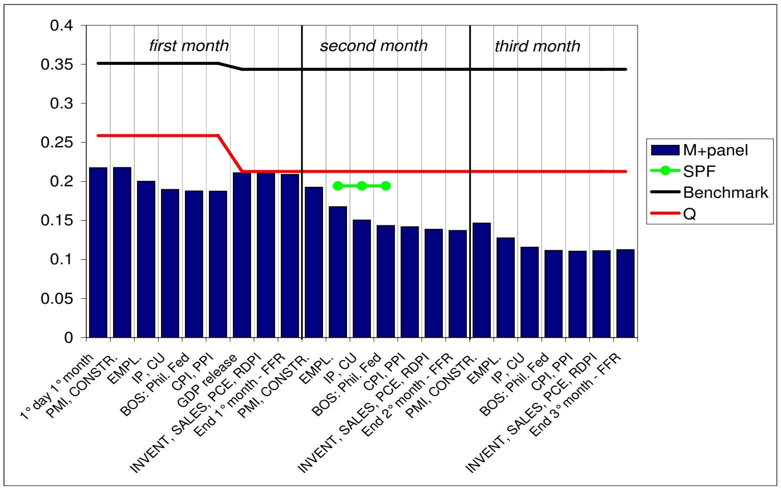

Figure 7: **NOWCAST TFP growth: MSFE across vintages throughout the month**

walk in levels) which is estimated as the mean of the last 10 years TFP growths. We also compare the performance of Q and M+panel with the performance of the SPF's nowcast of TFP growth, obtained as specified above.

Figure (8) depicts the nowcasts of TFP growth produced with by the naive model, the Q model and M+panel model, at different dates in the quarter, and hence with different information available to forecast. The top-left panel of each graph report the nowcasts generated with panel 6, *i.e.* in the first month of the quarter, right after the release of the data for prices. The top-right panel graphs the nowcasts produced with "panel 12", the day of the release of Industrial Production and Capacity utilization data in the second month of each quarter. Since the information, in this case, approximately matches the one available to the SPF when the produce their forecasts, we include the latter in the graph. The bottom-left panel of figures (8) plots the nowcasts generated by the various models the day of the release of employment data in the third month of the quarter. Finally, the bottom-right panel reports the nowcasts produced at the end of the third month of each quarter, once the information on that quarter's Fed Funds rate becomes available.

Table (6) reports the MSFE for nowcasts and forecasts up to 4 quarters ahead of TFP growth. We compare the naive benchmark of a constant growth model, the quarterly DSGE model (Q), the monthly DSGE model (M) and the monthly DSGE model that also exploits the information contained in the panel (M+panel) with the forecasts produced by SPF. Hence, in order to match the information available to them at the time of the forecast, we generate the forecasts of tables (3)-(5) with "panel 12", *i.e.* corresponding to the release

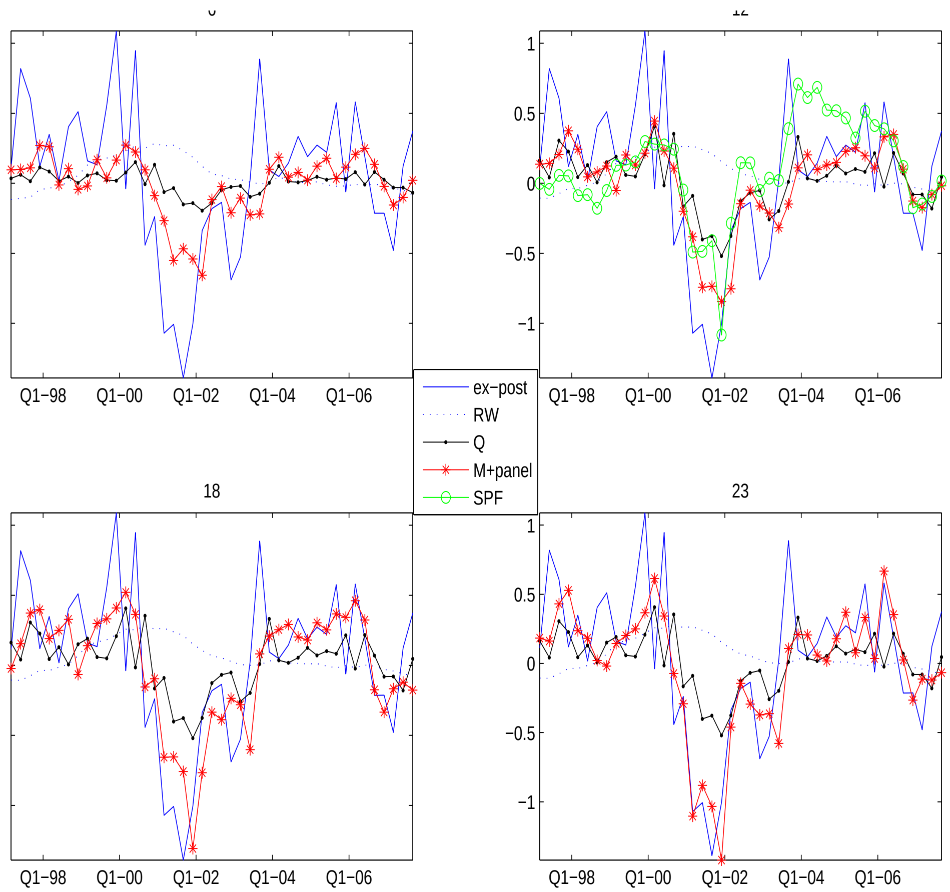

Figure 8: **realtime estimate of tfp growth**

Table 6: Mean square forecast errors of TFP growth forecasts with horizons 0 to 4 for the different model relative to the naive benchmark.

	NB	SPF	Q	M+panel
Q0	0.3437	0.1925	0.2128	0.1505
Q1	0.3639	0.302	0.2647	0.2425
Q2	0.3639	0.338	0.2839	0.2761
Q3	0.3686	0.343	0.2862	0.2846
Q4	0.3844	0.346	0.2951	0.2942

of Industrial Production and Capacity utilization data in the second month of each quarter.

The set of tables and graphs we present for TFP allow us to draw the following conclusions. First, the information extracted from the panel can only be exploited effectively when estimating current TFP growth. However, this information is very useful in the current quarter and allows to track very well the ex-post estimate of TFP, the better, the more advanced we are in the quarter. A part from the improvement in accuracy of the forecasts of the estimates, which was clear also from the previous subsection, the highlight of this exercise is that we are able to track - quite well and in real-time - an important unobservable quantity of the DSGE model. Hence, the model becomes a tool to interpret reality also within the quarter. In this sense, we have bridged conjunctural analysis with structural models.

5 Conclusions

This paper has proposed a formal method to link the real time flow of information within the quarter to quarterly structural DSGE models. Our procedure allows to obtain early estimates of key observable quantities considered in the model before they become available. It can also be used to obtain early estimates of unobserved key variables such as total factor productivity or the natural interest rate. We show how to define the monthly dynamics compatible with the model and how to expand its state space representation to incorporate information from monthly variables which are used in conjunctural analysis to derive early estimates. At the end of the quarter, when all quarterly variables included in the model are published, the quarterly DSGE estimates and the estimates obtained with the augmented monthly state space representation of the model are the same.

The empirical application is based on a prototypical three equations DSGE model à la Del Negro and Schorfheide (2004) which we augment with twenty three monthly variables. We show how the model-compatible estimates of GDP, inflation, the federal funds rate and total factor productivity evolve throughout the quarter and become more accurate as increasingly more information becomes available.

This method provides a useful framework to combine reduced form analysis for the short term and structural analysis for the medium and long term.

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6 Appendix A

We use a simple new-keynesian dynamic stochastic general equilibrium model, as the one used in Del Negro and Schorfheide (2004). The model consists of a representative household, a continuum of monopolistically competitive firms and a monetary policy authority that sets the nominal interest rate in response to deviations of inflation and output from their targets. The representative household derives disutility from hours worked and utility from consumption C relative to a habit stock and real money balances $\frac{M}{P}$. We assume that the habit stock is given by the level of technology A.[4] The representative household maximizes expected utility

$$E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{(C_s/A_s)^{1-\tau} - 1}{1-\tau} + \chi \log \frac{M_s}{P_s} - h_s \right) \right] \tag{10}$$

where β is the discount factor, τ the risk aversion parameter and χ is a scale factor. P is the economy-wide nominal price level that the household takes as given. The (gross) inflation rate is defined as $\pi_t = \frac{P_t}{P_{t-1}}$.

The household supplies perfectly elastic labor supply services to the firm period by period and receives in return real wage W. It also has access to a domestic capital market on which they can trade nominal government bonds B that pay gross interest rate R. Moreover, the household receives aggregate residual profits D and has to pay lump-sum taxes T. Hence, its budget constraint is:

$$C_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + \frac{T_t}{P_t} = W_t h_t + \frac{M_{t-1}}{P_t} + R_{t-1}\frac{B_{t-1}}{P_t} + D_t \tag{11}$$

The transversality condition on asset accumulation rules out Ponzi schemes.

On the production side, there is a continuum of monopolistically competitive firms, each facing a downward-sloping demand curve, derived in the usual way from Dixit-Stiglitz type of preferences, for its differentiated product

$$P_t(j) = \left(\frac{X_t(j)}{X_t} \right)^{-1/\nu} P_t, \tag{12}$$

where $P_t(j)$ is the profit-maximizing price that is consistent with production level $X_t(j)$, while P_t is the aggregate price level and X_t is aggregate demand (both beyond the control of the individual firm). The parameter ν is the elasticity of substitution between two differentiated goods. We assume that the firms face quadratic adjustment costs: that is, when a firm wants to change its price beyond the economy-wide inflation rate π^*, it incurs menu costs in terms of lost output:

$$AC_t(j) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi^* \right)^2 X_t(j). \tag{13}$$

The presence of these adjustment costs determines the presence of nominal rigidities, and the parameter $\phi \geqslant 0$ determines the degree of stickiness within the economy.

The production function is linear in labor, which is hired from the household:

$$X_t(j) = A_t h_t(j). \tag{14}$$

[4]This assumption ensures that the economy evolves along a balanced growth path.

Total factor productivity A_t follows a unit root process of the form:

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \hat{z}_t, \tag{15}$$

where

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}. \tag{16}$$

Hence, there will be a stochastic trend in the model. $\varepsilon_{z,t}$ can be broadly interpreted as a technology shocks that affects all firms in the same way.

The maximization problem faced the firm is the following:

$$\max E_t \left[\sum_{s=t}^{\infty} Q_s D_s(j) \right] \tag{17}$$

subject to (14) and (15), and where the j-th firm's profit $D_s(j)$ is

$$D_s(j) = \left(\frac{P_s(j)}{P_s} X_s(j) - W_s h_s(j) - \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi^* \right)^2 X_t(j) \right). \tag{18}$$

Q_s is the time-dependent discount factor that firms use to evaluate future profit streams. Although firms are heterogeneous ex-ante, we only consider the symmetric equilibrium in which all firms behave identically and can be aggregated into a single representative monopolistically competitive firm. Since the household is the recipient of the firms' residual payments, it directs firms to make decisions based on the household's intertemporal rate of substitution. Hence $Q_{t+1}/Q_t = \beta(C_t/C_{t+1})^{\tau}$.

The monetary policy authority follows an interest rate rule, such that it adjusts its instruments in response to deviations of inflation and output from their respective targets:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi^*} \right)^{\psi_1} \left(\frac{X_t}{X_t^*} \right)^{\psi_2} \right]^{1-\rho_R} e^{\varepsilon_{R,t}} \tag{19}$$

where R^* is the steady-state nominal interest rate and X_t^* is potential output, which we defined as $X_t^* = A_t$ after normalizing to one hours worked. The central bank supplies the money demanded by the households to support the desired nominal interest rate. The parameter $0 \leqslant \rho_R < 1$ governs the degree of interest rate smoothing, while $\varepsilon_{R,t}$ can be interpreted as an unanticipated deviation from the policy rule.

The government consumes a fraction ζ_t of each individual good and levies a lump-sum tax (or subsidy) T_t/P_t to finance any shortfall in government revenues (or to rebate any surplus), so its budget constraint is:

$$\zeta_t X_t + R_{t-1} \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = \frac{B_T}{P_t} + \frac{B_t}{P_t} + \frac{M_t}{P_t}. \tag{20}$$

The fiscal authority accomodates the monetary policy of the central bank and endogenously adjusts the primary surplus to changes in the government's outstanding liabilities. Finally, we define $g_t = 1/(1-\zeta_t)$ and assume that $\hat{g}_t = \ln(g_t/g^*)$ follows a stationary AR(1) process

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \tag{21}$$

where $\varepsilon_{g,t}$ can be broadly interpreted as a government spending shock.

To solve the model, we derive the optimality conditions from the maximization problem. Consumption, output, wages and the marginal utility of consumption are detrended by the total factor productivity A_t, in order to obtain a model that has a deterministic steady-state in terms of the detrended variables. The loglinearized system can be reduced to three equations in output inflation and the interest rate:

$$\begin{aligned}
\hat{y}_t &= E_t\hat{y}_{t+1} + \frac{1}{\tau}E_t\hat{\pi}_{t+1} - \frac{1}{\tau}\hat{r}_t + (1-\rho)\hat{g}_t + \frac{\rho_z}{\tau}\hat{z}_t \\
\hat{\pi}_t &= e^{\frac{\gamma - r^*/4}{100}}E_t\hat{\pi}_{t+1} + \kappa(\hat{y}_t - \hat{g}_t) \\
\hat{r}_t &= \psi_1(1-\rho_r)\hat{\pi}_t + \psi_2(1-\rho_r)\hat{y}_t + \rho_r\hat{r}_{t-1} + 0.25\varepsilon_{r,t}
\end{aligned}$$

The relation between logdeviations from steady state and observable output growth CPI inflation and the annual nominal interest rate is given by the following measurement equation.

$$\begin{aligned}
INFL_t &= \pi^* + 4\pi_t \\
RA_t &= \pi^* + r^* + r_t \\
\Delta \ln Y_t &= \ln\gamma + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t
\end{aligned}$$