# Forecasting the Frequency of Recessions*

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**Abstract**

The US economy has experienced only two recession in the last twenty four years. This follows a period of nearly 150 years when the US had experienced recessions on average every four to five years. Using Bayesian methods simple autoregressive models allowing for structural breaks are estimated. The implied frequency of future recessions from these models is considerably lower than past experience whereas the models match the historical frequency of recession.

Keywords: Recession, Business Cycle, Bayesian Methods.

# 1 Introduction

The US economy has experienced two recessions in the last twenty four years. Does this represent an incredibly lucky streak of good shocks or is there evidence that the economy has become more robust to the negative shocks that produce recessions? Further, if the economy is more robust to these

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negative shocks does this imply that the frequency of recessions in the future will be considerably lower? In the past, business cycle observers have become convinced that the US economy was more stable only for the economy to surprise them with either a mild recession (1969-1970) or a depression (1929 onwards).

The National Bureau of Economic Research has analyzed the US business cycle over a period of 152 years (1854 through 2005). In this time there were 32 recessions. This implies an average frequency of a recession every 4.75 years. In 1982 a similar calculation would have found the average frequency of recessions to be 4.3 years. Alternatively from 1946 to 1982 we had 7 recessions, a frequency of once every 4.5 years. Thus, using the whole historical record or records that end in 1982 it appears that we are short about 3 recessions in the last 24 years.

If we consider probability of a recession to be similar to the probability of observing heads while tossing a coin then we can quantify just how lucky recent history has been. In 24 tosses of a biased coin with probability of heads equal to 2/9, (i.e., a recession frequency of once every 4.5 years) such a run of good luck has probability $\binom{24}{2}\frac{2}{9}^2\left(\frac{7}{9}\right)^{22} = 0.054$. If we observe a recession in 2006 the probability of 3 recessions in 25 years is $\binom{25}{3}\left(\frac{2}{9}\right)^3\left(\frac{7}{9}\right)^{22} = 0.10$. Of course in 152 years of data one might expect such a run of good luck at least once. If we simulate many sequences of 152 coin tosses with probability of heads 2/9 then in approximately 32% of these sequences we have a 24 period consecutive stretch with 2 or fewer heads.

Thus, just taking the recent lack of recessions as the evidence it appears that a run of good luck is a reasonable explanation. On the other hand, we observe much more about the economy than just whether it is in recession or not. Recently, a number of papers have presented evidence using different methods that fluctuations in US output both in expansions and recessions are less volatile than they used to be (McConnell and Perez 2000, Kim and Nelson, 1999, Koop and Potter, 2000).

Following the lead of these papers one can estimate different statistical models for US output before and after the volatility break. These models can be used to simulate recession frequencies using a standard quantitative definition of a recession. If we assume that there was a break in the dynamic properties of US GDP in 1984 (as these papers have found) and one lag of growth is sufficient to capture dynamics then such methods imply that we should experience recessions only every **50 years** in the future. This number seems incredible, implying that the average person born today would only experience one recession in their lifetime.

Indeed if one conducts a similar exercise using data that ends in 1969 quarter 3, **assuming a break** in the dynamics of output growth in 1961, the implied frequency of recessions is every **70 years**. But we know that the US economy went into recession in 1969 quarter 4 and has experienced 6 recessions in the last 35 years. Thus, not only is the result for the US economy up to 1969 incredible, it is also wrong. Is there any reason to believe that the estimate produced for 2006 is any more reliable, when we do not have the luxury of checking our predictions against the next 35 years of data.

In order to answer this question I consider whether a more careful statistical analysis of the data up to 1969 would have found the unreliability in the estimated recession frequency. The estimate of 70 years is based on assuming a breakpoint occured in1961 and ignoring all uncertainty in parameter estimates. In practice one would needs to take into account uncertainty over the date and existence of a break and simultaneously the parameter uncertainty in the statistical model describing the dynamics after the break. I use ideas from the Bayesian literature that allow one to average over the properties of a particular model and across models to derive a forecast of the frequency of recession.

First, the estimate of low frequency of recession was based on point estimates of the parameters from a time series model. Given that there is some uncertainty about the true value of the parameters of such a model, then one can examine its effect on the frequency of recession. Taking into account the parameter uncertainty raises the frequency of recession dramatically to one every 20 years with a break in 1961. However, this is still far from an accurate prediction of the number of recessions that have occurred in the last 36 years.

Additional uncertainty surrounds the exact date when the break occurred. If we consider the possibility that the break could have occurred anywhere in the first half of the 1960s, then the frequency of recessions is estimated to be higher in years other than 1961 with the highest of once every 10 years if the break was in 1964.

But this could be something of a false increase in the frequency of recession if there is more evidence that the break in the dynamics of GDP occurred in 1961. We can deal with this issue by weighting the various estimates of recession frequency by the respective probability of a break at this date under the maintained assumption that a break did occur sometime in the sample.

Conducting this exercise reduces the estimated recession frequency to every 15 years.

This leaves the crucial question, just how much evidence is there in favor of a break in dynamics of US output in the first half of the 1960s using data through 1969 quarter 3? The answer in terms of probability is just over 1/10. If we take the 15 year recession frequency estimate and weight it by this probability and combine it with a frequency of recession estimate assuming no break of 3.5 years, this gives a frequency of recession of around 5 years - almost identical to the historical record.

If we conduct a similar analysis for the current estimate of recession frequency, the introduction of parameter uncertainty and uncertainty about the date of the break point combined together increase the estimated frequency of future recessions to every 20 years but there is much more evidence in favor of a structural break than there was in the 1960s. Indeed to raise the estimated frequency of recessions to one every 4/5 years would require an *a priori* belief in stability of almost 3 million to 1. This seems to be a very dogmatic viewpoint. Thus, unlike previous experiences it might actually be true that the traditional defined recession is on its deathbed.

The rest of the paper gives more details about how the calculations were carried out. Section 2 describes the construction of recession frequencies from an known time series model. Section 3 gives a description of the Bayesian techniques used to find various probabilities and recession frequencies. Section 4 gives a more detailed listing of the results and examines sensitivity to recession definitions, data and priors. Section 5 concludes with a discussion of the meaning of the business cycle.

# 2 Recession Frequencies from Time Series Models

This section describes some simple methods for deriving recession frequencies from time series models. I start by drawing some distinctions between the frequency of recession and the duration of expansions. I then turn to providing a quantitative definition of a recession and a combined measure of recession frequency and expansion duration. The section concludes by describing how to calculate the measure using simulation techniques for a know time series model.

## 2.1 Differences Between Recession Frequency and Expansion Duration

In the introduction some simple calculations of recession frequencies were made based on the number of observed recessions divided by the number of years observing the economy. It is natural to invert such statements to calculate how long expansions last. Or given a statement about the length of time to the next recession invert that to form a recession frequency. Consider the statement: "recessions occur once every 4 years on average". One interpretation of this statement is that the average duration of an expansion is 3 years. This interpretation would be exact if the average duration of a recession was 1 year. However, if recessions lasted less than one year on average then the duration of expansions would be longer than 3 years.

Alternatively consider the statement that the average duration of an expansion is 10 years. Does this imply that recessions occur every 10th year? The answer is no not necessarily. Suppose that recessions, if they occur, last for 10 years also on average. And further, that unconditionally recessions are as likely as expansions. Then the **frequency of years in which the economy is in recession** is about one out of two years. However, if we had just reached the trough of the business cycle it is true that we would expect the next recession to happen in 10 years. These ambiguities suggest using a combination of these two measures, which I introduce in the next subsection.

## 2.2 Calculating Time to Next Recession

Throughout I focus on probabilities generated by assuming the initial conditions of the time series are drawn from its stationary distribution. In order to start we need to convert the subjective criteria of the NBER into a quantitative statement about output growth. I follow a long literature and assume that a recession is equivalent to two **consecutive** quarters of negative output growth.[1] Below in the empirical work I will assess the sensitivity of the

[1] The NBER does not define a recession in terms of two consecutive quarters of decline in real GDP. Rather, a recession is a recurring period of decline in total output, income, employment,and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy. Further, if one used the two consecutive criterion to match the historical record, one would fail. However, the objective here is to examine the future likelihood of recession like events. And as we shall see, the likelihood of recession events does match well to the historical record.

results to a slightly weaker definition of a recession. Based on this criteria we need to distinguish between forecasting a recession at a certain date in the future and expected time to the next recession. The former is captured by statements such as the probability of being in a recession at the end of the year 2007 is 10%:

$$P[\text{Recession in 2007Q3/4}] = P[Y_{2007Q3} < 0, Y_{2007Q4} < 0] = 0.10$$

Notice that this leaves unanswered the question of whether there will be a recession in the year 2001. We are interested in the different exercise of finding the probability of the next recession. It is useful to define a sequence of binary random variable, $\text{rec}_t$ with the property that:

$$\text{rec}_t = \begin{cases} 1 & \text{if a recession occurs at time } t \text{ or earlier} \\ 0 & \text{if no recession has occurred by time } t \end{cases} .$$

Then the conditional probability of a recession at time $t$ given no previous recessions is given by:

$$\begin{aligned} & P[\text{Recession at time } t|\text{no recession before } t] \\ = & P[Y_{t-1} < 0, Y_t < 0|\text{rec}_{t-1} = 0]. \end{aligned}$$

Assuming that $t = 2, \ldots$ then the probability of no recession before a certain date is given by:

$$\begin{aligned} & P[\text{No Recession thru periods } 2, 3, \ldots, t] = P[\text{rec}_t = 0] \\ = & 1 - \sum_{t=2}^{s} P[Y_{t-1} < 0, Y_t < 0|\text{rec}_{t-1} = 0] P[\text{rec}_{t-1} = 0], \end{aligned}$$

and $rec_1 = 0$, with initial conditions for $Y_0, Y_1$ from the stationary distribution.

In particular, one might be interested in the value of $t$ for which $P[\text{rec}_t = 0] \leq 0.5$ and $P[\text{rec}_{t-1} = 0] > 0.5$, i.e., the median of the distribution of times to the next recession. Further, the expected waiting time to the next recession starting from the stationary distribution of the time series is then given by

$$\sum_{s=1}^{\infty} P[\text{rec}_s = 0].$$

note that the first term in the sum is 1 since by assumption the earliest a recession can happen is after two periods.

These probabilities and waiting times depend on the initial state. Again in a pure forecasting exercise the initial state would be determined by current conditions. In contrast, the initial state here is taken to be a draw from the stationary distribution of output growth rates. This assumption defines a unique sequence of the probabilities indexed by the forecast horizon $t$. One issue is that the stationary distribution will place some weight on starting from a recession position. If this probability was large then the calculations below would underestimate the length of expansions as described above. The alternative would be to consider the initial condition to be the turning point from a recession to an expansion: $Y_1 > 0$ and $Y_0 < 0$ and $Y_{-1} < 0$. This would provide a measure of the average length of an expansion. However, as discussed above this measure would tend to underestimate the frequency of recessions if the duration of recessions was similar to that of expansions. As recessions become less likely there will be little disagreement between the measures. Computationally in the case where recessions are infrequent starting from the stationary distribution is much easier.

## 2.3 Simulation Techniques

In order to calculate these quantities from a known time series model, for example a Gaussian autoregression of the form:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \sigma V_t,$$

simulation methods can be used. One starts by constructing initial conditions from stationary distribution of the time series. In the case of a Gaussian model this requires draws from a multivariate normal distribution, in more complicated time series a burn-in phase of realizations could be used.

Next, draws of the random shock, $V_t$ are propagated by the time series model for $N$ periods. Suppose that $K$ collections of length $N$ are simulated. One forms the time series:

$$R_n^k = 1[Y_{n-1}^k < 0, Y_n^k < 0 \text{ and } R_{n-1}^k = 0] + R_{n-1}^k,$$

with $R_1^k = 0$ and then the sequence of averages:

$$\overline{R}_n = \frac{1}{K} \sum_{k=1}^{K} R_n^k$$

which converge to $P[Y_{n-1} < 0, Y_{n-1} < 0|\text{rec}_{n-1} = 0]P[\text{rec}_{n-1} = 0]$ as $K \to \infty$.

# 3 Bayesian Methods

The estimated recession probability sequence is only as accurate as the underlying time series model is at describing the random fluctuations in output. Unfortunately, we do not know *ex ante* any of the parameters of the time series model nor do we know the appropriate lag length or even whether a constant parameter Gaussian linear time series is appropriate. As we shall see allowing for all these sources of uncertainty can have striking impacts on the results. This section describes Bayesian methods to deal with these issues and contrasts these methods with recently developed classical approaches (see Hansen 2001 for a review and references to the classical literature)

Suppose a range of univariate time series models are available to describe the fluctuations in GDP: constant parameter Gaussian autoregressions, Gaussian autoregressions where all the parameters change once, threshold autoregression models etc. In addition each class of models contains various sub-models based on the choice of lag length and in the case of structural break models the date of the break. One approach would be to find the best model according to some criterion and then evaluate its properties with respect to the frequency of recessions. Such an approach is common across economics but is ill-suited to the task at hand. First, we cannot expect that any of the models exactly captures the truth, they are all approximations. Secondly, the frequency of recessions is a complicated nonlinear function of the underlying parameters that can be very sensitive to small changes in parameters.

The Bayesian approach to these problems is to average across both parameters and models. In order for the averaging to make sense jointly across models and parameters we require proper prior distributions. Further, in order to reduce the computational burden I use priors from a particular parametric family. This of course leads to the concern the results are dependent on the prior rather than the observed data. Below we will use relatively uninformative priors that should be dominated by the data assuming relatively large sample sizes. Further, we have a consistency check from the observed frequency of recessions in the sample before the estimated breakpoint. If our estimate differs greatly from this historical evidence then it suggests that the prior is overly influencing the results.

On the other hand, the use of prior information is less of a concern if the sample size is small and the prior reproduces information from a larger sample available to the investigator. Further, in a small sample the the prior can be constructed to ensure that the estimated model is stationary.

I start by describing the structural break model considered and the prior distributions used. Next I discuss how to generate draws from the posterior distributions for the models for a fixed breakpoint. I then turn to constructing a posterior distribution across breakpoints and the an estimate of the expected recession frequency that averages across the breakpoints using the posterior distribution. Finally, I describe how one averages of different lag lengths and different models to produce an overall estimate of the the frequency of recession.

## 3.1 Alternative Models

In the empirical section we focus on two types of models: a constant parameter Gaussian autoregression and a Gaussian autoregression with a single structural break. Since the latter model nests the former we develop its estimation.

$$Y_t = \begin{cases} \phi_{10} + \phi_{1p}(L)Y_{t-1} + \sigma_1 V_t & \text{if } I_t = 1 \\ \\ \phi_{20} + \phi_{2p}(L)Y_{t-1} + \sigma_2 V_t & \text{if } I_t = 2, \end{cases}$$

where $V_t$ is an independent sequence of standard normals.

We define $I_t$ in two ways:

1. The linear Gaussian AR model is obtained if we set $I_t = 1$ for all $t$.

2. The structural break model is obtained if we set $I_t = 1$ if $t < \tau_1$ and $I_t = 2$ if $t \geq \tau_1$.

## 3.2 Natural Conjugate Priors and Posteriors

The major advantage of the simple classes of models considered is that analytical expressions for the posterior distribution are available if we use independent (across structural breaks) natural conjugate priors for $(\phi_{i1}, \phi_{i1}, \cdots, \phi i_p), \sigma_i$. The use of natural conjugate priors in time series models is controversial (see the 1991 special issue of the Journal of Applied Econometrics and Koop and Potter (2006)). Here their computational convenience makes up for some of their shortcomings and they have the advantage that they produce point estimates and measures of individual parameter uncertainty conditional on a breakpoint very similar to the classical approach.

If we treat initial conditions as fixed, it is well known that the AR model can be analyzed in a similar manner to the standard linear regression model and the natural conjugate prior is in the normal inverted gamma family. Thus, we assume an informative prior of the form

$$p(\boldsymbol{\phi}_0, \boldsymbol{\phi}_1, \sigma_0, \sigma_1|\text{breakpoint}) = \prod_{i=1}^{2} p(\boldsymbol{\phi}_i|\sigma_i)p(\sigma_i),$$

with a normal-inverted gamma form for $p(\boldsymbol{\phi}_i|\sigma_i)p(\sigma_i)$ (see, e.g., Judge, Griffiths, Hill, Lutkepohl and Lee, 1985, pp. 106-107 for further details about the normal-inverted gamma prior).

Conditional on $\sigma_i^2$ it is assumed that $\boldsymbol{\phi}_i$ is multivariate normal with mean vector $\mathbf{0}$ and diagonal variance covariance matrix $\sigma_i^2\underline{\mathbf{D}}$. That is, the prior is centered around a random walk with no drift for the growth rates. The weight that the prior view has on the posterior distribution will depend on the precision ($\underline{\mathbf{D}}^{-1}$). The degrees of freedom of the inverted gamma distribution is $\underline{\nu}$ and the mean is $\underline{s}^{-2}$.

It is assumed that the hyperparameters of the prior do not depend on breakpoint. This is a restriction that could be relaxed but it would it make more difficult to generate measures of model uncertainty. The priors for the breakpoint are discussed below. Let us begin by considering the linear AR model.

1. The sample size $N$

2. The ordinary least squares estimates of the parameter vector $\boldsymbol{\phi}$

$$\hat{\phi} = [\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{Y},$$

   where $\mathbf{Y} = [Y_{p+1}, \cdots, Y_{N+p}]'$ and

$$\mathbf{X} = \begin{bmatrix} 1 & Y_p & \cdots & Y_1 \\ 1 & Y_{p+1} & \cdots & Y_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & Y_{N+p-1} & \cdots & Y_{N-p} \end{bmatrix}.$$

3. The moment matrix $\mathbf{X}'\mathbf{X}$.

4. The sum of squared errors:

$$\nu s^2 = (\mathbf{Y} - \mathbf{X}\hat{\phi})'(\mathbf{Y} - \mathbf{X}\hat{\phi}).$$

Then draws from the posterior distribution are generated by:

1. Draw $\sigma^{-2}$ from the Gamma distribution with degrees of freedom:

$$\overline{\nu} = \underline{\nu} + N,$$

and mean $\overline{s}^{-2}$ obtained from

$$\overline{\nu s}^2 = \underline{\upsilon s}^2 + \nu s^2.$$

2. Draw $\boldsymbol{\phi}$ from the multivariate normal distribution with variance matrix:

$$\sigma^2 \overline{\mathbf{D}} = \sigma^2 \left[\underline{\mathbf{D}}^{-1} + \mathbf{X}'\mathbf{X}\right]^{-1},$$

and mean vector

$$\overline{\boldsymbol{\phi}} = \overline{\mathbf{D}}\mathbf{X}'\mathbf{X}\widehat{\boldsymbol{\phi}}.$$

Note that draws from this distribution do not have to satisfy the conditions for stationarity. To impose these conditions one must reject draws of $\boldsymbol{\phi}$ that imply explosive or unit root behavior. Again this is trivial to do if the focus is on a particular model but introduces complications in the case where multiple models are compared.

For each draw from the posterior distribution one can calculate the expected time to the next recession using the simulation methods described above. Let

$$\overline{R}_n(\tau, p) = \frac{1}{J}\sum_{j=1}^{J} \frac{1}{K}\sum_{k=1}^{K} R_n^k(\psi_j(\tau, p)),$$

be the estimate conditional on the lag length and breakpoint. The $J$ values of

$$\frac{1}{K}\sum_{k=1}^{K} R_n^k(\psi_j(\tau, p))$$

could be used to give a measure of uncertainty around this expected value.

In comparison classical methods would focus on the least squares estimators $\widehat{\boldsymbol{\phi}}$ and $\upsilon s^2/N$ to produce the estimated recession frequency. Measures of uncertainty around this point estimate could be produced in a number of ways. One common approach would be to use large sample methods to approximate the sampling distribution. In this case such methods are not directly useful since the frequency of recession is a complicated nonlinear function of the parameters and the size of the sample will vary depending on the location of the break. A simpler computationally intensive method would be to perform a bootstrap where the least squares estimates were used to generate artificial time series of the same length as the original sample. These artificial time series would be used to re-estimate the model and the bootstrap parameter values could then be used to calculate the recession frequencies. For my purposes I assume that the draws from the posterior distribution for a fixed breakpoint are a good representation of uncertainty in the classical estimates.

## 3.3 Marginal Likelihood and Posterior of Breakpoint

If we ignore the stationarity condition then we can integrate the likelihood with respect to the normal inverted gamma prior distribution to obtain what is called the marginal likelihood, i.e, the average height of the likelihood for the observed data.

Combining the sample information with the prior information, the average/marginal likelihood is:

$$\ell(\mathbf{Y}) = \frac{\Gamma(\bar{\nu}/2)(\underline{\nu s}^2)^{\underline{\nu}/2}}{\Gamma(\underline{\nu}/2)\pi^{N/2}} \frac{|\overline{\mathbf{D}}|^{1/2}}{|\underline{\mathbf{D}}|^{1/2}} (\bar{\nu}\bar{s}^2)^{-\bar{\nu}/2},$$

where

$$\bar{\nu}\bar{s}^2 = \underline{\nu s}^2 + \upsilon s^2 + (\overline{\boldsymbol{\phi}} - \hat{\phi})'\mathbf{X}'\mathbf{X}(\overline{\boldsymbol{\phi}} - \hat{\phi}) + \overline{\boldsymbol{\phi}}'\underline{\mathbf{D}}^{-1}\overline{\boldsymbol{\phi}}$$

and $\Gamma$ is the gamma function. The marginal likelihood is related to the Bayesian information criterion which is produced by considering the behavior of marginal likelihood as the sample becomes large under an improper prior.

Conditional on a known break point, the structural break model can be divided into two simple linear regression models and we can calculate the marginal likelihood across each sample of data. Hence, conditional on the break the marginal likelihood for structural break model is the product of the marginal likelihoods across the different periods, if the priors before and after the break are independent. The following sample information is required to calculate the marginal likelihood for the two regime model:

1. The sample size $N_i$ for each of the regimes.

2. The ordinary least squares estimates of the regime parameter vector $\boldsymbol{\phi}_i$:
$$\hat{\phi}_i = [\mathbf{X}_i'\mathbf{X}_i]^{-1}\mathbf{X}_i'\mathbf{Y}_i,$$
where $Y_{it} = 1(I_t = i)Y_t$ and $X_{it} = 1(I_t = i)X_t$.

3. The moment matrices $\mathbf{X}_i'\mathbf{X}_i$.

4. The sum of squared errors within each regime:
$$\upsilon s_i^2 = (\mathbf{Y}_i - \mathbf{X}_i\hat{\phi})'(\mathbf{Y}_i - \mathbf{X}_i\hat{\phi}).$$

In the case of the structural break model the marginal likelihood conditional on the break point is:

$$\ell_{\text{Break}}(\mathbf{Y}|\tau_i) = \prod_{i=0}^{1} \frac{\Gamma(\bar{\nu}_i/2)(\underline{\nu}\underline{s}^2)^{\underline{\nu}/2}}{\Gamma(\underline{\nu}/2)\pi^{N_i/2}} \frac{|\overline{\mathbf{D}}_i|^{1/2}}{|\underline{\mathbf{D}}|^{1/2}} (\bar{\nu}_i\bar{s}_i^2)^{-\bar{\nu}_i/2}.$$

If this was the only breakpoint of interest one could immediately assess the evidence in favor of a break by examining the ratio:

$$\frac{\ell(\mathbf{Y})}{\ell_{\text{Break}}(\mathbf{Y}|\tau_i)}.$$

If all parameters were known this would be equivalent to the likelihood ratio, however, in the more realistic case where parameters are unknown it differs from the likelihood ratio by integrating out unknown parameters over the observed sample using the prior rather than maximizing than out using the observed sample.

In the case of structural breaks perhaps the most important unknown parameter is the breakdate itself. In order to estimate the location of the breakpoint, the Bayesian approach is to calculate the posterior distribution over the possible breakpoints. First consider the case where there are two candidate breakpoints which are ex ante equally likely. The posterior probabilitity of breakpoint 1 would be given simply by

$$\frac{\ell_{\text{Break}}(\mathbf{Y}|\tau_1)}{\ell_{\text{Break}}(\mathbf{Y}|\tau_1) + \ell_{\text{Break}}(\mathbf{Y}|\tau_2)}$$

Thus, calculating marginal likelihoods at each possible breakpoint with each breakpoint equally likely a priori we obtain the posterior probability distribution for each possible breakpoint. An overall marginal likelihood can be obtained by using a discrete uniform prior over all possible sample breaks. Thus, no *a priori* information needs to be used about the location of the break point.

Thus, the marginal posterior over breakpoints $p(\tau|Y)$ is given by:

$$\frac{\ell_{\text{Break}}(\mathbf{Y}|\tau_i)}{\sum_{\tau\in\mathcal{T}}\ell_{\text{Break}}(\mathbf{Y}|\tau)}.$$

This posterior distribution provides information on the most likely locations of the breakpoints. From the classical perspective it is possible to provide confidence intervals over breakpoints (see Hansen 2001). This approach uses the shape of the likelihood in a similar way to the Bayesian approach.

The posterior also provides a method for calculating the expected time to the next recession that averages over possible breakpoints as well as over the model parameters at a particular breakpoint. First, one uses standard methods to draw a breakpoint from $f(\tau|Y)$.[2] For each draw one calculates $\overline{R}_n(\tau,p)$ as above. By generating a large number of draws from the posterior over the breakpoints one is able to to approximate the weighted sum:

$$\overline{R}_n(\mathcal{T},p)=\sum_{\tau\in\mathcal{T}}\overline{R}_n(\tau,p)f(\tau|Y),$$

which gives the recession hitting probabilities for a model with lag length $p$ and possible breakpoints $\mathcal{T}$.

## 3.4 Model Averaging

The final step is to weight the implied expected time to the next recession from of different types of models, in our case the constant parameter Gaussian autoregressions of various lags and the single structural break models of various lags. The weights are given by the relative marginal likelihoods scaled by a priori information on the types of models. In the case of the models without breaks this is simply the marginal likelihood from above.

[2] One could also just evaluate the expected recession time for each possible breakpoint and then weight the results together using the posterior distribution. In practice this is inefficient since most of the posterior probability is concentrated on a few dates.

For the structural break models the marginal likelihoods are given by the simple average of the marginal likelihoods for each break point:

$$\ell(\mathbf{Y}|p, \mathcal{T}) = \frac{1}{\#\mathcal{T}.} \sum_{\tau \in \mathcal{T}} \ell_{\text{Break}}(\mathbf{Y}|\tau)$$

In our case we limit the analysis to models of lag length 1 or 2 and assume each are equally likely ex-ante. Thus, the probability of lag 1 is given by

$$\pi(1, \mathcal{T}) = \frac{\ell(\mathbf{Y}|1, \mathcal{T})}{\ell(\mathbf{Y}|1, \mathcal{T}) + \ell(\mathbf{Y}|2, \mathcal{T})}$$

the conditional marginal likelihoods across all the values of $\tau$.

$$\overline{R}_n(\mathcal{T}) = \sum_{p=1}^{2} \overline{R}_n(\mathcal{T}, p)\pi(p, \mathcal{T}),$$

which gives the recession hitting probability for the class of structural break models considered. Alternatively, one could mix the posterior distributions of $\overline{ER}_n(\mathcal{T}, p)$ with weights $\pi(\mathcal{T}, p)$ to obtain a posterior distribution over the expected recession hitting time.

Finally we calculate the odds in favor of a structural break from

$$\frac{\pi_{\text{break}}}{\pi_{\text{no break}}} \frac{\ell(\mathbf{Y}|1, \mathcal{T}) + \ell(\mathbf{Y}|2, \mathcal{T})}{\ell(\mathbf{Y}|1, \varnothing) + \ell(\mathbf{Y}|2, \varnothing)}.$$

Using these probabilities we have the predicted recession probabilities:

$$\pi_{\text{break}} \overline{R}_n(\mathcal{T}) + \pi_{\text{no break}} \overline{R}_n(\varnothing).$$

## 3.5 Properties of the Prior Distribution

It remains to discuss the choice of the hyperparameters for the Normal-inverted gamma priors. These priors are important because in the finite samples available they can influence the results. One approach to the problem of prior sensitivity is to use ‘flat’ priors over the parameters of interest. In many cases this can be reasonable but here it automatically ensures that the more parsimonious constant parameter linear models receive 100% of the posterior probability.

I begin by eliciting a prior which is relatively noninformative in terms of the object of interest expected time to the next recession. To simplify matters, it is assumed the prior means and covariances are zero for all the regression parameters in all models (i.e. the $\phi_i$'s are centered over zero). The prior variance for the intercept is taken to be 4. We use a shrinkage prior on the autogressive coefficients with the first autogressive lag having a variance of 1 and the second lag when used 0.5. In particular, we assume the marginal prior variance for $\phi_i$ is $E(\sigma_i^2)cA_{p+1}$ where $A_{p+1}$is a $(p+1)\times(p+1)$ diagonal matrix with $(1,1)'th$ element 4 and all other diagonal elements given by the shrinkage prior. Hence in the notation above $\underline{\mathbf{D}} = cA_{p+1}$. Degrees of freedom ($\underline{\nu}$) for the inverted gamma priors are 3 for all models, which is very noninformative, but which allows for the first two marginal prior moments to exist for all parameters. The other hyperparameter of the inverted gamma prior is $\underline{s}$. This hyperparameter is defined so that $E(\sigma_i^2) = \frac{\underline{\nu}}{\underline{\nu}-2}\underline{s}^2$.

We set $\underline{s}^2 = 20/3$, $c = 1/20$. This implies a very flat prior for $\sigma_i^2$, but one that has mean 20. Since the data are measured as percentage changes (e.g. 1.0 implies a 1.0 percent change in GDP) and annualized, this choice of $\underline{s}^2$ is sensible in light of the typical fluctuation in quarterly GDP since the Second World War. The prior centers the AR coefficients over zero for all series, but allows for great prior uncertainty.

In order to understand the implications of the prior for the expected time to the next recession we simulate from it imposing the stationarity condition. Figure 1a and 1b contains the implied prior distributions over the probability of recessions at various horizons and the expected time to the next recession for lag lengths 1 and 2. Both priors concentrate most of their mass on the expected time being less than 5 years but they also give some weight to recessions lasting considerably longer.

# 4 Results

Two samples of data are used in the analysis. The first sample uses GDP growth rates from 1947Q2 through 1969Q3. The second extends this sample to 2006Q1. I use the most recent vintage of of GDP data. I also consider the sensitivity of the estimates to some earlier vintages of GDP data. Not only are these series shorter than the one currently available but even across common samples revisions can have some effect on the results. In particular, the 1999 benchmark revision to GDP that treated software as a final investment good tended to raise the growth rate of GDP towards the end of the sample with obvious consequences for for the measures calculated in this paper.

I present results for autoregressions of lag order 1 and 2. Longer autoregressions were not as strongly supported by the data. For the shorter sample some comparisons with nonlinear threshold models were included. There was some evidence that a nonlinear model should receive some weight but given the complexity of constructing the expected time to recession measure, their properties were not pursued. Most of the discussion will focus on the results for the structural break models.For both samples the restriction was imposed that there were at least 15 observations in each regime.

## 4.1 Results with Standard Recession Definition and Reference Prior

The initial analysis starts with the implications of least squares point estimates for the frequency of recession in both samples (see Figures 2a to d) using the structural break model. It is assumed that a break occurred in the first half of the 1960s for the shorter sample and in the 1980s for the longer sample. The results indicate that the expected time to the next recession is very sensitive to the particular choice of breakpoint and to some extent on the choice of lag length for the period after the break but much less so before the break. The highest estimates occur in 1961 and the lowest at the start and end of the sample. For the sample thru 2006Q1 once again the pre-break estimates are not sensitive to the assumed breakpoint but the post break ones are. The insensitivity of the pre-break estimates continued throughout the analysis.

The least squares estimates do not allow for uncertainty about the parameters. The exercise of calculating the expected time to the next recession was repeated using a 4000 draws from the posterior of the pre- and post-break models for each possible breakpoint considered in the previous analysis (see Figures 2a to d). This exercise would be very similar to a classical finite sample analysis since the prior distributions assumed are relatively uninformative. However, it would not necessarily be similar to a classical asymptotic analysis since one would make the large sample assumption that the estimate of the variance was normally distributed. Taking into account parameter uncertainty has a larger effect on the values of the expected time to a recession for the sample thru 1969. The effect is less dramatic for the longer sample since the post-break sample period is considerably longer but still produces a big reduction for lag length 1.

Next, the likelihood of breakpoints at particular time periods was used (see Figures 3a to d). For completeness breaks were allowed not only in the first half of the 1960s and the decade of the 1980s but at any date consistent with at least 15% of the sample being in both regimes. Thus, in the longer sample it was possible to find a break in the 1960s rather than in the 1980s as has been the assumption so far. Both samples show the strongest evidence for breaks around dates when the expected recession time was estimated to be high, 1961 and 1984 respectively.

Using the classical approach of Hansen (2001) very similar estimated breakdates were found. However, the confidence intervals on the location of the breakdates were wider than those in the posterior distributions. In the case of the 1960s break the confidence interval effectively covers the whole sample and in the whole sample the confidence interval covered the period from 1972 to 1995.

Thus, the average across breakpoints was more influenced by the larger values from the previous Bayesian analysis. In the sample ending in the 1960s conditional on a lag length of 1 the expected time to a recession after the break was around 15 years. Conditional on a lag length of 2 the expected time to recession was just under 20 years. However, the shorter lag length was almost 15 times more likely than the longer one, thus their average is about 15.5 years.

For the longer sample period the shorter lag length was considerably more likely than in the shorter sample, with probability 0.895 versus 0.105 for the longer lag length. The expected time to the next recession was 20 years with the shorter lag length and 40 years with the longer lag length. This averages out to 22 years. Thus, conditional on the existence of a break the sample up to the present suggest a considerable increase in the duration of expansions. Figures 4a to 4d show

There remains the major question: How likely is that there was really was a break? In the 1960s using the sample up to 1969 the evidence is weak: the probability attached to a break is just under 12 %. Further, if the sample is extended the evidence of a break in the 1960s disappears completely.

The evidence in favor of a break in the 1980s is much stronger. With the posterior probability of a break effectively 1. The results from the classical tests also agree with this assessment with the p values associated with the test statistics effectively 0.

Thus, at the end of 1969 if a similar statistical analysis had been carried out, the expected time to a recession could have been estimated as a weighted average of expected time assuming no break (4) and the expected time assuming a break of 15.5 years: $4 \times 0.88 + 16 \times 0.12 = 5.5$ years, very close to the observed outcome of 6 recessions over the next 36 years. A similar procedure applied to the data through the end of 2006Q1 implies that the expected recession time has increased considerably since the post break estimate receives a weight of 1.

It is interesting to examine this current prediction in more detail. First, because of positive skewness the expected time to a recession, is not the same as the horizon in which the recession probability goes above 50%. This is approximately $12 - 13$ years averaging over parameter uncertainty and both lag lengths. Further, we can consider the median of the distribution of expected weighting times from the simulated posterior draws across over the two lag lengths, which is around 15 years. Figure 4 contains the posterior distribution over the expected time to the next recession for the two different lags lengths. From this one can observe that the probability that the expected recession time is greater than 5 years is around 95%. Further the probability that the expected time to the next recession is more than 10 years is 70% for lag length 1and 75% for lag length 2.

## 4.2 Robustness of Results

### 4.2.1 Weaker Recession Definition

One possible explanation of the results is that the recession definition is too strong. A weaker recession definition that captures the most recent recession in the United States is (needs to be updated for longer sample):

1. $Y_{t-1} < 0, Y_t < 0$ or
2. $Y_{t-1} < 0, Y_t > 0, Y_{t+1} < 0.$[3]

The analysis was repeated for this recession definition.

Expected Recession Time Alternative Definition

[3] The Harding-Pagan definition $Y_{t-1} + Y_t < 0$ gives similar results and also (just) classifies 2001 as a recession in the United States.

| Lag Length | Before Break | After Break |
|---|---|---|
| $p = 1$ | 3.2 | 13.4 |
| $p = 2$ | 3.3 | 14.5 |

Although the numbers are lower by construction the average across lag lengths for the whole sample after the break is still much higher than historical experience Further, the expected recession time before the break is lower than the observed average of 4 years suggesting that the more traditional two consecutive quarter decline in GDP is more appropriate.

#### 4.2.2 Prior Sensitivity

The major problem with the Bayesian approach followed is that the results are sensitive to the prior assumed. Here the prior sensitivity could manifest itself in a number of ways: it might change the inference about the existence of a break; it might affect the most likely position of the break; or it might affect the expected time to the next recession given the location of a break. I note first that he results are very insensitive to the truncation used in the prior on the breakpoints as can seen from the plot of the posterior distribution in Figures $3a$ to $3d$.

The first two possibilities are relatively simple to assess computationally. As we vary the prior variance on the autoregressive coefficients there is little change in the high probability attached to the existence of a structural break but there are changes in the relative probabilities attached to the choice of one or two lags. Similar results occur as we change the prior variance on the intercept.

The probability of a break is more sensitive to the parameters of the inverted gamma distribution for the error variance. As the degrees of freedom are increased for a fixed mean of 20 the evidence for a break starts to disappear. With $\underline{\upsilon} = 50$ the probability of a break in the 1980s drops to about 3/4, at $\underline{\upsilon} = 66$ the probability of a break is only about one half. With the prior degrees of freedom set at 66 there is also some evidence of a break in the early 1960s in addition to the break in the mid-1980s. Note that with prior degrees of freedom at 66 the sample evidence from a break in the mid-1980s is given equal weight with the prior information in estimating the variance and in evaluating the probability of a break in. If we change the prior mean of the variance to 14 (the least squares estimate over the whole sample) and keep $\underline{\upsilon} = 66$ the probability of the break increases back over 98%, with the break located in the early 1980s but less precisely estimated than with $\underline{\upsilon} = 3$.

Most of the probability is given to the model with a single lag. Thus we focus on this lag length in evaluating the expected time to the next recession. With the prior degrees of freedom fixed at 66, a prior mean of 20 produces an expected recession time of 5 years after the break, with the prior mean at 14 it produces an expected recession time of 7 years after the break. Once again the pre-break expected recession times were insensitive to the changes in the prior. Examining the posterior mean of the variance we find that with $\underline{\upsilon} = 66, \underline{s}^{-2} = 14$ after the break it averages to 9 compared to a least squares estimate of 4 with a break in 1984.[4]

In contrast if we scale up the matrix $\underline{\mathbf{D}}$ by a factor of 100 (but return the prior hyperparameters of the inverted gamma to their original values) which virtually removes any influence of the prior on the draws of $\phi$ the expected recession time increases to over 30 years for lag length 1. Once again if we make the prior very informative by dividing $\underline{\mathbf{D}}$ by 100 then we get a drastic drop in the expected recession time to 1.4 years both before and after the break. Thus, it is clear that this result is produced by the prior imposing that GDP growth is approximately a driftless random walk and downweighting heavily the sample information.

The conclusion appears to be that, unless we have considerable prior information on the parameters of the time series model, the expected recession time has increased substantially since the 1980s.

#### 4.2.3 Earlier Vintages and Truncated Samples

First the effect on the probability and location of the break was examined. Two different sets of data were used. First we truncated the sample at 1998Q4. The results were even stronger in favor of a break and its the posterior distribution on the breakpoint did not change. The only minor change was some increase in relative support for the model with lag length 2. Next we used GDP data available in the first quarter of 1999 for the period 1947Q1 to 1998Q4. This data in addition to the more usual revisions differs from the current estimates of GDP by the treatment of software. In the summer of 1999 the national income accounts were revised to include software as a capital good. With this earlier vintage of data the probability of a break remains high.

[4] The implied prior for the expected recession time was calculated with prior degrees of freedom equal to 66. It was bimodal with one mode at 5 years or less and the other mode at 100 years or more.

Given these results the expected recession time for lag length 1 was calculated for these alternative data sets. For the truncated current sample the expected recession time was again 20 years. However, if we use GDP data of the earlier vintage the expected recession time falls to 12 years. This discrepancy is mainly associated with differing treatment of software in the two types of National Income accounts.

# 5 Conclusions

This paper has presented strong statistical evidence that the business cycle as traditionally described in terms of recession and expansions is dying out. As discussed in the paper, the end of the business cycle has often been declared dead before with embarrassing results. The statistical evidence presented here is designed to produce a more robust claim of the end of the business cycle by learning from previous false claims of the death of the business cycle. However, it is still dependent on history providing an accurate view of the future.

History can be interpreted in more than one way. A reasonable amount of evidence was presented above that the US economy was more stable in the 1960s than in the earlier periods (see also Blanchard and Simon, 2001) but this relatively stable period was followed by the turbulent 1970s. An observer of the US economy in the third quarter of 1969 would not have been able to predict, for example, the rise in power of OPEC. Similarly, there might be future shocks of a new origin hitting the US economy that produce a return to greater instability.

For example, consider the conjecture that there is a probability of $p$ in the next year that we will return to the greater instability of the pre-1984 period. Further, it makes sense to assume that return of instability will be coincident with a recession. Then, this gives a probability of recession in the year ahead of $p + (1 - p)0.05$, since the calculations above imply a probability of recession one year ahead of approximately 5%. If one has a subjective belief in a recession in the next year of $q$, this would be supported by $p = (q - 0.05)/0.95$. For example, if one thought the recession probability was 2/9 (the historical record), then the probability of returning to greater instability would have to be only around 18% each year. Alternatively if one thought recessions occured about every 6 years then the probability of returning to greater instability would have to around 12%. Neither of these probabilities are particularly incredible.

Another interpretation is that defining recessions in terms of declines in GDP is no longer appropriate. There has been considerable debate about the dating of the 2001 recession. One aspect of this debate has been that the labor market did not start improving until 20 months after the end of the NBER dated recession. Another has been that the NBER appeared to rely heavily on a proxy for monthly GDP in its assessment that the recession had ended.

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Figure 1a: Prior Distribution of Time to Recession

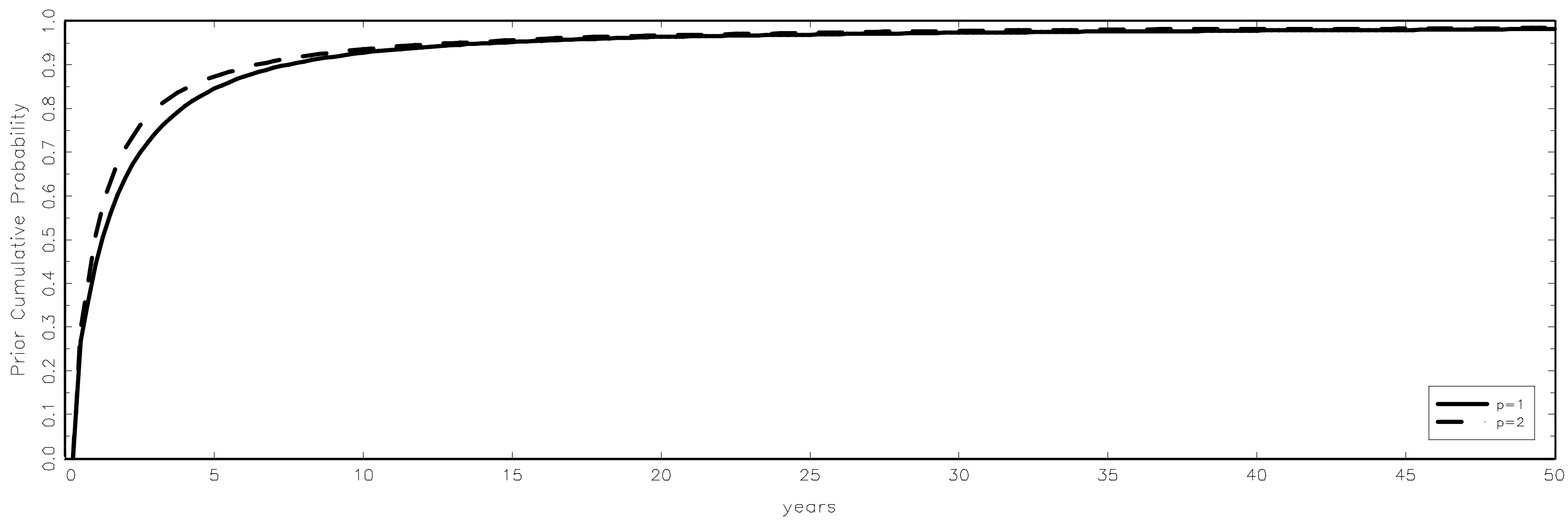


Figure 1b: Prior Distribution of Expected Time to Recession

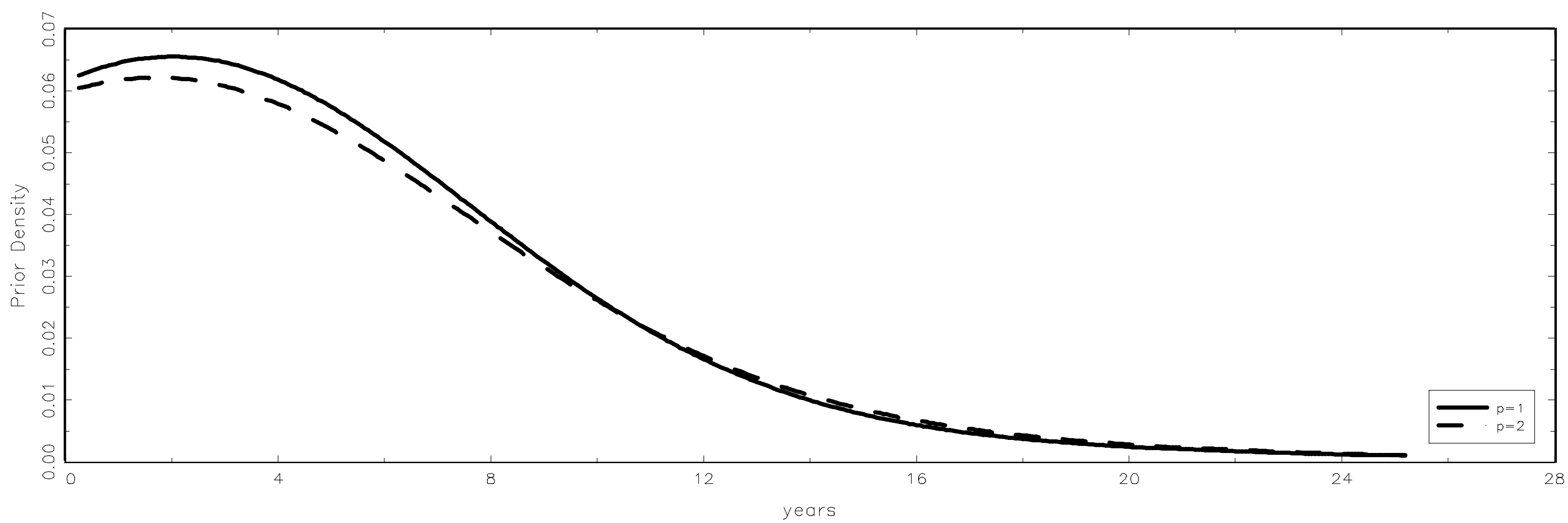

Figure 2.a: 1969 p=1

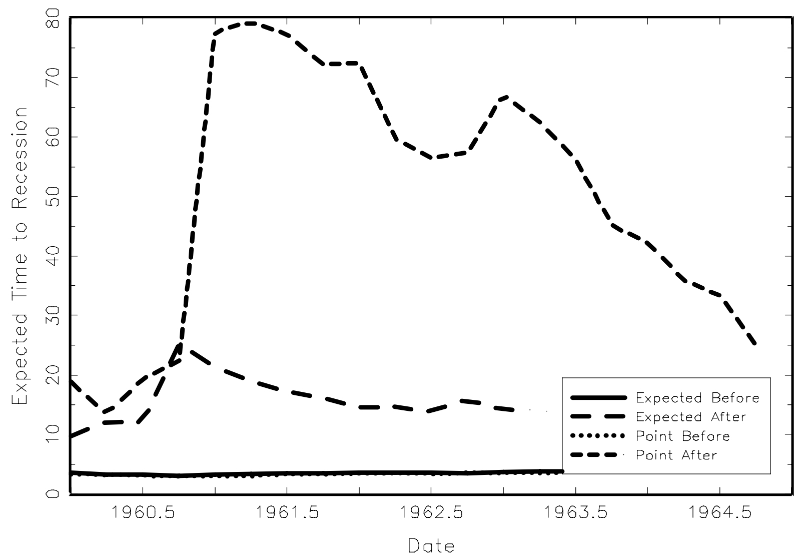


Figure 2.b: 1969 p=2

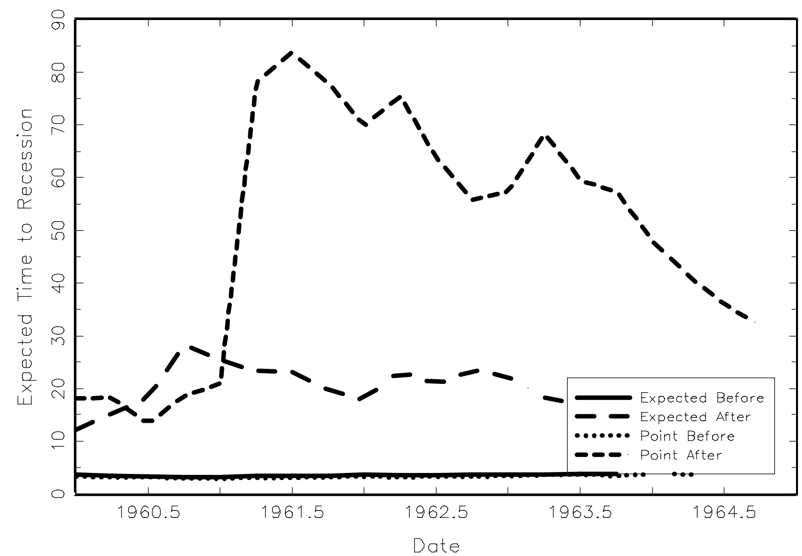


Figure 2.c: 2006 p=1

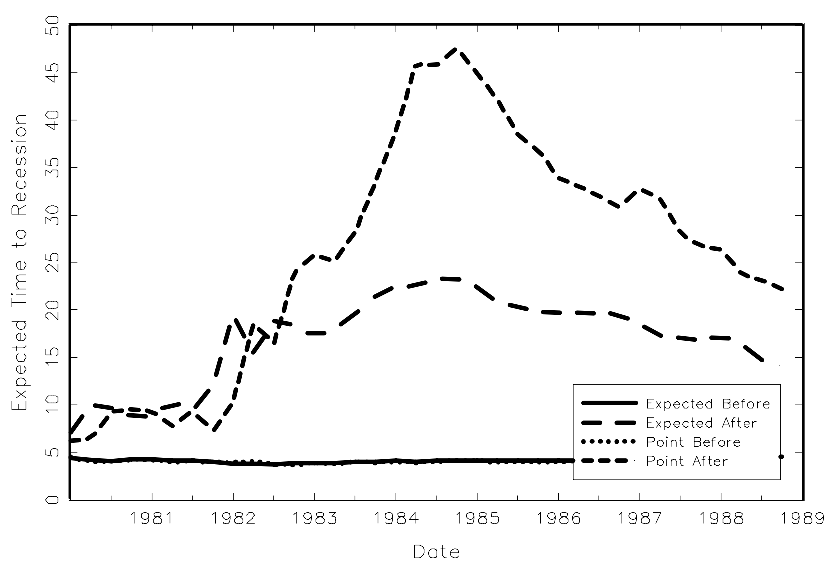


Figure 2.d: 2006 p=2

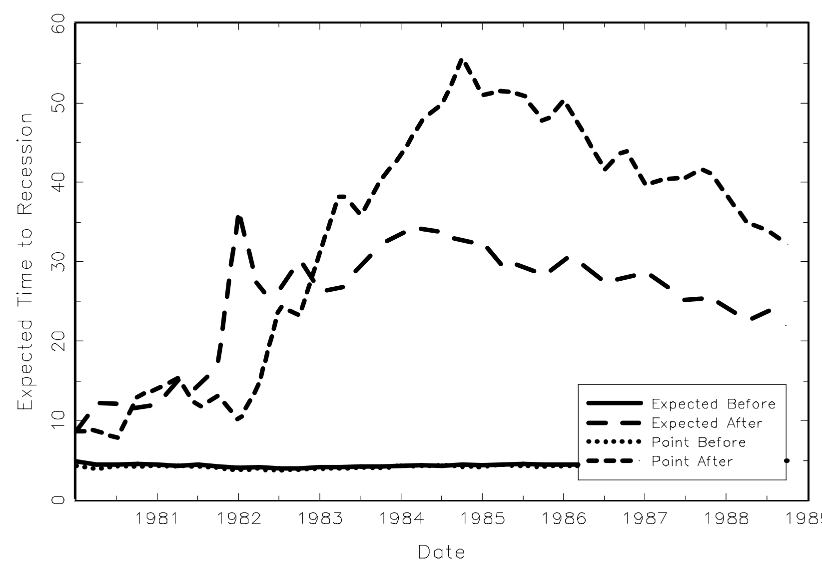

Figure 4.a: Distribution of Time to Recession 1969 p=1

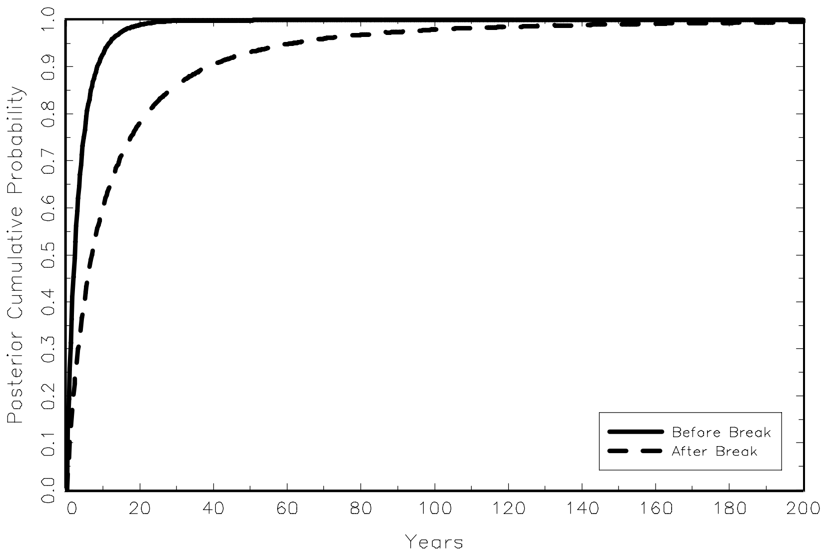


Figure 4.b: Distribution of Time to Recession 1969 p=2

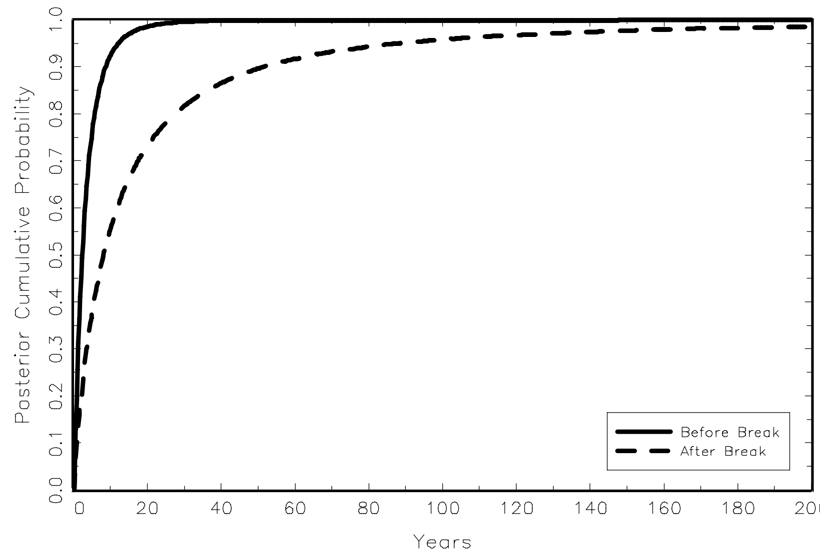


Figure 4.c: Distribution of Time to Recession 2006 p=1

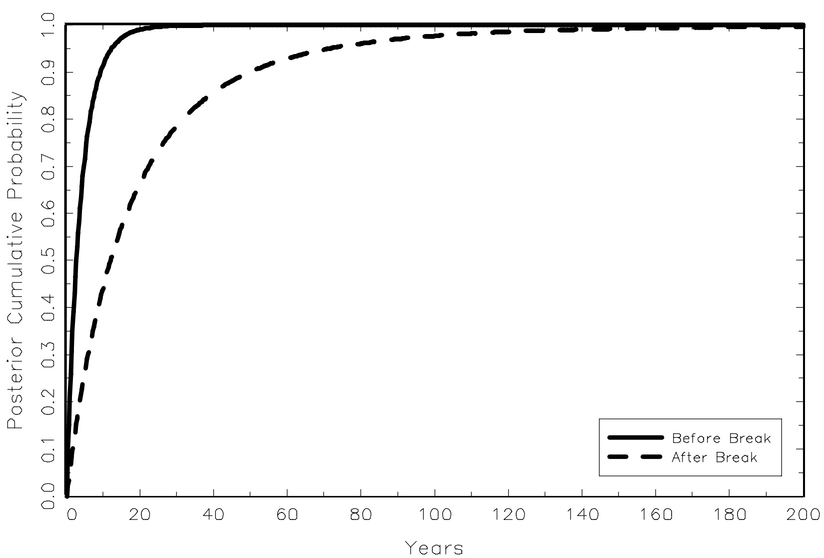


Figure 4.d: Distribution of Time to Recession 2006 p=2

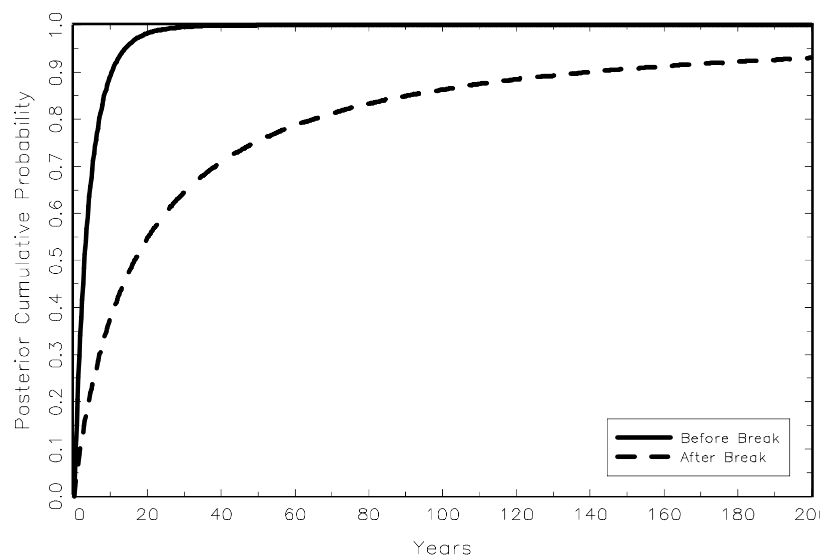

Figure 5a: Distribution of Expected Time to Recession 1969 p=1

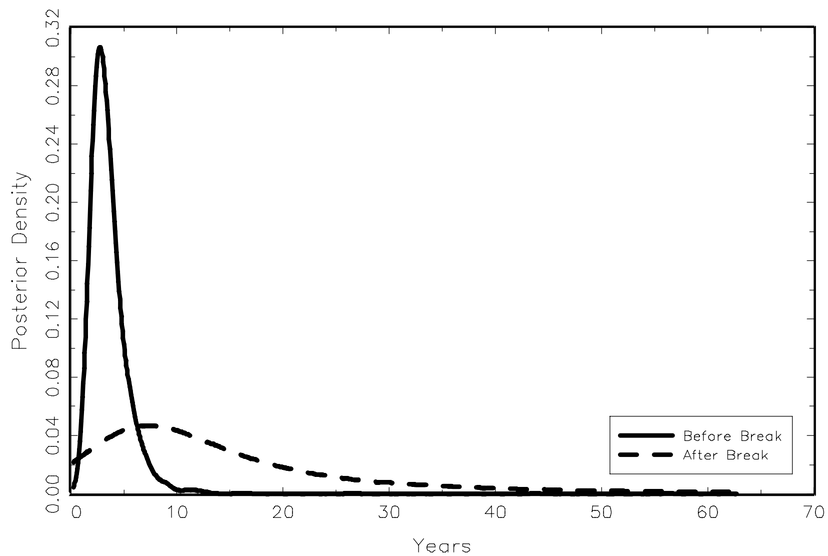


Figure 5b: Distribution of Expected Time to Recession 1969 p=1

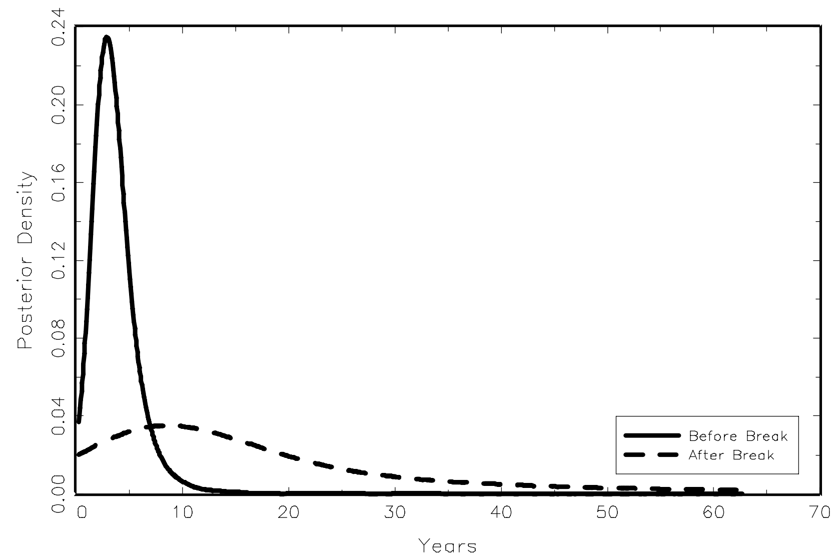


Figure 5c: Distribution of Expected Time to Recession 2006 p=1

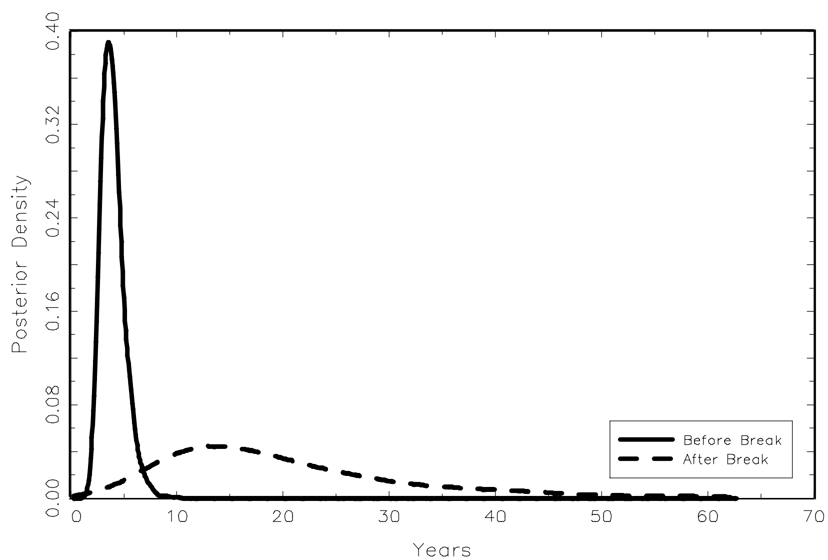


Figure 5d: Distribution of Expected Time to Recession 2006 p=2

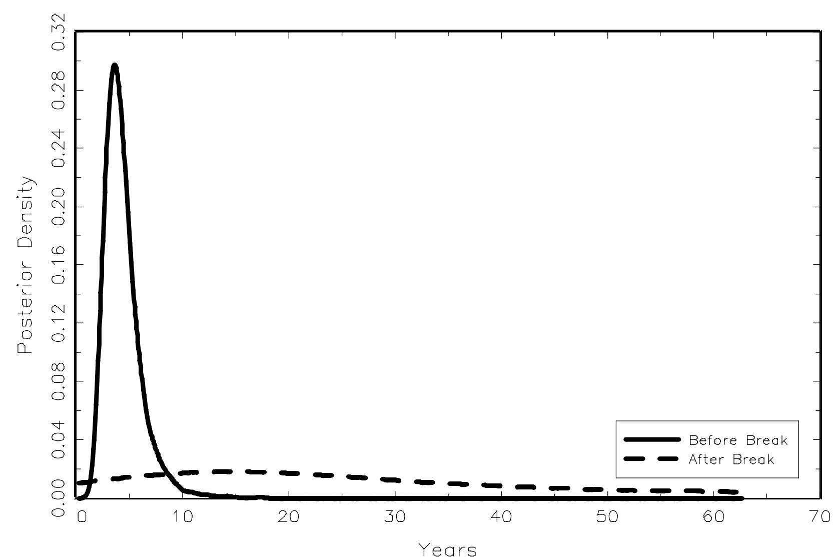

Figure 6a: Posterior Distribution of Time to Recession

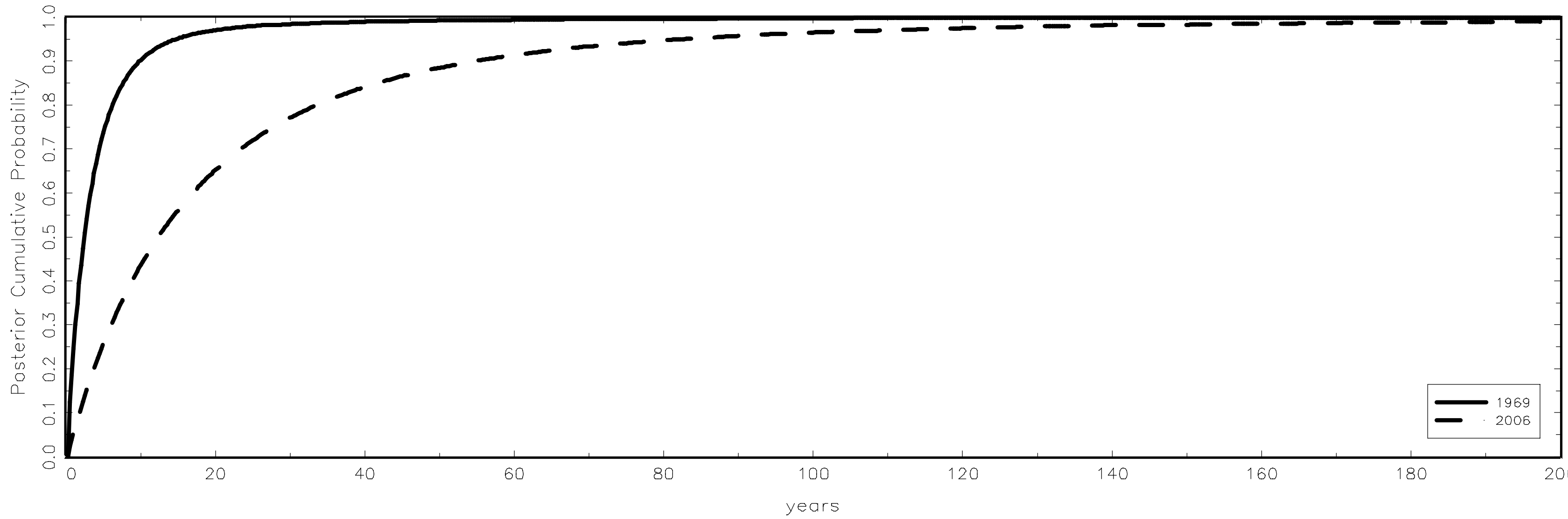


Figure 6b: Posterior Distribution of Expected Time to Recession