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**THE LONG-RUN VALUE OF CURRENCIES:
A BIG MAC PERSPECTIVE***

by

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Abstract

Purchasing power parity (PPP), the link between exchange rates and prices, is a fundamental building block of international finance, one which has been attracting increasing research interest during the past three decades. The Big Mac Index (BMI), invented by The Economist magazine in 1986, has played a major role in popularising PPP and bringing its practical implications to the attention of financial markets. The aim of this paper is to derive long-run equilibrium values of currencies using the Big Mac data from The Economist magazine. As there are only ten years of data available, we have to use a parsimonious approach to modelling the evolution of exchange rates. The stationarity of real exchange rates is tested using recently-developed panel unit root tests with modifications. Through Monte Carlo methods, we analyse the whole distribution of the estimated equilibrium exchange rates and derive the adjustment paths of actual rates into the future. The new approach we propose for equilibrium exchange rates seems to have a number of attractive features including its modest data requirements (Big Mac prices), the minimal economic structure on the problem and its simplicity. Interestingly, our estimate of the speed of adjustment of real exchange rates to shocks and of the equilibrium exchange rates are quite close to those of other studies using more complex methodologies.

* This paper is based on part of my PhD thesis. I would like to thank my supervisor Professor Ken Clements for his constant encouragement, stimulation, understanding and patience. I would also like to acknowledge the helpful comments of seminar participants at The University of Western Australia.

1. Introduction

The theory of purchasing power parity (PPP) is one of the fundamental principles of international economics. It states that prices across countries should be equal when converted to a common currency (absolute PPP), or less strictly, the change in the exchange rate should be equal to the difference between inflation at home and abroad (relative PPP). The theory has been extensively researched and consensus has been reached that PPP is not a theory of short-term exchange rate determination. However, it remains an open question whether PPP offers a long-run equilibrium relationship between relative prices and exchange rates (see, e.g., Edison, 1987, Frenkel, 1976, Froot and Rogoff, 1995, Rogoff, 1996).

Although the PPP doctrine has its roots with the writings of Gustav Cassel (1866-1945), it was not subjected to extensive analytical scrutiny until Balassa (1964) and Samuelson (1964) proposed the famous productivity bias hypothesis, which has obtained considerable empirical support over the past few decades. On the other hand, in the 1970s and 1980s, empirical tests using data from the onset of the floating exchange rate system in 1973 tended to strongly reject PPP except for hyperinflation countries. This stream of research reached its high-water mark in the early 1980s with a paper published by Frenkel (1981) entitled “The Collapse of Purchasing Power Parities during the 1970s”. It is now widely agreed that results during this period are subject to qualifications due to non-stationarity issues and the low power of tests (Froot and Rogoff, 1995). In the late-1980s and 1990s, the development of a variety of new test approaches, including the use of panel data, contributed to the rebirth of and substantial growth in the PPP literature with empirical evidence now accumulating in support of PPP as a long-run proposition.

This paper aims to contribute to the PPP literature by testing PPP using the Big Mac data published in The Economist magazine and then deriving equilibrium exchange rates for the 16 countries under investigation. Rather than employing conventionally-used CPIs, WPIs, or GDP deflators in testing PPP, we use McDonald’s Big Macs, which are made of roughly the same recipe in 120 countries, to form a single-good price index.

Previous research on the Big Mac index has evolved into an important strand of literature called “Burgernomics”. Despite some widely-acknowledged qualifications (e.g., the disregard of barriers to trade, pricing to market, etc.), this index performs at least as well as most other indexes used in tests for PPP (Click, 1996, Cumby, 1996, Ong, 1997, Pakko and Pollard, 1996), and is a valuable international asset allocator (Annaert and Ceuster, 1997). Other applications of Burgernomics include Ong (1998a, 1998b) and Ong and Mitchell (2000).

The organisation of the paper is as follows: Section 2 documents the rapid growth in the PPP literature. The descriptive features of the 16 Big Mac real exchange rates are examined in Section 3. In Section 4, we perform a preliminary analysis of the mean-reversion properties of the Big Mac exchange rates and carry out a simple unit root test. Section 5 explicitly recognises the cross-country dimension of the problem and applies a multivariate test. Section 6 explores the conceptual foundations of the equilibrium exchange rate, while Section 7 derives estimates of equilibrium exchange rates and examines their sampling variability using Monte Carlo methods. Section 8 illustrates the practical usefulness of Big Mac equilibrium exchange rates by deriving the adjustment path of exchange rates into the future. Our estimates of the speed of adjustment and equilibrium exchange rates are compared with those from other studies in Section 9. The final section summarises and concludes.

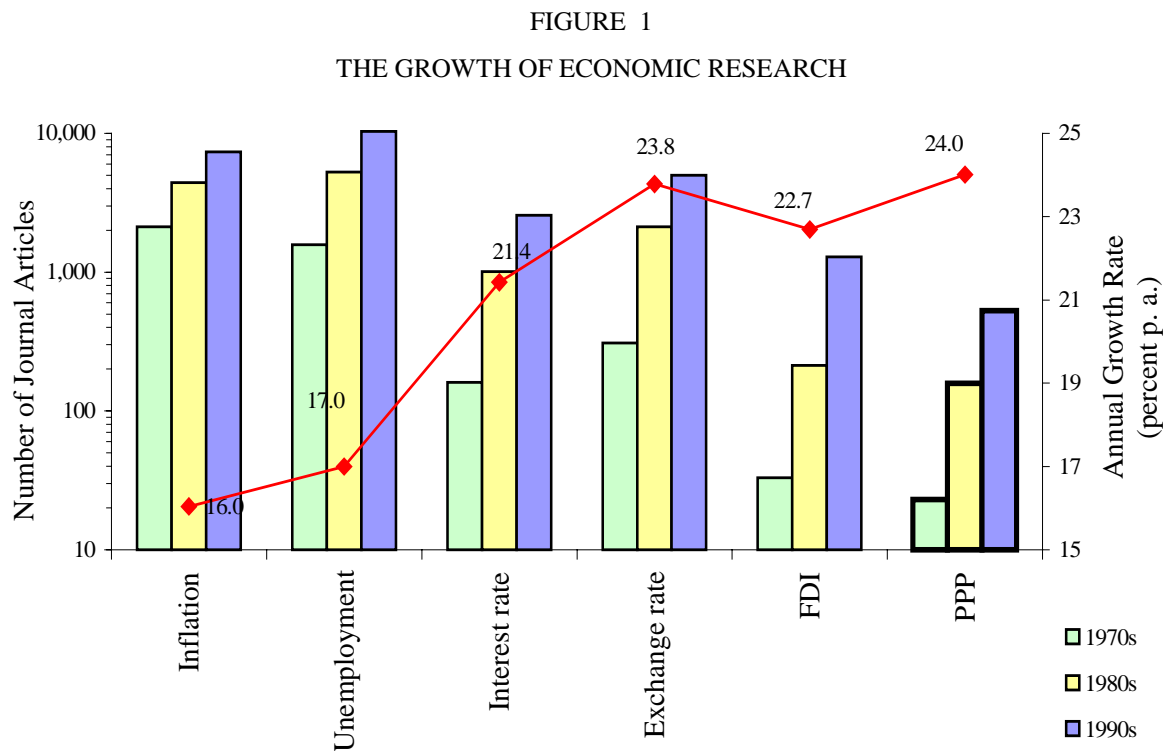
2. The Explosion of PPP

As a way of measuring the extent of professional interest in PPP, this section reports results of searching for (i) PPP in Econlit, a widely-used economic indexing database produced by the American Economic Association;¹ and (ii) the Big Mac Index in Google, a popular search engine on the world wide web.

¹ The source material of Econlit includes international economic journals, essays, research papers, books, dissertations, book reviews, and working papers. Years of coverage are from 1969 to the present with approximately 26,000 records added annually. Our search is done through the licensed Econlit website at The University of Western Australia: <http://ovid.library.uwa.edu.au/ovidweb/ovidweb.cgi>.

As we need to compare the amount published on PPP with something, in Econlit we also searched for four additional broad economic terms -- inflation, unemployment, interest rate and exchange rate -- and another relatively narrow term, foreign direct investment (FDI).² We recorded the number of research articles on each topic published in the 1970s, 1980s and 1990s. Figure 1 plots, in the logarithmic scale on the left-hand axis, the number of articles in each decade for the six topics. The change in the height of the bars for each term indicates the exponential rate of growth. The right-hand axis gives the growth rate, on an annual basis, for each topic. It can be seen that PPP has grown at an average rate of 24 percent p.a., which is the highest of the six topics. This growth rate clearly reflects the strong expansion of research interest in PPP over the past three decades, so that PPP research can be described as “exploding” rather than “collapsing”.

Since the introduction of the Big Mac Index (BMI) by The Economist in 1986, financial markets have become interested in PPP as a practical approach to valuing



² Foreign direct investment is chosen as a topic due to its extraordinary growth over the past few decades.

currencies and in making international price comparisons. As a way to measure the extent of this interest, we searched for the exact phrase “Big Mac Index” in a powerful web search engine -- Google.³ This resulted in 697 entries.⁴ We categorised these BMI web sites according to the language they use and their institutional domains and the results are presented in Table 1. The left panel shows that around 70 percent of the search results are written in the 17 frequently-used languages and the remaining 30 percent are in the language not specifically identified by Google. English web sites are the most frequent, followed by European languages. Developing countries are also aware of this widely-quoted invention from The Economist. From the right panel of Table 1, it can be seen that BMI is used in all sectors. About 40 percent of the BMI web pages are created by commercial institutions, suggesting the widespread practical usefulness of the BMI. Another 20 percent of the BMI pages are related to educational activities. Even serious international organisations (which have the domain .org) find this metric of relative currency values simple, timely, and, most importantly, accurate for policy-makings. In the subsequent sections, we examine BMI in terms of its time-series properties and its implications for the long-run value of a currency.

TABLE 1
BIG MAC INDEX SEARCH RESULTS

Language of web page	Number	Institutional Domain	Number
English	405	.com	271
Danish, French, German, Japanese, Swedish	66	.edu	131
Chinese, Dutch, Italian, Korea, Norwegian, Russian, Spanish	27	.org	45
Czech, Finnish, Hungarian, Portuguese	7	.net	38
Other	192	Other	260
Total	697	Total	697

Source: <http://www.google.com>.

³ The address of the Google search engine is <http://www.google.com>.

⁴ Note that (1) to narrow down the search, we use an additional constraint “not computer”; (2) all the search results in Google refer to the returned entries excluding pages similar to those displayed; and (3) another 18 entries from Google contain only the word “Burgernomics”, but not the phrase “Big Mac Index”.

3. The Big Mac Data

Let P_{ct} be the price of a Big Mac hamburger in country c in terms of domestic currency in year t and P_t^* the corresponding price in the US. Then, P_{ct}/P_t^* , the relative price of a Big Mac, is the PPP exchange rate, which can be directly compared with S_{ct} , the actual exchange rate, defined as the domestic currency cost of \$US1. The deviation of the PPP exchange rate from the actual rate is $P_{ct}/P_t^* - S_{ct}$, or in relative terms

$$r_{ct} = \frac{P_{ct}/P_t^* - S_{ct}}{S_{ct}}.$$

If $r_{ct} > 0$, then prices at home are too high relative to foreign prices and the exchange rate; and vice versa for $r_{ct} < 0$. Another way of visualising r_{ct} is to consider converting the domestic cost of a Big Mac into US dollars, P_{ct}/S_{ct} , which can then be compared with the US price in dollars, P_t^* . The relative difference between P_{ct}/S_{ct} and P_t^* is

$$\frac{P_{ct}/S_{ct} - P_t^*}{P_t^*} = r_{ct}.$$

It thus follows that $r_{ct} > 0$ also implies that the dollar price of a Big Mac in country c exceeds the corresponding US price, and vice versa if $r_{ct} < 0$. Accordingly, a positive (negative) value of r_{ct} indicates over (under) -valuation of the country's currency relative to the US dollar. When absolute PPP holds, the price of a Big Mac is equalised in terms of a common currency, so that $P_{ct} = S_{ct} P_t^*$ and $r_{ct} = 0$. As long as $P_{ct}/S_{ct}P_t^*$ is not too far away from unity (as suggested by the PPP theory), r_{ct} is approximately equal to $\log(P_{ct}/S_{ct}P_t^*)$,⁵ which is the conventionally-defined real exchange rate in terms of natural logarithms, hereafter denoted by q_{ct} . As before, $q_{ct} > 0$ implies that country c 's currency is overvalued.

⁵ Mathematically, $\log(1+x) \approx x$ when $x \approx 0$.

Table 2 presents q_{ct} for $c = 1, \dots, 16$ countries and $t = 1, \dots, 10$ years.⁶ It can be seen from a given row that q_{ct} for country c fluctuates a good deal over time. Averaging over time eliminates such short-term fluctuations and the last column of Table 2 shows that the currencies of a majority of countries are overvalued. Similarly, the average of q_{ct} over the 16 countries at time t , given in the second-to-last row of Table 2,

TABLE 2
THE VALUE OF CURRENCIES

Country	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	Mean
Australia	-18	-23	-15	-12	-26	-29	-25	-18	-22	-38	-23
Belgium	12	24	25	39	39	30	50	39	24	11	29
Britain	6	4	28	33	20	14	19	14	20	18	18
Canada	-11	-15	-10	6	-4	-11	-15	-12	-16	-26	-11
Denmark	51	60	62	68	62	51	75	62	49	28	57
France	32	36	35	40	42	32	51	37	23	10	34
Germany	12	15	13	23	24	16	40	31	17	5	20
Hong Kong	-73	-69	-68	-64	-67	-66	-64	-61	-64	-67	-66
Italy	17	37	26	42	26	19	13	21	12	-3	21
Japan	32	6	22	27	42	49	69	13	-4	-21	24
Netherlands	17	24	22	28	30	22	42	31	16	3	23
Singapore	-35	-46	-35	27	-27*	-19	-10	-9	-15	-32	-20
South Korea	58	30	26	30	24	21	25	22	6	-37	21
Russia	99*	104	94	-132	-70	-34	-36	-20	-23	-25	-4
Spain	17	24	41	34	22	8	21	20	7	-6	19
Sweden	48	58	65	67	41	33	42	50	33	16	45
Mean – All countries	17	17	21	16	15	8	19	14	4	-10	12
– Russia excluded	11	11	16	26	17	11	22	16	6	-9	13

- Notes: 1. The entries are 100 times the natural logarithms of the ratios of the actual exchange rate to its Big Mac PPP value. Accordingly the entries are approximately percentage differences between PPP and the actual exchange rate. A positive (negative) entry indicates that the domestic currency is over (under) -valued relative to the US dollar.
2. An asterisk (*) indicates that the entry is interpolated or extrapolated.
3. Data regarding Russia up to 1991 refer to the Soviet Union.

Source: The Economist, various issues.

⁶ It is appropriate to note here the relationship between logarithmic and percentage differences. As the lower bound of P_{ct}/P_t^* is zero (or arbitrarily close thereto), r_{ct} cannot fall below -1 , indicating that no currency can be undervalued by more than 100 percent. However, as illustrated in Table 2 by Russia in 1992 when its logarithmic difference between PPP and the actual exchange rate was -1.32 , q_{ct} can fall below -1 . But as $\{\exp(-1.32)-1\} = -0.73$, this value corresponds to the rouble being undervalued by 73 percent. On the other side of the coin, there is no upper bound to either q_{ct} or r_{ct} , but note that $r_{ct} > q_{ct}$ for all values of $q_{ct} \neq 0$.

indicates the average valuation of all currencies against the US dollar, which is the negative of the valuation of the US dollar against the 16 Big Mac currencies. Due to the turbulence in Russia in the first four years for this period, the last row gives the mean with Russia excluded. It can be seen that on average the US dollar was moderately undervalued in most years up to 1995, when it was undervalued by 19 percent; thereafter it gradually appreciated, so that by 1998 the US dollar was overvalued by 9 percent.

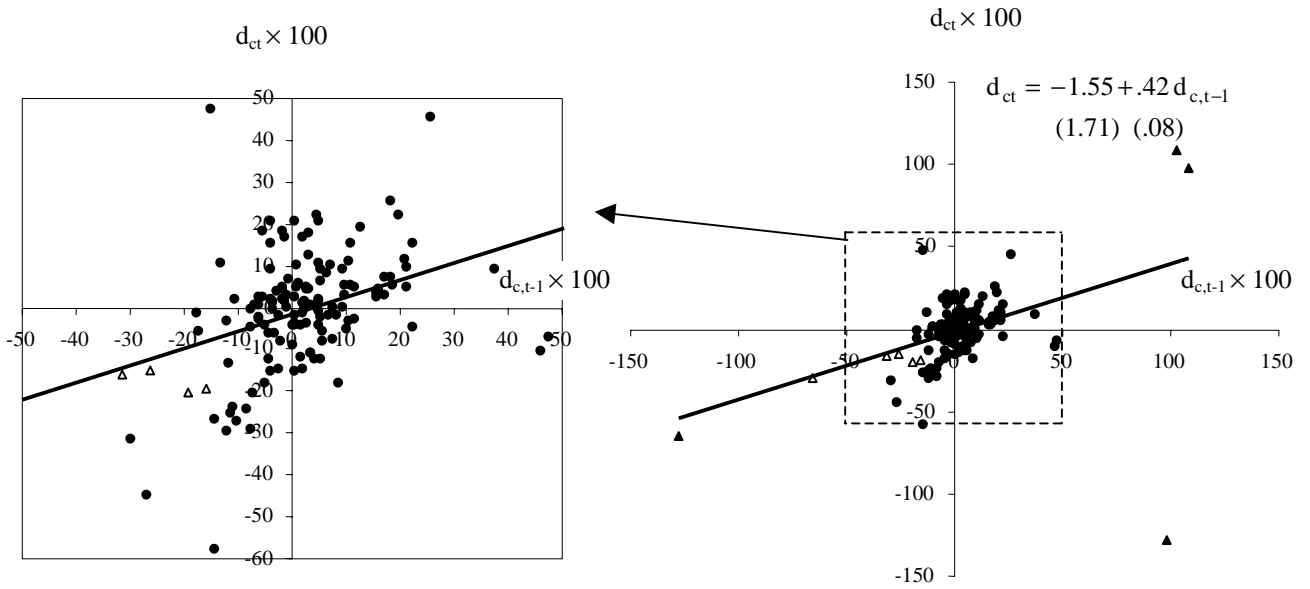
4. Preliminary Analysis of Stationarity

Stationarity of the real exchange rate is crucial to the validity of long-run PPP. When absolute PPP holds, $P_{ct} = S_{ct} P_t^*$ and the real exchange rate $q_{ct} = \log(P_{ct} / S_{ct} P_t^*)$ is equal to zero or has tendency to return to zero after shocks. If relative PPP holds, $P_{ct} = K_c S_{ct} P_t^*$, where $\log K_c$ is the real exchange rate for country c and deviations from this real exchange rate are temporary and disappear with time. Here, the real exchange rate is stationary, i.e., mean-reverting. Engle and Granger (1991, Chap. 1) describe a stationary time series as one that has a tendency to return to an “attractor” by some mechanism. At any point in time, there could be some shocks which may take a variable away from the attractor, but there will still be an overall tendency for the value of the variable to move back toward it. Thus the attractor can be thought of as the centre of gravity, or the equilibrium value. On the other side of the coin, if PPP does not hold, the real exchange rate will have a unit root, deviations from parity will be cumulative and the real exchange rate has no long-run mean. In this section, we conduct a preliminary analysis regarding the stationarity of real exchange rates for the 16 Big Mac countries.

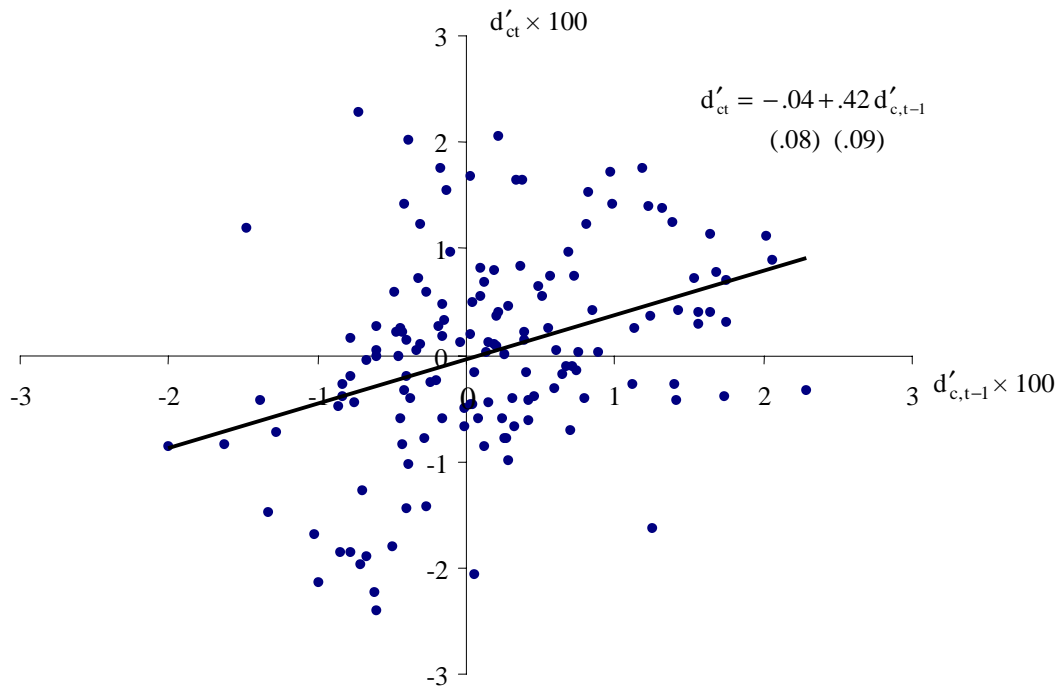
Define the deviation of country c 's real exchange rate as the difference of its value from its mean, $d_{ct} = q_{ct} - q_{c\bullet}$, and the standardised deviation as $d'_{ct} = d_{ct} / SD(d_c)$, where $q_{c\bullet}$ is the average of q_{ct} over the sample period and $SD(d_c)$ is the standard deviation. As the deviations d_{ct} and d'_{ct} both have zero means by construction, one way to examine the degree to which they have a tendency to revert to the mean is to investigate their serial properties. Panel A of Figure 2 contains a scatter of the pairs $\{d_{ct}, d_{c,t-1}\}$ for

FIGURE 2
SCATTER PLOTS OF DEVIATIONS AGAINST LAGGED VALUES

A. Non-Standardised



B. Standardised



$c = 1, \dots, 16$ countries and $t = 2, \dots, 10$ years, a total of $16 \times 9 = 144$ observations.⁷ Panel B contains the same scatter for the standardised deviations. The solid lines in Figure 2 are the least-square regression lines. The slope coefficient is estimated to be .42, which is significantly different from unity. Deleting four outliers (indicated by filled triangle markers) in Panel A (which all pertain to Russia) gives an estimated slope of .48 (with a standard error of .08). Furthermore, deleting all nine pairs of observations for Russia (triangle markers) completely gives an estimate in between the above two -- .44 (SE = .10). Accordingly, the deletion of Russia has the effect of slightly increasing the slope coefficient and decreasing the speed of adjustment. This is understandable as Russia has a relatively rapid rate of mean reversion: After an initial four years of large values of its real exchange rate, the rate then moves toward parity in a relatively short period of time (see the row for Russia in Table 2). As the estimate of the slope coefficient is less than unity, shocks to real exchange rates die out over time; that is, these shocks are temporary, not permanent, so that exchange rates revert to well-defined equilibrium values. It must be emphasised, however, that this is simply a preliminary analysis and more sophisticated tests are used subsequently. Interestingly however, these subsequent tests are not inconsistent with the findings of this preliminary analysis.

The regression line in Panel A of Figure 2 is of the form, $d_{ct} = \alpha + \beta d_{c,t-1} + u_{ct}$, where β is the speed-of-adjustment parameter which is common to all countries. Recalling that $d_{ct} = q_{ct} - q_{c\bullet}$, this can be reformulated as

$$(1) \quad q_{ct} = \alpha_c + \beta q_{c,t-1} + u_{ct},$$

where the country-specific intercept $\alpha_c = \alpha + (1-\beta)q_{c\bullet}$. Equation (1) is the basic model of the evolution of the real exchange rate used in this paper. Due to the short time span of the available Big Mac data, ten years, it is appropriate to be parsimonious, so we have only one lagged value on the right-hand side of equation (1). Under PPP, the real exchange rate is stationary, so that the speed-of-adjustment coefficient in equation (1) $\beta < 1$. If PPP

⁷ The equation corresponding to the regression line in Panel A is $d_{ct} = \alpha + \beta d_{c,t-1} + u_{ct}$, where α and β are coefficients which are taken to be the same in each of the $c = 1, \dots, 16$ countries. As we wish to compare d_{ct} with $d_{c,t-1}$ and not d_{ct} with $d_{c't}$ for $c \neq c'$, in the regression we omit the observations where the countries “overlap”, that is, the 15 values of $\{d_{c1}, d_{c-1,10}\}$ for $c = 2, \dots, 16$.

does not hold, the deviations are non-stationary, $\beta = 1$ and equation (1) is then a random walk with a drift.⁸ We estimate (1) for $c = 1, \dots, 16$ countries and $t = 2, \dots, 10$ years by the least squares dummy variables (LSDV) method.⁹ The test statistic for $H_0 : \beta = 1$ is the usual t-statistic, $\tau = (\hat{\beta} - 1)/SE(\hat{\beta})$, where $\hat{\beta}$ is the estimated coefficient and $SE(\hat{\beta})$ is its standard error. As is well known, the distribution of τ is non-standard and Levin and Lin (1992) give the critical values of τ . When $T = 10$, the 5 percent critical values are -6.28 for $N = 15$ and -7.06 for $N = 20$. Our test statistic is -7.40, suggesting that the common speed-of-adjustment parameter is significantly different from one. However, we have to exercise some caution when interpreting such results from the application of OLS. There are two issues here:

- (i) It is known that in the presence of lagged dependent variables, the OLS estimator based on T observations is biased, but it is consistent when $T \rightarrow \infty$ (see, e.g., Greene, 1997, p. 40). This is attributed to the fact that lagged real exchange rate is correlated with the past values of the disturbances in equation (1).
- (ii) If the disturbances in equation (1) are autocorrelated, the magnitude of bias depends on the specific properties of disturbance generating process (Maeshiro, 1999).

We will deal with bias in the next section. Regarding possible autocorrelation in the disturbances, as a preliminary analysis we simply regress the OLS residuals from equation (1) on their values lagged one year and find no evidence of serial correlation.

Although a parsimonious specification such as equation (1) is attractive, it does not account for possible cross-country correlation among exchange rates (O'Connell, 1998). For more efficient estimation and hypothesis testing, a multivariate methodology is adopted in the next section.

⁸ Strictly speaking, a random walk possesses a unit root, but a unit root is a necessary but not sufficient condition for the random walk hypothesis (Lee, 1992, Lo and MacKinlay, 1989). In addition, random-walk changes are required to be uncorrelated. Therefore, a random walk process is a subset of a unit root one. The existence of stationarity leads to the rejection of both a unit root and a random walk, but the latter can also be rejected because of correlated changes. However, such a fine distinction is often neglected in tests of PPP.

⁹ In a preliminary analysis, we allowed the speed-of-adjustment parameter to differ across countries, so that $q_{ct} = \alpha_c + \beta_c q_{c,t-1} + u_{ct}$. Country-by-country estimates of this equation reveal that many of the parameters are not precisely estimated, no doubt because there are only 10 observations per country. It is noteworthy, however, that 15 of the 16 point estimates of β_c are less than one. In addition, the unweighted and weighted (by the reciprocal of the standard errors) means of the 16 slope coefficients are .48 and .46 respectively, which are not too different from that reported in the text. In the next section, we formally test the null hypothesis of a common speed-of-adjustment in the multivariate setting.

5. More on Stationarity

In this section, we apply multivariate unit root tests by using the generalised least squares (GLS) method in the seemingly unrelated regression (SUR) setting for the system of equations comprising equation (1) for $c = 1, \dots, 16$ and propose a simple technique for the adjustment of biases and standard errors of estimates. Although SUR-GLS has been previously adopted in Flôres et al. (1999), Higgins and Zakrajšek (1999), O’Connell (1998), Papell (1997) and Papell and Theodoridis (1998) to test for PPP, none of these studies makes the necessary adjustments.

The application of SUR requires an estimate of the 16×16 contemporaneous covariance matrix of the disturbances Σ . Usually Σ is approximated by its unbiased estimator S , the matrix of mean squares and cross products of the OLS residuals. In our application, we have $T = 10$ years and $N = 16$ countries. As N is large relative to T , there is an undersized sample problem and S is singular.¹⁰ To get around the problem of singularity, we structure the covariance matrix by using two alternative assumptions.¹¹ We assume that real exchange rates are governed by:

- (1) A type of block-independence whereby countries are classified into three blocks: Asia Pacific, Europe and other. It is assumed that real exchange rate innovations between countries in the same block are correlated, while those for countries in different blocks are not. This model is called “block-sectional independence”, or BSI for short.¹²
- (2) A process which summarises the cross-country dependence in one factor common to all countries. This “common factor model” (CFM) originates from Sharpe (1964) and assumes that the real exchange rate innovations in all countries are driven by a single determinant. The common factor model is derived using two weighting schemes, (i) the share of each country in world trade; and (ii) the share of each country in world GDP.

For more details, see Lan (2001).

In Section 4, we assumed that the value of the speed-of-adjustment parameter β was the same across countries. To investigate whether this restriction is valid in the SUR

¹⁰ Although mathematically a matrix is either singular or it is not, numerically there are degrees of singularity. One way of measuring the degree of singularity of a matrix is by its condition number, which is defined as square root of the ratio of the maximum characteristic root to the minimum value. A condition number over 10^{10} is considered “large” and the matrix is difficult to invert accurately (Judd, 1998). With our data, the condition number of S is 6.7×10^{15} , suggesting the singularity of S .

¹¹ See Fiebig and Kim (2000) for an investigation of several alternative estimators of the near-singular covariance matrix in SUR models. Chen (1995) studies undersized samples in the context of demand analysis.

¹² More precisely, countries are “allocated” to these regions on this basis. See Lan (2001) for details.

setting, we follow Parkes and Savvides (1999) and conduct an F-test. The linear restrictions $\beta_1 = \beta_j$, $j = 2, \dots, 16$ can be written in matrix form as $\mathbf{R}\boldsymbol{\theta} = \mathbf{q}$, where $\boldsymbol{\theta} = [\beta_1, \dots, \beta_{16}]'$ is a $K = 16$ vector of unrestricted estimates of the slope coefficients, \mathbf{R} is a $J \times K = 15 \times 16$ matrix, and \mathbf{q} is a $J = 15$ vector. The F-type statistic for testing the validity of the J linear restrictions is $\lambda = (\mathbf{R}\hat{\boldsymbol{\theta}} - \mathbf{q})'[\mathbf{R}\mathbf{C}^{-1}\mathbf{R}']^{-1}(\mathbf{R}\hat{\boldsymbol{\theta}} - \mathbf{q})/J$, where $\mathbf{C} = \mathbf{X}'(\boldsymbol{\Omega} \otimes \mathbf{I})\mathbf{X}$, with \mathbf{X} an $N(T-1) \times K = 144 \times 16$ matrix of regressors, $\boldsymbol{\Omega}$ the 16×16 residual covariance matrix derived from BSI or CFM and \mathbf{I} an identity matrix of order $T-1 = 9$. The distribution of λ is unknown and the critical values of the test statistic are derived from Monte Carlo simulations. The quasi-F test results do not reject the hypothesis of a common speed of adjustment. See Lan (2001) for details.

We estimate equation (1) by O'Connell's (1998) GLS procedure.¹³ Recall from the end of Section 4 that the LSDV estimates of equation (1) are biased. It is thus appropriate to examine whether our GLS estimates are also biased, especially as the disturbance covariance matrix is patterned to accommodate the problems associated with the undersized sample.¹⁴ Suppose that the error terms in equation (1) for $c = 1, \dots, 16$ are drawn from a multivariate normal distribution with zero mean vector and the appropriate covariance matrix. The real exchange rates are then computed from equation (1) using the data-based GLS estimates as true values and these generated data are then used to re-estimate the model by GLS. This procedure is repeated 1,000 times and we compute the mean of 1,000 estimates. For any parameter θ_i , let $\hat{\theta}_i$ be its GLS estimate and $\bar{\theta}_i$ be the mean over 1000 trials. Then the proportionate bias for a GLS estimate is $p_i = (\bar{\theta}_i - \hat{\theta}_i)/\hat{\theta}_i$. It is found that the biases, $\bar{\theta}_i - \hat{\theta}_i$, are substantial. We propose a simple iterative scheme to adjust for bias:

- In each iteration, we use each data-based GLS estimate adjusted by an appropriate bias factor as the "true" value for the simulation. Denote the true value of θ_i in the k^{th} iteration by $\theta_i^{\text{True}(k)}$ and the adjustment factor computed from the last iteration by $b_i^{(k)}$, defined as the proportionate difference of $\hat{\theta}_i$ from $\theta_i^{\text{True}(k)}$. Accordingly, the

¹³ Using the same informal approach as in the last section and two formal tests (Dezhebakhah and Thursby, 1994, Rayner, 1993), we do not find evidence of residual autocorrelation based on the GLS estimates.

¹⁴ Bun and Kiviet (1999), Kendall (1954), Kiviet et al. (1995, 1999) Kiviet and Phillips (1993), Nickell (1981), Phillips (1977), Sawa (1978) and Sevestre and Trognon (1985) are several examples that examine the bias in regressions with a lagged dependent variable.

true value used in the k^{th} iteration is $\theta_i^{\text{True}(k)} = \hat{\theta}_i / (1 + b_i^{(k)})$. Note that in the first iteration, $b_i^{(1)} = p_i$.

- In each iteration, 1,000 Monte Carlo trails are performed as follows. In each trail s ($s=1, \dots, 1000$), we generate real exchange rates based on the true values of estimates, re-estimate equation (1) by GLS, and then adjust the GLS estimate of θ_i , $\hat{\theta}_i^{(s)}$, by $b_i^{(k)}$ (the same proportionate bias used to compute the true value) to yield the bias-adjusted estimate, $\hat{\theta}_i^{\text{ba}(s)} = \hat{\theta}_i^{(s)} / (1 + b_i^{(k)})$. Then we compute the mean of the bias-adjusted estimates over the 1,000 trials $\bar{\theta}_i^{(k)} = (1/1000) \sum_{s=1}^{1000} \hat{\theta}_i^{\text{ba}(s)}$ and compare it with $\theta_i^{\text{True}(k)}$.
- The proportionate bias at the k^{th} iteration is then $p_i^{(k)} = (\bar{\theta}_i^{(k)} - \theta_i^{\text{True}(k)}) / \theta_i^{\text{True}(k)}$. For the $(k+1)^{\text{th}}$ iteration, the adjustment factor used for adjusting the data-based GLS estimate $\hat{\theta}_i$ is computed as $b_i^{(k+1)} = (1 + b_i^{(k)})(1 + p_i^{(k)}) - 1$. This serves to “scale up” the true value at the k^{th} iteration, $\theta_i^{\text{True}(k)} = \hat{\theta}_i / (1 + b_i^{(k)})$, by the new proportionate bias $p_i^{(k)}$, so that the true value in the $(k+1)^{\text{th}}$ iteration is $\theta_i^{\text{True}(k+1)} = \theta_i^{\text{True}(k)} / (1 + p_i^{(k)})$.
- As the estimates of α_c seem to converge simultaneously with that of β , the iterative procedure ends when the change in the proportionate bias $p_i^{(k)}$ for the speed-of-adjustment parameter is sufficiently small.

To investigate the true sampling variability of the bias-adjusted estimates, in the above final iteration (which gives the converged estimates, i.e., when $\bar{\theta}_i \approx \theta_i^{\text{True}}$), we compute the root-mean-squared error (RMSE) and the root-mean-squared standard error (RMSSE) of $\hat{\theta}_i^{\text{ba}}$ over the 1,000 simulations as

$$\text{RMSE}_i = \sqrt{\frac{1}{1000} \sum_{s=1}^{1000} (\hat{\theta}_i^{\text{ba}(s)} - \bar{\theta}_i)^2}, \quad \text{RMSSE}_i = \sqrt{\frac{1}{1000} \sum_{s=1}^{1000} [\text{SE}(\hat{\theta}_i^{(s)})]^2},$$

where $\text{SE}(\hat{\theta}_i^{(s)})$ is the GLS standard error of $\hat{\theta}_i$ in trial s . The results shows that the ratios RMSE/RMSSE are quite close to one for the intercepts α_c , but are far above unity for the speed-of-adjustment coefficient β , suggesting that the conventionally-computed standard error for this parameter grossly understates its true sampling variability. We thus correct the standard error using a similar approach -- by multiplying the GLS standard error (SE) by the ratio RMSE/RMSSE . That is, if for the parameter θ_i , this ratio is defined by $\phi_i = \text{RMSE}_i / \text{RMSSE}_i$, we define the adjusted standard error as $\text{ASE}(\hat{\theta}_i) = \phi_i \times \text{SE}(\hat{\theta}_i)$. To evaluate the performance of this correction, we carry out another Monte Carlo simulation. In trial s of the 1,000 experiments, we multiply the GLS standard error by the ratio ϕ_i and then compute the root-mean-squared adjusted standard error as $\text{RMSASE}_i = \sqrt{(1/1000) \sum_{s=1}^{1000} [\text{ASE}(\hat{\theta}_i^{(s)})]^2}$. It is found that the

RMSASEs are very close to the corresponding RMSEs, suggesting that the correction procedure works satisfactorily.

The bias-adjusted estimates and their corrected standard errors are presented in Panel A of Table 3. Note that the estimates of β in columns 2 and 3 of Table 3 are fairly close to each other, while it is somewhat lower in the GDP-weighted case under CFM (column 4). In addition, a comparison of the corrected standard errors shows that the estimates based on BSI are not as efficient as those based on CFM. Therefore, in subsequent sections we use the estimates derived from CFM using trade weights (column 3) for further analysis.

To test whether the speed-of-adjustment parameter β is equal to one, we again use the t statistic $\tau = (\hat{\beta} - 1) / SE(\hat{\beta})$. As the distribution of τ is non-standard under the null, we derive critical values by using a Monte Carlo approach. We first obtain proportionate bias and the ratio applicable under the null (denoted by p_i^{Null} and ϕ_i^{Null} respectively) as they are different from those above. In each trial s ($s = 1, \dots, 1000$), the real exchange rates are computed from equation (1) with $\alpha_c = \hat{\alpha}_c^{\text{ba}}$ ($c = 1, \dots, 16$), $\beta = 1$ and multivariate normal error terms. Equation (1) is then re-estimated by GLS and the estimates and standard errors are adjusted using p_i^{Null} and ϕ_i^{Null} . This procedure is replicated 1,000 times and the sampling distribution of the test statistic τ under the null is obtained. Panel B of Table 3 presents the test statistics and the critical values derived from the simulation. It can be seen from column 2 that under BSI, the unit-root null cannot be rejected at the 10 percent level. For the CFM, the unit-root null is rejected at the 5 percent level no matter whether trade or GDP weights are used (columns 3 and 4). An examination of test power shows that the unit root null tends to be rejected only when a majority of real exchange rates series is stationary.¹⁵ When the null is not rejected, the low power of the tests is to blame. These results point to the conclusion that real exchange rates are stationary.

¹⁵ We examine the power of the tests through the design of two sets of alternative data-generating possibilities: (A) All Rates Stationary, whereby all exchange rates are assumed to be stationary, but the cross-sectional dimension of the panel successively rises from 1 to 2 to 3 ... to 16. The values of parameters for each rate are set equal to the estimates presented in Panel A of Table 3. (B) Sub-Sets of Rates Stationary. The first k ($k \leq 16$) series are stationary, and their speed of adjustment is set equal to the estimated value, whereas the remaining $16 - k$ are non-stationary and their speed-of-adjustment is thus one. It is found that when all rates are stationary, the power of the tests reach 95 percent when there are at least 3 (CFM) or 14 rates (BSI). When subsets of rates are stationary, the power of tests does not exceed 95 percent unless all rates are stationary. For details, see Lan (2001).

TABLE 3
BIAS-ADJUSTED ESTIMATES OF EXCHANGE RATE EQUATIONS
AND UNIT ROOT TESTS

$$q_{ct} = \alpha_c + \beta q_{c,t-1} + u_{ct}$$

(Standard errors in parentheses)

Parameter/ Test statistic	Covariance matrix specified as					
	Block sectional independence	Common factor model				
(1)	(2)	Trade-based		GDP-based		
(1)	(2)	(3)	(3)	(4)	(4)	
A. GLS estimates						
Intercept α_c						
Australia	-10.13	(3.88)	-9.93	(2.86)	-11.50	(2.79)
Belgium	13.06	(5.16)	13.13	(4.13)	15.39	(4.35)
Britain	8.31	(3.32)	8.47	(2.95)	9.62	(3.02)
Canada	-5.12	(3.13)	-5.04	(2.55)	-5.70	(2.54)
Denmark	21.97	(8.20)	22.23	(6.06)	26.41	(6.40)
France	12.63	(5.48)	12.80	(4.31)	15.29	(4.39)
Germany	8.22	(4.33)	8.14	(3.65)	9.81	(3.73)
Hong Kong	-25.20	(8.07)	-25.44	(4.85)	-30.08	(5.52)
Italy	7.66	(4.86)	7.81	(4.21)	9.51	(4.54)
Japan	6.47	(7.43)	6.92	(7.36)	8.91	(7.62)
Netherlands	9.24	(4.52)	9.41	(3.88)	11.07	(3.81)
Singapore	-6.63	(7.34)	-6.67	(6.98)	-7.69	(6.74)
South Korea	1.80	(3.13)	1.81	(2.80)	3.51	(3.70)
Russia	-15.34	(37.04)	-13.60	(29.79)	-17.59	(38.15)
Spain	6.81	(4.41)	6.91	(4.14)	8.45	(4.18)
Sweden	16.18	(7.19)	16.49	(5.42)	20.27	(6.27)
Speed-of-adjustment β	61.66	(28.79)	60.80	(17.69)	53.27	(19.87)
B. Monte Carlo tests						
Test statistic	-1.33		-2.22		-2.35	
Critical value 1%	-7.09		-2.41		-3.17	
5%	-2.83		-1.41		-1.65	
10%	-1.66		-1.05		-1.17	

Note: The entries in panel A are to be divided by 100.

6. Equilibrium Exchange Rates, Part I: Theory

In this section, we draw on the conceptual framework of MacDonald and Stein (1999) to examine the role of PPP in determining the equilibrium exchange rate. In the next section, we derive long-run equilibrium exchange rates for the 16 Big Mac countries.

Recall from Section 3 that the real exchange rate is defined as $q = p - s - p^*$, where lowercase letters denote natural logarithms. Under absolute PPP, $S = P/P^*$ and q is zero. Consider the situation when PPP is subject to a zero-mean shock, e , so that $q = p - s - p^* + e$. In this case, as $E(q) = E(p - s - p^* + e) = 0$, absolute PPP now implies that the average, or equilibrium, real exchange rate is zero. A weaker version of the above is to allow systematic deviations from absolute PPP, i.e., $P = K S P_t^*$ or $S = (P/P^*)/K$, where K is a constant. In logarithmic form, we have $s = p - p^* - k$. Adding a zero-mean disturbance term gives

$$(2) \quad s = p - p^* - k + e.$$

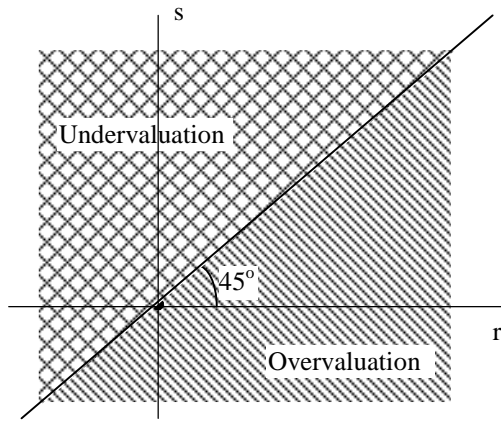
As $E(e) = 0$, k is interpreted as the equilibrium real exchange rate, while the term e can be identified with the transitory, or disequilibrium, component. Note also that the actual real exchange rate is $q = k - e$. Equation (2) encompasses the three versions of PPP:

- (i) The strictest version of PPP, absolute PPP, is when $k = e = 0$, so that $s = p - p^*$.
- (ii) Relative PPP holds when $k \neq 0$ and $e = 0$.
- (iii) Stochastic departures from relative PPP pertain when $k \neq 0$ and $e \neq 0$.

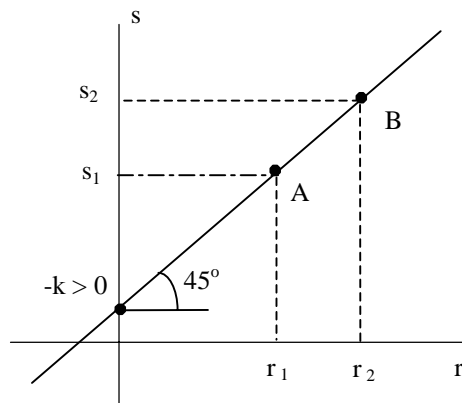
Figure 3 illustrates the above framework by plotting the nominal exchange rate s against relative prices, $r = p - p^*$. The three panels correspond to the three versions of PPP above. Panel A case presents the case when $k = e = 0$, so that the 45-degree line passing through the origin corresponds to absolute PPP. Any combination of s and r that lies above the line implies an undervaluation of the home country currency, while points below represent overvaluation. Panel B allows $k \neq 0$ and $e = 0$, which is relative PPP. Here the 45-degree line does not pass through the origin, but still an increase in the relative

FIGURE 3
EXCHANGE RATES AND RELATIVE PRICES

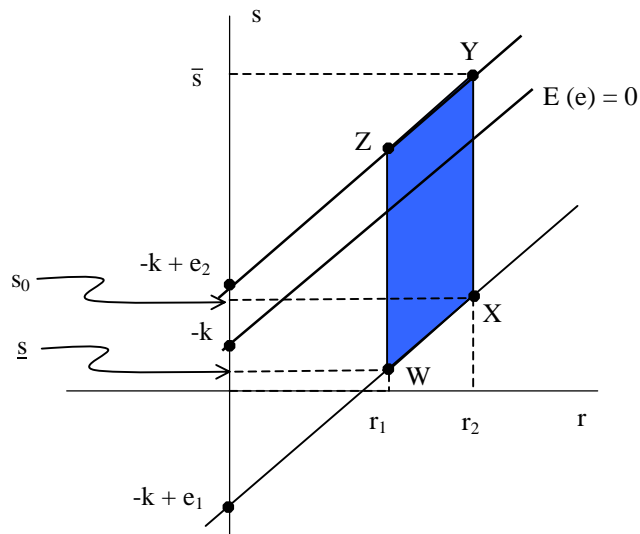
A. Absolute PPP



B. Relative PPP



C. Stochastic Deviations from Relative PPP



price leads to an equi-proportional depreciation of the currency, as is illustrated by the movement from the point A to B, whereby $s_2 - s_1 = r_2 - r_1$. The central line in Panel C corresponds to relative PPP and is the centre-of-gravity relationship when there are stochastic shocks in the short run. Suppose for simplicity that e is a discrete random variable and that $e_1 < 0$ and $e_2 > 0$ are its only possible values. When the shock is $e_1 < 0$, we obtain a new lower, 45-degree line, which has an intercept of $-k + e_1$; similarly, $e_2 > 0$ results in the upper line in Panel C. Consider the situation in which the exchange rate is \underline{s} and relative prices r_1 , so we are located at the point W in Panel C. If there is now the same increase in relative prices as before, from r_1 to r_2 , then, in the presence of the shock e_1 , we move to the point X with the rate depreciating to s_0 . With the shock e_2 , the same relative price r_2 leads to an exchange rate of \bar{s} , as indicated by point Y. More generally, if relative prices change within the range $[r_1, r_2]$ and if the shocks can now vary continuously within the range $[e_1, e_2]$, then the exchange rate lies somewhere in the shaded parallelogram WXYZ. It is to be noted that the height of this parallelogram exceeds its base and the range of the exchange rate, $\bar{s} - \underline{s}$, exceeds that of prices, $r_2 - r_1$. This “overshooting” accords with the idea that in the short run exchange rates are considerably more variable than relative prices. This contrasts with the situation in Panels A and B whereby the exchange rate is proportional to prices.

Much of what follows is devoted to estimating the equilibrium real exchange rate k for the 16 Big Mac countries.

7. Equilibrium Exchange Rates, Part II: Measurement

In this section, we compute equilibrium exchange rates for the 16 countries. Recall from Section 4 that the evolution of the real exchange rate for country c from year $t-1$ to t is described in equation (1). We proceed by taking expectation of equation (1). As $E(u_{ct}) = 0$ and noting that in the long run $E(q_{ct}) = E(q_{c,t-1}) = q_c^E$, it follows that the equilibrium exchange rate (EER) for country c is

$$(3) \quad q_c^E = \frac{\alpha_c}{1 - \beta}.$$

We use the estimates given in Table 3 to estimate the EERs and column 3 of Table 4 gives the results pertaining to the common factor model using trade weights; estimates using alternative covariance structures and weights are similar and are reported in Lan (2001). A positive (negative) number implies the equilibrium value of a currency is below (above) the ratio of domestic to foreign prices; i.e., in the long run, the currency is estimated to be over- (under-) valued. Column 2 in Table 4 gives the average of the real exchange rates over the ten years for each country, reproduced from the last column of Table 2 with two decimal places added. Compared to column 3, these “naive” estimates of EERs are not too far away from their GLS counterparts, except for South Korea and Russia. This suggests that the GLS estimation procedure does have some merit in picking up information that is missed by simple averaging.

The estimated EER is obtained by replacing the unknown parameters in equation (3) by their estimates, $\hat{q}_c^E = \hat{\alpha}_c^{ba} / (1 - \hat{\beta}^{ba})$, which involves a ratio of estimated parameters. Under normality, these ratios are typically not normally distributed and do not possess finite moments; see, e.g., Bewley and Fiebig (1990), Chen (1999) and Zellner (1978). Under such circumstances, the standard application of asymptotic theory can be risky and misleading. Accordingly, to measure the sampling variability of \hat{q}_c^E we employ Monte Carlo simulation.

We first generate error terms corresponding to the disturbance u_{ct} in equation (1), using a multivariate normal distribution. Then we use these generated errors, the value of simulated real exchange rates in the previous period, and data-based bias-adjusted estimates to form the simulated value of the current-period exchange rates as the dependent variables. Equation (1) is then re-estimated with the simulated data to yield the trial s estimates, $\alpha_c^{ba(s)}$ and $\beta^{ba(s)}$. These estimates are then used in equation (3) to yield the estimated EER for trial s , $q_c^{E(s)} = \alpha_c^{ba(s)} / (1 - \beta^{ba(s)})$. We perform 1,000 trials and obtain the mean $\bar{q}_c^E = (1/1000) \sum_{s=1}^{1000} q_c^{E(s)}$. In addition, we compute the root-mean-squared error (RMSE) for the EER of country c as $RMSE = \sqrt{1/1000 \sum_{s=1}^{1000} (q_c^{E(s)} - \bar{q}_c^E)^2}$. The simulation results, based on trade weights under CFM, are contained in columns 4–5 of Table 4.

TABLE 4
EQUILIBRIUM EXCHANGE RATES

Country	Mean of raw real exchange rate		Equilibrium exchange rate	Equilibrium rate from 1,000 Monte Carlo experiments					
				Real			Nominal		
				Mean	RMSE	p-value	Currency unit	Mean	RMSE
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
Australia	-22.57	(2.33)	-25.33	-26.22	7.37	.00	A\$	1.39	.11
Belgium	29.38	(3.95)	33.49	34.95	10.44	.00	Bfr	32.32	3.34
Britain	17.52	(2.83)	21.61	22.08	7.76	.00	£	0.59	.04
Canada	-11.42	(2.64)	-12.86	-13.32	6.47	.01	C\$	1.31	.09
Denmark	56.87	(4.08)	56.71	58.50	12.61	.00	Dkr	6.30	.78
France	33.68	(3.47)	32.65	33.57	9.53	.00	Ffr	5.63	.53
Germany	19.67	(3.26)	20.77	22.00	9.15	.01	DM	1.62	.15
Hong Kong	-66.20	(1.02)	-64.90	-67.01	8.84	.00	HK\$	7.82	.71
Italy	20.78	(4.05)	19.92	20.86	10.24	.02	Lire	1488.36	152.24
Japan	23.53	(8.38)	17.65	18.97	17.96	.14	Yen	128.77	23.47
Netherlands	23.34	(3.30)	24.01	24.21	9.09	.00	Fl	1.84	.17
Singapore	-20.08	(6.54)	-17.02	-16.63	17.63	.17	S\$	1.45	.26
South Korea	20.52	(7.59)	4.62	4.89	6.87	.24	Won	958.65	66.15
Russia	-4.20	(24.84)	-34.69	-52.18	92.83	.28	Rouble	7.92	12.87
Spain	18.93	(4.30)	17.63	18.34	9.53	.02	Pta	122.69	11.66
Sweden	45.36	(5.01)	42.07	44.42	12.25	.00	Skr	7.00	.84
Mean	11.57	(7.53)	8.52	7.96	25.50	-	-	-	-

- Notes: 1. Column 2 is based on the last column of Table 2. The figures in parentheses are standard errors of the means.
2. Columns 2 to 5 are to be divided by 100. The units of columns 8 and 9 are the domestic currency cost of \$US1; column 7 gives the domestic currency unit.
3. The nominal exchange rate of Russia is in terms of new roubles applicable since 1998.

As can be seen, the means of the equilibrium exchange rates are fairly close to the “true” values in column 3, indicating unbiasedness. The RMSE is largest for Russia; this is understandable as the Russian rouble has the highest volatility during the estimation period. The ratio of the point estimate of q_c^E to its RMSE provides a test of whether in the long run the country’s currency is over- or under-valued. The t-ratios for Japan, Singapore, South Korea and Russia are less than 1.5, indicating that the EER for these countries are not significantly different from zero. For these countries, it is not possible to reject the hypothesis that absolute PPP holds in the long run. The reverse is true for the remaining 12 countries. (Subsequently, we shall come back to the remaining columns of Table 4).

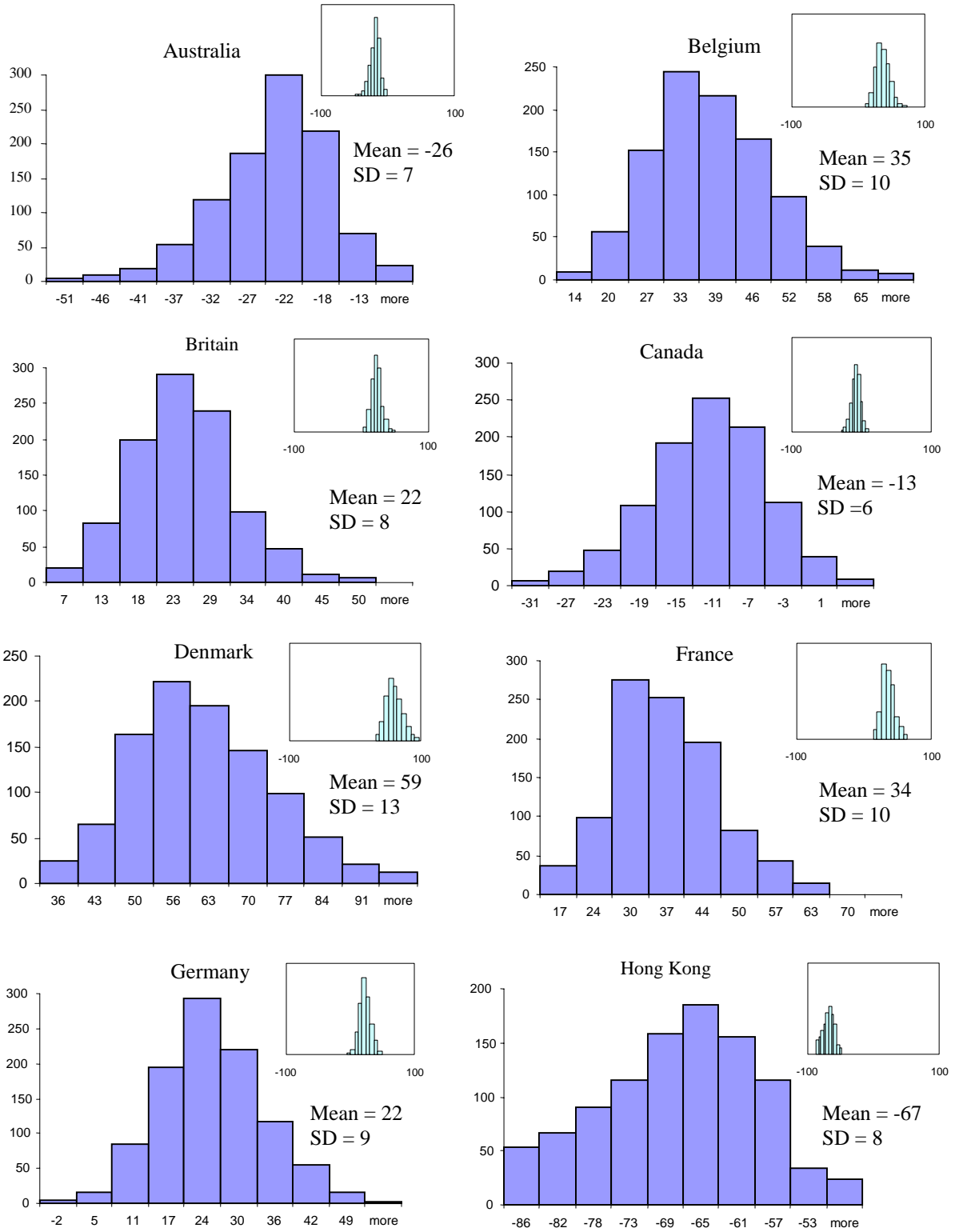
To examine further the properties of the simulated EERs, in Figure 4 we give histograms of the 1,000 simulated values of the EER for each of the 16 countries based on multivariate normal errors. It can be seen that the distributions are unimodal and some display a certain degree of asymmetry (especially for Australia, France and Hong Kong). The top-right corner of the graph for each country presents the same histogram, but now with the scale (of both axes) the same for each country. Therefore, the location and dispersion of the smaller histograms are directly comparable across countries. As the mid-point of the horizontal axis is zero, if a histogram is located towards the right (left) in the smaller graph, then according to the centre of the gravity of the simulation, the country’s currency is over (under)-valued in the long run. If the middle range of a histogram includes zero, the EER is likely to be insignificantly different from zero. This is the case for Japan, Singapore, South Korea and Russia, which agrees with the t-tests of the previous paragraph. Figure 4 also clearly shows again that Russia has the biggest variance, as expected.

For country c , we have 1,000 simulated values of its EER, $q_c^{E(1)}, \dots, q_c^{E(1,000)}$. Suppose that the mean over these 1,000 values happens to be positive; that is $\bar{q}_c^E > 0$. If the number of trails for which the value of $q_c^{E(s)}$ is negative is X , then $X/1,000$ can be considered as an estimate of the p-value for the null hypothesis that in the long run currency satisfies absolute PPP; that is, that $q_c^E = 0$. The procedure is modified in an

FIGURE 4

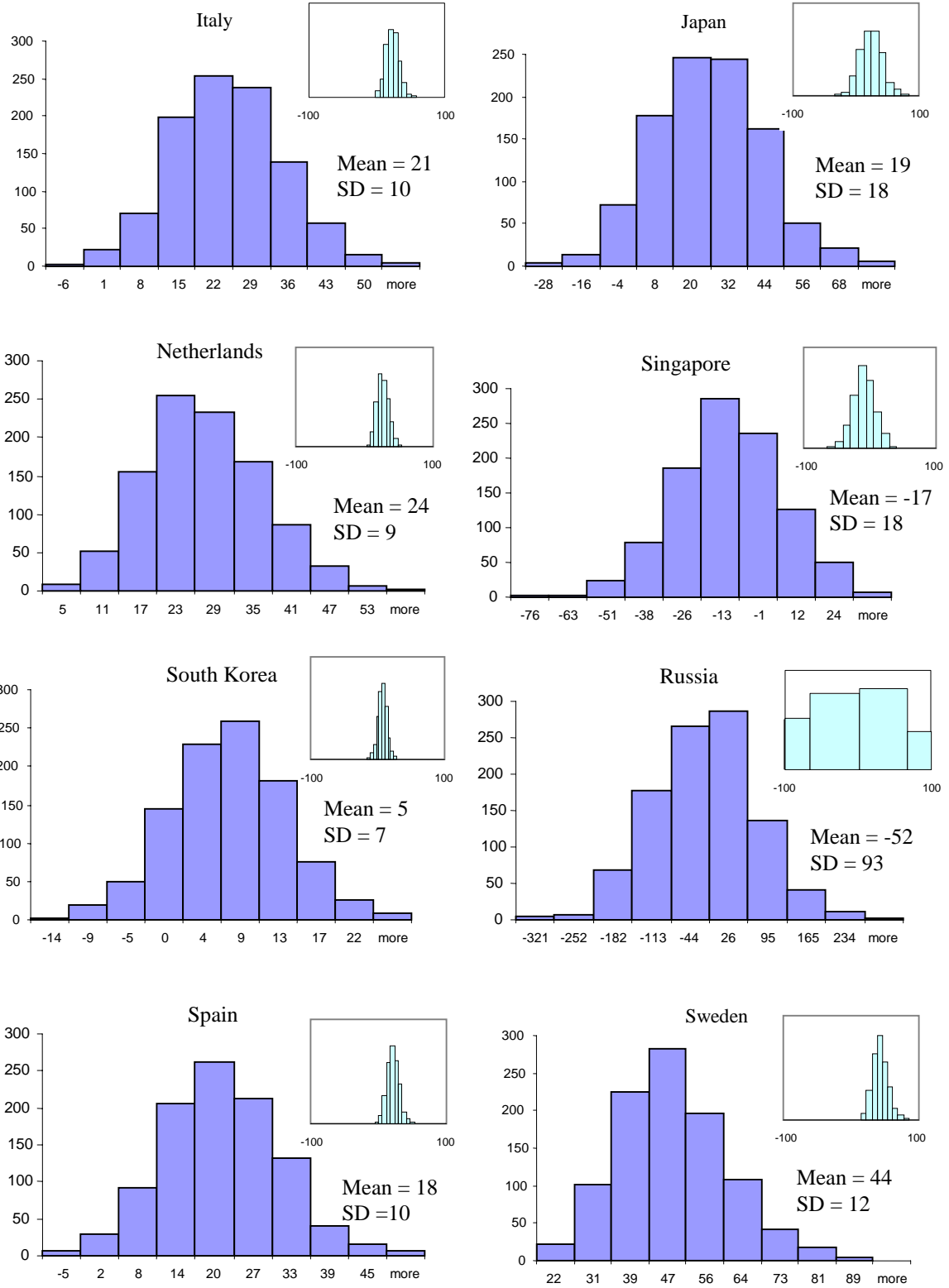
SIMULATED EQUILIBRIUM EXCHANGE RATES

(100 × logarithmic differences of nominal exchange rates from price ratios)



(continued on next page)

FIGURE 4 (continued)
SIMULATED EQUILIBRIUM EXCHANGE RATES



(100 × logarithmic differences of nominal exchange rates from price ratios)

obvious way for $\bar{q}_c^E < 0$. Column 6 of Table 4 present the results. It can be seen that for twelve countries we can reject the null of long-run parity at the 5 percent level. The p-values for the remaining four countries, Japan, Singapore, South Korea and Russia, range from around 14 to near 28 percent. This is consistent with Figure 4, where the histogram ranges for these four countries include zero. (Columns 7-9 of Table 4 are discussed in the next section.)

8. Convergence to Equilibrium

In this section, we illustrate the practical usefulness of the equilibrium exchange rate concept by showing how it can be applied in “real time” to analyse the future time path of the actual rate as it adjusts to the long-run equilibrium value.

The data-generating process of real exchange rate c is described by equation (1) as $q_{ct} = \alpha_c + \beta q_{c,t-1} + u_{ct}$, where $\beta < 1$.¹⁶ By successive substitution, it can be written as

$$q_{ct} = \alpha_c \frac{1-\beta^t}{1-\beta} + \beta^t q_{c1} + \sum_{\tau=1}^t \beta^{t-\tau} u_{c\tau}.$$

Note that $\alpha_c / (1-\beta)$ is the EER for country c , q_c^E . Denoting the number of years ahead of 1998 (the last year in the sample) by j , we can write the real-time version of the above process from the perspective of 1998 as

$$(4) \quad q_{c,1998+j} = q_c^E + (q_{c,1998} - q_c^E) \beta^j + \sum_{\tau=1}^j \beta^{j+\tau} u_{c,1998-\tau} + \sum_{\tau=1}^j \beta^{j-\tau} u_{c,1998+\tau}.$$

¹⁶ This equation also has another interpretation. Write it as an AR(1) process as $x_t = a + b x_{t-1} + \varepsilon_t$, where a and b are constants and ε_t is an error term. This is the discrete-time version of the Ornstein-Uhlenbeck (OU) model. In continuous time, the OU process is $dx = \eta(\bar{x} - x)dt + \sigma dz$, where \bar{x} is the mean of x , dz follows a Wiener process, η is the speed-of-adjustment parameter, and σ is a variance parameter. Given an initial value x_0 , the expected value of x at some future date, t , is

$$(a) \quad E(x_t) = \bar{x} + (x_0 - \bar{x})e^{-\eta t}.$$

Write x_t as the sum of its expected value and an error term (ε_t), $x_t = E(x_t) + \varepsilon_t$. Interpreting x_0 as x_{t-1} , we have $t=1$ in the exponential. Substituting the RHS of equation (a) for $E(x_t)$ yields the AR(1) model $x_t = a + b x_{t-1} + \varepsilon_t$, with $a = (1 - e^{-\eta})\bar{x}$ and $b = e^{-\eta}$. For details, see Dixit and Pindyck (1994, p. 76).

This equation defines the time path of the real exchange rate into the future. There are four components of equation (4):

- The equilibrium exchange rate q_c^E .
- The initial deviation from equilibrium $q_{c,1998} - q_c^E$. Given the speed of adjustment $0 < \beta < 1$, by the year $1998 + j$, this deviation will have declined to $(q_{c,1998} - q_c^E)\beta^j$, a fraction of its initial value. When $j \rightarrow \infty$, $\beta^j \rightarrow 0$ and $q_{ct} \rightarrow q_c^E$.
- The term $\sum_{\tau=1}^9 \beta^{j+\tau} u_{c,1998-\tau}$. As $u_{c,1997}, \dots, u_{c,1989}$ are the disturbances within the sample period, this term is a weighted sum of these disturbances. The weight $\beta^{j+\tau}$ is accorded to the disturbance $(j+\tau)$ years before the year $1998 + j$ and serves to dampen the impact of this disturbance on the value of the exchange rate in the year $1998 + j$.
- The term $\sum_{\tau=1}^j \beta^{j-\tau} u_{c,1998+\tau}$. This is the analogous weighted sum of disturbances in the forecasting period. The weight $\beta^{j-\tau}$ is given to the disturbance $(j-\tau)$ years before the year $1998 + j$. In particular, when $\tau = j$, the weight is $\beta^0 = 1$, so that the “contemporaneous” shock is given full weight in determining the forecast of that year’s exchange rate.

It is to be noted that a smaller value of β means that the convergence of $q_{c,1998+j}$ to q_c^E is faster. Equation (4) implies that if the deviation is positive (negative) at the end of 1998, i.e., $q_{c,1998}$ lies above (below) q_c^E , the time path of $q_{c,1998+j}$ will always be negatively (positively) sloped as $q_{c,1998+j}$ approaches q_c^E .

We derive the adjustment paths using a Monte Carlo approach similar to that employed above. Given the bias-adjusted estimate of the speed-of-adjustment coefficient of .53 - .62 from Section 5, the half-life of the real exchange rate is between 1.1-1.4 years.¹⁷ This means that real exchange rates will converge to their long-run equilibrium values fairly quickly, so we only look at six years ahead of 1998 (the last year in the sample), from 1999 to 2004 (i.e., $j = 1, \dots, 6$). In trials of the 1,000 simulation experiments, we draw $q_{c,1998-\tau}^{(s)}$ ($\tau = 1, \dots, 9$) and $u_{c,1998+\tau}^{(s)}$ ($\tau = 1, \dots, j$) for the 16 countries to generate real exchange rates. We then obtain bias-adjusted GLS estimates of equation (1), $\alpha_c^{ba(s)}$ and $\beta^{ba(s)}$, and compute $q_c^{E(s)}$. Finally, $q_{c,1998+j}^{(s)}$ is calculated via equation (4). This procedure is replicated 1,000 times. We finally obtain the mean

¹⁷ The concept of half-life originates in physics and measures the time taken by a large number of identical particles to decay to half its mass. It is an alternative measure of the speed-of-adjustment of the process to a shock. The relationship between the half-life H and speed-of-adjustment β is $H = -\log 2 / \log \beta$.

$\bar{q}_{c,1998+j}$ and the $2\frac{1}{2}$ and $97\frac{1}{2}$ percentiles of the 1,000 simulated values, denoted as $q_{c,1998+j}^L$ and $q_{c,1998+j}^U$.

Recall that the real exchange rate is defined in terms of natural logarithms, i.e., $q_{ct} = \log(P_{ct}/S_{ct}P_t^*)$. As it is more convenient to examine the paths of future exchange rates in terms of currency units, we convert the logarithmic value back to the domestic currency cost of one US dollar. Dropping the country subscript for convenience, we have

$$(5) \quad q_t = p_t - s_t,$$

where $p_t = \log(P_t/P_t^*)$ and $s_t = \log S_t$. Given an estimate of q_t for some year in the future, q_{1998+j} , the problem is to then infer the value of s_{1998+j} . As the future value of relative price p_{1998+j} is unknown, we treat this as a signal extraction problem, which originates from Lucas (1973). That is, we form the optimal forecast of p_{1998+j} conditional on q_{1998+j} and the past history of these variables. We assume that (i) p_t has mean \bar{p} and variance σ_p^2 ; (ii) s_t has mean \bar{s} and variance σ_s^2 ; and that (iii) p_t and s_t are orthogonal, i.e., $\text{cov}(p_t, s_t) = 0$.¹⁸ We wish to find an optimal forecast of p_{1998+j} , \hat{p}_{1998+j} , based on the available information I_{1998+j} , which consists of q_{1998+j} , \bar{p} and \bar{s} . That is, we seek to obtain $\hat{p}_{1998+j} = E(p_{1998+j} | I_{1998+j}) = E(p_{1998+j} | q_{1998+j}, \bar{p}, \bar{s})$, to infer the future values of the nominal exchange rate s_{1998+j} . In Lan (2001), we show that minimising the mean squared forecast error yields

$$(6) \quad \hat{p}_{1998+j} = (1-\lambda)\bar{p} + \lambda(\bar{s} + q_{1998+j}),$$

where $\lambda = \sigma_p^2 / (\sigma_p^2 + \sigma_s^2)$. As is well known, relative prices are much less variable than exchange rates (see, e.g., Mussa, 1986); i.e., the ratio σ_p / σ_s is likely to be very small. Thus, it is natural to consider the situation in which this ratio tends to zero. Write $\sqrt{\theta} = \sigma_p / \sigma_s$, so that $\lambda = \theta / (1+\theta)$. Then, $\lim_{\theta \rightarrow 0} \lambda = 0$. In words, the limiting value of the coefficient λ is zero when the variance of relative prices becomes very small (relative to the variance of the exchange rate). Equation (6) is thus simplified to $\hat{p}_{1998+j} = \bar{p}$, and equation (5) for the year $1998 + j$ becomes

$$(7) \quad q_{1998+j} = \bar{p} - s_{1998+j} .$$

¹⁸ The justification for the orthogonality of the exchange rate and the relative price is the empirical regularity that over the short term, exchange rates seem to be more or less unrelated to relative prices.

When the relative price is taken to be a constant, there is a one-for-one co-movement in real and nominal exchange rates. Such result is consistent with the findings on the behaviour of exchange rates in the short run (see, e.g., Obstfeld, 1995). Based on equation (7), we have $s_{1998+j} = \bar{p} - q_{1998+j}$, or in terms of domestic currency units,

$$(8) \quad S_{1998+j} = e^{\bar{p} - q_{1998+j}}.$$

We convert real rates to their nominal counterparts by using equation (8) as follows.¹⁹ Firstly, the 1,000 values of equilibrium rates q_c^E are used to compute 1,000 values of S_c^E by using $S_c^E = e^{\bar{p} - q_c^E}$. The means and RMSEs of the 1,000 values of S_c^E are contained in columns 8 and 9 of Table 4. Secondly, we compute the mean over the 1,000 simulated values of the future nominal exchange rate for country c , $\bar{S}_{c,1998+j}$, from $\bar{q}_{c,1998+j}$. These define the future time path and are indicated in Figure 5 by the middle curve in each panel. Finally, the upper and lower bounds of the 95 percent confidence interval are obtained from the nominal versions of $q_{c,1998+j}^U$ and $q_{c,1998+j}^L$. These are the upper and lower curves in each panel of Figure 5. Note that the three curves in each panel start with the same point -- the actual nominal exchange rate in 1998. It can be seen from Figure 5 that²⁰

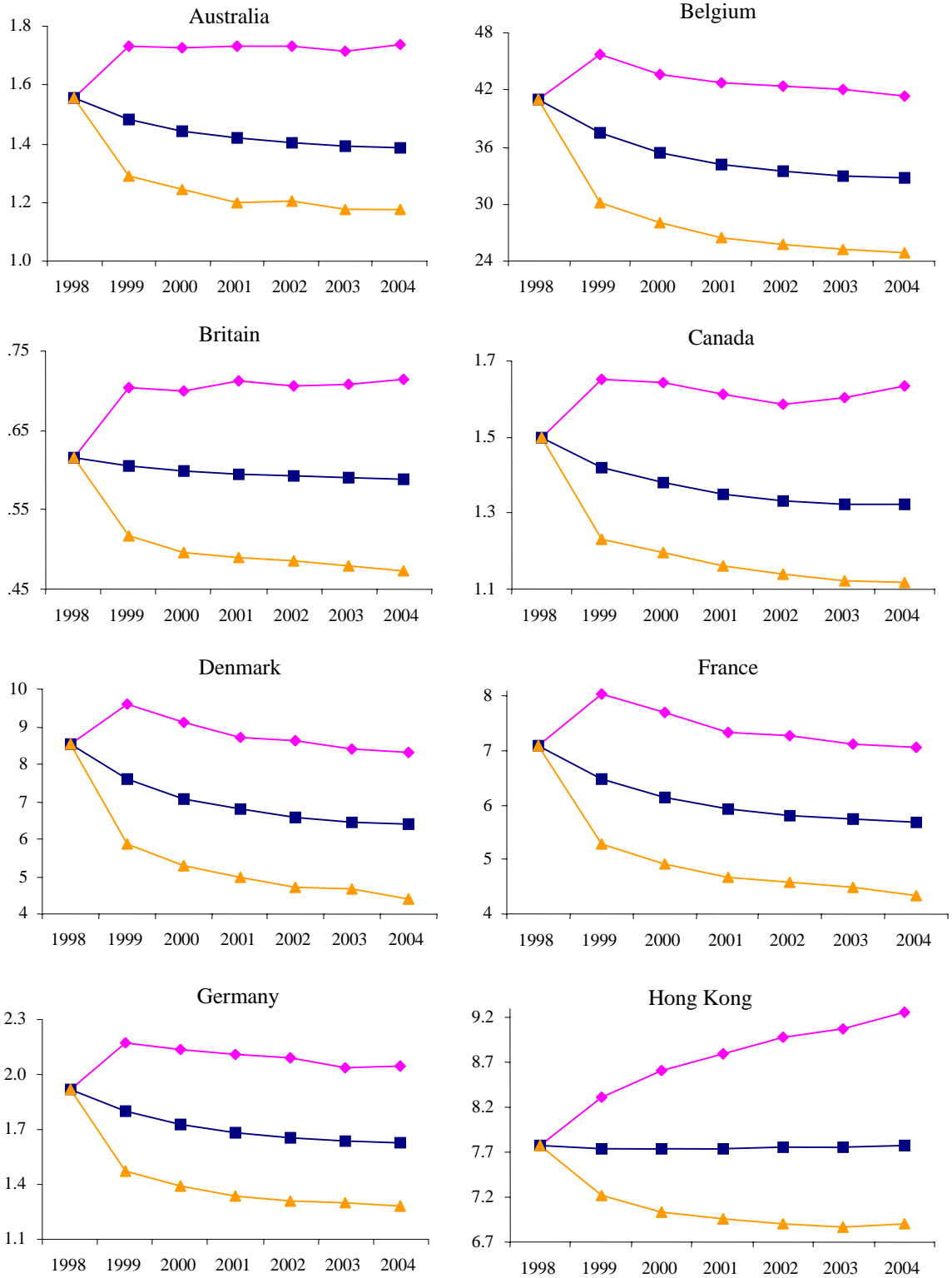
- In 1998, the nominal exchange rate in most countries is undervalued compared to their long-run equilibrium, except for Hong Kong and Russia. Thus as the rates adjust to their long-run equilibrium values, they mostly appreciate, so that the time paths are negatively sloped.
- The confidence intervals slightly widen as time moves further into the future. This makes sense as the more distant future is usually more uncertain.
- The confidence bands of most currencies are more or less symmetric, except for Russia.²¹

¹⁹ The relative price for Russia requires a special treatment. First, as the price level in that country for the first three years of the sample period seems to be unrealistically low, we compute its \bar{p} over the remaining seven years. Second, as the post-January 1998 rouble is equal to 1,000 old roubles, we divide the average relative price of Russia by 1,000 when using equation (8).

²⁰ It is worthwhile noting two issues here: (1) Currencies in the euro area are fixed vis-à-vis each other after 1999, but not with respect to the US dollar. Another reason for these currencies to have different under/overvaluations is because Big Mac prices vary within Euroland. Accordingly, for these countries the EERs and adjustment paths are not the same. (2) Hong Kong has (since 1983) operated under a currency board system whereby its exchange rate is fixed at \$HK7.80/\$US1. While there is no compelling reason for its real exchange rate to be constant, this in part explains why its EER has a relatively small standard error (8 percent), compared to other countries. As the deviation of \$HK from its EER in 1998 is quite small, its adjustment path is almost horizontal, as indicated by Figure 5.

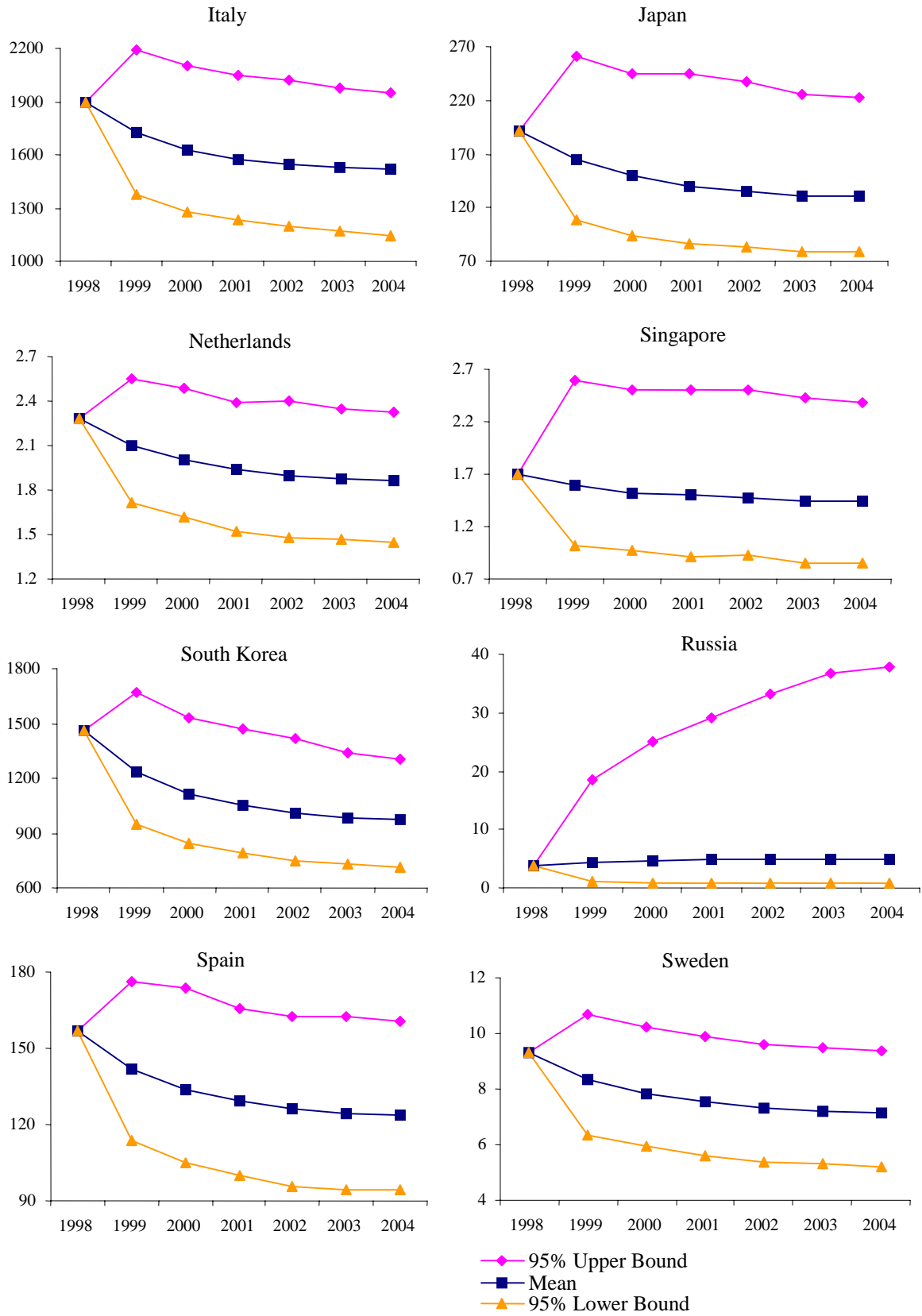
²¹ The asymmetry in the confidence bands in Figure 5 pertaining to Russia simply reflects the large standard deviation of its estimated EER. As $q = \log(P/SP^*)$, $q \in (-\infty, +\infty)$; but when we convert back to the nominal rate for the forecasts via equation (8), as $e^x \geq 0$ the range is necessarily asymmetric.

FIGURE 5
 FUTURE TIME PATHS OF EXCHANGE RATES
 (Domestic currency cost of \$US1)



(continued on next page)

FIGURE 5 (continued)
 FUTURE TIME PATHS OF EXCHANGE RATES
 (Domestic currency cost of \$US1)



Before concluding this section, as we now have realised exchange rates for April, 1999-2001, it is appropriate to say a few words about the *ex post* quality of the forecasts. Table 5 gives a comparison of actual and forecast exchange rates. It is fair to say that the point forecasts are not too close to the corresponding realisations. However, most are contained within the 95 percent confidence bands, especially for 1999 and 2000 when there is only one forecast outside the band each year. For the year 2001, 50 percent of the actual rates are outside (more precisely, above) their respective upper bound. These forecasting errors serve to remind us of the difficulties in forecasting nominal exchange rates.

To further analyse the discrepancy between the actual and forecast exchange rates, we compute the logarithmic forecast error for country c as $d_c = \log(\text{actual}_c / \text{forecast}_c)$. The results are given in column 5, 9 and 13. It can be seen that the differences are generally under 15 percent, except for Russia. Due to the great uncertainty regarding the estimate of Russia's EER and hence the forecasts of its nominal rates, we compute the weighted average of d_c with Russia included and excluded, $\bar{d}_c = \sum_c w_c d_c$, the weight w_c being the trade share of country c in the total for the group. The third- and fourth-to-last rows of Table 5 present \bar{d}_c for the years 1999-2001. It can be seen that (1) the inclusion of Russia increases the mean error by about 2 percent; (2) the actual rates are, on average, fairly close to their forecasts for the year 1999, but the discrepancies widen in the subsequent two years; and (3) the errors for 2001 are all positive, indicating that currencies are worth less than forecasted. This is clear evidence that the US dollar continued to be overvalued into 2001. The last two rows of Table 5 measure the dispersion of errors in the form of the weighted root-mean-squared error of the differences, $\sqrt{\sum_c w_c (d_c - \bar{d})^2}$. As can be seen, when Russia is included, the dispersion of the errors is between 9 and 14 percent and largest for 2000. The exclusion of Russia decreases the dispersion by 4-7 percent.

Even though our forecasts of nominal exchange rates are not entirely accurate, especially as the horizon becomes more distant, in the next section, we show that our estimated EERs are quite close to those obtained from other studies.

TABLE 5
A COMPARISON OF ACTUAL AND FORECAST EXCHANGE RATES

Country and currency units	1999				2000				2001				
	Actual	Forecast	Is actual in 95 percent confidence band?	Logarithmic error × 100	Actual	Forecast	Is actual in 95 percent confidence band?	Logarithmic error × 100	Actual	Forecast	Is actual in 95 percent confidence band?	Logarithmic error × 100	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	
Australia	A\$	1.56	1.48	√	2.29	1.68	1.44	√	6.69	1.99	1.42	x	14.66
Belgium	Bfr	37.71	37.45	√	.30	42.69	35.40	√	8.13	45.18	34.23	x	12.05
Britain	£	.62	.61	√	.71	.63	.60	√	2.12	.70	.60	√	6.69
Canada	C\$	1.49	1.42	√	2.09	1.47	1.38	√	2.74	1.56	1.35	√	6.28
Denmark	Dkr	6.95	7.61	√	-3.94	7.88	7.08	√	4.65	8.36	6.78	√	9.10
France	Ffr	6.13	6.49	√	-2.48	6.94	6.15	√	5.25	7.35	5.94	x	9.25
Germany	DM	1.83	1.80	√	.72	2.07	1.73	√	7.79	2.19	1.69	√	11.26
Hong Kong	HK\$	7.75	7.74	√	.06	7.79	7.73	√	.34	7.80	7.74	x	.34
Italy	Lire	1,810	1,725	√	2.09	2,049	1,627	√	10.02	2,169	1,573	x	13.95
Japan	Yen	120	164	√	-13.57	106	149	√	-14.79	124	120	√	1.42
Netherlands	Fl	2.06	2.11	√	-1.04	2.33	2.00	√	6.63	2.47	1.94	x	10.49
Singapore	S\$	1.71	1.59	√	3.16	1.71	1.52	√	5.12	1.81	1.50	√	8.16
South Korea	Won	1,205	1,233	√	-1.00	1,110	1,114	√	-1.16	1,327	1,049	√	10.21
Russia	Rouble	15.01	4.33	x	53.99	28.08	4.57	√	78.85	28.62	4.86	√	77.00
Spain	Pta	156	142	√	4.08	176	134	x	11.84	186	129	x	15.89
Sweden	Skr	8.33	8.34	√	-.05	8.74	7.82	√	4.83	10.20	7.52	x	13.24
Weighted mean	-- Russia included				-.07				4.59				10.28
	-- Russia excluded				-1.13				2.82				8.69
Weighted RMSE	-- Russia included				8.99				13.70				11.21
	-- Russia excluded				4.92				7.56				4.45

Notes: 1. The actual exchange rates in columns 2, 6 and 10 refer to monthly averages of the exchange rates in April of the corresponding year and are obtained from the University of British Columbia web site (<http://pacific.commerce.ubc.ca/xr/data.html>).

2. We use trade shares (imports plus exports) as weights, with the underlying data from International Monetary Fund International Financial Statistics. As the data for the year 2001 are not available, we use those for 2000.

9. Comparison with Other Studies

In this section, we compare our estimates of the speed-of-adjustment parameter and equilibrium exchange rates with those from other studies.

Panel A of Table 6 provides a summary of the estimates of β from previous sections of this paper and other Big Mac research. It can be seen that the bias-adjusted estimates from this paper fall in the range .53 – .62, indicating a half-life of 1.1 – 1.4 years. This relatively short half-life is reassuring as it implies a fairly rapid mean-reversion process. The bias-adjusted estimate from Cumby (1996) is reasonably close to ours and suggests a slightly lower half-life. In Panel B of Table 6, we present the speed-of-adjustment estimates derived from non-Big-Mac research on PPP. These estimates are usually more than .7, implying a half-life of around two years or more. The difference in adjustment speeds is marked and its explanation warrants further research.

As summarised in Montiel (1999), there are three approaches to estimate equilibrium exchange rates (EERs): (1) A relative PPP-based methodology; (2) a trade-equations, or a macroeconomic balance, approach; and (3) a general-equilibrium approach. Our use of the Big Mac data adds a new dimension to the first approach. How close are our estimates to those obtained from more complicated techniques? Table 7 makes a comparison of our estimates of the long-run equilibrium exchange rate, with similar concepts, viz., the fundamental equilibrium exchange rate (FEER) and the behavioural equilibrium exchange rate (BEER). Note that the FEERs in columns 3 and 7 refer to the years 1990 and 1995 respectively, and the BEERs in column 5 correspond to the year 1990. As the years to which the FEERs and BEERs apply are contained in our sample, we can make a direct comparison. We first compare our Big Mac estimates of the EER (column 2) with the FEERs in Williamson (1994) (column 3) for German mark and Japanese yen. It can be seen that Williamson's FEERs for both countries are different from our EER estimates by less than 10 percent (column 4). Our EERs are strikingly close to the BEERs in Clark and MacDonald (1998) for these two countries (columns 5 and 6). From columns 7 and 8, it can be seen that the FEERs for the five industrial countries of Wren-Lewis and Driver (1998) are less than 11 percent (in absolute value) different from our EERs, except for Japan. Including Japan, the average difference is an apparently

TABLE 6
ESTIMATES OF THE SPEED-OF-ADJUSTMENT PARAMETER

Estimate (standard errors in parentheses)	Source	Estimation method
<u>A. Big Mac Research</u>		
.42 (.08 - .09)	This paper	LSDV
.53 (.20) to .62 (.29)	This paper	Bias-adjusted GLS
.45 (.12)	Cumby (1996)	Bias-corrected LSDV
<u>B. Selected Non-Big-Mac Research</u>		
.85	Frankel and Rose (1996)	OLS with White (1980) standard errors
.79 to .92	Higgins and Zakrajšek (1999)	SUR-GLS and IPS-GLS
.78 and .89	Lothian and Taylor (1996)	OLS with White (1980) standard errors
.69 to .78	Papell (1997)	FGLS
.85 and .87	Wei and Parsley (1995)	Fixed effects models

- Notes: 1. Estimates of the speed of adjustments in non-Big-Mac research are sometimes reported in terms of the parameter ρ in the equation $\Delta q_{ct} = \alpha + \rho q_{c,t-1} + \varepsilon_{ct}$. In these cases, we transform the estimate of ρ into that of β using $\beta = \rho + 1$.
2. All adjustment speeds are expressed on an annual basis. In those cases where the underlying data are not annual and the parameter estimated is β , we compute the speed of adjustment per annum as β^n , where n is the number of periods per year.

TABLE 7
ALTERNATIVE EQUILIBRIUM EXCHANGE RATES
(Domestic currency cost of \$US1)

Country	Big-Mac estimates	Williamson (1994)		Clark and MacDonald (1998)		Wren-Lewis and Drive (1998)	
		FEER as at 1990	Percentage difference from EER	BEER as at 1990	Percentage difference from EER	FEER as at the first half of 1995	Percentage difference from EER
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Britain	0.59	-	-	-	-	.63	7
Canada	1.31	-	-	-	-	1.39	6
France	5.63	-	-	-	-	5.04	-10
Germany	1.62	1.49	-8	1.66	3	1.44	-11
Italy	1,488	-	-	-	-	1,654	11
Japan	129	117	-9	133	3	90	-30
Mean	-	-	-9	-	3	-	-5

- Notes: 1. Columns 3 and 5 are calculated from Clark and MacDonald (1998, Table 11, p. 32) based on the average of daily actual exchange rates for the first quarter of 1990.
2. Column 7 is taken from Wren-Lewis and Drive (1998, Table 5.4, p. 61).

Modest -5 percent. This brief comparison with more complex methodologies demonstrates the credible performance of the Big Mac Index in providing a new basis for estimating equilibrium exchange rates.

10. Summary and Conclusion

The literature on PPP has been growing rapidly over the past three decades. A prominent recent strand of this research tests for PPP by examining the time-series properties of the real exchange rate with the nominal rate deflated by price indexes based on a broad set of goods, such as the CPI, WPI, or GDP deflators. In this paper, we use real exchange rates in terms of a single good -- a McDonalds' Big Mac hamburger -- to test for PPP. We employ recently-developed panel unit root tests to control for cross-sectional dependence among exchange rates. As the time span of the Big Mac data is relatively short (1989-1998), we face an undersized sample problem. To overcome this problem, some structure is placed on the disturbance covariance matrix. We constrain the speed-of-adjustment parameter in real exchange rate equations to be the same across countries in the SUR setting and a Monte Carlo procedure indicates that this assumption is not rejected by the data. A simple iterative procedure is introduced to adjust for bias and correct standard errors of GLS estimates and it seems to work satisfactorily. We then carry out a multivariate unit root test and considerable evidence is found that real exchange rates exhibit mean reversion, so that the impacts of shocks die out over time. In comparison with much of the previous literature, we also find a faster speed of adjustment toward PPP, i.e., a half-life of 1.1-1.4 years.

As real exchange rates are stationary, they converge to equilibrium values in the long run. We clarify the theoretical concept of the equilibrium exchange rate and then apply it to the 16 Big Mac currencies. We analyse the whole distribution of the estimated equilibrium exchange rates and derive the future time paths of the actual exchange rates adjusting to their equilibrium values through Monte Carlo methods. A comparison of our estimated equilibrium exchange rates with those of others reveals that they are quite close to those derived from more complex methodologies, which further enhances the appeal of the Big Mac approach.

Overall, there are several attractions of our approach to estimating equilibrium exchange rates: (1) It only requires the Big Mac prices and nominal exchange rates, both readily available, as input; (2) the economic structure placed on the problem is simple, viz., each real exchange rate has some well-defined long-run equilibrium value; and (3) the implementation of the approach is relatively easy. The results of this paper lead to the conclusion that the Big Mac approach provides a convenient and valuable way of testing for PPP and estimating equilibrium exchange rates.

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