Modelling Social Infrastructure and Growth

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March 9, 2002

Abstract

This paper examines the impact of social infrastructure on economic growth by endogenously modelling its provision by a public sector in the context of a multi-sector growth model. Our model shows that not only is social infrastructure positively correlated with output per worker, countries that are more efficient in providing the infrastructure are able to limit the level of diversion while those that are not are unable to do so. Next, we augment the model with human capital which is endogenously determined by the education sector. The extended model indicates a positive link between the education and public sectors such that a shock to one of these sectors affects not only the immediate sector but the other as well. We also show that favourable social infrastructure can have positive long-term growth effects when the Lucas (1988) specification for the accumulation of human capital is adopted. Our results suggest that emphasis should be placed on raising the efficiency level of the public sector and productivity level of the education sector. Finally, the best way of combating diversion is to encourage individuals to adopt a higher degree of aversion to it.

1 Introduction

No individuals exist in a vacuum within a society. In every society, we can invariably identify some system that shapes its identity and determines the way individuals live and work. The system can come in the form of cultures such as customs and traditions inherited from our ancestors, or social

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codes and rules created as a result of the complex web of human interactions over long periods of time. The main function of a system, hence, is not only to enable individuals to identify themselves as a distinct community through their cultures but also to govern the way individuals interact and work with each other through social codes and rules. To a large extent, social codes and rules are created to minimise the incidents of diversive activities such as predation, squatting, rent seeking, cronyism and extortion. The fact that every society has its own list of social codes and rules suggests that individuals have a proclivity towards diversion; the degree of proclivity may, however, vary across individuals. Hence, unless some form of social action can be harnessed to counter this predisposition, there will be few or no incentives for production to take place. Authors who have studied the issue of social action and how it affects output per worker include Olson (1965), Olson (1982), Baumol (1990), North (1990), Greif and Kandel (1995), and Weingast (1995). Ideally, the government should be the most efficient agent for social action through its regulatory and legislative power. Ironically, the same power also enables it to be a chief instrument of diversion through expropriation and corruption, for example.

It is important, then, that for an economy to function well, some body of system must exist with the sole purpose of countering diversion. Hall and Jones (1999) defined this body as social infrastructure, which is "the institutions and government policies that make up the economic environment within which individuals and firms make investments, create and transfer ideas, and produce goods and services." In their empirical study involving cross-country regressions, they find that social infrastructure is strongly and positively correlated with rates of capital accumulation, educational attainment, and productivity. An economy with more social infrastructure enables more production of output to take place because diversive activities are discouraged. Hall and Jones (1999) identify two elements that are considered necessary before diversion can be successfully suppressed. The first involves some form of moral education where individuals learn to distinguish between good and evil. The second involves the threat of punishment. It is obvious that unless the threat is credible, no amount of legislation and policies that constitute the threat will suppress diversive activities. The credibility of the threat in turn depends on the level of efficiency in, and the quantity of resources devoted to, the provision of social infrastructure. Hence, there should be some optimal mix of resource allocation such that the right amount of social infrastructure is provided in every period without sacrificing too much of the final good. The optimal mix should depend on the various parameters that characterise the economy, such as the rate of time preference of households and the population growth rate.

This paper contributes to the existing literature of growth theory by extending the single-sector neoclassical growth model formulated by Cass (1965) and Koopmans (1965) to include a public sector with the sole function of providing social infrastructure. Although intangible, we model social infrastructure metaphorically as a stock variable that can be accumulated over time just like physical and human capital. According to the Cass-Koopmans model, the economy can be characterised by an aggregate production function for the final goods sector

$$Y = AK^\alpha L^{1-\alpha},$$

where Y, K and L denote aggregate output, total stock of physical capital and raw labour respectively, and an equation of motion for K

$$\dot{K} = Y - C - \delta K,$$

where C denotes aggregate consumption and δ measures the rate at which K depreciates. The model assumes a total absence of diversion such that the economy always lies on the production possibilities frontier. Note too that the differential equation for K assumes that the flow of savings, $Y - C$, is completely transformed into new physical capital in every period. With the introduction of the public sector, we modify the aggregate production function to

$$Y = AK^\alpha \left(u_Y mL\right)^{1-\alpha},$$

where $u_Y \in (0, 1)$ denotes the fraction of productive time allocated to the final goods sector, and the amount of productive time is measured by $m \in (0, 1)$. Note that time has been normalised to unity. The amount of time used up in diversive activities is therefore measured by $1 - m$. Hence, when no diversion takes place, the value of m becomes one and the economy will be producing on its production possibilities frontier. Any other values of m would mean a loss of potential output. We argue that the size of m depends on the stock of social infrastructure, amongst other things, which the public sector is solely responsible for its provision.

We will use the basic two-sector model to assess several issues such as the relative importance of the level of efficiency in the public sector versus the quantity of resources devoted to it. Next, we extend the basic model to include human capital and examine the link between the education and public sectors in relation to the provision of social infrastructure. Finally, we introduce a model of social infrastructure and human capital with endogenous growth effects. The rest of the paper is divided into five sections. In section 2,

the basic model is developed and explained in detail. In section 3, we present the steps involved in solving the model and discuss the implications of the model. In section 4, we augment the basic model with human capital and discuss the implications of the model. Section 5 introduces the endogenous growth model and examines the relationship between social infrastructure and the steady-state growth rate of the economy. Section 6 concludes the paper.

2   The Basic Model

2.1   Behaviour of a Representative Individual

The basic model consists of a final goods sector and a public sector. The first sector produces goods that can be used for either consumption or investment. The second provides social infrastructure that is aimed at deterring diversive activities. We assume that every individual is identical and engages in some level of diversive activity. We could instead develop a model with two groups of individuals where one is strictly productive and the other strictly diversive. Diversion then leads to wealth transfers from the former to the latter group. We make the assumption that every individual is likely to engage in diversion since we are more interested to examine the impact of social infrastructure on the economy as a whole. This assumption can be rationalised using a game-theoretic approach illustrated by the payoff matrix in Figure 1. The table captures the payoffs for two groups of players, i.e. a representative individual and other individuals, where each group may select from the same set of strategies. These strategies are either to not engage in diversive activities (No Diversion) or to engage in diversive activities (Diversion). The payoffs measure the marginal change to the groups' initial endowments.

There are three possible outcomes. In the first outcome, both groups select the 'No Diversion' strategy. There will be no change to their initial endowments, no diversion will occur and no social costs will be incurred. In the second outcome, one of the groups selects the 'Diversion' strategy while the other stays with the 'No Diversion' strategy. The former will gain $x - \tau$ units while the latter will lose $x + \tau$ units, where $x > 0$ and $\tau > 0$. The term x represents the private gain (or loss) to the group arising from the diversive activity while τ represents an explicit cost imposed on both players as a result of the social cost created by the diversive activity. This explicit cost could, for example, take the form of taxes raised by the government to fund anti-diversive activities. In this situation, diversion leads to a wealth transfer from one group to another. This, however, is not a zero-sum game

	Other Individuals	
	No Diversion	Diversion
A Representative No Diversion	$0 \, , 0$	$-x - \tau \, , x - \tau$
Individual Diversion	$x - \tau \, , -x - \tau$	$-\tau \, , -\tau$

Figure 1: Diversion payoffs matrix

since the two payoffs actually sum to -2τ. We can view the latter term as a measure of the social cost of diversion. Finally, in the third outcome, both players select the 'Diversion' strategy. We assume that the net private gain from diversion for each group is zero, but both groups will still incur the explicit cost of τ units each.

It is clear that the third outcome yields the unique Nash equilibrium with payoffs $(-\tau, -\tau)$ and the dominant strategy for both groups is 'Diversion'. The equilibrium outcome is worse than the case where both choose the 'No Diversion' strategy. This result reflects the famous Prisoners' Dilemma where the impossibility of the two prisoners to monitor each other's behaviour lead them to select the strategy of confession. Here, we assume that the population is large enough such that it is too costly for any given individual to monitor the behaviour of other individuals. Hence, it is not possible to sustain the first outcome as an equilibrium since the incentive to deviate from the strategy of 'No Diversion' to 'Diversion' without being observed is too great for any given individual. The problem of diversion is therefore modelled as a matter of how *much* diversion each individual chooses instead of how *many* individuals choose to engage in diversion.

We define $m \in (0, 1)$ as the fraction of time each individual chooses to engage in market, i.e. non-diversive, activities, where the total time endowment is normalised to unity. Thus, $1 - m$ measures the fraction of time devoted to diversive activities. We shall use $1 - m$ as a measure of the level of diversion. In the basic model, market activities comprise the production of final goods, the transformation of savings into productive capital, and the provision of social infrastructure. For simplicity, we assume that labour is required only in the first and last activity. We model m as a positive function of $q \in (0, 1)$, i.e. the probability that an individual gets caught for diversion. So, the higher the probability, the higher the level of market activities engaged in, and the lower the level of diversion. Specifically,
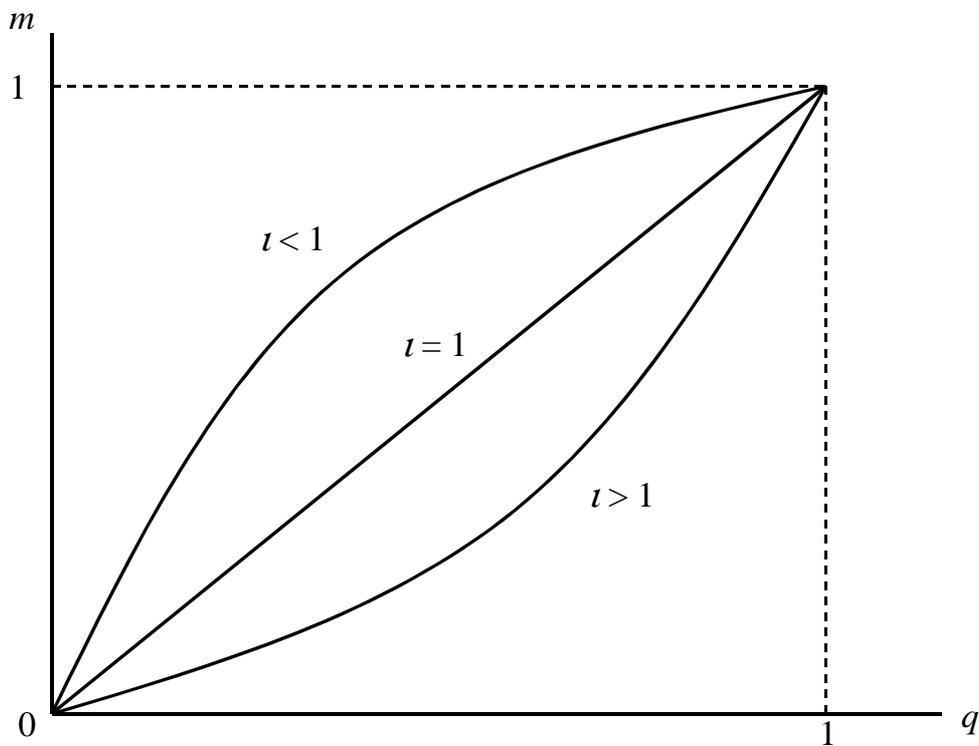
$$m = q^{\iota},\tag{1}$$

Figure 2: The relationship between the probability of getting caught and level of market activities under different values of the diversion aversion parameter

where $\iota > 0$ measures the representative individual's degree of aversion to diversive activities. We shall call this the diversion aversion parameter.

Definition 1 *We describe the representative individual as highly averse to diversion if he or she exhibits no proclivity towards diversion.*

A lower ι means that for a given value of q, individuals will choose a higher level of m and *vice versa* since $q \in (0, 1)$. Hence, low ι values reflect high levels of diversion aversion, and *vice versa*. We illustrate these effects in Figure 2. The figure shows that for low ι values, the level of m is high; the corollary of that is that the level of diversion, i.e. $1 - m$, is low. The size of ι could depend on factors that are intrinsic to the individual, regional, cultural, social, religious, or political, for example.

The size of q depends on the stock of social infrastructure (Q), the size of

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the population (L), and the level of diversion technology (D). Specifically,

$$q = \frac{Q}{DL^\kappa},\tag{2}$$

where $\kappa \in (0, 1)$ and $D \geq 1$. *Ceteris paribus*, a higher stock of social infrastructure will increase q while the converse is true for a higher level of diversion technology or population size. We argue that for a given size of Q, a larger L will lower the effectiveness of social infrastructure in suppressing diversive activities. This is because when L is larger, more resources are required to maintain the effectiveness of social infrastructure as it is now harder to monitor and detect diversive activities. Hence, it is not the size of Q *per se* but its size relative to L that determines q. This is comparable to the concept of congestion applied to physical infrastructure such as highways, power and water systems. The association between the rate of increase in Q and the rate of increase in L, however, is not one-for-one. That is, a one percent increase in L does not require the same percentage increase in Q to maintain the effectiveness of social infrastructure. It requires only κ percent. This is to allow for the possibility that Q, unlike a consumption or investment good, is partially non-rivalrous. In the extreme case where it is completely non-rivalrous or completely rivalrous, κ would be equal to zero or one respectively. We will show later that the mathematics for steady-state conditions provides an endogenous analytical expression for κ.

The diversion technology parameter, D, is an attempt to capture the level of technology used by individuals in diversive activities. An example which illustrates this parameter is the theft of cars. Recent reports have indicated a significant rise in the incidents of stolen cars. Several studies have shown that the rise can be attributed to the use of more sophisticated technology by perpetrators, which reduces the probability of getting caught for their criminal acts. Hence, a higher level of D will enable individuals to engage in more diversive activities since the chances of getting caught is lower.

2.2   The Public Sector

The evolution of Q in any given period is specified by the following differential equation:

$$\dot{Q} = G\left(u_Q mL\right)^\sigma Q^\varepsilon - \delta_Q Q,\tag{3}$$

where G is a constant efficiency parameter (or quality index of Q), u_Q is the fraction of m allocated to the public sector, δ_Q is the rate at which existing Q decays, and $\{\sigma, \varepsilon\} \in (0, 1)$ are elasticity parameters. The presence of the

decay parameter, δ_Q, captures the idea that social infrastructure is subject to 'wear and tear' like physical capital in a metaphorical sense if resources are not continuously channelled to the provision of new social infrastructure. An example of this is the area of law enforcement carried out by the police and other law enforcement agents. The services provided by these agents represent a constituent of social infrastructure that helps create the incentives for individuals to engage in market instead of diversive activities. In order for these services to remain effective, we need to supply resources continuously to its provision.

The efficiency parameter, G, includes factors such as government policies, judicial systems, and the level of bureaucratic corruption, which affect the efficiency level of the public sector in providing new social infrastructure. For example, a powerful judicial system that strives to protect property rights will enable law enforcement agents to carry out their duties more effectively since the threat of punishment becomes more credible. Another possible factor suggested in Hall and Jones (1999) that could be included in G is the extent of influence from Western Europe. This is because many factors that are conducive to having a favourable social infrastructure were discovered in that region, such as the ideas of Adam Smith, property rights, and the system of checks and balances in government. Equation (3) assumes that an economy with a larger Q possesses a better knowledge and experience in providing social infrastructure and hence is able to produce more Q in any given period. It is assumed that the provision of new Q is subject to diminishing returns with respect to $u_Q mL$ and Q individually. We will show later that the mathematics for a partially non-rivalrous Q necessitates diminishing returns to scale with respect to $u_Q mL$ and Q jointly as well, i.e. $\sigma + \varepsilon < 1$.

2.3   The Final Goods Sector

We assume that the economy produces only one type of final good where the amount produced in any given period is measured by Y. This final good can be used for either consumption or investment and it is assumed that no cost is incurred when it is transformed back into physical capital. The stock of physical capital in any given period is measured by K. Using only labour and physical capital, the production function for Y takes the following Cobb-Douglas form:

$$Y = AK^\alpha \left(u_Y mL\right)^{1-\alpha}, \tag{4}$$

where A is a constant technology parameter, u_Y is the fraction of m allocated to the final goods sector, and $\alpha \in (0,1)$ is an elasticity parameter

that measures the physical capital share of income. Equation (4) exhibits diminishing returns to scale with respect to K and $u_Y m L$ individually, and constant returns to scale with respect to K and $u_Y m L$ jointly. The above specification of the aggregate production function allows us to model Y as a positive function of m. When individuals engage less in diversive activities and thus more in market activities, more final goods can be produced. Furthermore, the assumption that diversion, when engaged in by every individual, leads to zero net private gains implies that only market activities matter when it comes to the production of real goods and the accumulation of wealth. Hence, *ceteris paribus*, an economy with a larger stock of social infrastructure will have a higher output per worker. Note, however, that the economy will be operating within its production possibilities frontier as long as $m < 1$.

2.4   The Evolution of Physical Capital

The change in physical capital in any given period is governed by the following differential equation:

$$\dot{K} = \xi\left(m\right)\left(Y - C\right) - \delta_K K, \tag{5}$$

where C denotes aggregate consumption, $Y - C$ measures the flow of savings, δ_K is the rate at which physical capital depreciates, and $\xi\left(m\right) \in (0, 1)$ is a transformation coefficient which determines the fraction of $Y - C$ that is effectively transformed into new productive capital. Chou and Chin (2001) argue that the size of the transformation coefficient depends positively on, *inter alia*, the stock of financial innovations (or products) produced in the financial sector. These innovations include automatic teller machines, phone and internet banking systems, financial derivatives, and initial public offerings. Although we do not model the financial sector endogenously here, we postulate that the transformation coefficient would be a positive function of the fraction of time engaged in market activities, i.e. m, which in turn is a positive function of the stock of social infrastructure. This is because a higher level of social infrastructure, *ceteris paribus*, suppresses more diversive activities and encourages people to engage more in market activities which include the transformation of savings into productive capital. Without any loss of generality, we assume that ξ is a simple linear function of m such that

$$\dot{K} = m\left(Y - C\right) - \delta_K K. \tag{6}$$

3 Solving the Basic Model

3.1 The Optimal Control Problem

The social planner framework is used to work out the optimal solutions to the model. We adopt the conventional constant-relative-risk-aversion utility function for the representative individual which the hypothetical social planner seeks to maximise by choosing the paths of consumption and the shares of labour time allocated to the two sectors. Hence, the social planner's objective is to

$$\max_{c,u_Y} \quad U_0 \equiv \int_0^\infty \frac{c^{1-\theta}-1}{1-\theta} e^{-(\rho-n)t} dt,$$

where $c \equiv C/L$ denotes consumption per worker, $\rho > 0$ denotes the individual's subjective rate of time preference, $\theta \geq 0$ is the risk-aversion parameter, and $n \geq 0$ is the rate of population growth, i.e. $\dot{L}/L$, subject to

$$\begin{aligned}
\dot{K} &= m(Y-C) - \delta_K K, \\
\dot{Q} &= G(u_Q mL)^\sigma Q^\varepsilon - \delta_Q Q, \\
1 &= u_Y + u_Q.
\end{aligned}$$

The Hamiltonian is

$$\mathsf{H} \equiv \frac{c^{1-\theta}-1}{1-\theta} e^{-(\rho-n)t} + \nu\left[m(Y-C) - \delta_K K\right] + \pi\left[G(u_Q mL)^\sigma Q^\varepsilon - \delta_Q Q\right],$$

where the control variables are c and u_Y; the state variables are K and Q; and the costate variables are ν and π.

The first-order conditions for the control variables, i.e. $\partial\mathsf{H}/\partial c = 0$ and $\partial\mathsf{H}/\partial u_Y = 0$, yield the following equations respectively:

$$\begin{aligned}
\frac{\dot{c}}{c} &= -\frac{1}{\theta}\left(\rho + \frac{\dot{\nu}}{\nu} + \iota\frac{\dot{q}}{q}\right), \\
\frac{\nu}{\pi} &= \frac{\sigma}{1-\alpha}\frac{G(u_Q mL)^\sigma Q^\varepsilon}{mAK^\alpha(u_Y mL)^{1-\alpha}}\frac{u_Y}{u_Q}.
\end{aligned}$$

The first-order conditions for the state variables K and Q are given by equations (6) and (3) respectively. The first-order conditions for the costate variables, i.e. $-\dot{\nu} = \partial\mathsf{H}/\partial K$ and $-\dot{\pi} = \partial\mathsf{H}/\partial Q$, yield the following equations

respectively:

$$-\frac{\dot{\nu}}{\nu} = mA\alpha k^{\alpha-1}(u_Y m)^{1-\alpha} - \delta_K,$$

$$-\frac{\dot{\pi}}{\pi} = \frac{\sigma\iota}{1-\alpha}\frac{G(u_Q mL)^\sigma Q^{\varepsilon-1}}{Ak^{\alpha-1}(u_Y m)^{1-\alpha}}\frac{u_Y}{u_Q}\left[(2-\alpha)Ak^{\alpha-1}(u_Y m)^{1-\alpha} - \frac{c}{k}\right]$$
$$+ (\sigma\iota + \varepsilon)G(u_Q mL)^\sigma Q^{\varepsilon-1} - \delta_Q,$$

where $k \equiv K/L$. Finally, the transversality conditions are

$$\lim_{t\to\infty} K(t)\nu(t) = 0,$$
$$\lim_{t\to\infty} Q(t)\pi(t) = 0.$$

3.2   Steady-State Conditions and Analytical Solutions

Definition 2 *The economy is on its balanced growth path (or in its steady state) if the variables $y \equiv Y/L$, c, k, m, q, u_Y and u_Q are all constant.*

This definition provides us the steady-state growth rate of the model. Namely, Y, C and K must all grow at rate n, and Q at rate κn.

Condition 3 *The parameter κ must be equal to $\sigma/(1-\varepsilon)$ in order for $\dot{q} = 0$ in the steady state.*

Proof. Using equation (2), we can write the growth rate of q as

$$\frac{\dot{q}}{q} \equiv \frac{\dot{Q}}{Q} - \kappa n.$$

Setting $\dot{q} = 0$ gives us

$$G(u_Q mL)^\sigma Q^{\varepsilon-1} = \kappa n + \delta_Q.$$

Differentiating the logarithm of the equation above with respect to time leads us with

$$\sigma\left(\frac{\dot{u}_Q}{u_Q} + \iota\frac{\dot{q}}{q} + n\right) - (1-\varepsilon)\frac{\dot{Q}}{Q} = 0.$$

Since $\dot{u}_Q = \dot{q} = 0$ in the steady state, it must be that

$$\frac{\dot{Q}}{Q} = \frac{\sigma n}{1-\varepsilon}.$$

Hence, κ must be equal to $\sigma/(1-\varepsilon)$. ∎

The model therefore provides us with an endogenous expression for κ. Next, we will see the implication for the elasticity parameters, σ and ε, with regard to the assumption of a partially non-rivalrous Q.

Condition 4 *The assumption of a partially non-rivalrous Q, i.e. $\kappa \in (0, 1)$, necessitates that $\sigma + \varepsilon < 1$.*

Proof. For κ to lie strictly within the open interval $(0, 1)$, we need

$$0 < \frac{\sigma}{1 - \varepsilon} < 1.$$

Multiplying the above inequality equation by $1 - \varepsilon$ and rearranging then gives us the diminishing returns to scale requirement. ∎

We can use the definition of the balanced growth path to help generate the four steady-state equations that enable us to solve for the four key variables in the model, namely, k, c, q and u_Q. These four equations, derived from the steady-state conditions $\dot{k}/k = 0$, $\dot{c}/c = 0$, $\dot{q}/q = 0$ and $\dot{u}_Y/u_Y = 0$ respectively, are

$$m \left[A k^{\alpha-1} \left(u_Y m \right)^{1-\alpha} - \frac{c}{k} \right] = n + \delta_K,$$

$$m A \alpha k^{\alpha-1} \left(u_Y m \right)^{1-\alpha} = \rho + \delta_K,$$

$$G u_Q^\sigma D^{\varepsilon-1} q^{\sigma\iota+\varepsilon-1} = \kappa n + \delta_Q,$$

$$-\frac{\dot{\pi}}{\pi} = -\frac{\dot{\nu}}{\nu} - (1 - \kappa) n.$$

The analytical solutions are

$$u_Q^* = \frac{\Gamma}{\Gamma + \Phi},$$

$$q^* = \left[\frac{G u_Q^{*\sigma}}{D^{1-\varepsilon} \left(\kappa n + \delta_Q \right)} \right]^{\frac{1}{(1-\varepsilon)(1-\kappa\iota)}},$$

$$k^* = \left(\frac{m^* A \alpha}{\rho + \delta_K} \right)^{\frac{1}{1-\alpha}} u_Y^* m^*,$$

$$c^* = \frac{\rho + \delta_K - \alpha \left(n + \delta_K \right)}{\alpha} \frac{k^*}{m^*},$$

where

$$\Gamma \equiv \sigma\iota \left(\kappa n + \delta_Q \right) \left[\alpha \left(n + \delta_K \right) + (1 - \alpha) \left(\rho + \delta_K \right) \right],$$

$$\Phi \equiv (1 - \alpha) \left(\rho + \delta_K \right) \left[\rho - n + (1 - \varepsilon) \left(1 - \kappa\iota \right) \left(\kappa n + \delta_Q \right) \right],$$

$$u_Y^* = 1 - u_Q^*,$$

$$m^* = q^{*\iota}.$$

Note that to get sensible predictions for q^*, the exponent in the solution for q^* needs to be strictly positive. Given that $\{\varepsilon, \kappa\} \in (0,1)$, the restriction implies that

$$0 < \iota < \frac{1}{\kappa}.$$

3.3 Model Implications

Proposition 5 *When the economy is on its balanced growth path, social infrastructure has no impact on the growth rate of Y. It does, however, have an impact when the economy is on its transitional path towards the steady state.*

Proof. Using the aggregate production function given by equation (4), we can write the growth rate of Y as

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + (1 - \alpha)\left(\frac{\dot{u}_Y}{u_Y} + \iota \frac{\dot{q}}{q} + n\right). \tag{7}$$

It is clear from this equation that the growth rate of Y will be affected by social infrastructure as long as $\dot{q} \neq 0$, which occurs when the economy is not in its steady state. ∎

Equation (7) shows that if q is increasing (i.e. $\dot{q} > 0$), which can arise from, say, a positive shock to the efficiency parameter, G, then *ceteris paribus*, the growth rate of Y can rise above n.

Proposition 6 *If individuals are highly averse to diversion, i.e. the diversion aversion parameter, ι, approaches zero, all labour time will be allocated to the final goods sector and no diversive activities will take place. On the other hand, if individuals are less averse, the optimal solution is to allocate more labour time to the public sector to raise the production of social infrastructure.*

Proof. We observe from the analytical solutions for u_Q^* that $\lim_{\iota \to 0} \Gamma = 0$. This implies that $\lim_{\iota \to 0} u_Q^* = 0$ and $\lim_{\iota \to 0} u_Y^* = 1$. Moreover, $\lim_{\iota \to 0} m^* = 1$, i.e. all labour time will be allocated to market activities only. Next, given that $\partial \Gamma / \partial \iota > 0$ and $\partial \Phi / \partial \iota < 0$, we then have $\partial u_Q^* / \partial \iota > 0$ and $\partial u_Y^* / \partial \iota < 0$. ∎

When the diversion aversion parameter approaches zero, the outcome is comparable to the first outcome illustrated in the payoff matrix in Figure 1, where both groups select the strategy of 'No Diversion'. No social costs will be created and the economy operates on its production possibilities frontier.

When individuals are less averse, the model attempts to increase the production of social infrastructure by allocating more labour time to the public sector in the hope of suppressing the possible surge in diversive activities, i.e. to prevent m^* from falling. The rise in ι affects m^* in two ways. We have seen in Figure 2 that for a given q, the rise will decrease m. However, the above proposition shows that a rise in ι will increase u_Q^*, and given the analytical solution for q^*, the rise in u_Q^* will in turn increase q^*. This implies that the net effect on m^* can be a fall or rise. The next proposition shows that whether m^* falls continuously or rises eventually as ι increases from its lower limit to its upper limit will depend on the size of the efficiency parameter, G.

Proposition 7 *If the efficiency parameter, G, is not sufficiently above the term $D^{1-\varepsilon}(\kappa n + \delta_Q)$, m^* will be a strictly monotonically decreasing function of ι. If G is sufficiently above $D^{1-\varepsilon}(\kappa n + \delta_Q)$, m^* at first falls but rises eventually as ι increases from zero to $1/\kappa$.*

Proof. From the first total derivative of m^* with respect to ι,

$$\frac{dm^*}{d\iota} = \frac{m^*}{(1-\varepsilon)(1-\kappa\iota)}\left[\frac{1}{1-\kappa\iota}\ln\frac{Gu_Q^{*\sigma}}{D^{1-\varepsilon}(\kappa n + \delta_Q)} + \frac{\sigma\iota}{u_Q^*}\frac{\partial u_Q^*}{\partial\iota}\right],$$

we need

$$\ln\frac{G}{D^{1-\varepsilon}(\kappa n + \delta_Q)} \lesseqgtr -\sigma\ln u_Q^* - \frac{\sigma\iota(1-\kappa\iota)}{u_Q^*}\frac{\partial u_Q^*}{\partial\iota},$$

if we want $dm^*/d\iota \lesseqgtr 0$ respectively. From the previous proposition, we know that $\partial u_Q^*/\partial\iota > 0$. Also, $\ln u_Q^* < 0$ since $u_Q^* \in (0,1)$. Consider the case where $\iota \to 0$. Taking limits on both sides, the inequality becomes

$$\ln\frac{G}{D^{1-\varepsilon}(\kappa n + \delta_Q)} \lesseqgtr \infty - \sigma.$$

For any given finite value of G, it is clear that only the less-than inequality sign can be satisfied. This means that $dm^*/d\iota < 0$ for low values of ι. Now consider the case where $\iota \to 1/\kappa$. Taking limits on both sides, we have

$$\ln\frac{G}{D^{1-\varepsilon}(\kappa n + \delta_Q)} \lesseqgtr -\sigma\ln u_Q^*.$$

Given that $-\sigma\ln u_Q^* > 0$, the less-than inequality sign can be satisfied *iff* G is not sufficiently above $D^{1-\varepsilon}(\kappa n + \delta_Q)$. Conversely, the greater-than inequality sign can be satisfied *iff* G is sufficiently above $D^{1-\varepsilon}(\kappa n + \delta_Q)$.

Hence, it must be the case that $dm^*/d\iota < 0 \; \forall \iota \in (0, 1/\kappa)$ if G is not sufficiently above $D^{1-\varepsilon}(\kappa n + \delta_Q)$. On the other hand, $dm^*/d\iota < 0$ for low values of ι and $dm^*/d\iota > 0$ for high values of ι if G is sufficiently above $D^{1-\varepsilon}(\kappa n + \delta_Q)$. ∎

The result of this proposition is important since it suggests that a limit exists as to how high the level of diversion, $1 - m^*$, can be over the range of values for the diversion aversion parameter on the proviso of a sufficiently high G. The proposition also highlights the importance of having an efficient public sector since an inefficient one will not be capable of countering the ill-effects of diversion following a rise in ι in spite of an increase in u_Q^*. Also, for the turning point in the level of diversion to exist, G has to be higher when the following parameters are higher: (i) diversion technology, D; degree of rivalry in social infrastructure, κ; population growth rate, n; and rate of decay of social infrastructure, δ_Q. We illustrate the effects of varying levels of ι on $1 - m^*$ in Figure 3. We also show the effects on output per worker, y^*, in Figure 4.

Assuming that G is sufficiently high, another implication of the above proposition is that the turning point in the level of diversion will occur at a lower value of ι when G is higher, *ceteris paribus*. This means that albeit a sufficiently high G is necessary for $dm^*/d\iota > 0$, it is, however, not a sufficient condition. This is because for any given value of ι, say ι^*, $dm^*/d\iota > 0 \; iff$ the turning point occurs at $\iota_0 \in (0, \iota^*)$. Hence, the likelihood that $dm^*/d\iota > 0$ is higher when G is higher. We illustrate this implication in Figure 5. This figure also shows that for a given value of ι, a higher G will lead to a lower level of diversion and higher level of market activities. Naturally, one would postulate, in theory at least, that the level of diversion can be completely suppressed by raising G to some threshold level. We illustrate this point in the next proposition.

Proposition 8 *In theory, although it is possible to suppress diversion to zero levels, i.e. $1 - m^* \to 0$, by raising the efficiency parameter, G, the outcome, however, will always be 'second best' (in terms of output per worker) relative to the case where the diversion aversion parameter approaches zero.*

Proof. We can infer from the analytical solution for q^* that $m^* \to 1$ if $G \to G^*$, where

$$G^* = \frac{D^{1-\varepsilon}(\kappa n + \delta_Q)}{u_Q^{*\sigma}}.$$

Hence, diversion levels will fall to zero as G increases to G^*. The analytical
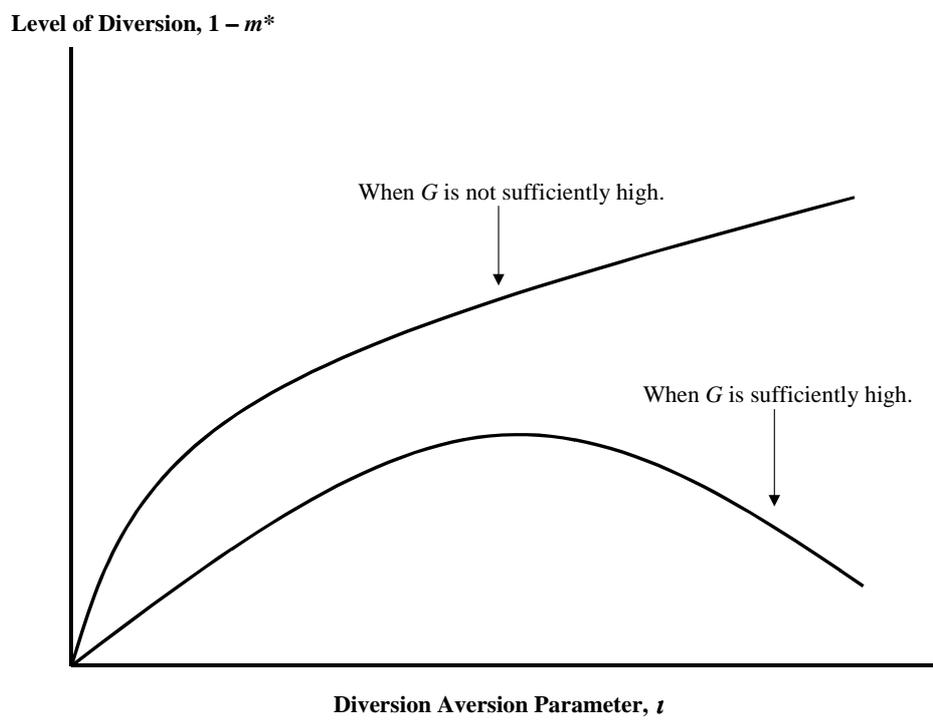
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Level of Diversion, 1 − *m**

When *G* is not sufficiently high.

When *G* is sufficiently high.

Diversion Aversion Parameter, *ι*

Figure 3: The relationship between the diversion aversion parameter and level of diversion

Output Per Worker, y^*

$y^*_{potential}$

When G is sufficiently high.

When G is not sufficiently high.

Diversion Aversion Parameter, ι
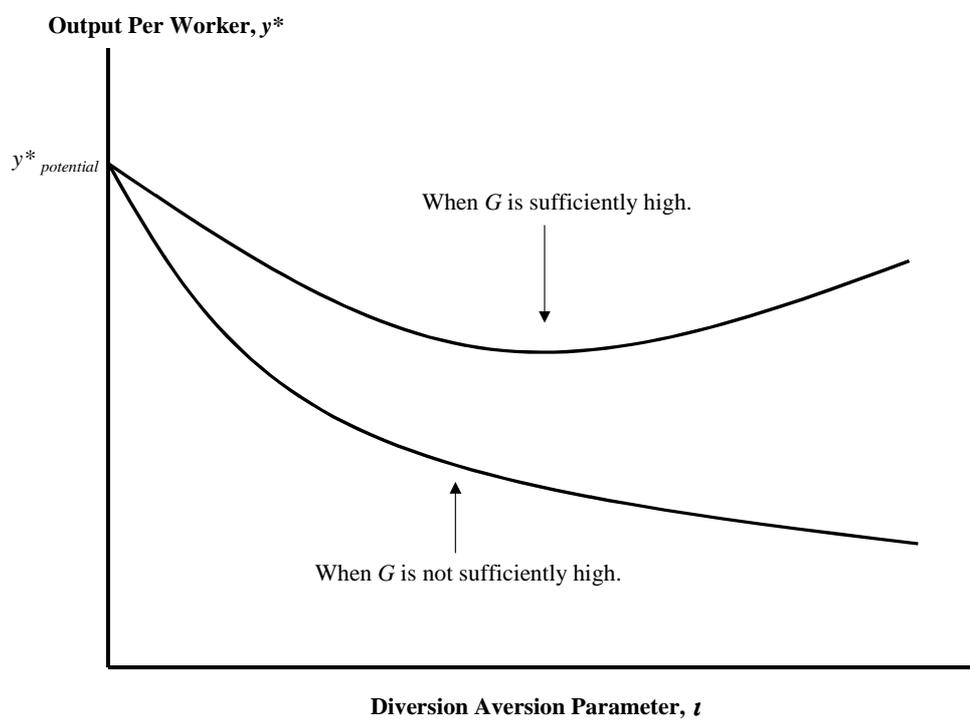
Figure 4: The relationship between the diversion aversion parameter and output per worker

Level of Diversion, $1 - m^*$

When G is sufficiently high
and equal to, say, G^*.

$dm^*/ d\imath < 0$

When G is sufficiently high
and equal to, say, $G^{**} > G^*$.

$dm^*/ d\imath > 0$

$\imath^*$

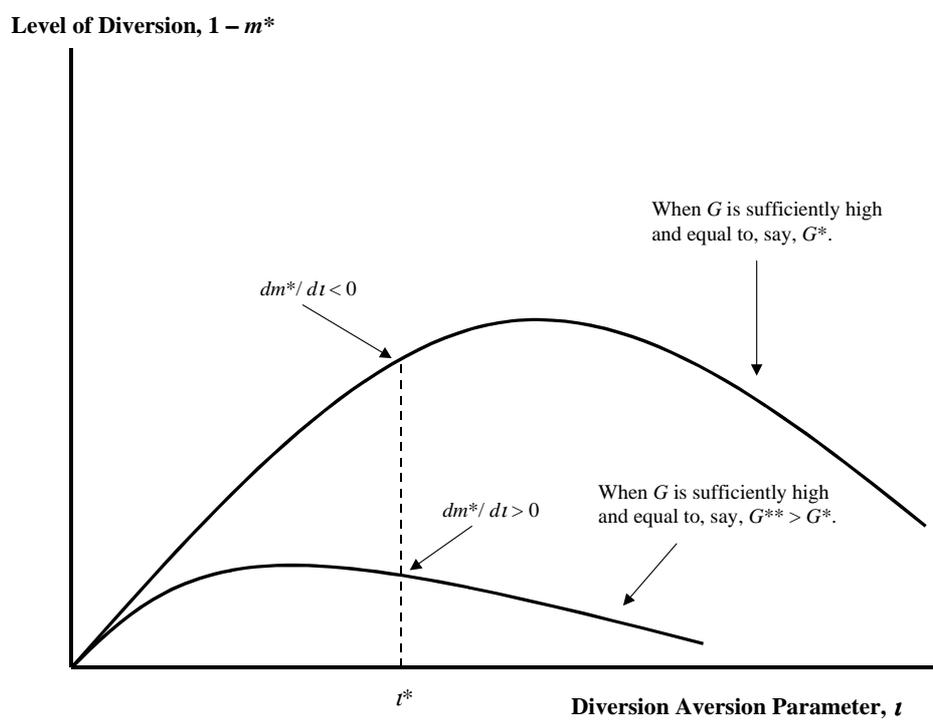**Diversion Aversion Parameter, $\imath$**

Figure 5: The relationship between the diversion aversion parameter and level of diversion under different values of the efficiency parameter

18

solution for output per worker would then be

$$y = \left[A \left(\frac{\alpha}{\rho + \delta_K} \right)^{\alpha} \right]^{\frac{1}{1-\alpha}} u_Y^*.$$

However, given that $u_Q^* \to 0$ if $\iota \to 0$, i.e. $u_Y^* \to 1$, output per worker will always be higher in the case where $\iota \to 0$ than where $G \to G^*$. ∎

This proposition says that despite the importance of having a high level of efficiency in the public sector, the ideal case will always be the one where individuals are highly averse to diversion. When that happens, there are no social costs. Hence, no labour time need be taken away from the final goods sector to the public sector to produce any social infrastructure. The economy will be producing at its potential capacity.

4   The Model with Human Capital

We now consider the case where human capital contributes as an additional factor in the production of final goods as well as new social infrastructure. We augment the basic model with a third sector, namely, the education sector. The production of new human capital requires raw labour as well as existing human capital. The equation for this sector is

$$\dot{H} = E \left(u_H m L \right)^{1-\eta} \left(u_H m H \right)^{\eta} - \delta_H H, \tag{8}$$

where E is a constant productivity parameter, u_H is the fraction of m allocated to the sector, δ_H is the rate at which the stock of human capital depreciates, and $\eta \in (0, 1)$ is an elasticity parameter. For technical reasons and simplicity, we let the two inputs, L and H, exhibit diminishing marginal returns individually but constant returns to scale jointly in the production process. Here, we model labour and human capital as distinct rather than joint inputs because we want to examine how the level of human capital is determined as well as the relationship between it and social infrastructure. We will discuss the case of modelling labour and human capital as joint inputs in the next section.

We argue that human capital aids the provision of social infrastructure by equipping workers in the public sector with better skills thereby raising their ability to formulate more effective policies or strategies to counter diversive activities. We believe that a more educated workforce can lower the level of diversion not directly but via its positive impact on the public sector. This suggests that a positive link between human capital and the efficiency parameter, G, should exist. We can therefore write the equation of motion for Q as

$$\dot{Q} = \tilde{G} \left(u_Q m L\right)^\sigma \left(u_Q m H\right)^\lambda Q^\varepsilon - \delta_Q Q, \tag{9}$$

where $G \equiv \tilde{G} \left(u_Q m H\right)^\lambda$ and $\lambda \in (0,1)$ is an elasticity parameter. The new efficiency parameter, $\tilde{G}$, now only captures factors that are not related to human capital. We will show later that the partially non-rivalrous assumption of Q requires that $\sigma + \lambda + \varepsilon < 1$.

We now write the aggregate production function as

$$Y = AK^\alpha \left(u_Y m L\right)^{1-\alpha-\beta} \left(u_Y m H\right)^\beta, \tag{10}$$

where $\beta \in (0,1)$ is an elasticity parameter. Again, constant returns to scale is assumed across the three inputs albeit each input experiences diminishing returns. Note that both equations (9) and (10) collapse to their counterparts in the basic model when λ and β are both equal to zero.

4.1 The Optimisation Problem

We adopt the social planner framework to work out the optimal solutions to the model. Under the same constant-relative-risk-aversion utility function for the representative individual, the hypothetical social planner seeks to maximise the individual's utility by choosing the paths of consumption and the shares of labour time allocated to the three sectors. Hence, the social planner's objective is to

$$\max_{c,u_Y,u_H} \quad U_0 \equiv \int_0^\infty \frac{c^{1-\theta}-1}{1-\theta} e^{-(\rho-n)t} dt,$$

subject to

$$\begin{aligned}
\dot{K} &= m\left(Y-C\right) - \delta_K K, \\
\dot{H} &= E u_H m L^{1-\eta} H^\eta - \delta_H H, \\
\dot{Q} &= \tilde{G}\left(u_Q m\right)^{\sigma+\lambda} L^\sigma H^\lambda Q^\varepsilon - \delta_Q Q, \\
1 &= u_Y + u_H + u_Q.
\end{aligned}$$

The Hamiltonian is

$$\begin{aligned}
\mathsf{H} \equiv{}& \frac{c^{1-\theta}-1}{1-\theta} e^{-(\rho-n)t} + \nu\left[m\left(Y-C\right) - \delta_K K\right] + \mu\left(E u_H m L^{1-\eta} H^\eta - \delta_H H\right) \\
& + \pi\left[\tilde{G}\left(u_Q m\right)^{\sigma+\lambda} L^\sigma H^\lambda Q^\varepsilon - \delta_Q Q\right],
\end{aligned}$$

where the control variables are c, u_Y and u_H; the state variables are K, H and Q; and the costate variables are ν, μ and π.

The first-order conditions for the control variables, i.e. $\partial \mathsf{H}/\partial c = 0$, $\partial \mathsf{H}/\partial u_Y = 0$ and $\partial \mathsf{H}/\partial u_H = 0$, yield the following equations respectively:

$$
\frac{\dot{c}}{c} = -\frac{1}{\theta}\left(\rho + \frac{\dot{\nu}}{\nu} + \iota\frac{\dot{q}}{q}\right),
$$

$$
\frac{\nu}{\pi} = \frac{\sigma + \lambda}{1 - \alpha}\frac{\tilde{G}\left(u_Q m\right)^{\sigma+\lambda} L^\sigma H^\lambda Q^\varepsilon}{mAK^\alpha\left(u_Y m\right)^{1-\alpha} L^{1-\alpha-\beta} H^\beta}\frac{u_Y}{u_Q},
$$

$$
\frac{\mu}{\pi} = \left(\sigma + \lambda\right)\frac{\tilde{G}\left(u_Q m\right)^{\sigma+\lambda} L^\sigma H^\lambda Q^\varepsilon}{EmL^{1-\eta}H^\eta}\frac{1}{u_Q}.
$$

The first-order conditions for the state variables K, H and Q are given by equations (6), (8) and (9) respectively. The first-order conditions for the costate variables, i.e. $-\dot{\nu} = \partial \mathsf{H}/\partial K$, $-\dot{\mu} = \partial \mathsf{H}/\partial H$ and $-\dot{\pi} = \partial \mathsf{H}/\partial Q$, yield the following equations respectively:

$$
-\frac{\dot{\nu}}{\nu} = mA\alpha k^{\alpha-1}\left(u_Y m\right)^{1-\alpha} h^\beta - \delta_K,
$$

$$
-\frac{\dot{\mu}}{\mu} = \left(\frac{\beta}{1-\alpha}u_Y + \eta u_H + \frac{\lambda}{\sigma+\lambda}u_Q\right) Emh^{\eta-1},
$$

$$
\begin{aligned}
-\frac{\dot{\pi}}{\pi} &= \frac{\left(\sigma + \lambda\right)\iota}{1-\alpha}\frac{\tilde{G}\left(u_Q m\right)^{\sigma+\lambda} L^\sigma H^\eta Q^{\varepsilon-1}}{Ak^{\alpha-1}\left(u_Y m\right)^{1-\alpha} h^\beta}\frac{u_Y}{u_Q}\left[\left(2-\alpha\right) Ak^{\alpha-1}\left(u_Y m\right)^{1-\alpha} h^\beta\right. \\
&\left. -\frac{c}{k}\right] + \tilde{G}\left(u_Q m\right)^{\sigma+\lambda} L^\sigma H^\eta Q^{\varepsilon-1}\left[\left(\sigma+\lambda\right)\iota\left(\frac{u_H}{u_Q}+1\right)+\varepsilon\right] - \delta_Q,
\end{aligned}
$$

where $h \equiv H/L$. Finally, the transversality conditions are

$$
\begin{aligned}
\lim_{t\to\infty} K\left(t\right)\nu\left(t\right) &= 0, \\
\lim_{t\to\infty} H\left(t\right)\mu\left(t\right) &= 0, \\
\lim_{t\to\infty} Q\left(t\right)\pi\left(t\right) &= 0.
\end{aligned}
$$

4.2 Steady-State Conditions and Analytical Solutions

Definition 9 *The economy is on its balanced growth path (or in its steady state) if the variables y, c, k, m, h, q, u_Y, u_H and u_Q are all constant.*

With this definition, we have Y, C, K and H all growing at rate n, and Q at rate κn.

Condition 10 *The parameter κ must be equal to $(\sigma + \lambda)/(1 - \varepsilon)$ in order for $\dot{q} = 0$ in the steady state.*

Proof. Using equation (2), we can write the growth rate of q as

$$\frac{\dot{q}}{q} \equiv \frac{\dot{Q}}{Q} - \kappa n.$$

Setting $\dot{q} = 0$ gives us

$$\tilde{G}\left(u_Q m\right)^{\sigma + \lambda} L^\sigma H^\lambda Q^{\varepsilon - 1} = \delta_Q + \kappa n.$$

Differentiating the logarithm of the equation above with respect to time leads us with

$$(\sigma + \lambda)\left(\frac{\dot{u}_Q}{u_Q} + \iota\frac{\dot{q}}{q} + n\right) - (1 - \varepsilon)\frac{\dot{Q}}{Q} = 0.$$

Since $\dot{u}_Q = \dot{q} = 0$ in the steady state, it must be that

$$\frac{\dot{Q}}{Q} = \frac{(\sigma + \lambda) n}{1 - \varepsilon}.$$

Hence, κ must be equal to $(\sigma + \lambda)/(1 - \varepsilon)$. ∎

The endogenous analytical expression for κ in the basic model can thus be viewed as a special case (i.e. $\lambda = 0$) of the more general expression in this model.

Condition 11 *The assumption of a partially non-rivalrous Q, i.e. $\kappa \in (0, 1)$, necessitates that $\sigma + \lambda + \varepsilon < 1$.*

Proof. For κ to lie strictly within the open interval $(0, 1)$, we need

$$0 < \frac{\sigma + \lambda}{1 - \varepsilon} < 1.$$

Multiplying the above inequality equation by $1 - \varepsilon$ and rearranging then gives us the diminishing returns to scale requirement. ∎

We can use the definition of the balanced growth path to help generate the six steady-state equations that enable us to solve for the six key variables in the model, namely, k, c, h, q, u_H and u_Q. These six equations, derived from

the steady-state conditions $\dot{k}/k = 0$, $\dot{c}/c = 0$, $\dot{h}/h = 0$, $\dot{q}/q = 0$, $\dot{u}_Y/u_Y = 0$ and $\dot{u}_H/u_H = 0$ respectively, are

$$m\left[Ak^{\alpha-1}\left(u_Y m\right)^{1-\alpha}h^\beta - \frac{c}{k}\right] = n + \delta_K,$$

$$mA\alpha k^{\alpha-1}\left(u_Y m\right)^{1-\alpha}h^\beta = \rho + \delta_K,$$

$$Eu_H m h^{\eta-1} = n + \delta_H,$$

$$\tilde{G}u_Q^{\sigma+\lambda}h^\lambda D^{\varepsilon-1}q^{(\sigma+\lambda)\iota+\varepsilon-1} = \kappa n + \delta_Q,$$

$$-\frac{\dot{\pi}}{\pi} = -\frac{\dot{\nu}}{\nu} - (1-\kappa)\,n,$$

$$-\frac{\dot{\pi}}{\pi} = -\frac{\dot{\mu}}{\mu} - (1-\kappa)\,n.$$

The analytical solutions are

$$u_Q^* = \frac{\Lambda_1\left(\Gamma\Upsilon_2 + 1\right)}{\Phi + \Gamma\Upsilon_1 - \Lambda_2\left(\Gamma\Upsilon_2 + 1\right)},$$

$$u_H^* = \Lambda_1 + \Lambda_2 u_Q^*,$$

$$q^* = \left\{\left[\frac{\tilde{G}u_Q^{*\sigma+\lambda}}{D^{1-\varepsilon}\left(\kappa n + \delta_Q\right)}\right]^{1-\eta}\left(\frac{Eu_H^*}{n + \delta_H}\right)^\lambda\right\}^{\frac{1}{(1-\eta)(1-\varepsilon)(1-\kappa\iota)-\lambda\iota}},$$

$$h^* = \left(\frac{Eu_H^* m^*}{n + \delta_H}\right)^{\frac{1}{1-\eta}},$$

$$k^* = \left(\frac{m^* A\alpha h^{*\beta}}{\rho + \delta_K}\right)^{\frac{1}{1-\alpha}}u_Y^* m^*,$$

$$c^* = \frac{\rho + \delta_K - \alpha\left(n + \delta_K\right)}{\alpha}\frac{k^*}{m^*},$$

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where

$$\Gamma \equiv \frac{\alpha\left(n+\delta_K\right)}{\left(1-\alpha\right)\left(\rho+\delta_K\right)}+1,$$

$$\Phi \equiv \frac{\rho-n+\left(1-\varepsilon\right)\left(1-\kappa\iota\right)\left(\kappa n+\delta_Q\right)}{\left(\sigma+\lambda\right)\iota\left(\kappa n+\delta_Q\right)},$$

$$\Upsilon_1 \equiv \frac{1-\alpha}{\beta}\frac{\lambda}{\sigma+\lambda},$$

$$\Upsilon_2 \equiv \frac{1-\alpha}{\beta}\left(\frac{\rho}{n+\delta_H}-\eta\right),$$

$$\Lambda_1 \equiv \frac{1}{1+\Upsilon_2},$$

$$\Lambda_2 \equiv \left(\Upsilon_1-1\right)\Lambda_1,$$

$$u_Y^* = 1-u_Q^*-u_H^*,$$

$$m^* = q^{*\iota}.$$

Note that to get sensible predictions for q^*, the exponent in the solution for q^* needs to be strictly positive. Given that $\{\eta,\varepsilon,\kappa\}\in(0,1)$, the restriction implies that

$$0<\iota<\frac{\left(1-\eta\right)\left(1-\varepsilon\right)}{\left(1-\eta\right)\left(1-\varepsilon\right)\kappa+\lambda}.$$

The interval restriction for ι in the basic model is thus a special case (i.e. $\lambda=0$) of the above.

4.3   Model Implications

Proposition 12 *A fall in the efficiency parameter, $\tilde{G}$, or the productivity parameter, E, has a negative impact on social infrastructure, human capital per worker and output per worker.*

Proof. We can infer from the analytical solution for q^* that a fall in $\tilde{G}$ will decrease q^* and hence m^*, which in turn decreases h^* and y^* given that

$$y^*=Ak^{*\alpha}\left(u_Y^*m^*\right)^{1-\alpha}h^{*\beta}.$$

The parameter E appears in the analytical solutions for h^* and q^* such that a fall in E will decrease these two variables and thus m^* and y^*. ∎

We see that a positive link exists between the education and public sectors. A shock to any of the two parameters, $\tilde{G}$ or E, affects not only the

immediate sector but also the other, which in turn affects the final goods sector. Hence, an economy with low social infrastructure can discourage investments in human capital which in turn affects the provision of social infrastructure and output. This result is consistent with the finding in Hall and Jones (1999) where social infrastructure is positively correlated with educational attainment rates.

Proposition 13 *If individuals are less averse to diversion, i.e. the diversion aversion parameter is higher, the optimal solution for u_Q^* will unambiguously rise. The effect on u_H^*, however, is likely to be positive for higher values of λ and vice versa.*

Proof. Of the six identities in the analytical solution for u_Q^*, ι appears only in Φ. Given that $\partial\Phi/\partial\iota < 0$ and $\partial u_Q^*/\partial\Phi < 0$, we then have $\partial u_Q^*/\partial\iota > 0$. Next, we can infer from the analytical solution for u_H^* that the sign of $\partial u_H^*/\partial u_Q^*$ depends critically on the model parameters in Λ_2. Assuming that $\rho > \eta\left(n + \delta_H\right)$ and given that

$$\Lambda_2 = \frac{\frac{1-\alpha}{\beta}\frac{\lambda}{\sigma+\lambda} - 1}{1 + \frac{1-\alpha}{\beta}\left(\frac{\rho}{n+\delta_H} - \eta\right)},$$

$\Lambda_2 > 0$ if $\lambda > \beta\sigma/\left(1 - \alpha - \beta\right)$. ∎

This proposition suggests that if returns to human capital in the public sector is not sufficiently high, then it will not be attractive for individuals to allocate more time to the education sector when they become less averse to diversion. As a result, despite the rise in u_Q^* in response to a rise in ι, the net effect on q^*, and hence m^*, could be negative if u_H^* falls by a sufficient amount. This highlights the importance of effective utilisation of skills of public servants in order for human capital to have a positive effect on social infrastructure.

5 The Model with Endogenous Growth

One of the useful things about the previous model is that it allows us to examine the relationship between the level of human capital and social infrastructure analytically. We are able to do this because human capital has been modelled as an input factor that can be identified distinctively from raw labour. Such a specification is similar to the aggregate production function augmented with human capital introduced by Mankiw, Romer, and Weil (1992) in their Neoclassical model of economic growth. As a consequence,

the model does not exhibit endogenous growth. In particular, key variables such as output, physical capital and consumption all grow at the same rate as the population growth rate. In this section, we adopt a specification that is similar to the Lucas (1988) model for the human capital accumulation equation. Namely,

$$\dot{H} = Eu_H mH - \delta_H H, \tag{11}$$

where, $H \equiv hL$. When $m = 1$, equation (11) becomes identical to Lucas's (1988) model. Here, human capital and raw labour are jointly supplied by workers such that one is not distinguishable from the other. Hence, an increase in the total stock of human capital, H, can come about through a rise in h or L or even both. Although equation (11) describes the evolution of H over time, it does not describe the individual time paths of h and L. It is because of this that an analytical study on the behaviour of h is precluded if the Lucas's (1988) specification is adopted. Notwithstanding that, the latter specification enables us to analyse the relationship between social infrastructure and the endogenous growth rate of the economy.

The equation of motion for Q now becomes

$$\dot{Q} = \tilde{G} \left(u_Q mH\right)^\lambda Q^\varepsilon - \delta_Q Q, \tag{12}$$

and the aggregate production function is

$$Y = AK^\alpha \left(u_Y mH\right)^{1-\alpha}. \tag{13}$$

Note that the function for the probability of getting caught, q, is now written as

$$q = \frac{Q}{DH^\kappa}.$$

Again, it can be shown that $\kappa = \lambda / (1 - \varepsilon)$, and the assumption of a partially non-rivalrous Q requires that $\lambda + \varepsilon < 1$.

The optimisation problem for the social planner now is to

$$\max_{C, u_Y, u_H} \quad \int_0^\infty \frac{C^{1-\theta} - 1}{1 - \theta} e^{-\rho t} dt,$$

subject to

$$
\begin{aligned}
\dot{K} &= m\left(Y - C\right) - \delta_K K, \\
\dot{H} &= Eu_H mH - \delta_H H, \\
\dot{Q} &= \tilde{G}\left(u_Q mH\right)^\lambda Q^\varepsilon - \delta_Q Q, \\
1 &= u_Y + u_H + u_Q,
\end{aligned}
$$

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where $Y = AK^\alpha (u_Y m H)^{1-\alpha}$. The Hamiltonian is

$$
\begin{aligned}
\mathsf{H} \equiv{} & \frac{C^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \nu \left[m \left(Y - C \right) - \delta_K K \right] + \mu \left(E u_H m H - \delta_H H \right) \\
& + \pi \left[\tilde{G} \left(u_Q m H \right)^\lambda Q^\varepsilon - \delta_Q Q \right],
\end{aligned}
$$

where the control variables are C, u_Y and u_H; the state variables are K, H and Q; and the costate variables are ν, μ and π.

The first-order conditions for the control variables, i.e. $\partial \mathsf{H}/\partial C = 0$, $\partial \mathsf{H}/\partial u_Y = 0$ and $\partial \mathsf{H}/\partial u_H = 0$, yield the following equations respectively:

$$
\begin{aligned}
\frac{\dot{C}}{C} &= -\frac{1}{\theta} \left(\rho + \frac{\dot{\nu}}{\nu} + \iota \frac{\dot{q}}{q} \right), \\
\frac{\nu}{\pi} &= \frac{\lambda}{1 - \alpha} \frac{\tilde{G} \left(u_Q m H \right)^\lambda Q^\varepsilon}{m A K^\alpha \left(u_Y m H \right)^{1-\alpha}} \frac{u_Y}{u_Q}, \\
\frac{\mu}{\pi} &= \lambda \frac{\tilde{G} \left(u_Q m H \right)^\lambda Q^\varepsilon}{E u_H m H} \frac{u_H}{u_Q}.
\end{aligned}
$$

The first-order conditions for the state variables K, H and Q are given by equations (6), (11) and (12) respectively. The first-order conditions for the costate variables, i.e. $-\dot{\nu} = \partial \mathsf{H}/\partial K$, $-\dot{\mu} = \partial \mathsf{H}/\partial H$ and $-\dot{\pi} = \partial \mathsf{H}/\partial Q$, yield the following equations respectively:

$$
\begin{aligned}
-\frac{\dot{\nu}}{\nu} ={} & m A \alpha k^{\alpha-1} \left(u_Y m \right)^{1-\alpha} - \delta_K, \\
-\frac{\dot{\mu}}{\mu} ={} & \frac{1}{1 - \alpha} \frac{E u_H m}{A k^{\alpha-1} \left(u_Y m \right)^{1-\alpha}} \frac{u_Y}{u_H} \left\{ \left[(1 - \alpha)(1 - \kappa \iota) - \kappa \iota \right] A k^{\alpha-1} \left(u_Y m \right)^{1-\alpha} \right. \\
& \left. + \kappa \iota \frac{c}{k} \right\} + (1 - \kappa \iota) E u_H m \left(1 + \frac{u_Q}{u_H} \right) + \delta_H, \\
-\frac{\dot{\pi}}{\pi} ={} & \frac{\lambda \iota}{1 - \alpha} \frac{\tilde{G} \left(u_Q m H \right)^\lambda Q^{\varepsilon-1}}{A k^{\alpha-1} \left(u_Y m \right)^{1-\alpha}} \frac{u_Y}{u_Q} \left[(2 - \alpha) A k^{\alpha-1} \left(u_Y m \right)^{1-\alpha} - \frac{c}{k} \right] \\
& + \tilde{G} \left(u_Q m H \right)^\lambda Q^{\varepsilon-1} \left[\lambda \iota \left(\frac{u_H}{u_Q} + 1 \right) + \varepsilon \right] - \delta_Q,
\end{aligned}
$$

where $k \equiv K/H$, $c \equiv C/H$. Finally, the transversality conditions are

$$
\begin{aligned}
\lim_{t \to \infty} K(t) \nu(t) &= 0, \\
\lim_{t \to \infty} H(t) \mu(t) &= 0, \\
\lim_{t \to \infty} Q(t) \pi(t) &= 0.
\end{aligned}
$$

5.1 Steady-State Analytical Solutions

Definition 14 *The economy is on its balanced growth path (or in its steady state) if the variables $y \equiv Y/H$, c, k, m, q, u_Y, u_H and u_Q are all constant.*

We now have Y, C and K all growing at rate γ_H^*, where γ_H^* is the steady-state value of $\gamma_H \equiv \dot{H}/H$, and Q at rate $\kappa\gamma_H^*$. The steady-state growth rate, γ_H^*, is endogenously determined in the model. There are five key variables in this model, namely, k, c, q, u_Y and u_H. Using the definition of the steady state, we derive the following five equations that correspond to the five steady-state conditions $\dot{k}/k = 0$, $\dot{c}/c = 0$, $\dot{q}/q = 0$, $\dot{u}_Y/u_Y = 0$ and $\dot{u}_H/u_H = 0$ respectively:

$$
\begin{aligned}
m\left[Ak^{\alpha-1}\left(u_Y m\right)^{1-\alpha} - \frac{c}{k}\right] &= \gamma_H + \delta_K, \\
mA\alpha k^{\alpha-1}\left(u_Y m\right)^{1-\alpha} &= \rho + \theta\gamma_H + \delta_K, \\
Gu_Q^\lambda q^{\lambda\iota+\varepsilon-1}D^{\varepsilon-1} &= \kappa\gamma_H + \delta_Q, \\
-\frac{\dot{\pi}}{\pi} &= -\frac{\dot{\nu}}{\nu} - (1-\kappa)\gamma_H, \\
-\frac{\dot{\pi}}{\pi} &= -\frac{\dot{\mu}}{\mu} - (1-\kappa)\gamma_H.
\end{aligned}
$$

The analytical solutions are

$$
\begin{aligned}
u_Q^* &= \frac{\Gamma\left(\Upsilon_2 + \Lambda_2\right) + \Upsilon_2\left(\Upsilon_1 - \Lambda_1\right)}{\Gamma\left(\Phi + \Upsilon_2 + \Lambda_2\right) + \left(\Phi + \Upsilon_2\right)\left(\Upsilon_1 - \Lambda_1\right)}, \\
u_H^* &= \frac{\Phi\left(\Upsilon_1 - \Lambda_1\right)}{\Gamma\left(\Upsilon_2 + \Lambda_2\right) + \Upsilon_2\left(\Upsilon_1 - \Lambda_1\right)}u_Q^*, \\
q^* &= \left[\frac{\tilde{G}u_Q^\lambda}{D^{1-\varepsilon}\left(\kappa\gamma_H^* + \delta_Q\right)}\right]^{\frac{1}{(1-\varepsilon)(1-\kappa\iota)}}, \\
k^* &= \left(\frac{m^*A\alpha}{\rho + \theta\gamma_H^* + \delta_K}\right)^{\frac{1}{1-\alpha}}u_Y^* m^*, \\
c^* &= \frac{\rho + \theta\gamma_H^* + \delta_K - \alpha\left(\gamma_H^* + \delta_K\right)}{\alpha}\frac{k^*}{m^*},
\end{aligned}
$$

where

$$\begin{aligned}
\Gamma &\equiv \rho + \theta\gamma_H^*, \\
\Phi &\equiv \rho + [\theta + \lambda(1 - \kappa\iota) - 1]\gamma_H^* + (1 - \varepsilon)(1 - \kappa\iota)\delta_Q, \\
\Upsilon_1 &\equiv (1 - \kappa\iota)(\gamma_H^* + \delta_H), \\
\Upsilon_2 &\equiv \lambda\iota(\kappa\gamma_H^* + \delta_Q), \\
\Lambda_1 &\equiv \frac{\alpha\kappa\iota(\gamma_H^* + \delta_H)(\gamma_H^* + \delta_K)}{(1 - \alpha)(\rho + \theta\gamma_H^* + \delta_K)}, \\
\Lambda_2 &\equiv \frac{\alpha\lambda\iota(\kappa\gamma_H^* + \delta_H)(\gamma_H^* + \delta_K)}{(1 - \alpha)(\rho + \theta\gamma_H^* + \delta_K)}, \\
u_Y^* &= 1 - u_Q^* - u_H^*, \\
m^* &= q^{*\iota}.
\end{aligned}$$

The key variables are all functions of the model parameters and the steady-state growth rate, $\gamma_H^* = Eu_H^*m^* - \delta_H$, which the model is unable to provide an analytical expression. We can, however, use numerical simulations to obtain values of γ_H^* that are consistent with the analytical solutions for given sets of parameter values. To do that, we first define the implicit function $f(\gamma_H^*) = 0$, where

$$f(\gamma_H^*) = \gamma_H^* + \delta_H - Eu_H^*m^*.$$

After substituting the analytical solutions u_H^* and m^* into the above, we then solve numerically for the value of γ_H^* that satisfies the implicit function.

5.2   Numerical Simulations and Model Implications

We illustrate the impact of the efficiency parameter, $\tilde{G}$, productivity parameter, E, diversion technology, D, and diversion aversion parameter, ι, on the steady-state growth rate, γ_H^*, of the economy in Figure 6. The numerical simulations are performed by varying the value of the relevant parameter holding other parameter values constant. The baseline values used for the model parameters are

θ	ρ	A	α	δ_K	E	δ_H	$\tilde{G}$	λ	ε	δ_Q	D	ι
1.5	0.02	0.35	$\frac{1}{3}$	0.005	0.13	0.005	0.03	$\frac{2}{3}$	0.2	0.005	3	0.3

The simulation results show that an increase in either $\tilde{G}$ or E will raise γ_H^* while an increase in D or ι will lower γ_H^*. In particular, the positive relationship between the efficiency parameter and steady-state growth rate suggests that social infrastructure can have positive long-term growth effects in this model.
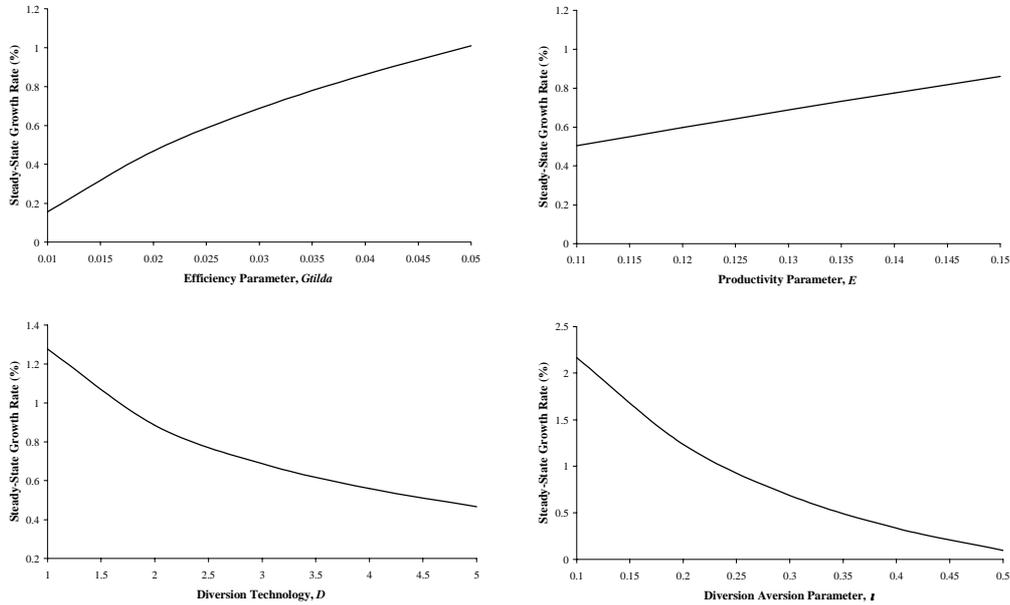
29

Figure 6: The impact of the efficiency parameter, productivity parameter, diversion technology, and diversion aversion parameter on the steady-state growth rate

6 Concluding Remarks

We have developed a basic two-sector model where social infrastructure is endogenously determined. We showed that unless the level of efficiency in the public sector is sufficiently high, raising the share of labour time allocated to that sector alone is incapable of countering the undesirable effects of a rise in the diversion aversion parameter. Although it is possible, in theory, to suppress diversion to zero levels, the model indicates that the outcome will still be 'second-best' to the case where individuals are highly averse to diversion. This is because no labour time is needed in the public sector in the latter case, and the economy will be producing at its potential capacity.

When the economy is on its balanced growth path, we showed in the basic model that social infrastructure does not affect the long-term growth rate at all. However, it does have an effect when the economy is on the transitional path towards its steady state. Notwithstanding that, we showed in our third model of human capital that social infrastructure can lead to positive long-term growth effects. In relation to output per worker, the models indicate that a rise in the efficiency parameter, $\tilde{G}$, will raise the portion of time engaged in market activities which in turn raises output per worker.

We also showed in our second model how human capital can affect social infrastructure and *vice versa*. An inefficient public sector will adversely affect the accumulation of human capital thereby lowering the level of human capital per worker. This in turn affects the provision of social infrastructure as well as the production of final goods. Alternatively, an unproductive education sector adversely affects the provision of social infrastructure thereby decreasing the fraction of time allocated to market activities. This in turn affects the accumulation of human capital as well as aggregate output. The model also shows the importance of effectively utilising the skills of public servants if the economy wants to encourage higher rates of human capital accumulation following a rise in the diversion aversion parameter.

We believe that the public sector can play a crucial role in improving the status of the economy via social infrastructure. In particular, it should concentrate its efforts on raising the levels of $\tilde{G}$ and E, and lowering the levels of D and ι. In particular, we find that the economy will benefit the most if individuals become more averse to diversion. When that happens, more resources can be devoted to the production of goods and services thereby pushing the economy closer to its production possibilities frontier. Finally, we suggest two ways where the paper can be extended in future research. One is to allow individuals to have heterogenous levels of diversion aversion. Another is to examine the links between social infrastructure and the financial and R&D sectors.

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