Modelling the costs and benefits of higher capital ratios

This document attempts to reproduce and extend previous work to model the costs and benefits of higher capital ratios.

Previous work was undertaken by the Basel Committee on Banking Supervision and by the Reserve Bank (the “Harrison model”), but parts of this were not well documented and the Reserve Bank work appears to contain some errors.

We have tried to remedy these problems by providing a more clearly documented analytical foundation for the model and identifying errors to be corrected.

A decision was made not to develop a different approach from the Harrison model, because it would be a distraction from more important work (the wider capital review) and it might not materially improve estimates of net benefit.

However, in the course of trying to work out how the Harrison model worked, we did find it necessary to extend the model in some ways to produce an internally consistent framework.

The rest of this document proceeds as follows: firstly, the Basel capital equation is used to calculate two benefits of fewer bank failures resulting from higher capital ratios. The benefits considered are the avoidance of recessions and the avoidance of bailout costs. This is closely aligned to the framework used in past Reserve Bank work, but there are some differences which (hopefully) will result in more correct estimate. The detailed calculation of benefits in the Harrison model (i.e. the past Reserve Bank work) is discussed at the end of the first part of the document.

Secondly, other costs and benefits are calculated within a supply and demand framework. One cost considered is lower GDP as a result of reduced bank lending and investment. As well, transfers to overseas residents and taxation effects are considered. While the elements which are taken into account are the same as those mentioned in past Reserve Bank work our approach differs in the detail, with a view to internal consistency. At the end of the section, the calculation of costs and benefits in the Harrison model is discussed.

The earlier Reserve Bank work can be found in the following:

The Regulatory Impact Statement which describes the previous Reserve Bank work (Ian Harrison’s Tuatara model) - [http://docs/webtop/drl/objectId/090000c3803c010b](http://docs/webtop/drl/objectId/090000c3803c010b)

A Powerpoint presentation about Ian Harrison’s model -

[http://docs/webtop/drl/objectId/090000c380352c80](http://docs/webtop/drl/objectId/090000c380352c80)

The spreadsheet with cost-benefit calculations in it (used in the Regulatory Impact Statement) -

[http://docs/webtop/drl/objectId/090000c3803b6929](http://docs/webtop/drl/objectId/090000c3803b6929)

A supporting spreadsheet -

[http://docs/webtop/drl/objectId/090000c3803b692e](http://docs/webtop/drl/objectId/090000c3803b692e)
Using the Basel capital equation to model bank failure
Assume there is single bank holding all loans.

Assume that the portfolio can be modelled using the corporate capital equation from BS2B (see BS2B 4.136):

$$K = LGD \times \left[ \Phi \left( \frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) + R \Phi^{-1}(0.999) \right) - PD \right] \times \frac{1 + b(M - 2.5)}{1 - 1.5b}$$

where $b = [0.11852 - 0.05478 \ln PD]^2$, $R = R(CR)$ (in BS2B, $R$ is a function of $PD$ but in the Harrison model it is defined to be a different function of $CR$), $\Phi$ is the standard normal cumulative distribution function, and $\Phi^{-1}$ is its inverse.

This gives the amount of capital ($K$) the bank needs to hold, per dollar of $EAD$, for there to be a 0.999 probability the bank will survive the next 12 months, unconditional on the state of the economy, assuming provisions have also been made for $PD \cdot LGD$ per dollar of exposure.\(^1\)\(^2\)\(^3\) 0.999 is the survival probability ($SP$), which we generalise from this point.

Assume the average risk weight for the portfolio is $RW$. The following equation shows the relationship between the regulatory capital ratio and the survival probability. To be precise, it shows the minimum capital ratio necessary to achieve the desired survival probability.

$$CR = \frac{LGD}{RW} \times \left[ \Phi \left( \frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) + R \Phi^{-1}(SP) \right) - PD \right] \times \frac{1 + b(M - 2.5)}{1 - 1.5b}$$

Solving for $SP$ allows the survival probability to be determined from a given capital ratio:

$$SP = \Phi \left[ \frac{1}{\sqrt{R}} \left( CR \times \frac{1 - 1.5b}{1 + b(M - 2.5)} \times \frac{RW}{LGD} + PD \right) - \frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) \right]$$

$SP$, with appropriate tweaks at the end-points, is a cumulative distribution function, and has a corresponding probability density function $sp$ (lower case). See the Appendix for more detail.

$SP(CR)$ is the probability that all shocks over the next year will be small enough to leave some capital intact, given a capital ratio of $CR$ (and also assuming provisions of $PD \cdot LGD$ per dollar of exposure).

Equivalently, $1 - SP(CR)$ gives the probability that a shock will be large enough to wipe out all capital when the capital ratio is $CR$, causing the bank to fail.

The probability of a shock that is big enough to wipe out a bank with a capital ratio between $CR$ and $CR + a$ (and not big enough to wipe out a bank with a capital ratio higher than $CR + a$) is:

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\(^2\) More properly, it is the probability of survival unconditional on the state of the economy.

\(^3\) Footnote: Strictly speaking, this equation is consistent with provisions of $PD \cdot LGD \cdot EAD \cdot maturity adjustment$, where maturity adjustment is the third factor in the equation for $K$ above.

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\[ P(\text{Shock is big enough to wipe out CR}) - P(\text{Shock is big enough to wipe out CR} + a) \\
= 1 - SP(CR) - (1 - SP(CR + a)) = SP(CR + a) - SP(CR) \]

Because \( SP \) is a cumulative distribution function and \( sp \) is the corresponding probability distribution function:

\[ SP(CR + a) - SP(CR) = \int_{CR}^{CR+a} sp(x)dx \]

**Benefits of higher capital (general description)**

Shocks that cause the bank to fail generate two sorts of losses: direct economic losses and bank losses. Direct economic losses, caused by disruption to production, are always socially costly. In the model bank losses are socially costly if the government bails out the bank, to the extent that bailout funds are paid to non-residents. Raising the capital ratio reduces both kinds of losses, so is beneficial (in a gross sense).

**Reduction of economic losses causes by crises**

The direct economic cost of a shock depends on the size of the shock. The expected direct economic cost of shocks, for a bank with a capital ratio of \( CR \), is given by\(^4\)

\[ \int_{CR}^{\infty} EC(x)sp(x)dx \]

where \( EC(x) \) is the direct economic cost of a shock that is just big enough to wipe out a bank with a capital ratio of \( x \).

This is just the ordinary expected value of \( EC(x) \), assuming economic costs are nil for shocks too small to cause the bank to fail:

\[ \int_{-\infty}^{\infty} EC(x)sp(x)dx = \int_{-\infty}^{CR} 0 \cdot sp(x)dx + \int_{CR}^{\infty} EC(x)sp(x)dx \]

The expected cost is reduced if the capital ratio is increased, and this is a benefit. When the capital ratio goes from \( a \) to \( b \), the benefit is:

\[ \int_{a}^{b} EC(x)sp(x)dx \]

This can be shown, for example, using Leibniz’s rule to find the derivative of the expected cost with respect to \( CR \) then integrating over \( a \) to \( b \). Intuitively, the benefit from moving to \( b \) is that there are some shocks big enough to cause failure with \( CR = a \) but not big enough to cause failure with \( CR = b \). It is the cost of these shocks that is avoided.

This is an annual cost; this cost is what is expected to be incurred every year, *on average* (in practice the cost is likely to be extremely lumpy). To find the total cost, the annual cost for each year \( j \) is divided by the appropriate discount rate for that year. Then the discounted costs are summed:

\(^4\) Direct economic and bailout costs are specified as positive quantities. In determining the net benefits of raising the capital requirement, these quantities are therefore to be subtracted.
Reduction of bailout costs

Similarly, the expected cost of bank losses, with a capital ratio of CR, is given by

\[ \int_{\text{CR}}^{\infty} BL(x) sp(x) dx \]

where \( BL(x, CR) \) is the bailout funds paid to foreigners, in the case of a shock that is just big enough to bring down a bank with a capital ratio of \( x \).

As with economic costs, bailout costs decrease when the capital ratio goes from \( a \) to \( b \). The benefit is given by:

\[ \int_{a}^{b} BL(x, a) sp(x) dx + \int_{b}^{\infty} (BL(x, a) - BL(x, b)) sp(x) dx \]

The first term is the saving from the cases where the larger capital buffer means the bank no longer fails. The second term reflects a lower bailout cost in the cases where there is still a bank failure, but the higher capital buffer has absorbed more losses.\(^5\)

The expression above gives an annual cost. Total costs are given by discounting and summing:

\[ \sum_{i=1}^{\infty} \frac{\int_{a}^{b} BL(x, a) sp(x) dx + \int_{b}^{\infty} (BL(x, a) - BL(x, b)) sp(x) dx}{\prod_{j=1}^{i} (1 + r_{BL,j})} \]

Recognising the value of certainty

As it stands, losses that have been avoided are all discounted at the same required rate of return, regardless of the size of the loss. But risk-averse consumers will place a higher value on the avoidance of large losses than on the avoidance of small ones, because of the concavity of their utility curves. The benefits above are therefore multiplied by a utility correction function \( UC(x) \), which can increase with the size of the loss.

\[ \sum_{i=1}^{\infty} \frac{\int_{a}^{b} UC(x) EC(x) sp(x) dx}{\prod_{j=1}^{i} (1 + r_{EC,j})} \]

\[ + \frac{\int_{a}^{b} UC(x) BL(x, a) sp(x) dx + \int_{b}^{\infty} UC(x) (BL(x, a) - BL(x, b)) sp(x) dx}{\prod_{j=1}^{i} (1 + r_{BL,j})} \]

\(^5\) The need for two terms will become clearer when the precise bailout function for the Harrison model is discussed. It would be possible to specify a different function (not necessarily a sensible one) and have only the first term. The economic loss function above has only one term because additional capital held by the bank is not seen as a buffer to an economic downturn once failure has occurred.
Benefits in the Harrison model
The Harrison model uses roughly the approach we have just described, but there are some
differences which will be discussed further below. Because the documentation for the model is
sparse, some of our discussion is speculative.

Discount rates
\( r_{EC,j} = r_{BL,j} = 3\% \) for all \( j \). The reason for this choice of discount rate is unclear.

Discrete approximations
The integrals used to calculate expected values of utility are approximated as finite sums, using
constant costs and utility corrections over the interval between capital ratios (capital ratios are all
expressed as whole numbers of percentage points). This allows \( EC, BL \) and \( UC \) to be taken outside
the integrals for each increment, leaving only the probability density which is trivially integrated
using the \( SP \) function described above.

For the density function, set \( PD = 1.5\%, \ \text{LGD} = 30\%, \ M = 2.5, \ \text{and} \ RW = 0.5 \). \( R \) is normally calculated
from \( PD \) and would be about 0.18 in this case. However, this was felt to be “too optimistic” (see
paragraph 24 of RIS), so higher values were imposed:
\[
R = \begin{cases} 
0.2000 + 1.25(CR - .02), & 0.02 \leq CR \leq 0.14 \\
0.21 + CR, & CR \geq 0.15 
\end{cases}
\]

Next evaluate the change in survival probability for each increment in the capital ratio, in steps of
0.01 starting from \( a: \int_{a}^{a+1} sp(x)dx = SP(a + 1) - SP(a) \) for \( a = 0 \) to \( 0.20 \). There is also a value
calculated for \( \int_{0.20}^{\infty} sp(x)dx \), which is \( 1 - SP(0.20) \). \( ^6 \)

\( EC(x) \) is zero when there is no bank failure, and is otherwise\( ^7 \)
\[
\text{GDP} \times \% \ \text{GDP loss due to single failure} = \text{GDP} \times \begin{cases} 
0.1, & 0.02 \leq x \leq 0.03 \\
0.12 + 2(x - 0.04), & 0.04 \leq x \leq 0.07 \\
0.2, & x \geq 0.08 
\end{cases}
\]

\( BL(x) \) is given by the expression

\[\text{Expected bailout costs for a capital ratio of CR} = \text{bank assets} \times \left( 0.00329 \times [\text{terminal loss} + \text{asset price discount} + RW \times (21 - CR)] + \sum_{b=CR}^{2.0} \left( [\text{asset price discount} + 0.25 + RW \times (b - CR)] \left[ \Delta_{b-1}^{b-1}[1 - SP] \right] \right) \right) \]

with \text{terminal loss} \ and \ \text{asset price discount} \ both set to 0.03 (3\% of assets).

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\( ^6 \) In the model previously implemented there is an unusual, hardcoded value of 0.089 for the change in
probability when the capital ratio goes from 0.01 (or possibly from 0.00) to 0.02. It seems this value should
properly be 0.070 (for 1.00 to 2.00) or 0.203 (for 0.00 to 2.00).

\( ^7 \) In the practical implementation of the Harrison model in 2011, a single GDP loss is calculated for the shift
from a capital ratio of 0.00 to 0.02 (or possibly 0.01 to 0.02) and then manually apportioned to \( CR = 0.01 \) and
\( CR = 0.02 \). The basis for the apportionment is unclear.
The assumption implicit in the expression is that when a bank fails, its losses depend on the size of the shock. If the shock would have just wiped out a bank with a capital ratio $a$, it will also wipe out the capital of a bank with capital ratio $a - b$ and put its capital ratio $b$ in the red. With a risk weight of 50%, each lost unit of capital below zero is worth 0.5 times the bank’s initial assets. The amount $0.5b$ is therefore funded by deposit-holder losses, which the government will need to pay for in the case of a bailout.

In addition to any capital losses, the bank is expected to lose money through fire sales (this is our interpretation of the asset price discount) and bailout administration costs (this is our interpretation of the 0.25 figure).

The expected loss from shocks which are very large indeed – big enough to wipe out a bank with a capital ratio higher than 20% – is a 21% reduction in the capital ratio, plus a further 6% of assets (the asset price discount and the terminal loss).

The probability of a government bailout is set at 50%, which halves the expected bailout costs.

The discrete approximation as implemented in the Harrison model in practice – losses are treated as constant over the range of capital ratios $[a, a+1]$ – means that the losses are all greater than in the continuous case.

For instance, the loss used when $CR = 19$ and $b = 20$ (ignoring terminal losses) is 3.25% + 0.5% of assets. Because of the discrete approximation, as well as applying to shocks which would just wipe out firms with $CR = 20$, this loss applies to all shocks which would just wipe out firms with $19 < CR < 20$. The actual loss should be smaller in this second set of cases, because the shocks are smaller (less equity is wiped out).

A potential remedy to this problem is to use a better discrete approximation. See the appendix for one possibility.

**Calculating UC(x)**

A utility adjustment factor, for a particular capital ratio, is calculated:

$$Utility\ adjustment = \frac{1}{5(1 - 5x \times 0.6)^6} + \frac{4}{5(1 - \frac{5}{4}x \times 0.4)^6}$$

with

$$x = (500 \times (CR - 0.01))^{0.47}$$

A possible derivation for the utility adjustment (we are guessing this is how the result was arrived at) follows.

Divide society into two groups – the bottom quintile by income and the rest.

Before a shock which causes a bank failure, annual GDP is $a + b$. This is entirely consumed, $a$ by the bottom quintile and $b$ by the rest.

Following a bank failure GDP is reduced by a fraction $x$, to $(a + b)(1 - x)$. The lost GDP is $x(a + b)$. The bottom quintile bears a fraction $y$ of the loss and the other quintiles bear the fraction $(1 - y)$. For the
purposes of the model, \( y \) is 0.6. This implies that when a shock hits, it mostly affects those on the lowest incomes (e.g. through unemployment, lack of savings etc.).

We end up with:

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Consumption before</th>
<th>Consumption after</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom</td>
<td>( a )</td>
<td>( a - (a + b)xy = a \left[ 1 - \left( 1 + \frac{b}{a} \right)xy \right] )</td>
</tr>
<tr>
<td>Others combined</td>
<td>( b )</td>
<td>( b - (a + b)x(1 - y) = b \left[ 1 - \left( 1 + \frac{a}{b} \right)x(1 - y) \right] )</td>
</tr>
</tbody>
</table>

If it is assumed that all quintiles initially have consumption equal to 1, then \( a = 1 \) and \( b = 4 \) and the post-shock consumption becomes \( [1 - 5xy] \) for the bottom quintile and \( [1 - \frac{5}{4}x(1 - y)] \) for each of the other quintiles. The assumption that all quintiles have equal consumption is very unlikely to be correct and is arguably inconsistent with the idea that those on low incomes are most affected by a shock, but the assumption simplifies the later mathematics.

For a given utility function \( U \), utility for the bottom quintile would be given by \( U(1 - 5xy) \) and the utility for each of the other quintiles would be given by \( U\left(1 - \frac{5}{4}x(1 - y)\right) \). Weighting these together using population weights gives “society’s” utility:

\[
U_{society} = \frac{1}{5}U(1 - 5xy) + \frac{4}{5}U\left(1 - \frac{5}{4}x(1 - y)\right)
\]

Society’s marginal utility would then be:

\[
U'_{society} = \frac{1}{5}U'(1 - 5xy) + \frac{4}{5}U'\left(1 - \frac{5}{4}x(1 - y)\right)
\]

Next assume that the marginal utility function is given by:

\[
U' = \frac{1}{\text{consumption}^6}
\]

This gives

\[
U'_{society} = \frac{1}{5(1 - 5xy)^6} + \frac{4}{5\left(1 - \frac{5}{4}x(1 - y)\right)^6}
\]

which is the function used by the model.

The functional form of \( U' \) is consistent with the isoelastic utility function with \( \gamma = 6 \), but also with other utility functions including, under certain conditions, the one specified on page 249 of Barro (2009), “Rare Disasters, Asset Prices, and Welfare Costs”, AER 99(1) (the article is referred to in the RIS). It is not clear why \( \gamma = 6 \) was chosen; the RIS argues for a “relatively high degree of risk aversion”, but no for a precise figure.

A feature of this marginal utility function is that marginal utility is 1 when consumption is 1. Since consumption is initially assumed to be 1 for each quintile, society’s marginal utility in the absence of
a shock is 1. So with no shock, the utility adjustment is zero. The utility of an additional dollar of consumption increases as consumption falls below 1 (which happens when there is a shock).

The model assumes that consumption losses – in the event of a bank failure – increase as the capital ratio increases. That is, the shocks that cause a loss of consumption at high capital ratios are larger, while shocks that cause a loss of consumption at low capital ratios are smaller (recall that society highly values avoiding the biggest shocks).

At a detailed level, the model has two apparently conflicting formulas for consumption (GDP) losses. The first – for the % GDP loss due to single failure – appears in an earlier section. The second, used only for the utility adjustment, is the expression for $x$ earlier in this section.

The expression for $x$ assumes losses (2-8% of GDP as far as I can discern) which are considerably smaller than those assumed elsewhere in the model. Using the losses of 10-20% of GDP that appear in other parts of the model would result in marginal utility adjustments of 50 times, rather than 1-2 times adjustment which is actually used.

It is unclear how the parameters in the expression for $x$ were decided, or why 0.01 is subtracted from $CR$. My guess is that the subtraction is a mistake – an off-by-one error in pasting from one spreadsheet to another.

Possibly, the model could be recalibrated to use the losses assumed elsewhere in the model but still produce reasonable results (e.g. by reducing $\gamma$).

The use of marginal utility seems to overstate the effect of risk aversion, particularly for larger losses. The marginal utility in the Harrison model should, strictly, apply only to the last dollar of the large loss, with progressively lower marginal utilities for earlier dollars of loss.

**Overall calculation of benefits**

In the overall calculation of benefits, it is assumed that a bank will fail when its capital ratio is 2%, rather than 0% (the Basel Committee assumed 4%). This means that the originally calculated benefits need to be shifted: a capital ratio of $a$ implies the benefits originally calculated for a capital ratio of $a-2$.

In the practical implementation of the model in 2011, this shift appears to have been done inconsistently. The GDP benefit is shifted by 3%, and the fiscal bail-out benefit is not shifted at all. This is assumed to unintended.

**Costs of higher capital (general description)**

Changing the capital ratio may shift the supply curve upwards in the domestic market for mortgages. This is because of corporate taxes and / or because the Modigliani Miller theorem does not hold.

In response to the upward shift, the mortgage interest rate will change, affecting GDP, transfers to foreigners, and taxes. How much it changes depends on the response of bank shareholders.

If bank shareholders recognise that a higher capital ratio has reduced the riskiness of their investment, the change in the mortgage rate will be modest. If, by contrast, they maintain that the investment is just as risky, the mortgage rate will change by a greater amount.
Higher capital ratio reduces riskiness of equity cashflows (Modigliani Miller case)

If the Modigliani Miller theorem holds, then end-of-period cashflows to shareholders and debtholders respectively, valued at the beginning of the period and after all taxes, are given by:

\[
CF_E = \left( \frac{A_0 r_0}{r_{du}} - \frac{D_0 r_d}{r_{dd}} \right) (1 - t_c)(1 - t_e)
\]

and

\[
CF_D = \frac{D_0 r_d}{r_{dd}} (1 - t_d)
\]

with the meaning of the variables in the following table:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>Mortgage balances</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Mortgage interest rate</td>
</tr>
<tr>
<td>$r_{du}$</td>
<td>Discount rate for unlevered bank equity</td>
</tr>
<tr>
<td>$D_0$</td>
<td>Bank liabilities (balance)</td>
</tr>
<tr>
<td>$r_d$</td>
<td>Interest rate on bank’s liabilities</td>
</tr>
<tr>
<td>$r_{dd}$</td>
<td>Discount rate for bank liabilities</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Effective corporate tax rate (New Zealand)</td>
</tr>
<tr>
<td>$t_e$</td>
<td>Effective tax rate on dividends and capital gains on shares (NZ or overseas)</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Effective tax rate on interest payment and capital gains on debt (NZ or overseas)</td>
</tr>
</tbody>
</table>

Following a change in the capital ratio, $A_0$ and $D_0$ change to $A_1$ and $D_1$ and $r_0$ changes to some value $r_1$ which is to be determined. For tractability it is assumed that $r_d, F, r_{du}$ and all the discount rates are constant.

The changes in cashflows to shareholders and debtholders are then given by:

\[
\Delta CF_E = \left( \frac{A_1 r_1 - A_0 r_0}{r_{du}} - \frac{(D_1 - D_0)r_d}{r_{dd}} \right) (1 - t_c)(1 - t_e) + \frac{(D_1 - D_0)r_d}{r_{dd}} (1 - t_d)
\]

and

\[
\Delta CF_D = \frac{(D_1 - D_0)r_d}{r_{dd}} (1 - t_d) - \frac{(D_1 - D_0)r_d}{r_{dd}} (1 - t_d) = 0
\]

In the case where debt decreases, shareholders are effectively purchasing a stream of income from debtholders. The second term in the shareholder equation is what is paid to debtholders for the stream of income – debtholders will choose to hold on to their securities if the amount paid is any less. The first term in the shareholder equation incorporates the purchased stream of income (after the subtraction sign). This new stream of shareholder income is less risky – it is plausibly assumed – than the stream of gross income, and so the shareholder is willing to accept a lower rate of return on it. This is the Modigliani Miller theorem in action.

Because

\[
A_1 r_1 - A_0 r_0 = r_1 (A_1 - A_0) + A_0 (r_1 - r_0) = r_1 \Delta A + A_0 \Delta r
\]
and

\[ D_1 - D_0 = \frac{D_1}{A_1} (A_1 - A_0) + A_0 \left( \frac{D_1}{A_1} - \frac{D_0}{A_0} \right) = \frac{D_1}{A_1} \Delta A + A_0 \Delta \left( \frac{D}{A} \right), \]

\(\Delta CF_E\) can be decomposed into changes due to the change in the capital ratio (with assets held constant) and changes due to movements in bank assets as a result of the interaction of supply and demand:

\[
\Delta CF_E = \Delta A \left( \frac{r_1 (1 - t_e)(1 - t_e) - \frac{D_1}{A_1} r_d [(1 - t_c)(1 - t_e) - (1 - t_d)]}{r_{du}} \right) - \frac{\Delta \left( \frac{D}{A} \right) r_d [(1 - t_c)(1 - t_e) - (1 - t_d)]}{r_{dd}} + A_0 \left( \frac{\Delta r (1 - t_c)(1 - t_e) - \Delta \left( \frac{D}{A} \right) r_d [(1 - t_c)(1 - t_e) - (1 - t_d)]}{r_{du}} \right) - \frac{r_{dd}}{r_{du}} \left( 1 - t_d \right). (1 - t_c) (1 - t_e)
\]

Suppose that, with assets held constant, the shareholder wishes \(\Delta CF_E\) to be zero and will adjust the mortgage interest rate to ensure this is the case. This amounts to recovering the additional tax costs of a higher capital ratio from mortgage borrowers. Then the change in the mortgage rate must be:

\[
\Delta r = \Delta \left( \frac{D}{A} \right) r_d \left[ r_{du} - \frac{1 - t_d}{(1 - t_c)(1 - t_e)} \cdot r_{du} \right]
\]

In reality, assets are likely to change. Depending on how they change – this depends on the precise specification of the supply and demand curves – some other value of \(\Delta r\) might be the equilibrium outcome (in the case of a higher capital ratio, the expression for \(\Delta r\) above is an upper bound).

**Higher capital does not reduce riskiness of equity cashflows**

In contrast to the previous case, suppose shareholders attribute the same riskiness to their cashflows no matter how indebted the bank. End-of-period cashflows, valued at the beginning of the period after all taxes, are then:

\[
CF_E^* = \left( \frac{A_0 r_0 - D_0 r_d}{r_{de}} \right) (1 - t_c)(1 - t_e)
\]

and

\[
CF_D^* = \frac{D_0 r_d}{r_{dd}} (1 - t_d)
\]

where \(r_{de}\) is the (assumed constant) discount rate for equity, whether levered or unlevered.

Changes in cashflows following a change in the capital ratio are then:

\[
\Delta CF_E^* = \left( \frac{r^* - A_0 r_0 - (D^*_1 - D_0) r_d}{r_{de}} \right) (1 - t_c)(1 - t_e) + \frac{(D^*_1 - D_0) r_d}{r_{dd}} (1 - t_d)
\]

and

\[\Delta A\] is normally correlated with \(\Delta r\), so the decomposition is not pure. It is still useful, particularly in the cases where the demand curve is vertical (\(\Delta A = 0\)) or the supply curve is horizontal (\(\Delta r\) does not depend on \(\Delta A\)).
\[ \Delta CF_D^* = 0 \]

As in the previous case, the second term on the right hand side of the first equation is the amount paid to debtholders in the case of a higher capital ratio to acquire their stream of cashflows.\(^9\)

Decomposing the changes\(^{10}\):

\[
\Delta CF_E^* = \Delta A^* \left( \frac{D_1}{A_1} \frac{(1-t_c)(1-t_e)}{r_{de}} - \frac{D_1}{A_1} \frac{1-t_d}{r_{dd}} \left( \frac{1-t_c(1-t_e)}{r_{de}} \right) \right)
+ \Delta A_0 \left( \frac{\Delta r^*}{r_{de}} \frac{1-t_d}{r_{dd}} - \Delta \left( \frac{D}{A} \right) r_d \left( \frac{1}{r_{dd}} - \frac{1-t_c(1-t_e)}{r_{de}} \right) \right)
\]

Holding assets constant and finding the change in mortgage rates which leaves discounted cashflows unchanged gives:

\[
\Delta r^* = \Delta \left( \frac{D}{A} \right) \frac{r_d}{r_{dd}} \left( \frac{1}{(1-t_c)(1-t_e)} \right) r_{de}
\]

As in the case where the Modigliani Miller theorem holds, \(\Delta r^*\) is an upper bound. Depending on the specification of the supply and demand curves, \(\Delta r^*\) could be lower.

**Changes in income and transfers**

A rise in the mortgage interest rate is expected to reduce GDP growth.\(^{11}\) This is modelled using the Basel Committee’s approach, which is discussed later in this document.

A rise in the interest rate and a changing capital ratio might also result in a transfer of money from mortgage borrowers to offshore bank shareholders and debtholders, which is a cost to New Zealand. The following tables show how the changes in cashflows are directed to various stakeholders.\(^{12}\) It is assumed that all shareholders are foreign and that all debtholders who sell back their debt (or take on new debt) are also foreign. Corporate taxes are imposed on the bank in New Zealand and apply to income net of interest payments. Dividend and interest taxation are imposed on foreign stakeholders only by foreign governments.\(^{13}\)

---

\(^9\) This amount is determined by the debtholder’s valuation of the cashflows. The debtholder’s valuation is the same in the Modigliani-Miller case and in this one, because the discount rate for debt is assumed constant. This ignores the likelihood that the discount rate for debt might decrease because debt claims on a more highly capitalised bank are less risky.

\(^{10}\) Assuming the bank is always at the prescribed capital ratio, then \(\frac{D_1}{A_1} = \frac{D_1^*}{A_1^*}\).

\(^{11}\) This is the assumption which has been made in the Basel modelling and earlier RBNZ work. It is inconsistent with a fixed value of bank assets, which has also been assumed in the past when working out the increase in the interest rate, unless the demand curve is vertical. In principle, the more general specification in this document allows for a decrease in GDP and a *consistent* increase in interest rates. In practice, it might be difficult to know the specifications for the supply and (especially) demand curves.

\(^{12}\) The initial retirement of existing debt is a payment to debtholders at the market price for the debt. The market price is determined by the discounted value of future cashflows. (In the case of a falling capital ratio, the payment is from debtholders to the bank, again at the market price for the stream of future interest cash flows). This initial payment is shown as a “one-time exchange” in the table). The initial exchange is assumed to have no tax consequences.

\(^{13}\) At the cost of some complexity, it should be straightforward to repeat the analysis with alternative assumptions.
<table>
<thead>
<tr>
<th>Stakeholder</th>
<th>Change in cash flows as the result of a change in the capital ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shareholder (perpetually)</td>
<td>[ A_0 \left( \Delta r - r_d \Delta \left( \frac{D}{A} \right) \right) + \Delta A \left( r_1 - r_d \frac{D_1}{A_1} \right) (1 - t_c)(1 - t_e) ]</td>
</tr>
<tr>
<td>Shareholder (one-time exchange with debtholder)</td>
<td>[ \frac{r_d}{r_{dd}} A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A (1 - t_d) ]</td>
</tr>
<tr>
<td>Debtholder (perpetually)</td>
<td>[ r_d A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A (1 - t_d) ]</td>
</tr>
<tr>
<td>Debtholder (one-time exchange)</td>
<td>[ - \frac{r_d}{r_{dd}} A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A (1 - t_d) ]</td>
</tr>
<tr>
<td>NZ fisc (perpetually)</td>
<td>[ A_0 \left( \Delta r - r_d \Delta \left( \frac{D}{A} \right) \right) + \Delta A \left( r_1 - r_d \frac{D_1}{A_1} \right) \frac{t_c}{t_d} ]</td>
</tr>
<tr>
<td>Foreign fisc (perpetually)</td>
<td>[ A_0 \left( \Delta r - r_d \Delta \left( \frac{D}{A} \right) \right) + \Delta A \left( r_1 - r_d \frac{D_1}{A_1} \right) (1 - t_c)t_e + r_d \left[ A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A \right] t_d ]</td>
</tr>
<tr>
<td>Total (perpetually)</td>
<td>[ A_0 \left( \Delta r - r_d \Delta \left( \frac{D}{A} \right) \right) + \Delta A \left( r_1 - r_d \frac{D_1}{A_1} \right) + r_d \left[ A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A \right] = A_0 \Delta r + \Delta A r_1 = A_1 r_1 - A_0 r_0 ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stakeholder</th>
<th>Change in discounted cash flows as the result of a change in the capital ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shareholder (perpetually)</td>
<td>[ A_0 \left( \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{du}} \right) + \Delta A \left( \frac{r_0}{r_{du}} + \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{dd} A_1} \right) (1 - t_c)(1 - t_e) ]</td>
</tr>
<tr>
<td>Shareholder (one-time exchange with debtholder)</td>
<td>[ \frac{r_d}{r_{dd}} A_0 \Delta \left( \frac{D}{A} \right) + r_{du} \left[ A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A \right] (1 - t_d) ]</td>
</tr>
<tr>
<td>Debtholder (perpetually)</td>
<td>[ r_d A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A (1 - t_d) ]</td>
</tr>
<tr>
<td>Debtholder (one-time exchange)</td>
<td>[ - \frac{r_d}{r_{dd}} A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A (1 - t_d) ]</td>
</tr>
<tr>
<td>NZ fisc (perpetually)</td>
<td>[ A_0 \left( \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{du}} \right) + \Delta A \left( \frac{r_0}{r_{du}} + \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{dd} A_1} \right) \frac{t_c}{t_d} ]</td>
</tr>
<tr>
<td>Foreign fisc (perpetually)</td>
<td>[ A_0 \left( \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{du}} \right) + \Delta A \left( \frac{r_0}{r_{du}} + \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{dd} A_1} \right) (1 - t_c)t_e + \frac{r_d}{r_{dd}} \left[ A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A \right] t_d ]</td>
</tr>
<tr>
<td>Total (perpetually)</td>
<td>[ A_0 \left( \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{du}} \right) + \Delta A \left( \frac{r_0}{r_{du}} + \frac{\Delta r - r_d \Delta \left( \frac{D}{A} \right)}{r_{dd} A_1} \right) + \frac{r_d}{r_{dd}} \left[ A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A \right] = \frac{A_0 \Delta r + \Delta A r_1}{r_{du}} = \frac{A_1 r_1 - A_0 r_0}{r_{du}} ]</td>
</tr>
</tbody>
</table>
The first table shows undiscounted cash flows. The second table shows discounted cash flows. The discount rates used are “NZ Inc”’s rates. These are assumed to be always the same as the discount rates used by investors in the case where Modigliani Miller holds.

The change in discounted cashflows that go offshore, given the assumptions above, is the sum of the shaded rows in the second table:

\[
\text{Transfers} = A_0 \left( \frac{\Delta r}{r_{du}} - \frac{r_d}{r_{dd}} \Delta \left( \frac{D}{A} \right) \right) + \Delta A \left( \frac{\eta_0 + \Delta r}{r_{du}} - \frac{r_d}{r_{dd}} \frac{D_1}{A_1} \right) \left( 1 - t_c \right) + \frac{r_d}{r_{dd}} \left[ A_0 \Delta \left( \frac{D}{A} \right) + \frac{D_1}{A_1} \Delta A \right]
\]

Assuming bank shareholders recognise changes in risk (the Modigliani Miller theorem holds), substitute in

\[
\Delta r = \Delta \left( \frac{D}{A} \right) r_d \left[ \frac{r_{du}}{r_{dd}} - \frac{1}{1 - t_c} \left( 1 - t_d \right) \frac{r_{du}}{r_{dd}} \right]
\]

and rearrange to obtain

\[
\text{Transfers} = A_0 \Delta \left( \frac{D}{A} \right) r_d \left[ \frac{r_{du}}{r_{dd}} \left( 1 - t_c \right) - \frac{1}{1 - t_c} \left( 1 - t_d \right) \frac{r_{du}}{r_{dd}} + \frac{r_d}{r_{dd}} \frac{D_1}{A_1} t_c \right]
\]

The first term has two components: the decrease in pre-tax interest payments to non-resident debtholders (in the case of a higher capital ratio), and the increase in post-New Zealand-tax dividends to non-resident shareholders. The non-resident shareholder wants the purchased stream of debtholder income to generate the same after-tax return as it would if it was held as debt. This means that the dividend is grossed up to reverse the effect of New Zealand company taxes (which do not apply to debt). A further grossing up (or, quite plausibly, down) is required to reflect any difference between foreign dividend and interest taxation.

If bank shareholders do not recognise changes in risk (the Modigliani Miller theorem does not hold even in part), then instead substitute in \(\Delta r^*\) and obtain

\[
\text{Transfers} = A_0 \left( \Delta \left( \frac{D}{A} \right) r_d \left[ \frac{r_{du}}{r_{dd}} \left( 1 - t_c \right) - \frac{1}{1 - t_c} \left( 1 - t_d \right) \frac{r_{du}}{r_{dd}} + t_c \right] \right) + \Delta A \left( \frac{T_0}{r_{du}} \left( 1 - t_c \right) + \Delta \left( \frac{D}{A} \right) r_d \left[ \frac{r_{du}}{r_{dd}} \left( 1 - t_c \right) - \frac{1}{1 - t_c} \left( 1 - t_d \right) \frac{r_{du}}{r_{dd}} + \frac{r_d}{r_{dd}} \frac{D_1}{A_1} t_c \right] \right)
\]
By way of illustrative example, assign some values to the parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>$1.000$</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>$A_1$</td>
<td>$1.000$</td>
<td>For the MMI case (noting $\Delta r$ is negligible)</td>
</tr>
<tr>
<td>or $A_1$</td>
<td>$0.999$</td>
<td>For the case without MM1 (arbitrarily less than $A_0$)</td>
</tr>
<tr>
<td>$\Delta A$</td>
<td>-0.001</td>
<td>Calculated directly</td>
</tr>
<tr>
<td>$D_0/A_0$</td>
<td>0.960</td>
<td>Rough estimate of current ratio</td>
</tr>
<tr>
<td>$D_1/A_1$</td>
<td>0.955</td>
<td>Assumed lower ratio</td>
</tr>
<tr>
<td>$\Delta (D/A)$</td>
<td>-0.005</td>
<td>Calculated directly</td>
</tr>
<tr>
<td>$r_0$</td>
<td>0.05500</td>
<td>Rough estimate of current interest rate on loan book</td>
</tr>
<tr>
<td>$r_d$</td>
<td>0.03000</td>
<td>Rough estimate of current interest rate on bank debt</td>
</tr>
<tr>
<td>$r_{du}$</td>
<td>0.02975</td>
<td>Rough estimate of current required return after all taxes</td>
</tr>
<tr>
<td>$r_{dd}$</td>
<td>0.02310</td>
<td>Rough estimate of current required return after all taxes</td>
</tr>
<tr>
<td>$r_{de}$</td>
<td>0.18935</td>
<td>Calculated from numbers above using Modigliani Miller II</td>
</tr>
<tr>
<td>$t_c$</td>
<td>0.30</td>
<td>Approximate corporate tax rate</td>
</tr>
<tr>
<td>$t_d$</td>
<td>0.30</td>
<td>Guessed foreign tax rate</td>
</tr>
<tr>
<td>$t_e$</td>
<td>0.15</td>
<td>Guessed foreign tax rate</td>
</tr>
</tbody>
</table>

The results are:

<table>
<thead>
<tr>
<th></th>
<th>With MMI</th>
<th>Without MMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta r$</td>
<td>0.341bp</td>
<td>13.0bp</td>
</tr>
<tr>
<td>Transfers (per $ of initial assets)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets held constant</td>
<td>-0.001</td>
<td>0.029</td>
</tr>
<tr>
<td>Due to asset change</td>
<td>0.000</td>
<td>-0.002</td>
</tr>
<tr>
<td>Total</td>
<td>-0.001</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Changes in income and transfers in the Harrison model
The Harrison model incorporates three effects of a higher capital ratio on income and transfers:

1. A reduction in GDP because of an increase in mortgage lending interest rates
2. An increase in transfers abroad because of a switch from debt to (higher-cost) equity
3. An increase in tax paid in New Zealand, owing to reduced interest deductions and increased company profits.

**GDP effect**
The GDP effect is taken directly from earlier work by the Basel Committee on Banking Supervision, which found that a one percentage point increase in the capital ratio would increase mortgage lending interest rates by 13 basis points and decrease GDP by 0.09%. In calculating the interest rate effect, the Basel Committee assumed that total bank assets would be unchanged. The Committee then relied on a suite of models to assess the likely effect on GDP.

Lost GDP per percentage point capital ratio = Initial GDP $\times$ 0.0009

$= 200 \text{ billion} \times 0.0009 = 180 \text{ million}$
The approach appears to be internally inconsistent because the decrease in GDP would be driven by lower investment and – logically – reduced bank assets. However, it has the advantage that nothing needs to be known about the precise specification of supply and demand in the mortgage market.

The Basel-determined effect is an annual effect. The Harrison model discounts the infinite stream of annual effects (assumed to be the same at all initial capital ratios) using a compounding 3% rate to determine the net present value of GDP losses. The Harrison model also reduces the discounted result by 85%, because the Basel Committee estimate assumes the Modigliani Miller theorem does not hold at all, which results in over-stated estimates.

The Harrison model doubles the Basel-determined effect, apparently because of an erroneous assumption that the effect of an increase in the leverage ratio – and not the capital ratio – was wanted.

**Transfers effect**

The transfers effect is calculated by assuming that when a dollar of equity is substituted for a dollar of debt, the interest rate on debt is saved and the rate of return on equity is instead paid.

The margin between the interest rate on debt and the return on equity (a risk premium) is assumed to be 10 percentage points, regardless of the level of capital.

The total amount of equity substituted for debt should be calculated as 1% of risk-weighted assets. Risk weighted assets are calculated indirectly, using tier 1 capital and the current capital ratio.

\[
\text{Additional transfers} = \text{change in debt} \times \text{return on equity} - \text{change in debt} \times \text{debt interest rate} \\
= \text{change in debt} \times (\text{return on equity} - \text{debt interest rate}) \\
= .01 \times \frac{\text{risk weighted assets}}{\text{initial capital ratio}} \times (\text{return on equity} - \text{debt interest rate}) \\
= .01 \times \frac{\text{initial tier one capital}}{\text{initial capital ratio}} \times .10
\]

But as with the GDP effect, the Harrison model apparently doubles the transfers effect by determining the change using the current leverage ratio rather than the current capital ratio.

The transfer effect is annual. The endless stream of transfers is discounted by a compounding 3% rate to determine a total cost. The Harrison model also reduces the discounted result by 85%, because assumption that the risk premium does not vary with the capital ratio is unrealistic and results in overstated estimates.

**Tax effect**

It is unclear exactly how the tax effect was calculated, but we can roughly reproduce the numbers.

That basic rationale is that as debt decreases there are fewer tax deductions allowed to the bank, and New Zealand tax increases. New Zealand does not capture all the benefit because foreign shareholders seek to recoup some of the additional tax costs they incur by raising mortgage interest rates for bank customers (an earlier presentation of the model indicated that half of the benefits are lost in this way).
Additional tax = change in debt × debt interest rate × company tax rate × \frac{1}{2} \\
= risk weighted assets × 0.01 × 0.05 × 0.28 × \frac{1}{2}

We have assumed an interest rate of 5% and a company tax rate of 28%. The effect is an annual one. Annual effects are summed and discounted to establish the net present value of tax increases. The net real discount rate on debt is assumed to be 5%, “consistent with a risky debt investment”.

We arrived at an estimate of $266 million, which is close to the $250 million the Harrison model should produce.

As in the cases of the GDP and transfer effects, the Harrison model appears to have incorrectly assumed that the effect should be calculated for a one percentage point change in the leverage ratio, rather than the capital ratio (it is a little more difficult to tell if this is so for the tax effect because there is less documentation, but we are assuming it is and that it erroneously doubles the size of the tax effect).

Comparing the Harrison approach and the new approach in this document

The combined effect of the tax and transfer effects after correcting for the doubling which we think is unintended is a net cost of roughly 0.2% of total bank assets. Using the illustrative numbers from the approach presented earlier produces a cost of about 0.4% of total bank assets (this uses the numbers from the case where Modigliani Miller does not hold, but then reduces them by 85% as Harrison has done).

Appendix

SP is a cumulative distribution function

From earlier results:

$$SP = \Phi \left[ \frac{1-R}{R} \left( \Phi^{-1} \left[ CR \times \frac{1-1.5b}{1+b(M-2.5)} \times \frac{RW}{LGD} + PD \right] - \frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) \right) \right]$$

As it stands, this is not defined everywhere, because the domain of $\Phi^{-1}$ is only [0,1].

So define $SP$ to be zero when

$$\frac{CR \times 1-1.5b}{1+b(M-2.5)} \times \frac{RW}{LGD} + PD \leq 0$$

or, equivalently when

$$CR \leq -PD \times \frac{1+b(M-2.5)}{1-1.5b} \times \frac{LGD}{RW} = \text{low}$$

That is, the best the bank can possibly do is to suffer no losses at all, in which case it can convert its provisions into equity. If it has negative equity which exceeds provisions in magnitude, the bank fails with certainty. (This explanation is slightly hand-wavy; I am ignoring the second factor, the term involving $b$).
Also, define SP to be one when
\[ CR \times \frac{1 - 1.5b}{1 + b(M - 2.5)} \times \frac{RW}{LGD} + PD \geq 1 \]
or
\[ CR \geq (1 - PD) \times \frac{1 + b(M - 2.5)}{1 - 1.5b} \times \frac{LGD}{RW} = \text{high} \]

That is, if the bank holds enough equity, together with provisions, to cover the loss of its entire portfolio, at the given LGD, then it is certain to survive. (Again, hand-wavy).

Then
\[ \lim_{CR \to \text{low}^+} SP(CR) = SP(\text{low}) = \lim_{CR \to \text{low}^-} SP(CR) = \lim_{CR \to -\infty} SP(CR) = 0 \]
(where the left hand side follows, at least intuitively, from the properties of the cumulative normal and inverse cumulative normal distributions) and
\[ \lim_{CR \to \text{high}^-} SP(CR) = SP(\text{high}) = \lim_{CR \to \text{high}^+} SP(CR) = \lim_{CR \to +\infty} SP(CR) = 1 \]

From this, and the continuity of the cumulative normal and inverse cumulative normal functions over their respective domains, SP is continuous in CR. (Actually, continuity depends on the functional form of R as well. R is assumed to be continuous or, if it is not, the lack of continuity is assumed to have negligible effects).

By the chain rule and the inverse function theorem:
\[
\frac{dSP}{dCR} = \frac{1 - 1.5b}{1 + b(M - 2.5)} \times \frac{RW}{LGD} \times \sqrt{\frac{1 - R}{R}} \times \phi \left[ \frac{1 - R}{R} \left( \Phi^{-1} \left( CR \times \frac{1 - 1.5b}{1 + b(M - 2.5)} \times \frac{RW}{LGD} + PD \right) \right) - \frac{1}{\sqrt{1 - R}} \Phi^{-1}(PD) \right] > 0
\]

where \( \phi \) is the standard normal probability distribution function and b is assumed to be less than \( \frac{2}{3} \).

From the properties above, \( SP(CR) \) is a cumulative distribution function.

\[
\frac{dSP}{dCR} = sp \ (\text{lower case}) \text{ is the corresponding probability density function (it is 0 where } SP(CR) \text{ is 0 or 1)}.\]

A better discrete approximation to the integral
This is one possibility for approximating the integral used to calculate bailout costs.

Recall the expected bailout costs at a (discrete) capital ratio of CR:
\[ BC = \text{Expected bailout costs for a capital ratio of CR} \]
\[ = \text{bank assets} \]
\[ \times \left( 0.00329 \times [0.06 + RW \times (21 - CR)] \right. \]
\[ \left. + \sum_{x=CR}^{20} \left( [0.0325 + RW \times (x - CR)][\Delta_x^{x-1}[1 - SP]] \right) \right) \]

That formulation assumes the capital ratios change in whole percentage point increments. But suppose instead that we allow any change \( \epsilon \):

\[ BC = \text{bank assets} \]
\[ \times \left( 0.00329 \times [0.06 + RW \times (21 - CR)] \right. \]
\[ \left. + \sum_{x=CR}^{20} \left( [0.0325 + RW \times (x - CR)][\Delta_x^{x-1}[1 - SP]] \right) \right) \]

It is implicit that the summation is over \( x, x + \epsilon, x + 2\epsilon, \ldots, 20 \). In the limit as \( \epsilon \to 0 \) we obtain bailout costs for a continuous capital ratio \( CR \):

\[ BC = \text{bank assets} \]
\[ \times \left( 0.00329 \times [0.06 + RW \times (21 - CR)] \right. \]
\[ \left. + \int_{x=CR}^{20} \left( [0.0325 + RW \times (x - CR)][sp(x)] dx \right) \right) \]

We would like to see how the bailout costs change as \( CR \) changes from some ratio \( a \) to another ratio \( b \). We will first work how the costs change for an infinitesimal change in \( CR \). Then we will sum up the effects of the infinitesimal changes as \( CR \) changes from \( a \) to \( b \).

To work out the effect of an infinitesimal change in \( CR \), we calculate the derivative, using Liebniz’s rule for differentiating under the integral:

\[ \frac{dBC}{dCR} = \text{bank assets} \times \left( -0.00329 \times RW \int_{x=CR}^{20} sp(x) dx - 0.0325sp(CR) \right) \]
\[ = \text{bank assets} \times \left( -0.00329 \times RW[SP(20) - SP(CR)] - 0.0325sp(CR) \right) \]
Integrating this expression gives the change in the bailout costs when moving from a CR of $a$ to a CR of $b$:

\[
\text{Costs of moving from } a \text{ to } b = \text{bank assets} \\
\begin{align*}
&\times \int_{CR=a}^{b} -0.00329 \times RW - RW[SP(20) - SP(CR)] - 0.0325 sp(CR)dCR \\
&= \text{bank assets} \\
&\times \left( -0.00329 \times RW \left( b - a \right) - 0.0325 [SP(b) - SP(a)] \\
&- RW \int_{CR=a}^{b} SP(20) - SP(CR)dCR \right)
\end{align*}
\]

The integral inside the parentheses is an area on the graph of $CR$ vs $SP(CR)$ (the cumulative distribution function).

Specifically, it is the area below $SP(20)$, above the cumulative distribution function, to the left of $b$ and to the right of $a$. For reasonably small values of $b-a$, a trapezoidal approximation should be adequate and will certainly be better than a rectangular one. The approximation is:

\[
\int_{CR=a}^{b} SP(20) - SP(CR)dCR \approx \frac{1}{2} \left( SP(20) - SP(a) + SP(20) - SP(b) \right) \left( b - a \right)
\]

\[
= \left[ SP(20) - \frac{1}{2} (SP(a) + SP(b)) \right] \left( b - a \right)
\]
which gives:

\[
\text{Costs of moving from } a \text{ to } b = \\
= \text{bank assets} \\
\times \left( -0.00329 \times RW \ (b - a) - 0.0325[SP(b) - SP(a)] \\
- RW \left[ SP(20) - \frac{1}{2}(SP(a) + SP(b)) \right] (b - a) \right)
\]

This form is inconvenient for finding the effect of capital ratios in excess of 20%. If we had taken the summation in the original (discrete) equation to infinity and dropped the terminal term, we would have instead ended up with:

\[
\text{Costs of moving from } a \text{ to } b = \\
= \text{bank assets} \\
\times \left( -0.0325[SP(b) - SP(a)] - RW \left[ SP(\infty) - \frac{1}{2}(SP(a) + SP(b)) \right] (b - a) \right)
\]

\(SP\) is not defined for an infinite capital ratio (or, indeed, any fairly high capital ratio). But if we set \(SP\) to 1 when the capital ratio is too high to allow for it to be calculated, we get, finally:

\[
\text{Costs of moving from } a \text{ to } b = \\
= \text{bank assets} \\
\times \left( -0.0325[SP(b) - SP(a)] - RW \left[ 1 - \frac{1}{2}(SP(a) + SP(b)) \right] (b - a) \right)
\]

Note that this is a cost, so a negative result is a negative cost (a benefit). If \(b\) is higher than \(a\) (moving from a low to a high capital ratio) then the cost is negative (there is a benefit).