A model to calculate the net benefits of changing bank capital requirements

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1 Purpose of this document

This document explains how a model (the 'V2' model) was built to estimate the costs and benefits of changing the regulatory capital ratio for banks. The intended audience for this document is a person seeking to understand the model in detail or to implement the model.

Caution: this model is flawed in many ways. At least in its current form, it should not be used to support arguments for a particular capital requirement.

2 Background

When New Zealand made the decision to adopt Basel III, Ian Harrison (and others) built a model – which we will refer to as the Harrison model – to assess the costs and benefits of adoption.

Descriptions of the Harrison model can be found in the regulatory impact statement that accompanied Basel III adoption (#4932427) and a Powerpoint presentation to FSO (#4484716). These are intuitive rather than technical explanations. The model itself can be found in two spreadsheets, one containing the main model (#4893410) and the other containing calculations for a utility adjustment (#4893409).

The Harrison model drew on other, international work by the Basel Committee on Banking Supervision [Basel (2006)] and the Bank of England [Bank of England], particularly to determine the effects of banking crises on GDP and the effects of higher capital ratios on retail interest rates (and, thereby, also GDP).

The Reserve Bank is currently undertaking a review of the capital regulations that apply to New Zealand-incorporated banks. As part of the review, a decision was taken to update the Harrison model to reflect current circumstances.

The update ended up being a re-implementation with some significant changes, because the initial model was poorly documented and its implementation error-ridden. The re-implementation is the V2 model described in this document.

3   Outline of the V2 model

The model estimates the costs and benefits of different regulatory capital requirements for banks. The idea is that an optimal capital requirement – where net benefits are greatest – can be determined from the model.

Costs and benefits are determined, crudely speaking, as the changes in national welfare caused by changing capital requirements.

Higher capital requirements reduce the chance of systemic banking crises. Since banking crises reduce GDP and can impose bailout costs on taxpayers, the avoidance of crises is a benefit.

At the same time, higher capital requirements can impose costs. They might raise banks' cost of capital, leading to reduced lending and economic activity, and to increased transfers to foreigners. The magnitude of costs depends crucially on the extent to which bank shareholders and debtholders respond to higher capital requirements. Simple economic theory suggests that bank shareholders and debtholders should be willing to accept reduced returns when capital requirements are high, because they face less risk, but the theory is disputed and the empirical evidence is mixed.

The V2 model assumes, as did the Harrison model, that the only form of capital is common equity. In a world where capital is only common equity, the capital requirement is both the "common equity tier 1" (CET1) requirement and the "total capital" requirement. In the real world, other forms of capital are permitted and used. When other forms of capital are used, the model should probably be regarded as producing a result which is somewhere between a CET1 requirement and a total capital requirement.

The banking sector is modelled as a single institution with a homogeneous portfolio that is fully diversified. That is, the idiosyncratic risk of individual exposures is eliminated at the portfolio level. In this framework a crisis occurs if unexpected portfolio losses exceed the level of capital; such a crisis is by definition systemic.

Further discussion of the V2 model, including the choice of parameters, may be found in the FSO document #6656095. Version 1.1 of that document (which is not the version which went to FSO) contains much additional discussion and analysis. #6567676 was an early version of this document. It contains errors but also contains additional discussion of the Harrison model and the differences between the Harrison and V2 models.

There is a spreadsheet implementation of the V2 model (#6700690).

3.1 Digression – the risk-weighted capital ratio

The model is formulated in terms of a capital requirement per dollar of exposure-at-default, but the capital requirement is more commonly expressed as a minimum *risk-weighted capital ratio.*

Dividing the capital requirement from the V2 model by an average risk weight would give the more familiar risk-weighted ratio. The conversion is not done in this document, but is carried out in our spreadsheet implementation.

A difficulty is that the Basel framework *defines* the risk weight to be 12.5 times the minimum capital requirement, so that the capital ratio is always 8%. So we had to take a different approach.

We crudely estimated the current average risk weight, using the current capital ratio of 8%, and used it as the "correct" weight rw. *We then held the estimated risk weight* rw *constant as we varied the capital requirement, so that the risk-weighted capital ratio changed linearly with the capital requirement* $\left(cr = \frac{k}{rw}\right)$.

3.2 Digression – failure threshold

The model described in this document assumes that a bank fails when it runs out of capital entirely. The Harrison model, and some international models, assume that banks fail when there is some small positive amount of capital left.

Harrison assumed a failure threshold of 2% of risk-weighted assets. This was not modelled explicitly. Rather, Harrison modelled the zero-capital threshold and then "shifted" all the results of the model. That is, the net benefits of moving from a risk-weighted capital ratio of a to a ratio of b were relabeled as the benefits from moving from a risk-weighted capital ratio of $a + .02$ to a ratio of $b + .02$.

In our spreadsheet implementation of the model, we copy Harrison's approach, but in this document, we assume a zero-capital threshold.

4 Components of the model

This section explains each of the parts of the model.

4.1 The Basel equation

We assume that banks are less likely to fail when they have a high capital ratio. We use the Basel capital equation [Basel (2006)] to work out *how much* less likely.

The Basel equation gives the amount of capital the bank has to hold, per dollar of exposure, to have a 99.9% chance of surviving the next year. 99.9% is the *survival probability (sp)*.

We interpret the equation as a function of sp, for given values of the other parameters:

$$k(sp) = lgd \times \left\{ \Phi\left[\sqrt{\tfrac{1}{1-r}}\left(\Phi^{-1}\left[pd\right] + \sqrt{r}\Phi^{-1}\left[sp\right]\right)\right] - pd \right\} \times \tfrac{1+(m-2.5)b}{1-1.5b} \qquad (1)$$

where k is the capital requirement per dollar of exposure-at-default, lgd is the (percentage) loss-given-default, $\Phi(x)$ is the standard normal cumulative distribution function and $\Phi^{-1}(x)$ is its inverse, r is the correlation parameter, pd is the unconditional probability of default, sp is the survival probability, m is maturity, and b is given by $(.11852 - .05478\ln pd)^2$.

4.1.1 Digression

In the standard Basel framework the correlation parameter is defined as a function of pd:

$$r = .12 \times \frac{1 - \mathrm{e}^{-50pd}}{1 - \mathrm{e}^{-50}} + .24 \times \left(1 - \frac{1 - \mathrm{e}^{-50pd}}{1 - \mathrm{e}^{-50}}\right) \qquad (2)$$

Harrison concluded that the Basel function is inconsistent with plausible rates of bank failure. So the Harrison model uses its own, higher correlation parameter. The V2 model uses a continuous analogue of Harrison's parameter:

$$r = \begin{cases} .2 + 1.25\left(\frac{k}{rw} - .02\right) & \frac{k}{rw} < .14 \\ .21 + \frac{k}{rw} & .14 \leq \frac{k}{rw} < .2 \\ .41 & \frac{k}{rw} \geq .2 \end{cases} \qquad (3)$$

which is depicted in figure 1. **End of digression.**

The Basel capital equation $k(sp)$ can be inverted so that survival probability is a function of capital.

$$sp(k) = \Phi\left[\sqrt{\tfrac{1}{r}}\left\{\Phi^{-1}\left[Z(k)\right]\sqrt{1-r} - \Phi^{-1}\left[pd\right]\right\}\right] \qquad (4)$$

where

$$Z(k) = \frac{k}{lgd}\frac{1+(m-2.5)b}{1-1.5b} + pd$$

Figure 2 shows what $sp(k)$ looks like, with the inset graph showing a zoomed portion of the curve.
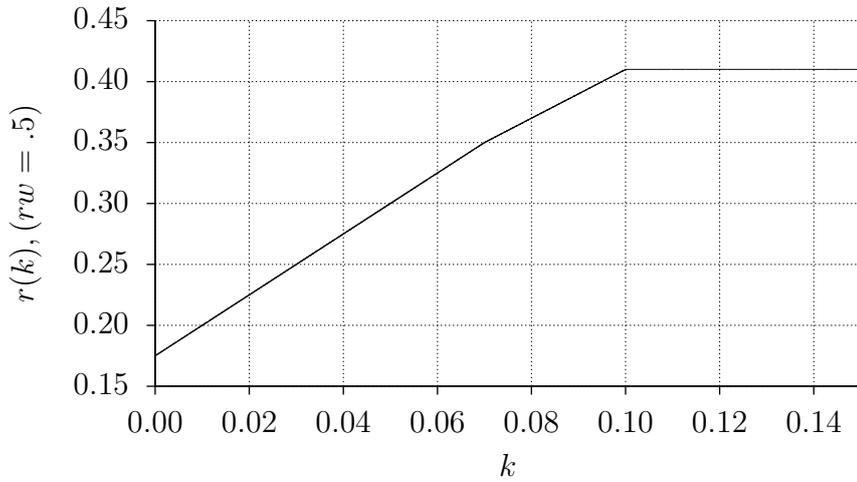
Figure 1: Correlation factor as a function of k with $rw = .5$
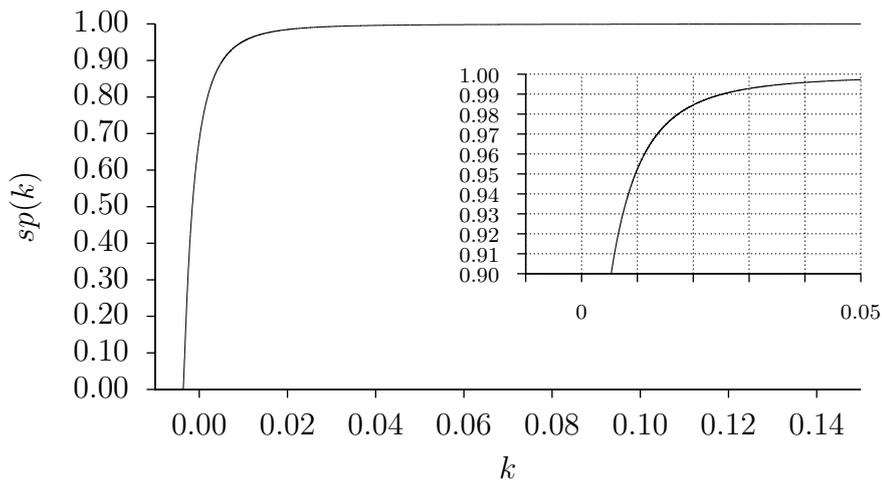
Figure 2: Survival probability (CDF) as a function of capital

To be clear, the inverted equation gives the probability the bank will survive the next year if it holds capital of exactly k per dollar of exposure.

Using the inverted equation, we can work out how much the survival probability changes as we increase the capital requirement. If the capital requirement changes from a to b, the probability of survival changes by $sp(b) - sp(a)$.

There is another way to think about sp which turns out to be helpful and important.

Suppose that $k = b$. Then the probability that the bank fails in the next year

is $1 - sp(b)$. The bank is buffeted by shocks and $1 - sp(b)$ of the time they are big enough to bring the bank down.

Now suppose that $k = a < b$. The probability that the bank fails is now $1 - sp(a) > 1 - sp(b)$. Any shock that would have wiped out the bank when k was equal to b will still wipe out the bank when $k = a$, but there are now some smaller shocks which will also wipe out the bank.

The probability of failure has gone up by $1 - sp(a) - [1 - sp(b)] = sp(b) - sp(a)$. $\text{sp(b)} - \text{sp(a)}$ *is the probability of a shock which is big enough to wipe out a bank with capital* a *but not big enough to wipe out a bank with capital* b.

4.1.2 Digression

As it stands, $sp(k)$ is not defined when $Z(k) = \leq 0$ or ≥ 1. So we choose to set $sp(k)$ to 0 when $Z \leq 0$ and 1 when $Z \geq 1$. This is mathematically convenient but also intuitively reasonable.

If the maturity correction term is ignored, then $Z = 0$ implies that the bank has a shortfall of capital which is equal to provisions. The best the bank can possibly do is to suffer nil losses, in which case all provisions can be reversed and the bank will be restored to a nil equity position. If $Z \leq 0$ then the bank fails with certainty. Similarly, $Z \geq 1$ implies that the bank holds enough capital to absorb the loss of its entire portfolio (taking into account provisions and the given lgd).

If we now assume that r and b, which can be non-constant functions, are continuous then $sp(k)$ is a cumulative distribution function and there is a corresponding probability density function $sp'(k)$. **End of digression.**

Using the chain rule and inverse function theorem, the derivative of $sp(k)$ is:

$$
\begin{aligned}
sp'(k) = & \phi \left[\sqrt{\frac{1}{r}} \left\{ \Phi^{-1} \left[Z(k) \right] \sqrt{1-r} - \Phi^{-1} \left[pd \right] \right\} \right] \\
& \times \left(\sqrt{\frac{1}{r}} \left\{ \frac{\sqrt{1-r}}{\phi \left[\Phi^{-1} \left[Z(k) \right] \right]} \frac{1 + (m-2.5)b}{(1-1.5b)lgd} - \frac{1}{2} \frac{dr}{dk} \frac{\Phi^{-1} \left[Z(k) \right]}{\sqrt{1-r}} \right\} \right. \\
& \left. - \frac{1}{2r^{3/2}} \frac{dr}{dk} \left\{ \Phi^{-1} \left[Z(k) \right] \sqrt{1-r} - \Phi^{-1} \left[pd \right] \right\} \right)
\end{aligned}
\tag{5}
$$

where $\phi(x)$ is the standard normal probability distribution function and we assume away the non-existence of derivatives between the cases in (3) so that:

$$
\frac{dr}{dk} = \begin{cases} \frac{1.25}{rw} & \frac{k}{rw} < .14 \\ \frac{1}{rw} & .14 \leq \frac{k}{rw} < .2 \\ 0 & \frac{k}{rw} \geq .2 \end{cases}
\tag{6}
$$

Figure 3 shows what $sp'(k)$ looks like.
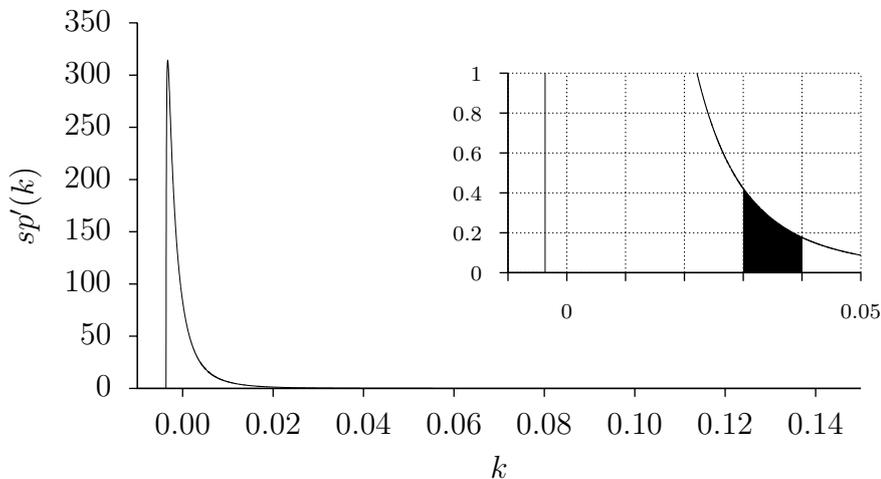
Figure 3: PDF corresponding to survival CDF

By the usual properties of a probability density function, $sp(b) - sp(a)$ is given by the area under sp' and between $k = a$ and $k = b$. For example, the shaded area in figure 3 is $sp(0.04) - sp(0.03) \approx 0.003040$.

If a and b are very close to each other (much closer than in figure 3), then the area under sp' is the probability of a shock that will *only just* wipe out a bank with $k = a$ and will wipe out almost no bank which has $k > a$. More formally, in the limiting case we have the differential:

$$d\left[sp(k)\right](a) = sp'(a)dk \tag{7}$$

For a bank with k in some infinitesimal range $[a, a + dk]$, the "probability" of a shock just big enough to cause the bank to fail is the expression on the right side of (7).

4.2   Utility adjustment

The value of an additional dollar in the midst of a systemic banking crisis is likely to be much higher than the value of a dollar in ordinary times.

The benefit of higher capital is the avoidance of costs which are borne only in a crisis – GDP losses and bailout costs due to bank failures. These avoided losses should therefore be valued more highly than the costs of higher capital, which are borne primarily in non-crisis times.

We have taken the formula for the utility adjustment directly from the Harrison model. Our derivation is inferred from the formula.

The adjustment assumes that society is divided into quintiles by income. Each quintile has the same isoelastic utility function $U(y) = -\frac{1}{7y^7}$.

Each quintile initially has income $y = 1$. When there is a loss l of income, the bottom quintile bears a disproportionate share $p = 0.6$ of the loss, so that the weighted-average utility of "New Zealand" following the loss is given by:

$$U_{NZ} = \frac{1}{5}\left[U\left(1 - pl\right) + 4U\left(1 - \frac{1}{4}(1-p)l\right)\right] \tag{8}$$

Since total initial income was 5, the nominal loss l can be expressed as a percentage loss $l^* = \frac{l}{5}$, and we have utility of:

$$U_{NZ} = \frac{1}{5}\left[U\left(1 - 5pl^*\right) + 4U\left(1 - \frac{5}{4}(1-p)l^*\right)\right]$$
$$= -\frac{1}{5}\left[\left(\frac{1}{7\left(1 - 5 \times .6l^*\right)}\right)^7 + 4\left(\frac{1}{7\left(1 - \frac{5}{4}.4 \times l^*\right)}\right)^7\right] \tag{9}$$

and marginal utility equal to:

$$U'_{NZ} = \frac{1}{5}\left[\left(\frac{1}{1 - 5 \times .6l^*}\right)^6 + 4\left(\frac{1}{1 - \frac{5}{4}.4 \times l^*)}\right)^6\right] \tag{10}$$

Figure 4 depicts U_{NZ} and U'_{NZ} and shows that marginal utility is significantly higher when income is lower.

We need to determine the size of l^* for a particular capital requirement. GDP losses given a bank failure, which seem a reasonable proxy for income losses, are already related to capital requirements in other parts of the model; see figure 7. However, using the relationship in figure 7 with U creates implausibly large utility adjustments. Instead, therefore, we use the Harrison assumption that

$$l^*(k) = \frac{1}{100}\left(500\frac{k}{rw}\right)^{.47} \tag{11}$$

and then we define the utility adjustment:

$$ua(k) = \frac{1}{5}\left[\left(\frac{1}{1 - 5 \times .6l^*(k)}\right)^6 + 4\left(\frac{1}{1 - \frac{5}{4}.4 \times l^*(k))}\right)^6\right] \tag{12}$$

noting that l^* is now explicitly given by (11). Equation (11) is depicted in figure 5. Figure 6 shows how ua is related to k.

In the model, the benefits of moving from $k = a$ to some nearby $k = b$ are multiplied by $ua(b)$. Note that $ua(k) = 1$ when $k = 0$, which corresponds to the case of no loss; the utility adjustment tends to multiplication by 1 as the loss goes to zero.
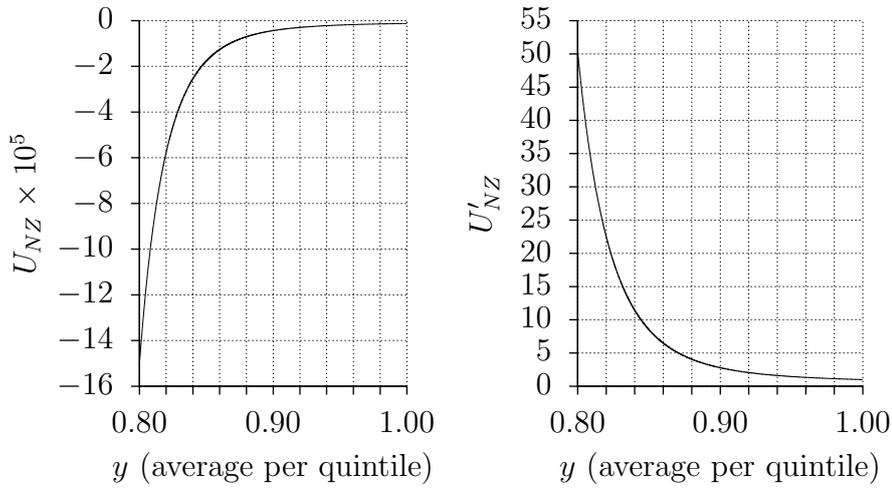
Figure 4: Utility and marginal utility functions used for the utility adjustment, as a function of average quintile income
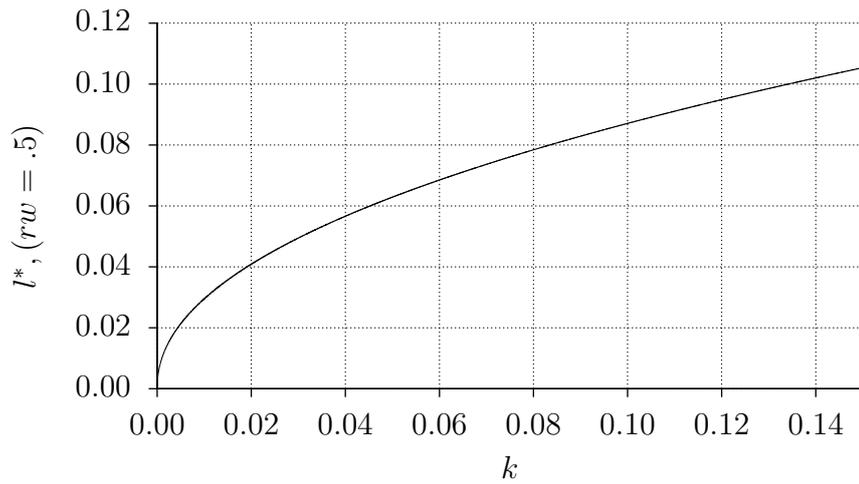
Figure 5: Relating income losses to the capital requirement, for utility adjustment

4.3 Reduced GDP losses, owing to fewer crises

If there is a shock and the bank fails, then we assume GDP is reduced. If the bank does not fail, we record no GDP loss due to the shock.

Our goal is to work out the expected losses from all shocks that the bank may experience.

Say our bank (Bank A) has $k = a$. We have seen that the "probability" of a
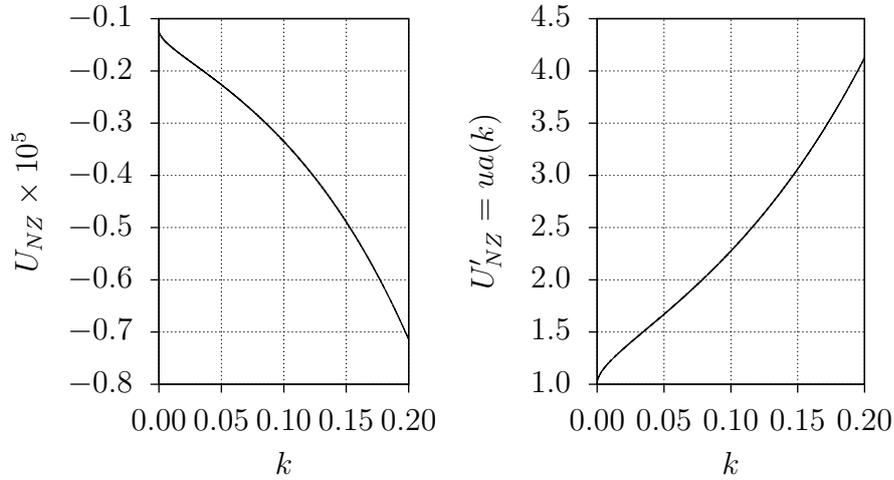
Figure 6: Utility and marginal utility functions used for the utility adjustment, as a function of k.

shock which only just wipes out this bank is $sp'(a)dk$ (see (7)). We assume that if the bank fails due to this shock then the cost, as a percentage of GDP, is given by some non-negative function $gc(k)$. We then apply the utility adjustment in (12) to this cost. So the expected GDP loss from a shock which is just big enough to wipe out our bank is

$$GDP \times gc(a)ua(a)sp'(a)dk \tag{13}$$

where GDP is the current (nominal) level of annual GDP. Now suppose there is some other, hypothetical, Bank B with $k = a + \epsilon > a$, with ϵ small. The "probability" of B only just being wiped out is $sp'(a + \epsilon)dk$, the GDP cost if it fails due to the shock is $GDP \times gc(a + \epsilon)$, and the expected GDP loss from this shock is

$$GDP \times gc(a + \epsilon)ua(a + \epsilon)sp'(a + \epsilon)dk \tag{14}$$

The shock which wipes out B will clearly wipe out A as well. So if we want to work expected losses from all shocks affecting Bank A we will have to add equations (13) and (14). We'll also need to add the loss from a shock which just wipes out Bank C, having $k = a + 2\epsilon$, the loss from a shock which just wipes out Bank D, having $k = a + 3\epsilon$, and so on.

In the limiting case where $\epsilon \to 0$, we end up with an expected GDP loss (gl):

$$E[gl|k = a] = GDP \int_a^\infty sp'(x)ua(x)gc(x)dx \tag{15}$$

10

where x is a dummy variable to avoid confusion with k on the left hand side of the equation.

If capital changes from a to b then the expected loss changes and we record a net benefit:

$$gb(a,b) = - \left(E\left[gl|k=b\right] - E\left[gl|k=a\right] \right) \tag{16}$$

We have chosen a relatively simple function for our loss given bank failure, gc:

$$gc(k) = \begin{cases} .1 & \frac{k}{rw} \le .03 \\ .1 + 2\left(\frac{k}{rw} - .03\right) & .03 < \frac{k}{rw} \le .08 \\ .2 & \frac{k}{rw} > .08 \end{cases}$$

where rw is the average risk weight for the bank's portfolio. $gc(k)$ is the continuous analogue of the the discrete function used in the Harrison model. The intuition is that if a bank has a lot of capital (k high) and still collapses, the shock must have been very large and the disruption caused to economic activity will also be large. If a bank has only a little capital and collapses, it is more likely that the collapse was caused by a small (common) shock than a large (rare) one, and the economic disruption will be less. Figure 7 depicts gc.
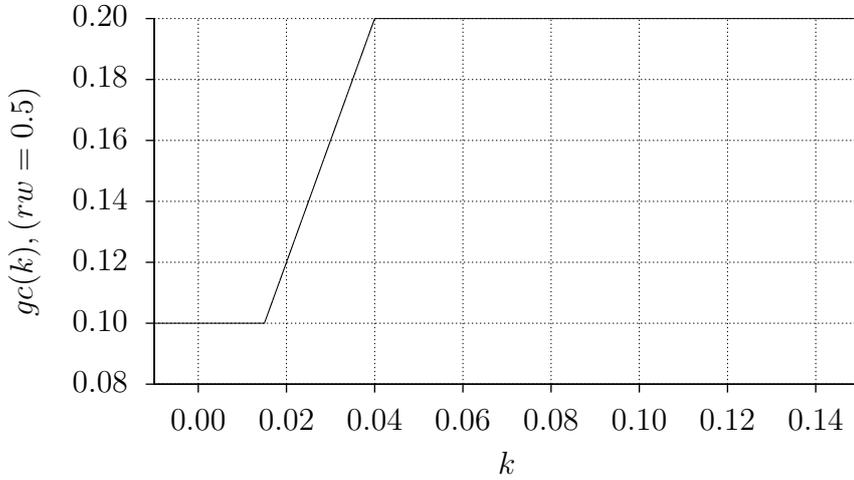
Figure 7: The cost function gc.

We make the simplifying assumption that a and b are not too far apart, so that we can treat $ua(k)$ as a constant and remove it from the integral. Then, with the simple form of gc that we have chosen, the net benefit of moving from $k = a$ to

$k = b$ is straightforwardly

$$gb(a, b) = ua(b) \times GDP \int_a^b sp'(x)gc(x)dx \tag{17}$$

We evaluate gb numerically (see figure 8).
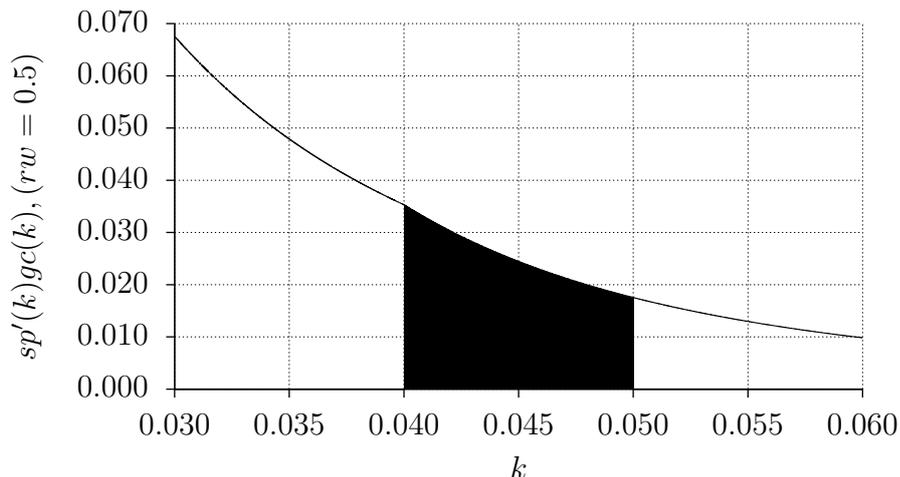
Figure 8: Calculation of gb. The area below the curve is the expected benefit, per dollar of nominal GDP, from moving from $k = .04$ to $k = .05$, before the utility adjustment is applied. i.e. the area is $gb(.04, .05) \div ua(.05)$.

Although we have not expressed it as an expected value, gb is an *expected* net benefit; the right side of (16) is the difference between two constants so its expected value is the same difference. gb is an *annual* expected net benefit. To work out the total expected future cost we must find the discounted sum of the annual benefits.

4.4 Reduced bailout costs, owing to fewer crises

We assume that when the bank fails there is some constant probability pb of the government bailing out creditors (other than equity holders). If the bank doesn't fail, we record no bailout cost.

We use the same underlying approach as we used to calculate reduced GDP costs, but with a different cost function. The probability of the bank (Bank A) only just being wiped out with $k = a$ is $sp'(a)dk$. If Bank A fails because of the shock and the government bails it out, there is a cost (as a percentage of the bank's initial assets) given by:

$$ua(k) \times [.0325 + (k - a)] \tag{18}$$

12

where a is the capital the bank actually holds and we have used the utility adjustment discussed earlier in this document.

The idea is that there is some deadweight bailout cost, being 0.0325 times the bank's assets, due to fire sales and administration costs. The 0.0325 constant is taken from the Harrison model. There is also the cost of making depositors whole, which is the capital shortfall. The shock, by definition, wipes out capital of k, so the loss which cannot be absorbed by actual capital and is borne by depositors is $k - a$.

$k = a$ for a shock which is just big enough to wipe out Bank A, but k grows with the size of the shock. For instance, if the shock is big enough to wipe out (hypothetical) Bank B which holds capital of $a + \epsilon > a$, this will also wipe out Bank A. For bank A there will be a capital shortfall of $k - a = a + \epsilon - a = \epsilon$. Integrating over k gives an expected bailout cost (bc):

$$E\left[bc|k = a\right] = pb \times \text{initial assets} \times \int_a^\infty ua(k)sp'(x)\left(.0325 + x - a\right)dx \quad (19)$$

When capital changes from a to b, the expected bailout cost also changes and we recognise a net benefit bb:

$$bb(a,b) = -\left(E\left[bc|k = b\right] - E\left[bc|k = a\right]\right) \quad (20)$$

Equation (20) is an *expected* net benefit, and can be evaluated numerically. bb is an *annual* expected net benefit. To work out the total expected future cost we must find the discounted sum of the annual benefits.

4.4.1 Digression

Finding bb is not as straighforward as finding gb, because a appears as a limit in the integration *and* in the integrand in bb. We first make the simplifying assumption that we will be working with small incremental changes in capital requirements so that, for one of these small incremental changes from a to b we can treat $ua(k)$ as a constant and remove it from the integral. Now note that that:

$$\int_b^\infty sp'(x)dx = 1 - sp(b) \quad (21)$$

and also that, by integration by parts:

$$\int_b^\infty x \cdot sp'(x)dx - \int_a^\infty x \cdot sp'(x)dx = -\int_a^b x \cdot sp'(x) = -xsp(x)\big|_a^b + \int_a^b sp(x)dx \quad (22)$$

Then we have:

$$(E[bc|k=b] - E[bc|k=a]) \div (pb \times ua(a) \times \text{initial assets})$$

$$= \int_b^\infty sp'(x)(.0325 + x - b)\,dx - \int_a^\infty sp'(x)(.0325 + x - a)\,dx$$

$$= .0325\,(sp(a) - sp(b)) - \int_a^b x \cdot sp'(x)\,dx + a\int_a^\infty sp'(x)\,dx - b\int_b^\infty sp'(x)\,dx$$

$$= .0325\,(sp(a) - sp(b))) - b \cdot sp(b) + a \cdot sp(a) + \int_a^b sp(x)\,dx + a\,(1 - sp(a)) - b\,(1 - sp(b))$$

$$= .0325\,(sp(a) - sp(b))) + \int_a^b sp(x)\,dx - 1\,(b - a)$$

$$= .0325\,(sp(a) - sp(b))) - \int_a^b 1 - sp(x)\,dx$$

(23)

and

$$-\,(E[bc|k=b] - E[bc|k=a])$$

$$= pb \times ua(b) \times \text{initial assets}\left\{ .0325\,(sp(b) - sp(a)) + \int_a^b 1 - sp(x)\,dx \right\}$$

(24)

The last term on the right is the area on the graph of k versus $sp(k)$ which is contained by the lines $sp(k) = 1$, $sp(k)$, $k = a$ and $k = b$ (see figure 9). It can be approximated by a trapezium, for small $a - b$, or otherwise evaluated numerically. **End of digression.**

4.5 Higher interest costs

Banks argue that higher capital requirements are costly because they must issue more equity and equity investors demand a higher required rate of return than debt-holders. To cover the higher funding cost, the argument continues, banks must increase retail interest rates. In turn, these higher interest rates depress economic activity.

The Basel Committee on Banking Supervision calculated that a one percentage point increase in the (risk-weighted) capital ratio would translate into a 9 basis point decrease in GDP [Basel (2010)]. The calculations assumed that required rates of return on bank equity and bank debt (individually) would not change and that banks would pass on all additional funding costs to their customers. The Committee acknowledged that these were extreme assumptions.

The Modigliani Miller theorems (MM) show that, under certain restrictive assumptions, higher capital requirements are *not* more costly; shareholders are

14
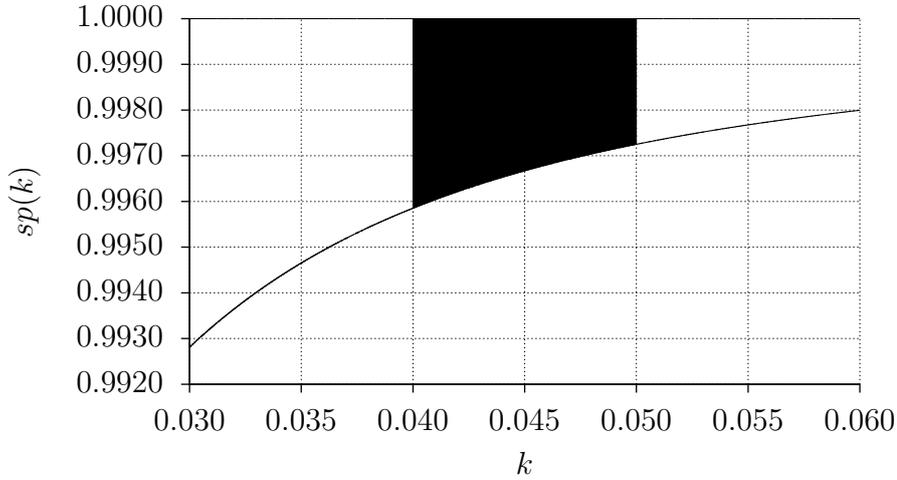
Figure 9: Calculation of *bb*. The shaded area shows $\int_{.04}^{.05} 1 - sp(x)dx$.

willing to accept a lower rate of return when the bank has a lower (less risky) debt burden.

Harrison assumed, though noting it was inconsistent with theory, that MM holds only after a delay. He reduced the effect estimated by the Basel Committee by 85% to account for the delay. We have continued to use this 85% figure.

When the bank holds capital k, the penalty from higher interest rates is given by

$$ip(k) = (1 - .85) \times .0009 \times 100 \times GDP \times \tfrac{k}{rw} \tag{25}$$

noting that $\frac{k}{rw}$ is the risk-weighted capital ratio when the average risk weight is rw and is measured as a decimal, not in percentage points. When capital changes from $k = a$ to $k = b$ we record a net benefit equal to

$$ic(a,b) = ip(b) - ip(a) \tag{26}$$

ic is an *annual* expected net cost. To work out the total expected future cost we must find the discounted sum of the annual costs.

4.6 Additional transfers

Changes in capital structure can lead to transfers between shareholders, debtholders, depositors, and governments. We ignore these transfers if they are domestic, since national welfare is (very crudely) unaffected, but count them as costs or benefits when they are international.

15

Our approach is to value changes in the stream of net cashflows going abroad and count this as a cost. The cost depends on:

- the reaction of equity-holders to changes in capital requirements;

- the bank's response to any change in overall funding costs;

- the response of bank borrowers to any change in retail interest rates; and

- how "New Zealand" values the net cash flows.

4.6.1 Determinants of cost

We use two scenarios for equity-holder behaviour. In one, we calculate costs assuming that equity-holders demand a fixed rate of return which does not vary with the bank's capital k. In the other scenario, we calculate costs assuming that equity-holders accept a lower rate of return when k is high, consistent with the Modigliani Miller theorems. We use a weighted average of the results from the two scenarios as our final cost, with an 85% weight given to the second (MM) scenario. The 85% figure comes from the original Harrison model.

We assume that banks pass on any changes in overall funding costs to customers, by raising the retail interest rate.

We do not explicitly model the response of retail borrowers to changes in retail interest rates, but the overall level of bank assets may be changed arbitrarily to reflect estimates of borrower response (the modeller may input any change she wishes). Note that only the effect on international transfers is reflected in this part of the model. Domestic effects of changes in bank assets such as decreases in economic activity could, with appropriate recalibration, be picked up in equation (26).

We assume that New Zealand always values net future cash flows in a completely rational way, consistent with MM (with taxes). That is, New Zealand accepts that future cashflows to bank shareholders should be discounted less when the bank's debt-to-equity ratio falls. We do this even in the case where bank shareholders do not behave consistently with MM.

4.6.2 Further assumptions

We assume that all bank shareholders are non-residents, which is a reasonable approximation for the foreign-owned New Zealand-incorporated banks which account for most of the banking sector. We also assume that *changes* in bank debt are changes in debt held by non-residents.

We assume that the required rate of return on bank debt is fixed. This simplifies the analysis.

4.6.3 Determining the value of claims on the bank

The bank charges a retail interest rate rr on the face value of its assets a. The bank has issued debt with a face value of d, and the bank pays tax-deductible interest to debtholders at an interest rate rd. Foreign debtholders pay tax on interest only in their own country, at a rate td. Corporate tax is levied in New Zealand on net profits at a rate tc, and after-tax profits are distributed as dividends. Shareholders pay dividend tax only in their own country, at a rate te, with no credit for corporate tax paid in New Zealand.

Required rates of return, after all taxes, are rdd for bank debtholders and red for bank shareholders. red is the required return on *levered* equity at the initial debt-to-equity ratio; the required return on unlevered equity is rud.

The net present value of debtholders' claims is given by:

$$v_d = \sum_{t=1}^{\infty} \frac{rd\,(1-td)\,d}{(1+rdd)^t} = \frac{rd\,(1-td)\,d}{rdd} \tag{27}$$

The value of shareholders' claims is given by:

$$\begin{aligned} v_e &= \sum_{t=1}^{\infty} \frac{(a \times rr - d \times rd)\,(1-tc)\,(1-te)}{(1+red)^t} \\ &= \frac{(a \times rr - d \times rd)\,(1-tc)\,(1-te)}{red} \end{aligned} \tag{28}$$

4.6.4 Determining the effect of a change in the capital requirement

We first assume that there is no change in the bank's assets a.

Then in response to a change in capital requirements, there must be some conversion between debt and equity. For example, if capital requirements increase some debt must be converted to equity. To achieve a conversion, we assume that either shareholders inject money into the bank to buy back debt from debtholders, or debtholders inject money into the company to buy back shares.

Say the amount of debt converted is Δd (a positive value indicates equity is converted to debt). The change in the value of debtholders' claims is:

$$\frac{rd(1-td)\Delta d}{rdd} \tag{29}$$

Transactions are assumed to be at market value, so equation (29) is also the amount of cash that debtholders pay if $\Delta d > 0$, or are paid if $\Delta d < 0$, for the change in

their claim. There is an exchange of equal values and the net effect on debtholders is nil.

Via the bank, shareholders receive the payment from debtholders if $\Delta d > 0$, or fund the payment to debtholders if $\Delta d < 0$. In exchange there is a change in future dividends, because the company's future interest payments increase (if $\Delta d > 0$) or decrease (if $\Delta d < 0$) and this affects profits.

As well as the change in debt, we allow for the retail interest rate to change by Δrr. Then the change in per-period dividends, after taxes, is:

$$(a\Delta rr - rd\Delta d)\,(1 - tc)\,(1 - te) \tag{30}$$

To recap, using the case in which $\Delta d < 0$ to illustrate, shareholders have paid, once only, the amount in (29). In return they get the perpetual annual amount given by (30). The values, in the eye of the shareholder, of the two amounts must be equal. Otherwise, there is either an arbitrage opportunity or shareholders will refuse to exchange shares.

4.6.5 A change in the capital requirement with MM

Suppose first that shareholders value the future cash flow in (30) "properly", according to the predictions of MM. Then, recalling that rud is the required rate of return for unlevered equity, the requirement is that:

$$-\frac{rd\,(1 - td)\,\Delta d}{rdd} = \left(\frac{a\Delta rr}{rud} - \frac{rd\Delta d}{rdd}\right)(1 - tc)\,(1 - te) \tag{31}$$

Solving for Δrr gives

$$\Delta rr = \frac{rud \times rd}{rdd}\left[1 - \frac{1 - td}{(1 - tc)\,(1 - te)}\right]\frac{\Delta d}{a} \tag{32}$$

4.6.6 A change in the capital requirement without MM

Now suppose that shareholders value the cash flow in (30) using the initial required rate of return red, not recognising that the bank has changed its capital ratio. Then the requirement is that:

$$-\frac{rd\,(1 - td)\,\Delta d}{rdd} = \left(\frac{a\Delta rr^* - rd\Delta d}{red}\right)(1 - tc)\,(1 - te) \tag{33}$$

where we have used the notation Δrr^* to indicate that the change in the interest rate will be different from the change in the MM case (Δrr). Solving for Δrr^* gives

$$\Delta rr^* = rd\left[1 - \frac{red}{rdd} \times \frac{1 - td}{(1 - tc)\,(1 - te)}\right]\frac{\Delta d}{a} \tag{34}$$

4.6.7 Allowing for asset change

When the retail interest rate changes it is likely that there will be some sort of demand response, and the bank's assets a will change. This will in turn affect transfers.

We assume that the bank issues debt and equity to fund an increase in assets, and retires debt and equity when assets decrease. We assume it does this proportionately, so that the change in assets does not affect the capital ratio. The change in assets therefore has no further effect on retail interest rates or stakeholders' unit returns.

To be clear, suppose that regulatory capital requirements increase. The bank adjusts to the new capital requirement with initial assets unchanged, converting Δd of debt to equity. With the new requirement achieved, the bank allows assets to change, but in a way which maintains the new debt-to-asset ratio. There is therefore a further change in debt equal to the change in assets Δa times the new debt-to-asset ratio $\frac{d+\Delta d}{a}$.

We denote the change in assets Δa when shareholders behave in accordance with MM. When they do not, we use Δa^*, recognising that the effects of rr and rr^* on assets are likely to differ.

4.6.8 Final effect on transfers

Recall that "New Zealand" always values cash flows according to MM. Initially, New Zealand sees cashflows going abroad – a cost to New Zealand – which are equal to the dividend and interest streams, after New Zealand tax:

$$\left(\frac{a \times rr}{rud} - \frac{d \times rd}{rdd} \right)(1 - tc) + \frac{d \times rd}{rdd} = \frac{a \times rr}{rud}(1 - tc) + \frac{d \times rd}{rdd}tc \qquad (35)$$

After capital requirements change, New Zealand sees either:

$$\frac{(a + \Delta a)(rr + \Delta rr)}{rud}(1 - tc) + \frac{\left(d + \Delta d + \frac{d+\Delta d}{a}\Delta a\right)rd}{rdd}tc \qquad (36)$$

or, if shareholders don't behave as MM suggests:

$$\frac{(a + \Delta a^*)(rr + \Delta rr^*)}{rud}(1 - tc) + \frac{\left(d + \Delta d + \frac{d+\Delta d}{a}\Delta a^*\right)rd}{rdd}tc \qquad (37)$$

The difference between (36) and (35) is the change in transfers when MM holds:

$$ta = \frac{(a + \Delta a)\Delta rr + rr\Delta a}{rud}(1 - tc) + \frac{\left(\Delta d + \frac{d+\Delta d}{a}\Delta a\right)rd}{rdd}tc \qquad (38)$$

19

and the change in transfers when MM does not hold is given similarly by:

$$ta^* = \frac{(a + \Delta a^*) \Delta rr^* + rr\Delta a^*}{rud}(1 - tc) + \frac{\left(\Delta d + \frac{d + \Delta d}{a}\Delta a^*\right)rd}{rdd}tc \tag{39}$$

Assuming that MM holds "85% effectively" (consistent with the assumption in the Harrison model) we get an overall transfer cost due to a change in capital requirements:

$$tc = .85ta + .15ta^* \tag{40}$$

tc, unlike the other costs and benefits of capital requirements which are annual, is already a discounted sum of all future net transfers. No further discounting or aggregation is required.

5 Calculation of total net benefits

The total net benefits of changing from a capital ratio of a to b are given by:

$$tb(a, b) = \frac{gb(a, b) + bb(a, b) - ic(a, b)}{\text{discount rate}} - tc(a, b) \tag{41}$$

where the modeller may choose any discount rate she wants.

Note that a and b must not be too far apart for gb and bb to be calculated using the methods we have suggested. If the net benefit is wanted for a large change, then the change should first be broken into small parts, with the results for each added to find the final result.

6 Known weaknesses

The V2 model, like the Harrison model which it is based on, is complex. This increases the likelihood of errors in the model itself or its implementation. In addition, there are several other problems:

- There is no documented basis for the assumption that MM holds with 85% effectiveness, and international empirical evidence suggests a lower number might be appropriate.

- We have not seriously reconsidered the assumptions about GDP losses when banks fail. The numbers used come from the Harrison model, and are low compared to numbers used in other studies. There were two arguments for using low numbers: studies of losses fail to separate the specific effects of

bank failure from the general effects of shocks that caused the failure; and shocks should not have permanent effects on GDP. We now have a decade of GDP data following the GFC, a shock which was almost entirely a financial system crisis, that we could use to examine whether these arguments remain convincing.

- The correlation function used in the model is arbitrary. It is correct that the Basel capital equation produces an implausibly low systemic failure rate with the standard correlation, but it is not obvious that changing the correlation is the solution to this problem (possibly the equation is just the wrong one to use).

- The utility adjustment is arbitrary. We agree that some adjustment is called for, but there are problems with the one that is used. The relationship between capital and income losses which feeds into the adjustment is inconsistent with the relationship between the same two variables which is used elsewhere in the model. As well, because the marginal utility for the last dollar of loss/cost is applied to the entire loss/cost, the adjusted figure is overstated, possibly materially.

- The model assumes that all capital is common equity. It is unclear how to regard the output of the model when Additional Tier 1 and Tier 2 capital are permitted. If AT1 and T2 are not as effective as common equity in reducing the costs of crises, the output of the model will tend to be lower than a true optimal *total* capital requirement. At the same time, some costs are likely to be overstated compared to true costs, because the return on, and tax treatment of, AT1 and T2 instruments is not the same as for common equity.

- The method used in our spreadsheet implementation to move from a zero-capital threshold for failure to a 2% risk-weighted capital ratio threshold, is crude. In particular, it implies that when a government bails out a bank it restores the risk-weighted capital ratio to 2% but receives nothing in return (not even shares in the recapitalised bank).

- Our spreadsheet implementation calculates the initial debt-to-asset ratio (for use in calculating transfer costs) using real-world bank asset and equity data. This is not really appropriate, since we are calculating changes for given (not risk weighted) capital ratios which differ from actual ratios. Some initial analysis suggests the effect on model outputs is insignificant, but this is a very tentative conclusion.

21

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