The implications of uncertainty for monetary policy
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A simple model of the Australian economy is used to empirically examine the consequences of parameter uncertainty for monetary policy. Optimal policy responses are computed for a monetary authority that targets inflation and output stability. Parameter uncertainty is characterised by the estimated distribution of the model coefficient estimates. Learning is ruled out, so monetary authorities are able to commit to their ex ante policy responses. Taking account of parameter uncertainty generally recommends more activist use of the policy instrument. However, this finding is specific to the economy, model specification and shock.

1.0 Introduction
Monetary authorities aim to achieve low and stable inflation while keeping output at capacity. To achieve these goals they manipulate the policy instrument which has an effect on economic activity through one or more transmission mechanisms. Monetary authorities face many difficulties in achieving these goals. The current state of the economy, for example, is not known with certainty. Moreover, the response of the economy to demand and supply shocks are difficult to quantify and new shocks are arriving all the time. As if these problems are not enough, the transmission channels from the policy instrument to the objectives are complex and imprecisely estimated.

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Economic models are useful tools for dealing with these uncertainties. By abstracting from less important uncertainties, models provide a framework within which the workings of the economy can be quantified. In doing so, models generally reduce the complexity of the policy decision-making process and go some way towards helping monetary authorities achieve their goals. However, to the extent that models are only an approximation to the ‘true’ economy, there will always be uncertainty about the correct structure and parameters of an economic model.

Blinder (1995), commenting in his capacity as a central banker, observed that model uncertainty can have important implications for policy. In particular, uncertainty about the model may make monetary authorities more conservative in the sense that they determine the appropriate policy response ignoring uncertainty, “and then do less.” This conservative approach to policy was first formalised by Brainard (1967). Although Blinder views the Brainard conservatism principle “as extremely wise,” he admits that the result is not robust. For practical purposes, this recommendation leaves two questions unanswered. First, how much should policy be adjusted to account for model uncertainty? Second, is conservatism always appropriate for the economic models in current use?

This paper addresses both of these questions by generalising the Brainard model to a multi-period horizon and a multivariate model. A small data-consistent model of the Australian economy is used to illustrate the effect of parameter uncertainty on policy responses. We show that parameter uncertainty actually induces greater policy activism for most types of shocks. We argue that this increased activism is a consequence of uncertainty about the persistence of shocks to the economy. This type of uncertainty cannot be incorporated into the static model of Brainard.

The remainder of the paper is structured as follows. In section 2, we discuss the various sources of forecasting error which lie behind model uncertainty. Section 3 summarises the specification of a small macroeconomic model used in the remainder of the paper. Section 4 shows how sensitive policy responses are to parameter uncertainty when the uncertainty has been ignored by the policymaker. Section 5 demonstrates how parameter uncertainty can be accommodated in the solution to a monetary authority’s optimal policy problem and section 6 illustrates the difference between naïve policy, that ignores parameter uncertainty, and policy that explicitly takes parameter uncertainty into account. Section 7 concludes and summarises the implications for monetary policy.

2.0 Sources of forecast uncertainty

Hendry and Clements (1994) provide a taxonomy of forecast error sources for an economic system that can be characterised as a multivariate, linear stochastic process. This model can generally be represented as a vector autoregressive system of linear equations. Furthermore, most models of interest to policymakers can be characterised by a set of trends that are common to two or more variables describing the economy. In these cases the economic models can be written as vector error-correction models.
For these models, forecast errors can come from five distinct sources:

1. Structural shifts;
2. Model misspecification;
3. Additive shocks affecting endogenous variables;
4. Mismeasurement of the economy; and
5. Parameter estimation error.

The first source of forecasting error arises from changes in the economic system during the forecast period. The second source of forecasting error may arise if the model specification does not match the actual economy. This may arise, for example, if the long-run relationships or dynamics have been incorrectly specified. Forecasting errors will also arise when unanticipated shocks affect the economic system. These shocks accumulate, increasing uncertainty with the length of the forecast horizon. If the initial state of the economy is mismeasured to begin with then this will cause persistent forecast errors. Finally, forecast errors may also arise because finite-sample parameter estimates are random variables, subject to sampling error.

By definition, without these sources of forecast error, there would be no uncertainty attached to the forecasts generated by a particular system of equations. Recent research by Clements and Hendry (1993, 1994 and 1996) explores the relative importance of each of these sources of forecasting error. They find that structural shifts in the data generating process or model misspecifications, resulting in intercept shifts, are the most persistent sources of forecasting error in macroeconomic models.

Rather than comparing the relative importance of each of these sources of uncertainty for forecasting, this paper makes a contribution toward understanding the implications of parameter uncertainty for monetary policy decision-making. We solve for ‘optimal policy’ explicitly taking into account uncertainty about the parameter estimates.

Why focus only on uncertainty arising from sampling error in the parameter estimates? This question is best answered by referring to each of the remaining sources of forecast error. Firstly, structural change is the most difficult source of uncertainty to deal with because the extent to which the underlying structure of the economy is changing is virtually impossible to determine in the midst of the changes. Clements and Hendry have devoted considerable resources toward exploring the consequences of such underlying structural change. However, this type of analysis is only feasible in a controlled simulation. In the context of models that attempt to forecast actual economic data, for which the ‘true’ economic structure is not known, this type of exercise can only be performed by making assumptions about the types and magnitudes of structural shifts that are likely to impact upon the economy. With little or no information on which to base these assumptions, it is hazardous to attempt an empirical examination of their consequences for the conduct of monetary policy.\footnote{However, it may be possible to exploit the work of Knight (1921) to generalise the expected utility framework for policymakers that are not certain about the true structure and evolution of the economy. See, for example, Epstein and Wang (1994).}
Similarly, it is difficult to specify the full range of misspecifications that may occur in a model. Without being able to justify which misspecifications are possible and which are not, analysis of how these misspecifications affect the implementation of policy must be vague at best.

Turning to the third source of forecast errors, it is well known that normally distributed shocks affecting the system in a linear fashion have no impact on optimal policy when the policymaker’s objective function is quadratic.\(^2\) To the extent that a linear model is a sufficiently close approximation to the actual economy and given quadratic preferences, this implies that the accumulation of unanticipated shocks are of no consequence to policymakers until they actually occur. For this reason, forecast uncertainty caused by the impact of random shocks to the economic system is also ignored in this paper.

The initial state of the economy, which provides the starting point for forecasts, is often mismeasured, as reflected in subsequent data revisions. It is possible to assess the implications of mismeasurement for forecast uncertainty by estimating the parameters of the model explicitly taking data revisions into account (Harvey, 1989). However, this requires a complete history of preliminary and revised data and is beyond the scope of this paper.

In light of these issues, this paper focuses on parameter estimation error as the only source of uncertainty. From a theoretic perspective, this issue has been dealt with definitively by Brainard (1967). Brainard developed a model of policy implementation in which the policymaker is uncertain about the impact of the policy instrument on the economy. With a single policy instrument (matching the problem faced by a monetary authority, for example) Brainard showed that optimal policy is a function of both the first and second central moments characterising the model parameters.\(^3\) Under certain assumptions about the cross correlations between parameters, optimal policy under uncertainty was shown to be more conservative than optimal policy generated under the assumption that the true parameters are actually equal to their point estimates. However, Brainard also showed that it is possible for other assumptions about the joint distribution of the parameters to result in more active use of the policy instrument than would be observed if the policymaker ignored parameter uncertainty.

Resolving the implications of parameter uncertainty for monetary policy then becomes an empirical issue. To this end, the next section describes the impact of this Brainard-type uncertainty on monetary policy decision making in a small macroeconomic model of the Australian economy.

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\(^2\) This is the certainty equivalence result discussed, for example, in Kwakernaak and Sivan (1972).

\(^3\) Only the first and second moments are required because of the assumption that the policymaker has quadratic preferences. Otherwise it is necessary to maintain the assumption that the parameters are jointly normally distributed.
3.0 An economic model

The model we use is built around two identities and five estimated relationships determining key macroeconomic variables in the Australian economy: output, prices, unit labour costs, import prices and the real exchange rate. The equations in the model were developed in previous Bank research and were most recently applied in de Brouwer and Ellis (1998). The equations are estimated separately, using quarterly data from September 1980 to December 1997, except for the real exchange rate equation, which was estimated using quarterly data from March 1985. The specification of the model is summarised in table 1. Appendix A contains a more complete description of the model.

Table 1
The specification of the model

Output

\[ \Delta y_t = \alpha_1 - 0.23(y_{t-1} + 0.10 r_{t-3} - 0.05 nftot_{t-1}) + 0.27 y^v_{t-1} + 0.29 \Delta y^v_{t-3} + 0.01 \Delta y^v_{t-3} \]

\[ + 0.01 \Delta y^v_{t-2} - 0.09 r_{t-1} - 0.01 r_{t-3} + 0.01 r_{t-4} - 0.09 r_{t-5} - 0.07 r_{t-6} \]

Prices

\[ \Delta p_t = \alpha_2 + 0.04 (ulc_{t-1} - p_{t-1}) + 0.04 (p^v_{t-1} - p_{t-1}) + 0.08 \Delta ulc_t + 0.02 \Delta p^v_{t-3} + 0.12 \text{gap}_{t-3} \]

Unit Labour Costs

\[ \Delta ulc_t = 0.50 \Delta p_{t-3} + 0.50 \Delta p_{t-1} + 0.09 \text{gap}_{t-1} \]

Import Prices

\[ \Delta p^v_t = \alpha_4 - 0.18 (p^v_{t-1} - p^v_{t-1} + ner_{t-1}) - 0.51 \Delta ner_t \]

Real Exchange Rate

\[ \Delta rer_t = \alpha_5 - 0.31 \text{rer}_{t-1} + 0.19 \text{tot}_{t-1} + 0.72 (r_{t-1} - r^*_{t-1}) + 1.39 \Delta \text{tot}_{t-1} \]

Nominal Exchange Rate

\[ ner_t = rer_t - p_t + p^*_{t-1} \]

Real Interest Rate

\[ r_t = i_t - (p_t - p_{t-4}) \]
It is also necessary to specify the preferences of the policymaker, in this paper, the monetary authority. Specifically, we assume that the policymaker sets the profile of the policy instrument, the nominal cash rate, to minimise the loss function:

\[
\text{Loss} = E_t \left[ \alpha \sum_{j=1}^{k} \text{gap}^2_{t+j} + \beta \sum_{j=1}^{k} (\pi_{t+j} - \pi^*)^2 + \gamma \sum_{j=1}^{k} (i_{t+j} - i_{t+j-1})^2 \right]
\]  \hspace{1cm} (1)

Notes: All variables except interest rates are expressed in log levels.

Figures in brackets ( ) are standard errors (adjusted for residual heteroscedasticity or autocorrelation where necessary).

(a) The coefficients on the lagged level of rer and nftot were calibrated so that an equal simultaneous rise in both the terms of trade and the real exchange rate results in a net contraction in output in the long run.

(b) As specified, this equation implies linear homogeneity in the long-run relationship between prices, nominal unit labour costs and import prices (this restriction is accepted by the data).

(c) The restriction that the coefficients on lagged inflation sum to unity was imposed (this restriction is accepted by the data) and the equation was estimated by generalised least squares to correct for serial correlation.
where $\pi$ is the year-ended inflation rate; $\pi^*$ is the inflation target and $E_t$ is the expectations operator conditional on information available at time $t$.

The first two terms in this objective function describe the policymaker’s preference for minimising the expected output gap and deviations of expected inflation from target. The third term in the loss function represents the penalty attached to volatility in the policy instrument. This term is included to reduce the monetary authority’s freedom to set policy in a way that deviates too far from the observed behaviour of the policy instrument. By penalising volatility in the policy instrument, this term imposes a degree of interest-rate smoothing (Lowe 1997).

At each point in time, optimal policy is achieved by minimising the loss function with respect to the path of the policy instrument over the forecast horizon, $t+1$ to $t+h$, subject to the system of equations described in table 1.

The coefficients, $\alpha$, $\beta$ and $\gamma$ are the relative weights (importance) attached to minimisation of the output gap, deviations of inflation from target and movements in the policy instrument. In this paper, $\alpha$, $\beta$ and $\gamma$ are 0.02, 0.98 and 0.02 respectively. These particular weights characterise a monetary authority with a strong emphasis on keeping inflation close to target. For most kinds of shocks, the weights were selected so that the optimal policy response brings inflation back to target within a reasonable number of years. There is a much higher weight on inflation relative to the output gap because the output gap is an important determinant of inflation. Therefore, a policy which concentrates on getting inflation back to target indirectly aims to close the output gap. While optimal policy is certainly sensitive to the choice of weights, they do not affect the qualitative implications of parameter uncertainty for monetary policy.

This completes the description of the model and the objectives of the monetary authority. The simulations in the following sections report optimal policy responses to shocks that move the model from its steady-state, where the steady-state is characterised by the error correction terms in the equations.

The steady-state conditions are satisfied by normalising the constants and variables to zero and assuming that the exogenous variables have zero growth rates. This particular steady-state has two advantages. First, the zero-growth assumption eliminates the need for more sophisticated modelling of the exogenous variables. Second, different parameter estimates imply different long-run relationships, but the zero-level steady-state is the only one that satisfies each of these estimated long-run relationships. This means that the results which we present later can be interpreted as deviations from baseline.

\[\text{However, in practice, generating forecasts and optimal policy responses from this model would require explicit models for the exogenous variables, which would introduce additional parameter uncertainty.}\]
4.0 Optimal policy ignoring parameter uncertainty

All of the estimated regression parameters in table 1 are point estimates of the true parameter values and, as such, are random variables. The uncertainty surrounding these point estimates is partly reflected in their associated standard errors. This section highlights the consequences for monetary policy when the monetary authority assumes that these point estimates accurately describe the true economy. We generate a range of model-optimal policy responses and associated forecast profiles that would obtain under different draws of the parameter estimates from their underlying distribution. We describe these optimal policy responses as ‘naïve’ because they ignore parameter uncertainty. In section 6, we show how the policy responses change when the optimal policy problem is solved recognising parameter uncertainty.

For each equation, we assume that the parameters are normally distributed with first moments given by their point estimates in table 1 and second moments given by the appropriate entries in the estimated variance-covariance matrix of the parameter vector. This approach to defining a distribution from which to draw the parameters of the model also ignores uncertainty about the estimates of the variance-covariance matrices themselves.

Because each equation is estimated separately, there is no information available concerning the cross-correlations between the parameters in the different equations. This implies that the variance-covariance matrix of the entire parameter vector is block diagonal, with each block given by the variance-covariance matrix of an individual equation.

While we characterise parameter uncertainty as being caused by sampling error, this understates the variety of factors that can contribute to imprecision in the parameter estimates. When we estimate each of the five behavioural equations, we assume that the parameters do not change over time. Any changes in the parameters must then be partially reflected in the parameter variance-covariance matrix. This contribution to parameter uncertainty actually derives from model misspecification rather than sampling error. While we ignore these distinctions for the remainder of the paper, we acknowledge that the sampling error interpretation of the variance-covariance matrices will overstate the true sampling error problems and understate the problems of model misspecification and structural breaks in the model.

Taking the estimated parameter distributions as given, each draw of the model parameters requires pre-multiplication of a vector of independent standard normal variates by the lower triangular Cholesky decomposition of the full variance-covariance matrix.

The remainder of this section presents forecasts that arise when the monetary authority faces a given parameter-estimate draw and ignores the distribution from which it is drawn. By solving the same optimal-policy problem for a large number of parameter draws, we obtain a range of forecasts which indicate the consequences of ignoring parameter uncertainty. Starting from the

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5 This approach to defining a distribution from which to draw the parameters of the model also ignores uncertainty about the estimates of the variance-covariance matrices themselves.

6 For the real exchange rate and terms of trade parameters in the output equation, which have been calibrated, the appropriate terms in the variance-covariance matrix have been approximated by the corresponding terms in the variance-covariance matrix of the unconstrained output equation.
steady-state defined in the previous section, we assume that the system is disturbed by a single one percentage point shock to one of the five estimated equations. Then, for one hundred different parameter draws, the naïve optimal policy problem is solved to generate the path of the nominal cash rate and the corresponding forecast profiles over a seven-year horizon. Each of the simulations that follow are based on the same one hundred draws from the underlying parameter distribution.

For example, consider a one percentage point shock to real output. Figure 1 summarises the results of this simulation. In this figure, the maximum, minimum and median along with the first and third quartiles illustrate the dispersion of forecast profiles generated by the different parameter draws. The thicker, central black line denotes the median, while the thinner lines marking the limits of the light shaded regions are the maximum and minimum. The dark shaded region is the inter-quartile range.

Note that the spread of forecasts around the median need not be symmetric. This is because asymmetries result from non-linearities in the way that the model parameters enter the construction of the forecasts. Although the model is linear in each of the variables, forecasts can be high-order polynomials in the lag coefficients.

The output shock opens up a positive output gap which generates inflationary pressures in the economy. Feedback between wages and prices means that this inflationary pressure eventually feeds into unit labour cost growth. Consistent with the monetary authority’s objectives, the optimal response to this shock is to initially raise the nominal cash rate. However, the size of this initial tightening can vary by up to three-quarters of a percentage point, depending on the parameter draw. With backward-looking inflation expectations, the rise in nominal interest rates raises the real cash rate, which has a dampening effect on output and eventually reverses the upward pressure on unit labour costs and inflation. The higher real interest rate also appreciates the real and nominal exchange rate, lowering inflation directly by reducing the Australian dollar price of imports and indirectly by reducing output growth.

Over time, the initial tightening is reversed and eventually policy follows a dampening cycle as the output gap is gradually closed and wage and inflation pressures subside. In the limit, all real variables and growth rates return to target and the system returns to the steady-state.\(^7\)

What is most striking about these simulations is the range of different forecast profiles caused by parameter uncertainty. For example, depending on the parameter draw, the optimal policy response at any time during the forecast horizon can vary by as much as one and a half percentage points. This variation demonstrates that naïve policy responses are not robust across parameter draws.

\(^7\) Whilst this is true for the model which we are using in this paper, after the 28 periods shown in figure 1, some of the variables do not completely return to steady-state. This is because the mean parameter draw results in a model which is quite persistent anyway and furthermore, some of the more extreme parameter draws can generate larger and more long-lasting cyclical behaviour in the variables. Eventually, however, all of the real variables and growth rates will return to steady-state.
It should be stressed that the optimal policy responses in figure 1 assume no learning on the part of the monetary authority. Although the monetary authority may set interest rates according to a calculated optimal policy path, the economy will only ever evolve according to the true parameter draw. Generically, the forecasts of the monetary authority will be proved wrong ex post, providing a signal that the initial parameter estimates were incorrect. If the monetary authority learns more about the true model parameters from this signal then Brainard-type uncertainty will become gradually less relevant over time. However, in the naïve policy responses shown in figure 1, this type of learning is ruled out because we assume that the policymaker always believes that the given parameter estimates are the true parameter values. In this case, any deviation between the actual and forecast behaviour of the economy would be attributed to unanticipated shocks.
We also examine the range of forecast profiles obtained under shocks to the other endogenous variables. Figure 2 shows the optimal response of the nominal cash rate to various other one percentage point shocks. These simulations are similar to that shown for the output shock in the sense that they all exhibit considerable variation in the optimal policy response across different parameter draws. However, in all cases, the optimal policy response drives the economy back into equilibrium with real variables trending back to their baseline values and nominal growth rates stabilising in accordance with the inflation target.

These simulations show that, where there is uncertainty regarding the true model parameters, the naïve optimal policy response can vary quite considerably with observed parameter estimates. There are certainly considerable risks involved in implementing policy assuming that the estimated parameters are equal to their true values. In the next section, we demonstrate how the optimal policy problem can be modified to explicitly take into account parameter uncertainty. Rather than solving for the optimal path of the cash rate for a particular set of parameter estimates, the monetary authority takes into account the uncertainty associated with the distribution of parameter estimates.

Figure 2
Optimal policy responses ignoring parameter uncertainty
(Deviations from equilibrium, percentage points)
5.0 Characterising parameter uncertainty

The main contribution of this paper is to solve for optimal policy in the context of an empirical model of the Australian economy, taking into account parameter uncertainty. This section generalises the Brainard formulation of optimal policy under uncertainty to accommodate multivariate models and multi-period time horizons. In the following section, we apply this formulation to the model described in section 3.

The original Brainard model assumed that the monetary authority minimises squared deviations of a variable $y$ from a target (normalised to zero) by controlling a policy instrument $i$, where the economy is described by:

$$y_t = \theta i_t + \epsilon_t$$

where $\theta$ is an unknown parameter and $\epsilon$ is an independently and identically distributed white-noise process. To explore the issues involved in generalising this specification, it is useful to consider an economy that also includes a role for dynamics:

$$y_t = \theta i_t + \rho y_{t-1} + \epsilon_t$$

where $\rho$ is another unknown parameter.

In this model, parameter uncertainty arises because $\rho$ and $\theta$ can only be imprecisely estimated. The central message of this section and the next is that the implications of parameter uncertainty depend crucially upon the relative uncertainty about policy effectiveness ($\theta$) and persistence ($\rho$). If uncertainty about policy effectiveness dominates, then the usual Brainard conservatism result obtains. However, if uncertainty about persistence is more important then policy may be more aggressive.

To contrast the policy implications of these two types of uncertainty, consider a monetary authority which sets the policy instrument to affect the target variable over two periods. For a monetary authority which is aware of the uncertainty surrounding the model parameters, it is possible to analyse the costs of implementing a policy response that does not take this uncertainty into account. Denote the monetary authority’s parameter estimates as $\hat{\rho}$ and $\hat{\theta}$. Then, in the first two periods after the initial shock, $y$ will be related to the policy instrument by:

$$
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} = \begin{bmatrix}
\hat{\theta} & 0 \\
\hat{\rho} \hat{\theta}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix} + \begin{bmatrix}
\hat{\rho} y_0 \\
\hat{\rho}^2 y_0
\end{bmatrix}
$$

where $y_0$ is the deviation from target in the previous period. The policy response
sets the expected values of $y_1$ and $y_2$ to zero, minimising the sum of squared deviations of $y$ from target for a monetary authority which ignores uncertainty. Substituting this ‘naïve’ policy response back into (1), the monetary authority induces a distribution of possible outcomes for the target variable:

$$
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} = 
\begin{bmatrix}
\left(\rho - \frac{\theta}{\hat{\theta}}\right)y_0 \\
\rho \left(\frac{\theta}{\hat{\theta}}\right)y_0
\end{bmatrix}
$$

(6)

Two extreme cases make the intuition behind conservative and aggressive policy responses more clear. First, consider the case where $\rho$ is known to equal 1, while the monetary authority estimates $\hat{\theta} = 0.05$. In this case, persistence is known but the monetary authority is uncertain about policy effectiveness. If the monetary authority perceives that $\theta$ is equally likely to take one of three possible values; 0, 0.5 or 1, then the possible outcomes for $y$, in periods one and two, are shown in figure 3. The left panel shows the outcomes under the naïve policy response while the right panel shows the outcomes if the naïve policy response is halved (a conservative policy response). In this example, the more conservative policy response has a lower expected sum of squared deviations of $y$ from target. This response dominates the naïve response because it reduces the spread of possible outcomes.

**Figure 3**
Forecasts of the Target Variable (Uncertain $\theta$)
(Deviations from equilibrium, percentage points)
Next, consider the case where $\theta$ is known to equal 1 while the monetary authority estimates $\hat{\rho} = 0.5$. In this case, policy effectiveness is known but the monetary authority is uncertain about persistence. If the monetary authority perceives that $\rho$ is equally likely to take one of three possible values, 0, 0.5 or 1 then the possible outcomes for $y$ are shown in figure 4.

**Figure 4**
**Forecasts of the Target Variable (Uncertain $\rho$)**
(Deviations from equilibrium, percentage points)

In this example, the more aggressive policy response has a lower expected loss. The monetary authority would prefer the more aggressive policy response because any overshooting in period one is rapidly unwound while the magnitude of an undershooting is reduced.

The same motivation for aggressive policy responses applies more generally. The target variables overshoot if persistence is lower than expected, implying that the overshooting will be rapidly unwound. In contrast, if the target variables undershoot then persistence must be higher than expected so the undershooting will erode relatively slowly. A monetary authority which is aware of parameter uncertainty will take this asymmetry into account, moving the policy instrument more aggressively. This will ensure that more persistent outcomes are closer to target at the cost of forcing less persistent outcomes further from target. This reduces expected losses because the outcomes furthest from target unwind most quickly. However, in this case it is important to recognise that if the loss function contained a discount factor then this could reduce the costs of conservative policy. For example, with discounting, the conservative policymaker in figure 4 is less concerned about the bigger actual losses sustained in the second period. Therefore, if policymakers do not care as much about the future as the present, then they may prefer less activism rather than more.

To summarise, the implications of parameter uncertainty are ambiguous. Policy should be more conservative when the effectiveness of policy is relatively imprecisely estimated while policy may be...
more aggressive when the persistence of the economy is less precisely estimated.

This example also highlights the importance of assuming that monetary authorities never learn about the true model parameters. The monetary authority always observes the outcome in period one. If the extent of the policy error in period one conveys the true value of \( \rho \) or \( \theta \), then policy could be adjusted to drive \( y \) to zero in period two. Ruling out learning prevents these ex post policy adjustments, making the initial policy stance time consistent. To the extent that uncertainty is not being resolved through time, this is a relevant description of policy. Additional data may sharpen parameter estimates but these gains are often offset by instability in the underlying parameters themselves. Sack (1997) explored the case in which uncertainty about the effect of monetary policy is gradually resolved through time by learning using a simple model in which the underlying parameters are stochastic.

Having used examples to identify the influences of uncertainty on policy, the general problem is stated in Proposition 1. Matrix notation makes this statement of the policy problem considerably more transparent.

**Proposition 1:**
The optimal policy problem for a monetary authority with quadratic preferences given by equation 1 and a backward-looking multivariate model of the economy (that is linear in both the variables and shocks) can be written in the form:

\[
\min_{R} \text{Loss} = E[T'\Omega T]
\]

subject to:

\[
T = FR + G
\]

\(T\) is a vector of policy targets in each period of the forecast horizon; \(R\) is the vector of policy instruments; the matrices, \(F\) and \(G\) are functions of both history and the parameter estimates while \(\Omega\) summarises the penalties contained in the objective function. The time subscript has been omitted for simplicity. The proof is left to Appendix B. The form of the problem in Proposition 1 highlights its similarity to the problem solved by Brainard. The loss function continues to be quadratic and the constraint set remains linear. Consequently, the usual optimal policy response under parameter uncertainty will apply, as stated in the Proposition 2.
Proposition 2:

If $F$ and $G$ are stochastic, the solution to the optimal policy problem in Proposition 1 is:

$$\mathbf{R}^* = -\left(\mathbf{F}' \Omega \mathbf{F} + \mathbb{E} \left[ (\mathbf{F} - \mathbb{E} \mathbf{F})' \mathbf{F} (\mathbf{F} - \mathbb{E} \mathbf{F}) \right] \right)^{-1} \left(\mathbf{G}' \Omega \mathbf{F} + \mathbb{E} \left[ (\mathbf{G} - \mathbb{E} \mathbf{G})' \mathbf{F} (\mathbf{F} - \mathbb{E} \mathbf{F}) \right] \right) \quad (9)$$

If $F$ and $G$ are deterministic, the solution to the optimal policy problem in Proposition 1 is:

$$\mathbf{R}^* = -\left(\mathbf{F}' \Omega \mathbf{F} \right)^{-1} \mathbf{G}' \Omega \mathbf{F} \quad (10)$$

$\mathbb{F}$ is the expectation of $F$ and $\mathbb{G}$ is the expectation of $G$. $F$ and $G$ are stochastic if they contain parameter estimates. The proof is given in Appendix B. The deterministic case in Proposition 2 describes the naïve policy response, when the monetary authority ignores parameter uncertainty. The difference between the optimal policy responses with and without parameter uncertainty can be ascribed to the variance of $F$ and the covariance between $F$ and $G$. Brainard’s policy conservatism result depends crucially on the independence of $F$ and $G$. However, $F$ and $G$ will not be independent if they are derived from a model that exhibits persistence.

To make the optimal-policy definition in (9) operational, it is necessary to compute the variance and covariance terms. This can be done using a relative frequency estimator of the expectations terms. Instead of working with the loss function in (7), we estimate it using:

$$\text{Loss} = \frac{1}{N} \sum_{i=1}^{N} T_i' \Omega T_i \quad (11)$$

where $N$ is the number of parameter draws. The first-order necessary condition for this loss function to be minimised subject to the set of constraints in (8) is:

$$\hat{\mathbf{R}}^* = \left( \frac{1}{N} \sum_{i=1}^{N} \mathbf{F}_i' \Omega \mathbf{F}_i \right)^{-1} \left( \frac{1}{N} \sum_{i=1}^{N} \mathbf{G}_i' \Omega \mathbf{F}_i \right) \quad (12)$$

By summing across a large number of parameter draws, optimal policy can be computed from the linear relationship between the target variables of policy and the policy instrument. As the number of draws from the estimated parameter distribution increases, this approximation will tend to the true optimal policy.

The next section presents these optimal policy computations and compares them with naïve policy responses. Both types of policy responses are estimated using the same one thousand draws from the underlying parameter distribution. A large number of draws is required because of the large number of parameter estimates.
6.0 Optimal policy acknowledging parameter uncertainty

Beginning with a shock to output, figure 5 compares optimal policy responses. The darker line represents optimal policy when parameter uncertainty is acknowledged, while the lighter line represents the naïve optimal policy response.

When computing the naïve policy response, it is necessary to specify the parameter vector that the monetary authority interprets as being the true parameter values. Usually this vector would contain the original parameter estimates. However, we compute the naïve policy response using the average value of the parameter vector draws. This prevents differences between the two policy profiles being driven by the finite number of draws which are taken from the parameter distribution.

Figure 5
Real output shock
(Deviations of the nominal cash rate from equilibrium, percentage points)

The key feature of interest in figure 5 is the larger initial policy response when uncertainty is taken into account. While later oscillations in the cash rate are similar, the initial response of the cash rate is more than ten percent larger when policy takes parameter uncertainty into account. The finding, that policy should react more aggressively because of parameter uncertainty, is specific to the output shock used to generate figure 5. In contrast, the naïve policy response is relatively more aggressive for a one percentage-point shock to the real exchange rate as shown in figure 6. This implies that the persistence of a real exchange rate shock is more precisely estimated than the persistence of an output shock. With less uncertainty about the persistence of a real exchange rate shock, the optimal policy response taking into account uncertainty is more conservative because it is dominated by uncertainty about policy effectiveness.
Figures 7 to 9 present the policy responses to shocks to import prices, consumer prices and unit labour costs respectively. In each case, the naïve policy response is initially more conservative than the policy response that accommodates parameter uncertainty.

Figure 7
Import price shock
(Deviations of the nominal cash rate from equilibrium, percentage points)
Figure 8
Price inflation shock
(Deviations of the nominal cash rate from equilibrium, percentage points)

Figure 9
Unit labour cost shock
(Deviations of the nominal cash rate from equilibrium, percentage points)
These three types of nominal shocks to the model confirm that conservatism is by no means the generic implication of parameter uncertainty. In our model, it appears that uncertainty about the effectiveness of the policy instrument is generally dominated by uncertainty about model persistence and this explains the more aggressive policy response to most of the shocks which we examined.

7.0 Conclusion
This paper extends Brainard’s formulation of policy making under uncertainty in several directions. First, it generalises the solution to the optimal policy problem to accommodate multiple time-periods and multiple objectives for policy. This generalisation develops the stochastic properties of the equations relating target variables to the policy instrument from the estimated relationships defining the underlying economic model.

Whereas uncertainty about the effectiveness of policy tends to recommend more conservative policy, we explore the intuition for why other forms of parameter uncertainty may lead to more aggressive policy. In a simple example, we show that uncertainty about the dynamics affecting the economy can be a source of additional policy activism. However, this consequence of parameter uncertainty is only relevant in a multi-period generalisation of the Brainard model.

In the context of any specific model, it is an empirical issue to determine the exact implications of parameter uncertainty for monetary policy. We examine this using a small linear model of the Australian economy that captures the key channels of monetary policy within an open-economy framework. Optimal policy responses, ignoring parameter uncertainty, are compared to optimal responses that explicitly take parameter uncertainty into account. While the differences between these policy responses vary with the source of shocks to the economy, the evidence suggests that, for most shocks, parameter uncertainty motivates more aggressive use of the short-term interest rate.

Although the results in this paper are reported as deviations from equilibrium, the method used to construct optimal policy responses under parameter uncertainty is also applicable in a forecasting environment where past data must be taken into account. The approach is applicable to all backward-looking linear models in which the objectives of the policymaker are quadratic.

The simulations also demonstrate how frequency-sampling techniques can be used to evaluate the analytic expression for optimal policy under parameter uncertainty, despite the presence of complex expectations terms. This approach to policy determination is as practical, and more theoretically appealing, than the application of alternative rules-based approaches.

While the findings of the paper are of considerable interest, they should not be overstated. In particular, the implications of parameter uncertainty are dependent upon the type of shock being accommodated. They are also dependent upon the specification of the model. For example, changes to the model specification will substantially alter the measured uncertainty attached to the effectiveness of policy instruments relative to the measured uncertainty associated with the model’s dynamics. If the techniques developed in this paper are to be of use in implementing monetary
policy, the underlying model must first be well understood and carefully specified. Also, it is worth remembering that, although our model suggests that optimal monetary policy taking account of uncertainty is more activist for most kinds of shocks, the difference in policy response is quite small relative to the degree of conservatism that is actually practiced by most central banks.

In this paper we have not sought to argue that conservative monetary policy is not optimal. In fact, there are probably a number of good reasons why conservative policy is optimal. Instead, the central message of the paper is that, if we are to justify conservative monetary policy, then explanations other than Brainard’s are required.
Appendix A

Model description

Unless otherwise specified, the equations were estimated by ordinary least squares. When necessary, the variance-covariance matrices of the coefficients were estimated using White’s correction for residual heteroscedasticity and the Newey-West correction for residual autocorrelation.

For each equation, we report the coefficient estimates, their corresponding (adjusted) standard errors, standard diagnostic tests and the full correlation matrices for the coefficient estimates.

Output equation

$\Delta y_t = \alpha_0 - 0.23(y_{t-1} + 0.10r_{t-1} - 0.05r_{t-1} + 0.27y_{t-1} + 0.29\Delta y_{t-1} + 0.01\Delta y_{t-1} + 0.01\Delta y_{t-2} - 0.09r_{t-2} - 0.01r_{t-3} + 0.05r_{t-4} - 0.09r_{t-5} - 0.07r_{t-6}$

Diagnostic tests

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<thead>
<tr>
<th>Test</th>
<th>Significance level</th>
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<tr>
<td>Lagrange Multiplier test for third-order serial correlation</td>
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<tr>
<td>Test for third-order ARCH properties in the residuals</td>
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<tr>
<td>Jarque-Bera test for residual non-normality</td>
<td>0.28</td>
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</table>
Collinearity between the terms of trade and the real exchange rate causes these coefficients to be imprecisely estimated. Consequently, the coefficients on the lagged levels of the terms of trade and real exchange rate were calibrated so that an equal simultaneous rise in both the terms of trade and the real exchange rate results in a net contraction of output in the long run. The output equation is based on Gruen and Shuetrim (1994).
Price equation

\[ \Delta p_t = \alpha_2 + 0.04 (ulc_{t-1} - p_{t-1}) + 0.04 (p^m_{t-1} - p_{t-1}) + 0.08 \Delta ulc_t + 0.02 \Delta p^m_{t-3} + 0.12 \text{gap}_{t-3} \]

Diagnostic tests

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Parameter correlation matrix

<table>
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<tr>
<th></th>
<th>ulc_{t-1} - p_{t-1}</th>
<th>p^m_{t-1} - p_{t-1}</th>
<th>\Delta ulc_t</th>
<th>\Delta p^m_{t-3}</th>
<th>gap_{t-3}</th>
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<tr>
<td>ulc_{t-1} - p_{t-1}</td>
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<tr>
<td>p^m_{t-1} - p_{t-1}</td>
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<td>1.00</td>
<td>-0.89</td>
<td>-0.82</td>
<td>1.00</td>
</tr>
<tr>
<td>\Delta ulc_t</td>
<td>-0.89</td>
<td>-0.82</td>
<td>1.00</td>
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</tr>
<tr>
<td>\Delta p^m_{t-3}</td>
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<td>-0.014</td>
<td>-0.05</td>
<td>1.00</td>
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<tr>
<td>gap_{t-3}</td>
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<td>0.27</td>
<td>0.25</td>
<td>0.16</td>
<td>1.00</td>
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</tbody>
</table>

As specified, this equation implies linear homogeneity in the long run relationship between prices, unit labour costs and import prices. The equation is based on de Brouwer and Ericsson (1995).

Unit labour cost equation

\[ \Delta ulc_t = 0.50 \Delta p_{t-1}^{(0.05)} + 0.50 \Delta p_{t-2}^{(0.05)} + 0.09 \text{gap}_{t-1}^{(0.03)} \]
Diagnostic tests

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<td>Jarque-Bera test for residual non-normality</td>
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Parameter correlation matrix

\[
\begin{pmatrix}
\Delta p_{t-1} & 1.00 \\
gap_{t-1} & -0.16 & 1.00
\end{pmatrix}
\]

The restriction that the coefficients on lagged inflation sum to unity was imposed. This explains why the coefficient on inflation lagged two periods is not included in the parameter correlation matrix. The equation was estimated by generalised least squares to correct serial correlation in the residuals.

Import price equation

\[
\Delta p_t^{mp} = \alpha_4 - 0.18 \left( p_t^{mp} - p_{t-1}^{mp} + n_{t-1} - 0.51 \Delta n_{t-1} \right)
\]

Diagnostics tests

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<thead>
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<td>Breusch Pagan test for heteroscedasticity</td>
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<td>Jarque-Bera test for residual non-normality</td>
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</tr>
</tbody>
</table>
Parameter correlation matrix

\[ p_{t-1} - p_{t-1}^{\text{shadow}} + \text{ner}_{t-1} \]
\[ \Delta \text{ner}_t \]

1.00 .
-0.16 1.00

As specified, the equation implies linear homogeneity between import prices, the exchange rate and the foreign price of Australian imports. This is equivalent to assuming purchasing power parity and full first-stage pass-through of movements in the exchange rate and foreign prices to domestic prices.

Real exchange rate equation

\[ \Delta \text{rer}_t = \alpha_5 - 0.31 \text{rer}_{t-1} + 0.19 \text{tot}_{t-1} + 0.72 (r_{t-1} - r_{t-1}^{*}) + 1.39 \Delta \text{tot}_t \]

Diagnostics tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Significance level</th>
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<td>Jarque-Bera test for residual non-normality</td>
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</tr>
</tbody>
</table>

Parameter correlation matrix

\[ \text{rer}_{t-1} \]
\[ \text{tot}_{t-1} \]
\[ r_{t-1} - r_{t-1}^{*} \]
\[ \Delta \text{tot}_t \]

1.00 .
0.05 1.00 .
-0.26 -0.70 1.00 .
0.25 -0.46 0.30 1.00
A Hausman test failed to reject the null hypothesis that the terms of trade was exogenous so results have been reported using OLS estimates.\textsuperscript{8} OECD industrial production was used to instrument the contemporaneous difference in the terms of trade when conducting the Hausman test. The equation is based on Gruen and Wilkinson (1991), Blundell-Wignall, Fahrer and Heath (1993) and Tarditi (1996).

\textsuperscript{8} The p-value of the test statistic was 0.99 indicating that the test fails to reject at any reasonable level of significance.
Appendix B
Proofs

Proposition 1:
The optimal policy problem for a monetary authority with quadratic preferences given by (1) and a backward-looking multivariate model of the economy (that is linear in both the variables and shocks) can be written in the form:

\[
\min_{\mathbf{R}} \text{Loss} = E \left[ \mathbf{T}' \mathbf{\Omega} \mathbf{T} \right]
\]

subject to:

\[
\mathbf{T} = \mathbf{F} \mathbf{R} + \mathbf{G}
\]

Proof:
The loss function in (1) can be rewritten using matrix notation as:

\[
\text{Loss} = E_t \left[ \alpha \mathbf{Y}_t' \mathbf{Y}_t + \beta \mathbf{\Pi}_t' \mathbf{\Pi}_t - \gamma \mathbf{R}_t' (\mathbf{I} - \mathbf{\Gamma})' (\mathbf{I} - \mathbf{\Gamma}) \mathbf{R}_t \right]
\]

where:

\[
\mathbf{Y}_t = \begin{bmatrix} \text{gap}_{t+1} & \text{gap}_{t+2} & \ldots & \text{gap}_{t+h} \end{bmatrix}'
\]

\[
\mathbf{\Pi}_t = \begin{bmatrix} (\pi_{t+1} - \pi^*) & (\pi_{t+2} - \pi^*) & \ldots & (\pi_{t+h} - \pi^*) \end{bmatrix}'
\]

\[
\mathbf{R}_t = \begin{bmatrix} i_{t+1} & i_{t+2} & \ldots & i_{t+h} \end{bmatrix}'
\]

and \( \mathbf{\Gamma} \) is the matrix that lags the nominal interest rate vector \( \mathbf{R}_t \) by one period; and \( \mathbf{I} \) is an \( (h \times h) \) identity matrix. The subscript \( t \) denotes the current date from which forecasts are being generated.
Given that the model of the economy is linear, the policy target variables are affine transformations of the forecast profile for the policy instrument:

\[ \mathbf{Y}_t = \mathbf{A} \mathbf{R}_t + \mathbf{B} \]

and

\[ \mathbf{Π}_t = \mathbf{C} \mathbf{R}_t + \mathbf{D} \]

\( \mathbf{A} \), \( \mathbf{B} \), \( \mathbf{C} \) and \( \mathbf{D} \) are stochastic matrices constructed from the parameters of the model and the history of the economy. The structure of these stochastic matrices is determined by the relationships laid out in the definition of the model’s equations. Matrices \( \mathbf{B} \) and \( \mathbf{D} \) are the impulse response functions of the output gap and inflation to a shock at time \( t \). Likewise, matrices \( \mathbf{A} \) and \( \mathbf{C} \) are the marginal impact of the nominal cash rate on the output gap and inflation respectively.

Defining \( \mathbf{Δ}_t = (\mathbf{I} - \mathbf{Γ}) \mathbf{R}_t \), the vector of first differences in the nominal cash rate over the forecast horizon, it is possible to specify the full set of policy targets as:

\[ \mathbf{T}_t = \left[ \mathbf{Y'}_t \quad \mathbf{Π'}_t \quad \mathbf{Δ'}_t \right] \]

Then, upon dropping time subscripts, the optimal policy problem can be restated succinctly as:

\[ \min_{\mathbf{R}} \text{Loss} = E \left( \mathbf{T'} \mathbf{Ω} \mathbf{T} \right) \]

subject to:

\[ \mathbf{T} = \mathbf{F} \mathbf{R} + \mathbf{G} \]
where the matrices $F$ and $G$ are defined in terms of $A, B, C, D$ and $(I - \Gamma)$ as:

\[
F = \begin{bmatrix}
A & 0 & 0 \\
0 & C & 0 \\
0 & 0 & I - \Gamma
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
B \\
D \\
0
\end{bmatrix}
\]

(24)

and the weights on the different components of the loss function $\alpha, \beta$ and $\gamma$, have been subsumed into the diagonal matrix $\Omega$ according to:

\[
\Omega = \begin{bmatrix}
\alpha & 0 & 0 \\
0 & \beta & 0 \\
0 & 0 & \gamma
\end{bmatrix} \otimes I
\]

(25)

where $I$ is the same identity matrix used to define the first differences in the cash rates, $\Delta$. Ignoring the fact that it is in matrix notation and observing that the target values of the target variables have been normalised to zero, this problem is exactly the same as that on page 413 of Brainard (1967) as required.

**Proposition 2:**

If $F$ and $G$ are stochastic, the solution to the optimal policy problem in Proposition 1 is:
\[ \mathbf{R}^* = -\left( \mathbf{F}' \Omega \mathbf{F} + E\left[ (\mathbf{F} - \mathbf{F})' \Omega (\mathbf{F} - \mathbf{F}) \right] \right)^{-1} \left( \mathbf{G}' \Omega \mathbf{F} + E\left[ (\mathbf{G} - \mathbf{G})' \Omega (\mathbf{F} - \mathbf{F}) \right] \right) \]

(26)

If \( \mathbf{F} \) and \( \mathbf{G} \) are deterministic, the solution to the optimal policy problem in Proposition 1 is:

\[ \mathbf{R}^* = -\left( \mathbf{F}' \Omega \mathbf{F} \right)^{-1} \mathbf{G}' \Omega \mathbf{F} \]

(27)

Proof:

The loss function in (7) can be rewritten by adding and subtracting the expected values of \( \mathbf{T} \) from itself, yielding:

\[ \min_{\mathbf{R}} \text{Loss} = E\left[ (\mathbf{T} - \mathbf{T} + \mathbf{T})' \Omega (\mathbf{T} - \mathbf{T} + \mathbf{T}) \right] \]

(28)

Upon expanding, this loss function can be alternatively be expressed as:

\[ \min_{\mathbf{R}} \text{Loss} = E\left[ (\mathbf{T} - \mathbf{T})' \Omega (\mathbf{T} - \mathbf{T}) \right] + \mathbf{T}' \Omega \mathbf{T} + 2 \mathbf{T}' \Omega E[\mathbf{T} - \mathbf{T}] \]

\[ = E\left[ (\mathbf{T} - \mathbf{T})' \Omega (\mathbf{T} - \mathbf{T}) \right] + \mathbf{T}' \Omega \mathbf{T} \]

(29)

Taking advantage of the fact that \( E(\mathbf{T}) \equiv \mathbf{T} \).

Substituting in (8) and simplifying then yields:

\[ \min_{\mathbf{R}} \text{Loss} = E\left[ (\mathbf{F} - \mathbf{F})' \mathbf{R} + \mathbf{G} - \mathbf{G} \right] (\mathbf{F} - \mathbf{F})' \mathbf{R} + \mathbf{G} - \mathbf{G} + \mathbf{R}' \mathbf{F}' \Omega \mathbf{F} \mathbf{R} + 2 \mathbf{R}' \mathbf{F}' \Omega \mathbf{F} \mathbf{G} + \mathbf{G}' \Omega \mathbf{G} \]

(30)
The first order necessary condition is obtained by differentiating with respect to $\tilde{R}$. 

$$2(F - F')\Sigma (F - F) \tilde{R} + 2(F - F')\Sigma (G - G) \tilde{R} + 2 F' \Sigma F \tilde{R} + 2 F' \Sigma G = 0$$

(31)

Solving for $\tilde{R}^*$ then gives optimal policy when taking uncertainty into account, as expressed in (26). Given that the loss function is strictly convex, this first order necessary condition is also sufficient for a minimum of the expected loss function.

The naïve optimal policy response obtains as a simplification of this formulation in which $F$ is set to $\overline{F}$ and $G$ is set to $\overline{G}$. 
References


