Abstract

We examine a general equilibrium model with collateral constraints and increasing returns to scale in production. The utility function is nonseparable, with no income effect on the consumer’s choice of leisure. Unlike this model without a collateral constraint, we find that indeterminacy of equilibria is possible. Hence, business cycles can be driven by self-fulfilling expectations. This is the case for more realistic parametrizations than in previous, similar models without these features. We propose that this model can be applied to the Great Recession, in which speculative asset price movements, driven by increasing debt, led to a credit crisis.

*Keywords: Business cycles, Credit markets, Collateral Constraint, Sunspots. JEL Classification: E32
1 Introduction

The Great Recession of 2008-9 has renewed interest in the role of credit and financial markets in macroeconomic fluctuations. In particular, as are many macroeconomists of late, we are interested in the role that speculative asset market bubbles and the functioning of credit markets play in determining business cycle fluctuations. An extensive review of the existing literature is beyond the scope of this paper, but the idea that economies are inherently subject to bouts of speculation that lead to alternating booms and crises is not new.\(^1\) The current paper presents a general equilibrium model that addresses these issues. In our model, there is an interaction between credit constraints and self-fulfilling expectations.

Starting with Benhabib and Farmer (1994) and Farmer and Guo (1994), a large literature now exists in which a standard real business cycle model is modified to include sufficiently high increasing returns to scale, resulting in indeterminate equilibria driven by sunspot shocks.\(^2\) Jaimovich (2008) and Meng and Yip (2008) (hereafter MYJ) add non-separable preferences as in Greenwood, Hercowitz, and Huffman (1988) (hereafter GHH) to the one-sector model with increasing returns to scale. With no income effects on leisure, they find that indeterminacy is ruled out, no matter the size of returns to scale. Here, we overturn MYJ’s results by adding a collateral constraint as in Kiyotaki and Moore (1997). The addition of a collateral constrained borrowing limit allows for the existence of indeterminacy and hence endogenous, expectations-driven business cycles. In addition, this indeterminacy results with empirically plausible returns to scale.

In our model, land is the only asset and credit takes on the following form. Firms must pay for some of their inputs before production takes place, and so they must borrow within the period. This assumption appears to be relevant for the workings of typical firms. For example, evidence in Devereux and Schiantarelli (1989) suggests that firms fund a significant part of their current expenses with external funds. In our model, we restrict this funding to credit from the banking sector. The credit constraint stands in for a lack of contract enforcement, which limits the amount of credit firms can obtain with its collateral. If firms fail to repay debt, the creditors will seize the collateralized land and sell it.

In previous one-sector models, whether or not indeterminacy results can be understood as follows. Upon optimistic expectations about the future, agents act upon (subjectively) higher wealth. Hence, labor supply shifts

---

\(^1\)See for example, Minsky (1985) and Kindleberger (2000). See also Akerlof and Shiller (2009), along with Farmer’s (2009) critical review.

\(^2\)See Benhabib and Farmer (1999) for an excellent survey.
inwards. In the Benhabib-Farmer-Guo set-up, with increasing returns high enough to make the reduced form labor demand upward sloping, employment rises and the expectation is self-fulfilled. In MYJ’s set-up, there is no wealth effect on leisure, and so such expectations cannot be self-fulfilling.

On the contrary, in our model, with a collateral constraint, indeterminacy can result with downward sloping labor demand and no income effect. If people become optimistic about the prospects of the economy, their demand for assets, in particular, land, rises. As it is in fixed supply, the price of land surges, relaxing the credit constraint. The amount of borrowing goes up proportionately with the prices of the collateralizable land. This starts a positive feedback: firms are able to expand production as labor demand shifts out. Real economic activity increases, and the optimistic expectation is self-fulfilled.

We examine two versions of the model. In both, indeterminacy is possible. In the first, output is produced with labor, land and capital. In addition, the utilization rate of capital is allowed to vary. As in Wen (1998), including capital utilization simply serves to lower the threshold level of returns to scale necessary for indeterminacy, by increasing the elasticity of output with respect to labor. To further lower this threshold, the second version adds material inputs into the production function, so that firms produce gross output. This increases this elasticity even more.

Indeterminacy in the second version of our model is empirically plausible in two important ways. First, the threshold value of returns to scale is within the range estimated by many authors (for example Harrison, 2003). Second, this is the case for much lower labor supply elasticities than are typically assumed in previous work, where an infinite labor supply elasticity is often used (for example Farmer and Guo, 1994 and Wen, 1998). In sum, the lack of an income effect, the binding collateral constraint and the inclusion of materials in the production function, all make sunspot equilibria easier to obtain.

The rest of this paper proceeds as follows. In Section 2 we outline the basic model, without material inputs. Section 3 describes the steady state and calibration, In Section 4, we quantify the regions in which indeterminacy exists. Section 5 adds material inputs to the model. Section 6 applies the model to two historic episodes. Section 7 summarizes.

---

3Cordoba and Ripoll (2004) obtain indeterminacy in a version of Kiyotaki and Moore (1997). However, they focus on parametrizations with determinacy.
2 The artificial economy

Our economy is based on Kiyotaki and Moore’s (1997) model of credit frictions.\textsuperscript{4} There is no fundamental uncertainty. The economy is populated by a continuum of (measure one) agents who have identical preferences. Each agent has access to a production technology. Let $c_t$ be consumption and $h_t$ the agents’ own labor supply. Then, lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

with period utility

$$u(c_t, h_t) = \frac{1}{1-\sigma} \left( c_t - \frac{\phi}{1+\varepsilon} h_t^{1+\varepsilon} \right)^{1-\sigma}$$

which corresponds to Greenwood, Hercovitz and Huffman’s (1988) quasi-convex utility. GHH preferences have the property that the supply of labor is immune to income effects. $\beta$ denotes the discount factor. The disutility of working is measured by $\phi$, $1/\varepsilon$ corresponds to the intertemporal elasticity of substitution in labor supply and $1/\sigma$ measures the elasticity of substitution of consumption. We restrict $0 < \beta < 1$, $\gamma > 0$, $\varepsilon > 0$, and $\sigma > 0$.

The period budget constraint is

$$w_t h_t + y_t + q_t (a_{t-1} - a_t) \geq w_t n_t + c_t + k_{t+1} - (1-\delta_t)k_t$$

with income on the left hand side and spending on the right hand side. Here, $w_t$ is the real wage, and $k_t$ stands for the stock of capital. We assume that agents do not work for themselves, and so each hires labor services $n_t$. In equilibrium, $h_t = n_t$. The agents enter the period with land holdings $a_{t-1}$ while $a_t$ denotes land holdings during production in period $t$. Each agent sells labor, land and capital services, and hires these services in the respective markets. Land is traded at the beginning of the period at price $q_t$, and is in fixed supply, which we normalize to one. Land is of the utmost importance in our model. It is required to be able to undertake production, which is the source of its demand.

As in most studies with variable capital utilization, $u_t$, the rate of deprecation, $\delta_t$, is an increasing function of the utilization rate:

$$\delta_t = \frac{1}{\theta} u_t^\theta \quad \theta > 1.$$  

\textsuperscript{4}Some of its simplifications follow Kobayashi and Nutahara (2007),
Finally, $y_t$ is output; and its production is described by

$$y_t = a_t^\gamma (u_t k_t)^{\alpha} n_t^{1-\alpha - \nu} Y_t^{\frac{\gamma}{1+\gamma}}.$$  

Note that the producers face constant returns to scale at the private level. $Y_t$ stands for aggregate output; and the externality in production is of size $\gamma$. All markets are perfectly competitive.

Each period, the price of land is determined in trade. Next, wages have to paid. Because the other transactions and production have not yet taken place, agents are forced to borrow working capital in advance in the form of (intratemporal) loans from financial intermediaries. To prevent default, the availability of credit is restricted by the endogenous borrowing constraint

$$\psi q_t a_t \geq w_t n_t \quad 0 \leq \psi \leq 1.$$  

As in Kiyotaki and Moore (1997), this constraint stands in for limited contract enforcement. Borrowers must put up valuable collateral against debt. The only asset accepted by creditors is land. If borrowers fail to repay their debt, the creditors will seize the land and sell it. Hence, an agent’s loans cannot exceed the fraction $\psi$ of the market value of the collateral. Repayment of the debt is assumed to occur within the current period such that there is a unit opportunity cost to funds. If $\psi$ is large, the collateral constraint will not bind and the model reduces to the standard model. Here, we assume that this constraint is binding at all times (by choice of parametrization and its effect on $\psi$).

The program can be compactly written as

$$\max_{\{c_t, h_t, a_{t+1}, n_t, a_t\}} \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left( c_t - \frac{\phi}{1+\varepsilon} h_t^{1+\varepsilon} \right)^{1-\sigma}$$

$$+ \lambda_t \left[ w_t h_t + a_t^\gamma (u_t k_t)^{\alpha} n_t^{1-\alpha - \nu} Y_t^{\frac{\gamma}{1+\gamma}} + q_t (a_{t-1} - a_t) - w_t n_t - c_t - k_{t+1} + (1-\delta_t) k_t \right]$$

$$+ \mu_t \left[ \psi q_t a_t - w_t n_t \right].$$

In equilibrium, $n_t = h_t$, and the optimality conditions imply that

$$\left( c_t - \frac{\phi}{1+\varepsilon} h_t^{1+\varepsilon} \right)^{-\sigma} = \lambda_t$$  

$$- \left( c_t - \frac{\phi}{1+\varepsilon} h_t^{1+\varepsilon} \right)^{-\sigma} \phi h_t^\gamma = \lambda_t w_t$$  

$$(1 - \alpha - \nu) \frac{y_t}{h_t} = \frac{\lambda_t + \mu_t}{\lambda_t} w_t = 0$$  

5
\[
\beta \lambda_{t+1} q_{t+1} + \lambda_t \nu \frac{y_t}{a_t} + \psi \mu_t q_t - q_t \lambda_t = 0
\] (4)

\[
\beta \lambda_{t+1} \left[ \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta_{t+1} \right] - \lambda_t = 0
\] (5)

\[
\psi q_t a_t - w_t h_t = 0
\] (6)

\[
\alpha \frac{y_t}{u_t} = \alpha^{a-1} k_t
\] (7)

and

\[
y_t = a_t^{\nu(1+\gamma)} u_t^{a(1+\gamma)} k_t^{\alpha(1+\gamma)} h_t^{(1-a-\nu)(1+\gamma)}
\] (8)

Equations (1) and (2) describe the consumption leisure trade-off. Equation (3) denotes the demand for labor; this condition is distorted by the borrowing constraint. In the absence of credit frictions, \( \mu_t = 0 \), and the standard condition equating the marginal product and wage results. Equation (4) describes the intertemporal demand for land. Integrating (4) forward gives the current price of land which is equal to the present discounted value of current and future marginal products augmented by a credit constraint effect (in parenthesis):

\[
q_t = \sum_{\tau=0}^{\infty} \left( \prod_{i=0}^{\tau} \frac{1}{1 - \psi \lambda_{i+1}} \right) \beta^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \nu \frac{y_{t+\tau}}{a_{t+\tau}}.
\]

Equation (5) is the usual intertemporal Euler equation and (6) repeats the borrowing constraint. Equation (7) equates the marginal product and marginal cost of raising the utilization rate. Equation (8) is the production technology in symmetric equilibrium. Finally, we have the aggregate resource constraint

\[
y_t - c_t - k_{t+1} + (1 - \delta) k_t = 0
\] (9)
as well as the equilibrium condition

\[a_t = 1.\] (10)

3 The steady state and calibration

Let us now turn to the unique stationary state of this economy. Since it is not standard, we describe it in some detail. Below we discuss our calibrations of \( \bar{\lambda}, \bar{\nu}, \bar{k}, \bar{wh}, \bar{\beta}, \sigma \) and \( \delta \). Once these are set, the steady state can be computed. The intertemporal Euler equation for capital,

\[
1 - \beta \left[ \alpha (1 - \eta) \frac{y}{k} + 1 - \delta \right] = 0
\]
determines $\alpha$. Next (3)

$$
(1 - \alpha - \nu) \frac{y}{wh} = 1 + \frac{\mu}{\lambda}
$$

yields a value for $\nu$. The consumption share, $c/y$, is determined by (8)

$$
\delta = \frac{y}{k}(1 - \frac{c}{y}).
$$

The intertemporal Euler equation for capital and the collateral constraint become

$$
\frac{\nu}{1 - \beta - \frac{a}{k}\psi} = \frac{aq}{y}
$$

and

$$
\psi \frac{qa}{y} - \frac{wh}{y} = 0.
$$

These form a system of two equations in two unknowns which can be solved for $\frac{aq}{y}$ and $\psi$. Lastly,

$$
\theta = \frac{1 - \beta(1 - \delta)}{\beta \delta}.
$$

We calibrate our economy to averages of the US economy. The fundamental period is a quarter. In our benchmark calibration, we assume $\sigma = 1$, $\beta = 1.03^{-1/4}$ and $\delta = 0.02$. The capital to GDP ratio is set to 9, which corresponds to Maddison’s (1991) annual value for the US ratio of gross nonresidential capital stock to GDP. Empirical measures of the intertemporal elasticity of labor supply, $1/\varepsilon$, are largely varied. Kimball and Shapiro (2009) suggest that the Frisch elasticity of labor supply is about one. Gourinchas and Parker (2002) present estimates that range from 0.7 to 2.2. We set $1/\varepsilon = 2.2$. Hence, labor supply is significantly less elastic than is typical of models with indeterminacy (i.e. infinity), and it also less elastic than is normally assumed in real business cycle models (e.g. King, Plosser and Rebelo, 1988, use 4).\(^5\)

Lastly, we set $\mu/\lambda = 0.255$, to ensure that the credit constraint is binding. In doing this, we choose the smallest possible value of $\mu/\lambda$ given the rest of our calibration. One way to interpret $\mu/\lambda$ is as an endogenous theory of the labor wedge. We can rewrite our first order condition for labor, (3), as:

$$
\frac{\lambda}{\lambda + \mu} (1 - \alpha - \nu) \frac{y}{h} = w
$$

\(^5\)GHH preferences do not nest indivisible preferences, hence $\varepsilon = 0$ cannot be interpreted as Hansen-Rogerson. Jaimovich (2007) shows that the Frisch labor supply elasticity is given by $(1 - \varepsilon h)/\varepsilon h$. 
and consider the Chari, et al. (2007) interpretation of the wedge as a tax. That is, in

\[(1 - \tau) MPL = w\]

we have \(\frac{\lambda}{\lambda + \mu} = 1 - \tau\), and hence a wedge of about 20 percent.\(^6\)

We then log-linearize the model, reducing it to:

\[
\begin{bmatrix}
\hat{k}_{t+1} \\
E_t \hat{y}_{t+1} \\
E_t \hat{c}_{t+1}
\end{bmatrix} = M
\begin{bmatrix}
\hat{k}_t \\
\hat{y}_t \\
\hat{c}_t
\end{bmatrix}.
\]

Indeterminacy requires that at least two eigenvalues of the matrix \(M\) are smaller than unity. We look for indeterminacy in the next Section.

## 4 Indeterminacy

Our main result is that indeterminacy can in fact result. This stands in sharp contrast to MYJ, in which indeterminacy is ruled out in the one-sector model with no income effect and no collateral constraint.

**Result 1:** In sharp contrast to MYJ, indeterminate equilibria result in our model with sufficiently high returns-to-scale.

In order to quantify "sufficiently high," we first examine our benchmark calibration versus empirical estimates. In the empirical literature, the estimate most relevant is that of Harrison (2003, Table 6), who estimates production functions for gross output, and corrects for utilization. She estimates internal returns (standard error) to be 0.98 (0.01) and the externality to be 0.165 (0.035). Basu and Fernald (1997, Table 3) also estimate overall returns for gross output, but without adjusting for utilization. They get 1.08 (0.04).\(^7\) This brings us to our next result.

**Result 2:** In our benchmark calibration, with an elasticity of substitution of consumption of \(\sigma = 1\) and \(1/\varepsilon = 2.2\), indeterminacy arises at returns to scale of 1.915 and higher.

---

\(^6\)To make the model and implied calibrations comparable, we normalize the parameter \(\nu\) so that it approaches zero (in the benchmark case \(\nu = 0.000024\)). This is motivated by observing that for a positive \(\nu\), the fixed amount of land introduces *de facto* decreasing returns at the private (and social) level.

\(^7\)Burnside, Eichenbaum and Rebelo (1995) adjust for utilization, but do not include material inputs in their production function.
Clearly, increasing returns of 1.915 are not plausible. Lowering $\sigma$ and/or $\varepsilon$, however, does reduce this threshold, as illustrated in Figure 1. The highest threshold that we would accept as empirically plausible is 1.235, which assumes internal constant returns plus an externality two standard deviations above Harrison’s (2003) estimate ($0.165 + 2 \times 0.035 = 0.235$). With $\sigma = 1$ (0.5), this requires a labor supply elasticity well above 10 (of about 3).

Therefore, we seek to find more empirically plausible parameterizations. Hence, in the next subsection, we examine the requirements for indeterminacy when we move away from this basic model.

5 Adding material inputs

In this Section, we modify the production function to account for materials usage. The addition of materials adds another margin along which firms adjust their production, and increases the elasticity of output with respect to labor. This is analogous to Wen’s (1998) addition of varying capital utilization to the standard model. We denote by $m_t$ intermediate goods input. Therefore, technology is now described by

$$y_t = m_t a_t^{(1-\eta)\nu} k_t^{\alpha(1-\eta)} n_t^{(1-\alpha-\nu)(1-\eta)} L_t^{\frac{\nu}{1-\eta}}.$$ 

Furthermore, we assume that the endogenous borrowing constraint becomes

$$\psi q_t a_t \geq w_t n_t + m_t$$

which implies that to be able begin production, agents have to obtain working capital for both the wage bill and materials (see also Kobayashi, Nakajima, and Inaba, 2007). Essentially all first-order conditions carry through. In addition we have

$$\eta \frac{y_t}{m_t} - \frac{\lambda_t + \mu_t}{\lambda_t} = 0,$$

which determines the demand for the intermediate good. We adjust our benchmark calibration as follows: Jaimovich (2007) suggests a materials share of gross output of around 50 percent and a labor plus materials share of around 85 percent. In this vein, we fix the share of materials in the model to be 50 percent of gross output and target the wage share at 35 percent. We set $\mu/\lambda = 0.03$, to ensure that, given the calibration, the credit constraint binds. Then, $\eta = 0.515$, $\nu = 0.0023^8$, and $\psi = 0.2351^9$.

---

^8 Though Kocherlakota (2000) states that the sum of capital and land shares is approximately 40% in the US data, see also our footnote 6.

^9 In 2000, Total Loans and Leases at Commercial Banks were 3.488 billion of current US dollars (source: Federal Reserve’s FRED). The sum of aggregate market values of residential
Figure 2 plots the indeterminacy regions for the extended model. Result 4 summarizes the results for our benchmark parameterization, and for \( \sigma = 0.5 \).

**Result 4:** In our benchmark calibration of the credit model with material usage, with \( \sigma = 1 \) (\( \sigma = 0.5 \)), indeterminacy arises at returns to scale of 1.205 (1.155) and higher.

Both of these values are within two standard deviations of Harrison’s (2003) estimate. In particular, even with \( \sigma = 1 \), we obtain indeterminacy with plausible returns to scale. In addition, recall that we have \( \varepsilon = 1/2.2 \), while in all existing work that we know of, infinitely elastic labor supply is used to achieve indeterminacy with plausible returns to scale.

Infinitely elastic labor supplies are hard to defend from a microeconometric perspective (see for example Gourinchas and Parker, 2002). Moreover, Lubik (2007) estimates a sunspot model (a version of Wen, 1998) using US data. His estimates do suggest degrees of return to scale used in the theoretical literature; but these are too small for sunspot equilibria to arise, given his estimated labor supply elasticity: he finds that labor supply slopes up. That is, he rejects the indivisible labor argument of a completely elastic supply schedule. In the present model, indeterminacy can arise with downward sloping labor demand and realistic labor supply elasticities.

For more perspective on this, and for a better comparison with Wen (1998) and Benhabib and Farmer (1996), Figure 3 looks also at very high labor supply elasticities in the present model, from 10 to 1000. For \( \sigma = 1 \) (0.5), with \( \varepsilon = 1/100 \), minimum returns to scale are 1.072 (1.028), and when \( \varepsilon = 1/1000 \), they are 1.069 (1.025). These values are quite close to constant returns. Moreover, comparing our results with very high labor supply elasticity to previous work, our model obtains indeterminacy at lower returns to scale. Wen (1998) for example needs returns to scale of about 1.11 to obtain indeterminacy.

### 5.1 Intuition

Overall, our results demonstrate that indeterminacy appears at reasonable levels of both returns to scale and labor supply elasticities. But what drives our results? In particular, how can indeterminacy occur here when it is ruled out in Jaimovich? In the Benhabib-Farmer-Wen models, sunspots work as

---

land, values of homes and replacement cost of residential structures amounted to 27.500 billion (source Lincoln Institute of Land Policy). Hence, our value of \( \psi \) does not seem unreasonable. In addition, allowing it to increase leads to implausible values of other parameters in our model.
follows. Upon optimistic expectations about the future, agents act upon (subjectively) higher wealth. Hence, labor supply shifts inwards. With increasing returns high enough to make the reduced form labor demand upward sloping, employment rises and the expansionary sunspot cycle starts. In MYJ’s set-up, there is no income effect on leisure, and so such expectations cannot be self-fulfilling.

However, in our model, with a collateral constraint, income effects are not necessary for indeterminacy to arise. Indeterminacy arises as a consequence of a relaxed credit constraint induced by optimistic expectations that drive up land prices. In particular, if people become optimistic about the prospects of the economy – for whatever extrinsic reason – their demand for assets, in particular, land, rises. As it is in fixed supply, the price of land rises, relaxing the credit constraint. Firms are able to expand production as distortions shrink: even though technology is unaffected, the downward-sloping labor demand shifts out. Real economic activity increases, and the optimistic expectation is self-fulfilled.

The behavior of aggregate variables under indeterminacy can be seen in Figure 4, which plots the impulse responses of the benchmark model with material usage to a one time pessimistic shock to expectations. Increasing returns are set at 1.22, which is just above the minimum requirement for indeterminacy. GDP, hours, consumption and the price of land all fall on impact (i.e. are procyclical) and all variables are very persistent. The drop in consumption is smaller than output’s, reflecting a lower relative volatility of consumption, consistent with the data.\textsuperscript{10} Hours also fall, and their deviation from steady state appears to be more persistent than the other variables. The sunspot shock affects the financing possibilities of firms, and therefore their ability to hire new labor. Since the borrowing limit depends on asset values, the sharp fall in land prices reduces available credit.

It should be noted that inclusion of a income effect will actually counter this mechanism. That is, a leftward shift of labor supply would diminish the expansionary nature of the sunspot shock. It may, in fact, prevent the expectation from being realized. For example, consider a special case of King, Plosser and Rebelo (1988) preferences that is often used in previous work:

\[ u(c_t, h_t) = \ln c_t - \phi h_t. \]

This corresponds to Hansen’s (1985) real business cycle model. In this case, with infinitely elastic labor supply, the minimum increasing returns required for indeterminacy in the benchmark parametrization with materials would rise to 1.567. Here, the marginal rate of substitution between consumption

\textsuperscript{10}Investment is more volatile than output.
and leisure is not independent of consumption and thus income effects are present in the labor supply decision. Hence, an optimistic sunspot, which can be interpreted as a positive wealth shock, causes a rise in leisure and a decline in labor. Under constant or low increasing returns, this leads to a decrease in output, thus generating a recession. Put another way, the model requires much larger externalities to overturn the shift of labor supply.

6 Real sunspots

In this Section, we follow Harrison and Weder’s (2006) work on whether nonfundamental changes in expectations can explain the fluctuations that occurred during the Great Depression era. In the context of our model, among the infinite number of possible sequences of the expectational error, we seek the one that best describes the behavior of agents’ extrinsic uncertainty during this historic period and during the Great Recession. We then evaluate the validity of our model by comparing the resulting model-generated sequences of output and other variables to true data from this era.\textsuperscript{11}

6.1 Method and data

By definition, sunspot shocks must be purely nonfundamental, they must be serially uncorrelated and have mean zero. In order to find a measure of nonfundamental confidence, we follow methods similar to those used in Matsusaka and Sbordone (1995) and Chauvet and Guo (2003). In particular, we construct a VAR with a measure of confidence and several measures of fundamentals. The residual from the confidence equation will serve as our sunspot shock. Residuals from a regression are by definition mean zero; and testing for serial correlation will be straightforward.

The key variable of interest is a measure of confidence to use in the VAR system. While Matsusaka and Sbordone and Chauvet and Guo use the index published by the Survey Research Center at the University of Michigan, unfortunately such data is not available for the period of the Great Depression. Therefore, we must instead use another proxy. Harrison and Weder (2006) draw on Temin (1976) and Kindleberger (1989). Temin quantifies the pessimism felt during the beginning years of the Great Depression. In particular, he interprets extensive net downgradings as reflecting greater anticipated risk and therefore predicting bad economic times. Kindleberger

\textsuperscript{11}Note that we are therefore working under the assumption that fluctuations are driven only by sunspot shocks.
also suggests that interest rate spreads quantify confidence. We therefore argue that an interest rate spread is an ideal candidate to measure confidence: if people fear a recession, for example, the spread would widen, since the anticipated risk of default on average hits lower rated companies first and foremost. Therefore, a rise in the spread represents a fall in confidence. In order to determine which interest rate spread to use, we focus on finding the difference between the returns on a low and a higher risk asset. We use the difference between the returns on Baa and Aaa rated bonds to eliminate any potential noise that may result from using different types (e.g. government and private) of assets. Indeed, Bernanke (1990) and Friedman and Kuttner (1993) use the same quality spread as an instrument for perceived default risk. This spread increases by more than 400 basis points during the Great Depression. A similar (but somewhat smaller) increase arises from 2007 to 2008. These two spikes are outliers.

The idea of using the VAR is to filter out fundamental influences. We choose our measures of fundamentals with the following in mind: our variables must be good predictors of the spread and if the residuals from the spread equation are to be taken to measure sunspot shocks, they must, by definition, be orthogonal to past values of the chosen fundamentals. We must therefore choose variables that capture as thoroughly as possible the state of economic activity over the sample period, so that changes in expectations due to fundamentals are accounted for in the regressions. We use the following variables in the VAR (quarterly 1920:I to 2009:II): the growth rate in real gross national product \((y)\); the growth rate in real money supply, as measured by M2 \((m)\); the rate of change of the GNP deflator \((p)\); and the absolute change in the nominal return on prime commercial paper \((cp)\). We use the widely defined real money supply and interest rates to capture the effects of monetary policy and financial intermediaries. Data on \(y\) provide a measure of the overall performance of the real economy. Then our model is as follows:

\[
\begin{bmatrix}
m_t \\
ct_t \\
p_t \\
y_t \\
S_t
\end{bmatrix}
= \begin{bmatrix} P_1(L) \end{bmatrix} \begin{bmatrix}
m_t \\
ct_t \\
p_t \\
y_t \\
S_t
\end{bmatrix} + \begin{bmatrix} P_2 \end{bmatrix} \begin{bmatrix}
\epsilon^{m}_t \\
\epsilon^{c}_t \\
\epsilon^{p}_t \\
\epsilon^{y}_t \\
\epsilon^{s}_t
\end{bmatrix},
\]

where \(S\) is the interest rate spread. Dickey-Fuller-tests indicate that each of these variables exhibits stationarity over the considered sample. The matrix \(P_1(L)\) is of polynomials of length 4. We also include a constant in each regression. We consider the following specification of \(P_2\): the matrix is upper triangular (UT). In other words, the innovations to the spread are first in the causal chain so that other variables may respond contemporaneously to
them, but they are exogenous. Here the spread innovations are in a sense the primary cause of fluctuations. This is motivated by not wanting to rule out contemporaneous sunspot effects to fundamentals. Figure 5 displays the sequence of sunspots. Positive innovations indicate pessimistic shocks. 1930-33 and 2007-09 stand out with highest volatility and large pessimistic shocks.

6.2 The Great Depression in the model

Figure 6 displays output in the U.S. data with that implied by the artificial economy from 1921 to 1939. This essentially repeats the Harrison and Weder (2009) findings. The model economy predicts well the size and duration of the Depression. It predicts a tepid recovery. This stands in sharp contrast to the prediction of the real business cycle model, in which there is a much faster recovery, due to the presence of large positive technology shocks (Cole and Ohanian, 1999).

6.3 The Great Recession in the model

Figure 7 shows the level of confidence (generated by a first-difference filter of shocks) for the US economy over the last ten years. There has been a slow but steady increase from 2002 onwards. During 2007 attitudes changed and confidence dropped dramatically. Figure 8 plots artificial output generated from above series of sunspot shocks and compares it to actual US data. The model replicates the steady boom after 2002 and the large fall end of 2007.

7 Conclusion and future work

We have examined a general equilibrium model with collateral constraints and increasing returns to scale in production. With nonseparable utility, no income effect on the consumer’s choice of leisure, and materials in the production function, indeterminacy of equilibria arises for more realistic parametrization than in previous, similar models without these features.

In examining this model, we had in mind an explanation of the recent financial crisis, as well as other episodes in the economic history of the US, in which the role of self-fulfilling expectations is evident. We apply the model directly to the Great Depression and the current crisis. We find evidence that sustained pessimism turned what might have been a recession into the Great Depression. We also find evidence that the Great Recession was (in parts) driven by sunspots.
References


Figure 1: Indeterminacy zones
Figure 2: Indeterminacy zones
Figure 3: Indeterminacy zones
Figure 4: Impulse responses

Figure 5: Sunspots
Figure 6: Artificial and US output

Figure 7: Sunspot confidence
Figure 8: Artificial and US output