Inflation and Unemployment Gaps in U.S. Business Cycles

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Preliminary - Comments welcome

Abstract

This paper estimates a DSGE model with sticky prices and equilibrium unemployment on post-1984 US data by using a limited-information method that matches theoretical with empirical spectral densities. The model points toward neutral technology shocks, investment-specific technology shocks and monetary policy shocks as the main sources of business-cycle fluctuations. I use the estimates of the natural and potential rates of unemployment implied by the model to construct two theory-based measures of unemployment gaps and compare them to a conventional statistical measure of the unemployment gap. Finally, I examine the correlations between these alternative unemployment gaps and (1) actual price inflation, (2) actual nominal wage inflation and (2) actual real output growth at various frequencies. Strong correlations between the model’s natural unemployment gap and both the actual price- and wage- inflation rates emerge at medium and high frequencies.

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1 Introduction

Recently, a rapidly growing body of literature has emerged that combines monetary DSGE models of the business cycle with the modern theory of equilibrium unemployment, i.e. the Search and Matching model. The present paper uses this theoretical framework to undertake a structural analysis of some empirical regularities that involve the unemployment gap such as the Phillips curve and the Okun’s law.

Apart from the labor market block, the model is similar to the one estimated by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). Firms adjust labor exclusively through job creation and face convex hiring costs as in Yashiv (2006). This feature helps the model to capture the high persistence in vacancies and unemployment. Fluctuations are driven by seven disturbances: a neutral technology shock, an investment-specific technology shock, a risk-premium shock, a price-markup shock, a bargaining-power shock which is similar to a wage-markup shock, an exogenous spending shock and a monetary policy shock.

I use post-1984 quarterly US data on seven key aggregate variables to estimate the DSGE model. The estimation technique matches the theoretical spectral densities with their empirical counterparts and maximizes the fit at business-cycle frequencies. This approach is appealing to estimate misspecified business-cycle model. The estimated model fits several business-cycle stylized facts and points towards technology and monetary policy shocks as the main sources of fluctuations.

The DSGE model allows me to back out the natural and the potential rates of unemployment. Following Justiniano and Primiceri (2008), the natural rate is defined as the rate of unemployment in the absence of nominal rigidities, i.e. when nominal prices and wages are perfectly flexible. Instead, the potential rate corresponds to the unemployment rate in the absence of both nominal rigidities and inefficient price- and wage-markup shocks. I use the natural and the potential rates to construct the potential and the natural unemployment gaps. These two model-based concepts unemployment gaps are compared to a standard atheoretical measure obtained by applying the Hodrick-Prescott filter to actual unemployment. I then examine the correlations between each of the three unemployment gaps and (1) actual price inflation, (2) actual wage inflation and (3) actual output growth. A strong correlation between the model’s natural unemployment gap and the actual inflation rate emerges at medium frequencies. In addition, the natural unemployment gap exhibits the strongest correlation with nominal wage inflation at all frequencies.

My work is closely related to Sala, Söderström and Trigari (2008) and Justiniano and Primiceri (2008). Differently from Justiniano and Primiceri (2008), my model features equilibrium unemployment and I focus on the unemployment gap instead of the output gap. Sala et al. (2008) use a DSGE model with labor market frictions to estimate a theory-based unemployment gap. However, they do not examine the relationship between actual inflation and their inferred unemployment gap. Moreover, both Justiniano and Primiceri (2008) and Sala et al. (2008) estimate

\[ \text{See for example Trigari (2009), Walsh (2005), Krause and Lubik (2007), Sveen and Weinke (2008), Gertler, Sala and Trigari (2008).} \]
their DSGE models with Bayesian techniques while I opt for a limited-information strategy.

2 Model

The economy consists of a representative family, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by \( i \in [0, 1] \), a central bank and a government that sets monetary and fiscal policy respectively. I now describe the behavior of these agents.

2.1 The representative household

There is a continuum of identical households of mass one. Each household is a large family, made of a continuum of individuals of measure one. Family members are either working or searching for a job.\(^2\) Following Merz (1995), I assume that family members pool their income before the head of the family chooses optimally per capita consumption.

The representative family enters each period \( t = 0, 1, 2, \ldots \), with \( B_{t-1} \) bonds and \( K_{t-1} \) units of physical capital. At the beginning of each period, bonds mature, providing \( B_{t-1} \) units of money. The representative family uses some of this money to purchase \( B_t \) new bonds at nominal cost \( B_t/r_tB \), where \( r_tB \) denotes the gross nominal interest rate between period \( t \) and \( t+1 \).

The representative household owns capital and chooses the capital utilization rate, \( u_t \), which transforms physical capital into effective capital according to

\[
K_t = u_tK_{t-1}. \tag{1}
\]

The household rents \( K_t(i) \) units of effective capital to intermediate-goods-producing firm \( i \in [0, 1] \) at the nominal rate \( r^K_t \). The household’s choice of \( K_t(i) \) must satisfy

\[
K_t = \int_0^1 K_t(i) \, di. \tag{2}
\]

The cost of capital utilization is \( a(u_t) \) per unit of physical capital. I assume the following functional form for the function \( a \),

\[
a(u_t) = \phi_u (u_t - 1) + \frac{\phi_u^2}{2} (u_t - 1)^2, \tag{3}
\]

and that \( u_t = 1 \) in steady state.

Each period, \( N_t(i) \) family members are employed at intermediate goods-producing firm \( i \in [0, 1] \). Each worker employed at firm \( i \) works a fixed amount of hours and earns the nominal wage \( W_t(i) \). \( N_t \) denotes aggregate employment in period \( t \) and is given by

\[
N_t = \int_0^1 N_t(i) \, di. \tag{4}
\]

\(^2\)The model abstracts from the labor force participation decision.
The remaining \((1 - N_t)\) family members are unemployed and and each receives nominal unemployment benefits \(b_t\), financed through lump-sum taxes.

During period \(t\), the representative household receives total nominal factor payments \(r_t^K K_t + W_t N_t + (1 - N_t) b_t\). In addition, the household also receives nominal profits \(D_t (i)\) from each firm \(i \in [0, 1]\), for a total of

\[
D_t = \int_0^1 D_t (i) \, di. \tag{5}
\]

In each period \(t = 0, 1, 2, \ldots\), the family uses these resources to purchase finished goods, for both consumption and investment purposes, from the representative finished goods-producing firm at the nominal price \(P_t\). The law of motion of physical capital is

\[
\bar{K}_t \leq (1 - \delta) \bar{K}_{t-1} + \mu_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t, \tag{6}
\]

where \(\delta\) denotes the depreciation rate. The function \(S\) captures the presence of adjustment costs in investment, as in Christiano et al. (2005). I assume the following functional form for the function \(S\),

\[
S \left( \frac{I_t}{I_{t-1}} \right) = \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - g_I \right)^2, \tag{7}
\]

where \(g_I\) is the steady-state growth rate of investment. Hence, along the balanced growth path, \(S (g_I) = S' (g_I) = 0\) and \(S'' (g_I) = \phi_I > 0\). \(\mu_t\) is an investment-specific technology shock affecting the efficiency with which consumption goods are transformed into capital. The investment-specific shock follows the exogenous stationary autoregressive process

\[
\ln (\mu_t) = \rho_\mu \ln (\mu_{t-1}) + \epsilon_{\mu_t}, \tag{8}
\]

where \(\epsilon_{\mu_t}\) is \(i.i.d. N \left( 0, \sigma^2_\mu \right)\).

The family’s budget constraint is given by

\[
P_tC_t + P_t I_t + \frac{B_t}{\epsilon_{bt} r_t^B} \leq B_{t-1} + W_t N_t + (1 - N_t) b_t + r_t^K u_t \bar{K}_{t-1} - P_t a (u_t) \bar{K}_{t-1} - T_t + D_t \tag{9}
\]

for all \(t = 0, 1, 2, \ldots\). As in Smets and Wouters (2007), the shock \(\epsilon_{bt}\) drives a wedge between the central bank’s instrument rate \(r_t^B\) and the return on assets held by the representative family. As noted by De Graeve, Emiris and Wouters (2009), this disturbance works as an aggregate demand shock and generates a positive comovement between consumption and investment. The risk-premium shock \(\epsilon_{bt}\) follows the autoregressive process

\[
\ln \epsilon_{bt} = \rho_b \ln \epsilon_{bt-1} + \epsilon_{bt}, \tag{10}
\]

where \(0 < \rho_b < 1\), and \(\epsilon_{bt}\) is \(i.i.d. N \left( 0, \sigma^2_b \right)\).
The family’s lifetime utility is described by

\[
E_t \sum_{s=0}^{\infty} \beta^s \ln (C_{t+s} - hC_{t+s-1})
\]  

(11)

where \(0 < \beta < 1\). When \(h > 0\), the model allows for habit formation in consumption and consumption responds gradually to shocks.

The head of the family chooses \(C_t, B_t, u_t, I_t,\) and \(K_t\) for each \(t = 0, 1, 2, \ldots\) to maximize the expected lifetime utility (11) subject to the constraints (6) and (9).

2.2 The representative finished goods-producing firm

During each period \(t = 0, 1, 2, \ldots\), the representative finished goods-producing firm uses \(Y_t(i)\) units of each intermediate good \(i \in [0, 1]\), purchased at the nominal price \(P_t(i)\), to manufacture \(Y_t\) units of the finished good according to the constant-returns-to-scale technology described by

\[
\left[ \int_0^1 Y_t(i)^{\theta_t-1} d\bar{i} \right]^{\theta_t/(\theta_t-1)} \geq Y_t,
\]  

(12)

where \(\theta_t\) translates into a random shock to the markup of price over marginal cost. This markup shock follows the autoregressive process

\[
\ln (\theta_t) = (1 - \rho_\theta) \ln (\theta) + \rho_\theta \ln (\theta_{t-1}) + \varepsilon_{\theta t},
\]  

(13)

where \(0 < \rho_\theta < 1, \theta > 1\), and \(\varepsilon_{\theta t}\) is \(i.i.d.\ N(0, \sigma_\theta^2)\).

Intermediate good \(i\) sells at the nominal price \(P_t(i)\), while the finished good sells at the nominal price \(P_t\). Given these prices, the finished goods-producing firm chooses \(Y_t\) and \(Y_t(i)\) for all \(i \in [0, 1]\) to maximize its profits

\[
P_t Y_t - \int_0^1 P_t(i) Y_t(i) d\bar{i},
\]  

(14)

subject to the constraint (12) for each \(t = 0, 1, 2, \ldots\). The first-order conditions for this problem are (12) with equality and

\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t
\]  

(15)

for all \(i \in [0, 1]\) and \(t = 0, 1, 2, \ldots\).

Competition in the market for the finished good drives the finished goods-producing firm’s profits to zero in equilibrium. This zero-profit condition determines \(P_t\) as

\[
P_t = \left[ \int_0^1 P_t(i)^{1-\theta_t} d\bar{i} \right]^{1/(1-\theta_t)}
\]  

(16)

for all \(t = 0, 1, 2, \ldots\).
2.3 The representative intermediate goods-producing firm

Each intermediate goods-producing firm \( i \in [0, 1] \) enters in period \( t \) with a stock of \( N_{t-1}(i) \) employees carried from the previous period. At the beginning of period \( t \), before production starts, \( \rho N_{t-1}(i) \) old jobs are destroyed, where \( \rho \) is the job destruction rate.\(^3\) The pool of workers \( \rho N_{t-1} \) who have lost their job at the beginning of period \( t \) start searching immediately and can possibly be hired in period \( t \). \( N_t(i) \) denotes the pool of employees taking part to production at firm \( i \) in period \( t \). The law of motion of the stock of productive workers at firm \( i \) is

\[
N_t(i) = (1 - \rho) N_{t-1}(i) + m_t(i).
\]

(17)

\( m_t(i) \) denotes the flow of new employees hired by firm \( i \) in period \( t \), and is given by

\[
m_t(i) = q_t V_t(i),
\]

(18)

where \( V_t(i) \) denotes vacancies posted by firm \( i \) in period \( t \) and \( q_t \) is the aggregate probability of filling a vacancy in period \( t \). Workers hired in period \( t \) take part to period \( t \) production. Employment is therefore an instantaneous margin. However, each period some vacancies and job seekers remain unmatched. As a consequence, a firm-worker pair enjoys a joint surplus that motivates the existence of a long-run relationship between the two parties.

Aggregate employment \( N_t = \int_0^1 N_t(i) di \) evolves over time according to

\[
N_t = (1 - \rho) N_{t-1} + m_t,
\]

(19)

where \( m_t = \int_0^1 m_t(i) di \) denotes aggregate matches in period \( t \). Similarly, the aggregate vacancies is equal to \( V_t = \int_0^1 V_t(i) di \). The pool of job seekers in period \( t \), denoted by \( S_t \), is given by

\[
S_t = 1 - (1 - \rho) N_{t-1}.
\]

(20)

The matching process is described by the following aggregate CRS function

\[
m_t = \zeta S_t^\sigma V_t^{1-\sigma},
\]

(21)

where \( \zeta \) is a scale parameter that captures the efficiency of the matching technology. The probability \( q_t \) to fill a vacancy in period \( t \) is given by

\[
q_t = \frac{m_t}{V_t}.
\]

(22)

The probability, \( s_t \), for a job seeker to find a job is

\[
s_t = \frac{m_t}{S_t}.
\]

(23)

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\(^3\) The rate of match dissolution is exogenous. This is consistent with Hall (2005) and Shimer (2005)’s finding that recent business cycle fluctuations in the U.S. labor market mostly come from the job creation margin.
Finally aggregate unemployment is defined by

\[ U_t = 1 - N_t. \tag{24} \]

During each period \( t = 0, 1, 2, \ldots \), the representative intermediate goods-producing firm combines \( N_t(i) \) homogeneous employees with \( K_t(i) \) units of efficient capital to produce \( Y_t(i) \) units of intermediate good \( i \) according to the constant-returns-to-scale technology described by

\[ Y_t(i) = A_t^{1-\alpha} K_t(i)^\alpha N_t(i)^{1-\alpha}. \tag{25} \]

\( A_t \) is an aggregate labor-augmenting technology shock whose growth rate, \( z_t = A_t / A_{t-1} \), follows the exogenous stationary stochastic process

\[ \ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}, \tag{26} \]

where \( z > 1 \) denotes the steady-state growth rate of the economy and \( \varepsilon_{zt} \) is \( i.i.d. N(0, \sigma_z^2) \).

Following Yashiv (2006), intermediate goods-producing firms face convex hiring costs, measured in terms of the finished good and given by

\[ \frac{\phi_N}{2} \left[ \frac{q_t V_t(i)}{N_t(i)} \right]^2 Y_t, \tag{27} \]

where \( \phi_N \) governs the magnitude of these costs.

Intermediate goods substitute imperfectly for one another in the production function of the representative finished goods-producing firm. Hence, each intermediate goods-producing firm \( i \in [0, 1] \) sells its output \( Y_t(i) \) in a monopolistically competitive market, setting \( P_t(i) \), the price of its own product, with the commitment of satisfying the demand for good \( i \) at that price. Firms take the nominal wage as given when maximizing the discounted value of expected future profits.

Each intermediate goods-producing firm faces costs of adjusting its nominal price between periods, measured in terms of the finished good and given by

\[ \frac{\phi_P}{2} \left[ \frac{P_t(i)}{\pi_{t-1}^\zeta \pi^{1-\zeta} P_{t-1}(i)} - 1 \right]^2 Y_t. \tag{28} \]

\( \phi_P \) governs the magnitude of the price adjustment cost. \( \pi_t = \frac{P_t}{P_{t-1}} \) denotes the gross rate of inflation in period \( t \). \( \pi > 1 \) denotes the steady-state gross rate of inflation and coincides with the central bank’s target. The parameter \( 0 \leq \zeta \leq 1 \) governs the importance of backward-looking behavior in price setting.\(^4\)

Each intermediate goods-producing firm faces quadratic wage-adjustment costs which are

\[^4\text{See Ireland (2007).}\]
proportional to the size of its workforce and measured in terms of the finished good

\[
\frac{\phi_W}{2} \left[ \frac{W_t(i)}{\left( \pi_{t-1}^w \right)^{\varphi} (\pi^w)^{1-\varphi} W_{t-1}(i) } - 1 \right]^2 N_t(i) Y_t,
\]

(29)

where \( \phi_W \) governs the magnitude of the wage adjustment cost and where \( \pi_t^w = \frac{W_t}{W_{t-1}} \) denotes the gross wage inflation rate in period \( t \). \( \pi^w > 1 \) denotes the steady-state gross rate of nominal wage inflation \( (\pi^w = z \pi) \). The parameter \( 0 \leq \varphi \leq 1 \) governs the importance of employers’ backward-looking behavior in wage setting.

Adjustment costs on the hiring rate, price and wage changes make the intermediate goods-producing firm’s problem dynamic. It chooses \( K_t(i), N_t(i), V_t(i) \) and \( Y_t(i) \) for all \( t = 0, 1, 2, ... \) to maximize its total market value, given by

\[
E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[ \frac{D_{t+s}(i)}{P_{t+s}} \right]
\]

(30)

where \( \beta \Lambda_t/P_t \) measures the marginal utility to the representative household of an additional dollar of profits during period \( t \) and where

\[
D_t(i) = P_t(i) Y_t(i) - W_t(i) N_t(i) - r_t^K K_t(i) - \frac{\phi_N}{2} \left( \frac{q_t V_t(i)}{N_t(i)} \right)^2 P_t Y_t
\]

\[
- \frac{\phi_P}{2} \left( \frac{P_t(i)}{\pi_{t-1}^w \pi^{1-\varphi} P_{t-1}(i) } - 1 \right)^2 P_t Y_t - \frac{\phi_W}{2} \left( \frac{W_t(i)}{(\pi_{t-1}^w)^{\varphi} (\pi^w)^{1-\varphi} W_{t-1}(i) } - 1 \right)^2 N_t(i) P_t Y_t,
\]

subject to the constraints

\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t,
\]

(32)

\[
Y_t(i) \leq K_t(i)\alpha [A_t N_t(i)]^{1-\alpha},
\]

(33)

\[
N_t(i) = \chi N_{t-1}(i) + q_t V_t(i),
\]

(34)

where \( \chi \equiv 1 - \rho \) is the job survival rate.

### 2.4 Wage setting

Unemployment benefits \( b_t \) are proportional to the value of the nominal wage along the balanced growth path, \( W_{ss,t} \),

\[
b_t = \tau W_{ss,t},
\]

(35)

where \( \tau \) is the replacement ratio. The fact that hiring costs and unemployment benefits both share the common stochastic trend ensures that the unemployment rate is stationary.

Jobs and workers at a given intermediate goods-producing firm are homogeneous. \( W_t(i) \) denotes the nominal wage paid for any job at firm \( i \) in period \( t \). Each period \( t \), the representative
intermediate goods-producing firm bargains with each of its employees separately over $W_t(i)$. The nominal wage is determined through bilateral Nash bargaining,

$$W_t(i) = \arg \max \left[ S_t(i)^{\eta_t} J_t(i)^{1-\eta_t} \right].$$

(36)

$S_t(i)$ denotes the surplus of the representative worker at firm $i$ while $J_t(i)$ is the surplus of firm $i$. Both $S_t(i)$ and $J_t(i)$ are expressed in real terms. $\eta_t$ denotes the worker’s bargaining power which evolves exogenously according to

$$\ln \eta_t = (1 - \rho) \ln \eta + \rho \ln \eta_{t-1} + \varepsilon_{\eta t},$$

(37)

where $0 < \eta < 1$ and $\varepsilon_{\eta t}$ is i.i.d. $N(0, \sigma^2_{\eta t})$.

The worker’s surplus in terms of final consumption goods is given by

$$S_t(i) = \frac{W_t(i)}{P_t} - \frac{b_t}{P_t} + \beta E_t \left[ \chi (1 - s_{t+1}) \right] \left( \frac{\Lambda_{t+1}}{\Lambda_t} \right) S_{t+1}(i).$$

(38)

The surplus of firm $i$ expressed in real terms is given by

$$J_t(i) = \xi_t(i) (1 - \alpha) \frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} + \frac{\phi_x Y_t x_t(i)^2}{N_t(i)}
- \frac{\phi_W}{2} \left( \frac{W_t(i)}{\pi_t^{w(i)}} \right)^{\theta_t} \left( \frac{W_t(i)}{\pi_t^{w(i)}} \right)^{1-\theta_t} \left( \frac{W_t(i)}{\pi_t^{w(i)}} - 1 \right) Y_t + \beta \chi E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1}(i) \right).$$

(39)

Nash bargaining requires that the equilibrium nominal wage $W_t(i)$ satisfies the following first-order condition

$$\eta_t J_t(i) \frac{\partial S_t(i)}{\partial W_t(i)} = -(1 - \eta_t) S_t(i) \frac{\partial J_t(i)}{\partial W_t(i)},$$

(40)

where

$$\frac{\partial S_t(i)}{\partial W_t(i)} = \frac{1}{P_t},$$

(41)

$$\frac{\partial J_t(i)}{\partial W_t(i)} = \left\{ \frac{1}{P_t} + \phi_W Y_t \left( \left( \frac{W_t(i)}{\pi_t^{w(i)}} \right)^{1-\theta_t} W_t(i) - 1 \right) \left( \frac{W_t(i)}{\pi_t^{w(i)}} \right)^{\theta_t} \left( \frac{W_t(i)}{\pi_t^{w(i)}} \right)^{1-\theta_t} W_t(i) \right\}. \right\}$$

(42)

When $\phi_W = 0$, adjusting nominal wages is costless for the firm. In that case, the effects of a marginal increase in the nominal wage on the worker’s surplus and on the firm’s surplus have the same magnitude (with opposite signs):

$$\text{if } \phi_W = 0, \text{ then } \frac{\partial S_t(i)}{\partial W_t(i)} = - \frac{\partial J_t(i)}{\partial W_t(i)} = \frac{1}{P_t}.$$ 

(43)

In the absence of nominal wage-adjustment costs, Nash bargaining over the nominal wage implies
the usual first-order condition

$$S_t(i) = \left(\frac{\eta_t}{1 - \eta_t}\right) J_t(i).$$

Thus, as pointed out by Arsenau and Chugh (2008), Nash bargaining over the nominal wage
when there are no nominal wage adjustment costs is equivalent to Nash bargaining over the real
wage. The presence of nominal wage-adjustment costs affects the effective bargaining powers
of the firm and the worker respectively. In the presence of nominal wage adjustment costs, the
first-order condition from Nash bargaining is given by

$$S_t(i) = \frac{\eta_t}{(1 - \eta_t)} \left[ \frac{\partial S_t(i)}{\partial W_t(i)} \right] J_t(i),$$

$$S_t(i) = \Omega_{it} J_t(i),$$

where we have introduced the notation

$$\Omega_{it} \equiv \left(\frac{\eta_t}{1 - \eta_t}\right) \left(\frac{\partial S_t(i)}{\partial W_t(i)}\right) \left(-\frac{\partial J_t(i)}{\partial W_t(i)}\right).$$

Finally, the equation governing the dynamics of the real wage at firm $i$ is given by

$$\frac{W_t(i)}{P_t} = \left(\frac{\Omega_{it}}{1 + \Omega_{it}}\right) \left[ \frac{\xi_t(i)}{1 - \alpha} Y_t(i) + \frac{\phi_N Y_t x_t(i)^2}{N_t(i)} \right]$$

$$+ \frac{\phi_{\text{mp}}}{2} \left(\frac{P_{t-1}}{P_t}\right)^{\gamma} W_t(i) - \beta \chi E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \left(\frac{\phi_N Y_{t+1} x_{t+1}(i)^2}{N_t+1(i)}\right)$$

$$+ \frac{b_t}{P_t} - \beta \chi E_t \Omega_{it+1} (1 - s_{t+1}) \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \left(\frac{\phi_N Y_{t+1} x_{t+1}(i)^2}{N_t+1(i)}\right).$$

### 2.5 Government

The central bank adjusts the short-term nominal gross interest rate $r_t^B$ by following a generalized
Taylor type rule (Smets and Wouters (2007))

$$\ln \left(\frac{r_t^B}{r_{t-1}^B}\right) = \rho_r \ln \left(\frac{r_{t-1}^B}{r_{t-1}^B}\right) + (1 - \rho_r) \left[ \rho_x \ln \left(\frac{\pi_t}{\pi_t}\right) + \rho_y \ln \left(\frac{Y_t}{Y_{t-1}^p}\right) \right]$$

$$+ \rho_{gy} \ln \left(\frac{Y_t}{Y_{t-1}^p}\right) + \ln \epsilon_{\text{mp}}.$$

$$\ln \epsilon_{\text{mp}} = \rho_{\text{mp}} \ln \epsilon_{\text{mp}-1} + \epsilon_{\text{mp}}.$$

where $\pi_t = P_t/P_{t-1}$. $Y_{t-1}^p$ is the potential level of output, i.e. the level of output in the flexible
price - flexible wage economy ($\phi_P = 0$ and $\phi_W = 0$) in the absence of markup and bargaining-
power shocks $\left(\tilde{\theta}_t = 0 \text{ and } \tilde{\eta}_t = 0\right)$. The monetary policy shock $\epsilon_{\text{mp}}$ follows an AR(1) process
with $0 \leq \rho_{\text{mp}} < 1$ and $\epsilon_{\text{mp}} \sim i.i.d. N \left(0, \sigma_{\text{mp}}^2\right)$. The degree of interest-rate smoothing $\rho_r$ and the
reaction coefficients $\rho_x, \rho_y$, and $\rho_{gy}$ are all positive.
The government budget constraint is of the form

\[ P_t G_t + (1 - N_t) b_t = \left( \frac{B_t}{B_{t-1}} - B_{t-1} \right) + T_t, \quad (48) \]

where \( T_t \) denotes total nominal lump-sum transfers. Public spending is an exogenous time-varying fraction of GDP

\[ G_t = \left( 1 - \frac{1}{\varepsilon_{gt}} \right) Y_t, \quad (49) \]

where \( \varepsilon_{gt} \) evolves according to

\[ \ln \varepsilon_{gt} = (1 - \rho_g) \ln \varepsilon_g + \rho_g \ln \varepsilon_{gt-1} + \varepsilon_g, \quad (50) \]

with \( \varepsilon_{gt} \sim i.i.d. N (0, \sigma^2_g) \).

### 2.6 Symmetric equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that \( Y_t(i) = Y_t, P_t(i) = P_t, N_t(i) = N_t, V_t(i) = V_t, K_t(i) = K_t \) for all \( i \in [0, 1] \) and \( t = 0, 1, 2, \ldots \). Moreover, workers are homogeneous and all workers at a given firm \( i \) receive the same nominal wage \( W_t(i) \), so that \( W_t(i) = W_t \) for all \( i \in [0, 1] \) and \( t = 0, 1, 2, \ldots \). The aggregate resource constraint is obtained by aggregating the household budget constraint over all intermediate sectors \( i \in [0, 1] \),

\[ \left[ \frac{1}{\varepsilon_{gt}} - \phi_N \frac{\sigma^2_t}{2} - \phi_P \left( \frac{\pi_t}{\pi_{t-1}^{\lambda-\kappa}} - 1 \right)^2 - \phi_W \left( \frac{\pi_t^{\lambda}}{(\pi_{t-1}^{\lambda})^{\eta} (\pi_{t-1}^{\lambda})^{1-\eta} - 1} \right)^2 \right] N_t \]

\[ Y_t = C_t + I_t + a(u_t) K_{t-1}. \quad (51) \]

### 2.7 Model solution

Real output, consumption, investment, capital and wages share the stochastic trend induced by the unit root process for neutral technological progress. In the absence of shocks, the economy converges to a steady-state growth path in which all stationary variables are constant. I first rewrite the model in terms of stationary variables, and then loglinearize the transformed economy around its deterministic steady state. The approximate model can then be solved using standard methods.
3 Econometric strategy

3.1 Data

The estimation is based on quarterly U.S. data on seven key aggregate variables: the yearly growth rate of real output, the yearly growth rate of real consumption, the yearly growth rate of real investment, the yearly growth rate of real wages, the yearly inflation rate, the short-term nominal interest rate and the vacancy/unemployment ratio which summarizes the tightness of the labor market and plays an important role in the Mortensen-Pissarides model. The sample period runs from 1984:Q1 to 2006:Q1. The appendix describes the dataset in details.

Two reasons motivate my choice of using the vacancy/unemployment ratio as an observable variable. First, in the model, labor adjusts exclusively along the extensive margin. Data on employment or unemployment seem therefore better suited than data on total hours. Second, unemployment and vacancies are very persistent and strongly negatively correlated (the Beveridge curve). By considering the vacancy/unemployment ratio, I exploit the Beveridge curve to remove the trend shared by unemployment and vacancies.

3.2 Calibrated parameters

Because of weak identification problems, I calibrate seven parameters prior to estimation. Table 1 summarizes the calibration. The quarterly depreciation rate $\delta$ is set equal to 0.025, a value commonly used in the literature. The calibration of the vacancy filling rate $q$ is just a normalization as $q$ is not identified. I set the government spending/output ratio $G/Y = 0.20$. Finally, the steady-state values of the unemployment rate $U$, the rate of inflation $\pi$, the nominal interest rate $r^B$, and the growth rate of output $z$, are set equal to their respective sample averages.

3.3 Estimation technique

The spectrum and the covariance generating function are two alternative ways to summarize the complete set of second moments. The spectrum is convenient to analyze cyclical fluctuations. This paper applies an estimation technique which minimizes a distance between the spectrum of the model and the spectrum of the data. This spectra-matching approach makes it straightforward to weight frequencies differently in estimation. As stressed by Diebold, Ohanian and Berkowitz (1999), this feature is appealing to estimate a stylized model which is designed to explain primarily some frequencies of interest. The implementation of spectra matching in this paper follows Wen (1998).

Let $\theta$ denotes a column vector stacking the DSGE model’s parameters to be estimated. The spectra-matching estimator is defined by

$$\hat{\theta}_W = \arg \min_{\theta} [G_W (\theta)]$$

(52)
where the distance $G_W(\theta)$ is given by,

$$G_W(\theta) = \text{tr} \left[ D(\theta) \right],$$  \hspace{1cm} (53)

with

$$D(\theta) = \sum_{j=1}^{T} W(\omega_j) \odot \left| F_m(\omega_j; \theta) - \hat{F}_d(\omega_j) \right|, \quad \omega_j \in (0.01, \pi).$$  \hspace{1cm} (54)

Here $\odot$ denotes the element-by-element multiplication, $T$ denotes the sample size and $\{ \omega_1, \ldots, \omega_T \}$ is a grid of equidistant points over $(0.01, \pi)$. The function $W(\omega_j)$ weights the absolute value of the difference between the model spectrum, $F_m(\omega_j; \theta)$, and a consistent estimate of the data spectrum, $\hat{F}_d(\omega_j)$, across frequencies $\omega_j \in (0.01, \pi)$. I compute the theoretical spectrum $F_m(\omega; \theta)$ directly from the state-space representation of the solution to the log-linearized DSGE model. The spectrum of the data is estimated using a VAR with four lags. Importantly, by taking the trace of matrix $D(\theta)$, I disregard the off-diagonal elements which contain the information about the cross-covariances.

The weighting function $W(\omega_j)$ takes the following expression

$$W(\omega_j) = \hat{F}_d(\omega_j) \odot \left[ \sum_{j=1}^{T} \hat{F}_d(\omega_j) \right],$$  \hspace{1cm} (55)

where $\odot$ denotes the element-by-element division. The first term $\hat{F}_d(\omega_j)$ weights frequency $\omega_j$ according to its contribution to the total variance of the data. The term in squared brackets approximates the covariance matrix of the data. It acts as a scaling factor that prevents the distance function to be mainly influenced by the variables with the largest variances.

4 Results

4.1 Parameter estimates

Table 2 reports the parameter estimates. The estimate of the elasticity of output to capital ($\alpha = 0.15$) is very close to the one obtained by Smets and Wouters (2007). The estimate of the elasticity of substitution between intermediate goods ($\phi = 5$) implies a steady-state markup of 25 percents. The estimate of the habit persistence parameter ($h = 0.84$) of the investment adjustment cost parameter ($\phi_I = 4.3$) and of the elasticity of the rental rate ($\phi_{\pi R} = 67.2\%$) are in line with the ones obtained by Smets and Wouters (2007). The price adjustment cost parameter ($\phi_p = 149$) is relatively high and implies roughly that prices are reoptimized once every 6 quarters. Turning to the labor market, the elasticity of the matching function to unemployment is $\sigma = 0.79$. This estimate slightly exceeds the range of plausible values reported by Petrongolo and Pissarides (2001). The job destruction rate ($\rho = 0.08$) is equal to the value used by Yashiv (2006). The estimated replacement rate ($\tau = 0.67$) is relatively high. The output
share of hiring costs is $(\phi_N x^2/2) = 1\%$.

### 4.2 Model fit

Figure 1 plots the theoretical spectral densities of the seven observable variables against the bootstrapped empirical confidence bands. The model captures well the shape and magnitude of the seven empirical spectral densities. The model however significantly overestimates the variance of consumption growth at low frequencies as well as the variance of the interest rate at business-cycle and high frequencies.

Figure 2 compares some theoretical coherences with the bootstrapped empirical confidence bands. Since the estimation technique disregards cross-spectra, looking at coherences provides a way to evaluate the model. The model matches well the coherences between output growth and inflation, inflation and the interest rate, inflation and tightness and wage growth and tightness (although the model slightly overpredicts the last two coherences at high frequencies). The model overpredicts the coherence between output growth and tightness at high frequencies. This finding suggests that allowing for variations in hours worked per worker may improve the specification of the model. The model fails to replicate the shape of the coherence between output growth and the interest rate and of the coherence between tightness and the interest rate. The model also fails to match the joint behavior of output growth, consumption growth and investment growth. At this stage, it is not clear to which extent these failures can be imputed to the limited-information estimation technique. In future research, it would be interesting to estimate the model using a likelihood-based approach and to compare the fit obtained with the two estimation strategies.

### 4.3 Sources of business cycles

Figure 3 shows the contribution of each shock to the theoretical spectral densities. The neutral technology shock, followed by the investment-specific-technology shock and the monetary policy shock emerge as the main sources of business cycles. Government spending shocks play a non-negligible role for variations in inflation and the interest rate.

Figures 4 to 10 plot the impulse responses to each of the seven disturbances.

### 4.4 The relationship between inflation and the natural rate of unemployment

Following Justiniano and Primiceri (2008), I compute the potential and the natural rates of unemployment. The natural rate corresponds to the unemployment rate under flexible prices and wages while the potential rate is defined as the unemployment rate under flexible prices and wages and no price-markup and wage-markup disturbances. Figure 11 compares the actual unemployment rate with the smoothed estimates of the potential rate and the natural rate of unemployment. Figure 11 also shows the unemployment trend extracted with the HP filter.

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5The coherence decomposes the squared correlation between two variables $X$ and $Y$, frequency by frequency.
In line with Justiniano and Primiceri (2008), figure 11 shows that the natural rate has been extremely volatile, especially during the first half of the nineties as the economy was buffeted by price-markup shocks and wage-markup shocks (see Figure 13). The variability of the potential rate is about the same as the variability of the HP trend. However, the potential rate spiky whereas the HP trend is smooth. Figure 12 suggests that mainly risk-premium shocks have been responsible for the bulk of fluctuations in the potential rate of unemployment.

The potential unemployment gap is defined as the log-deviation of the actual unemployment rate from the potential rate. Similarly, the natural gap and the HP gap are the log-deviation of the actual rate from the natural rate and the HP trend respectively. Figure 14 shows the coherences between each of the three unemployment gap measures and (1) price inflation, (2) wage inflation and (3) output growth. At business-cycle frequencies, the HP gap exhibits the strongest correlation with output growth. Instead, the HP gap is the most weakly correlated with wage inflation at all frequencies. The natural unemployment gap exhibits the strongest total correlation with price- and wage- inflation. In particular, the natural gap appears to be especially useful to understand nominal wage inflation dynamics at medium and high frequencies.

5 Concluding remarks

This paper estimates by spectra matching a medium-scale DSGE model with sticky prices and equilibrium unemployment on post-1984 quarterly US data. The estimated model fits well the spectral densities of the seven key macroeconomic variables used in estimation and points toward technology and monetary policy shocks as the main source of business cycles. The model however fails to capture the pattern of comovements between output growth, the interest rate, consumption growth and investment growth. At this stage, it is not clear whether these failures are due to some specification errors or are a consequence of the limited-information estimation technique.

The model allows us to estimate the path of the potential and natural rates of unemployment and to construct time series for a economically meaningful measures of the unemployment gap. The natural rate of unemployment seems particularly useful to understand price- and wage-inflation dynamics at medium and high frequencies.

6 Appendix: Description of the database

The estimation is based on seven variables: per capita output, consumption and investment in real terms, real wages, labor market tightness (i.e. the ratio of vacancy over unemployment), inflation and the nominal short-term interest rate. I use quarterly U.S. data. The dataset spans a sample from 1984:Q1 to 2006:Q1. All series are downloaded from the FRED database. Following Justiniano, Primiceri and Tambalotti (2009), I measure nominal consumption using data on nominal personal consumption expenditures of nondurables and services. Nominal investment corresponds to the sum of personal consumption expenditures of durables and gross private domestic investment. Nominal output is measured by nominal GDP. Per capita real
GDP, consumption and investment are obtained by dividing the nominal series by the GDP deflator and population. Real wages corresponds to nominal compensation per hour in the non-farm business sector, divided by the GDP deflator. Consistently with the model, I measure population by the labor force which is itself defined as the sum of official unemployment and official employment. The vacancy rate is measured by the index of Help wanted advertising in newspapers divided by labor force. The unemployment rate is the ratio of official unemployment over labor force. Labor market tightness is the ratio of the vacancy rate over the unemployment rate. Year-on-year inflation corresponds to the year-on-year difference of the log of the GDP deflator. The nominal interest rate is measured by the effective Federal Funds rate.

References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>0.0250</td>
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<tr>
<td>Probability to fill a vacancy within a quarter</td>
<td>$q$</td>
<td>0.7000</td>
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<tr>
<td>Exogenous spending/output ratio</td>
<td>$g/y$</td>
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<tr>
<td>Unemployment rate</td>
<td>$U$</td>
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<tr>
<td>Quarterly growth rate</td>
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<tr>
<td>Quarterly inflation rate</td>
<td>$\pi$</td>
<td>1.0062</td>
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<tr>
<td>Quarterly nominal interest rate</td>
<td>$r^B$</td>
<td>1.0129</td>
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Table 2: Estimated parameters

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Capital share</td>
<td>$\alpha$</td>
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<tr>
<td>Elasticity of substitution btw goods</td>
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<tr>
<td>Job destruction rate</td>
<td>$\rho$</td>
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<tr>
<td>Replacement rate</td>
<td>$\tau$</td>
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<tr>
<td>Hiring cost/output ratio</td>
<td>$\frac{\phi_{W}}{2} x^2$</td>
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<tr>
<td>Habit persistence in comsum.</td>
<td>$h$</td>
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<tr>
<td>Elasticity of matches to unemp.</td>
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<tr>
<td>Investment adjustment cost</td>
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<tr>
<td>Capital utilization cost</td>
<td>$\phi_{u2}$</td>
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<td>Price adjustment cost</td>
<td>$\phi_P$</td>
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<tr>
<td>Wage adjustment cost</td>
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<td>Backward-looking price setting</td>
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<tr>
<td>Backward-looking wage setting</td>
<td>$\varrho$</td>
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<td>Interest rate smoothing</td>
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<td>Response to inflation</td>
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<td>Response to output gap</td>
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<td>Response to output-gap growth</td>
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<td>Autocorrelation of technology growth shock</td>
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<td>Std dev of technology growth shock</td>
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<td>Standard deviation of risk-premium shock</td>
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<td>Autocorrelation of markup shock</td>
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<td>Std dev of markup shock</td>
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<td>Autocorrelation of exog. spend. shock</td>
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</tr>
<tr>
<td>Std dev of exog. spending shock</td>
<td>$\sigma_g$</td>
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<tr>
<td>Parameter</td>
<td>Formula</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
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<td>Employment adjustment cost</td>
<td>$\phi_N = \frac{2\times(\frac{N^2}{x^2})}{x^2}$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = \frac{z\pi}{\tau N}$</td>
</tr>
<tr>
<td>Job survival rate</td>
<td>$\chi = 1 - \rho$</td>
</tr>
<tr>
<td>Employment rate</td>
<td>$N = 1 - \rho$</td>
</tr>
<tr>
<td>Hiring rate</td>
<td>$x = \rho$</td>
</tr>
<tr>
<td>Mean of exogenous spending shock</td>
<td>$\epsilon_g = \frac{1}{1 - y/y}$</td>
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<td>Real marginal cost</td>
<td>$\xi = \frac{\theta-1}{\theta}$</td>
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<tr>
<td>Quarterly net real rental rate of capital</td>
<td>$\bar{r}^K = \frac{z}{\beta} - 1 + \delta$</td>
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<td>Capital utilization cost first parameter</td>
<td>$\phi_{u1} = \bar{r}^K$</td>
</tr>
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<td>Capital/output ratio</td>
<td>$\frac{k}{y} = \frac{\alpha_0}{\bar{r}^K}$</td>
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<tr>
<td>Investment/capital ratio</td>
<td>$\frac{i}{k} = z - 1 + \delta$</td>
</tr>
<tr>
<td>Investment/output ratio</td>
<td>$\frac{i}{y} = \frac{i}{k} \frac{k}{y}$</td>
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<tr>
<td>Consumption/output ratio</td>
<td>$\frac{c}{y} = \frac{1}{\epsilon_g} - \frac{\phi_N x^2}{x^2} - \frac{i}{y}$</td>
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<tr>
<td>Vacancies</td>
<td>$V = N^\frac{\epsilon}{q}$</td>
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<tr>
<td>Pool of job seekers</td>
<td>$S = 1 - \chi N$</td>
</tr>
<tr>
<td>Matching function efficiency</td>
<td>$\zeta = q \left( \frac{V}{S} \right)^\sigma$</td>
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<tr>
<td>Job finding rate</td>
<td>$s = \zeta \left( \frac{V}{S} \right)^{1-\sigma}$</td>
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<td>Employees’ share of output</td>
<td>$\bar{w}N/y = \xi (1 \alpha) - (1 - x - \beta \chi) \phi_N x$</td>
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<td>Bargaining power</td>
<td>$\eta = \frac{1-\tau}{\theta - \tau}$ where $\theta \equiv \frac{\xi(1-\alpha+\phi_N x^2+\beta \chi\phi_N x^2)}{\bar{w}N/y}$</td>
</tr>
<tr>
<td>Effective bargaining power</td>
<td>$\bar{\eta} = \frac{\eta}{1-\eta}$</td>
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Figure 1. Bootstrapped confidence bands for empirical spectra (90%, 75%), median empirical spectra (thin blue line), and theoretical spectra (thick red line).
Figure 2. 90% and 75% bootstrapped confidence bands of empirical coherences, median empirical coherences (thin blue line) and theoretical coherences (thick red line).
Figure 3. Theoretical spectra conditional on one shock at a time.
Figure 4. Impulse responses to a one-standard-deviation NEUTRAL TECHNOLOGY shock.
Figure 5. Impulse responses to a one-standard-deviation MONETARY POLICY shock.
Figure 6. Impulse responses to a one-standard-deviation INVESTMENT-SPECIFIC TECHNOLOGY shock.
Figure 7. Impulse responses to a one-standard-deviation RISK-PREMIUM shock.
Figure 8. Impulse responses to a one-standard-deviation PRICE MARKUP shock.
Figure 9. Impulse responses to a one-standard-deviation WAGE MARKUP shock.
Figure 10. Impulse responses to a one-standard-deviation EXOGENOUS SPENDING shock.
Figure 11. TOP panel: Unemployment rates: actual, potential, natural and HP-trend. BOTTOM panel: Unemployment gaps: potential, natural and HP.
Figure 12. Historical decomposition of the POTENTIAL rate of unemployment.
Figure 13. Historical decomposition of the NATURAL rate of unemployment.
Figure 14. TOP panel: Coherences between actual PRICE INFLATION and: [1] the POTENTIAL unemployment gap (thick red); [2] the NATURAL unemployment gap (blue); [3] the HP unemployment gap (thin black).
