Monetary Policy, Heterogeneity, and the Housing Channel*

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Abstract

We investigate the role of housing and mortgage debt in the transmission and effectiveness of monetary policy. First, monetary policy induced-movements in house prices translate into consumption changes because of wealth effects. Second, a contractionary monetary shock raises the cost of borrowing which reduces the demand and as a result the liquidity of the housing market, further depressing house prices and further increases the cost of borrowing. Furthermore, nominal long-term mortgage debt implies that changes in monetary policy result in redistribution between lenders and borrowers and generate cash-flow effects that are larger for borrowing constrained households. We build a heterogenous agent New Keynesian model with a frictional housing market to quantify the various mechanisms. The model is able to match the rich empirical heterogeneity in home ownership, leverage and MPC across households. In particular, our model is consistent with the significant difference in MPC between low- and high-LTV households that we document in the data. Our quantitative findings are as follows: First, we find that about 20% of the drop in aggregate consumption against a contractionary monetary shock is due to declining house prices. Second, we find asymmetric responses of the economy to shocks, with contractionary shocks yielding a larger response of all variables. Finally, we investigate how the transmission of monetary policy depends on the distribution of mortgage debt and find that monetary policy is more effective in stimulating the economy in an high-LTV environment.

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1 Introduction

The recent Great Recession has brought to light the importance of housing and household debt for the propagation of shocks in the macroeconomy. For a majority of US households, owner-occupied housing represents the single most important asset in the household portfolio and is tied to the single largest liability—the mortgage. In this paper, we investigate the rich role that housing and nominal mortgage borrowing jointly play in the transmission of monetary policy.

In addition to the direct intertemporal substitution effect of monetary policy on consumption, changes in interest rates generate several indirect effects related to housing and mortgages. First, changes in interest rates can generate movements in real house prices. A reduction in the interest rate reduces the cost of borrowing, alleviates credit constraints and increases the demand for housing. The increase in demand for housing increases real house prices. This indirect effect on house prices can translate into significant movements in household consumption, as recent empirical research finds large effects of house prices changes on household durable spending and non-durable consumption (for example, Mian et al. (2013); Kaplan et al. (2016a)). We refer to this as the house price channel. Furthermore, an increase in demand makes the housing market more liquid, meaning that households can sell their homes faster and at a higher price. As a result, financially distressed households are more likely to sell their homes and thus less likely to default. This, in turn, further alleviates borrowing constraints as banks internalize the decline in the riskiness of mortgage lending. This liquidity channel disproportionally affects households with high-leverage, who have a higher marginal propensity to consume (MPC) relative to those with less mortgage debt.

Moreover, the fact that houses are financed with long-term nominal debt implies that declines in interest rates may affect disposable incomes through increased refinancing activity, i.e., cash-flow channel (see, e.g., Flodén et al. (2016)). Furthermore, a change in real interest rate results in significant redistribution between households that are net debtors in nominal assets and net lenders. If debtors and lenders have different propensities to consume, this redistribution channel through balance sheets could have large effects on aggregate consumption (see, e.g., Auclert (2015)).

Heterogeneity is crucial for accurately evaluating the importance of these different channels in understanding the role of monetary policy on aggregate demand. The fact that some households have borrowed more than others is not random, and clearly not
orthogonal to their MPC. Thus, it is the joint distribution of mortgage debt and MPC that determines the relative contribution of the redistribution, cash-flow, and liquidity channels. Similarly, the distribution of liquid wealth is a key driver of the responsiveness of housing demand with respect to financing costs. Finally, generating a realistic distribution of marginal propensities to consume enables us to accurately capture indirect effects of monetary policy through the income channel—the consumption response to changes in labor demand (see, e.g. Kaplan et al. (2016b)).

The goal of this paper is to understand and quantify the extent to which the joint distribution of housing and mortgage debt affects the transmission of monetary policy through the above mechanisms. Answering this question requires developing a new framework that combines elements from heterogeneous-agent macro models, models with nominal rigidities and frictional models of the housing market. In particular, our framework features (1) incomplete financial markets with uninsurable idiosyncratic labor income risk; (2) a frictional housing market that endogenizes the price and liquidity of the housing market;\(^1\) (3) long-term nominal mortgages that generate nominal rigidity in household budget constraint and redistribution across borrowers and lenders; (4) price rigidities due to monopolistically competitive producers that face quadratic price adjustment costs à la Rotemberg (1982).

We calibrate the steady state of the model to match United States microeconomic and macroeconomic data over the past twenty years. Given the importance of housing and debt in the mechanisms we emphasize in this paper, the calibration pays particularly close attention to matching the important dimensions of the joint distribution of assets, housing wealth, and mortgage debt as well as key moments related to household price posting and selling behavior in the housing market, households choices related to mortgage debt and default. The model generates the empirically relevant difference in MPC between low- and high-loan-to-value (LTV) households. Namely, using data from the Panel Study of Income Dynamics (PSID) we estimate the consumption response of households to transitory income shocks conditional on their LTV ratios. We find that the elasticity of consumption with respect to income shocks to be 0.27 for households with a LTV ratio above 85 percent, whereas the corresponding figure for households with a LTV ratio below 85 percent is significantly lower (in a statistical sense) at 0.19.

\(^1\)We employ directed search in the housing market and show that our formulation admits block recursivity in the spirit of Menzio and Shi (2010), greatly increasing the computational tractability and allowing us to conduct rich experiments to isolate the economic forces.
The model generates several quantitatively important deviations from standard incomplete markets models where households are able to easily access their entire portfolio of wealth. First, heavily leveraged homeowners experience long selling delays in the housing market because their outstanding debt acts as a binding lower bound on the list price. This inability to quickly sell increases the exposure of such homeowners to idiosyncratic and aggregate risk, thereby making mortgage default more likely. This elevated default risk from housing illiquidity causes access to mortgage credit to tighten, thereby further impeding consumption smoothing. This impaired consumption smoothing generates as a result significant dispersion in the distribution of the marginal propensity to consume. Renters, and homeowners with substantial equity are relatively insensitive to income fluctuations, while heavily indebted homeowners stuck with houses they cannot quickly sell are far more responsive to shocks.

We then use this model to study the transmission of the monetary policy. Our quantitative findings are as follows: First, house prices decline in response to a monetary tightening, which causes an endogenous cascade of declining housing liquidity, elevated foreclosure risk, tighter credit, and further price declines. As this cycle unfolds, the consumption of leveraged homeowners responds strongly, which then impacts aggregate demand, income, and output. We find that about 20% of the consumption response is due to the movements in house prices.

Second, we investigate asymmetric effects of policy. We find that contractionary monetary shocks have larger effects on aggregate demand than expansionary shocks. This is mostly due to the shape of the leverage distribution: When rates do down, this relaxes fewer households’ budget constraint than the number of households that fall into trouble with their mortgages when rates go up. Because of this underlying heterogeneity, the foreclosure rate disproportionately increases in response to monetary tightening, causing an asymmetric response in house prices. The asymmetry in house prices then feeds back into consumption.

Lastly, we investigate how the effectiveness of the monetary policy depends on the distribution of mortgage debt. For this purpose, we conduct a simple experiment and reduce the maximum LTV households are allowed to take from 125%, which is not binding in the calibration, to 85%. This generates nontrivial changes to the LTV distribution. The high-LTV economy implies a wider distribution of MPC with more households having high MPCs. As a result we find that expansionary monetary policy is more effective in an high-LTV economy.
Related Literature  This paper is related to several strands of the literature. Several papers examine the role of house prices in driving aggregate fluctuations through consumption. Mian et al. (2013) and Kaplan et al. (2016a) study the response of spending on cars and non-durable consumption, respectively, to housing net worth prices by exploiting geographical variation in the housing collapse of 2006—2009. They find an elasticity in the range of 0.6 to 0.8 for spending on cars and 0.2 to 0.4 for non-durable expenditures. Furthermore, they show that this elasticity is higher for poorer households and those with higher loan-to-value on their mortgages. In addition, Kaplan et al. (2016a) decompose the response of non-durable spending into changes in prices versus quantities and show that price movements can account for one fifth of the consumption response.

The elasticities of consumption to house price changes documented in the Great Recession, however, are much larger than previously estimated in the literature using different data and over different time periods that generally found an elasticity in the range of 0 to 0.10 (Carroll et al. (2011); Attanasio et al. (2009); Calomiris et al. (2009); Browning et al. (2013); Case et al. (2011); Campbell and Cocco (2007)).

There is a nascent empirical literature that investigates how household balance sheets affect the transmission of monetary policy. Di Maggio et al. (2014) show a significant consumption response to monetary policy, but they highlight how voluntary deleveraging attenuates the effect of rate reductions on consumption. Consistent with other work, they show that the marginal propensity to consume is higher for low income or underwater borrowers, and that the effect is larger in counties with a greater fraction of adjustable rate mortgages. In other words, debt rigidity reduces the effectiveness of monetary policy in their setting. Keys et al. (2014) also show that voluntary deleveraging mutes the response of consumption to lower mortgage rates. They also show that regions that were more exposed to mortgage rate declines saw faster recovery of house prices, consumption, and employment, particularly in the non-tradable sector. In short, these papers provide empirical support for the key result that the effectiveness of monetary policy crucially depends on the distribution of liquid wealth and mortgage debt. Auclert (2015) documents the importance of the redistribution channel and develops a sufficient statistic approach for quantifying it, but abstracts from housing in the household portfolio. A number of other papers have focused on how different dimension of heterogeneity such as age (Wong (2015)), debt-to-income ratio (Flodén et al. (2016)), and housing and

\[^2\text{Stroebel and Vavra (2014) have previously shown that these movements in prices should be interpreted as changes in local mark-ups to demand shocks.}\]
mortgage tenure (Cloyne et al. (2015)).

Our paper also contributes to several strands of the modeling literature. First, our framework is the first to introduce housing, long-term debt and a frictional housing maker with the new class of heterogeneous-agent New-Keynesian (HANK) models (Challe et al. (2015); Gornemann et al. (2014); Kaplan et al. (2016b); McKay and Reis (Forthcoming); Luetticke (2015)). Second, our paper contributes to the modeling literature that has investigated the role of housing and household debt in understanding the consumption and foreclosure dynamics (e.g. Garriga and Hedlund (2016); Garriga et al. (2015); Huo and Rios-Rull (2013); Kaplan et al. (2015); Favilukis et al. (2010); Corbae and Quintin (2015); Chatterjee and Eyigungor (2011); Hedlund (2015)) by jointly modeling heterogeneity, monetary policy and long-term debt. Our paper is closely related to Hedlund (2015), which looks at what impact higher inflation could have had in potentially mitigating the Great Recession by inflating away nominal mortgage debt. But that paper abstracts from the conduct of monetary policy, nominal rigidities and production in the economy.

2 Model

In this section, we develop a heterogeneous agents New Keynesian model with housing and mortgages. To investigate the effect of the housing channel in the transmission of monetary policy, we include the following key ingredients: (1) a frictional housing market that endogenizes the price and liquidity of the housing market; (2) long-term nominal mortgages that generate nominal rigidity in household budget constraint and redistribution across borrowers and lenders; (3) sticky prices in goods market à la Rotemberg (1982), so as to provide a role for aggregate demand in determining output.

The economy is populated by (i) a measure one of infinitely lived households, (ii) real estate brokers that facilitate transactions in the housing market, (iii) banks that issue long-term adjustable rate mortgages, (iv) government sponsored enterprises that provide insurance to the banks against the default risk of mortgages, (v) a set of intermediate goods producers in a monopolistically competitive market producing a product of variety \( j \in (0, 1) \), (vi) a representative final goods producer aggregating intermediate goods into the final consumption good in a competitive market, (vii) a government that sets fiscal policy, and (viii) a monetary authority that sets the nominal interest rate. Time is discrete.
Below, we present the details of the model. Wherever non-essential, the technical details and equations are relegated to appendix A.

2.1 Households

There is a continuum of infinitely lived households that are subject to uninsurable idiosyncratic labor productivity risk. Their labor productivity \( z_t \) follows an exogenous finite state Markov according to the transition matrix \( \Gamma(z_{t+1} \mid z_t) \). Households have preferences over non-durable consumption \( c \), leisure \( l \), and housing services \( s \) and are endowed with one unit of time every period, which they allocate between market work and leisure. Preferences are time-separable and the future is discounted at rate \( \beta \). The expected lifetime utility of a household is given by:

\[
E_0 \sum_{t=0}^{\infty} \beta^{t-1} \left[ \frac{(1 - \phi_h) c_t^{1-\gamma_h} + \phi_h s_t^{1-\gamma_h}}{1-\sigma} \right] - g(l_t), \quad \text{with} \quad g(l_t) = \psi \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}.
\]

Furthermore, we assume that the utility from leisure is proportional to labor productivity so that only aggregate movements in the wage rate affect the labor supply of households. We use the CES aggregator between the consumption-leisure bundle and the housing services. The parameters \( \beta, \sigma_L, \gamma_h \), and \( \varphi \) measure the discount factor and relative risk aversion, elasticity of substitution between consumption and housing services, and Frisch elasticity of labor supply, respectively. \( \phi_h \) determines the share of housing services in total consumption.

The households enjoy housing services \( s \) either by owning and occupying a house or by renting from a competitive market in the form of apartment space. Owner-occupied housing comes in a set of discrete sizes, \( h \in \mathcal{H} = \{h_1, h_2, \ldots, h_n\} \). Living in an owner-occupied house of size \( h \) generates a service flow of \( s = h \omega_h \), whereas living in a rental unit of the same size generates a service flow of \( s = h \). \( \omega_h \geq 1 \) captures the motives towards ownership beyond those that are explicitly modeled. Homeowners are not allowed to rent out their housing unit to a tenant.

Households can save in a risk free bond, \( b_{t+1} \) at a price of \( q_t^B \). Renters are not allowed to borrow. Homeowners can borrow in the form of long term, adjustable rate nominal mortgage contracts. In other words, homeownership allows households to extend their borrowing limit. Mortgage size and the mortgage interest rate in period \( t \) are denoted by \( M_t \) and \( r_{mt} \), respectively.
2.2 Real estate brokers and the housing market

There is a fixed supply of housing in the economy that does not depreciate. Homeowners are allowed to sell their houses provided that they have the ability to pay off any outstanding mortgage debt. Houses are sold in a decentralized market subject to search frictions. Search is directed: A seller with a house size of \( h \) decides on the posting price \( x_s \) to put it on the market. She then meets a real estate broker who has entered the \((x_s, h)\)-submarket by paying an entry cost of \( \kappa h \). A meeting between a real estate broker and a seller happens with probability \( \tilde{p}_t(\theta_t(x_s, h)) \), where \( \theta_t(x_s, h) \) is the tightness of the submarket for houses of size \( h \) that are listed at a price of \( x_s \) in period \( t \). Homeowners that try and fail to sell their houses pay a utility cost \( \xi \).

After buying the houses from sellers, real estate brokers turn around and sell them to buyers in a centralized market. Real estate brokers have access to a technology that allows them to splice houses into a few smaller houses as well as to combine several into a larger unit. The implication of this assumption is that houses in the buying market are priced per unit, i.e., renters (including people who just sold their house) can purchase houses from the real estate brokers at a unit price \( p^H_t \). Therefore, a renter buys a house of size \( h \) at price \( p^H_t h \). To keep the problem simple, we do not allow homeowners to own multiple houses at the same time. If they want a house of different size, they must first sell their existing house in the frictional market. Buyers immediately move into their house and switch from apartment-dweller (“renter”) status to homeowner status. Note that brokers are not permitted to carry housing inventories into future periods, but inventories do arise in equilibrium from the portion of the housing stock that owners put on the market but fail to sell.

**Directed search and block-recursive structure** We assume free entry of real estate brokers in every market. Letting \( \alpha_t(\theta_t(x_s, h)) = \frac{\tilde{p}_t(\theta_t(x_s, h))}{\theta_t(x_s, h)} \) denote the probability that a broker finds a seller in period \( t \), the free entry condition can be written as:

\[
\kappa h = \alpha_t(\theta_t(x_s, h)) (p^H_t h - x_s)
\]

for all markets with \( \theta_t(x_s, h) > 0 \). The revenue to a broker of purchasing a house is \( p^H_t h - x_s \). Therefore, brokers continue to enter the submarket \((x_s, h)\) until the cost \( \kappa h \) exceeds the expected revenue.

The use of directed search and real estate brokers borrows a key idea from the labor
search literature. As in Menzio and Shi (2010) and Menzio and Shi (2011), directed search with risk-neutral agents on one side of the market (real estate brokers in this model) gives rise to the simple condition in (1). This condition pins down the tightness of each market independent of household characteristics that decide to trade in that market, as shown in (2). The latter is important, because it allows us to solve market tightnesses as a function of $p^H_t$, without having to solve the maximization problem of households.

$$\theta_t(x_s, h) = \alpha_t^{-1} \left( \frac{\kappa h}{p^H_t h - x_s} \right)$$  \hspace{1cm} (2)

This feature would not arise in random search models with bargaining. In such models, the outcome of bargaining depends on the characteristics of market participants (e.g. wealth and income of households). Price posting solves this problem. Free entry of risk-neutral agents insures a simple relationship between price and liquidity that is independent of household characteristics. These insights were previously used in Hedlund (2016b), Hedlund (2016a), Karahan and Rhee (2013), and Garriga and Hedlund (2016) to study different issues about housing.

**Apartments** We assume that apartment space can be produced using the final good. In particular, landlords have access to a linear technology that converts one unit of the final good into $A_h$ units of apartment space (and vice versa). Market for apartments is competitive. Letting $r_h$ denote the rental price of a unit of apartment, the technology implies that $r_h = \frac{1}{A_h}$. Thus a renter who rents $a_h$ units of apartment space pays $r_h a_h$ units of final good as rent. It is important to note that the rental rate is pinned down by the technology and does not respond, among others, to changes in house prices. We also assume that the largest apartment one can rent is smaller than the maximum size of owner-occupied house, which implies a partial segmentation in the housing market. This is going to be one of the reasons why households would become home owners.

**2.3 Mortgages and banks**

Banks offer long-term, fixed-rate mortgage contracts. The contract is characterized by a nominal face value, $M_{t+1}$, the price at origination, $q^0_{mt}$, and the amortization rate $r_{mt}$. A household receives nominal funds $q^0_{mt} M_{t+1}$ when taking out the loan and could pay it off the subsequent period by paying $M_{t+1}$. However, mortgages are long-term and have no pre-defined maturity date. Borrowers are free to choose how quickly to pay down
their mortgage so long as they keep making a minimum payment, which is a pre-specified fraction of the loan; i.e. \( M_{t+1} \leq (1 - \chi)M_t \). Thus, \( 1/\chi \) controls the effective duration of the mortgage. To summarize, a borrower with an existing contract amount of \( M_t \) that chooses \( M_{t+1} \leq (1 - \chi)M_t \) has to make a mortgage payment of \( (1 + r_{mt})M_t - M_{t+1} \). In addition to making a payment (or paying off) the loan, borrowers also have the option to refinance by originating a new loan and paying off the existing one.

Banks take into account two types of risks when issuing loans. First, borrowers have the default option, in which case they lose their house, incur a utility cost \( f \), and have their debt discharged.\(^3\) In the event of foreclosure, the bank sells the repossessed house (REO properties) in the frictional decentralized housing market (as individual sellers do) and incurs a loss \( \gamma^{REO} \), proportional to the selling price\(^4\). When the bank sells a foreclosed house, it absorbs all losses but must pass along all profits to the borrower in the (unlikely) event that sales revenues exceed the remaining mortgage balance. The value to a lender of repossessing a house of size \( h \) is given by equation (5) in appendix A.

Second, as explained above, households also have the option of prepayment and refinancing the loan by paying off their old mortgage and taking out a new one. Banks have to price in these risks and determine the price of mortgage \( q_{mt}^0 \) accordingly. Thus, mortgage prices depend on borrower characteristics. The recursive equation that determines the mortgage pricing is given by equation (6) in appendix A.

Banks finance themselves by bundling the future streams of payments net of servicing costs into mortgage backed securities (MBS). A mortgage backed security is indexed by its date of issuance, \( MBS_r \), and has a price in period \( t \) given by \( q_t^{MBS_r} \) (we normalize the face value of the MBS to 1). Banks compete for consumers loan-by-loan, and so make zero expected (and ex-post) profit on each loan. Aggregate losses are reflected in the returns to the MBS. The proceeds from the issuance of the MBS need to satisfy all of the funds required to originate the mortgages in period \( t \):

\(^3\)The utility cost is meant to represent, among other things, the stigma associated with foreclosure and non-monetary moving costs.

\(^4\)This cost is meant to capture the various costs to the banks of selling foreclosed houses.
\[
q_t^{MBS_t} \ MBS_t = \int \text{Buy}(X_t^{\text{RENT}}) \times q^0_{mt}(X_{t+1}^{\text{OWN}})M_{t+1} \Phi_t^{\text{RENT}}(dX_t^{\text{RENT}})
+ \int \text{Refi}(X_t^{\text{OWN}}) \times q^0_{mt}(X_{t+1}^{\text{OWN}})M_{t+1} \Phi_t^{\text{OWN}}(dX_t^{\text{OWN}})
+ \int (\text{Sell}(X_t^{\text{OWN}}) \times \text{Buy}(X_t^{\text{OWN}})) \times q^0_{mt}(X_{t+1}^{\text{OWN}})M_{t+1} \Phi_t^{\text{OWN}}(dX_t^{\text{OWN}})
\]

where the first line are new owners who take on a mortgage, the second line are for households who refinance and the third line are owners who sell and buy a new house in the same period. The MBS will all be held by the mutual fund, and thus the payout of the MBS can be written recursively as:

\[
q_t^{MBS_{\tau}} \ MBS_{\tau} = \frac{1}{1 + r_{SDF}}(q_{t+1}^{MBS_{\tau}} \ MBS_{\tau})
+ \int \text{Pay}(X_t^{\text{OWN}_{\tau}})(M_t(X_t^{\text{OWN}_{\tau}}) - M_{t+1}(X_t^{\text{OWN}_{\tau}}))\Phi_t^{\text{OWN}}(dX_t^{\text{OWN}_{\tau}})
+ \int \text{Sell}(X_t^{\text{OWN}_{\tau}})M_t(X_t^{\text{OWN}_{\tau}})\Phi_t^{\text{OWN}}(dX_t^{\text{OWN}_{\tau}})
+ \int \text{Refi}(X_t^{\text{OWN}_{\tau}})M_t(X_t^{\text{OWN}_{\tau}})\Phi_t^{\text{OWN}}(dX_t^{\text{OWN}_{\tau}})
+ \int \text{Fore}(X_t^{\text{OWN}_{\tau}})P_{t+1J_{REO}}(h_t(X_t^{\text{OWN}_{\tau}}))\Phi_t^{\text{OWN}}(dX_t^{\text{OWN}_{\tau}}))
\]

where \(X_t^{\text{OWN}_{\tau}}\) are the individual state variables for households in period \(t\) who originated a mortgage in period \(\tau\) and still have a positive mortgage value outstanding. The price of an MBS originated in period \(\tau\) has a face value 0 in all periods beginning with the period in which the final mortgage is repaid. In order to compute the transition, what we need to figure out is how the prices of the MBS jump when the shock occurs. Basically, in the period of the shock we need to solve for the fixed point for the return on the previous period’s savings that is also consistent with the return on the mortgage portfolio. From the backsolve, we know exactly the \(q_2^{MBS_{\tau}} \ MBS_{\tau}\) component, since we can use \(q_{0m,1}\) to price the continuations. But the payouts in that period will depend on the loss. But, then, going forward, we know that the return will just be given by the risk-free rate, so there will be no additional fixed points in the future periods, so we know exactly what the sequence of \(q\)’s will be.
2.4 Recruiting Firm

We follow the New Keynesian literature and assume that households provide differentiated labor services, $l_{it}$. We assume that there exists a representative recruiting firm that aggregates the differentiated labor services into the aggregate labor services $L_t$ with a CES aggregator:

$$L_t = \left( \int_0^1 z_{it} \left( l_{it} \right)^{\frac{\epsilon_w - 1}{\epsilon_w}} \, di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

where $\epsilon_w$ is the elasticity of substitution across labor services.

We assume that there exist a continuum of competitive unions that sell households’ labor services to the recruiting firm. Given aggregate labor demand $L$ by the intermediate goods firms, the recruiting firm minimizes costs

$$\int_0^1 W_{it} z_{it} l_{it} \, di$$

implying a demand for the labor services of household $i$:

$$l_{jt} = l(W_{it}; W_t, L_t) = \left( \frac{W_{it}}{W_t} \right)^{-\epsilon_w} L_t$$

where $W_t$ is the (equilibrium) nominal wage which can be expressed as

$$W_t = \left( \int_0^1 z_{it} W_{it}^{1-\epsilon_w} \, di \right)^{\frac{1}{1-\epsilon_w}}$$

The middleman sets a nominal wage $\hat{W}_t$ for an effective unit of labor so that $W_{it} = \hat{W}_t$ to maximize profits subject to wage adjustment costs similar to the price adjustment costs as in Rotemberg (1982). These adjustment costs are proportional to idiosyncratic productivity $z$ and are measured in units of aggregate output and are given by a quadratic function of the change in wages above and beyond steady state wage inflation $\Pi^w$,

$$\Theta \left( z_{it}, W_{it} = \hat{W}_t, W_{it-1} = \hat{W}_{t-1}; y_t \right) = z_{it} \frac{\theta_w}{2} \left( \frac{W_{it}}{\hat{W}_{it-1}} - \Pi^w \right)^2 L_t = z_{it} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1} - \Pi^w} \right)^2 L_t.$$
The middleman’s wage setting problem is to maximize

$$V^w_t (\hat{W}_{t-1}) \equiv \max_{\hat{W}_t} \int \left( \frac{z_{it}(1 - \tau_t)}{P_t} l(\hat{W}_t W_t, L_t) - \frac{g(h(\hat{W}_t W_t, L_t))}{u'(C_t, H_t)} \right) di - \int z_{it} \frac{\theta_w}{2} \left( \frac{\hat{W}_t}{\hat{W}_{t-1}} - \Pi^w \right)^2 L_t di + \beta \theta_w \hat{W}_t.$$

where $C_t$ is aggregate non-durable consumption and $H_t$ is aggregate consumption of housing services. Some algebra (delegated to the appendix) yields, using $l_{it} = L_t$ and $\hat{W}_t = W_t$ and defining the real wage $w_t = \frac{W_t}{P_t}$, the wage inflation equation

$$\theta_w (\pi^w_t - \Pi^w) \pi^w_t = (1 - \tau_t)(1 - \epsilon_w) w_t + \epsilon_w \frac{g'(l(\hat{W}_t W_t, L_t))}{u'(C_t, H_t)} + \beta u'(C_{t+1}, H_{t+1}) \theta_w (\pi^w_{t+1} - \Pi^w) \pi^w_{t+1} \frac{L_{t+1}}{L_t}.$$

Note, that if we do not want sticky wages, but just monopolistic competition and differentiated labor services, we obtain

$$w_t = \frac{\epsilon_w}{(1 - \tau_t)(\epsilon_w - 1)} \frac{g'(l(\hat{W}_t W_t, L_t))}{u'(C_t, H_t)}.$$

### 2.5 Final good producer

A competitive representative final goods producer aggregates a continuum of intermediate goods indexed by $j \in [0, 1]$ and with prices $p_{jt}$:

$$Y_t = \left( \int_0^1 \frac{\epsilon_y - 1}{y_{jt}^{\epsilon_y - 1}} dj \right) \frac{1}{\epsilon_y},$$

where $\epsilon_y$ is the elasticity of substitution across goods. Given a level of aggregate demand $Y$, cost minimization for the final goods producer implies that the demand for the intermediate good $j$ is given by:

$$y_{jt} (p_{jt}) = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon_y},$$

where $P_t$ is the (equilibrium) price of the final good in period $t$ and can be expressed as:

$$P_t = \left( \int_0^1 \frac{1}{p_{jt}^{1 - \epsilon_y}} dj \right)^{\frac{1}{1 - \epsilon_y}}.$$
2.6 Intermediate goods producers

Each intermediate good $j$ is produced by a monopolistically competitive producer using labor input $n_{jt}$. Production technology is linear.

$$y_{jt} = Z_t n_{jt},$$

where $Z_t$ is aggregate productivity in period $t$. Intermediate producers hire labor at (real) wage $w_t$ in a competitive labor market. With this technology, the marginal cost of a unit of intermediate good is $w_t/Z_t$.

Each firm chooses its price to maximize profits subject to price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic function of last period’s and this period’s prices, $(p_{jt-1}, p_{jt})$:

$$\Theta(p_{jt}, p_{jt-1}) = \frac{\theta}{2} \left( \frac{p_{jt}}{p_{jt-1}} - (1 + \bar{\pi}) \right)^2 P_t Y_t,$$

where $\bar{\pi}$ is the steady state (target) inflation rate.

Given last period’s individual price $p_{jt-1}$ and the aggregate price level $P_{t-1}$, and a rational expectation function for aggregate price $P_t = G_t(P_{t-1})$ the firm chooses this period’s price $p_{jt}$ to maximizes the present discounted value of future profits:

$$V_t(p_{jt-1}, P_{t-1}; G_t) \equiv \max_{p_{jt}, n_{jt}} \left\{ \frac{p_{jt} y_{jt}(p_{jt}) - P_t w_t n_{jt} - \Theta(p_{jt}, p_{jt-1})}{P_t} \right\} + \frac{\beta u'(C_{t+1}, H_{t+1})}{u'(C_t, H_t)} V_{t+1}(p_{jt}, P_t; G_{t+1}).$$

In the steady state, the value of an intermediate firm is given by:

$$V(p, P; G) \equiv \underbrace{PY/\epsilon_y + \beta V(p, P; G)}_{\text{profit}} \Rightarrow V = \frac{PY}{\epsilon_y (1 - \beta)}$$

2.7 Mutual Fund Sector

There is a large measure of competitive mutual funds with ownership of the intermediate good producers, government bonds, and MBS’s (which also absorbs profits/losses due to unanticipated aggregate shocks). Each of these funds issues shares $A$ with return $r^a$. 
\[ V(S, A, B; P_s, P) = \max_{S', A', B', D} D + \frac{1}{1 + r_{SDF}} V'(S', A', B'; P'_s, P') \]

subject to

\[ PD + P_s S' + A' + B' + \sum q^{MBS_r} MBS_r \leq (P_s + P_d) S + (1 + r_a) A + (1 + i) B + \sum (q^{MBS_r} + \text{coupon}) \]

where \( m_{SDF} = \frac{1}{1 + r_{SDF}} \) is the stochastic discount factor. The optimality conditions are:

\[
\begin{align*}
V'_1 &= P_s (1 + r_{SDF}) / P \\
V'_2 &= (1 + r_{SDF}) / P \\
V'_3 &= (1 + r_{SDF}) / P \\
V_1 &= (P_s + P_d) / P \\
V_2 &= (1 + r_a) / P \\
V_3 &= (1 + i) / P \\
\end{align*}
\]

which implies:

\[
\begin{align*}
P'P_s (1 + r_{SDF}) &= P(P'_s + P'_d) \\
P'(1 + r_{SDF}) &= P(1 + r'_a) \\
P'(1 + r_{SDF}) &= P(1 + i') \\
\end{align*}
\]

which implies:

\[
\begin{align*}
r'_a &= i' \\
1 + i' &= \left( \frac{P'_s}{P_s} + \frac{P'}{P_s} d' \right) \\
\end{align*}
\]

Now, in steady state we know \( P'_s / P_s = P' / P = \Pi \). Which implies \( 1 + i' = \Pi' (1 + d' / P_s) \leftrightarrow \frac{1 + i'}{1 + \pi'} = 1 + d' / P_s = 1 + r' = 1 + r_{SDF} \), thus, in equilibrium the intermediate goods
producing firms and the banks should discount the future at the risk-free rate. Note, that in the transition, nothing is changed in terms of the forward looking behavior. The only thing that changes will be the paths of \( i \) and \( d \) (and thus \( r_a \)). Now, it is true that on impact, \( P_s \) and \( P \) will (potentially) jump. Note, that in equilibrium the firm will have 0 net wealth, such that \(-A' = B' + P_s S'\), i.e. all of its investments are financed by issuing shares. Further, in equilibrium all \( S' = 1 \) (in the aggregate across all mutual funds).

Calibrating stuff: So, in the new SS I think we should target gov’t debt/annual GDP = 0.6 (to line up with 2000s). In order to make sure we have enough “wealth” in the economy, I think that we should target that the value of the shares of the intermediate goods firms are worth 2 times annual GDP (this is a bit lower than the capital stock, but doesn’t seem to crazy). Now, in order to implement this we need to introduce a fixed cost of production for the intermediate goods producers \( \Phi \). Now, using the above equation \( 1 + d'/P_s = 1 + r' \), we know that \( d' = Y/\epsilon - \Phi \), \( r' = 0.01 \), and we want \( P_s = 8Y \) (remember \( Y \) is quarterly GDP). Substituting and rearranging, this means that \( \Phi = Y/\epsilon - 0.08 \) (effectively, we need to set \( \Phi = Y/\epsilon - 0.01X \) where \( X \) is the multiples of quarterly GDP we want the shares of the firms to be worth.

Now, in terms of the government, we set \( B = 0.6 \), we can set \( G \) relatively low, say, 6% of GDP, so quarterly would be 0.06\( Y \), and we set transfers residually to clear the govt budget constraint.

The steady state algorithm would then works as follows:

1. Guess the discount factor \( \beta \)
   (a) Guess aggregate consumption \( C \)
      i. Use the recruiting firm problem and the fact that SS wage = \((\epsilon - 1)/\epsilon\) to back out \( L \) and \( Y \)
         A. Use the government budget constraint to determine \( T \)
         B. Guess the house price \( p_h \)
         C. Solve the HH problem given prices
         D. Compute the invariant distribution
         E. Check if housing market clears, if so step out, otherwise update \( p_h \)
      ii. Compute the aggregate consumption demand for households
         iii. If demand is consistent with the guess, step out, otherwise, update \( C \)
2. Compute the total supply and demand for the mutual fund shares
   
   (a) Note supply is just given from the household problem
   
   (b) Demand is \( B + P_s + MBS \) where MBS are the endogenous total amount of mortgage-backed securities, and \( B \) and \( P_s \) are given as above

### 2.8 Foreign Asset Demand

In order to embed possible exogenous movements in the risk-free rate, we assume that the rest of the world demands a fraction of U.S. safe assets. We denote this \( \text{RoW}_t \).

### 2.9 Government

The government fully taxes intermediate firm profits, \( P_t d_t \) and levies a progressive tax on household labor income \( y \) that consists of a lump-sum transfer \( T_t \) and a proportional tax \( \tau \):

\[
\tilde{T}_t(w_t z_t l_t) = -T_t + \tau P_t w_t z_t l_t.
\]

The government issues nominal bonds denoted by \( B^g_t \), with negative values corresponding to government saving. The government finances exogenous nominal government expenditures, \( G_t \), and interest payments on bonds.

The government budget constraint is therefore given by:

\[
B^g_{t+1} = (1 + i_t)B^g_t + G_t - P_t d_t - \int \tilde{T}_t(w_t z_t l_t)dl_t.
\]  \( \text{(3)} \)

Furthermore, there is a monetary authority that sets the nominal interest rate on bonds, \( i_t \) according to a Taylor rule:

\[
(1 + i_{t+1}) = (1 + i_t) \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\phi_T} e^{\epsilon_t},
\]

where \( \pi_t \) is the inflation rate in period \( t \), \( \bar{\pi} \) is the inflation target, and \( \phi_T \) is the Taylor rule coefficient. We assume the economy is at its steady state in period \( t = 0 \) with \( \epsilon_0 = 0 \). Later on, we analyze the response of this economy to policy shocks, where we allow nonzero shocks (\( \epsilon_t \neq 0, t > 0 \)) to the Taylor rule. In this case, we assume that \( \epsilon_t \) follows an AR(1) process.
In this model Ricardian equivalence breaks down because of the heterogeneity across households. As a result, the specifics of the monetary-fiscal coordination is not an innocuous assumption and affects the allocation and prices. Here we assume that the government keeps the real government debt constant and adjusts lumpsum transfers in order to balance the budget every period.

Finally, the real interest rate follows from the Fisher equation:

\[(1 + i_t) = (1 + r_t)(1 + \tau_t).\]

### 2.10 Timing of events

Below, we describe the timing of the events within a period.

1. **Shocks**: In the beginning of the period households learn about their idiosyncratic productivity and choose how much to work.

2. **Market for house selling**: Homeowners decide whether to put their houses up for sale and if so, at what price. Real estate brokers enter the various submarkets by paying the entry cost. Sellers meet with real estate brokers and transfer the ownership of their house.

3. **Default decisions**: Homeowners who have not sold their houses make default decisions. Banks take the ownership of the foreclosed houses and try to sell it to real estate brokers. Homeowners that do not default on their mortgage debt, choose the mortgage payment for the current period. This decision also encompasses the refinancing decision.

4. **Market for house buying**: Renters, including those that recently sold their houses, decide whether to buy a house. Buyers also decide on how much mortgage debt to take out, if any.

5. **Production, consumption and savings**: Intermediate goods producers make production decisions. These goods are then aggregated into the final good by final goods producers. Households choose how much to consume and how much to save using the risk free bonds.
2.11 Equilibrium

For given level of real government debt, exogenous government expenditures $G_t$, proportional tax rate levied on labor income $\tau$ and monetary policy, an equilibrium path for this economy is given by a set of paths of prices $\{P_t, w_t, p_t^H, r_{ht}, r_{mt}, q_{it}^B, r_t\}_{t \geq 0}$, mortgage price functions $\{q_{mt}(\cdot)\}_{t \geq 0}$, lumpsum transfers $\{T_t\}_{t \geq 0}$, distribution of households over labor productivity, bond holdings, owned house size, mortgage debt $\Omega_t(z, b, h, M)$ and corresponding quantities, such that at every period $t$ given aggregate prices:

(i) households, firms, banks, and real estate brokers maximize their objective value,

(ii) the government budget holds

(iii) the mutual fund budget holds

(iv) the financial intermediary budget holds

(v) the following markets clear:

1. Labor market: $\int n_t dj = \int z_t d\Omega_t$,

2. Housing market: $\int h_t d\Omega_t = 1$,

3. Shares market: $\int S_t^{MF} = 1$

4. Asset market: $A_t + \int b_t d\Omega_t = 0$

5. Bond market: $B_t^{MF} + RoW_t = B_t^{g} + MBS_t$

6. Final good market: $\int P_t c_t d\Omega_t + \int r_{ht} a_{ht} d\Omega_t + S_{ot} + S_{ot} + G_t + i_t RoW_t = Y_t$,

where $S_{ot}$ and $S_{ot}$ denote the aggregate spending on service and origination costs by the banks in period $t$, respectively.

3 Calibration

We assume the economy is in steady state in period $t = 0$ and calibrate it to the US economy prior to the Great Recession during 2003 – 2005. The calibration puts emphasis on matching key housing moments related to sales, time on the market, and foreclosures,
as well as important dimensions of the joint distribution of assets, housing wealth, and mortgage debt. Some parameters are drawn from the literature or from external sources, but the remainder are determined jointly in the calibration.

### 3.1 Parameters set outside the model

**Income process**  Using administrative data, Guvenen et al. (2016) estimate a canonical income process for the life cycle annual earnings risk which consists of a random walk and an i.i.d transitory component. Since our model is not a life-cycle model, we cannot directly use this income process in our calibration. Instead, we target the variance of log earnings in the working age population implied by this income process, which is 80 log points. More specifically, we first set the variance of transitory shocks to 0.16.\(^5\) Second, in order to have a stationary distribution of earnings, we set the persistence parameter of persistent component \(\rho\) to 0.99, which is close to a random walk. Next, we choose the variance of innovations to the persistent component \(\sigma^2\) by targeting the variance of log earnings in the population. In particular, out of 80 log points, transitory shocks account for 16 log points. Then, variance of persistent shocks is chosen to generate 0.64 unconditional variance:

\[
0.64 = \frac{\sigma^2}{1-\rho^2} \Rightarrow \sigma^2 = 0.0127.
\]

However these values are for annual earnings whereas our model period is a quarter. Therefore, we convert these annual values to their corresponding quarterly values. In particular, we estimate a quarterly income process such that the resulting aggregated annual income process has a transitory component with variance 0.16 and a persistent component with persistence 0.99 and variance 0.0127. Finally, we employ Rouwenhorst (1995) method to discretize the persistent and transitory components of the income process. We use 3 grid points for the transitory component and 7 for the persistent component.

**Preference parameters**  Risk aversion is set to \(\sigma = 2\). The Frisch elasticity of labor supply \(\phi\) is set to 0.33 (Chetty 2008). Preference parameter governing housing \(\phi_h\), the value of housing services \(\omega_h\), the intratemporal elasticity of substitution \(\gamma_h\), and the discount factor \(\beta\) are determined jointly in the calibration.

\(^5\)Guvenen et al. (2016) estimate this parameter to be 0.25 for annual earnings. Since our model generates endogenous fluctuations in labor hours, and in turn in annual earnings, we calibrate this parameter to a smaller value of 0.16.
Technology  Steady state TFP in the consumption good sector is set to normalize mean quarterly earnings to 0.25. The apartment technology $A_h$ is set to generate an annual rent-price ratio of 3.5%.

Housing Market  Matching function in the selling markets is governed by a Cobb Douglas function, i.e. $\tilde{p}(\theta) = \min\{\theta^\gamma, 1\}$. Substituting in the equation for market tightnesses gives

$$\tilde{p}(\theta) = \begin{cases} 0 & \text{if } x_s > p^H h \\ \left(\frac{p^H h - x_s}{\kappa h}\right)^{\frac{\gamma}{1-\gamma}} & \text{if } (p^H - \kappa)h \leq x_s \leq p^H h \\ 1 & \text{if } x_s < (p^H - \kappa)h \end{cases}$$

The joint calibration determines the parameters $\kappa$, and $\gamma$. Holding costs (maintenance, property taxes, etc.) are set to 0.007 ($\eta = 0.007$).

Financial Markets  To match values in the U.S. over the period 2003–2005, the real risk-free rate is set to 1%, and the mortgage origination cost is set to 0.4%. The mortgage servicing cost $\phi$ is set to generate a 2.5% spread between the real mortgage rate and risk-free rate. The minimum payment is calibrated so that households can rollover their mortgage debt by paying interest only ($\chi = 0$). Lastly, the exogenous LTV limit is set to 125%, which makes it non-binding in the steady state.\(^6\)

3.2 Joint calibration and model fit

The remaining parameters to be calibrated are the discount factor $\beta$, the share of housing in the utility function $\phi_h$, the intratemporal elasticity of substitution $\gamma_h$, matching function elasticity $\gamma$, real estate market entry fee $\kappa$, size of the largest rental unit, smallest house size, utility cost of foreclosure $\xi_f$, and the efficiency loss that accrues to banks ($\gamma^{REO}$) are determined jointly in the calibration. The calibration targets various moments in the data that we explain below.

First, we target selected household portfolio moments calculated from the 2004 Survey of Consumer Finances (SCF) for prime-age households. Specifically, the calibration aims to match a homeownership rate of 62.7%, a median net worth of 1.06 (in terms of annual mean income), a mean value of housing (in terms of annual median income) of 3.37, a mean mortgage debt (in terms of annual median income) of 1.87, and a median

\(^6\)At the peak of the housing boom in 2005, the popularity of cash-out refinancing led to many instances of new mortgages with loan-to-value ratios in excess of 100%. The foreclosure penalty and the REO discount $\gamma^{REO}$ are determined in the joint calibration.
mortgage debt (in terms of annual median income) of 1.55. In addition, we also target a host of moments about mortgage holders. More specifically, we target the fraction of homeowners that have a mortgage (82%), median loan-to-value ratio (LTV), the fraction of mortgage holders with an LTV larger than 70%, 80%, 90%, and 95%. We also target a quarterly foreclosure rate of 0.4%. Lastly, we target a set of moments about the housing market. More specifically, we target the (mean) time it takes to buy a house, the time it takes to sell a house, and the fraction of the housing stock that is transacted every quarter. These set of moments are obtained from the National Association of Realtors. The calibration minimizes the percentage deviation from these moments and their model counterparts. The values of the targeted moments and the model’s fit are reported in table I. Values of parameters that are set outside the model are shown in table II, and the values of internally calibrated parameters are reported in table III.

4 Steady-state properties of the calibrated model

We start by investigating the steady state properties of the calibrated model. We focus on two aspects of our model that are important for the transmission of monetary policy. The first is about the liquidity of the housing market induced by search frictions
Table II – Externally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Interpretation</th>
<th>Value(s)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>Income process</td>
<td>see text</td>
<td>Guvenen et al. (2016)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Frisch elasticity</td>
<td>0.33</td>
<td>Chetty (2008)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Mortgage servicing cost</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>Mortgage initiation cost</td>
<td>0.4%</td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>Maximum LTV</td>
<td>125%</td>
<td></td>
</tr>
<tr>
<td>$\phi_T$</td>
<td>Taylor rule coefficient</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax rate</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Government spending (quarterly)</td>
<td>0.0425</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table reports the values for the parameters that are jointly calibrated within the model.
### Table III – Internally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Interpretation</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>Taste for housing</td>
<td>0.4244</td>
</tr>
<tr>
<td>$\gamma_h$</td>
<td>Elasticity of substitution $c, h$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>Elasticity of matching fnc. in selling market</td>
<td>0.8922</td>
</tr>
<tr>
<td>$\lambda_b$</td>
<td>Elasticity of matching fnc. in buying market</td>
<td>0.0894</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>Minimum house price that sells w. prob 1</td>
<td>0.7538</td>
</tr>
<tr>
<td>$h$</td>
<td>Size of smallest house</td>
<td>2.9486</td>
</tr>
<tr>
<td>$h_r$</td>
<td>Size of largest rental apartment</td>
<td>2.4287</td>
</tr>
<tr>
<td>$\xi_F$</td>
<td>Utility cost of foreclosure</td>
<td>0.0153</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency loss due to foreclosure</td>
<td>1.53%</td>
</tr>
</tbody>
</table>

Note: This table reports the values for the parameters that are jointly calibrated within the model.
and how this liquidity affects consumption. The second is about the distribution of consumption propensities of households.

4.1 Housing illiquidity and consumption behavior

The illiquidity of the housing market induced by search frictions has several effects on the behavior of consumption at the individual level. First, illiquidity creates selling risk for homeowners, and in particular for homeowners with substantial outstanding mortgage debt. Panel 1 of figure 1 displays the optimal listing price as a function of leverage for a homeowner with zero liquid assets, a low level of income so that cash at hand is about one quarter’s income, and a median sized house (worth 2.9 times annual income) and panel 2 shows the corresponding expected time to sell. This homeowner has a motive for selling and downsizing to smooth consumption. In case of an unsuccessful search for a buyer, the homeowner has the option to go to the bank, obtain a new mortgage, and extract equity, the value of which crucially depends on the outstanding mortgage debt. Panel 1 of figure 1 shows that, for low levels of mortgage debt (leverage ratio below 70%), the equity cushion allows this household to be patient and look for a good price for the house.

However, as leverage rises close to 75%-80%, the default risk rises (shown on panel 3), which manifests itself in the rising mortgage premium (panel 4), reducing the option value of going to the banks and asking to refinance. As a result, these homeowners become distressed sellers who optimally choose lower list prices (“fire sales”) in search of liquidity. For homeowners with higher debt values (above 80% leverage), the constraint \( x_s + y \geq \frac{m}{1+i} \) starts binding. As their equity cushion is non-existent, these debt-constrained sellers don’t have the flexibility to price their house to sell quickly and end up having to list their unit at high prices. These homeowners experience debt overhang as their houses take longer to sell. Quantitatively, time on the market can exceed one year for the most indebted homeowners.

In fact, these debt-induced selling delays have ramifications for mortgage default behavior, the supply of credit, and ultimately, for consumption behavior. By increasing the risk that a financially distressed and indebted homeowner fails to sell their house, selling delays spill over into higher foreclosure risk. Panel 3 shows a heat map for mortgage default as a function of leverage and cash at hand. In standard Walrasian models of housing, only the “negative equity default” region exists, because having negative equity is a necessary condition for default. Intuitively, as long as the market clearing price is
sufficiently high to repay the mortgage, owners with even a sliver of equity can always instantly sell their house to avoid default. However, longer endogenous selling delays in the model with housing search frictions have a direct impact on foreclosure activity. In short, having equity on paper is less relevant than being able to actualize that effort by selling at a particular price in a short time window.

As a result, endogenous housing illiquidity creates a new, graduated region of mortgage default, labeled as “illiquidity default” in panel 3. Notice that, depending on an owner’s asset position, even having 6%, 10%, or 15% equity does not inoculate owners against the risk of default. This behavior is consistent with empirical evidence on subprime mortgage data that finds repossession rates of 50% for delinquent mortgages with LTV ratios between 80% - 90% and 55% for delinquent mortgages with LTV ratios between 90% - 100%. In a Walrasian world, one would expect 0% repossession rates for such mortgages.

The heightened foreclosure risk that arises from housing illiquidity has a substantial effect on the supply of mortgage credit. Panel 4 of figure 1 plots sample default premia for new loans as a function of leverage. For clarity, a 10% default premium adds 10% to the cost of a loan over its lifetime. Default premia in the baseline economy with illiquid housing considerably exceed those in the Walrasian economy. This link between housing illiquidity and the availability of mortgage credit creates a powerful channel in the dynamic economy. If a shock hits the economy and drives down house prices, housing illiquidity will deteriorate endogenously and create longer selling delays. In
Figure 2 – Simulations over time of different individual homeowners who (1) pay down their mortgage; (2) use their equity cushion to cut their price; (3) engage in “distressed borrowing,” raise their price, experience selling delays, and then default.

Figure 2 plots some individual-level simulations to illustrate the mechanisms described above. In each case, we simulate the behavior of a homeowner who receives a constant stream of income. Panel 1 shows the path of mortgage leverage over time for a typical homeowner who does not experience any financial difficulty. Panel 2 shows the dynamic selling behavior of a homeowner who receives a sequence of low income realizations. This “distressed seller” lacks access to credit to extract equity but has sufficient equity to gradually lower the list price as financial duress intensifies. Lastly, panel 3 plots an example of “distressed borrowing” followed by debt overhang and default.

The illiquidity of housing, in conjunction with endogenous credit supply, generates substantial deviations in consumption behavior from standard incomplete markets models that assume households can costlessly access all of their wealth to smooth consumption. By explicitly modeling these important features of housing and mortgage markets, we allow monetary policy to affect consumption and output through changes in the liquidity of the housing market.

4.2 MPC Heterogeneity and household leverage

As shown in Kaplan et al. (2016b), the marginal propensity to consume out of liquid wealth and its distribution in the economy is important for the transmission of monetary policy. In our setup, housing is an illiquid asset that is hard to adjust due to various reasons discussed above. This feature generates a nontrivial fraction of homeowners that...
act as hand-to-mouth consumers, thereby allowing our model to generate a rich distribution of consumption propensities across households. Figure 3 plots the distribution of marginal propensities to consume, and it is evident that a non-trivial percentage of households respond strongly to changes in income.

To illustrate how the model is able to generate this rich heterogeneity, we now show that the MPC is systematically related to mortgage debt. Furthermore, we estimate empirically how the MPC is linked to mortgage debt and compare the model to the data. This comparison allows us to test an untargeted feature of the data. We first start by describing how we estimate the empirical relationship.

![Figure 3 - The distribution of marginal propensities to consume.](image)

### 4.2.1 MPC and leverage in the data

In this section, we estimate the consumption response of highly leveraged households to transitory income shocks. We show evidence that, consistent with the model presented in the next section, such households have a larger marginal propensity to consume out of transitory income shocks. We also estimate the elasticity of consumption with respect to house prices.

**Data and sample** We use data from the Panel Study of Income Dynamics, which is a panel of households and, starting in 1999, contains information on income, consumption and wealth. The PSID started collecting information on a sample of about 5,000 households in 1968. These families along with their split-offs have been interviewed annually.
until 1996. Starting in 1996, the survey became biennial. Further, the information collected about consumption expenditures was enriched to cover about 70 percent of all consumption items available in the Consumer Expenditure Survey (CEX).

From the PSID Core sample, we drop households with missing race, education or state of residence information. We also drop those whose annual income is below $100 and changes by more than a certain threshold.\(^7\) We also drop households with top-coded income and consumption. Our identification, as we discuss below, requires us to have at least three consecutive observations on each household. Therefore, we also drop those that do not satisfy this criterion. Our baseline sample consists of 25 to 60 year olds. Our sample and its construction closely mimics that of Kaplan et al. (2014).\(^8\)

Our consumption measure follows Blundell et al. (2012) and includes food at home, food away from home, utilities, gasoline, car maintenance, public transportation, child-care, health expenditures, and education. Household income is measured as the sum of labor earnings of the household and government transfers. The categorization of wealthy and poor hand to mouth households follows Kaplan et al. (2014), therefore we refer the reader to this paper. The PSID contains questions about the value of home equity as well as a subjective assessment of the value of the house. These questions allow us to construct measures of home equity ratios.

**Methodology** To estimate the consumption response to transitory income shocks, we follow the methodology of Blundell et al. (2008). Here, we provide an overview of the methodology and refer the reader to Blundell et al. (2008) for a detailed description.

First, we strip log consumption and log income of observable differences and obtain residuals. To this end, we regress log consumption and log income on a full set of year and cohort dummies, college dummy, race dummies, employment, geographic variables, and allow education, race, employment, and region to have year-specific effects. We then construct changes of residuals of log consumption and log income, \(\Delta_c_{it}\) and \(\Delta y_{it}\), respectively.

We postulate that log income comprises of a permanent component \(z_{it}\), and a transitory component, \(\varepsilon_{it}\):

\[
z_{it} = z_{it-1} + \eta_{it}
\]

\(^7\)More specifically, we drop households whose income grows by more than 500 percent or declines by more than 80 percent.

\(^8\)We are grateful to the authors for sharing their dataset.
With this structure, the change in log income is given by

\[ \Delta y_{it} = \eta_{it} + \Delta \varepsilon_{it} \]

The true marginal propensity to consume is given by

\[ MPC_i = \frac{cov(\Delta c_{it}, \varepsilon_{it})}{var(\varepsilon_{it})} \]

We use the following estimator of the transmission of transitory income shocks to consumption derived in Blundell et al. (2008):

\[ MPC_i = \frac{cov(\Delta c_{it}, \Delta y_{i,t})}{cov(\Delta y_{it}, \Delta y_{i,t+1})} \]

We implement this estimator with an IV regression of \( \Delta c_{it} \) on \( \Delta y_{it} \), with \( \Delta y_{i,t+1} \) as an instrument. The main idea behind this instrument is that \( \Delta y_{i,t+1} \) is correlated with the transitory shock at time \( t \), but is uncorrelated with the permanent shock. Kaplan and Violante (2010) study the robustness of this to the presence of borrowing constraints and find that tight borrowing constraints do not bias the estimate of the transmission coefficient. All regressions use PSID weights.

**Results** We are mainly interested in understanding differences between households with different leverage ratios. Using the information in the PSID on self-reported house values and housing equity, we construct household-level measures of leverage ratio. We classify households into two groups, depending on this measure: Households with a leverage ratio above 85 percent are labeled high leverage and those with below this threshold are labeled low leverage households.

The first column in table IV shows the results for our baseline sample. We estimate an elasticity of consumption with respect to income shocks of 0.27 for high leverage households. The corresponding figure for low leverage households is considerably lower at 0.19. This difference is economically (and statistically) sizable. It suggests that the distribution of housing debt and wealth in the economy could be an important determinant of the economy’s response to shocks. In this paper, we propose a model that is consistent with this form of MPC heterogeneity and then study the response of the economy to monetary policy shocks with different distributions of wealth.

We now investigate the robustness of this main result. One simple explanation for
our main finding is that high leverage households have little liquid wealth to smooth transitory income fluctuations. In column 2, we restrict our sample to only such households and find the high leverage households within this group to have higher MPCs.\(^9\)

Finally, the last column repeats the same estimation for a sample of wealthy hand to mouth households; i.e. those that have a net worth about a certain threshold but little wealth in liquid assets.

Lastly, we estimate consumption elasticity to house price changes. We estimate the following equation:

\[ \Delta c_{it} = \alpha + \beta X_{it} + \delta \Delta h_{it} + \xi_{it} \]  

(4)

Column (1) in table V estimates equation (4) without additional controls. The estimates suggest that the elasticity of nondurable consumption with respect to house prices is around 0.2. Column (2) controls for gender, a full set of age controls and education dummies, and shows that the resulting elasticity is essentially the same.

Our main findings in this section can be summarized as follows: i) Household leverage ratios are an important determinant of how much consumption responds to transitory income changes, with highly-levered households having the largest responses, ii) as estimated by a body of literature, consumption responds strongly to changes in house prices, with an elasticity around 0.2. We now turn to the model counterparts of these objects and measure them in the model.

\(^9\)The classification of hand to mouth households in the PSID follows Kaplan et al. (2014).
### 4.2.2 MPC and leverage in the model

We initialize the economy to its steady state and simulate a sample of 100,000 individuals for 500 periods. We separate the sample of homeowners in the model into two groups as those with below and above 85% LTV ratio. Note that a period in the model is a quarter, whereas data is measured every other year and asks about annual measures of income and consumption. To mimic the empirical setup as close as possible, we aggregate quarterly measures to annual frequency. We then throw out every other year to mimic the biannual nature of the PSID data. Finally, for each group of households, we compute the average MPC following the same methodology used in estimating its empirical counterpart.

Table VI compares the model generated MPCs against their empirical counterparts. The model generates substantial differences in the MPCs between these groups, consistent with the data. In particular, households with a high LTV ratio respond about two times stronger than low LTV households.

#### Table VI – Model vs. Data: MPC Heterogeneity by leverage ratio

<table>
<thead>
<tr>
<th></th>
<th>Data (All Sample)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>High LTV (≥ 0.85)</td>
<td>0.27</td>
<td>0.17</td>
</tr>
<tr>
<td>Low LTV (&lt; 0.85)</td>
<td>0.19</td>
<td>0.07</td>
</tr>
</tbody>
</table>

### 5 Distributional Consequences of Monetary Policy

The effects of a shock to monetary policy, and the subsequent adjustments in taxes and transfers and the equilibrium responses of hours and prices operate through various distributional channels in addition to the direct effect of the change in the nominal interest rate. An increase in the price level and of labor income leads to a redistribution from households who finance their consumption more from asset income to households who rely more on labor income. Changes in interest rates redistribute between debtors and lenders. Increases in house prices redistributes towards home owners.

These redistributions matter due to the endogenous heterogeneity in the MPCs in the data and in our model. This heterogeneity together with the redistribution determines the aggregate consumption response, and since output is demand determined due to price rigidities, also determines output. Individual household consumption $c_t$ depends on
transfers $T_t$, tax rates $\tau_t$, labor income $w_t l_t$, product prices $P_t$, house prices $p^h_t$, mortgage prices $q^m_t$ and nominal interest rates $\bar{i}_t$, so that aggregate private consumption

$$C_t(\{T_t, \tau_t, w_t l_t, P_t, p^h_t i_t, q^m_t\}_{t \geq 0}) = \int c_l(b, h, m, s; \{T_t, \tau_t, w_t l_t, P_t, p^h_t i_t, q^m_t\}_{t \geq 0}) d\Omega_t.$$ 

In our model, households household labor supply depends both on the preferences of household and total aggregate demand. In equilibrium consumption and hours will be determined jointly, but to understand the demand response of a monetary policy shock it turns out to be useful to consider $w_t l_t$ as exogenous for consumption decisions here. In particular it allows us to distinguish between the initial impact, “first round”, demand impulse due to the shock and “second, third ... round” due to equilibrium responses. Those arise in our model since an initial expansionary policy shock leads to more employment by firms, and so higher labor income and higher house prices, which in turn implies more consumption demand, which again leads to more employment and so on until an equilibrium is reached where all variables are mutually consistent. Denoting pre-shock variables by a bar, we can now decompose the aggregate consumption response,

$$(\Delta C)_t = C_t(\{T_t, \tau_t, w_t l_t, P_t, p^h_t i_t, q^m_t\}_{t \geq 0}) - C_t(\{\bar{T}_t, \bar{\tau}, \bar{w} \bar{l}, \bar{P}, \bar{p}^h_t, \bar{i}, \bar{q}^m_t\}_{t \geq 0})$$

into its different channels:

$$(\Delta C)_t = C_t(\{\bar{T}_t, \bar{\tau}, \bar{w} \bar{l}, \bar{P}, \bar{p}^h_t, \bar{i}, \bar{q}^m_t\}_{t \geq 0}) - C_t(\{\bar{T}_t, \bar{\tau}, \bar{w} \bar{l}, \bar{P}, \bar{p}^h_t, \bar{i}, \bar{q}^m_t\}_{t \geq 0}) + \]

Direct Impact of Interest Rate

$$C_t(\{T_t, \tau_t, w_t l_t, P_t, p^h_t i_t, q^m_t\}_{t \geq 0}) - C_t(\{\bar{T}_t, \bar{\tau}, \bar{w} \bar{l}, \bar{P}, \bar{p}^h_t, \bar{i}, \bar{q}^m_t\}_{t \geq 0}) + \]

Indirect Equilibrium Effect: Labor Income

$$C_t(\{T_t, \tau_t, w_t l_t, P_t, p^h_t i_t, q^m_t\}_{t \geq 0}) - C_t(\{\bar{T}_t, \bar{\tau}, \bar{w} \bar{l}, \bar{P}, \bar{p}^h_t, \bar{i}, \bar{q}^m_t\}_{t \geq 0}) + \]

Indirect Equilibrium Effect: House Prices

$$C_t(\{T_t, \tau_t, w_t l_t, P_t, p^h_t i_t, q^m_t\}_{t \geq 0}) - C_t(\{\bar{T}_t, \bar{\tau}, \bar{w} \bar{l}, \bar{P}, \bar{p}^h_t, \bar{i}, \bar{q}^m_t\}_{t \geq 0}) + \]

Indirect Equilibrium Effect: Taxes and Transfers

$$C_t(\{T_t, \tau_t, w_t l_t, P_t, p^h_t i_t, q^m_t\}_{t \geq 0}) - C_t(\{\bar{T}_t, \bar{\tau}, \bar{w} \bar{l}, \bar{P}, \bar{p}^h_t, \bar{i}, \bar{q}^m_t\}_{t \geq 0})$$

Price and Mortgage rate Adjustment
6 Effectiveness of monetary policy and the role of housing and heterogeneity

We now study the effectiveness of monetary policy in our setting and focus on the role of housing in transmitting policy shocks. We first start with a contractionary shock and then turn to an expansionary one. We assume the economy is at its steady state, following a Taylor rule with $\epsilon_t = 0$, $t < 1$, where $\epsilon_t$ is the innovation to the Taylor rule assumed to follow an AR(1) process with persistence $\rho\epsilon = 0.6$.

$$\epsilon_t = \rho \epsilon_{t-1} + \eta_t$$

In period 1, this economy is hit by a one-time unexpected monetary policy shock so that the nominal rate goes up by 100 basis points. The nominal rate then recovers according to the dynamics dictated by the Taylor rule. We compute the perfect foresight transition of the economy in response to this shock.

Figure 4 shows the response of various variables. Similar to representative agent New Keynesian models, a fall in the nominal rate in the model generates a fall in inflation and real wages, a rise in the real rate, and a contraction in consumption. There are various channels that account for the fall in aggregate consumption. The first is the usual New Keynesian channel that amplifies declines in consumption coming through the intertemporal substitution channel. Note that, in addition to this standard transmission channel of monetary policy, housing market plays a rich and important role, which we explain below.

First, the increase in the nominal rate increases the financing costs of houses, which puts a downward pressure on house prices. The decline in house prices then generates a decline in nondurable consumption for homeowners, because it restricts the equity cushion that homeowners occasionally tap into (“the collateral channel”). The second channel is the so-called cash-flow channel. The fact that all mortgage contracts have adjustable rates implies that an increase in the nominal rate maps into an increase in financing costs. Lastly, the decline in house prices, along with an increase in the average time it takes to sell a house, increases default risk, and through a precautionary motive contributes to the contraction of consumption.

Note that the various mechanisms embedded in the model have different implications
Figure 4 – Contractionary policy shock
across the LTV distribution. For example, the collateral channel implies that people with more equity (i.e. lower LTV) are more affected, whereas the cash-flow channel implies the opposite. To better understand which force is stronger, figure 5 plots the consumption response for three LTV groups. We find that the high LTV households respond the strongest to a contractionary monetary policy shock.

**Figure 5 – Heterogeneous effects of monetary policy**

While it is a nontrivial task to decompose the quantitative magnitudes of all the various interplays of housing and monetary policy, we conduct a simple exercise to quantify the role of house prices. Simply put, we ask by how much would have consumption moved, had house prices remained fixed. To get at this, we exogenously fix house prices at the steady state values and resolve the model. Figure 6 shows the response of aggregate consumption under this setting. We find that the endogenous response of house prices accounts for about 20 percent of the response of aggregate consumption.
6.1 Asymmetric effects of monetary policy

One advantage of our model (and of our solution technique) is to allow for nonlinear dynamics. This allows us to contrast the response of the economy to contractionary and expansionary shocks. We compute the transition path of the economy in response to a 100bp expansionary shock. Figure 7 contrasts the response of consumption and house prices to the contractionary case.

We find important asymmetries: For example, the top left panel in figure 7 shows that consumption increases by about 20% less in response to an expansionary shock than it falls in response to a contractionary shock of the same size. A similar asymmetry exists for house prices (top right panel) and foreclosures (bottom right panel).

What explains these asymmetries? There are several forces through which this happens, but to facilitate understanding, we will explain them in two steps. The first is that the heterogeneity in the joint distribution of liquid and housing wealth generates an asymmetric response in the demand for housing, which then generates an asymmetric
Asymmetric effects of monetary policy

Response in house prices. Consider a decline in the nominal interest rate. This decline stimulates the demand for housing due to falling financing costs. If renters are liquidity constrained, they won’t be able to obtain decent mortgages, which limits their ability to take advantage of lower rates. Similarly, existing homeowners that might be interested in upgrading to larger units may be constrained by existing mortgage debt if they have high leverage ratios. Thus, the joint distribution of liquid and housing net wealth affects how much housing demand (and thus house prices) respond to changes in interest rates. A similar argument can be made for why foreclosure rates respond asymmetrically (bottom right panel of figure 7). In short, the precise shape of this distribution generates an asymmetric response in house prices, which then generates an asymmetric consumption response through the collateral channel.

However, even fixing house prices, changes in consumption are asymmetric. This is
because the extent of transmission from housing wealth to consumption also depends on mortgage debt. As we have shown above, MPC out of net worth is much higher for high debt households. Figure 8 shows the shifts in the LTV distribution under contractionary and expansionary shocks. Changes in the LTV distribution, coupled with the heterogeneity in the MPCs generates an asymmetric response of consumption to house price changes.

These discussions highlight the role of heterogeneity in the transmission mechanism and the effectiveness of monetary policy.

6.2 The role of heterogeneity [in progress]

Given the role of heterogeneity in the transmission of monetary policy, we ask if standard monetary policy would be more or less effective in an economy with lower mortgage debt. To answer this question, we conduct a simple experiment and reduce the maximum LTV households are allowed to take from 125%, which is not binding
in the calibration, to 85%. This generates nontrivial changes to the LTV distribution. We then compare the effect of an expansionary monetary policy in this economy to the original calibration. Figure 9 shows the response of consumption and house prices. Interestingly, we find that the response of house prices is nearly identical across the two economies, whereas the consumption response is drastically higher in the high LTV economy (benchmark). This result is consistent with the previous explanation that the cash-flow channel is quantitatively very important in the transmission of monetary policy. A decline in mortgage financing costs generates a relaxation in the budget constraint of households in both economies. However, these changes in the cash flow map into much larger consumption responses in the high LTV economy because of the higher average MPC in this economy.

7 Conclusion

In this paper we study the role of the joint distribution of housing and mortgage debt in the transmission of monetary policy using a quantitative heterogeneous agents New Keynesian model with a frictional housing market and and long-term nominal mortgates. In addition to the direct intertemporal substitution effect of monetary policy on consumption, our model allows for various indirect mechanisms through housing and mortgages. We calibrate this model to match the US macroeconomic statistics from over the past twenty years, specifically, the joint distribution of assets, housing wealth, and mortgage debt as well as key housing moments related to sales, time on the market, and
foreclosures. The model is able to capture the rich heterogeneity in MPCs between low- and high-LTV households observed in the data.

We then use this model to study the transmission of the monetary policy. We, first, find that decline in house prices explain 20% of the drop in aggregate consumption against a contractionary monetary shock. Second, in our model contractionary monetary shocks have larger effects on aggregate demand than expansionary shocks. This is mostly because house prices and foreclosures respond to contractionary shocks more due to the shape of the leverage distribution: When rates do down, this relaxes fewer households’ budget constraint than the number of households that fall into trouble with their mortgages when rates go up. Finally, we investigate how the effectiveness of the monetary policy depends on the distribution of mortgage debt and find that expansionary monetary policy is more effective in an high-LTV economy. These results highlight the role of heterogeneity in balance sheets of households in the transmission mechanism and the effectiveness of monetary policy.
References


A Details of the model

This section presents the technical details that were omitted in the main text.

Value to banks of repossessing a house  The value to a lender of repossessing a house of size $h$ is

\[
J^t_{\text{REO}}(h) = R^t_{\text{REO}}(h) = \max \left\{ 0, \max_{x_s \geq 0} \tilde{p}_t(\theta_t(x_s, h)) \left[ (1 - \gamma^{\text{REO}}) x_s - \left( -\eta h + \frac{1}{1 + r_{t+1}} J^{t+1}_{\text{REO}}(h) \right) \right] \right\}
\]

(5)

where $\eta$ is the cost of holding onto the house (maintenance, property taxes, etc.) and $R^t_{\text{REO}}(h)$ is the option value of trying to sell the house in period $t$.

Mortgage pricing  Mortgage prices for the amount of $M_{t+1}$ for the household with risk-free saving $b_{t+1}$, house size $h_t$, and idiosyncratic labor productivity $z_t$ satisfy the following recursive relationship:

\[
q_{mt}(M_{t+1}, b_{t+1}, h_t, z_t) M_{t+1} = \frac{1}{1 + r_{mt}} \mathbb{E} \left\{ \begin{array}{l}
\text{sell + repay} \\
\text{no sale (do not try/fail)} \\
\text{default + repossession} \\
\text{borrower payment net of servicing costs} \\
\text{continuation value of new } M''
\end{array} \right\}
\]

(6)

where $P_{t+1}$ is the price level and $x_{st+1}, d_{t+1}, b_{t+2}$, and $M_{t+2}$ are the policy functions for list price, mortgage default ($\in \{0, 1\}$), bonds, and new mortgage balance next period, respectively. At origination, $q^0_{mt} = \frac{1}{1 + r_{mt}} q_{mt}$. If the borrower never sells or defaults, mortgage prices in the steady state reduce to $q_{mt}(M_{t+1}, b_{t+1}, h_t, z_t) = \frac{1}{1 + r_{mt}}$.

Balance sheet of the GSEs  The balance sheet of the GSEs is given by:
\[ B_{t+1}^m + T_{GSE}^t + \int \left[ \begin{aligned} & \text{Refi}(b_t, M_t, h_t, z_t)M_t \\ & + \text{Sell}(b_t, M_t, h_t, z_t)M_t \\ & + \text{Fore}(b_t, M_t, h_t, z_t)P_{t+1}J_{REO}(h_t) \\ & + \text{Pay}(b_t, M_t, h_t, z_t)(M_t - M_{t+1}(b_t, M_t, h_t, z_t)) \end{aligned} \right] d\Phi^t_{OWN} \]

\[ (1 + r)B_t^m + \int \left[ \begin{aligned} & \text{Refi} \times q^0_{mt}(M_{t+1}, b_{t+1}, h_{t+1}, z_{t+1})M_{t+1} \\ & + \text{Sell} \times \text{Buy} \times q^0_{mt}(M_{t+1}, b_{t+1}, h_{t+1}, z_{t+1})M_{t+1} \\ & + \text{Buy} \times q^0_{mt}(M_{t+1}, b_{t+1}, h_{t+1}, z_{t+1})M_{t+1} \\ & + \int \text{Buy} \times q^0_{mt}(M_{t+1}, b_{t+1}, h_{t+1}, z_{t+1})M_{t+1}d\Phi^t_{RENT} \end{aligned} \right] d\Phi^t_{OWN} \]

where \( B_t^m \) are the MBS issued by the GSE and \( T_{GSE}^t \) are transfers from the government in period \( t \). \( \text{Refi} \), \( \text{Fore} \), and \( \text{Pay} \) denote the indicator functions for households who refinanced, foreclosed, and pre-paid their mortgages in period \( t \), respectively. Similarly, \( \text{Sell} \) and \( \text{Buy} \) denote the indicator functions for households who successfully sold and bought their houses in period \( t \), respectively.

**Household’s Dynamic Problem**  Households take the paths of prices \( \{P_t, w_t, p^H_t, r_{ht}, r_{mt}, r_t\}_{t \geq 0} \), mortgage price functions \( \{q_{mt}(.)\}_{t \geq 0} \), lumpsum transfers \( \{T_t\}_{t \geq 0} \) as given. Household dynamic problem can be summarized by the timeline shown in Figure 10.

Value function of a household who owns a house in the beginning of the period and has already decided to keep it in her portfolio involves decisions of consumption, labor supply, bond holdings, refinance, and mortgage debt payment:

\[ V^t_{OWN}(a_t, M_t, h_t, z_t) = \max_{M_{t+1}, b_{t+1}, c_{t, t+1}, l_0} \left[ \begin{array}{l} u(c_t, h_t, l_t) + \beta_t \mathbb{E} [V^{t+1}_{OWN}(a_{t+1}, M_{t+1}, h_{t+1}, z_{t+1})] \end{array} \right] \]

subject to

\[ P_t c_t + q^B_t b_{t+1} + M_t \leq a_t + \frac{M_{t+1}}{1 + r_{mt}} \text{ if } M_{t+1} \leq (1 - \chi)M_t \]

\[ P_t c_t + q^B_t b_{t+1} + M_t \leq a_t + q^0_{mt} M_{t+1} \text{ if } M_{t+1} > (1 - \chi)M_t \]

\[ a_{t+1} = P_{t+1} w_{t+1} z_{t+1} l_{t+1} + b_{t+1} \]

\[ h_{t+1} = h_t \]
Value function of a household who buys a house:

\[
V^t_{\text{Buy}}(a_t, z_t) = \max_{h_t, M_{t+1}, t_{t+1}, c_t, l_t \geq 0} u(c_t, h_t, l_t) + \beta_L \mathbb{E} \left[ V^{t+1}_{\text{OWN}}(a_{t+1}, M_{t+1}, h_{t+1}, z_{t+1}) \right]
\]

subject to

\[
P_t c_t + q^B_t b_{t+1} + p_t h_t \leq a_t + q^0_{mt} M_{t+1}
\]

\[
a_{t+1} = P_{t+1} w_{t+1} z_{t+1} l_{t+1} + b_{t+1}
\]

Value function of a household who rents:

\[
V^t_{\text{Rent}}(a_t, z_t) = \max_{b_{t+1}, s_t, c_t, l_t \geq 0} u(c_t, h_t, l_t) + \beta_L \mathbb{E} \left[ V^{t+1}_{\text{NO Own}}(a_{t+1}, z_{t+1}) \right]
\]

subject to

\[
P_t c_t + q^B_t b_{t+1} + P_t r_t s_t \leq a_t
\]

\[
s_t \leq \bar{s}
\]

\[
a_{t+1} = P_{t+1} w_{t+1} z_{t+1} l_{t+1} + b_{t+1}
\]

Default decision (when household defaults it is always foreclosed and banks do not
come after liquid assets of the household):

$$V_{NSell}^t(a_t, M_t, h_t, z_t) = \max_{d_t \in \{0,1\}} d_t(V_{Rent}^t(a_t, z_t) - \chi_f) + (1 - d_t)V_{OO}^t(a_t, M_t, h_t, z_t)$$

Buying decision a household who doesn’t own a house (either recently sold or never owned):

$$V_{NOwn}^t(a_t, z_t) = \max_{Buy_t \in \{0,1\}} Buy_tV_{Buy}^t(a_t, z_t) + (1 - Buy_t)V_{Rent}^t(a_t, z_t)$$

Choosing a list price $x_s$ for a household who wants to sell her house:

$$V_{SELL}^t(a_t, M_t, h_t, z_t) = \max_{x_{st}} \xi + \tilde{p}_t(\theta_t(x_s, h))V_{NOwn}^t(a_t + x_{st} - M_t, z_t)$$

$$+ \tilde{p}_t(\theta_t(x_s, h))V_{NSell}^t(a_t, M_t, h_t, z_t)$$

Selling decision of a household which defines $V_{OWN}$:

$$V_{OWN}^t(a_t, M_t, h_t, z_t) = \max_{\sigma_t \in \{0,1\}} \sigma_tV_{SELL}^t(a_t, M_t, h_t, z_t) + (1 - \sigma_t)V_{NSell}^t(a_t, M_t, h_t, z_t)$$