

Housing Finance, Boom-Bust Episodes, and the Macroeconomy*

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Abstract

This paper analyzes how arrangements in the in the mortgage market impact the dynamics of housing (boom-bust episodes) and the economy using a structural equilibrium model with incomplete markets and endogenous adjustment costs. In response to mortgage rates and credit conditions, the model can generate movements in house prices, residential investment, and homeownership consistent with the U.S. housing boom-bust. The propagation to the macroeconomy is asymmetric with much higher consumption sensitivity during the bust than the boom due to the endogenous fragility caused by mortgage debt. Mortgages with adjustable-rate increase the sensitivity of house prices to credit conditions relative to an economy with fixed-rate loans without refinancing. Macro prudential policies can mitigate fragility by reducing the magnitude of house price movements without curtailing homeownership.

Keywords: Housing; Consumption; Liquidity; Debt; Great Recession

JEL Classification Numbers: D31, D83, E21, E22, G11, G12, G21

1 Introduction

Between 2002 and 2009, real house prices in the United States soared over 50 percent before collapsing. Homeownership followed a very similar pattern, with a large number of households entering the housing market during the boom and exiting during the bust. The

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increased access to owner-occupied housing was partially fueled by easy credit conditions in the mortgage market such as low mortgage rates and relaxed loan-to-values (LTV). These also allowed exiting homeowners to refinance their mortgage, extract home equity, and re-balance the household portfolio, or permit them to trade the existing housing unit for a different one. As a result, credit conditions not only affected new homebuyers, but also existing buyers as suggested by Greenspan and Kennedy (2007).

From a macroeconomic perspective, easy conditions in mortgage finance have a direct effect on residential investment, as new housing units are constructed when prices increase, and build-up mortgage debt as housing becomes more expensive. The expansion of mortgage debt not only increases the size of outstanding mortgage debt relative to household income, but also increases the fraction of households with high LTV making the economy more vulnerable or fragile to changes in aggregate conditions such as income/employment risk or the cost of borrowing. Under this conditions, a decline in house prices can generate episodes with sizeable decline in economic activity followed by a slow recovery as the demand for new housing is low, but also the demand for non-housing goods as the existing homeowners have to adjust their balance sheets to reduce the debt burden. Some of the households reduce their debt balances by defaulting on their mortgage obligations, making credit on new borrowers more expensive. This narrative suggests that recessions that include sizeable decline in house prices, after a period of debt buildup, can be deeper and more prolonged is consistent with the empirical work of Martin, Munyan, and Wilson (2015) that analyze the path of recovery of developed economies (i.e. United States, European Union, Japan, etc) depending on the magnitude of the recession or the type of shock.

This paper explores the contribution of housing finance and mortgage arrangements in driving the housing market (i.e. house prices, homeownership) and the macroeconomy during the boom-bust episodes using a general equilibrium model. In order to provide a meaningful answer, it is important to depart from the canonical macro housing model with complete markets, as in this framework the tenure decision (i.e. renting vs owning) is indetermined as well as the capital structure of the households balance sheet (i.e. mortgage debt vs. home equity). To overcome a Modigliani-Miller irrelevance on the contract structure in the household sector it is important to introduce some important frictions such as incomplete markets,

mortgage default, and endogenous adjustment costs. In the economy, there is a continuum of individuals that in the tradition of models with incomplete markets face uninsurable income risk. The individuals need to purchase consumption goods and housing services. While consumption goods are purchased in the market every period, housing can be rented each period or purchased as an investment good. Consistent with the evidence in the U.S., the units that can be rented each period come in smaller sizes than owner-occupied housing. As a result, households with more resources will need to purchase the large units to enjoy more services. The house purchase can be financed using a long-term mortgage collateralized loan with a default option. In the baseline economy, the mortgage loan has a fixed interest rate (FRM) determined at origination, but alternative arrangements that allow adjustable interest rates (ARM) are also considered. To distinguish the importance of downpayment constraints on new purchases from collateral constraints on existing loans it is necessary to introduce refinancing. The baseline mortgage arrangements allow households to refinance their mortgage and withdraw home equity but this is costly. In the presence of income shocks and no unsecured credit, the option to refinance provides an additional motive to own a house as it provides insurance against transitory income shocks. Everytime a loan is originated, in addition to the downpayment constraint, the borrowers face a payment-to-income constraint as in Greenwald (2016). This constraint ensures that the size of mortgage obligations does not exceed a fraction of the homeowners resources. The default option is priced by the lenders and depends on the individual risk at origination but also on the aggregate conditions on the housing market (i.e. the perspective to resale the reposed unit) . The housing market is subject to endogenous transaction costs formalized by a trading friction. As a result, the liquidity properties of the housing stock are endogenously determined allowing to capture extreme liquidity (or very low time in the market) during the peak of the housing market, and the illiquidity during the credit driven recessions generating an asymmetry between boom and bust.

The baseline version of the model is calibrated to replicate key features of the United States economy prior to the housing boom (circa 1998). The calibration puts heavy emphasis on matching key housing moments related to homeownership, sales, and foreclosures, but also important dimensions of the joint distribution of assets, housing wealth, and mortgage

debt. This allows to capture aggregate wealth in terms of financial assets and housing net of mortgages but also its distribution across households.

To evaluate the importance of housing finance and mortgage arrangements is necessary to shock the economy from the initial steady state. To generate movements in house prices the model is exposed to two series of unanticipated shocks on the real economy (i.e. productivity and income risk) and financial conditions (i.e. low mortgage rates and loose LTV and payment-to-income constraints). The initial shock displays a positive real shock and easy credit conditions in the mortgage market. Agents perceive these conditions as permanent, but they are again surprised by a reversal. Afterwards, the agents face a perfect foresight path.

In response to these shocks, the model can rationalize the performance of the housing market during the boom and the bust replicating the dynamics and magnitude of house prices, home ownership rates, housing defaults, and endogenous housing liquidity measured in terms of time-on-the-market (TOM). Analyzing this particular episode through the lens of the model provide some important lessons in terms of the quantitative importance of the various mechanism at play.

During the housing boom, the low mortgage rates, access to home equity and the ability to collateralize made homes a very attractive asset for many households that previously rented. Improvements in the mortgage market (i.e., lower mortgage rates and downpayment limits) drive all the income savings into housing as opposed to consumption. The collapse of the housing market wiped out the home equity of many homeowners, but also reduced the liquidity properties of the house. As a result, a significant number of households exited the owner-occupied housing market, via selling or defaulting, and had to adjust their consumption expenditures. Housing has favorable risk-sharing benefits in good times by allowing owners to extract equity through refinancing or selling, but it reverses when home equity and liquidity evaporate. This mechanism is the main driver of the asymmetric behavior of aggregate consumption dynamics, as aggregate consumption responds more strongly to house price movements during the crisis when equity extraction becomes more difficult/costly than during boom periods. During the housing bust, the model matches the consumption elasticity to house price movements as estimated by Mian, Rao and Sufi (2013).

In the baseline economy households use FRM with very low refinancing costs. This allows existing homeowners to take advantage of low mortgage without having to sell the house and/or withdraw equity during the housing boom. Eliminating the ability to refinance unables homeowners to exercise this option, and as a result dampens the size of the housing boom, but dramatically increases foreclosures and slows the recovery despite reducing the magnitude of the housing bust. This is an endogenous outcome as both economies are exposed to the same sequence of unanticipated shocks.

When the cost of refinancing FRM is low, there are minimal differences with ARM contracts during the boom episode as the passthrough of interest rates is very similar. To generate differences it is necessary to increase the cost of refinancing, hence reducing the fraction of homeowners that take advantage of lower rates in the FRM economy. In general, the presence of ARM contracts exposes homeowners to interest rate risk, therefore, recession with a tightening of interest rates exacerbate the crisis. This impact is particularly negative among homeowners with high-LTV.

The nature of long-term mortgages allows separating the stock of mortgage debt from the flow of new originations. Most macro models of housing assume one-period loans (see Piazzesi and Schneider, 2016 for a summary of the literature). A version of the model with one-period loans generates similar price dynamics but larger propagation to sales, foreclosures, and consumption during the bust as rollover costs spike. This arrangement forces everyone to refinance-rollover the loan, but pay higher premiums when default risk is high. This risk is sizeable for high-LTV owners during the crisis period.

Macro prudential policies are often advocated as a tool that can/should mitigate the macroeconomic impact of housing crises. The model suggests that tighter LTV requirements significantly dampen the boom and the bust. This policies are particularly effective when the initial mortgage rate is low suggesting that the optimal LTV should not be invariant to the underlying cost of borrowing as this policy operates by reducing the financial fragility of the economy. The model also highlights that tightening payment-to-income constraints can dampen the appreciation of house values without curtailing homeownership.

1.1 Related Literature

There is a growing literature that emphasizes the connection between the housing market and the macroeconomy. Some examples include Iacoviello (2005), Davis and Heathcote (2005), Leamer (2007). An extensive summary of the literature is provided by Davis and Van Nieuwerburgh (2015) and Piazzesi and Schneider (2016). While these papers measure the contribution of housing to the traditional business cycle, none of them specifically addresses the episode of the Great Recession.

One of the main challenges to understand this episode was the dramatic boom-bust in valuation of the housing stock and leverage cycle of mortgage debt. With this regard, traditional macroeconomic models of housing have serious challenges to replicate the observed patterns of prices and quantities during this episode. As a result, the majority of the research on the Great Recession is making advances by analyzing different aspects of this event.

To understand the dynamics of house prices during the boom and the bust Garriga, Manuelli, and Peralta-Alva (2012) develop a stylized macroeconomic model of market segmentation that generates sizable movement in house values, about 50 percent, driven by changes in housing finance. In their economy, the collapse of house prices, inducing a large and persistent recession through the deleveraging process and decline in non-housing consumption. This paper shares similar features in the process of engineering a housing crisis as unanticipated set of events, but the mechanisms are different allow the intensive and extensive margin of homeownership are considered. In addition, homeowners can choose to deleverage by repaying the loan or default. The choice of deleverage has important implications for the path the consumption of the homeowners during the boom and the bust.

One can interpret the decline in house prices as a shock to households net worth. There is also an extensive literature that analyzes the response of consumption to negative shocks in the balance sheet or income. For example, Iacoviello and Pavan (2013) argue that a tightening of households budget, due to the drop in real estate wealth, can generate a sharp decline in aggregate consumption. Huo and Ros-Rull (2016) also analyze this issue in an economy with a continuum of agents and frictions on the goods market. In their economy goods are produced in a market with frictions and as a result, a negative wealth effects

effectively reduces aggregate demand generating a significant decline in consumption and output. However, households can readjust their portfolios instantly without incurring a cost and the houses not subject to any form of transaction costs.

To amplify the response to shocks recently Kaplan and Violante (2014) have argued that in the presence of illiquid assets, the response of consumption to unanticipated shocks can be substantially larger. When households have a substantial fraction of their wealth tied up in an illiquid asset, they behave as wealthy hand-to-mouth agents with relatively high marginal propensities to consume. This sensitivity affects income shocks but also shocks to interest rate as discuss by Kaplan, Moll and Violante (2016). The notion of liquidity in these models is not tight to the macroeconomic performance, rather exogenous transaction costs. In this paper, a decline in the house price endogenously reduces the liquidity properties of some assets, in this case homes. This mechanism significantly amplifies the response of consumption to house price shocks.

There is an important literature that explores the increase in foreclosure dynamics during the Great Recession. To simplify the problem a number of papers consider an exogenous change in house prices to analyze the dynamics of defaults (i.e. Such as Guler (2014), Corbae and Quintin (2014), Campbell and Cocco (2014), and Hatchondo et. al. (2014)). Other papers endogenize both Garriga and Schlagenghauf (2009), Chatterjee and Eyigungor (2014), Arslan, Guler, and Temel (2015), but housing liquidity is exogenous.

The heterogeneity in the model has clear testable data implications. The ability of the model to match the empirical counterparts as suggested by the works of Mian, Rao, and Sufi (2013), Mian and Sufi (2014), Petev, Pistaferri, and Eksten (2011), and Parker and Vissing-Jorgensen (2009) among other is discussed in the results section.

2 The Model

2.1 Households

Households are infinitely lived and have preferences over consumption c and housing services c_h . Agents obtain housing services either as homeowners or apartment dwellers. Apartment

dwellers, or “renters,” purchase apartment space $a \leq \bar{a}$ and consume $c_h = a$ each period at a cost of r_a per unit. Agents become homeowners by purchasing a house $h \in H$ that generates $c_h = h$ housing services each period. The housing market is physically segmented, i.e. $\bar{a} < \underline{h}$. In other words, large units are only available for purchase.¹ Owners are not permitted to possess multiple houses or to have tenants.

Households supply a stochastic labor endowment $e \cdot s$ to the labor market. The persistent component $s \in S$ follows a Markov chain $\pi_s(s'|s)$, and households draw the transitory $e \in E \subset \mathbb{R}_+$ from the distribution $F(e)$.

2.2 Technology

The economy has a production sector for consumption goods and for houses. In the consumption sector, goods are produced according to a linear technology using labor, $Y_c = A_c N_c$.

A linear reversible technology converts consumption into apartment services at the rate A_a . Thus, apartment services have price $r_a = 1/A_a$.²

Builders construct new houses using land L , structures S_h , and labor N_h using a constant returns to scale technology $Y_h = F_h(L, S_h, N_h)$. Builders purchase structures S_h from the consumption sector, and as in Favilukis, Ludvigson and Van Nieuwerburgh (2016), the government supplies new permits $\bar{L} > 0$ each period and consumes the revenues. Houses depreciate with probability δ_h , and there are no construction delays. Thus, the end of period stock of housing H follows

$$H' = (1 - \delta_h)H + Y'_h.$$

2.3 Housing Market

Buyers and sellers of houses trade in a decentralized housing market and direct their search by house size and transaction price. Sellers of house $h \in H$ choose a list price p_s and face an equilibrium trade-off between higher prices and longer expected time on the market. Buyers who direct their search to house h and price p_b face an equilibrium trade-off between *lower*

¹This segmentation is consistent with the empirical evidence in the U.S. showing that the average rental unit is approximately half the size of the average owner-occupied unit.

²Sommer, Sullivan and Verbrugge (2013) and Davis, Lehnert and Martin (2008) report that rents have remained flat over the past 30 years, independent of house price swings.

prices and longer expected time searching. **Housing illiquidity** is reflected by the trade-off between price and trading probability and the presence of failures to trade.

In general, the presence of heterogeneous buyers and sellers (in terms of assets, income, and debt) with directed search creates an intractable dynamic sorting problem. To circumvent this issue, market makers, referred to here as real estate brokers, are introduced as a modeling device. These brokers intermediate trades by first matching with sellers, purchasing their houses, and then matching with buyers who purchase the houses. Brokers can frictionlessly trade houses with each other at cost $p(h) = ph$ and purchase newly built housing.³ Brokers do not have the ability to speculate against housing dynamics, as they are not permitted to hold onto housing inventories. The only inventories are houses that owners and banks fail to sell.

2.3.1 Directed Search in the Housing Market

Buyers direct their search by choosing a submarket $(p_b, h) \in \mathbb{R}_+ \times H$. With probability $\eta_b(\theta_b(p_b, h))$, the buyer matches with and purchases house $h \in H$ from a broker at cost p_b , where $\theta_b(p_b, h)$ is the ratio of brokers to buyers, i.e. the market tightness. Each period, sellers of house $h \in H$ choose a list price $p_s \geq 0$ and enter selling submarket (p_s, h) . With probability $\eta_s(\theta_s(p_s, h))$, the seller matches with and sells their house to a broker for p_s , where θ_s is the ratio of brokers to sellers. To prevent excessive time on the market, owners that try and fail to sell pay a small utility cost ξ .

Brokers find buyers and sellers with probabilities α_b and α_s , respectively, which are both decreasing functions of the market tightness. Brokers incur entry costs each period of $\kappa_b h$ and $\kappa_s h$ in the buying and selling submarkets, respectively. On both sides of the market, all participants take submarket tightnesses as given.

The profit maximization conditions of the real estate brokers (some of whom meet with

³Here, brokers trade discrete houses with buyers and sellers but divisible units of housing stock with each other. A generalized case would segment by h , in which case $p(h) = p_h h$.

sellers, and some of whom meet with buyers) are

$$\kappa_b h \geq \overbrace{\alpha_b(\theta_b(p_b, h))}^{\text{prob of match}} \overbrace{(p_b - p(h))}^{\text{broker revenue}} \quad (1)$$

$$\kappa_s h \geq \overbrace{\alpha_s(\theta_s(p_s, h))}^{\text{prob of match}} \overbrace{(p(h) - p_s)}^{\text{broker revenue}} \quad (2)$$

where the conditions hold with equality in active submarkets.

The revenue to a broker that purchases a house from a seller is $p(h) - p_s$. Therefore, brokers continue to enter submarket (p_s, h) until the cost $\kappa_s h$ exceeds the expected revenue. An analogous process occurs for buyer-brokers.

2.3.2 Block Recursivity

In [Menzio and Shi \(2010\)](#), block recursivity completely eliminates the need to keep track of the cross-sectional distribution when solving for equilibrium labor market dynamics. However, in this framework with housing, the presence of brokers as market makers simplifies the dynamic sorting problem but still leaves some dependence of market tightnesses θ_s and θ_b on the distribution Φ of income, assets, and debt, i.e. $\theta_b(p_b, h; \Phi)$ and $\theta_s(p_s, h; \Phi)$. With brokers, however, market tightnesses only depends on the distribution through its impact on p , i.e. $p(h)(\Phi) = p(\Phi)h$.

$$\theta_b(p_b, h; \Phi) = \alpha_b^{-1} \left(\frac{\kappa_b h}{p_b - p(h)(\Phi)} \right) \quad (3)$$

$$\theta_s(p_s, h; \Phi) = \alpha_s^{-1} \left(\frac{\kappa_s h}{p(h)(\Phi) - p_s} \right) \quad (4)$$

Absent the brokers, market tightnesses would depend nonparametrically on Φ , and households would need to forecast the evolution of each tightness independently. Thus, block recursivity simplifies the problem to solving for the dynamics of $p(h)(\Phi)$ and substituting into (3) – (4), all without altering the underlying economics of household buying and selling behavior.

2.4 Financial Markets

Households save using one period bonds which trade in open financial markets at an exogenous risk-free rate r . In addition, homeowners can borrow in the form of long term, fixed rate mortgage contracts with a default option where housing serves as collateral.⁴

2.4.1 Mortgages

Banks price default risk into new mortgage contracts. As such, this economy features **credit illiquidity**. Specifically, when a borrower with bonds b' , house h , and persistent labor efficiency s takes out a mortgage of size m' at rate r_m , the bank delivers $q_m^0((r_m, m'), b', h, s)m'$ units of the composite consumption good to the borrower at origination, where r_m remains fixed for the duration of the loan. Mortgages in the model stand in for all forms of mortgage debt (beyond 30-year first liens) by not having a predefined maturity date, and as a result, amortization is endogenous. Homeowners can prepay without penalty but must pay a cost to extract equity through refinancing.

Banks incur an origination cost ζ and servicing costs ϕ over the life of each mortgage. During repayment, banks have exposure to two risks. First, if the house depreciates with probability δ_h , the bank must forgive the loan.⁵ Second, homeowners can default in a given period by not making a payment. In this situation, the lender forecloses on the borrower with probability φ and repossesses the house. With probability $1 - \varphi$, the lender ignores the skipped payment until the next payment comes due.

Perfect competition assures zero ex-ante profits loan-by-loan. Banks price all individual default risk into q_m^0 at origination, but the fixed rate \bar{r}_m reflects depreciation risk, servicing costs, and long-term financing costs r^* , which depend on the future path r_t of the short term rate. A borrower with contract (\bar{r}_m, m) that chooses a new balance of $m' > m$ pays off m and refinances to a new, re-priced loan of balance m' . Otherwise, borrowers with debt m choose a payment $l \geq \frac{\bar{r}_m}{1+\bar{r}_m}m$, and their debt evolves according to $m' = (m - l)(1 + \bar{r}_m)$.

⁴Section 5.3 explores the implications of fixed vs. adjustable rate mortgages.

⁵This assumption prevents the model from generating artificially high foreclosure rates.

The fixed rate satisfies

$$1 + r_m = \underbrace{\left(\frac{1 + \phi}{1 - \delta_h} \right)}_{\text{spread}} \underbrace{1 + r^*}_{\text{long term risk-free rate}} \quad (5)$$

Mortgage prices satisfy the following recursive relationship:

$$\begin{aligned} q_m^0((\bar{r}_m, m'), b', h, s)m' &= \frac{1 - \delta_h}{(1 + \zeta)(1 + \phi)(1 + r)} \mathbb{E} \left\{ \underbrace{\eta_s(\theta_s(p'_s, h))m'}_{\text{sell + repay}} + \underbrace{[1 - \eta_s(\theta_s(p'_s, h))]}_{\text{no sale (do not try/fail)}} \right. \\ &\times \left[\underbrace{d' \varphi \min \{J_{REO}(h), m'\}}_{\text{default + repossession}} + \underbrace{d'(1 - \varphi)}_{\text{no repossession}} \left(\underbrace{-\phi m' + (1 + \zeta)(1 + \phi)q_m^0((\bar{r}_m, m'), b'', h, s')m'}_{\text{continuation value of current } m'} \right) \right. \\ &\left. \left. + (1 - d') \left\{ m' \mathbf{1}_{[\text{Refi}]} + \mathbf{1}_{[\text{No Refi}]} \left(\underbrace{l - \frac{\phi}{1 + \bar{r}_m} m''}_{\text{payment - servicing costs}} + \underbrace{(1 + \zeta)(1 + \phi)q_m^0((\bar{r}_m, m''), b'', h, s')m''}_{\text{continuation value of new } m''} \right) \right\} \right] \right\} \quad (6) \end{aligned}$$

where p'_s , d' , b'' , l , and m'' are the policies for list price, default, bonds, payment, and debt, respectively, and J_{REO} is the value of reposessed housing.

The long term nature of the contract is apparent in the continuation values, although the refinance option shortens the effective duration. Default risk depresses mortgage prices to the extent that $J_{REO}(h)$ falls below m' after foreclosure, and because delinquent borrowers are not immediately evicted. Lastly, illiquidity from selling delays increases the risk of default.

2.4.2 Foreclosure Process

Banks sell reposessed houses (REO properties) in the decentralized housing market and lose a fraction χ of proceeds as the cost of selling foreclosed houses. Banks absorb losses but must pass profits to the borrower.

The value to a lender in reposseing a house h is

$$\begin{aligned} J_{REO}(h) &= R_{REO}(h) - \gamma p(h) + \frac{1 - \delta_h}{1 + r} J_{REO}(h) \\ R_{REO}(h) &= \max \left\{ 0, \max_{p_s \geq 0} \eta_s(\theta_s(p_s, h)) \left[(1 - \chi)p_s - \left(-\gamma p(h) + \frac{1 - \delta_h}{1 + r} J_{REO}(h) \right) \right] \right\} \quad (7) \end{aligned}$$

where γ represents holding costs (maintenance, property taxes, etc.).

The forgiveness of debt from foreclosure entails other penalties besides the repossession of

the house. Specifically, defaulters receive a flag $f = 1$ on their credit record that shuts them out of the mortgage market. Flags persist to the next period with probability $\gamma_f \in (0, 1)$.

2.5 Household Problem

Each period contains three subperiods. First, households learn their labor efficiency $e \cdot s$ and their flag $f \in \{0, 1\}$. An owner's state is cash at hand y , mortgage rate \bar{r}_m and balance m , house h , and labor shock s . A renter's state is (y, s, f) . The household problem is solved backwards:

2.5.1 Subperiod 3: Consumption/Saving

End-of-period owner expenditures consist of consumption, holdings costs, bond purchases, and mortgage payments. Household resources come from labor income, savings, and equity extraction. Owners with good credit ($f = 0$) who refinance have value function

$$V_{own}^R(y, (\bar{r}_m, m), h, s, 0) = \max_{m', b', c \geq 0} u(c, h) + \beta \mathbb{E} \left[\begin{array}{l} (1 - \delta_h)(W_{own} + R_{sell})(y', (r_m, m'), h, s', 0) \\ + \delta_h(V_{rent} + R_{buy})(y', s', 0) \end{array} \right]$$

subject to

$$c + \gamma p(h) + q_b b' + m \leq y + q_m^0((r_m, m'), b', h, s) m'$$

$$q_m^0((r_m, m'), b', h, s) m' \leq \vartheta p(h)$$

$$y' = w e' s' + b'$$
(8)

where ϑ is the collateral constraint for new loans, q_m^0 reflects the mortgage re-pricing, and the updated rate is r_m . The terms $W_{own} + R_{sell}$ and $V_{rent} + R_{buy}$ are subperiod 1 utilities for owners and renters, respectively.

Owners who make a payment l on their existing mortgage solve

$$V_{own}^C(y, (\bar{r}_m, m), h, s, 0) = \max_{l, b', c \geq 0} u(c, h) + \beta \mathbb{E} \left[\begin{array}{l} (1 - \delta_h)(W_{own} + R_{sell})(y', (\bar{r}_m, m'), h, s', 0) \\ + \delta_h(V_{rent} + R_{buy})(y', s', 0) \end{array} \right]$$

subject to

$$c + \gamma p(h) + q_b b' + l \leq y$$

$$l \geq \frac{\bar{r}_m}{1 + \bar{r}_m} m$$

$$m' = (m - l)(1 + \bar{r}_m)$$

$$y' = w e' s' + b'$$
(9)

Borrowers must make at least an interest payment, and any larger payment reduces principal m' . Owners with bad credit solve a similar problem but lack access to mortgages. Renters face the following constraint: $c + r_a a + q_b b' \leq y$. Appendix A gives their detailed optimization problem.

2.5.2 Subperiod 2: House Buying

Buyers direct their search by choosing a submarket (p_b, h) . Buyers with bad credit are bound by the constraint $y - p_b \geq 0$, while buyers with good credit are bound by $y - p_b \geq \underline{y}(s, (h, 1))$, where $\underline{y} < 0$ captures their ability to take out a mortgage in subperiod 3. The option value R_{buy} of buying is as follows:

$$R_{buy}(y, s, 0) = \max\{0, \max_{\substack{h \in H, \\ p_b \leq y - \underline{y}}} \eta_b(\theta_b(p_b, h)) [V_{own}(y - p_b, 0, h, s, 0) - V_{rent}(y, s, 0)]\} \quad (10)$$

$$R_{buy}(y, s, 1) = \max\{0, \max_{\substack{h \in H, \\ p_b \leq y}} \eta_b(\theta_b(p_b, h)) [V_{own}(y - p_b, 0, h, s, 1) - V_{rent}(y, s, 1)]\} \quad (11)$$

2.5.3 Subperiod 1: Selling and Default Decisions

An owner deciding whether to default, refinance, or make a payment has utility

$$W(y, (\bar{r}_m, m), h, s, 0) = \max \{ \varphi(V_{rent} + R_{buy})(y + \max\{0, J_{REO}(h) - m\}, s, 1) \\ + (1 - \varphi)V_{own}^d(y, (\bar{r}_m, m), h, s, 0), V_{own}(y, (\bar{r}_m, m), h, s, 0) \} \quad (12)$$

where the value associated with defaulting but not being foreclosed on is

$$V_{own}^d(y, (\bar{r}_m, m), h, s, 0) = \max_{b', c \geq 0} u(c, h) + \beta \mathbb{E} \left[\begin{array}{l} (1 - \delta_h)(W_{own} + R_{sell})(y', (\bar{r}_m, m), h, s', 0) \\ + \delta_h(V_{rent} + R_{buy})(y', s', 0) \end{array} \right] \\ \text{subject to} \\ c + \gamma p(h) + q_b b' \leq y \\ y' = w e' s' + b' \quad (13)$$

Owners of house h who wish to sell choose a list price p_s . The option value R_{sell} of selling for an owner with good credit is

$$R_{sell}(y, (\bar{r}_m, m), h, s, 0) = \max\{0, \max_{p_s} \eta_s(\theta_s(p_s, h)) [(V_{rent} + R_{buy})(y + p_s - m, s, 0) \\ - W_{own}(y, (\bar{r}_m, m), h, s, 0)] + [1 - \eta_s(\theta_s(p_s, h))](-\xi)\} \text{ subject to } y + p_s \geq m \quad (14)$$

Debt overhang emerges when highly leveraged owners are forced to set high prices to pay off their debt, thereby resulting in long selling delays.

2.5.4 Equilibrium

A stationary equilibrium is value/policy functions for households and banks; market tightness functions θ_s and θ_b ; prices w , p_h , q_m^0 , q_b , and r_a ; and stationary distributions Φ of households and H_{REO} of REO housing stock that solve the relevant optimization problems and clear the markets for housing and factor inputs. Appendix A provides the detailed equilibrium conditions.

3 Parametrizing the Model

The model is calibrated to replicate key features of the United States economy during 2003 – 2005, prior to the Great Recession. The calibration puts heavy emphasis on matching key housing moments related to sales, time on the market, and foreclosures, as well as important dimensions of the joint distribution of assets, housing wealth, and mortgage debt.

3.1 Independent Parameters

The first set of parameters come from the literature or other external sources. On the household side, the labor efficiency process is adapted from Storesletten, Telmer and Yaron (2004) in the same way as done in Garriga and Hedlund (2017). In addition, households have constant relative risk aversion preferences with $\sigma = 2$ and CES period utility with an intratemporal elasticity of substitution of $\nu = 0.13$. The discount factor β and weight ω on non-housing consumption are determined jointly.

In terms of production, total factor productivity is set to normalize annual earnings to 1. Housing construction is Cobb-Douglas with a structures share of $\alpha_S = 0.3$ and a land share of $\alpha = 0.33$, consistent with evidence from the Lincoln Institute of Land Policy. Meanwhile, housing depreciates at an annual rate of 1.4%, and the apartment technology A_h is set to generate an annual rent-price ratio of 5%, consistent with Sommer et al. (2013).

Matching is Cobb-Douglas in the frictional housing market, and the joint calibration determines the entry costs, Cobb Douglas parameters, and disutility of attempting to sell. Holding costs (maintenance, property taxes, etc.) are $\eta = 0.007$.

Pertaining to financial markets, the real risk-free rate is set to 2%, the mortgage origination cost is 0.4%, and the mortgage servicing cost ϕ is set to bring the real mortgage rate to 5%. Furthermore, the exogenous LTV limit is $\vartheta = 1.25$ (125%), which makes it *non-binding* initially.⁶ Lastly, the persistence of bad credit flags is $\gamma_f = 0.95$, and the REO discount χ is determined in the joint calibration.

⁶See Herkenhoff and Ohanian (2015) for discussion of cash-out refinancing in the 2000s.

Table 1: Model Calibration

Description	Parameter	Value	Target	Model	Source/Reason
Calibration: Independent Parameters					
Autocorrelation	ρ	0.952			Storesletten et al. (2004)
SD of Persistent Shock	σ_ϵ	0.17			Storesletten et al. (2004)
SD of Transitory Shock	σ_e	0.49			Storesletten et al. (2004)
Intratemp. Elas. of Subst.	ν	0.13			Flavin and Nakagawa (2008)
Risk Aversion	σ	2			Various
Structure Share	α_S	30%			Favilukis et al. (2016)
Land Share	α_L	33%			Lincoln Inst Land Policy
Holding Costs	γ	0.7%			Moody's
Depreciation (Annual)	δ_h	1.4%			BEA
Rent-Price Ratio (Annual)	r_h	5%			Sommer et al. (2013)
Risk-Free Rate (Annual)	r	2.0%			Federal Reserve Board
Servicing Cost (Annual)	ϕ	3.1%			5.0% Real Mortgage Rate
Mortgage Origination Cost	ζ	0.4%			FHFA
Maximum LTV	ϑ	125%			Fannie Mae
Prob. of Repossession	φ	0.5			2008 OCC Mortgage Metrics
Credit Flag Persistence	λ_f	0.9500			Fannie Mae
Calibration: Jointly Determined Parameters					
Homeownership Rate	\bar{a}	2.005	66.7%	66.7%	Census
Starter House Value	h_1	2.4250	1.75	1.75	American Housing Survey
Housing Wealth (Owners)	ω	0.8177	2.49	2.49	1998 SCF
Median LTV	β	0.9657	62.90%	63.38%	1998 SCF
Months of Supply*	ξ	0.0016	5.30	5.32	Nat'l Assoc of Realtors
Avg. Buyer Search (Weeks)	γ_b	0.0940	10.00	10.04	Nat'l Assoc of Realtors
Maximum Bid Premium	κ_b	0.0171	2.5%	2.5%	Gruber and Martin (2003)
Maximum List Discount	κ_s	0.1029	15%	15%	RealtyTrac
Foreclosure Discount	χ	0.0980	21%	21%	Pennington-Cross (2006)
Foreclosure Starts (Annual)	γ_s	0.6550	1.60%	1.61%	Nat'l Delinquency Survey
Model Fit					
Borrowers with $LTV \geq 70\%$			40.00%	40.61%	1998 SCF
Borrowers with $LTV \geq 80\%$			25.00%	22.81%	1998 SCF
Borrowers with $LTV \geq 90\%$			14.50%	11.31%	1998 SCF
Borrowers with $LTV \geq 95\%$			9.20%	9.15%	1998 SCF
Median Owner Liq. Assets			0.25	0.23	1998 SCF

*Months of supply is inventories divided by the sales rate and proxies for time on the market.

3.2 Joint Calibration

The joint calibration determines the remaining parameters to match key aggregates, such as the homeownership rate, the value of gross housing wealth to income, median liquid assets, and the foreclosure rate. In addition, it is important that the model reasonably approximate the distribution of mortgage leverage, particularly at the upper end, as these homeowners are the most borrowing constrained and susceptible to shocks. Table 1 shows that the model successfully matches the targets and replicates other untargeted portfolio statistics from the 1998 Survey of Consumer Finances.

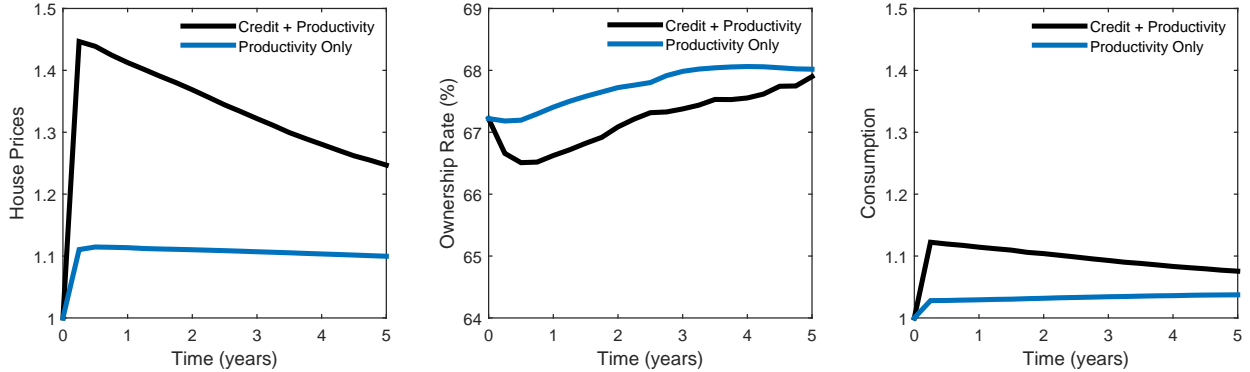


Figure 1: The effect of a 5% productivity boom for high rates/tight down payments (productivity boom only) vs. low rates/loose down payments (credit boom).

4 Anatomy of the Housing Boom

Although the U.S. has witnessed considerable regional swings in real house prices, the pronounced boom in national house prices from 2001 – 2006 stands out as unique and bears exploring. During this period, the national economy was in an expansion period, both in real activity and in the availability of cheap credit.

4.1 Productivity Booms vs. Credit Booms

To disentangle the economic expansion from the credit expansion, the structural model is used to assess the relative contributions of higher productivity and cheaper credit to the housing boom. Figure 1 shows that typical business cycles do not produce large booms in house prices. By itself, even a large, permanent 5% increase in wages from higher productivity causes only a 10% rise in house prices. However, if accompanied by a 200 basis point mortgage rate decline and lax down payment requirements—consistent with the U.S. experience in the early 2000s—the model matches the 45% house price boom from the data.⁷

Contrary to conventional wisdom, looser credit need not stimulate higher homeownership. Inspection of the middle panel of figure 1 shows that the productivity boom drives an increase in homeownership from 67% to 68% *with or without* the credit expansion. In a partial equilibrium sense, looser credit does indeed make homeownership cheaper and more

⁷In the productivity-only boom, real mortgage rates are 5.6% and households face a 20% down payment requirement. In the full boom, mortgage rates fall to 3.6% and households do not have to make any down payment (and can even engage in cash-out refinancing at up to 125% cumulative loan-to-value).

Table 2: The Broad-Based Expansion of Credit

	Low Income	Middle Income	High Income
<i>Average Borrower LTV</i>			
Pre-Boom	59.3%	61.3%	70.3%
Productivity Only	56.4%	58.9%	57.1%
Productivity + Credit	60.9%	65.8%	69.3%
$\Delta Credit$	+4.5%	+6.9%	+12.2%
<i>High-LTV Share*</i>			
Pre-Boom	13.9%	14.6%	36.3%
Productivity + Credit	16.7%	22.7%	31.1%
<i>Consumption Change</i>			
Productivity Only	4.8%	4.2%	1.3%
Productivity + Credit	6.0%	11.7%	13.3%
$\Delta Credit$	+1.2%	+7.5%	+12.0%

*The percentage of borrowers with mortgage debt exceeding 80% loan-to-value.

attractive. However, the dramatic equilibrium increase in house prices neutralizes the direct effect of cheaper credit on homeownership and even creates an initial dip.

The credit expansion also has broader macroeconomic consequences by amplifying the boom in aggregate consumption from 3% to over 10%. As section 5 discusses, the ability to use houses as an ATM is an important driver of housing market and consumption dynamics.

4.2 Credit Booms and the “New Narrative”

Consistent with the “new narrative” of Adelino, Schoar and Severino (2016), Foote, Loewenstein and Willen (2016), and Albanesi, DeGiorgi and Nosal (2016), the credit boom fuels a broad-based increase in borrowing, homeownership, and consumption across the income distribution that differs from the subprime narrative popularized by Mian and Sufi (2009). According to table 2, the productivity boom causes a modest *decrease* in leverage across the income distribution for low-income and middle-income borrowers and a steep drop for those with high incomes. Furthermore, when higher incomes are accompanied by lower mortgage rates and loose down payments, leverage increases across all income groups and not just among “subprime” borrowers.

Effects on the Housing Ladder As described previously, the credit boom does not lead to any additional homeownership on the extensive margin relative to the productivity boom alone. However, the credit boom amplifies the shift in ownership toward larger houses. The one percentage point increase in the homeownership rate masks the fact that 13% of small-house owners move up the ladder in response to the productivity boom, and the credit boom raises that share to 22% of small-house owners.

Consumption Spillovers Contrary to the subprime narrative, the credit boom actually increases consumption disproportionately among middle-income and high-income households. For low-income households, the productivity boom alone fuels a 4.8% rise in consumption, and the expansion of credit creates an additional 1.2% jump in consumption. The corresponding impact on consumption of the credit expansion for middle-income and high-income households is a much larger 7.5% and 12%, respectively.

5 The Non-Neutrality of Mortgage Structure

Under certain assumptions, the Modigliani-Miller theorem shows the irrelevance of corporate capital structure, but less is known about the importance of contract structure in the household sector. In the United States, thirty-year fixed-rate mortgages have traditionally been predominant, but alternative products gained in popularity during the housing boom. Furthermore, there is considerable cross-country variation in the design of mortgage contracts. This section analyzes the macroeconomic importance of these institutional arrangements through the lens of the recent U.S. experience.

5.1 The Boom, Bust, and Recovery

The housing boom, bust, and recovery are simulated using the model in three steps. Starting from the steady state calibrated to the late 1990s, the economy is shocked by the productivity boom and credit boom described in section 4. Agents perceive these shocks to be permanent but are surprised five years into the housing boom when the economy is hit by a surprise sequence of negative shocks that create a temporary but deep housing crash and

Table 3: The Housing Boom and Bust

	$\Delta\text{Prices}_{boom}$	ΔC_{boom}	Own_{boom}	$\Delta\text{Prices}_{bust}$	ΔC_{bust}	Own_{bust}
Model	+44.6%	+12.2%	68.1%	-24.5%	-18.5%	64.3%
Data	+41.9%	+5.1%	69.2%	-25.9%	-15.0%	64.2%

Sources: (House Prices) FHFA purchase index. (Consumption) Detrended per-capita nondurable consumption. (Ownership) Census Bureau.

recession. Shortly after the onset of the recession, agents are surprised one last time by an unexpected decrease in mortgage rates corresponding to the unprecedented mortgage interventions undertaken during the crisis.

Downside Uncertainty and Tighter Credit Garriga and Hedlund (2017) show that tighter down payment constraints and higher downside uncertainty in the form of left tail labor income shocks are the two key drivers of the housing crash. Other shocks, such as a large productivity decline or rise in interest rates, cannot reproduce the steep decline in house prices, sales, and homeownership or the spike in foreclosures. Thus, these same shocks are used here with one key difference. Whereas Garriga and Hedlund (2017) initialize the economy in 2006, here the model is calibrated to the 1990s, which means that the state of the economy (e.g. the leverage distribution) when the recession strikes is endogenous.

Asymmetric Balance Sheet Effects Figure 2 shows the impact of house price movements on consumption during the boom and bust. During the boom, the vast majority of the increase in consumption occurs because of the direct impact of higher productivity and looser credit, even if house prices were to remain flat (dashed line). The 45% equilibrium jump in house prices causes only a modest further increase in consumption, which is manifested by the 0.13 elasticity of consumption to house prices shown in the top-right panel.

However, consumption becomes much more sensitive to house prices during the bust. The 25% drop in house prices almost doubles the decline in consumption relative to the isolated direct effect of higher uncertainty and tighter credit. As a result, the elasticity of consumption to house prices is 0.3—over double the elasticity in the boom.

This asymmetry in balance sheet effects arises from state-dependent nonlinearities in the

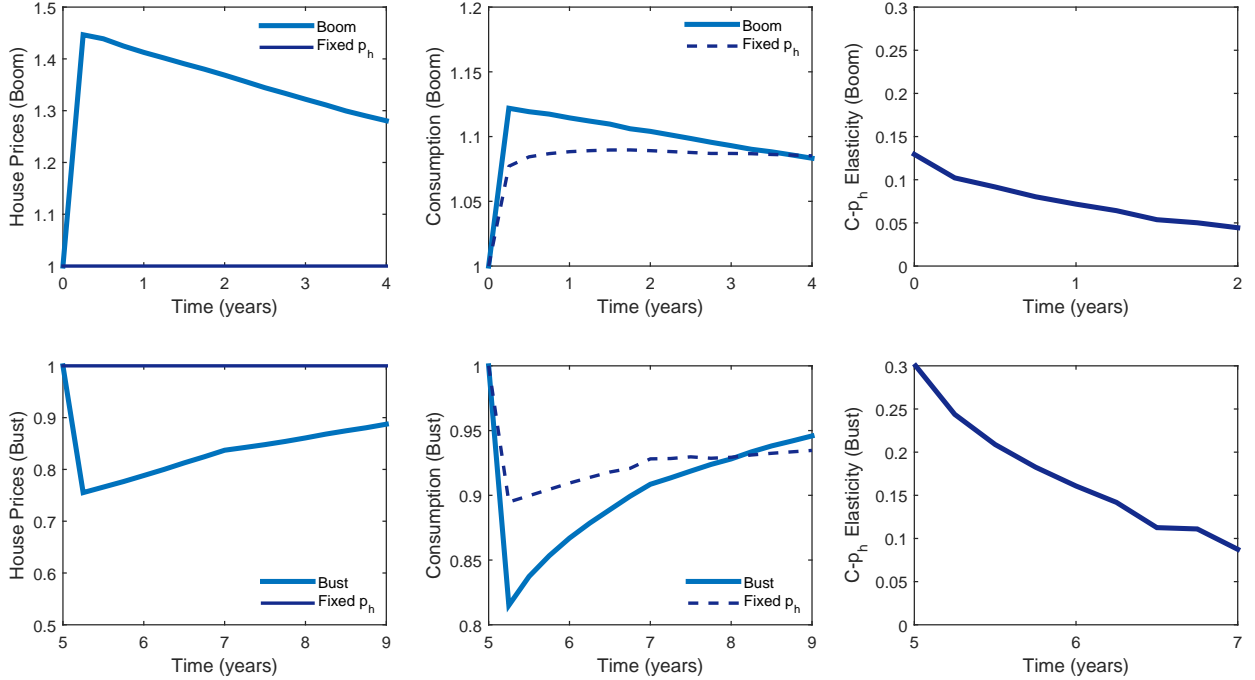


Figure 2: The asymmetric effect of house price movements on consumption during the boom and bust. Prices and consumption are re-normalized at the onset of the crash.

response of liquidity in the housing and credit markets. Housing *illiquidity*, as measured by average selling delays for houses on the market, is already low in the initial steady state and falls by only a few weeks during the boom. Similarly, pre-boom foreclosure activity is already quite low, which means that banks are willing to lend at a low default premium. However, during the bust, debt overhang pushes up time on the market from under 20 weeks to nearly a full year, and the annual foreclosure rate spikes from less than 0.5% to over 3.5%. The combined difficulty of selling and inability to extract equity at a reasonable cost pushes households to more severely cut their consumption. If house prices were to remain stable despite these shocks, homeowners could use the equity to better smooth consumption. Thus, the evaporation of equity during the bust has a much larger impact on consumption than does the increase in equity during the boom.

5.2 Equity Extraction and the Ability to Refinance

Not only does the relaxation in credit facilitate the purchase of larger houses during the boom, but it also allows new and existing owners to extract equity to fuel greater consumption.

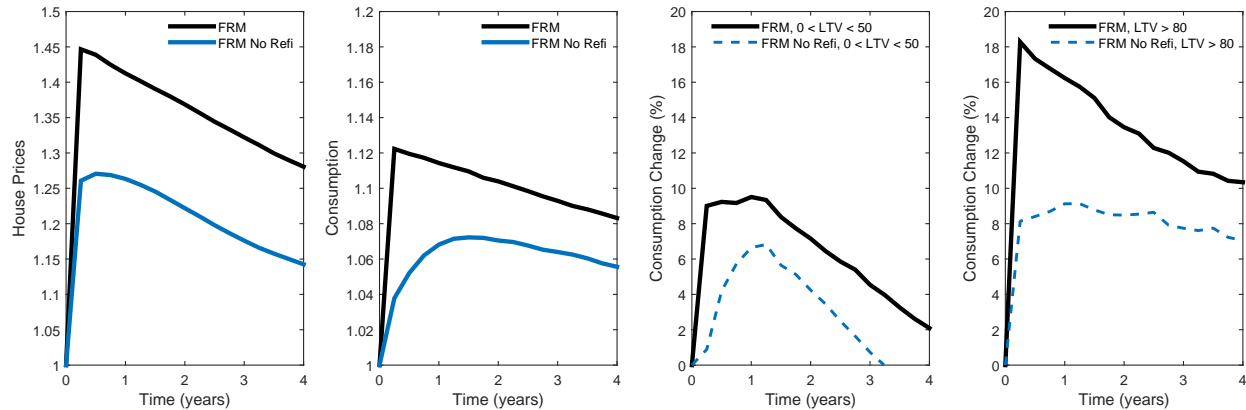


Figure 3: How the ability to refinance affects house price and consumption dynamics.

In fact, Gerardi, Lehnert, Sherlund and Willen (2008) document a substantial rise in the use of secondary “piggyback loans” with high leverage ratios during this period. By 2006, this type of lending accounted for approximately 50% of new originations and featured an average cumulative loan-to-value of 98.8%. Prior to the recent housing boom, owners did not have the ability to engage in such high leverage cash-out refinancing, and in some other countries, refinancing of any form is extremely rare and difficult.

The model predicts that removing the option to refinance cuts the boom in house prices nearly in half from 45% to just 27%, as shown in figure 3. The consumption boom would also become much smaller and more gradual. Removing the ability to refinance moderates the house price boom for two reasons. First, the value of housing as an ATM is diminished. Second, because housing and consumption are complements, a smaller consumption boom causes homeowners to demand less housing. This smaller house price boom further reduces the increase in consumption because of the previously described balance sheet channel. Note that the difference in consumption dynamics between the baseline economy and the economy with no refinancing is concentrated among homeowners. Consumption of renters is unaffected by the ability of homeowners to refinance, whereas the boom in consumption for highly leveraged homeowners shrinks by over 50%.

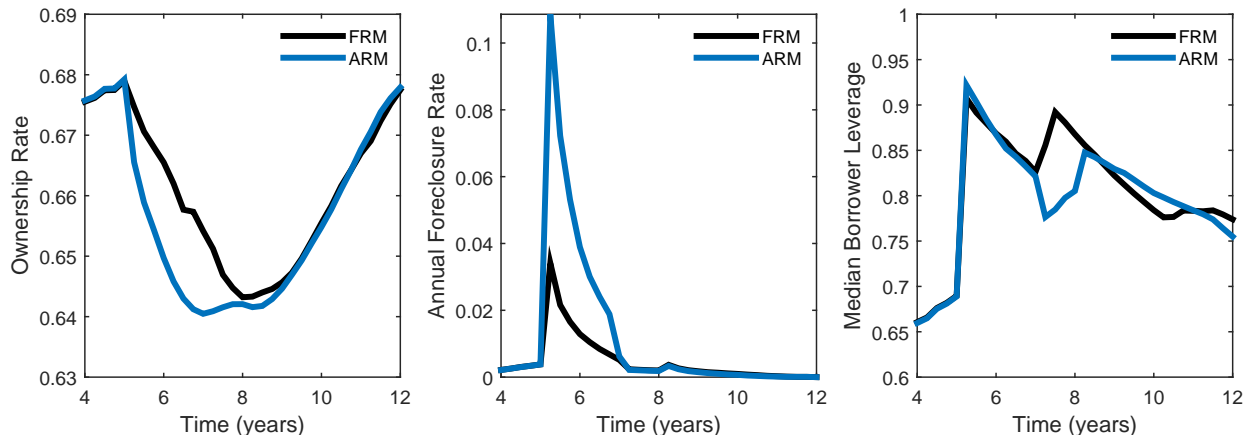


Figure 4: Fixed-rate vs. adjustable-rate regime.

5.3 Fixed-Rate vs. Adjustable-Rate Mortgages

The prevalence of the 30-year, fixed-rate mortgage is a unique staple of the United States housing market. In many other countries, adjustable-rate mortgages are the dominant contract. The advantage to fixed-rate contracts is that they provide insurance to borrowers during times of rising rates, but costly refinancing increases the difficulty for borrowers to take advantage of declining rates. The model approaches the comparison of fixed-rate and adjustable-rate mortgages by simulating two different regimes. In one regime, only fixed-rate mortgages are available, and in the other, only adjustable-rate mortgages are available.

The Boom Note that fixed-rate mortgages are only a one-sided commitment, because borrowers have the option to prepay their existing loan and take out a new mortgage at a lower rate if rates are falling. Because of this ability to refinance, the housing boom is identical in the fixed-rate and adjustable-rate regimes. If refinancing is not allowed, however, adjustable rate mortgages amplify the house price boom by just under 9%.

The Bust One component of the credit tightening that precipitates the housing bust is an initial increase in short term interest rates. In the fixed-rate regime, borrowers are shielded from higher borrowing costs, as the temporary increase in short rates does not pass through to mortgage rates. However, in the adjustable-rate regime, borrowers are faced with a rate reset that leads to a step increase in monthly payments. Therefore, unlike during the

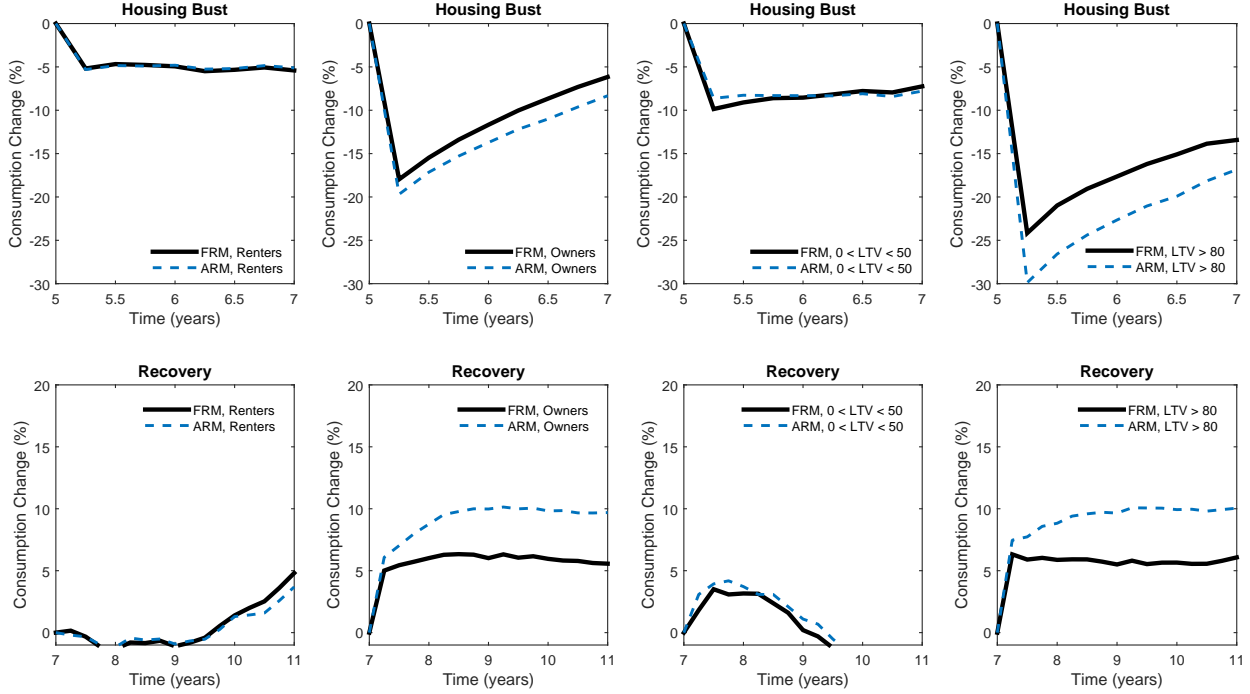


Figure 5: Consumption with FRMs and ARMs.

boom, the adjustable-rate economy responds much differently than the fixed-rate economy. As shown in figure 4, homeownership falls much more rapidly with adjustable rates, in no small part because there is nearly triple the amount of foreclosure activity. The result is that adjustable rates magnify the house price decline by 8.8%.

The mortgage rate structure also impacts consumption behavior. In the aggregate, adjustable rates magnify the consumption decline by almost 13%, but the amplification is not uniform. Renters, naturally, are indifferent to whether mortgages are fixed-rate or adjustable-rate contracts. However, the same is also true of homeowners with significant equity. By contrast, highly leveraged homeowners respond strongly to the rate resets in the adjustable rate consumption by substantially cutting consumption, as shown in figure 5.

The Recovery The downside of adjustable rate mortgages is that they increase the sensitivity of consumption to interest rate hikes. However, their advantage is that interest rate declines also immediately pass through to borrowers' balance sheets. With fixed-rate mortgages, the only way borrowers can take advantage of lower rates is to engage in costly refinancing. This distinction explains the divergent paths of leverage shown in the third

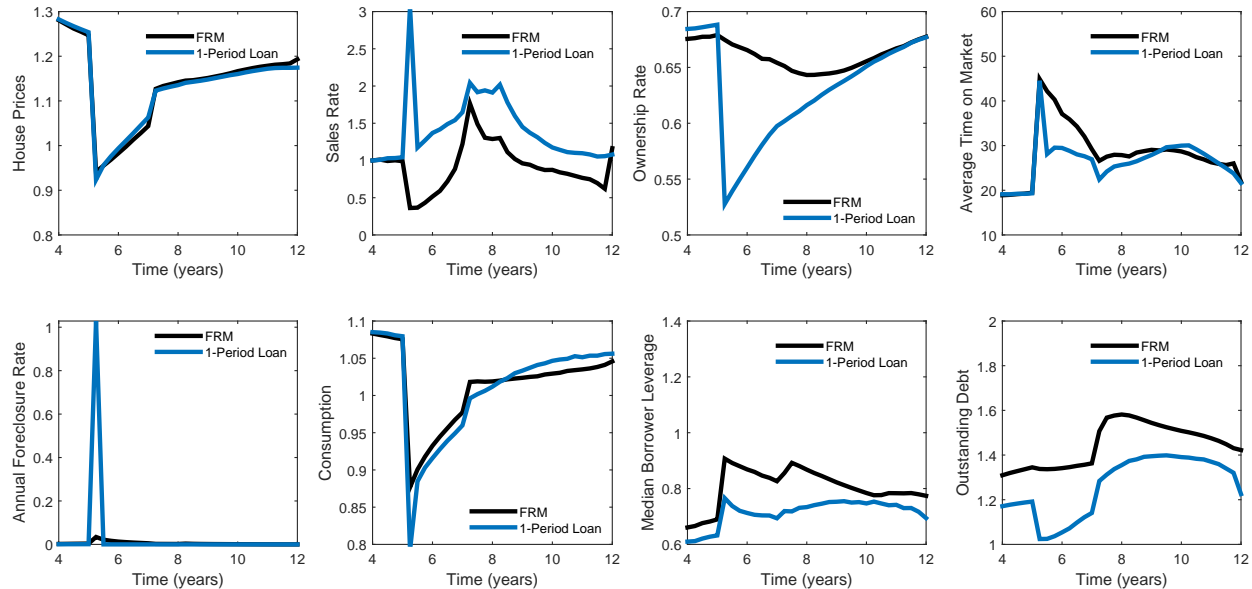


Figure 6: The impact of rollover risk with short-term debt.

panel of figure 4 following the post-intervention decline in mortgage rates. Leverage falls mechanically in the adjustable rate economy because equilibrium house prices increase in response to lower borrowing costs. In the fixed rate economy, a subset of homeowners responds to the decline in mortgage rates by refinancing and extracting equity. Already highly leveraged homeowners, however, are unable to extract any additional equity, which explains the larger consumption response shown in the bottom right panel of figure 5.

5.4 Rollover Risk and Mortgage Duration

Besides providing protection against interest rate risk, thirty-year mortgages also provide important insurance against rollover risk. Whenever households take out a new loan, banks set the cost of credit to correspond with the borrower's expected default risk. With short-term debt, borrowers who wish to roll over their existing balance into a new loan must go through underwriting again. If that period of underwriting happens to coincide with an unexpected negative income shock or drop in house prices, it is possible that the borrower may not be able to take out a new loan large enough to cover their existing debt. Long term debt, however, allows borrowers to lock-in their default premium at origination.

To assess the economic importance of this rollover insurance, the baseline economy is com-

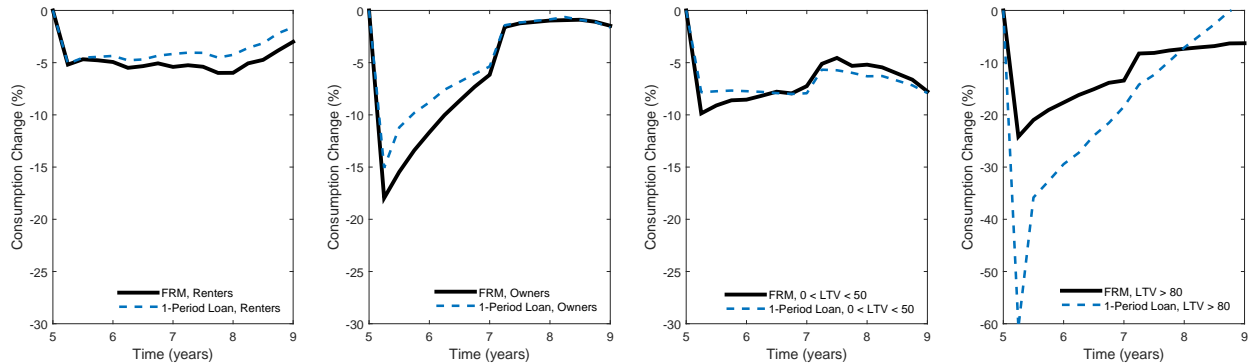


Figure 7: Consumption with rollover risk.

pared to a version of the model with short-term debt. During the boom, the two economies perform identically because rising home equity from high house prices render default risk nearly nonexistent. However, during the housing bust the two economies behave quite differently along certain margins, as shown in figure 6. In the economy with short-term debt, homeowners who find themselves underwater are unable to roll over their debt and immediately go into default, which causes the homeownership rate to plunge. By contrast, in the baseline economy with long-term debt, sales fall as houses become more illiquid and sit on the market for an extended period of time. Because sellers do not face rollover risk in the baseline economy, they can afford to ride out the crisis for longer in the hopes that they find a willing buyer. Thus, extended time on the market from debt overhang only exists when debt is long term. With short term debt, overhang is immediately resolved through default. This divergence explains why mortgage debt remains steady in the baseline economy (bottom-right panel) but falls with short-term debt.

The inability to roll over debt with short-term mortgages amplifies the consumption decline by 44% during the crisis. Again, this amplification is not uniform across households. For homeowners with substantial equity, the consumption response is nearly indistinguishable between the two economies. In fact, because homeowners endogenously increase savings during the boom to partially self-insure against rollover risk, their consumption actually falls by less than in the baseline economy. However, figure 7 reveals that highly leveraged homeowners experience a consumption disaster in the economy with short-term debt.

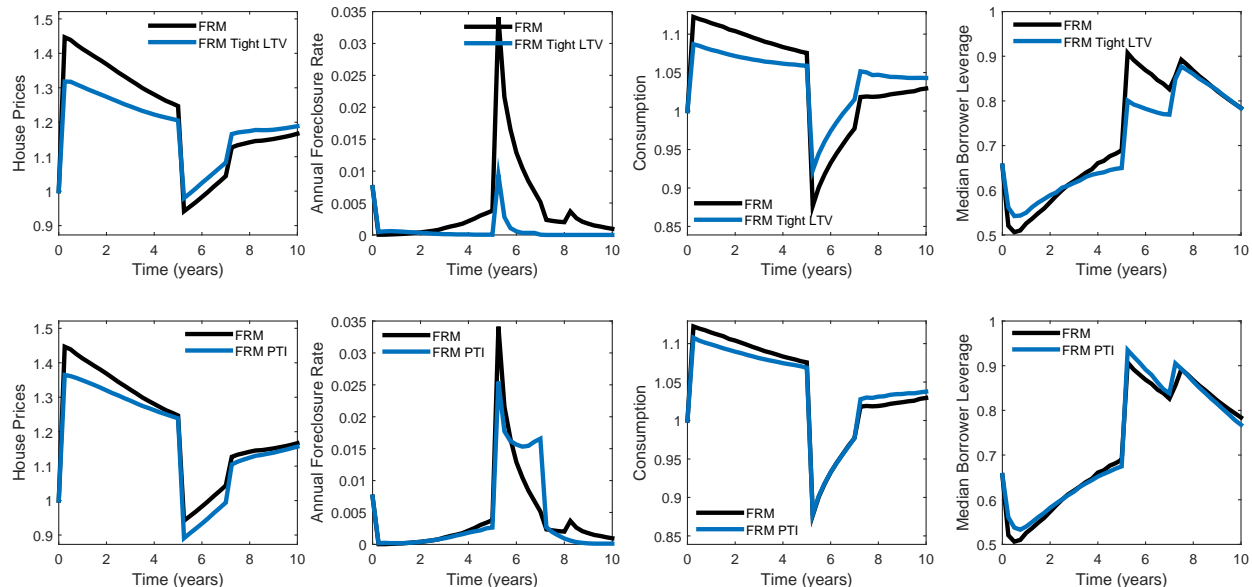


Figure 8: The effect of loan-to-value and payment-to-income requirements.

6 The Impact of Macroprudential Policies

During the housing boom, mortgage borrowing generates a new distribution of leverage that makes the economy more exposed and fragile to unanticipated credit reversals. Macro prudential policies are often advocated as a tool that can/should mitigate the macroeconomic impact of housing crises by taking the appropriate action, so in response to the same shocks the outcomes are different. Ensuring that homeowners have enough wealth or income, to absorb a negative house price shock, can reduce the foreclosure rate during a housing crash retaining a larger number of homeowners in their property. In equilibrium, the lower foreclosure rates and reduce number of units for sale could reduce, relative to the baseline, the default premiums and the endogenous transaction costs associated to trade houses (i.e. TOM). Two distinct macroprudential tools are considered: LTV caps to limit the amount of borrowing and payment-to-income (PTI) caps to ensure that the fraction of income allocated to meet mortgage obligations is not too high.

6.1 Loan-to-Value Constraints

A direct implication of introducing LTV caps is that significantly dampens the size of the housing boom. Households face the same initial drivers higher permanent income and lower

mortgage rates but the tighter LTV limits their ability to capitalize it by borrowing and spend it on housing. The size of the boom is reduced by a third (30 percent instead of 45 percent) making housing relatively more affordable thus increasing the homeownership rate. The tighter credit limits combined with a more modest housing appreciation reduces the response of non-housing consumption. The model points out that expanding credit is not a necessary condition to increase homeownership, what it matters its the levels of borrowing relative to the dynamics of house prices. Notice that both economies share a very similar leverage distribution (mortgage debt to house values), however, the level of mortgage debt relative to income is lower making the economy less fragile and exposed to credit reversals.

In response to the same negative credit contraction, the endogenous decline in house prices is substantially reduced. The combination of a smaller bust, relative to the baseline case without macroprudential policy, and less exposure to shocks in the households balance sheet substantially reduces the foreclosures spike during the crisis. The implications are not limited to the housing market, as more households stay in their house, but also for the macroeconomy, as aggregate consumption does not fall as much with the credit contraction. The combined macro effects are driven by a lower sensitivity of aggregate consumption to income increases and cheap credit easing during the boom, and less credit outstanding (new loans and home equity lines of credit) during the bust.

LTV caps are particularly effective when the initial mortgage rate is low suggesting that the optimal LTV should not be invariant to the underlying cost of borrowing as this policy operates by reducing the financial fragility of the economy.

6.2 Payment-to-Income Constraints

A complementary macroprudential policy could place limits in the fraction of income devoted to mortgage payments. In the baseline economy, the cap is set to 50 percent, but the fraction of homeowners that exceed more than 40 percent is smaller than XX percent. To explore the direct impact of payment-to-income constraints as a policy tool, it is useful to reduce the size of the cap to 35 percent but maintaining the LTV constraint limit in the baseline level. As can be seen in figure 8, the tightening payment-to-income constraints can dampen the appreciation of house values by 22 percent but the magnitude is not quite as large as with

LTV caps. The smaller boom is clearly driven by limiting the size of mortgage borrowing via payments. Lower mortgage rates and higher income reduce the severity of the constraint, hence the house price appreciation, but the size of mortgage debt is curtail relative to the baseline. Perhaps surprisingly, this policy significantly increases the homeownership rate due to the broad limitations in housing spending.

However, the payment-to-income limit still induces too much credit relative to LTV caps (i.e. 22 percent increase in outstanding mortgage debt instead of a 12 percent), and does not reduce the fragility to credit reversals. While the endogenous response in aggregate credit is not as large as the baseline, it is certainly spread out over a larger fraction of households because at the peak the fraction of homeowners is 70 percent instead of 68 percent. As a result, the same credit tightening generates a sizeable decline in house prices and a large spike in foreclosure. The lower fraction of income devoted to mortgage payments imposed at origination is not sufficient to discourage indebted homeowners to default. Even when homeowners have a relatively small commitment in terms of mortgage payments and flexible repayment options, as the minimum requirement is to pay interest on the principal, a large fraction still finds beneficial to adjust their balance sheet by defaulting instead of reducing the mortgage balance. The aggregate deleverage is a combination of defaults, mortgage liquidations via selling the houses, and portfolio rebalances. The payment-to-income constraint is insufficient to deter the decline in aggregate consumption during the housing bust.

The macroprudential effect of both policies is clearly distinct, as LTV caps have a more direct effect on the expansion of credit along the extensive and intensive margins. Nevertheless, the equilibrium feedback, in each case, generates a smaller housing boom that does not prevent homeownership.

7 Conclusion

This paper shows that arrangements in the mortgage market have a substantial impact on the dynamics of housing and the macroeconomy during episodes of booms and busts. Shocks to the cost and availability of credit fuel much larger housing booms than do typ-

ical productivity shocks, and these large swings in the housing market feed through into consumption. During downturns, the balance sheet channel that connects housing market behavior to consumption is even more sensitive to credit conditions. There are also several important lessons to be learned about mortgage design. First, the ease of equity extraction has first-order implications for the size of housing booms and busts. Second, economies with a high concentration of adjustable rate mortgages experience large house price swings and are more likely to go through periods of high foreclosure activity. However, these economies are also more responsive to policy interventions to stimulate the housing market and consumption. Third, long-term provides substantial insurance against rollover risk in a way that significantly mitigates the response of homeownership, foreclosures, and consumption during a housing bust. Lastly, by altering the endogenous fragility of the economy, macroprudential policies like loan-to-value and payment-to-income constraints are effective at moderating swings in the housing market and consumption.

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A Summary of Equilibrium Conditions

This section gives the complete definition of equilibrium from section 2.5.4.

A.1 Household Value Functions

A.1.1 Subperiod 3 Value Functions

Homeowners with good credit who refinance:

$$V_{own}^R(y, (\bar{r}_m, m), h, s, 0) = \max_{m', b', c \geq 0} u(c, h) + \beta \mathbb{E} \left[\begin{array}{l} (1 - \delta_h)(W_{own} + R_{sell})(y', (r_m, m'), h, s', 0) \\ + \delta_h(V_{rent} + R_{buy})(y', s', 0) \end{array} \right]$$

subject to

$$c + \gamma p(h) + q_b b' + m \leq y + q_m^0((r_m, m'), b', h, s) m'$$

$$q_m^0((r_m, m'), b', h, s) m' \leq \vartheta p(h)$$

$$y' = w e' s' + b'$$
(15)

Homeowners with good credit who make a regular payment:

$$V_{own}^C(y, (\bar{r}_m, m), h, s, 0) = \max_{l, b', c \geq 0} u(c, h) + \beta \mathbb{E} \left[\begin{array}{l} (1 - \delta_h)(W_{own} + R_{sell})(y', (\bar{r}_m, m'), h, s', 0) \\ + \delta_h(V_{rent} + R_{buy})(y', s', 0) \end{array} \right]$$

subject to

$$c + \gamma p(h) + q_b b' + l \leq y$$

$$l \geq \frac{\bar{r}_m}{1 + \bar{r}_m} m$$

$$m' = (m - l)(1 + \bar{r}_m)$$

$$y' = w e' s' + b'$$
(16)

Homeowners with bad credit:

$$\begin{aligned}
V_{own}(y, 0, h, s, 1) &= \max_{b', c \geq 0} u(c, h) + \beta \mathbb{E} \left[\begin{array}{l} (1 - \delta_h)(W_{own} + R_{sell})(y', 0, h, s', f') \\ + \delta_h(V_{rent} + R_{buy})(y', s', f') \end{array} \right] \\
&\text{subject to} \\
c + \gamma p(h) + q_b b' &\leq y \\
y' &= w e' s' + b'
\end{aligned} \tag{17}$$

Apartment-dwellers with good credit:

$$\begin{aligned}
V_{rent}(y, s, 0) &= \max_{b', c \geq 0, a \leq \bar{a}} u(c, a) + \beta \mathbb{E} [(V_{rent} + R_{buy})(y', s', 0)] \\
&\text{subject to} \\
c + q_b b' + r_a a &\leq y \\
y' &= w e' s' + b'
\end{aligned} \tag{18}$$

Apartment-dwellers with bad credit:

$$\begin{aligned}
V_{rent}(y, s, 1) &= \max_{b', c \geq 0, a \leq \bar{a}} u(c, a) + \beta \mathbb{E} [(V_{rent} + R_{buy})(y', s', f')] \\
&\text{subject to} \\
c + q_b b' + r_a a &\leq y \\
y' &= w e' s' + b'
\end{aligned} \tag{19}$$

A.1.2 Subperiod 2 Value Functions

The value of searching to buy a house:

$$R_{buy}(y, s, 0) = \max\{0, \max_{\substack{h \in H, \\ p_b \leq y - \underline{y}}} \eta_b(\theta_b(p_b, h)) [V_{own}(y - p_b, 0, h, s, 0) - V_{rent}(y, s, 0)]\} \quad (20)$$

$$R_{buy}(y, s, 1) = \max\{0, \max_{\substack{h \in H, \\ p_b \leq y}} \eta_b(\theta_b(p_b, h)) [V_{own}(y - p_b, 0, h, s, 1) - V_{rent}(y, s, 1)]\} \quad (21)$$

A.1.3 Subperiod 1 Value Functions

The utility associated with the default/refinance/payment decision:

$$W(y, (\bar{r}_m, m), h, s, 0) = \max\{\varphi(V_{rent} + R_{buy})(y + \max\{0, J_{REO}(h) - m\}, s, 1) \\ + (1 - \varphi)V_{own}^d(y, (\bar{r}_m, m), h, s, 0), V_{own}^R(y, (\bar{r}_m, m), h, s, 0), V_{own}^C(y, (\bar{r}_m, m), h, s, 0)\} \quad (22)$$

Utility of default conditional on no repossession:

$$V_{own}^d(y, (\bar{r}_m, m), h, s, 0) = \max_{b', c \geq 0} u(c, h) + \beta \mathbb{E} \left[\begin{array}{l} (1 - \delta_h)(W_{own} + R_{sell})(y', (\bar{r}_m, m), h, s', 0) \\ + \delta_h(V_{rent} + R_{buy})(y', s', 0) \end{array} \right] \\ \text{subject to} \\ c + \gamma p(h) + q_b b' \leq y \\ y' = w e' s' + b' \quad (23)$$

The value of attempting to sell a house for a (possibly indebted) owner:

$$R_{sell}(y, (\bar{r}_m, m), h, s, 0) = \max\{0, \max_{p_s} \eta_s(\theta_s(p_s, h)) [(V_{rent} + R_{buy})(y + p_s - m, s, 0) \\ - W_{own}(y, (\bar{r}_m, m), h, s, 0)] + [1 - \eta_s(\theta_s(p_s, h))] (-\xi)\} \text{ subject to } y + p_s \geq m \quad (24)$$

The value of attempting to sell a house for an owner with bad credit:

$$R_{sell}(y, 0, h, s, 1) = \max\{0, \max_{x_s} \eta_s(\theta_s(p_s, h)) [(V_{rent} + R_{buy})(y + p_s, s, 1) - W_{own}(y, 0, h, s, 1)] + [1 - \eta_s(\theta_s(p_s, h))] (-\xi)\} \quad (25)$$

A.2 Firms

A.2.1 Composite Consumption

The profit maximization condition of the composite good firm is

$$w = A_c \quad (26)$$

A.2.2 Apartments

The profit maximization condition of landlords is

$$r_a = \frac{1}{A_h} \quad (27)$$

A.2.3 Housing Construction

The relevant profit maximization conditions of home builders are

$$1 = p \frac{\partial F_h(\bar{L}, S_h, N_h)}{\partial S_h} \quad (28)$$

$$w = p \frac{\partial F_h(\bar{L}, S_h, N_h)}{\partial N_h} \quad (29)$$

A.3 Banks

Bond prices satisfy

$$q_b = \frac{1}{1 + r} \quad (30)$$

Mortgage rates satisfy

$$1 + r_m = \frac{(1 + \phi)(1 + r)}{1 - \delta_h} \quad (31)$$

The value to the bank of repossessing a house h is

$$J_{REO}(h) = R_{REO}(h) - \gamma p(h) + \frac{1 - \delta_h}{1 + r} J_{REO}(h) \quad (32)$$

$$R_{REO}(h) = \max \left\{ 0, \max_{p_s \geq 0} \lambda \eta_s(\theta_s(p_s, h)) \left[(1 - \chi) p_s - \left(-\gamma p(h) + \frac{1 - \delta_h}{1 + r} J_{REO}(h) \right) \right] \right\}$$

Mortgage prices satisfy the following recursive relationship:

$$q_m^0((\bar{r}_m, m'), b', h, s) m' = \frac{1 - \delta_h}{(1 + \zeta)(1 + \phi)(1 + r)} \mathbb{E} \left\{ \underbrace{\eta_s(\theta_s(p'_s, h)) m'}_{\text{sell + repay}} + \underbrace{[1 - \eta_s(\theta_s(p'_s, h))]}_{\text{no sale (do not try/fail)}} \right.$$

$$\times \left[\underbrace{d' \varphi \min \{J_{REO}(h), m'\}}_{\text{default + repossession}} + \underbrace{d'(1 - \varphi)}_{\text{no repossession}} \left(-\phi m' + \underbrace{(1 + \zeta)(1 + \phi) q_m^0((\bar{r}_m, m'), b'', h, s') m'}_{\text{continuation value of current } m'} \right) \right.$$

$$\left. \left. + (1 - d') \left\{ m' \mathbf{1}_{[\text{Refi}]} + \mathbf{1}_{[\text{No Refi}]} \left(\underbrace{l - \frac{\phi}{1 + \bar{r}_m} m''}_{\text{payment - servicing costs}} + \underbrace{(1 + \zeta)(1 + \phi) q_m^0((\bar{r}_m, m''), b'', h, s') m''}_{\text{continuation value of new } m''} \right) \right\} \right] \right\} \quad (33)$$

A.4 Housing Market Equilibrium

A.4.1 Market Tightnesses

Market tightnesses satisfy

$$\kappa_b h \geq \underbrace{\alpha_b(\theta_b(p_b, h))}_{\text{prob of match}} \underbrace{(p_b - p(h))}_{\text{broker revenue}} \quad (34)$$

$$\kappa_s h \geq \underbrace{\alpha_s(\theta_s(p_s, h))}_{\text{prob of match}} \underbrace{(p(h) - p_s)}_{\text{broker revenue}} \quad (35)$$

with $\theta_b(x_b, h) \geq 0$, $\theta_s(x_s, h) \geq 0$, and complementary slackness.

A.4.2 Determining the Shadow Housing Price

Housing supply $S_h(p)$ equals the sum of new and existing sold housing,

$$S_h(p) = \overbrace{Y_h(p)}^{\text{new housing}} + \overbrace{S_{REO}(p)}^{\text{REO housing}} + \overbrace{\int h\eta_s(\theta_s(x_s^*, h; p))\Phi_{own}(dy, dm, dh, ds, df)}^{\text{sold by owner}} \quad (36)$$

The supply of REO housing is given by

$$S_{REO}(p) = \sum_{h \in H} h\lambda\eta_s(\theta_s(x_s^{*REO}, h; p)) \left[\underbrace{H_{REO}(h)}_{\text{existing REOs}} + \underbrace{\int [1 - \eta_s(\theta_s(x_s^*, h; p))]d^*\Phi_{own}(dy, dm, dh, ds, 0)}_{\text{new foreclosures from failing to sell and then defaulting}} \right] \quad (37)$$

Housing demand $D_h(p)$ equals housing purchased by matched buyers,

$$D_h(p) = \int h^*\eta_b(\theta_b(x_b^*, h^*; p))\Phi_{rent}(dy, ds, df) \quad (38)$$

The per unit shadow housing price p (recall that $p(h) = ph$) equates these Walrasian-like equations,

$$D_h(p) = S_h(p) \quad (39)$$

A.5 Detailed Equilibrium Definition

Definition 1 *Given interest rate r and permits \bar{L} , a stationary recursive equilibrium is*

1. Household value and policy functions
2. Intermediary value and policy functions J_{REO} and x_s^{REO}
3. Market tightness functions θ_b and θ_s
4. A mortgage pricing function q_m^0
5. Prices w , q_b , q_m , r_h , and p

6. Quantities K_c , N_c , S_h , and N_h

7. Stationary distributions $\{H_{REO}\}_{h \in H}$, Φ_{own} , and Φ_{rent}

such that

1. **Household Optimality:** The value/policy functions solve (15) – (25).

2. **Firm Optimality:** Condition (29) is satisfied.

3. **Bank Optimality:** Conditions (30) – (33) are satisfied.

4. **Market Tightnesses:** $\{\theta_b(x_b, h)\}$ and $\{\theta_s(x_s, h)\}$ satisfy (34) – (35).

5. **Labor Market Clears:** $N_c + N_h = \sum_{s \in S} \int_E e \cdot sF(de)\Pi_s(s)$.

6. **Shadow Housing Price:** $D_h(p) = S_h(p)$.

7. **Stationary Distributions:** the distributions are invariant with respect to the Markov process induced by the exogenous processes and all relevant policy functions.

B Computation

The computational algorithm to find the stationary equilibrium is as follows:

1. Given r , calculate q_b and q_m using (30) – (31).

2. **Loop 1** – Make an initial guess for the shadow housing price p .

(a) Solve for market tightnesses $\{\theta_b(x_b, h; p)\}$ and $\{\theta_s(x_s, h; p)\}$ using (34) – (35).

(b) Calculate the wage w and housing construction Y_h using (26) – (29).

(c) **Loop 2a** – Make an initial guess for the bank’s REO value function, $J_{REO}^0(h)$.

i. Substitute J_{REO}^0 into the right hand side of (32) and solve for $J_{REO}(h)$.

ii. If $\sup(|J_{REO} - J_{REO}^0|) < \epsilon_J$, exit the loop. Otherwise, set $J_{REO}^0 = J_{REO}$ and return to (i).

(d) **Loop 2b** – Make an initial guess for mortgage prices $q_m^{0,n}(m', b', h, s)$ for $n = 0$.

- i. Calculate the lower bound of the budget set for homeowners with good credit entering subperiod 3, $\underline{y}(m, h, s)$, by solving

$$\underline{y}(m, h, s) = \min_{m', b'} [\gamma p(h) + q_b b' + m - \widetilde{q}_m(m', b', h, s) m'], \text{ where}$$

$$\widetilde{q}_m(m', b', h, s) = \begin{cases} q_m^0(m', b', h, s) & \text{if } m' > m \\ q_m & \text{if } m' \leq m \end{cases}$$

- ii. **Loop 3** – Make an initial guess for $V_{rent}^0(y, s, f)$ and $V_{own}^0(y, m, h, s, f)$.
- A. Substitute V_{rent}^0 and V_{own}^0 into the right hand side of (20) – (21) and solve for R_{buy} .
 - B. Substitute V_{rent}^0 , V_{own}^0 , and R_{buy} into the right hand side of (22) and solve for W_{own} .
 - C. Substitute W_{own} , V_{rent}^0 , and R_{buy} into the right hand side of (24) – (25) and solve for R_{sell} .
 - D. Substitute W_{own} , V_{rent}^0 , R_{sell} , and R_{buy} into the right hand side of (15) – (19) and solve for V_{rent} and V_{own} .
 - E. If $\sup(|V_{rent} - V_{rent}^0|) + \sup(|V_{own} - V_{own}^0|) < \epsilon_V$, exit the loop. Otherwise, set $V_{rent}^0 = V_{rent}$ and $V_{own}^0 = V_{own}$ and return to A.
- iii. Substitute $q_m^{0,n}$, J_{REO} , and the household's policy functions for bonds, mortgage choice and selling and default decisions into the right hand side of (33) and solve for q_m^0 .
- iv. If $\sup(q_m^0 - q_m^{0,n}) < \epsilon_q$, exit the loop. Otherwise, set $q_m^{0,n+1} = (1 - \lambda_q) q_m^{0,n} + \lambda_q q_m^0$ and return to (i).
- (e) Compute the invariate distribution of homeowners and renters, Φ_{own} and Φ_{rent} , and the stock of REO houses, $\{H_{REO}\}_{h \in H}$.
 - (f) Calculate the excess demand for housing using (36) – (39).
 - (g) If $|D_h(p) - S_h(p)| < \epsilon_p$, exit the loop. Otherwise, update p using a modified bisection method and go back to (a).

The state space (y, m, h, s) for homeowners is discretized using 275 values for y , 131 values for m , 3 values for h , and 3 values for s . Homeowners with bad credit standing ($f = 1$) have state (y, h, s) , and renters have state (y, s) . To compute the equilibrium transition path, the algorithm starts with an initial guess for the path of shadow house prices, $\{p_{h,t}\}_{t=1}^T$. The algorithm then does backward induction on the REO value function, mortgage price equation, and the household Bellman equations before forward iterating on the distribution of households and REO properties. Equilibrium house prices (which depend on the current guess for the house price trajectory) are calculated period by period during the forward iteration. The initial guess is then compared with these equilibrium prices, and a convex combination of these sequences is used for the next guess. The process continues until convergence.

C Calibrating Labor Efficiency

As explained in section 3, it is impossible to estimate quarterly income processes from the PSID because it is annual data. Instead, a labor process is specified like that in [Storesletten et al. \(2004\)](#), except without life cycle effects or a permanent shock at birth. Their values are adopted for the annual autocorrelation of the persistent shock and for the variances of the persistent and transitory shocks and transformed into quarterly values.

Persistent Shocks It is assumed that in each period households play a lottery in which, with probability 3/4, they receive the same persistent shock as they did in the previous period, and with probability 1/4, they draw a new shock from a transition matrix calibrated to the persistent process in [Storesletten et al. \(2004\)](#) (in which case they still might receive the same persistent labor shock). This is equivalent to choosing transition probabilities that match the expected amount of time that households expect to keep their current shock. [Storesletten et al. \(2004\)](#) report an annual autocorrelation coefficient of 0.952 and a frequency-weighted average standard deviation over expansions and recessions of 0.17. The Rouwenhorst method is used to calibrate this process, which gives the following transition matrix:

$$\tilde{\pi}_s(\cdot, \cdot) = \begin{pmatrix} 0.9526 & 0.0234 & 0.0006 \\ 0.0469 & 0.9532 & 0.0469 \\ 0.0006 & 0.0234 & 0.9526 \end{pmatrix}$$

As a result, the transition matrix is

$$\pi_s(\cdot, \cdot) = 0.75I_3 + 0.25\tilde{\pi}_s(\cdot, \cdot) = \begin{pmatrix} 0.9881 & 0.0059 & 0.0001 \\ 0.0171 & 0.9883 & 0.0171 \\ 0.0001 & 0.0059 & 0.9881 \end{pmatrix}$$

Transitory Shocks [Storesletten et al. \(2004\)](#) report a standard deviation of the transitory shock of 0.255. To replicate this, it is assumed that the annual transitory shock is actually the sum of four, independent quarterly transitory shocks. The same identifying assumption as in [Storesletten et al. \(2004\)](#) is used, namely, that all households receive the same initial

persistent shock. Any variance in initial labor income is then due to different draws of the transitory shock. Recall that the labor productivity process is given by

$$\ln(e \cdot s) = \ln(s) + \ln(e)$$

Therefore, total labor productivity (which, when multiplied by the wage w , is total wage income) over a year in which s stays constant is

$$(e \cdot s)_{\text{year 1}} = \exp(s_0)[\exp(e_1) + \exp(e_2) + \exp(e_3) + \exp(e_4)]$$

For different variances of the transitory shock, total annual labor productivity is simulated for many individuals, logs are taken, and the variance of the annual transitory shock is computed. It turns out that quarterly transitory shocks with a standard deviation of 0.49 give the desired standard deviation of annual transitory shocks of 0.255.