The Yield Curve in a Small Open Economy

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Correlations with US rates (1993-2009)
Alan Bollard (2007), Easy Money: Global Liquidity and its Impact on New Zealand
Recursive Correlations between Australian and US rates
From 1993 Q1

Source: RBA, FRED. * HP filtered data
Has the yield curve decoupled?
In this paper,

1. we set up a two-country DSGE model with long-term interest rates;
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2. we explore if the model can produce the pattern of Figure 1;
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In this paper,

1. we set up a two-country DSGE model with long-term interest rates;

2. we explore if the model can produce the pattern of Figure 1;

3. then, we estimate the model;

4. and ask if the estimated model is consistent with the pattern of Figure 1.
The Model: Large Economy

\[ y_t^* = E_t y_{t+1}^* - \sigma^{-1} (R_{1,t}^* - E_t \pi_{t+1}^*) + \sigma^{-1} (1 - \rho_g) g_t^* \]  

(1)

\[ R_{m,t}^* = \frac{1}{m} E_t \sum_{j=1}^{m} R_{1,t+j-1}^* \quad m = 2, 3, 4, \ldots \]

(2)

\[ \pi_t^* = \lambda (\sigma + \varphi) y_t^* + \beta E_t \pi_{t+1}^* - \lambda (1 + \varphi) a_t^* \]

(3)

\[ R_{1,t}^* = \rho_r R_{1,t-1}^* + \alpha_{\pi}^* \pi_t^* + \alpha_y^* y_t^* + \varepsilon_{r,t}^* \]

(4)

\[ a_t^* = \rho_a^* a_{t-1}^* + \varepsilon_{a,t}^* \]

(5)

\[ g_t^* = \rho_g^* g_{t-1}^* + \varepsilon_{g,t}^* \]

(6)
The Model: Small Open Economy

\[ c_t = E_t c_{t+1} - \sigma^{-1} (R_{1,t} - E_t \pi_{t+1}) + \sigma^{-1} (1 - \rho_g) g_t \]  \hspace{1cm} (7)

\[ y_t = c_t + \frac{\alpha \omega}{\sigma} s_t \]  \hspace{1cm} (8)

\[ \pi_t = \pi_{h,t} + \alpha \Delta s_t \]  \hspace{1cm} (9)

\[ R_{m,t} = \frac{1}{m} E_t \sum_{j=1}^{m} R_{1,t+j-1} \hspace{1cm} m = 2, 3, 4, ... \]  \hspace{1cm} (10)

\[ \pi_{h,t} = \lambda [\varphi y_t + \sigma c_t + \alpha s_t] + \beta E_t \pi_{h,t+1} - \lambda (1 + \varphi) a_t \]  \hspace{1cm} (11)

\[ R_{1,t} = \rho_r R_{1,t-1} + \alpha \pi \pi_t + \alpha_y y_t + \varepsilon_{r,t} \]  \hspace{1cm} (12)
The Model: Small Open Economy II

\[ \Delta e_t = \Delta q_t + \pi_t - \pi_t^* \] \hspace{1cm} (13)

\[ \Delta q_t = (1 - \alpha) \Delta s_t \] \hspace{1cm} (14)

\[ c_t = y_t^* + \frac{1}{\sigma}(g_t - g_t^*) + \frac{(1 - \alpha)}{\sigma} s_t \] \hspace{1cm} (15)

\[ a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \] \hspace{1cm} (16)

\[ g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \] \hspace{1cm} (17)
Yield curves in the model economy

- Euler equations and risk sharing imply UIP:

\[ R_{1,t} = R^*_1 + E_t \Delta e_{t+1} \]
Yield curves in the model economy

- Euler equations and risk sharing imply UIP:

\[ R_{1,t} = R_{1,t}^* + E_t \Delta e_{t+1} \]

- and the expectations hypothesis equations imply UIP for all \( m \)

\[ R_{m,t} = R_{m,t}^* + \frac{1}{m} \sum_{j=1}^{m} E_t \Delta e_{t+j} \]
Monetary transmission and the yield curve (I)

\[ c_t = E_t c_{t+1} - \sigma^{-1} (R_{1,t} - E_t \pi_{t+1}) \]

\[ c_t = -\sigma^{-1} (R_{1,t} - E_t \pi_{t+1}) - \sigma^{-1} (E_t R_{1,t+1} - E_t \pi_{t+2}) + E_t c_{t+2} \]

\[ c_t = -\sigma^{-1} m \left( R_{m,t} - \frac{1}{m} E_t \sum_{j=1}^{m} \pi_{t+j} \right) \]
Monetary transmission and the yield curve (II)

$R_{1,t}$ → Expectations hypothesis → $R_{m,t}$ → Price stickiness

$\pi_t$ → $y_t$ → Monopolistic Competition → $AD_t$

$\epsilon_t$ shock

$rr_{m,t}$
A look at the model’s mechanisms...
# Symmetric Parametrization

<table>
<thead>
<tr>
<th>$\sigma^{-1}$</th>
<th>1.0</th>
<th>$\beta$</th>
<th>0.99</th>
<th>$\lambda$</th>
<th>0.3</th>
<th>$\alpha$</th>
<th>0.4</th>
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<tbody>
<tr>
<td>$\varphi$</td>
<td>0.9</td>
<td>$\omega$</td>
<td>1.0</td>
<td>$\rho^*_r$</td>
<td>0.8</td>
<td>$\rho_r$</td>
<td>0.8</td>
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<tr>
<td>$\alpha^*_\pi$</td>
<td>0.5</td>
<td>$\alpha_\pi$</td>
<td>0.5</td>
<td>$\alpha^*_y$</td>
<td>0.01</td>
<td>$\alpha_y$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho^*_g$</td>
<td>0.9</td>
<td>$\rho_g$</td>
<td>0.9</td>
<td>$\rho^*_a$</td>
<td>0.9</td>
<td>$\rho_a$</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Impulse response to a domestic shock to TFP
Impulse response to a domestic shock to TFP (II)

High Persistence: $\rho_a = 0.90$

$R_{1,t}$

$R_{40,t}$

$\pi_t$

$y_t$
Impulse response to a domestic shock to TFP (III)

Low Persistence: \( \rho_a = 0.50 \)

High Persistence: \( \rho_a = 0.90 \)

\( R_{1,t} \)

\( R_{40,t} \)

\( \pi_t \)

\( y_t \)
Intuition

- A positive technology shock increases the marginal product of labour.
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- If technology is persistent, real wages will stay relatively high for a long time.
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- If the shock’s persistence falls, then the expected income stream from higher real wages in the future would be lower and consumption would increase by less than it would otherwise.
Intuition

▶ A positive technology shock increases the marginal product of labour.

▶ If technology is persistent, real wages will stay relatively high for a long time.

▶ Consumption depends on permanent income.

▶ If the shock’s persistence falls, then the expected income stream from higher real wages in the future would be lower and consumption would increase by less than it would otherwise.

▶ For markets to clear, prices have to fall by more than they would otherwise to induce households to purchase the increased output.
Impulse response to a foreign demand shock

Low Persistence: $\rho^* g = 0.50$

High Persistence: $\rho^* g = 0.90$
Intuition

- A positive demand shock increases marginal utility.
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\[ E_0 \sum_{t=0}^{\infty} \beta^t e^g_t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right) \]

- If the shock is persistent the difference between the marginal utilities of \( t \) and \( t + 1 \) is small.
Intuition

- A positive demand shock increases marginal utility.

\[ E_0 \sum_{t=0}^{\infty} \beta^t e^{g_t} \left( \frac{C_{t}^{1-\sigma}}{1-\sigma} - \frac{N_{t}^{1+\varphi}}{1+\varphi} \right) \]

- If the shock is persistent the difference between the marginal utilities of \( t \) and \( t + 1 \) is small.

- If the shock has little persistence, households would have a strong incentive to consume more goods and have more leisure today.
Some observations about yield curve correlations

- Domestic shocks reduce the correlation between domestic and foreign variables.
- Foreign shocks increase the correlation between domestic and foreign variables.
- The larger the variance of domestic shocks, the lower the correlations between domestic and foreign interest rates at all maturities.
- More persistent shocks move long-term rates by more than less persistent shocks would; and less persistent shocks move short-term rates by a greater degree than more persistent shocks would.
Some observations about yield curve correlations

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2-Step Bayesian Estimation

- $\beta$ is set at 0.99, and $\alpha$ is set at 0.4.
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- First step: estimate foreign parameters using US data on $y_t^*$, $\pi_t^*$, and $R_{1,t}^*$ for the sample period 1983Q1-2008Q4.
2-Step Bayesian Estimation

- $\beta$ is set at 0.99, and $\alpha$ is set at 0.4.

- Specify priors for the model’s parameters.

- First step: estimate foreign parameters using US data on $y_t^*$, $\pi_t^*$, and $R_{1,t}^*$ for the sample period 1983Q1-2008Q4.

- Second step: taking the posterior mode of the foreign parameters as given from the first step, estimate the domestic parameters using data on $y_t$, $\pi_t$, and $R_{1,t}$ and $y_t^*$, $\pi_t^*$, and $R_{1,t}^*$ for the sample period 1993Q1-2008Q4.
2-Step Bayesian Estimation: the data

- Inflation
- Real GDP per capita
- 3-month nominal interest rate
- HP-Filtered data

Australia
United States
First Step: Large Economy

Table 2: **Large Economy**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior Mean</th>
<th>Posterior Mean</th>
<th>Posterior Mean</th>
<th>90 percent C.I.</th>
<th>Prior Density</th>
<th>Prior Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^{-1}$</td>
<td>1.500</td>
<td>1.485</td>
<td>1.786</td>
<td>[0.926 2.597]</td>
<td>Gamma</td>
<td>0.500</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.300</td>
<td>1.073</td>
<td>1.103</td>
<td>[0.822 1.381]</td>
<td>Normal</td>
<td>0.250</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.900</td>
<td>1.236</td>
<td>1.318</td>
<td>[0.913 1.725]</td>
<td>Gamma</td>
<td>0.200</td>
</tr>
<tr>
<td>$\rho^*_r$</td>
<td>0.900</td>
<td>0.906</td>
<td>0.903</td>
<td>[0.838 0.970]</td>
<td>Beta</td>
<td>0.050</td>
</tr>
<tr>
<td>$\alpha^*_\pi$</td>
<td>0.500</td>
<td>0.463</td>
<td>0.521</td>
<td>[0.382 0.659]</td>
<td>Normal</td>
<td>0.200</td>
</tr>
<tr>
<td>$\alpha^*_y$</td>
<td>0.250</td>
<td>-0.036</td>
<td>-0.041</td>
<td>[-0.069 -0.012]</td>
<td>Normal</td>
<td>0.100</td>
</tr>
<tr>
<td>$\rho^*_g$</td>
<td>0.500</td>
<td>0.858</td>
<td>0.862</td>
<td>[0.795 0.927]</td>
<td>Beta</td>
<td>0.150</td>
</tr>
<tr>
<td>$\rho^*_a$</td>
<td>0.500</td>
<td>0.909</td>
<td>0.891</td>
<td>[0.832 0.956]</td>
<td>Beta</td>
<td>0.150</td>
</tr>
</tbody>
</table>

**Standard Deviations**

| $\sigma_{\varepsilon^*_a}$ | 0.015 | 0.004 | 0.004 | [0.003 0.005] | Uniform | .009 |
| $\sigma_{\varepsilon^*_g}$ | 0.020 | 0.010 | 0.011 | [0.007 0.015] | Uniform | .011 |
| $\sigma_{\varepsilon^*_r}$ | 0.015 | 0.002 | 0.002 | [0.0017 0.003] | Uniform | .009 |
Second Step: Small Economy

Table 3: Small Economy

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior</th>
<th>Posterior</th>
<th>Posterior</th>
<th>90 percent C.I.</th>
<th>Prior Density</th>
<th>Prior Std.</th>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Mode</td>
<td>Mean</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>2.000</td>
<td>1.141</td>
<td>1.179</td>
<td>[0.855 1.496]</td>
<td>Gamma</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.900</td>
<td>0.929</td>
<td>0.915</td>
<td>[0.855 0.977]</td>
<td>Beta</td>
<td>0.050</td>
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<tr>
<td>$\alpha_\pi$</td>
<td>0.500</td>
<td>0.565</td>
<td>0.622</td>
<td>[0.441 0.796]</td>
<td>Normal</td>
<td>0.200</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>0.250</td>
<td>0.080</td>
<td>0.077</td>
<td>[0.014 0.139]</td>
<td>Normal</td>
<td>0.100</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.500</td>
<td>0.570</td>
<td>0.574</td>
<td>[0.426 0.726]</td>
<td>Beta</td>
<td>0.150</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.500</td>
<td>0.615</td>
<td>0.615</td>
<td>[0.467 0.767]</td>
<td>Beta</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Standard Deviations

| $\sigma_{\varepsilon_a}$ | 0.015 | 0.004 | 0.005 | [0.004 0.005] | Uniform | 0.009 |
| $\sigma_{\varepsilon_g}$ | 0.015 | 0.011 | 0.012 | [0.009 0.015] | Uniform | 0.009 |
| $\sigma_{\varepsilon_r}$ | 0.005 | 0.002 | 0.002 | [0.002 0.003] | Uniform | 0.003 |

studies. The persistence parameters, as we emphasized above, are of particular importance for the model to be able to match the pattern of Figure 1; our estimates for these parameters are similar to those of other studies on estimated small open economy models.

Figure 5 shows the prior and posterior densities for the AR(1) parameters that govern exogenous persistence. Our choice of prior distributions is the same for foreign and domestic exogenous processes. In this way, our priors are silent about differences in exogenous persistence. Notice, however, that the data shift the prior distributions of the foreign AR(1)s towards more persistent processes; as shown, the posterior distributions are fairly tight around highly persistent values. And although for the small economy, the data too prefer more persistent processes relative to the priors, the posterior distributions are centered around less persistent processes than those of the foreign economy.

4.2 Independent Evidence of Differences in Persistence

The estimates from Tables 2 and 3 suggest that US exogenous shock processes are considerably more persistent than their Australian counterparts. This result plays an important role, as we discussed above, in determining the model’s ability to match the reduced form cross-country interest rate correlations.

As additional evidence to support the plausibility of this result, we estimate the shock processes of Equations (5) (6), (16), and (17), directly from the data. We focus first on technology shocks. Using the fact that the model’s production function implies that output per capita is the product of hours worked per capita and technology, we construct a technology series for the US and Australia. We then regress the HP-filtered technology series...
Priors and Posteriors for the Large Economy

![Graphs showing priors and posteriors for various economic variables](image)

- invs
- lambda
- phi
- roers
- alpis
- alxs
Priors and Posteriors of the Small Economy

omega

roer

alpid

alx

roea

roex
Foreign and domestic persistence

Large economy

Small economy

posterior

prior

\( \rho_g \)

\( \rho_a \)
### Table 4: Independent evidence

<table>
<thead>
<tr>
<th>United States</th>
<th>Australia</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a^*_t$</td>
<td>0.83</td>
<td>0.81</td>
</tr>
<tr>
<td>$a_{t-1}$</td>
<td>(15.69)</td>
<td>(11.00)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.71</td>
<td>0.66</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g^*_t$</td>
<td>0.71</td>
<td>0.69</td>
</tr>
<tr>
<td>$g_{t-1}$</td>
<td>(10.19)</td>
<td>(7.35)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.50</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Note: t-statistics in brackets
### Table 5: **Standard Deviations**

<table>
<thead>
<tr>
<th></th>
<th>Model (posterior mode)</th>
<th>90 per cent Confidence Interval</th>
<th>Data (1993:1 - 2008:4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>0.0064</td>
<td>[0.0053 0.0087]</td>
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<td>$\pi_t$</td>
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<td>[0.0030 0.0052]</td>
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<td>$\Delta e_t$</td>
<td>0.0087</td>
<td>[0.0077 0.0097]</td>
<td>0.0465</td>
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<tr>
<td>$R_{1,t}$</td>
<td>0.0023</td>
<td>[0.0021 0.0026]</td>
<td>0.0019</td>
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<td>$R_{20,t}$</td>
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<td>[0.0004 0.0006]</td>
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<tr>
<td>$R_{40,t}$</td>
<td>0.0002</td>
<td>[0.0002 0.0003]</td>
<td>0.0018</td>
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Table 7: Statistically Significant Correlations

<table>
<thead>
<tr>
<th>Data (1993:Q1–2008:Q4)</th>
<th>$y_t$</th>
<th>$\pi_t$</th>
<th>$\Delta e_t$</th>
<th>$R_{1,t}$</th>
<th>$R_{20,t}$</th>
<th>$R_{40,t}$</th>
<th>$R^*_{1,t}$</th>
<th>$R^*_{20,t}$</th>
<th>$R^*_{40,t}$</th>
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</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>1.00</td>
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<tr>
<td>$\pi_t$</td>
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<td>1.00</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\Delta e_t$</td>
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<td></td>
<td>1.00</td>
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<td>$R_{20,t}$</td>
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<tr>
<td>$R_{40,t}$</td>
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<td>0.72</td>
<td>0.98</td>
<td>1.00</td>
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<tr>
<td>$R^*_{1,t}$</td>
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<tr>
<td>$R^*_{40,t}$</td>
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<td>0.71</td>
<td>0.64</td>
<td>0.96</td>
<td>1.00</td>
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<table>
<thead>
<tr>
<th>Model (posterior mode)</th>
<th>$y_t$</th>
<th>$\pi_t$</th>
<th>$\Delta e_t$</th>
<th>$R_{1,t}$</th>
<th>$R_{20,t}$</th>
<th>$R_{40,t}$</th>
<th>$R^*_{1,t}$</th>
<th>$R^*_{20,t}$</th>
<th>$R^*_{40,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
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</tr>
<tr>
<td>$\pi_t$</td>
<td></td>
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<td></td>
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<tr>
<td>$R_{40,t}$</td>
<td></td>
<td>0.26</td>
<td></td>
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<td>1.00</td>
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<tr>
<td>$R^*_{1,t}$</td>
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<td></td>
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<td>0.71</td>
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<td>$R^*_{20,t}$</td>
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<td>0.71</td>
<td>0.71</td>
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<td>0.70</td>
<td>0.71</td>
<td>0.99</td>
<td>1.00</td>
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Back to Figure 1: Model at posterior mode

Data (1993:Q1 --- 2008:Q4)
Model (at posterior mode)

3-month
5-year
10-year

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Prior and Posterior
Deviations from UIP

- Take \( R_{1,t} = R^*_{1,t} + E_t \Delta e_{t+1} + \phi_t \)
Deviations from UIP

- Take $R_{1,t} = R_{1,t}^* + E_t \Delta e_{t+1} + \phi_t$

- with $\phi_t = \rho_{\phi} \phi_{t-1} + \varepsilon_{\phi,t}$ and $\varepsilon_{\phi,t}$ is iid zero-mean $\sigma_{\varepsilon_{\phi}}$
Deviations from UIP

- Take \( R_{1,t} = R_{1,t}^* + E_t \Delta e_{t+1} + \phi_t \)

- with \( \phi_t = \rho \phi \phi_{t-1} + \varepsilon_{\phi,t} \) and \( \varepsilon_{\phi,t} \) is iid zero-mean \( \sigma_{\varepsilon_{\phi}} \)

- At \( m \) we have \( R_{m,t} = R_{m,t}^* + \frac{1}{m} \sum_{j=1}^{m} E_t \Delta e_{t+j} + \frac{1}{m} \frac{1-\rho^m}{1-\rho} \phi_t \).
Deviations from UIP

- Take $R_{1,t} = R_{1,t}^* + E_t \Delta e_{t+1} + \phi_t$

- with $\phi_t = \rho_{\phi} \phi_{t-1} + \varepsilon_{\phi,t}$ and $\varepsilon_{\phi,t}$ is iid zero-mean $\sigma_{\varepsilon_{\phi}}$

- At $m$ we have $R_{m,t} = R_{m,t}^* + \frac{1}{m} \sum_{j=1}^{m} E_t \Delta e_{t+j} + \frac{1}{m} \frac{1-\rho_{\phi}^m}{1-\rho_{\phi}} \phi_t$

- For all $\rho_{\phi} \in (0, 1)$, $\lim_{m \to \infty} \frac{1-\rho_{\phi}^m}{m} \frac{1-\rho_{\phi}}{1-\rho_{\phi}} = 0$. 
Conclusion

- The model explains the co-movement of interest rates of different maturities and currencies that we observe in the data.
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- And in the background, the expectations hypothesis and uncovered interest rate parity hold!
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- Yield curve data were not used for estimation.
Is *theory* consistent with the pattern of Figure 1?

Yes.