Understanding Homeland Security: Theory and UK-Evidence

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Abstract

Recent years have seen governments restricting civic freedoms and legislating significant increases in spending to combat terrorist activities. In this paper we investigate the relationship between anti-terror spending and terrorism. In line with previous findings in the empirical literature on terrorist activity, our game-theoretic model of the interaction between a benevolent government and a terrorist organization is suggestive of a non-linear relation between terrorism and counter-terrorism spending. Using UK data, our empirical Markov-switching implementation provides evidence in favor of this approach.

Keywords: Defense Spending; Markov Switching; Terrorism; VAR

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1 Introduction and Literature

There is no meeting of minds, no point of understanding with such terror. Just a choice, defeat it or be defeated by it. And defeat it we must.
(Tony Blair)

Terrorism is without doubt one of the most serious security issues in the world today. Devastating events including the recent attacks in New York, London, Madrid, and Mumbai have placed the fight against terrorism highly on the political agenda. As a consequence, many governments reacted not only by limiting civil rights and individual freedoms but also by raising their budgets for anti-terror spending. The United Kingdom's security service MI5, for instance, reports that total annual expenditure on counter-terrorism and intelligence is scheduled to increase from £2.5 billion in 2007/08 to £3.5 billion by 2010/11.[1] Being far from constituting an isolated case, the UK example raises the question about the relationship between the extent of terror activities and the level of anti-terror spending by governments. We analyze this issue by setting up a game-theoretic model of the interaction between a terrorist organization and a benevolent government and subject the model's predictions to time series data on defense expenditures and terror events. In particular, we accommodate existing evidence of non-linearities in univariate time-series data on the intensity of terrorism in both, our theoretical and empirical modeling of the relationship between terror and defense spending.[2]

The theoretical model distinguishes between two different states of terror, a high and a low terror state, whose probabilities of materializing are determined by the representative terrorist organization's investment in costly terror activities. The government receives an informative but noisy signal about imminent terror events and subsequently decides on how to counteract terror in choosing between a high and a low level of anti-terror spending. This generates four states of the world, featuring high or low terror paired with either high or low anti-terror spending. We determine the endogenous probabilities of transiting between the different states from one period to the next. These transition probabilities are all positive, and they depend on model parameters, as given by the quality of the signal the government receives, the extent of damages from a successful terror attack, and terrorist's marginal costs of investment.

[1]The data were taken from the MI5's web presence (www.mi5.gov.uk). Another case in point is US anti-terror spending. Quoting a report by the Congressional Budget Office, the NY Times writes that the federal government had spent $37 billion in response to the Sept. 11 attacks and was on track to spend more than 10 times that much to counter terrorism over the decade (NYT, Sep 06 2002).

[2]See Enders et al. (1992) as well as Enders and Sandler (2000, 2005) for such evidence.

For example, if the signal quality improves, the government's policy response is more likely to be in line with the actual terror situation. This reduces the expected benefits from terrorist investments and thereby lowers the likelihood of terror. Our model also incorporates an intertemporal element of anti-terror spending in that a part of this spending is preemptive and, as such, only becomes effective with a one-period lag. As a consequence, high government spending in period t raises the probability of transiting to a low terror state in period $t+1$. The terror state, in turn, does not influence transition probabilities. Accounting for potential inertia in political decision making, we generalize our baseline model in allowing for persistence in the level of defense spending.

In our empirical implementation, we apply a Markov-switching model, which matches the structure of the theoretical set-up. We use quarterly data on terrorist activity and defense spending in the United Kingdom from 1983 to 2006. The empirical framework controls not only for the potential endogeneity between terrorism and government policy (see e.g. Enders and Sandler, 1991) but also for the aforementioned likely non-linear relationship between the two variables. The empirical findings generally support the predictions of the theoretical model. We identify states of high and low terror as well as states of high and low spending, and the probabilities of transiting between these states correspond well with the model's predictions. In particular, higher defense spending seems to translate into a decline in the probabilities of states with high terror, whereas we cannot identify a sizable influence of terror on the different transition probabilities. We also find that the levels of terror in the UK over the observed period differ substantially between phases of high and of low terrorism.

The economics literature has analyzed anti-terror policies in a variety of ways, with some approaches being of a decision theoretic nature, whereas others allow for strategic interaction between the government and terrorists in a game theoretic setting. Rübbelke (2005), for example, belongs to the first group of models. He employs various specifications of a potential terrorist's decision problem to explore the government's options in combating terrorism. In a similar vein, Islam and Shahin (1989) as well as Shahin and Islam (1992) study hostage taking. Of those papers which allow for strategic interaction between terrorists and the government, Selten (1977) and Lapan and Sandler (1988) investigate sequential games between kidnappers and the government. Faria (2003), building on Feichtinger et al. (2001), presents a theoretical model in which a terrorist group and the government solve dynamic optimization problems. He shows that a cyclical pattern of enforcement and terrorist activities may emerge in this framework. Das (2008) also studies a dynamic model of terror cycles introducing 'fear' as a stock variable, which is influenced by activities of terrorists and security deterrence measures by the government.

On the empirical side, studies on the effects of terror on economic outcomes suggest a large and negative impact, which may be explained by the fact that terror not only destroys the domestic capital stock, but also harms foreign capital inflows and trade.[3] Other papers have examined the effect of economic activity on terrorism or both causal directions.[4] Enders and Sandler (2005) use a threshold autoregression (TAR) model to show that the autoregressive nature of casualties from terror depends on the level of terrorism at the time of a shock. In their study, the TAR model outperforms a standard autoregressive representation. This suggests that the effects of terrorism on defense spending and vice versa are also better investigated by using non-linear models. Drakos and Giannakopoulos (2009) examines the success of anti-terror spending by estimating the impact of the level of government spending on the probability of a terror attack being stopped.

Our study contributes to the literature by incorporating the aforementioned non-linear evidence into a novel theoretical approach, which motivates the empirical Markov-switching model, and apply empirical method to examine the relationship between terror activities and the anti-terror spending. The paper proceeds as follows: Section 2 presents the theoretical model, section 3 describes the empirical analysis, and section 4 concludes.

2 Theoretical Model

2.1 Model Set-up

This section provides a theoretical foundation for the subsequent empirical analysis. We set up a game-theoretic model with imperfect information, which determines the relationship between the levels of terrorism and anti-terror spending by the strategic interaction of a terrorist organization and the government. In order to accommodate the structure of the Markov-switching model employed in the empirical analysis, we construct a stochastic set-up that allows for four different states of the world to arise in equilibrium. Thereby the economy's state is a consequence of the government's choice of either high or low anti-terror spending and the terror organization's level of

[3]See Abadie and Gardeazabal (2003, 2008), Nitsch and Schumacher (2004), Eckstein and Tsiddon (2004), and Blomberg et al. (2004).

[4]See, for instance, Enders and Sandler (1991), Faria (2003), Abadie (2006), or Santos-Bravo and Mendes-Dias (2006).

investment in terror activities, which may result in either a high or low terror state.

More specifically, we study a two period game $t \in \{1, 2\}$ between a government G, standing in for the representative agent, a terrorist organization T, and intermittent moves by nature N. The government's per period payoff is given by

$$U_t^G = 1 - b_t - d_t \ , \tag{1}$$

where b_t denotes the budget for anti-terror spending and d_t the damage from terror attacks in period t. In each period, the government chooses between two spending levels: high spending and low spending; that is, $b_t \in \{\underline{b}; \bar{b}\}$ and $0 \le \underline{b} < \bar{b} \le 1$.

We model terror as a binary random variable $\omega_t \in \{\omega^h; \omega^l\}$, with ω^h and ω^l respectively denoting the high and the low terror state. In the high terror state, a terror attack occurs with certainty, whereas there is no terror in the low terror state.[5] For the damage from terror d_t, we assume that

$$d_t(\omega_t) = \begin{cases} [1 - (1-\gamma)\, b_t - \gamma b_{t-1}]\, D & \text{if } \omega_t = \omega^h \\ 0 & \text{if } \omega_t = \omega^l, \end{cases}$$

where D denotes the exogenous gross damage of a terror attack. Anti-terror spending is assumed to reduce the damage resulting from a terror attack. Thereby we allow it to have defensive and proactive effects. The parameter $\gamma \in (0, 1)$ measures the share of preemptive spending in the government's anti-terror budget. In particular, we assume that a fraction $(1 - \gamma)b_t$ of the spending in period t is effective contemporaneously, whereas the other part γb_t only becomes effective with a one-period lag and therefore diminishes the impact of terror in the following period. The government's defensive $(1-\gamma)b_t$ and preemptive γb_{t-1} anti-terror spending from today and yesterday, respectively, reduce the gross damage D in the high terror state to a net damage of $d_t(\omega^h)$.[6]

The probability of $\omega_t = \omega^h$ is denoted by π_t, and it is determined by terrorists' investment $I_t \in [0, 1]$ in costly terror related activities. These activities may include building up and maintaining a terror network, training of terrorists, working out strategies and preparing attacks etc.[7] The higher is I_t, the higher is the probability of a terror attack in period t. For analytical convenience, we assume that $\pi_t = I_t$ and choose a

[5]The normalization of low terror to zero and high terror to one is made to simplify the exposition. It does not drive any of our results.

[6]Instead of this specification, we could equally well assume that government spending reduces the likelihood of a successful terror attack, given that terror is high. In this case, the term $d_t(\omega^h)$ would be the expected damage from terror in the high terror state.

[7]We abstract from the build-up of a terror capital stock implicitly assuming a rate of depreciation equal to one.

quadratic specification for the costs of terror investment $C(I_t) = c[I_t]^2/2$. The per-period payoff of the terrorist organization is equal to the difference between the damage d_t inflicted on the public by a terror attack and the costs of investment $C(I_t)$; that is,

$$U_t^T = d_t - c[I_t]^2/2 \; . \tag{2}$$

When deciding about its anti-terror budget, the government cannot directly observe the terror state ω_t. However, before the government sets b_t, it receives a signal $\tilde{\omega}_t \in \{\tilde{\omega}_t^h, \tilde{\omega}_t^l\}$ about the realization of ω_t. The signal may be interpreted as a secret service report on the activities and plans of the terror organization. With probability $\alpha \in (1/2; 1)$ the signal $\tilde{\omega}$ is correct, such that it represents the true state of ω; with probability $1-\alpha$ the government receives a false signal.[8] The higher is α, the more informative is the signal. For $\alpha \to 1$ the signal reveals the terror regime with certainty, whereas for $\alpha \to 1/2$ the signal becomes uninformative. After having observed the signal, the government updates its information regarding the occurrence of a terror attack according to Bayes' Rule. Hence, given its prior π_t, the posterior $\hat{\pi}_t(\tilde{\omega}_t)$ is determined by

$$\hat{\pi}_t(\tilde{\omega}^h) = \frac{\pi_t \alpha}{\pi_t \alpha + (1-\pi_t)(1-\alpha)} \qquad \text{or} \qquad \hat{\pi}_t(\tilde{\omega}^l) = \frac{\pi_t (1-\alpha)}{\pi_t (1-\alpha) + (1-\pi_t)\alpha} \; , \tag{3}$$

where π_t depends on the level of investment I_t by the terror organization. The probabilities π_t and α are common knowledge.

The previous paragraphs describe a sequential game between the government and the terrorist organization with intermittent moves by nature. Figure 1 illustrates the sequence of moves in period 1. At the beginning of the period, terrorists T decide about the continuous variable I_1. Subsequently nature N determines the terror regime $\omega_1 \in \{\omega_1^h, \omega_1^l\}$, with the probability of the high terror state arising given by $\pi_1 = I_1$. A second move by nature N determines the signal $\tilde{\omega} \in \{\tilde{\omega}^h, \tilde{\omega}^l\}$ that the government G receives. Upon observing the signal, the government chooses the level of anti-terror spending $b_1 \in \{\underline{b}, \bar{b}\}$. Towards the end of period 1, the state of terror is revealed and first period payoffs are realized. In the second period, the sequence of moves is identical to that in the first period.[9]

[8]That is, $Prob(\tilde{\omega} = \tilde{\omega}^h | \omega = \omega^h) = \alpha$ and $Prob(\tilde{\omega} = \tilde{\omega}^l | \omega = \omega^h) = 1 - \alpha$.

[9]The complete game tree is an extension of the tree in figure 1 in which a copy of the period 1 tree is attached to every ultimate branch of the tree in figure 1.

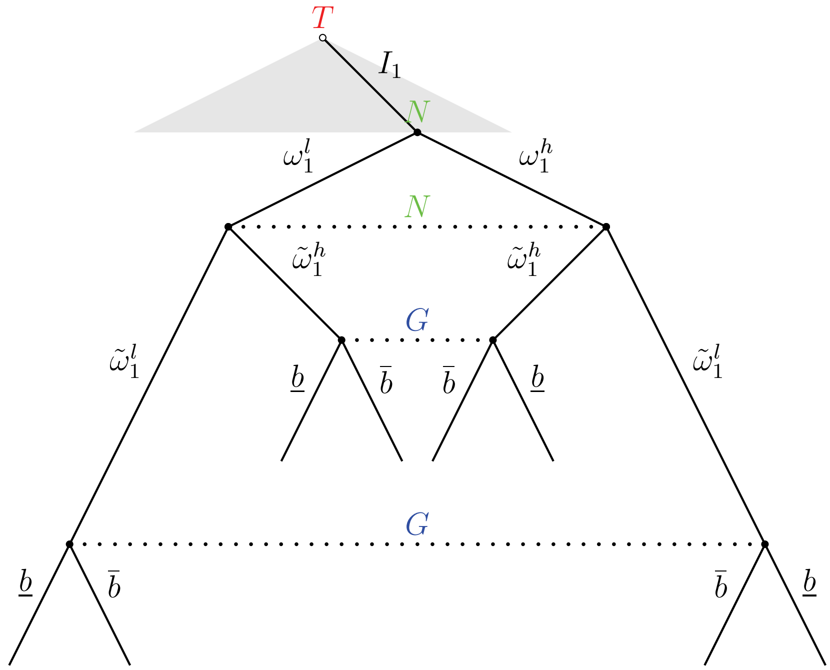

Figure 1: Sequence of Moves in Period 1

2.2 Equilibrium

In the spirit of backward induction, we solve this game from the end. Hence, in period 2, upon observing the signal indicating the current-period terror regime, the government chooses the optimal level of anti-terror spending:

$$\max_{b_2} \; E\left[U_2^G(b_2)|\,\tilde{\omega}\right] = 1 - \hat{\pi}(\tilde{\omega})\left[1 - \gamma b_1 - (1-\gamma)b_2\right] D - b_2 \quad \text{s.t.} \quad b_2 \in \{\underline{b}, \bar{b}\}\ . \tag{4}$$

The optimal spending level b_2 is signal-dependent and generates policies $b_2 = b_2(\tilde{\omega})$. In particular, the government sets $b_2 = b_2(\tilde{\omega}^h)$ if the signal is $\tilde{\omega}^h$ and $b_2 = b_2(\tilde{\omega}^l)$ upon observing $\tilde{\omega}^l$. Depending on parametric conditions, there are three conceivable policies: low spending $b_2(\tilde{\omega}^h) = b_2(\tilde{\omega}^l) = \underline{b}$, high spending $b_2(\tilde{\omega}^h) = b_2(\tilde{\omega}^l) = \bar{b}$, as well as mixed spending policies $b_2(\tilde{\omega}^h) = \bar{b}$ and $b_2(\tilde{\omega}^l) = \underline{b}$. In light of the following empirical application, we focus on the mixed spending case in which the government chooses high spending after having received a high terror signal and low spending after a low terror signal. For mixed spending policies to arise in equilibrium, it is necessary that $\pi_2 \in [\underline{\pi}_2, \bar{\pi}_2]$, where the reservation priors leave the government indifferent between low spending and mixed spending in the case of $\underline{\pi}_2$, and between mixed spending and high spending in the case of $\bar{\pi}_2$.[10] For a high prior ($\pi_2 > \bar{\pi}_2$), the government chooses a high level of anti-terror spending no matter what the signal. Similarly, for a low prior ($\pi_2 < \underline{\pi}_2$) the government chooses a low level of anti-terror spending. Only for intermediate values of the prior does the signal have an influence on the government's policy choice. The reservation priors are given by

$$\underline{\pi}_2 = \frac{1-\alpha}{1-\alpha+\alpha(x-1)} \quad \text{and} \quad \bar{\pi}_2 = \frac{\alpha}{\alpha+(1-\alpha)(x-1)}\ .^{11} \tag{5}$$

The constant $x \equiv (1-\gamma)D$ in (5) denotes the marginal benefit of defensive spending in period 2 conditional on a terror attack occurring. We assume that $x > 1$, i.e. we consider a setting in which the marginal benefit of defense spending exceeds its marginal costs of 1.[12] This assumption ensures that $0 < \underline{\pi}_2 < \bar{\pi}_2 < 1$ and provides the government with incentives to engage in defensive spending. We summarize, stating the following result:

$$b_2(\tilde{\omega}^l) = \underline{b} \quad \text{and} \quad b_2(\tilde{\omega}^h) = \bar{b} \quad \text{iff} \quad \underline{\pi}_2 \leq \pi_2 \leq \bar{\pi}_2\ . \tag{6}$$

Figure 2 illustrates the three cases which may result with respect to government spending.

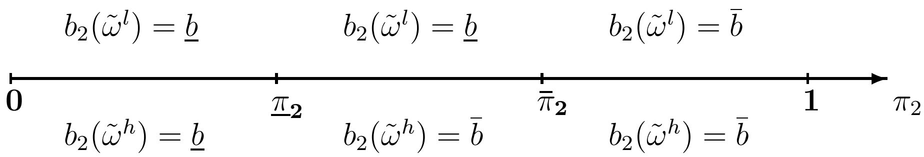

Figure 2: Prior Terror Probabilities and Anti-Terror Spending

As intuition suggests, the range of prior probabilities $[\underline{\pi}_2, \bar{\pi}_2]$ consistent with mixed spending policies in the second period increases in the quality of the signal α. In the limit $\alpha \to 1$, as the signal becomes state revealing, we have $\underline{\pi}_2 = 0$ and $\bar{\pi}_2 = 1$ such that only the mixed spending equilibrium exists. In contrast, as the signal becomes

[10] We assume that the government chooses mixed spending in the case of indifference between two policies. Therefore, we have a mixed spending equilibrium if $E[U_2^G(\bar{b})|\,\tilde{\omega}^l] \leq E[U_2^G(\underline{b})|\,\tilde{\omega}^l]$ and $E[U_2^G(\bar{b})|\,\tilde{\omega}^h] \geq E[U_2^G(\underline{b})|\,\tilde{\omega}^h]$ are satisfied.

[11] The probability $\bar{\pi}_2$ is obtained by inserting from (3) into $E[U_2^G(\bar{b})|\,\tilde{\omega}^l] = E[U_2^G(\underline{b})|\,\tilde{\omega}^l]$ and solving for π_2. The probability $\underline{\pi}_2$ can be derived in an analogous manner.

[12] With $x < 1$, the government would never choose to defend itself, even if it expected high terror with certainty.

uninformative, i.e. for $\alpha \to 1/2$, we have $\underline{\pi}_2 = \bar{\pi}_2$ in the limit such that there is a unique value of the prior for which the government is indifferent between high, low and mixed spending policies. An increase in the marginal benefit of defense spending x shifts both critical probabilities to the left, as it renders high defense spending more attractive for the government.

The terrorist organization, anticipating the behavior of the government, chooses the investment level I_2 to maximize its expected payoff. Employing $\pi_2 = I_2$ and assuming, for the time being, that the parametric condition (6) is satisfied, the terrorist organization's problem at the beginning of period 2 is

$$\max_{\pi_2} E\left[U_2^T(\pi_2)\right] = \pi_2\left[1 - \gamma b_1 - (1-\gamma)\left(\alpha \bar{b} + (1-\alpha)\underline{b}\right)\right] D - \frac{c\left[\pi_2\right]^2}{2} . \tag{7}$$

The first order condition for the terrorists then determines the period 2 equilibrium investment level and prior probability $\pi_2^* = I_2^*$ as

$$\pi_2^* = [1 - \gamma b_1 - (1-\gamma)(\alpha \bar{b} + (1-\alpha)\underline{b})]D/c = (\delta_1 - \gamma b_1)D/c , \tag{8}$$

where $\delta_1 \equiv 1 - (1-\gamma)\left(\alpha \bar{b} + (1-\alpha)\underline{b}\right)$ and $\gamma \bar{b} < \delta_1 < 1$. This interior solution is, of course, only feasible if $\underline{\pi}_2 \leq \pi_2^* \leq \bar{\pi}_2$, and we assume parameters such that this is the case.[13]

Terror investments and thereby the probability of a terror attack increase in the potential damage D and decrease in the marginal cost parameter c. An improvement in the signal quality α also reduces terror investments. Better information improves the reaction of the government to a state of high terror such that the incentives for terrorists to invest in terror activities decline.

Note also that the terror probability in the second period π_2^* depends on the level of defense spending in the first period, that is, $\pi_2^* = \pi_2^*(b_1)$. A higher spending level in period 1 reduces the amount of investment in terror related activities and therefore the probability of a terror attack in period 2. Thus, by influencing the terrorists' behavior tomorrow, government spending today has a positive effect on expected welfare today and tomorrow. The beneficial effect from preemptive spending in period 1 is also

[13]However, fulfillment of the first order condition is not sufficient for an equilibrium. The terrorists might have an incentive to reduce π_2 to a value below $\underline{\pi}_2$ such that the government chooses a low spending level $b_2 = \underline{b}$ irrespective of the signal $\tilde{\omega}_2$. Appendix A discusses this case in more detail.

evident from the expected indirect utility evaluated on the eve of period 2:

$$E[V_2^G(b_1)] = \pi_2^*(b_1)\left[\alpha E[U_2^G(b_1)|\,\tilde{\omega}^h] + (1-\alpha)E[U_2^G(b_1)|\,\tilde{\omega}^l]\right] + \\ [1-\pi_2^*(b_1)]\left[\alpha E[U_2^G(b_1)|\,\tilde{\omega}^l] + (1-\alpha)E[U_2^G(b_1)|\,\tilde{\omega}^h]\right] \; . \qquad (9)$$

More preemptive spending in period 1 increases tomorrow's expected utility in two ways: First, it lowers the damages from terror, given a terror attack occurs, and second, it reduces the likelihood of the high terror state through its effect on terrorists' investment in period 2. This can be seen from inserting for the terror probability π_2^* and the spending policy in period 2 into (9):

$$E\left[V_2^G(b_1)\right] = 1 - (\delta_1 - \gamma b_1)^2 D^2/c - E\left[b_2\right] \; , \quad \text{where} \qquad (10)$$

$$E\left[b_2\right] = \underline{b} + (1-\alpha)\left(\bar{b} - \underline{b}\right) + (\delta_1 - \gamma b_1)(2\alpha - 1)\left(\bar{b} - \underline{b}\right) D/c \; .$$

From (10) we see that the government's expected payoff in period 2 increases in the spending level b_1 in period 1. In light of solving the government's optimization problem in period 1, we may write the difference in expected utilities in period 2 from high $b_1 = \bar{b}$ and low $b_1 = \underline{b}_1$ anti-terror spending in period 1 as

$$\begin{aligned} \Delta E[V_2^G] =& E[V_2^G(\bar{b})] - E[V_2^G(\underline{b})] \\ =& \gamma\left[\bar{b} - \underline{b}\right]^2 (2\alpha - 1) D/c \\ &+ \left[2 - 2\underline{b} - \gamma(\bar{b} - \underline{b}) - 2\alpha(1-\gamma)(\bar{b} - \underline{b})\right]\gamma(\bar{b} - \underline{b}) D^2/c \; , \end{aligned}$$

where the first term captures tomorrow's expected defense expenditure savings, and the second term shows tomorrow's expected reduction in damages from a terror attack. The difference in expected utilities can be written more compactly as

$$\Delta E[V_2^G] = \gamma\left[\delta_2 + \delta_3\right](\bar{b} - \underline{b}) > 0 \; , \qquad (11)$$

where δ_2 and δ_3 are positive constants.[14] Consistent with the previous discussion, we find that $\Delta E[V_2^G]|_{\gamma=0} = 0$ and $\partial \Delta E[V_2^g]/\partial\gamma > 0$; that is, the higher is the share of preemptive spending in today's defense budget, the larger is tomorrow's welfare gain from raising the spending level. We restrict our attention to parametric conditions such that $\gamma(\delta_2 + \delta_3) < 1$. This inequality is satisfied if, for instance, γ is sufficiently

[14] We have $\delta_2 \equiv \left[2\delta_1 - \gamma(\bar{b} + \underline{b})\right] D^2/c > 0$ and $\delta_3 \equiv (2\alpha - 1)(\bar{b} - \underline{b}) D/c > 0$.

small. In that case, the additional benefits for the government in period 2, which result from high spending in period 1, are smaller than the additional costs. Otherwise, the effects on the outcome in the subsequent period alone would induce the government to choose high spending in period 1 irrespective of the terror signal.

Turning to the determination of the government's optimal spending decisions in the first period, the government's optimization problem after having observed the first period signal is given by

$$\max_{b_1} E\left[U_1^G(b_1)|\tilde{\omega}\right] + E\left[V_2^G(b_1)\right] \qquad \text{s.t.} \quad b_1 \in \{\underline{b}, \bar{b}\} \ .$$

In analogy to the government's second period problem, there are three possible first period policies – low, mixed, and high spending – depending on parametric conditions. Hence, there are first period reservation priors $\underline{\pi}_1$ and $\bar{\pi}_1$, where

$$\underline{\pi}_1 \equiv \frac{1-\alpha}{1-\alpha+\alpha(y-1)} \quad \text{and} \quad \bar{\pi}_1 \equiv \frac{\alpha}{\alpha+(1-\alpha)(y-1)} \ , \tag{12}$$

such that the government chooses mixed spending policies for values $\pi_1 \in [\underline{\pi}_1, \bar{\pi}_1]$ of the terror prior.[15] The constant y in (12) is defined by $y \equiv x/[1-\gamma(\delta_2+\delta_3)]$, where x still captures the marginal benefit of period 2 defense spending. Our assumption $\gamma(\delta_2+\delta_3) < 1$ implies $y > x$ and $0 < \underline{\pi}_1 < \bar{\pi}_1 < 1$. Interpreting y in analogy to x, we find that the marginal benefit of defense spending in the first period is larger than in the second period.[16] Given the augmented marginal benefit from defense spending in the first period, it is clear that $\underline{\pi}_1 < \underline{\pi}_2$ and $\bar{\pi}_1 < \bar{\pi}_2$ for $\gamma > 0$. As a consequence, the set of priors consistent with high anti-terror spending decisions is larger in the first than in the second period; that is, the government has stronger incentives to choose a high spending level in the first period. Figure 2.2 illustrates this result.

Turning to the terrorist organization's first period investment decision, we can determine the terrorists' expected indirect utility evaluated on the eve of period 2 as follows:

$$\begin{aligned} E\left[V_2^T(b_1)\right] &= \pi_2^*(b_1)\,(\delta_1-\gamma b_1)\,D - c\,[\pi_2^*(b_1)]^2\,/2 \\ &= (\delta_1-\gamma b_1)^2\,D^2/(2c) \ . \end{aligned} \tag{13}$$

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[15] More precisely, for the government to choose mixed spending policies, it is necessary that $E\left[U_1^G(\bar{b})|\tilde{\omega}^h\right] - E\left[U_1^G(\underline{b})|\tilde{\omega}^h\right] \geq -\Delta E\left[V_2^G\right]$ and $E\left[U_1^G(\underline{b})|\tilde{\omega}^l\right] - E\left[U_1^G(\bar{b})|\tilde{\omega}^l\right] \geq \Delta E\left[V_2^G\right]$; the former condition implies $b_1(\tilde{\omega}^h) = \bar{b}$ and the latter yields $b_1(\tilde{\omega}^l) = \underline{b}$. The reservation priors $\bar{\pi}_1$ and $\underline{\pi}_1$ are obtained from replacing the inequalities with equalities and solving the equations for π_1.

[16] Clearly, the fact that we restrict the model to two periods drives this finding.

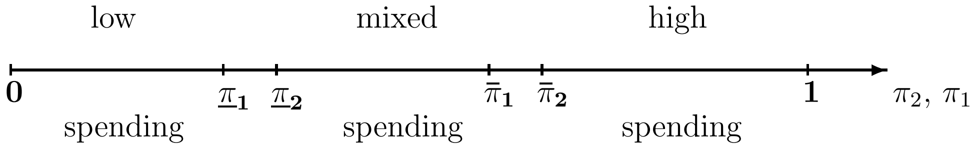

Figure 3: Situation in Period 1

Defining $\Delta E\left[V_2^T\right]$ as the difference in expected utilities of the terrorist organization, that is, $\Delta E\left[V_2^T\right] \equiv E\left[V_2^T(\bar{b})\right] - E\left[V_2^T(\underline{b})\right]$, we obtain

$$\Delta E\left[V_2^T\right] = -\gamma\delta_2\left(\bar{b} - \underline{b}\right)/2 < 0\ . \tag{14}$$

An increase in government spending in the first period lowers the effectiveness of terror attacks in the second period and hence reduces the expected payoff of the terrorists.

Assume now that the terrorist organization's optimal choice of π_1 satisfies $\pi_1^* \in (\underline{\pi}_1, \bar{\pi}_1)$. In analogy to the second period treatment, we focus on a prior choice by the terrorists implementing mixed spending policies by the government. The terrorist organization's optimization problem, using $\pi_1 = I_1$, at the beginning of period 1 is given by[17]

$$\max_{\pi_1}\ E\left[U_1^T(\pi_1)\right] + E\left[V_2^T(b_1)\right], \tag{15}$$

and the following equilibrium prior probability π_1^* results from the corresponding first order condition:

$$\pi_1^* = (\delta_1 - \gamma b_0)\,D/c - \gamma\delta_2\left(\bar{b} - \underline{b}\right)(2\alpha - 1)/2\ . \tag{16}$$

From the comparison of the terror probabilities in (16) and (8) across periods, it is evident that an additional effect influences the terrorists' behavior in the first period: In choosing a low level of π_1^*, terrorists reduce the likelihood of a high terror signal in the first period and thereby also the expected level of defensive spending in the second period. This effect lowers the terror investment in the first period compared to the second period.

[17]Note that $E\left[U_1^T(\pi_1)\right] = \pi_1\alpha\left[1 - \gamma b_0 - (1-\gamma)\bar{b}\right]D + \pi_1(1-\alpha)\left[1 - \gamma b_0 - (1-\gamma)\underline{b}\right]D - c\pi_1^2/2$, and that the expected payoff in period 2 can be written as $E\left[V_2^T(b_1)\right] = E\left[V_2^T(\underline{b})\right] + \left[\alpha\pi_1 + (1-\alpha)(1-\pi_1)\right]\Delta E\left[V_2^T\right]$.

2.3 Implications for the Empirical Model

This section derives the transition matrices implied by our theoretical model. The matrices can be read as follows:

$$\Pi = \begin{array}{c} \\ LL \\ HL \\ LH \\ HH \end{array} \begin{array}{c} \begin{array}{cccc} LL & HL & LH & HH \end{array} \\ \begin{pmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & \Pi_{34} \\ \Pi_{41} & \Pi_{42} & \Pi_{43} & \Pi_{44} \end{pmatrix} \end{array} . \tag{17}$$

Rows correspond to first period states of the economy, while columns collect the second period states. The expression LL indicates a state of low terror and low defense spending, HL corresponds to high-terror and low defense spending, LH denotes low terror and high defense spending, and finally HH accounts for the high terror and high spending state. Correspondingly, element Π_{22} of matrix (17) denotes the probability of transiting from high terror and low spending today to high terror and low spending tomorrow, and element Π_{23} is defined as the probability of going from high terror and low spending to low terror and high spending.

The following equation (18) shows the transition matrix based on our theoretical model outlined in section 2.2:

$$\Pi_1 = \begin{pmatrix} (1-\pi_1^h)\alpha & \pi_1^h(1-\alpha) & (1-\pi_1^h)(1-\alpha) & \pi_1^h\alpha \\ (1-\pi_1^h)\alpha & \pi_1^h(1-\alpha) & (1-\pi_1^h)(1-\alpha) & \pi_1^h\alpha \\ (1-\pi_1^l)\alpha & \pi_1^l(1-\alpha) & (1-\pi_1^l)(1-\alpha) & \pi_1^l\alpha \\ (1-\pi_1^l)\alpha & \pi_1^l(1-\alpha) & (1-\pi_1^l)(1-\alpha) & \pi_1^l\alpha \end{pmatrix} . \tag{18}$$

The probabilities π_1^h and π_1^l in (18) are defined by $\pi_1^h \equiv \pi_1^*(b_0 = \underline{b})$ and $\pi_1^l \equiv \pi_1^*(b_0 = \bar{b})$. Since π_1^* declines in b_0 we have $\pi_1^h > \pi_1^l$. We see from equation (18) that, conditioning on a low-spending state, the transition probabilities are identical (rows 1 and 2). Conditioning on a high-spending state, the same observation holds (rows 3 and 4). That is, our model predicts that the terror state in period t (high terror or low terror) has no consequences for the probabilities of reaching the four different states in $t+1$. The state of spending, in contrast, influences the transition probabilities. In particular, the probability of reaching a low terror state tomorrow is higher when today's anti-terror spending level is high ($\Pi_{31} + \Pi_{33} > \Pi_{11} + \Pi_{13}$). Similarly, our model predicts $\Pi_{12} + \Pi_{14} > \Pi_{32} + \Pi_{34}$.

In appendix B we also derive the equilibrium for a generalized version of our model that allows for a certain degree of persistence in the level of defense spending. As our empirical analysis is based on quarterly data, we believe that such persistence is necessary for a realistic description of government policy making. More precisely, we assume that with a given probability μ the level of defense spending in period 2 is the same as in period 1, i.e. $b_2 = b_1$. Matrix Π_1^b in equation (19) refers to this version of our model.

$$\Pi_1^b = \begin{pmatrix} \begin{array}{c}(1-\mu)(1-\pi_1^h)\alpha \\ +\mu(1-\pi_1^h)\end{array} & \begin{array}{c}(1-\mu)\pi_1^h(1-\alpha) \\ +\mu\pi_1^h\end{array} & \begin{array}{c}(1-\mu)(1-\pi_1^h) \\ \cdot(1-\alpha)\end{array} & (1-\mu)\pi_1^h\alpha \\ \begin{array}{c}(1-\mu)(1-\pi_1^h)\alpha \\ +\mu(1-\pi_1^h)\end{array} & \begin{array}{c}(1-\mu)\pi_1^h(1-\alpha) \\ +\mu\pi_1^h\end{array} & \begin{array}{c}(1-\mu)(1-\pi_1^h) \\ \cdot(1-\alpha)\end{array} & (1-\mu)\pi_1^h\alpha \\ (1-\mu)(1-\pi_1^l)\alpha & (1-\mu)\pi_1^l(1-\alpha) & \begin{array}{c}(1-\alpha+\mu\alpha) \\ \cdot(1-\pi_1^l)\end{array} & \begin{array}{c}(1-\mu)\pi_1^l\alpha \\ +\mu\pi_1^l\end{array} \\ (1-\mu)(1-\pi_1^l)\alpha & (1-\mu)\pi_1^l(1-\alpha) & \begin{array}{c}(1-\alpha+\mu\alpha) \\ \cdot(1-\pi_1^l)\end{array} & \begin{array}{c}(1-\mu)\pi_1^l\alpha \\ +\mu\pi_1^l\end{array} \end{pmatrix}. \quad (19)$$

The term $\mu \in [0, 1]$ in matrix (19) measures the degree of persistence in government spending. For $\mu = 1$, the level of defense spending today is fully determined by the spending level yesterday, whereas for $\mu = 0$ we are in our baseline model in which the government can freely set the spending level irrespective of previous spending. Comparing (18) with (19) reveals the differences between the predictions of the model versions with and without spending persistence: In our baseline model without persistence – conditioning on tomorrow's state of terror – all transition probabilities into states in which terror and spending are in the same mode are higher than the probabilities of transiting into "mismatched" states. This result follows from comparing the first with the third column and the second with the fourth column in matrix (18).The positive correlation ($\alpha > 1/2$) between the actual terror event and the terror signal drives this prediction: the government spends more on anti-terror measures when a crisis is likely – that is, after having received a high terror signal.

With spending persistence, in contrast, the policy choice of the previous period determines the current level of anti-terror spending with probability μ. Hence, in matrix (19), the influence of the current terror signal on current spending becomes weaker, and, in particular, $\Pi_{12} > \Pi_{14}$ and $\Pi_{33} > \Pi_{31}$ if $2\alpha(1-\mu) < 1$. This means that the probability of moving, say, from a low terror – low spending state to a situation characterized by high terror and low spending exceeds the probability of ending up in a state with high terror and high spending if the signal α is sufficiently weak or the degree of persistence μ is sufficiently high.

3 Empirical Analysis

Based on the theoretical model outlined in the previous section, we investigate the mutual relationship between terrorism and defense spending, referring to quarterly data from the UK, which provide a unique case study in this respect. The following section 3.1 briefly describes the methodology and data used, before we turn to the results of the empirical analysis in section 3.2.

3.1 Methodology and Data

Non-linear Markov-Switching Model. We consider a parameterization of the Markov switching model used in Philips (1991) and Ravn and Sola (1995) which allows for four possible states of the world consisting of combinations of either a low or a high mean of the two variables. A key feature of this approach is that it accounts for the fact that a crisis of high terrorism may better be characterized as a sporadic event, which only takes place a few times in a sample, rather than a structural relationship between terror events and public spending.

Consider the following model for the 2×1 vector $z_t = [Ter_t, Def_t]'$,

$$z_t = \mu_{s_t} + u_t, \tag{20}$$

where $\mu_{s_t} = [\mu^i_{Ter}, \mu^i_{Ter}]'$, the index i ($i = H, L$ refers to a low or a high mean of the variables, and u_t is a Gaussian process with zero mean and positive-definite covariance matrix Ω. The state of the system s_t is modeled as a first order, time-homogeneous Markov chain independent of u_t. The time series z_t, therefore, satisfies a four-state Markov process

$$z_t|(s_t = s) \sim N(\mu_{s_t}, \Omega) \ , \tag{21}$$

for $s \in \{LL, HL, LH, HH\}$. The transition matrix is given by a 4×4 matrix Π as depicted in equation (17) in which each row sums to unity and all elements are nonnegative.

Our data set consists of quarterly observations over the period 1983:Q1-2006:Q4, for a total of 93 observations. As a proxy for counter-terrorism expenditures, we employ the variable defense spending (Def), defined as the log of real defense expenditures.[18] Following Eckstein and Tsiddon (2004), the quarterly terror index (Ter) is defined as

[18] We are aware that defense expenditures are only an imperfect measure for anti-terror spending. As Drakos and Giannakopoulos (2009) we have refer to this variable due to data limitations (see also Drakos, 2009).

the log of (e + number of human casualties + number of people injured + number of terrorist attacks). Data for the terror events are collected from the MIPT (National Memorial Institute for Terrorism) database; GDP and defense spending data are taken from Datastream.

The choice of the UK as a case study seems to be well justified, given the long history of terror in the country. The majority of the terror events included in our sample were carried out by the Irish Republican Army (IRA, hereafter). However, our data series for terror does not only include terror caused by the IRA. Probably the most important terror event in recent UK history took place on the 7th of July 2005 in London. The London bomb attacks were a series of coordinated suicide attacks on London's public transport system carried out by British Islamist extremists, which killed 56 and injured hundreds.

Figure 4 shows the plots for the terror index and defense spending for our sample period. Visual inspection of the terror index shows two major spikes, one in 1987-1988, and one in 2005. As mentioned earlier, the 2005 spike is due to the 7/7 bombings, while the 1987-88 spike can be explained with an aggravation of the IRA conflict between 1987 and 1988, which claimed more than 200 lives (McKittrick and McVea, 2001).

3.2 Findings

Table 1 reports maximum likelihood (ML) estimates for the Markov-switching model. The model appears to be well identified in that parameters are statistically significant, and the standardized residuals exhibit no signs of linear or nonlinear dependence. The estimated conditional means show that *(i)* the mean of the terror variable is approximately three times larger in a high terror state ($\mu^h_{Ter} = 1.4619$) than in a low terror state ($\mu^l_{Ter} = 0.4519$) and *(ii)* the mean spending level in a state with high spending ($\mu^h_{Def} = 4.3479$) exceeds the spending level in a low spending state ($\mu^l_{Def} = 4.1430$) only slightly by approximately 5%. All these parameter estimates are highly significant indicating that the model captures the features of the data well. The correlation $\rho_{Ter,Def} = 0.2743$ is positive and significant.

TABLE 1

Maximum Likelihood Estimation Results for UK			
Ter		Def	
μ^l_{Ter}	0.4519 (0.0551)	μ^l_{Def}	4.1430 (0.0105)
μ^h_{Ter}	1.4619 (0.0731)	μ^h_{Def}	4.3479 (0.0117)
σ^2_{Ter}	0.1734 (0.0269)	σ^2_{Def}	0.0048 (0.0007)
		$\sigma_{Ter,Def}$	0.0079 (0.0001)
$LogLik$	219.97		
$LB_{Ter,(10)}$	5.6423	$LB_{Def,(10)}$	10.6082
$LB^2_{Ter,(10)}$	8.2552	$LB^2_{Def,(10)}$	4.5783

Note: Autocorrelation and heteroscedasticity-consistent standard errors, computed using the Newey and West variance-covariance matrix, are reported in brackets. LB and LB^2 are, respectively, the Ljung-Box test statistics of significance of autocorrelations of ten lags in the standardized and standardized squared residuals.

Table 2 reports the estimated transition probabilities. The probability mass in each row is highly concentrated. In particular, *(i)* Low Ter - Low Def at time t is followed by High Ter - Low Def at time $t+1$ with probability $\Pi_{12} = 0.9452$, *(ii)* High Ter - Low Def at time t is followed by High Ter - Low Def at time $t+1$ with probability $\Pi_{22} = 0.9754$, *(iii)* Low Ter - High Def at time t is followed by Low Ter - High Def at time $t+1$ with probability $\Pi_{33} = 0.9473$, and finally *(iv)* High Ter - High Def at time t is followed by Low Ter - High Def at time $t+1$ with probability $\Pi_{43} = 0.9306$.

In line with our model predictions, high spending in period t lowers the probability of transiting to a high terror state in $t+1$. Moreover, the empirical results suggest a considerable degree of persistence in defense spending.[19] If spending is low today, the estimated probability of transiting to a state with low spending and high terror tomorrow is close to one. Similarly, if spending is high today, the estimated probability

[19]Recall that our generalized model with spending persistence predicts $\Pi_{12} > \Pi_{14}$ and $\Pi_{33} > \Pi_{31}$ if and only if $2\alpha(1-\mu) < 1$. Our empirical estimations suggest that this is the case.

of ending up in a state with high spending and low terror tomorrow is close to one. We can also infer from Table 2 that the terror state in period t does not seem to have a sizable impact on the terror probability in $t + 1$.[20]

In summary, these observations correspond well with the generalized version of the theoretical model, which incorporates spending persistence .

TABLE 2

General Model Transition Probabilities

	LL	HL	LH	HH
LL	0.0071 (0.0189)	0.9452 (0.0503)	0.0456 (0.0379)	0.0021 (0.0009)
HL	0.0192 (0.0203)	0.9754 (0.0207)	0.0006 (0.0048)	0.0048 (0.0018)
LH	0.0002 (0.0028)	0.0497 (0.0722)	0.9473 (0.0585)	0.0028 (0.0011)
HH	0.0036 (0.0125)	0.0526 (0.0387)	0.9306 (0.2477)	0.0132 (0.0056)

Note: Standard errors are reported in brackets.

Figure 4 shows the plots for the terror index Ter_t and defense expenditure Def_t along with the corresponding filter probabilities. These are the probabilities of being in each of the four regimes allowed by the model (i.e. LL, HL, LH and HH). The four states are all clearly identified and separated by the filter, confirming the non-linear structure present in the data. In particular, a low terror – low defense spending regime is associated with the period ranging from late '94 to early '97 with a probability close to one. From late '98 to early '05 we identify a regime with high terror and low spending with a probability close to one. Furthermore, periods from the beginning of the sample to early '88 and from early '90 to early '93 are identified as low terror – low defense spending regimes. Finally, late '88 is the only period in the sample which features high terror and high defense spending.

As a comparison to our methodology, Appendix C presents the results of a standard linear vector-autoregressive model (VAR). This approach was used in the previous literature, for instance, by Enders and Sandler (2000) to investigate the interdependence between terror events of different categories and by Eckstein and Tsiddon (2004)

[20] Almost all corresponding elements in rows 1 and 2 as well as rows 3 and 4 do not differ significantly from each other. For instance, element $\Pi_{12} = 0.9452$ is within one standard deviation of element $\Pi_{22} = 0.9754$.

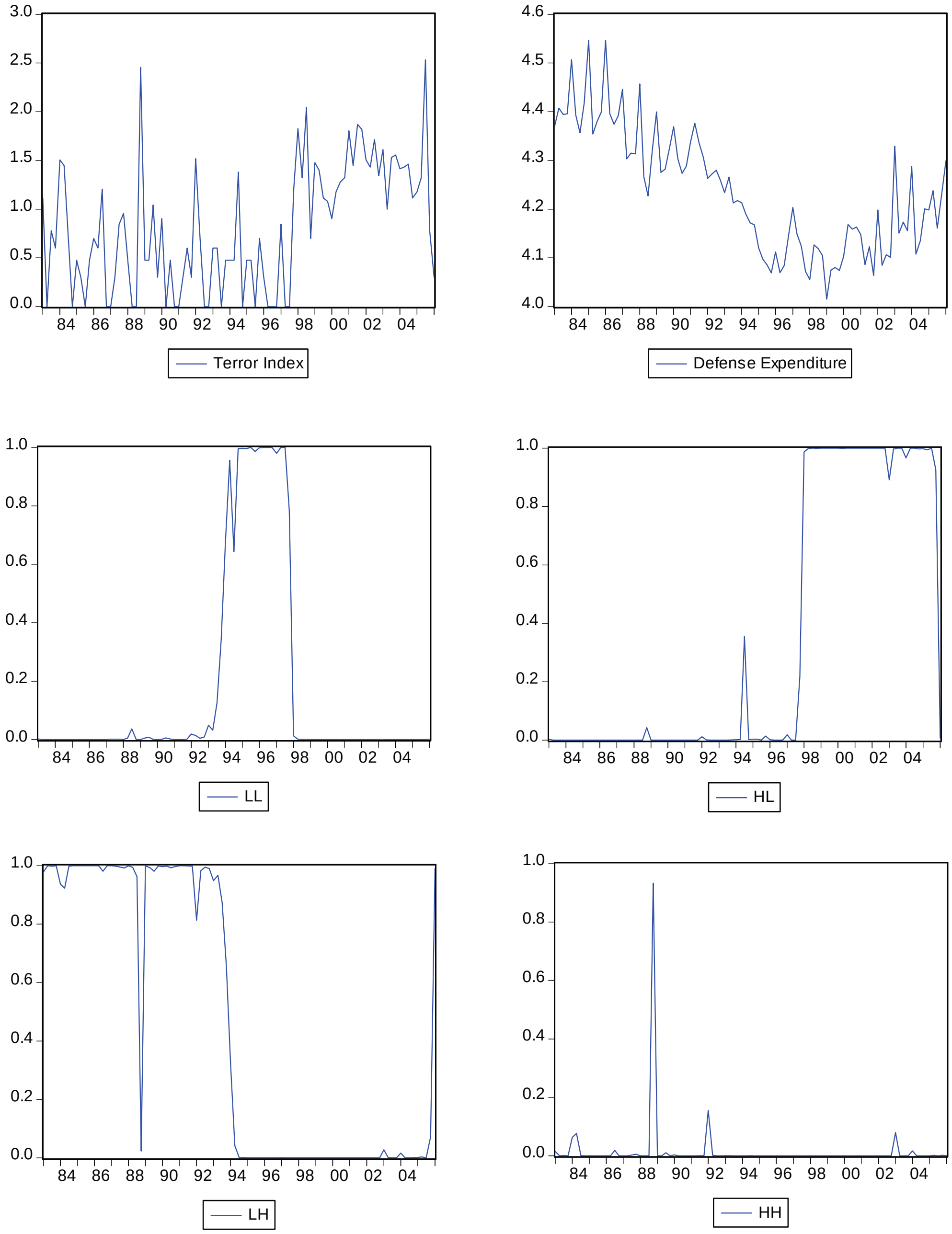

Figure 4: Defense Expenditure, Terror Index and Filter Probabilities

to study the relationship between the level of terror and aggregate economic activity. In contrast to our Markov-switching approach, the VAR-model does not show a relationship between spending and terror.

4 Conclusion

In this paper, we investigate the relationship between anti-terror policies and terrorism. Our theoretical model incorporates the particular structure of the Markov-switching approach into a game-theoretic setting. According to the model the probability of transiting to a high terror state in period $t+1$ declines in the spending level in period t. To explain this pattern, we refer to the partially preemptive nature of anti-terror spending. In contrast to this, the terror state in period t has no consequences for the transition probabilities in our model.

Our empirical analysis for the UK identifies four different states of the world, distinguishing between high and low terror activities as well as between high and low levels of defense spending. Employing a Markov-switching model we can estimate the transition probabilities between these states. The empirical results are consistent with the predictions of the generalized theoretical model that allows for persistence in defense spending.

Further avenues for research may include using our framework to test different types of defense spending as well as other measures that can be employed to counter terrorism. Needless to say, the validity of our empirical results should be tested for other countries, given data availability. The theoretical approach may be generalized in several directions: For example, one may extend the dynamic model to more than two periods and, in particular, to an infinite time-horizon. We expect, however, that the basic insights of our model carry over to such an extension. In addition, some of the exogenous variables in our model may be endogenized in future work. Promising candidates in this regard are, for instance, the share of preemptive spending or the quality of the terror signal the government receives.

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A Global Conditions for Terror Investment

This appendix briefly discusses global properties which have to be satisfied for the terror policy as determined in (8) to be an optimal strategy. Instead of π_2^* the terrorists may set $\pi_2 = \underline{\pi}_2 - \epsilon$ (with $\epsilon \to 0$) and thereby induce a low spending policy by the government. The expected payoff for the terrorists from choosing such a low terror strategy is given by

$$\lim_{\epsilon \to 0} E\left[U_2^T\left(\underline{\pi}_2 - \epsilon\right)\right] = \underline{\pi}_2\left[1 - \gamma b_1 - (1-\gamma)\underline{b}\right] D - \frac{c\left[\underline{\pi}_2\right]^2}{2} \,. \tag{22}$$

This has to be compared with

$$E\left[U_2^T\left(\pi_2^*\right)\right] = \pi_2^*\left(1 - \gamma b_1 - (1-\gamma)\underline{b} - (1-\gamma)\alpha\left(\bar{b} - \underline{b}\right)\right) D - \frac{c\left[\pi_2^*\right]^2}{2} \,. \tag{23}$$

From (22) and (23), we see that the low terror strategy $\underline{\pi} - \epsilon$ may pay off if $\bar{b} - \underline{b}$ is large, in which case the choice of π_2^* can not be part of an equilibrium strategy. However, if the difference between $\bar{b}$ and $\underline{b}$ is sufficiently small, then the fact that π_2^* maximizes $E\left[U_2^T(\pi_2^*)\right]$ also implies that π_2^* leads to a higher payoff than the low terror strategy $\underline{\pi}_2 - \epsilon$. In a similar fashion, global conditions for π_1^* can be checked.

B Spending Persistence

In our baseline model, the government can set the spending level in each period without restrictions. In the following we consider a generalized version of our model with spending persistence: We assume that the government is not necessarily free in determining the spending level in period 2. Instead, with an exogenously given probability μ, the spending level in the second period is predetermined by the level of spending in the first period. That is, with probability μ we impose the restriction $b_2 = b_1$. Apart from this restriction we keep the structure of our model unchanged.

If spending is predetermined, the government has nothing to decide in $t = 2$. The spending level is high ($b_2 = \bar{b}$) if it was high in the previous period and low ($b_2 = \underline{\mathrm{b}}$) if it was low in the previous period. If spending is not predetermined, the government sets b_2 as described in section 2.2.

The terrorists maximize the following expected payoff in period 2:

$$E\left[U_2^T\left(\pi_2\right)\right] = \pi_2\{1 - \gamma b_1 - (1-\gamma)\left[\mu b_1 + (1-\mu)\left(\alpha\bar{b} + (1-\alpha)\underline{b}\right)\right]\} D - \frac{c\left[\pi_2\right]^2}{2} \,. \tag{24}$$

This yields an optimal terror probability of

$$\pi_2^* = \left[\delta_1'(b_1) - \gamma b_1\right] D/c \,, \tag{25}$$

where $\delta_1'(b_1) = 1 - (1-\gamma)\left[\mu b_1 + (1-\mu)\left(\alpha\bar{b} + (1-\alpha)\underline{b}\right)\right]$. Comparing (8) with (25) shows that changes in government spending in the first period now have a stronger effect on the terror probability in the second period than in the baseline model. The explanation for this result is quite straightforward: The government in period 1 is now able to discourage terror investments not only via preemptive spending, but also to a certain degree by predetermining the level of defense spending in the next period. The expected welfare in the second period, evaluated at the end of period 1, is then

$$E\left[V_2^G(b_1)\right] = 1 - \left(\delta_1'(b_1) - \gamma b_1\right)^2 D^2/c - Eb_2\ , \quad \text{where} \tag{26}$$
$$Eb_2 = \mu b_1 + (1-\mu)\left[\underline{\mathrm{b}} + (1-\alpha)(\bar{b} - \underline{b}) + \pi_2^*(b_1)\,(2\alpha - 1)\left(\bar{b} - \underline{b}\right)\right]\ .$$

The terror probability $\pi_2^*(b_1)$ is determined in (25). The welfare difference from high spending versus low spending is given by

$$\Delta E\left[V_2^G\right] = -\left[\pi_2^*(\bar{b})^2 - \pi_2(\underline{\mathrm{b}})^2\right] c - \Delta Eb_2\ , \quad \text{where} \tag{27}$$
$$\Delta Eb_2 = \mu(\bar{b} - \underline{b}) + (1-\mu)(2\alpha - 1)(\bar{b} - \underline{b})\left[\pi_2^*(\bar{b}) - \pi_2^*(\underline{\mathrm{b}})\right]\ .$$

For the terrorists we obtain

$$\Delta E\left[V_2^T\right] = \left[\pi_2^*(\bar{b})^2 - \pi_2^*(\underline{\mathrm{b}})^2\right] c/2\ . \tag{28}$$

The government in $t = 1$ sets $b_1(\tilde{\omega}^h) = \bar{b}$ and $b_1(\tilde{\omega}^l) = \underline{b}$ if the following inequalities are satisfied: $E[U_1^G\left(\bar{b}\right) | \tilde{\omega}^h] + \Delta E\left[V_2^G\right] \geq E[U_1^G\left(\underline{b}\right) | \tilde{\omega}^h]$ and $E[U_1^G\left(\bar{b}\right) | \tilde{\omega}^l] + \Delta E\left[V_2^G\right] \leq E[U_1^G\left(\underline{b}\right) | \tilde{\omega}^l]$ with $E[U_1^G(b_1)] = 1 - \hat{\pi}_1\left(1 - \gamma b_0 - (1-\gamma)b_1\right) D - b_1$. This leads to the following critical values of π_1:

$$\underline{\pi}_1' = \frac{1-\alpha}{1-\alpha+\alpha(y'-1)} \quad \text{and} \quad \bar{\pi}_1' = \frac{\alpha}{\alpha+(1-\alpha)(y'-1)}\ , \tag{29}$$

where $y' \equiv x/\left[1 - \Delta EV_2^G/\left(\bar{b} - \underline{b}\right)\right]$. As before, we assume $y' > 1$ to ensure that $0 < \underline{\pi}_1' < \bar{\pi}_1' < 1$.

Finally, we have to solve the terrorists' problem in period 1. Their objective function is $E\left[U_1^T(\pi_1)\right] + E\left[V_2^T(b_1)\right]$, where

$$E\left[U_1^T(\pi_1)\right] = \pi_1\{1 - \gamma b_0 - (1-\gamma)\left[\mu b_0 + (1-\mu)\left(\alpha\bar{b} + (1-\alpha)\,\underline{b}\right)\right]\}D - \frac{c\,[\pi_1]^2}{2} \quad \text{and}$$
$$E\left[V_2^T(b_1)\right] = \mu E\left[V_2^T(b_0)\right] + (1-\mu)\left\{E\left[V_2^T(\underline{\mathrm{b}})\right] + \left[\alpha\pi_1 + (1-\alpha)(1-\pi_1)\right]\Delta E\left[V_2^T\right]\right\}.$$

The first order condition yields π_1^*.

C Linear VAR Model

This appendix reports the results of a standard vector autoregression (VAR) estimation. Generally speaking, variables in a VAR are treated symmetrically. For each variable, an equation explains its evolution based on its own lags and the lags of the other variables in the model. This allows us to account for a mutual causality between the model variables. As mentioned in the introduction, the VAR methodology has been applied in other contexts in the literature on the economics of terrorism. Following in the footsteps of previous studies, both variables are in natural logarithms, and the Akaike Information Criterion is used to determine the appropriate lag length, which was found to be two. The choice of the lag length happens to be consistent with the following non-linear estimation. Following Sims (1980), the model is estimated in levels, in order to avoid discarding information, for instance, about possible cointegrating relationships.

The parameter estimates of this model are presented in Table 3. The results do not show any statistically significant causality in any directions. This outcome may be explained with the fact that the non-linear effects suggested by our theoretical model are not captured by the VAR model. The autoregressive coefficients, however, are significant, pointing to persistence in the series of interest.

TABLE 3

Vector Autoregression Estimates		
	Terror (Ter)	Defense Spending (Def)
Ter_{-1}	$\mathbf{0.332}^{***}$ (0.105)	0.005 (0.013)
Ter_{-2}	$\mathbf{0.179}^{*}$ (0.103)	−0.004 (0.013)
Def_{-1}	−1.215 (0.818)	$\mathbf{0.550}^{***}$ (0.102)
Def_{-2}	0.274 (0.831)	$\mathbf{0.322}^{***}$ (0.104)
$const.$	$\mathbf{4.388}^{**}$ (2.242)	$\mathbf{0.542}^{**}$ (0.280)
$Obs.$	91	91
$\bar{R}^2$	0.258	0.695
$F - Stat$	8.803	52.338
$LogLik$	-72.432	116.820

Note: Standard errors in parentheses. Statistically significant coefficient estimates appear bold-faced. *, **, and *** respectively indicate statistical significance at the 10%, 5%, and 1% levels.