Endogenous Growth, Population Growth and the Repugnant Conclusion

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Abstract

Really few papers analyze the relationship between optimal endogenous growth models and population growth, when demographic change is endogenously determined. This paper try to fill this gap by analyzing the simplest endogenous growth model, an AK type model, driven by human capital accumulation. We show that in steady state both population growth and economic growth are constant, but the rate of these growth can be positive, negative or null accordingly to parameter values. Population dynamics is determined by the difference between the stationary fertility rate and the exogenous mortality rate: if this is positive population size indefinitely increases, otherwise it reaches a stationary level, which can be positive (if the difference is null) or null (if it is negative). If fertility is strictly lower than mortality, population size will constantly decrease in finite time and end up with a complete collapse of the economy, characterized by the full disappearance of the population. Moreover, the planner can intervene in the economy to avoid this result, through policies oriented to affect the fertility or the mortality rate. We also analyze the problem of optimal population size and its relationship with growth. The seminal work of Parfit (1984) suggests that total utilitarianism leads to increase population size indefinitely, even if it the average welfare tends to zero. It has been shown (Razin and Sadka, 1995) that in a growing framework, this repugnant conclusion, as Parfit defined it, does not hold. We show that in our model economy, under certain parametric conditions, the repugnant conclusion holds; in particular, this happens when consumption growth is negative and the stationary fertility rate is higher than the exogenous mortality rate.

Keywords: Economic Growth, Human Capital Accumulation, Endogenous Fertility, Optimal Population, Repugnant Conclusion

JEL Classification: O40, O41, J13

1 Introduction

The literature on population and economic growth is at least as old as economic science itself: Adam Smith and others before him understood that the relevant growth measure concerns per-capita rather than aggregate variables while Malthus dramatized the concept by identifying population as a possible threat to growth. However, despite the huge body of theoretical and empirical research, economists and demographers still do not have a shared view on the connections between population change and economic growth, as Bloom et al. (2003), clearly summarize: *"...Though countries with rapidly growing populations tend to have more slowly growing economies..., this negative correlation typically disappears (or even reverses direction) once other factors... are taken into account"*.

Three approaches have been proposed in the literature in order to study this issue: an optimistic, a pessimistic and a neutral view (see Bloom et al., 2003). The most probably diffused one is pessimistic (Solow, 1956, Becker and Barro, 1988, and Barro and Becker, 1989) and sees population as detrimental for growth. This effect works through two different channels: the pressure exerted by population growth on natural resources and the required diversion of investments to satisfy population needs. The proponents of this view base their argument on the idea that an increase in the population size leads to a dilution of reproducible resources. Accordingly to this view, we explicitly introduce a dilution function into the equation of human capital accumulation (as in Bucci, 2008), to take into account the effects of population change.

Most of the papers investigating the impact of population in the framework of endogenous growth, however, analyze the issue in a context of exogenous population change and only few studies do it when population growth is endogenous (Razin and Sadka, 1995; and Palivos and Yip, 1993). We aim to fill this gap studying the simplest endogenous growth model, an AK type model, where endogenous fertility choices affect the overall economy and how they relates to the problem of optimal population size. Respect to Razin and Sadka (1995) and Palivos and Yip (1993), in our model economy the social welfare function depends only on consumption and not on the fertility rate (in both of them the objective function shows both per-capita consumption and number of children as arguments) and in particular, respect to Palivos and Yip (1993), the accumulation of human capital explicitly takes into account the dilution effect, not present in their work. Moreover, the aim of the paper is different; in Palivos and Yip (1993) the main goal is showing that in a dynamic framework the Benthamite criterion leads to smaller population size than the Millian one; in Razin and Sadka (1995) it is showing that Parfit's repugnant conclusion does not hold in a dynamic context. In our model, instead, the objective is normative rather than descriptive: we aim to characterize the possible outcomes we can have in a framework of endogenous growth where population change, endogenously determined, represents only a cost for the society since it does not enter the objective function and it negatively affects human capital accumulation.

In particular, we consider an optimal growth model, driven by human capital accumulation, where both per-capita consumption and fertility rate are endogenous choice variables. The determination of the population size negatively affects human capital accumulation because of the dilution effect: an higher number of children lowers the quality level of the average individual in the economy and therefore it determines a cost for the society to bring the newcomers up to the average quality. It is quite standard to assume that this cost is linear in the fertility rate, even if a so simple specification risks to under estimate the complex interaction between many factors which ultimately determine the fertility rate. We consider instead that it is non-linear: this assumption has foundation in the empirical evidence on the relationship between population growth and economic growth. In fact, a diffused view sees the relationship between population and economic growth as non-monotonic and the absence of monotonicity means that human capital and population growth are non-linearly related.

In section 2 we briefly review the literature about the relationship between population and economic growth. A special emphasis is assigned to the empirical evidence supporting the idea of non-linear dilution effects, which ultimately moves our research. Section 3 instead analyzes some of the most important topics concerning the problem of optimal population size, both from a philosophical and ethical point of view and from an economical one. The attention is focused on the choice of the social welfare function and its main effects on the models outcome, in particular on what Parfit (1984) defines as repugnant conclusion. In section 4 we explicitly introduce the model and we characterize the optimal paths, while in section 5 we perform steady state analysis. The model is characterized by a balanced growth path equilibrium (BGP), along which the fertility rate is constant; moreover, we show that in steady state Parfit's repugnant conclusion holds under certain parametric conditions. Section 6 shows this results in a particular case of the model, that is when the dilution effect in human capital is quadratic. We study the main implications of such a formulation and show when, under general parametric values, the repugnant conclusion holds. Then, we perform a comparative statics exercise, underlying how population change (and therefore total welfare) can be affected through certain kind of policies. Section 7 instead concludes and suggests possible extensions for further research on the topic.

2 Population Growth and Economic Growth

The literature on the relationship between economic performance and population growth has really ancient roots, but a unique view has not arisen yet. Three main approaches have been proposed: an optimistic, a pessimistic and a neutral view (see Bloom et al., 2003). The optimistic view (Kuznets, 1960 and 1967, and Boserup, 1989; most recent analysis can be found in Jones, 2001, and Tamura, 2002) considers population growth as a fuel for economic performance; this may be the result of

knowledge production (since population is an input of this process, more researchers produce more knowledge[1]) or technical change (since population growth raises the returns to innovation). The neutralist view (Bloom et al., 2003) instead has empirical foundation: there exists little cross-country evidence that population growth might either slow down or encourage economic growth, when other factors are taken into account. The most probably diffused one is pessimistic (Solow, 1956, Becker and Barro, 1988, and Barro and Becker, 1989) and sees population as detrimental for growth. This effect works through two different channels: in an economy with fixed resources and without any source of technological progress, the (food) production activity is overwhelmed by the pressures of population growth, and this can lead the available diet to fall below the subsistence level, lowering productivity growth rate (Malthus[2], 1798); in an economy with rapid population growth, a large part of investment is used to satisfy the needs of the growing population (*investment-diversion effect* - Kelley, 1988), rather than to increase per-capita capital endowments, leading to a negative impact on capital intensity. The proponents of this view base their argument on the idea that an increase in the population size leads to a dilution of reproducible resources. Accordingly to this view, we explicitly introduce a dilution function into the equation of human capital accumulation, to take into account the effects of population change.

However, the literature concerning optimal growth and population change is quite limited and has mainly focused on the case of exogenous population change. Only few studies analyze this issue when demographic change is endogenous[3], namely Barro and Becker (1989), Palivos and Yip (1993) and Razin and Sadka (1995). While the first paper considers a neoclassical framework, where parents care of their offspring's future utility, permitting to aggregate a dynastic utility function, the others analyze endogenous growth models. But, while the aim in Palivos and Yip (1993) is to study how the choice of the Benthamite rather than the Millian criterion affects the outcome of the model, that of Razin and Sadka (1995) is to show that Parfit's repugnant conclusion does not hold. Therefore, the analysis of the relationship between endogenous (optimal) growth and endogenous fertility is still an open question and deserves more attention. Respect to previous works, our goal is normative, aiming to characterize the possible outcomes we can have in a framework of endogenous growth and

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[1]Jones (2005) summarizes the concept as: *"... just as the total output of any good depends on the total number of workers producing the good, more researchers produce more new ideas. A larger population means more Mozarts and Newtons, and more Wright brothers, SamWaltons, and William Shockleys"*

[2]Malthus (1798) famously concludes: *"Taking the population of the world at any number, a thousand millions, for instance... the human species would increase in the ratio of 1, 2, 4, 8, 16, 32, 64, 128, 256, 516, etc. and subsistence as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc. In two centuries and a quarter the population would be to the means of subsistence as 512 to 10; in three centuries as 4096 to 13, and in two thousand years the difference would be incalculable"*. More recent studies supporting this conclusion can be found, for example, in Ehrlich (1968): *"The battle ... is over. In the 1970s hundreds of millions of people are going to starve to death"*

[3]Many papers study this topic with overlapping-generation models, but they endogenize fertility in order to deal with inequalities and other positive issues

endogenous population change.

Respect to Razin and Sadka (1995) and Palivos and Yip (1993), in our model economy the social welfare function depends only on consumption and not on the fertility rate (in both of them the objective function shows both per-capita consumption and number of children as arguments) and in particular, respect to Palivos and Yip (1993), the accumulation of human capital explicitly takes into account the dilution effect, not present in their work. Moreover, the aim of the paper is different; in Palivos and Yip (1993) the main goal is showing that in a dynamic framework the Benthamite criterion leads to smaller population size than the Millian one; in Razin and Sadka (1995) it is showing that Parfit's repugnant conclusion does not hold in a dynamic context. In our model, instead, the objective is normative rather than descriptive: we aim to characterize the possible outcomes we can have in a framework of endogenous growth where population change, endogenously determined, represents only a cost for the society since it does not enter the objective function and it negatively affects human capital accumulation.

The empirical literature is critical on the existence of a relationship between population and human capital accumulation. Many papers analyze it at households level, but empirical evidence supporting both the pessimistic and the optimistic view can be found (for a survey see Kelley, 1998). Also at country level, empirical studies fail to show the existence of a clear effect of population on human capital accumulation (see Schulz, 1987; and Kelley, 1996), concluding that the sign of this relation is uncertain. For example, Kelley (1996) summarizing his findings states: *"...the net impact of this quantity-quality tradeoff in educational achievements [...] is highly uncertain"*.

The same is true for the relationship between population growth and economic growth. In fact, several studies on cross-sectional data fail to find a significant correlation between the growth rate of population and per-capita output (see Bloom et al., 2003). In fact, as noticed before, the neutralist view, widely diffused since the mid-1980s, arises as a result of such studies. The main conclusion of this theory does not concern the absence of a correlation between population change and economic performance but rather the presence of a different impact varying from a country to another one. In fact, while the National Research Council (1986) affirms *on balance ... slower population growth would be beneficial to economic development of most developing countries*, some World Bank economists suggest that in some countries bigger populations can boost economic growth (see Kelly and Schmidt, 1994, for a survey).

This idea is further analyzed and developed in Kelly and Schmidt (1995), which shows that the impact of population on the economy depends on the level of economic development: the impact of population growth is negative for less developed countries, while it is positive for developed ones. Therefore, this impact can change over time as the development proceeds. They clearly conclude: *"...population growth is not all good or all bad for economic growth: it contains both elements, which can and [...] do changes over time"*. According to this result, therefore, the relationship

between population and economic growth is non-monotonic. Non-monotonicity implies that such a relationship is non-linear: we formalize this by introducing a non-linear dilution function in the law of motion of human capital.

3   The Repugnant Conclusion

The issue on optimal population size dates back to Wicksell[4]. Later economists and philosopher were interested in the question raised firstly by Wicksell, that is, what is the optimal, or the most advantageous, number of lives in a population under given circumstances? The typical view on such a question is that the optimal population is the population that, given some predetermined conditions, ensures the largest social welfare. But this is not so easy as it may firstly seem; in fact, defining the social welfare function means to adopt a certain criterion, which implicitly requires to assign a value to human life.

On the philosophical side, the seminal work of Parfit (1984) has raised a warm debate on the problem of optimal population size. In fact, traditional ethical theories have counterintuitive and paradoxical implication with respect to fertility issues and moral duties towards next generations. For example, total utilitarianism, which aims to maximize the total well-being in the society, leads to Parfit's *Repugnant Conclusion*: *"for any perfectly equal population with very high positive welfare, there is a population with very low positive welfare which is better"*[5].

If you want to answer the question whether it is better to have 1 or 2 people in the population, you need before to understand what better means, and in particular to whom better has to refer to. Such an issue is what philosophers call *person-affecting* morality issues: an outcome can be better than another if it is better for at least one person. However this is not an easy evaluation: a larger population includes people who would not exist in a scenario with a lower population, and for these people the word better is not meaningful: the alternative for them is not to exist at all. Therefore, this kind of comparison implicitly requires to assign a value to the non-living state, raising not simple

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[4]The problem of the optimum population is much older than Wicksell: even in Plato (*"The number of our citizens shall be 5040 - this will be a convenient number... which can be divided by exactly fifty-nine divisors... will furnish numbers for war and peace, and for all contracts and dealings, including taxes and divisions of the land"*, Laws, 737-738) and Aristotle (*"A great state is not the same thing as a state with a large population. But certainly experience also shows that it is difficult and perhaps impossible for a state with too large a population to have a good legal government"*, Politics, 1326) writings we can find discussions on this topic

[5]See Parfit (1984, p. 388). The formulation of the Repugnant Conclusion we present here is from Arrhenius (2000), which is more general than Parfit's one, since Parfit does not require that people with very high welfare are equally well off. In fact, Parfit wrote: *"For any possible population of at least ten billion people, all with a very high quality of life, there must be some much larger imaginable population whose existence, if other things are equal, would be better even though its members have lives that are barely worth living"*.

ethical problems.

As noticed before, determining the optimal population size generates the necessity of defining a social welfare function. The standard approach deals with static framework[6] where the number of agents is not a choice variable and does not vary through time. The ethical problem implied in the issue derives from the comparison of allocations with different population sizes, because it requires to evaluate the utility of people alive in one allocation but not in the other: this implicitly means evaluating the utility of not being born. Consider now the probably most known criteria[7] that have been proposed: average utilitarianism and total utilitarianism.

In average utilitarianism, the objective is to maximize the average or per-capita welfare of the society (this is often associated with Mill, who used it as an argument for limiting the size of the optimal population). Implicit in this criterion is that the utility of not being born is equal to the average utility. In fact, considering two allocations, one in which there is one individual with a utility of two, and one in which a second person with utility of one is added to the previous one, average utilitarianism suggests that the first allocation has to be preferred, since average utility, coinciding with that of the single-living individual, is higher. But the addition of the second person in our society does not lead any member of the first allocation to be worse off: it only adds another person with utility of one. Preferring the first allocation means assuming that the second person is better off not being born: clearly, this is arbitrary.

In total utilitarianism, instead, the objective is to maximize total wellbeing in the economy, that is the average welfare multiplied by the size of population (this has been argued both by some ethicists, as Singer, and some economists, as Meade and Dasgupta, following the eighteen century philosopher Bentham). Implicit in this criterion is that the utility of not being born is zero (this is no more arbitrary than any other real number, as for example the average utility, as proposed by average utilitarianism). According to total utilitarianism, in the previous example the second allocation is to be preferred. Total utilitarianism is part of a more general class of welfare criteria known as critical-level utilitarianism (Blackorby, Bossert and Donaldson, 2005). With this criterion adding a person always improves welfare if his utility exceeds a critical level, which is zero in total

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[6]It deals with what Dasgupta calls a *Genesis Problem*. "*In the Genesis Problem there are no actual people. All persons are potential. In its purest form, the Genesis Problem asks how many lives there should be, enjoying what living standards*" (Dasgupta 1993, p. 381). Dasgupta (1993, p. 385) argues that this is not the right approach to the problem, since the current generation is trying to choose the future number of people and their living standard. This is called the *Actual Problem*: the actual people are what count when determining the well-being of a household, not the potential people that might become part of the family later. Therefore, potential people have no special claim on the rest of the family, since they are not part of the family

[7]Another very well-known criterion is the *max-min principle* (Rawls, 1971), which proposes to maximize the welfare of the poorest. However, it has not been used to any great extent in formal modeling of economic growth, so we do not analyze it

utilitarianism; it therefore implies that population should be increased indefinitely, even if average utility may approach zero. This is what Parfit defined as repugnant conclusion, since it implies a very low standard of living. Average utilitarianism, instead, suggests that population should be as small as possible, since this maximizes average welfare: there is no space for the repugnant conclusion in average utilitarianism.

If the parents care (or the planner) of their offspring's (next generations) future utility, then it is possible to aggregate a dynastic utility function[8]. The dynastic utility function can be seen as mix of the Benthamite and the Millian social welfare function, according to the weight attached to later generations utility. This allows us to extend the previous analysis also to a dynamic framework, where we need to assume discounted utilitarianism as optimality criterion, both in its average or total specification. Using discounted total utilitarianism, it is no longer certain that the size of population will be high and average utility will tend towards zero, both in an endogenous growth framework (Razin and Sadka, 1995, and Palivos and Yip, 1993) and in a neoclassical one (Dasgupta). Razin and Sadka (1995, pp. 175-179) show this in a setting with human capital, where human capital is produced sufficiently efficiently. Also Palivos and Yip (1993), demonstrating that the Benthamite criterion leads to smaller population size and higher economic growth, end up with the somewhat surprising result that the lower is the birth rate and the higher is consumption per-capita. However, one could claim that it is not surprising that utility does not approach zero in these two examples since resources are unlimited. Nevertheless, Dasgupta (1969) finds, for a production function that is homogeneous of degree less than one and an explicit subsistence level, that in steady state population will be stationary and consumption per-capita will be above the subsistence level; this point to the fact that if a parent cares of his children, it is not optimal to give birth to so many children so that their utility approaches zero, and this is why the repugnant conclusion does not hold, even if resources are fixed.

4 The Model

The economy is closed and composed of households that receive wages and interest income, purchase the consumption good and choose how much consuming and how many children to have. Population coincides with the available number of workers, so that there is no unemployment and the labor supply is inelastic (no leisure-work choice), and it grows in accordance to household decisions. The aggregate production function uses human capital to produce one homogeneous final good, that can be consumed or invested in human capital. The accumulation of human capital depends on output, consumption and the quality level of the average individual in the society.

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[8]Barro and Becker (1989) show that finitely living consumers behave as infinitely living ones if each generation takes into account next one

The representative household wants to maximize its lifetime utility function, which is the sum of its instantaneous utility function, which is assumed to be iso-elastic:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $\sigma \in (0,1)$. It depends only on its individual consumption level (it is not interested in aggregate consumption, but only in per-capita consumption) and does not depend on the fertility rate (having children or not does not affect the utility level).

Population grows over time at a non-constant rate, given by the difference between the natality level, n_t, and the constant and exogenous mortality level, d:

$$\dot{N}_t = (n_t - d)N_t, \tag{2}$$

where both n_t and d are strictly positive.

The production function depends linearly on human capital:

$$Y_t = AH_t \tag{3}$$

and human capital accumulation is given by the difference between production, the consumption level and quality level of the average individual in the population:

$$\dot{H}_t = Y_t - C_t - \phi(n_t)H_t, \tag{4}$$

where $\phi(n_t)$ represents the dilution function. The dilution effect in human capital accumulation represents the cost of bringing the level of human capital of the newcomers up to the average level of the existing population: population growth tends, ceteris paribus, to reduce the quality level of the average individual in the population. Intuitively, if the rate of investment in human capital were equal to zero, then per-capita human capital stock would diminish over time because of the population growth.

It has often been assumed that a linear function can be used to describe this kind of effect. We consider instead a non-linear function[9]:

$$\phi(n_t) = an_t^{\eta} \tag{5}$$

where $a > 0$ and for simplicity, we assume the function to be convex, that is $\eta > 1$. This non-linear term relies on the empirical evidence on the relationship between economic growth and demographic

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[9]Probably a better assumption would be $\phi(n_t) = a_1 n_t - a_2 n_t^{\eta}$, such that the dilution function is increasing for small n_t and becomes decreasing for n_t higher than a certain values. In our specification, non-monotonicity is limited by the fact that the fertility rate cannot be negative. However, such a different assumption would complicate too much the model, therefore it seems convenient to illustrate it in its simplest form

change. As mentioned before, a diffused view considers this relation to crucially depend on the phase of economic development. This implies the dilution function, which aim is to capture such a relationship, should be non-monotonic: non-monotonicity clearly means it is non-linear.

The social planner maximizes total welfare in the society, that is, he maximizes the welfare of the representative agent multiplied by the population size, under the economy resource constraint, the law of motion of demography and the initial conditions for human capital and population:

$$\max_{c_t, n_t} \quad \int_0^\infty u(c_t) N_t e^{-\rho t} dt \tag{6}$$
$$s.t. \quad \dot{H}_t = AH_t - N_t c_t - a n_t^\eta H_t$$
$$\dot{N}_t = (n_t - d) N_t$$
$$H_0, \ N_0 \ \text{given}$$

The planner objective function takes into account the size of current and future generations, showing inter-temporal altruism, represented by ρ, the rate of time preference (the lower the rate of time preference, the higher the planner's altruism towards later generations), and full intra-temporal altruism (it means that the weight assigned by the planner to each member of the same generation is the same: the weight of each individual is independent of the size of the generation).

4.1 Optimal Paths

From the social planner maximization problem, we can derive the Hamiltonian function:

$$\mathcal{H}_t(\cdot) \ = \ \frac{c_t^{1-\sigma}}{1-\sigma} N_t e^{-\rho t} + \lambda_t [AH_t - N_t c_t - a n_t^\eta H_t] + \mu_t [(n_t - d) N_t]$$

and the first order necessary[10] conditions:

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial c_t} = 0 \quad \rightarrow \quad c_t^{-\sigma} N_t e^{-\rho t} = \lambda_t N_t \tag{7}$$

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial n_t} = 0 \quad \rightarrow \quad \eta \lambda_t a n_t^{\eta-1} H_t = \mu_t N_t \tag{8}$$

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial H_t} = -\dot{\lambda}_t \quad \rightarrow \quad \lambda_t [A - a n_t^\eta] = -\dot{\lambda}_t \tag{9}$$

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial N_t} = -\dot{\mu}_t \quad \rightarrow \quad \frac{c_t^{1-\sigma}}{1-\sigma} e^{-\rho t} - \lambda_t c_t + \mu_t (n_t - d) = -\dot{\mu}_t \tag{10}$$

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[10]The Mangarasian condition for checking the sufficiency of the Pontryagin necessary conditions for a maximum requires that the Hamiltonian function, $\mathcal{H}_t(\cdot)$, is jointly concave in the control and state variables, (c_t, n_t, H_t, N_t), for all t. This condition is satisfied if $\eta \leq 5$. Therefore, for any value of η less or equal to 5, the Pontryagin first order necessary conditions are also sufficient for a maximum

together with the initial conditions $H(0)$ and $N(0)$, the state equations:

$$\dot{H}_t = AH_t - N_t c_t - an_t^\eta H_t \tag{11}$$

$$\dot{N}_t = (n_t - d)N_t \tag{12}$$

and the transversality conditions:

$$\lim_{t \to \infty} H_t \lambda_t = 0 \tag{13}$$

$$\lim_{t \to \infty} N_t \mu_t = 0. \tag{14}$$

Solving the system, we can obtain the optimal paths of consumption and fertility growth:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}[A - an_t^\eta - \rho] \tag{15}$$

$$\frac{\dot{n}_t}{n_t} = \frac{1}{\eta - 1}\frac{c_t N_t}{H_t}\left[1 - \frac{\sigma}{\eta(1 - \sigma)an_t^{\eta - 1}}\right] \tag{16}$$

Equations (16) is constant unless the term representing the dilution effect in human capital accumulation and it shows an inverse relation between the growth rate of consumption and the fertility rate. In particular, a rise in the fertility rate leads to a non proportional reduction in the growth rate of per-capita consumption and this is due to the dilution effect in human capital accumulation, which is non-linear. Notice that, the growth rate of consumption is a concave function of the fertility rate and the fertility rate maximizing consumption growth results to be null.

Equation (17) instead depends on the ratio between aggregate consumption and human capital and on the rate of fertility. The signs of the relations cannot be determined a priori: in fact, they crucially depend on the term in the squared bracket. It is interesting to notice that the choice of the consumption level, determining the size of the ratio term, affects the growth rate of fertility. This is not surprising since the lifetime utility function is formed in a setting of altruistic parents, that have to determine both the number of children and their welfare level.

The TVC (14) implies that the growth rate of human capital is bounded above:

$$\gamma_H < A - an_t^\eta \tag{17}$$

while the TVC (15) implies that the ratio aggregate consumption-aggregate human capital is positive.

5   Steady State Analysis

The growth rates of the per-capita consumption, human capital, population and fertility rate are:

$$\gamma_c = \frac{1}{\sigma}[A - an_t^\eta - \rho] \tag{18}$$

$$\gamma_H = A - \frac{c_t N_t}{H_t} - a n_t^\eta \tag{19}$$

$$\gamma_N = n_t - d \tag{20}$$

$$\gamma_n = \frac{1}{\eta - 1} \frac{c_t N_t}{H_t} \left[1 - \frac{\sigma}{\eta(1 - \sigma) a n_t^{\eta-1}} \right] \tag{21}$$

We now analyze possible equilibrium paths, considering a balanced growth path, along which the growth rate of fertility is null.

Definition 1: (*Balanced Growth Path, BGP*) a balanced growth path, BGP, or steady state equilibrium, $(\bar{c}, \overline{H}, \overline{N}, \overline{n}, \gamma_c, \gamma_H, \gamma_N, \gamma_n)$, is a sequence of time paths, $\{c_t, H_t, N_t, n_t\}_{t \geq 0}$, along which all economic variables grow at constant rates. A BGP is said non-degenerate if c_t and H_t grow at non negative rates, while it is said degenerate (DBGP) if c_t and/or H_t grow at non negative rates.

Along the BGP, the fertility rate, n_t, must be constant, $n_t = \overline{n}$: this means that the growth rate of fertility is null:

$$\overline{n} = \left[\frac{\sigma}{\eta(1 - \sigma) a} \right]^{\frac{1}{\eta-1}} \tag{22}$$

and the growth rate of population is:

$$\gamma_N = \overline{n} - d. \tag{23}$$

Consequently, the growth rate of per-capita consumption is:

$$\gamma_c = \frac{1}{\sigma} [A - a\overline{n}^\eta - \rho] \tag{24}$$

and that of human capital is:

$$\gamma_H = A - \frac{\overline{c}\overline{N}}{\overline{H}} - a\overline{n}^\eta \tag{25}$$

since the growth rate of aggregate human capital must equalize the growth rate of aggregate consumption[11], otherwise $\gamma_H = -\infty$ or it would violate the TVC (14):

$$\gamma = \gamma_H = \gamma_C = \gamma_c + \gamma_N. \tag{26}$$

This implies that per-capita variables, consumption and human capital, grow at the same rate $\gamma = \gamma_c$.

In order for endogenous growth to be verified, we need to have a lower bound for A:

$$A > a\overline{n}^\eta + \rho, \tag{27}$$

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[11]In fact, if $\gamma_C > \gamma_H$ we would asymptotically have $\gamma_H = -\infty$, while if $\gamma_C < \gamma_H$, we would have $\gamma_H = A - a n_t^\eta$, violating equation (21)

and in order to ensure bounded objective functioninstead, we have an upper bound for A:

$$A < a\overline{n}^{\eta} + \frac{\rho}{1-\sigma} - \frac{\sigma(\overline{n}-d)}{1-\sigma}. \tag{28}$$

Therefore, along the (non-degenerate) BGP the fertility is constant, the economy growth rate is positive and that of population can be positive, negative or null:

$$\overline{n} = \left[\frac{\sigma}{\eta(1-\sigma)a}\right]^{\frac{1}{\eta-1}} \tag{29}$$

$$\gamma = \frac{1}{\sigma}[A - a\overline{n}^{\eta} - \rho] \tag{30}$$

$$\gamma_N = \overline{n} - d. \tag{31}$$

The steady state of population size, $\overline{N}$, depends on the difference between the stationary fertility and the exogenous mortality rate. If the fertility is lower than mortality rate, the population growth rate is negative and in the long-run all individuals will disappear: the population will continue to decrease until its complete disappearance ($\overline{N} = 0$) but this would lead to have an high rate of growth during the life of the economy. If natality and mortality rate perfectly offset, the population will reach a positive stationary equilibrium level, coinciding with its initial level[12] ($\overline{N} = N_0$). If, instead, birth rate is higher than death rate, the population size will continue to rise ($\overline{N} = \infty$), leading to a lower growth rate.

Proposition 1: *along the BGP, the following results hold:*
(i) the stationary fertility level is a positive function of the elasticity of substitution, σ, while it is a negative function of the dilution effect parameter, a;
(ii) the growth rate of the economy depends negatively on the stationary fertility rate, $\overline{n}$;
(iii) population growth is a positive function of the stationary fertility level, $\overline{n}$ and a decreasing function of the mortality rate, d.

Proof: The result just derives from the partial derivatives of (29), (30) and (31), respect to the main parameters. ∎

Proposition 1 says that in our model economic growth is negatively affected by demographic growth (fertility), independently of the gap between fertility and mortality. Notice that such a result is a novelty in the literature: in standard growth models, the Benthamite criterion implies that the economic growth is independent of population variations; this result is due to the presence of

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[12]This result is due to the fact that the model does not show any transitional dynamics, as it will discussed at the end of this section

non-linear dilution effects. Moreover, when demographic change is positive (null) population size continues to increase (reaches a stationary level) along the BGP, while when population growth is negative, its size converge to zero. Along the BGP, it can be optimal for the planner (if $\overline{n} < d$) to exploit an high growth and decreasing population in finite time and end up with a complete collapse of the economy, characterized by the full disappearance of the population.

Parfit (1984) defines as repugnant the outcome where an increase in population is accompanied by a decrease in per-capita consumption. Therefore along the BGP, where per-capita consumption is increasing, Parfit's repugnant conclusion cannot hold. However, when condition (27) is not met, the model is consistent with such an outcome if population growth is positive and per-capita consumption growth is negative, that is when the balanced growth path is degenerate (DBGP).

Proposition 2: *along the BGP, Parfit's repugnant conclusion does not hold; along the DBGP (where $A < a\overline{n}^{\eta} + \rho$), instead, if $\overline{n} > d$ then Parfit's repugnant conclusion holds; otherwise, it does not.*

Proof: the condition $\overline{n} > d$ ensures population size is indefinitely increasing in steady state while $A < a\overline{n}^{\eta} + \rho$ ensures consumption growth is negative (and therefore average utility is decreasing) in steady state. ∎

Along the DBGP, when the repugnant conclusion does not hold, that is $\overline{n} \leq d$, total welfare tends to zero, since consumption (and therefore utility) asymptotically approaches zero and population size tends to a finite value (zero as well if $\overline{n} < d$ or a positive number if $\overline{n} = d$).

We can notice that along the BGP, when the fertility rate is constant, the dynamic behavior of the economy is the same as in a standard AK model. Moreover, as in standard AK model, the economy lies along its (D)BGP since time 0. In fact, at time 0 if the fertility rate is chosen equal to $\overline{n}$, the growth rate of the economy, given is equation (30) is constant. Therefore, there do not exist any transitional dynamics[13] and the steady state outcome characterizes the entire dynamic path. Notice that also Palivos and Yip (1993) find that an AK-type model with endogenous fertility does not show any transitional dynamics. This means both population growth and economic growth are constant as in standard economic growth models, but the rate of these growth can be positive, negative or null accordingly to parameter values. If condition (27) is verified, we have endogenous growth and the economy lies on the BGP; otherwise consumption gets smaller and smaller and the economy lies on the DBGP. Equation (31) instead characterize the sign of population change, which ultimately determines stationary population size.

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[13]See Appendix A for mote details

6   A Specific Case: Quadratic Dilution Effect

We now consider a specific case, which can be considered as a benchmark case, of the dilution function in order to exemplify the model and its steady state implications with refers to Parfit's conclusion. In particular we consider the dilution function to be quadratic in the fertility rate:

$$\phi(n_t) \;\; = \;\; a n_t^2.$$

This corresponds to the specification, $\eta = 2$. Notice that such a case satisfies the requirement for sufficiency.

In such a case, along the BGP, the growth rate of the economy, population growth and the stationary fertility rate becomes:

$$\begin{aligned}
\gamma &= \frac{1}{\sigma}\left[A - a\bar{n}^2 - \rho\right] \\
\gamma_N &= \bar{n} - d \\
\bar{n} &= \frac{\sigma}{2(1-\sigma)a}.
\end{aligned}$$

We now illustrate the growth rate of population implied by such a specification, under a given set of parameter values. In choosing such values we rely on existing empirical estimates or on baseline specifications coming from previous works. The main interest concerns verifying under which parameter values Parfit's repugnant conclusion holds. Proposition 2 tells us the only case where this can arise is along the DBGP. Therefore, we concentrare only on such a case and for simplicity we set $A = 0$. Since $\sigma \in (0, 1)$ for the fertility rate to be positive, we set it equal to 0.4 , following Arrow (1971), and Hansen and Singleton (1982,1984), who estimate the degree of relative risk aversion[14] (RRA) to be lower than 1. We set $\rho = 0.04$, as in Mulligan and Sala-i-Martin (1993). We set $d = 0.08$, which represents the average among OECD countries of the United Nation (2009) crude death rate[15]. Then, we let a vary.

Under these assumptions boundedness of the objective function is always verified and the economic growth rate is negative. These considerations, jointly with Proposition 2, imply that Parfit's repugnant conclusion holds if the rate of population growth in steady state is positive. This is given by the difference between the stationary fertility and mortality rates. The mortality rate is exogenous while the stationary fertility rate is endogenously determined in our model. In particular, $\bar{n}$ crucially depends on a, the parameter representing the intensity of the dilution effect in human capital accumulation. Figure 1 shows how fertility changes as the parameter a varies; we let it vary between 0.5

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[14]Also other works confirm such estimates. For example, Epstein and Zin (1991) conclude the relative risk aversion can span the range from 0.4 to 1.4

[15]The crude death rate is the number deaths per thousand persons in the same year, based on mid-year population
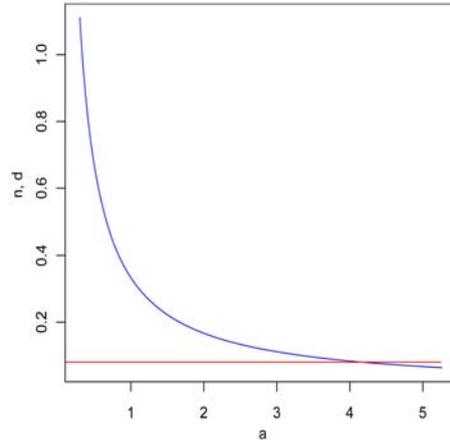
Figure 1: Population growth in steady state. The rate of population change is difference between the stationary fertility rate (the blue curve) and the mortality rate (the red line). The figure shows the values of the stationary fertility rate, as the parameter a changes

and 5.5. If a is high the repugnant conclusion does not hold, while when it is low Parfit's conclusion applies also to our framework. In fact, the population growth (the difference between the the blue curve and the red line) is positive for any a lower than 4.17. This means that Parfit's repugnant conclusion generally holds, unless the case where the dilution effect in human capital is particularly high. In such a case, however, population size decreases over time and it leads asymptotically the population to vanish, and therefore the economy to collapse.

In our model economy, along the DBGP, population growth plays a central role in determining the total welfare in the society. Average welfare is constantly decreasing and therefore total welfare crucially depends on population growth. If it is positive, population size increases indefinitely and this can compensate the constant decline in average utility; if it is negative or null, total welfare, reaches zero asymptotically while population size asymptotically converges to zero as well in the former case or to a positive constant in the latter one. Therefore, policies oriented to affect population change are crucial in order to reach any desired level of welfare. This can be done especially with two different kind of policies: one consists of manipulating the mortality rate while the other deals with the dilution effect parameter. In the last part of this section, we performs a comparative statics exercise, studying the impact of such kind of policies on the economic performance and population growth.

6.1   Changes in the Mortality Rate

Suppose a shock decreases permanently the exogenous mortality rate, represented by the parameter d, without affecting any other parameters of the model. Some economic shocks amount to changes

in the mortality rate; for example, some economic policies (supposing they can be achieved at zero cost), like public expenditures in health or incentives to private health care, can modify the mortality rate.

Suppose a new policy has just been introduced and its effect is to lower the mortality rate, affecting therefore the population growth. If the economy were initially along the DBGP, this shock has the effect[16] of shifting the economy from a DBGP to another one. Along the new DBGP the economic growth rate and the stationary fertility rate would remain unaffected while net population growth changes.

If originally the population growth rate were positive, along the new DBGP population growth would be higher and this leads to a faster increase in the population size. The same comments apply if the the original population growth rate were null.

If instead originally the population growth were negative, along the new DBGP population growth would increase but it could be positive, negative or null, in accordance to the magnitude of the change. If the new population growth were positive we would be driven back in the previous case. If it were still negative, population would continue to decrease at an higher rate implying a faster decline in the population size. If instead it were null, population size would reach a stationary level.

Therefore the introduction of a new policy aiming to increase public or private spending in health care, through the effect of lowering the mortality rate, can be used in order to reach a desired level of population, without encountering any change in the economic growth rate. Notice that the repugnant conclusion can arise or disappear in the economy as a result of such a kind of shock.

Suppose instead a shock, like the appearance of a new disease, increases the mortality rate. Its effect is just the opposite of that of a policy promoting health care: along the new DBGP, the economic growth rate remains unaffected while the population growth changes have opposite signs respect what just discussed.

6.2   Shifts in the Dilution Parameter

Some economic policies (at zero cost) can affect the parameter a, representing the intensity of the dilution effect. Such a policy can be, for example, the introduction of public expenditure to improve the efficiency of teachers and teaching and its effect would be to decrease the cost for the society to bring children up to average level of human capital (decreasing a).

Suppose such a disturbance affects the economy. If the economy were initially along the DBGP, along the new one the economic growth, population growth and the steady state fertility level change.

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[16]Short-run and long-run effects coincide, as in the standard AK model: every shock in the economy translates in a jump of the (economic and population) growth rates, of the stationary fertility level or both. In fact, at any shock the fertility rate can be adjusted in order to lie directly on the new (D)BGP

A drop in a leads first of all to a shift from a fertility rate to an higher new one, while the fertility growth continues to be null. This implies an higher population growth and a lower economic growth.

Also the introduction of a policy aiming to affect the efficiency of teachers can be used to affect the population size and the existence of the repugnant conclusion. But differently from a policy oriented to the health care, it can also be used to foster growth.

7 Conclusion

Really few papers analyze the relationship between economic and population growth, when demographic change is endogenously determined. This paper try to fill this gap by analyzing an optimal growth model, driven by human capital accumulation, where fertility choice is endogenous and it is taken jointly with consumption decisions: not only consumption choices, but also fertility ones affect human capital accumulation through a dilution effect. Increasing the population size dilutes human capital, that is, the larger the population, the lower average human capital. This is due to the fact that newcomers human capital cannot be brought to the average level without costs.

We show that in steady state both population growth and economic growth are constant as in standard economic growth models, but the rate of these growth can be positive, negative or null accordingly to parameter values. If per-capita consumption is increasing, we have endogenous growth and the economy lies on the BGP; otherwise consumption gets smaller and smaller and the economy lies on the DBGP. Population dynamics is determined by the difference between the stationary fertility rate and the exogenous mortality rate: if this is positive population size indefinitely increases, otherwise it reaches a stationary level, which can be positive (if the difference is null) or null (if it is negative). In this last case, where fertility is strictly lower than mortality, population size will constantly decrease in finite time and end up with a complete collapse of the economy, characterized by the full disappearance of the population. Moreover, the planner can intervene in the economy to avoid this result, through policies oriented to affect the fertility or the mortality rate. For example, the fertility rate can be increased through policies oriented to affect the efficiency of teachers; the mortality rate, instead, can be lowered increasing public expenditures in health or incentives to private health care.

We also study the implications of the model for the optimal population size problem, which deals with the identification of the most advantageous number of lives in a population and strictly relates to the choice of the social welfare function. In a static framework, total utilitarianism implies Parfit's repugnant conclusion: it is always worthwhile to add a person in the society if his lifetime utility is higher than zero; this means that a society indefinitely increases its population size, even if the average welfare is close to zero. This according to Parfit is repugnant since it leads to a mass of poverty. In a dynamic framework, we cannot use any more total utilitarianism but we need to rely

on discounted total utilitarianism. According to this criterion, the repugnant conclusion does not hold, both in an neoclassical context, as demonstrated in Dasgupta (1969), and in an endogenous growth framework, as in Palivos and Yip (1993) and Razin and Sadka (1995). We show that in our model, where the dynamic social welfare function is defined accordingly to the discounted total utilitarianism criterion, in steady state along the DBGP (when consumption growth is negative), the repugnant conclusion holds if the stationary fertility rate is higher than the exogenous mortality rate.

For further research we suggest to extend the analysis along two different directions: considering a multi sectorial economy and the effect of different welfare criterion. In our model, the population size, endogenously determined by fertility choices, explicitly shows only a negative effect in the economic dynamics through its link with consumption choices. In fact, in our one sector economy, an increase in the stock of population lowers human capital accumulation, which is the engine of growth. Therefore, the result of our model, that is decreasing the number of persons in the economy is positive for growth, is implied in such a formulation. Probably a better specification of the model could be a two sectors economy, á-la Uzawa-Lucas, where population size negatively affects human capital accumulation, still representing the engine of growth, but has also positive effects on the accumulation of physical capital. Such a model could probably be more realistic in describing the role of row population: a higher number of persons leads to accumulate more physical capital, but it also has high costs in terms of the necessity to bring the under average skilled people up to the average level in order to promote growth. This double effect of population size on the economy can generate a more complicated dynamics, whose net effect can be not obvious and more interesting.

Moreover, in our model, the social welfare function is defined accordingly to the Benthamite criterion. As previously seen, depending on the degree of altruism of the planner, the dynastic welfare function can be represented as a mix of the Benthamite and Millian criterion. It could be interesting to investigate whether the choice of the welfare function affects the outcome of the model, in particular how it relates to population and economic growth.

A On the Transitional Dynamics

The four dimensional dynamic system in (c_t, n_t, H_t, N_t), given in equations (18) - (21), can be reduced in a two dimensional one, by introducing the intensive variable $\varphi_t = \frac{c_t N_t}{H_t}$, and studying the planar system in (φ_t, n_t):

$$\frac{\dot{\varphi}_t}{\varphi_t} = \frac{1-\sigma}{\sigma}(A - an_t^\eta) - \frac{\rho}{\sigma} + (n_t - d) + \varphi_t \tag{32}$$

$$\frac{\dot{n}_t}{n_t} = \frac{1}{\eta - 1}\varphi_t\left[1 - \frac{\sigma}{\eta(1-\sigma)an_t^{\eta-1}}\right]. \tag{33}$$

The non trivial steady state is $E = (\overline{\varphi}, \overline{n})$, where:

$$\overline{\varphi} = \frac{\rho}{\sigma} - \frac{1-\sigma}{\sigma}(A - a\overline{n}^\eta) - (\overline{n} - d) \tag{34}$$

$$\overline{n} = \left[\frac{\sigma}{\eta(1-\sigma)a}\right]^{\frac{1}{\eta-1}}. \tag{35}$$

Notice that $\overline{n}$ is positive if $0 < \sigma < 1$ while $\overline{\varphi}$ is if $\overline{n} > \frac{\eta[(1-\sigma)A-\rho-\sigma d]}{\sigma(1-\eta)}$. This last condition is automatically ensured[17] by equation (28). Notice that, since $\overline{n} > 0$, a sufficient condition for this inequality to hold is that $\frac{\eta[(1-\sigma)A-\rho-\sigma d]}{\sigma(1-\eta)} \leq 0$, that is $A \geq \frac{\rho+\sigma d}{1-\sigma}$.

We can study the stability of the steady state, by linearizing:

$$J(\varphi_t, n_t) = \begin{bmatrix} \frac{1-\sigma}{\sigma}(A - an_t^\eta) - \frac{\rho}{\sigma} + (n_t - d) + 2\varphi_t & \varphi_t[1 - \frac{\eta(1-\sigma)a}{\sigma}n_t^{\eta-1}] \\ \frac{1}{\eta-1}[n_t - \frac{\sigma}{\eta(1-\sigma)an_t^\eta}] & \frac{1}{\eta-1}\varphi_t[1 - \frac{\sigma(2-\eta)}{\eta(1-\sigma)an_t^{\eta-1}}] \end{bmatrix} \tag{36}$$

and evaluating the Jacobian matrix at steady state E,

$$J(E) = \begin{bmatrix} \overline{\varphi} & 0 \\ 0 & \overline{\varphi} \end{bmatrix}. \tag{37}$$

Since $\overline{\varphi}$ is positive, it is straightforward to notice that both eigenvalues are positive, implying that the system never reaches the steady state (namely its BGP or DBGP), unless the initial choice for n_t is such that the fertility rate coincides with its stationary level from time 0.

Therefore, the economy lies immediately on the (D)BGP, otherwise will never converge to it. If $n(0) = \overline{n}$, the model behaves as a standard AK model: there is no transitional dynamics and from $t = 0$ the model is in its steady state. This means that the steady state outcome of the model describes the entire dynamic path.

References

1. Arrhenius, G. (2000), *An Impossibility Theorem for Welfarist Axiologies*, Economics and Philosophy 16, 247-266

2. Arrow K. (1971), *Essays in the Theory of Risk Bearing* (North Holland: Amsterdam)

3. Barro, R.J. and Becker G.S. (1989), *Fertility Choice in a Model of Economic Growth*, Econometrica 57(2), pp. 481-501

4. Barro, R.J. and Sala-i-Martin, X., 2004, *Economic Growth*, second ed. (Cambridge, Massachusetts: MIT Press)

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[17]The condition to impose boundedness of the objective function, given by equation (28) is equivalent to that for ensuring positivity of the consumption-human capital ratio

5. Becker G.S. and Barro, R.J. (1988) *A Reformulation of the Economic Theory of Fertility*, Quarterly Journal of Economics 108, pp. 1-25

6. Blackorby, C., Bossert, W., and Donaldson D. (2005), *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*, Econometric Society Monographs, Cambridge: Cambridge University Press

7. Bloom D.E., Canning D., Sevilla J. (2003), *The Demographic Dividend: a New Perspective on the Economic Consequences of Population Change* (Rand, MR-1274: Santa Monica, CA)

8. Bucci A. (1998), *Population in Factor Accumulation-Based Growth*, Departmental Working Papers 2008-17, Department of Economics, University of Milan, Italy

9. Dasgupta P.S. (1969), *On the Concept of Optimum Population*, Review of Economic Studies 36(3), pp. 295-318

10. Dasgupta P.S. (1993), *An Inquiry into Well-Being and Destitution* (Oxford: Clarendon Press)

11. Ehrlich, P. (1968), *The Population Bomb* (New York: Ballantine)

12. Ehrlich I. and Lui F. (1997), *The Problem of Population and Growth: a Review of the Literature from Malthus to Contemporary Models of Endogenous Population and Endogenous Growth*, Journal of Economic Dynnamics and Control 21, pp. 205-242

13. Epstein L.G. and Zin S.E. (1991), *Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: an Empirical Analysis*, Journal of Political Economy 99(2), pp. 339-357

14. Hansen L.P. and Singleton K.J. (1982), *Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models*, Econometrica 50(5), pp. 1269-1286

15. Hansen L.P. and Singleton K.J. (1984), *Errata: Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models*, Econometrica 52, pp. 267-268

16. Jones C.I. (2005), *Growth and Ideas*, in Aghion P. and Durlauf S. eds, Handbook of Economic Growth (North-Holland: Amsterdam)

17. Kelley A.C. (1988), *Economic Consequences of Population Change in the Third World*, Journal of Economic Literature 26(4), pp. 1685-1728.

18. Kelley A.C. and Schmidt R.M. (1995), *Population and Economic Growth Correlations: the Role of the Components of Demographic Change*, Demography 32(4), pp. 543-555

19. Mulligan C.B. and Sala-i-Martin X. (1993), *Transitional Dynamics in Two-Sector Models of Endogenous Growth*, Quarterly Journal of Economics, 108(3), pp. 739-73

20. National Research Council (1986), *Population Growth and Economic Development: Policy Questions* (Washington, D.C.: National Academy Press)

21. Palivos T. and Yip C.K. (1993), *Optimal Population Size and Endogenous Growth*, Economics Letters 41, pp. 107-110

22. Parfit D. (1984), *Reasons and Persons* (Oxford: Oxford University Press)

23. Ramsey F. 1928. *A Mathematical Theory of Saving*, Economic Journal 38, pp. 543-559

24. Rawls J. (1971), *A Theory of Justice* (Cambridge: Harvard University Press)

25. Razin A. and Sadka E. (1995), *Population Economics* (Cambridge: The MIT Press)

26. Solow R.M. (1956), *A Contribution to the Theory of Economic Growth*, Quarterly Journal of Economics 70, pp. 65-94

27. Tamura, R. (1994), *Fertility, Human Capital, and the Wealth of Families*, Economic Theory 4, pp. 593-603

28. United Nations (2009): *Social Indicators* [Data available on-line at http://unstats.un.org/unsd/demographic/products/socind]