Growth, Inflation, and Inequality in a Monetary Two-Sector Endogenous Growth Model*

Yi Jin†
Department of Economics
Monash University

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Abstract

This paper develops a monetary, two-sector, heterogeneous-agent endogenous growth model with both physical and human capital. We find that monetary frictions on human capital investment and cross-sector factor intensity differentials play important roles in generating the non-superneutrality of money with respect to growth, the relative price of sectoral outputs, cross-sector allocations of resources, and the inequality of income across households. In particular, when the education sector is more intensive in human capital, an increase in money growth lowers the aggregate growth, raises inflation, and reduces the fraction of factor inputs allocated to the education sector. It enlarges (mitigates) income inequality when physical-capital heterogeneity dominates (is dominated by) human-capital heterogeneity. Absent cross-sector factor intensity differentials, money growth is neutral with respect to income inequality.

JEL Classification: O41, O42, D31, E21, E25, E31, J24.

Keywords: Growth; Inflation; Income inequality; Physical and human capital heterogeneity; Superneutrality of money.

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†Department of Economics, Monash University, Caulfield East, VIC 3145, Australia. Tel: +61-3-99032042, Fax: +61-3-99031128, Email: yi.jin@buseco.monash.edu.au, URL: http://people.ku.edu/ ~yjin.

1 Introduction

This paper undertakes a theoretical exploration of the effects of monetary frictions and monetary policy on growth, inflation, and income inequality, as well as the relations among the latter three objects induced by variations in monetary policy, within a two-sector model of endogenous growth with accumulation of both physical and human capital. Our purpose is to provide a unified framework where the three important measures of macroeconomic performance—growth, inflation, and income inequality—are determined simultaneously in an endogenous fashion. In an environment where agents are heterogeneous with respect to non-human wealth as well as human capital, both of which they accumulate optimally over time, monetary policy potentially affects income inequality by altering the effectiveness of the translation of these heterogeneities into income inequality at the same time it affects growth and inflation. In such an environment, the inflation-growth, inflation-inequality, and growth-inequality relationships are interrelated.

The key features of our model are: (1) factor intensities differ across two production sectors—a goods producing sector and an education sector, (2) investment in human capital is subject to severer liquidity constraint than investment in physical capital, (3) households are heterogeneous with respect to their holdings of physical and human capital and receive different levels of income. These features give rise to a number of interesting results, especially that the effects of money growth on inflation, output growth, and inequality depend on the cross-sector factor intensity differentials, the comparison of the two types of household heterogeneities, one regarding the distribution of physical capital and the other regarding the distribution of human capital, as well as the extent to which monetary frictions distort investment in human capital and sectoral allocations of resources.

That investment in human capital faces severer liquidity constraint is captured in our model by the imposition of a cash-in-advance (CIA) constraint on household's education purchase and the absence of such a constraint on investment in physical capital. This assumption should not be interpreted literally. It is not intended to assert that investment in physical capital is exempted from monetary frictions of any kind. Rather, it is meant to describe starkly the relatively severer liquidity constraint on human capital investment. In reality, investment in physical capital is undertaken by business firms and can be financed by various means of credit. It is, however, relatively more difficult for people to borrow against their future returns on human capital to finance their current investment in it, a point highlighted earlier by Becker (1975) and Atkinson (1983) and later taken up by Galor and Zeira (1993).[1] Becker (1975) states that, "... it is difficult to borrow funds to invest in human capital because such capital cannot be offered as collateral, and courts have frowned on contracts that even indirectly suggest involuntary servitude." In Atkinson (1983), it is stated that "difficulties in borrowing arise particularly because human capital cannot be held as collateral, since that would be equivalent to a person's selling himself into slavery,

[1] Galor and Zeira (1993)'s analysis is conducted within a real environment while ours is a monetary one.

so that the lender has no security, as he would have if he held the title to a house or a claim against other physical assets."

We regard it empirically plausible to assume that the education sector is less intensive in the usage of physical capital and more intensive in human capital compared to the goods producing sector and focus our analysis on this case. (In contrast to goods production, a critical input in education is teachers that embody a great deal of human capital.) This assumption allows an analysis of the model's balanced growth equilibrium that is aided by the celebrated Stolper-Samuelson Theorem in international trade theory.[2] Along the balanced growth path (BGP) of a corresponding real environment (i.e., without monetary frictions), the return to physical capital is its rental rate while the return to human capital is the wage rate, adjusted for the relative price of education.[3] The Stolper-Samuelson Theorem implies that the return on physical (resp. human) capital is a decreasing (increasing) function of the relative price of education. No-arbitrage requires equalization of the returns on these two types of capital, which determines the relative price of education. With the presence of monetary frictions, the return on human capital has to be further adjusted for the gross nominal interest rate, which reflects the opportunity cost of education purchase inflicted by the CIA constraint. As long as the net nominal interest rate is positive, the presence of monetary frictions reduces the return on human capital for all

[2] Bond et al. (1996) use similar techniques from international trade theory. But their focus is on the characterization of transitional dynamics of a two-sector (real) endogenous growth model.

[3] Off the balanced growth path, there is also a term reflecting the appreciation of the relative price of human capital.

possible values of the relative price of education. As a result, the equilibrium relative price of education is higher.

The above outcome also has implications for the model's BGP growth and sectoral allocations of factor inputs. In particular, the presence of monetary frictions reduces the long-run rate of aggregate growth and the fraction of factor inputs allocated to the education sector, as compared to the non-monetary environment. When money grows faster, the distortions caused by monetary frictions become more pronounced. In particular, an increase in the money growth rate raises inflation and the nominal interest rate while reducing the aggregate growth and the allocation of resources to the education sector. Money is therefore non-superneutral in our model.

Perhaps an even more interesting result is that money growth is non-neutral with regard to income inequality in our analysis. The concept of the super-neutrality of money is thus expanded to the domain encompassing inequality. We decompose our measure of income inequality into a component associated with physical-capital heterogeneity and the other associated with human-capital heterogeneity. The weight attached to either type of heterogeneity is simply the share of total income that goes to that factor of production and depends solely on the relative size of the two sectors. A higher money growth rate leads to an expansion of the goods producing sector relative to the education sector, which in turn raises the income share of physical capital since the former sector is more intensive in it. It hence raises the contribution of physical-capital heterogeneity to the overall income inequality while reducing the contribution of human-capital heterogeneity. The overall effect depends on the relative size of the two types of heterogeneity. It is positive (resp. negative) when the heterogeneity in physical (resp. human) capital is the dominant force.

Our analysis also carries implications for the relations among growth, inflation, and income inequality. Specifically, variations in money growth trace out a negative relation between inflation and growth, a positive (resp. negative) relation between inflation and inequality, as well as a negative (resp. positive) relation between growth and inequality when physical (resp. human) capital heterogeneity dominates.

To highlight the role played by the cross-sector factor intensity differentials it would be useful to contrast the results described above with the case where the factor intensities are identical across the two sectors. We show that in the latter case an increase in money growth lowers the aggregate growth but is neutral with regard to the income distribution. Therefore sectoral allocations matter for income inequality only when the two production sectors are different in a nontrivial way.

Our paper is related to the theoretical work on the relations among growth, inflation, and inequality, including studies relating growth to inequality such as Bertola (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), Garćia-Peñalosa and Turnovsky (2006), Viane and Zilcha (2003), and Jin (2009), studies connecting inflation with inequality such as Albanesi (2007) and Heer and Süssmuth (2003), and those on the growth-inflation relation such as Stockman (1981), Dotsey and Sarte (2000), Gomme (1993), Jones and Manuelli (1995), Marquis and Reffert (1995), Haslag (1997), and Gillman and Kejak (2004). One of the major differences of our paper from existing works is that we incorporate all the three objects—growth, inflation, and inequality in a consistently specified framework where all of them appear as endogenously determined variables, making possible the study of the growth-inflation-inequality relations.

The rest of the paper is organized as follows. Section 2 describes the setup of our model. Section 3 characterizes the equilibrium, including the cross-sector allocations of factor inputs, the aggregate dynamics, and the measure of income inequality. The effects of monetary frictions and changes in money growth on growth, inflation, and inequality are analyzed in Section 4, followed by a discussion of the case of no factor intensity differential in Section 5. The last section concludes.

2 The Model

There are two sectors in the economy—a goods producing sector (f) and an education sector (e). In each sector there are a continuum of identical firms with unit mass. Firms produce according to the following standard Cobb-Douglas technologies:

$$Y^f = A^f \left(K^f\right)^{\alpha} \left(L^f\right)^{1-\alpha}, \tag{1}$$

$$Y^e = A^e \left(K^e\right)^{\beta} \left(L^e\right)^{1-\beta}, \tag{2}$$

where $Y^f\,(Y^e)$, $K^f\,(K^e)$ and $L^f\,(L^e)$ represent output, physical capital input, and effective labor used in sector f (e), respectively. A^f and A^e are the fixed sectoral productivities, and $\alpha \in (0,1)$ and $\beta \in (0,1)$ are capital shares.

Let p, w, and r denote the price of education, the wage rate, and the rental rate, respectively, the price of goods being normalized to be unity. Profit maximization by firms in each sector implies the following relationships between factor prices and sectoral capital-labor ratios:

$$w = (1-\alpha)\,A^f\left(K^f/L^f\right)^{\alpha} = p\,(1-\beta)\,A^e\,(K^e/L^e)^{\beta}, \tag{3}$$

$$r = \alpha A^f\left(K^f/L^f\right)^{\alpha-1} = p\beta A^e\,(K^e/L^e)^{\beta-1}. \tag{4}$$

There are a continuum of households with unit mass, indexed by $i \in [0,1]$. They differ in initial endowments of physical and human capital. Let K_i (resp. H_i) represent household i's stock of physical (resp. human) capital, with the initial value given by K_{i0} (resp. H_{i0}). The evolution of household i's capital stock is governed by

$$\dot{K}_i = I_i, \tag{5}$$

where I_i is the investment flow. Its human capital evolves according to

$$\dot{H}_i = e_i, \tag{6}$$

where e_i is the amount of education service purchased. For simplicity, we assume that the rates of depreciation for both physical and human capital are zero.

At every point in time, household i rents out its physical capital K_i to firms at rental rate, r. We assume that each household supplies inelastically its labor endowment, which is normalized to be one. Thus household i provides H_i units of effective labor. In addition, household i receives real lump-sum transfer τ_i from the government. It faces the following budget constraint at each point in time:

$$C_i + pe_i + I_i + \dot{M}_i = rK_i + wH_i + \tau_i - \pi M_i, \tag{7}$$

where C_i is consumption, M_i is the *real* balance, and π is the inflation rate. The initial value of household i's money holding is given.

To make money essential, we impose a cash-in-advance (CIA) constraint on household i's purchase of consumption and education services:[4]

$$C_i + pe_i \leq M_i. \tag{8}$$

Household i solves the following dynamic optimization problem:

$$\max_{C_i, e_i, K_i, H_i, M_i} \int_0^\infty \frac{C_i^\gamma}{\gamma} e^{-\rho t} dt, \tag{9}$$

$\gamma \leq 0$, subject to (5)-(8). In the objective function, $\rho > 0$ is the rate of time preference, and $1/(1-\gamma)$ is the elasticity of intertemporal substitution. As we are only interested in equilibria where the nominal interest rate is positive, we will treat (8) as binding in the sequel.

Let λ_i and μ_i be the co-state variables associated with constraints (5) and (6), in the current-value Hamiltonian, with e_i eliminated using the binding

[4] Note that real cash balances, a stock, constrain consumption and education services, a flow. The justification is provided by Feenstra (1985) for the CIA constraint in a continuous-time economy. Formally, one can write $\int_t^{t+\eta} [C(s) + p(s)e(s)]\, ds \leq M(t)$, where the integral can be (first-order) approximated as $\eta [C(t) + p(t)e(t)]$ and η is the length of time money must be held to finance consumption and education services. By normalizing η to one, we obtain the expression $C(t) + p(t)e(t) \leq M(t)$. This CIA constraint binds the rate at which cash goods can be purchased.

CIA constraint. (It can be easily shown that at the optimum the co-state variable associated with (7) equals λ_i.) The Pontriyagin Maximum Principle gives the following first-order conditions for household i's problem,

$$
\begin{aligned}
C_i &: \quad C_i^{\gamma-1} = \mu_i \frac{1}{p}, && (10)\\
K_i &: \quad \dot{\lambda}_i - \rho\lambda_i = -\lambda_i r, && (11)\\
H_i &: \quad \dot{\mu}_i - \rho\mu_i = -\lambda_i w, && (12)\\
M_i &: \quad \dot{\lambda}_i - \rho\lambda_i = \lambda_i (1+\pi) - \mu_i \frac{1}{p}, && (13)
\end{aligned}
$$

and transversality conditions,

$$
\begin{aligned}
\lim_{t\to\infty} \lambda_i K_i e^{-\rho t} &= 0, && (14)\\
\lim_{t\to\infty} \mu_i H_i e^{-\rho t} &= 0, && (15)\\
\lim_{t\to\infty} \lambda_i M_i e^{-\rho t} &= 0. && (16)
\end{aligned}
$$

To close the model we need to specify the market clearing conditions and the process of money supply. The factor markets clear when supply of factor inputs equal their demand:

$$
\begin{aligned}
K &\equiv \int K_i = K^f + K^e, && (17)\\
H &\equiv \int H_i = L^f + L^e. && (18)
\end{aligned}
$$

Furthermore, markets in the final goods and education clear when

$$
\begin{aligned}
Y^f &= \int \left(c_i + \dot{K}_i\right), && (19)\\
Y^e &= \int e_i. && (20)
\end{aligned}
$$

Finally, let money supply grow at the rate θ and let π be the inflation rate. Then real money supply follows

$$\frac{\dot{M}}{M} = \theta - \pi. \tag{21}$$

For simplicity, we assume that the rates of real transfers are proportional to individual real balances, i.e., $\tau_i = \theta M_i$, $i \in [0,1]$. This assumption is made to prevent monetary policies from imposing direct distributional effects on the economy.

3 Characterization of Equilibrium

3.1 Cross-Sector Allocations

The production side of our economy resembles the standard setup of two-sector economies as analyzed in Meade (1961), which in turn is often adopted in international trade theory. The setup has been adopted by many two-sector growth models, including Uzawa (1961, 1963), Bond et al. (1996), and Mulligan and Sala-i-Martin (1993). As Bond et al. (1996) did, we follow the standard practice in international trade theory to characterize sectoral capital-labor ratios (ratios of physical capital to effective labor) and factor prices as functions of relative output price. From (3) and (4), we obtain

$$\frac{K^f}{L^f} = \kappa p^{\frac{1}{\alpha-\beta}}, \qquad \frac{K^e}{L^e} = \frac{(1-\alpha)\beta}{\alpha(1-\beta)} \kappa p^{\frac{1}{\alpha-\beta}}, \tag{22}$$

and

$$w = (1-\alpha) A^f \kappa^\alpha p^{\frac{\alpha}{\alpha-\beta}}, \qquad r = \alpha A^f \kappa^{\alpha-1} p^{\frac{\alpha-1}{\alpha-\beta}}, \tag{23}$$

where

$$\kappa \equiv \left(\frac{A^e}{A^f}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}} \left(\frac{1-\alpha}{1-\beta}\right)^{\frac{\beta-1}{\alpha-\beta}}.$$

Define $u \equiv L^e/H$ as the fraction of the economy's aggregate effective labor allocated to the education sector, and $v \equiv K^e/K$ as the fraction of physical capital allocated to that sector. Equation (22) can be rewritten as

$$\frac{v}{u} = \frac{(1-\alpha)\beta}{\alpha(1-\beta)} \frac{1-v}{1-u}, \tag{24}$$

implying that the capital-labor ratios in the two sectors are proportional to each other. The proportion coefficient depends on the relative factor intensities in the two sectors. The sector that is more capital intensive will have a higher capital-labor ratio. From (24) the following monotonic relationship between the shares of capital and labor allocated to a given sector obtains:

$$v = \frac{\beta(1-\alpha)u}{\alpha(1-\beta)(1-u) + \beta(1-\alpha)u}. \tag{25}$$

Hence it suffices to describe the sectoral allocation of factors of production solely in terms of the variable u.

Let $k \equiv K/H$ be the economy-wide ratio of physical and human capital. The factor market clearing conditions (17) and (18) imply

$$u = \frac{K^f/L^f - k}{K^f/L^f - K^e/L^e} = \frac{\alpha(1-\beta)}{\alpha-\beta}\left(\kappa - kp^{\frac{1}{\beta-\alpha}}\right). \tag{26}$$

The sectoral outputs, normalized by the stock of human capital, are

$$y^f \equiv \frac{Y^f}{H} = (1-u)A^f\left(\frac{K^f}{L^f}\right)^{\alpha}, \tag{27}$$

$$y^e \equiv \frac{Y^e}{H} = uA^e\left(\frac{K^e}{L^e}\right)^{\beta}. \tag{28}$$

They can be regarded as functions of p and k in the light of (22) and (26).

For future references, we write the factor prices and sectoral allocations as functions of p and k: $r(p)$, $w(p)$, $u(p,k)$, $y^f(p,k)$, and $y^e(p,k)$. To be concrete, we assume that the education sector is less intensive in physical capital, i.e., $\alpha > \beta$. The sole dependence of factor prices on the relative output price is known as the *factor price equalization* property in international trade theory. It can be shown that $r'(p) < 0$ and $w'(p)\, p/w(p) > 1$, which is precisely the content of the *Stolper-Samuelson Theorem*: An increase in the relative price of education, p, leads to a more-than-proportional increase (resp. decrease) in the wage (rental) rate, that is, the price of the factor used more (resp. less) intensively in that sector.[5] Furthermore, the capital-labor ratios in both sectors are increasing in p. The sectoral allocation, u, increases in p and decreases in k. These imply that y^e also increases in p and decreases in k. The negative dependence of y^e on k is a revelation of the *Rybczynski effect*: an increase in the quantity of human capital (i.e., a fall in k) leads to a more-than-proportional expansion of the output of the education sector, which uses human capital more intensively.

3.2 Dynamics

We are interested in two aspects of the behavior of our dynamic system. First, we would like to derive the aggregate dynamics from the dynamics of individual households. Second, we intend to characterize the income

[5] Note that to obtain an interpretation consistent with the Stolper-Samuelson Theorem, the inequality $r'(p) < 0$ can regarded as indicating that the rental rate in units of education output, r/p, increases more than proportionately than the price of goods in terms of education, $1/p$, that is, $(r/p)/(1/p) = r$ is increasing in $1/p$.

distribution across households along the model's dynamic path. We now deal with the first issue.

Define $k_i \equiv K_i/H_i,\ i \in [0,1]$, as household i's ratio of physical and human capital stock. It is straightforward to show that the evolutions of variables pertaining to individual households are given by

$$\frac{\dot{C}_i}{C_i} = \frac{1}{1-\gamma}\left(r-\rho-\frac{\dot{\pi}+\dot{r}}{1+\pi+r}\right), \tag{29}$$

$$\frac{\dot{K}_i}{K_i} = r+w\frac{1}{k_i}+[\theta-(1+\pi)]\frac{M_i}{K_i}-\frac{\dot{M}_i}{M_i}\frac{M_i}{K_i}, \tag{30}$$

$$\frac{\dot{H}_i}{H_i} = \frac{e_i}{H_i}, \tag{31}$$

$$\frac{\dot{M}_i}{M_i} = \frac{\dot{C}_i}{C_i}\frac{C_i}{M_i}+\left(\frac{\dot{p}}{p}+\frac{\dot{e}_i}{e_i}\right)\frac{pe_i}{M_i}, \tag{32}$$

$i \in [0,1]$. The evolution of C_i in (29) is derived by taking time derivative of equation (10) and using (11)-(13). Equation (30) relates the growth of household i's physical capital stock to the growth of its real balance. It is obtained by dividing both sides of the household budget constraint by K_i and using (5) along with $\tau_i = \theta M_i$. Equation (31) is simply (6) rewritten. Finally, equation (32) is obtained by taking time derivative of household i's CIA constraint.

Let variables without subscript "i" denote aggregate quantities. Let $c \equiv C/H$, $k \equiv K/H$, $m \equiv M/H$. It turns out we can characterize the aggregate dynamics of our system by the following set of first-order differential equations in five variables, p, c, k, m, and π:

$$\frac{\dot{p}}{p} = r(p) - \frac{w(p)}{p}\frac{1}{1+r(p)+\pi} - \frac{r'(p)\dot{p}+\dot{\pi}}{1+r(p)+\pi}, \tag{33}$$

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{H}}{H} = \frac{1}{1-\gamma}\left[r(p) - \rho - \frac{r'(p)\dot{p}+\dot{\pi}}{1+r(p)+\pi}\right] - y^e(p,k), \tag{34}$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{H}}{H} = \left[y^f(p,k) - c\right]\frac{1}{k} - y^e(p,k), \tag{35}$$

$$\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \frac{\dot{H}}{H} = \frac{\dot{c}}{c}\frac{c}{m} + \left[\frac{\dot{p}}{p} + \frac{y^e_p\dot{p} + y^e_k\dot{k}}{y^e(p,k)}\right]\left(1 - \frac{c}{m}\right), \tag{36}$$

$$\pi = \theta - \frac{\dot{m}}{m} - y^e(p,k), \tag{37}$$

where y^e_p and y^e_k are the partial derivatives of y^e with respect to p and k.

Equation (33) , obtained from combining household first-order conditions (11)-(13), is the intertemporal no-arbitrage condition concerning investment to physical versus human capital. It gives the intertemporal adjustment of the relative price of education that is necessary to bring the net return on human capital equal to the net return on physical capital. The instantaneous return on physical capital is the rental rate, $r(p)$. Absent monetary considerations, the instantaneous return on human capital is just $w(p)/p$. Therefore the capital gain on human capital, $\dot{p}/p$, must equal the difference between $r(p)$ and $w(p)/p$ in order for investment in both types of capital to take place. However, with purchase of education services subject to the CIA constraint, the instantaneous return on human capital has to be adjusted for the gross nominal interest rate, which is given by $R \equiv 1 + r(p) + \pi$. The possible change of R over time further adjusts the return on human capital. If $\dot{R}/R = [r'(p)\dot{p} + \dot{\pi}] / [1 + r(p) + \pi]$ is positive, then return to current human capital investment is adjusted upward because of an intertemporal substitution effect: now is a better time to invest than later.

The growth of aggregate consumption, normalized by the human capital stock, is described by equation (34). The term $y^e(p,k)$ is simply the growth rate of aggregate human capitals stock, $\dot{H}/H$, which results from aggregating (31). From (29) we see that the growth rate of individual consumption is identical across households, and is thus equal to the growth rate of aggregate consumption.[6] Equation (34) is a standard Euler equation for consumption/saving choices except for the presence of nominal interest rate changes, which simply reflects our assumption that consumption purchase is also subject to the CIA constraint.

Equation (35) specifies the evolution of aggregate physical capital, relative to aggregate human capital. It results from substituting the goods market clearing condition into $\dot{K}/K$. Equation (36) comes from differentiating the CIA constraint. Finally, equation (37) obtains from (21).

Our system differs from standard non-monetary, two-sector endogenous growth models with physical and human capital in two important respects. First, the presence of money augments the standard dynamic system with the additional equations pertaining to the real balance and inflation. Second, as already discussed, monetary frictions inflicted by the CIA constraint cause the nominal interest rate and its growth to appear in the intertemporal no-arbitrage condition (33) and the consumption/saving Euler equation (34).

Although one of our ultimate goals is to analyze the distribution of in-

[6] Note that households face the same growth rate for their individual shadow values of capital.

come across households, our model possesses the feature that the model's aggregate dynamics can be tracked independently of the distribution of individual characteristics, despite the presence of household heterogeneity. This feature is dubbed the "representative-consumer theory of distribution" by Caselli and Ventura (2000), and shared by Garćia-Peñalosa and Turnovsky (2006). It arises because of the iso-elasticity of households' felicity function, which greatly facilitates aggregation.[7] Nevertheless, the distribution of income is affected by the aggregate variables. The main thrust of our analysis is that cross-sector allocation of resources, represented by the variable u, i.e., the fraction of aggregate effective labor used in the education sector, is a sufficient statistic for characterizing the distribution of income, an issue we now turn to.

3.3 Income Inequality

Household i's total income, denoted by Y_i, consists of capital income, rK_i, and labor income, wH_i. Its income share, s_i^y, is defined to be the ratio of Y_i to the aggregate income (output) Y:

$$s_i^y \equiv \frac{Y_i}{Y} = \frac{rK_i + wH_i}{Y^f + pY^e}.$$

Obviously s_i^y has mean one. Denote its variance by σ_y^2. Following Garćia-Peñalosa and Turnovsky (2006), we use σ_y^2 as our measure of income inequality as it represents the dispersion of households' relative income shares. Sim-

[7] See Deaton (1992) and Bertola et al. (2006) for details on the aggregation issue. Also notice that human capital investment is divisible in our model. Galor and Zeira (1993) presents a model with indivisible human capital investment (and credit market imperfections) where the distribution of wealth affects long-run growth.

ilarly, define household i's physical and human capital shares at any point in time as

$$s_i^k \equiv \frac{K_i}{K}, \quad s_i^h \equiv \frac{H_i}{H}.$$

Again, both s_i^k and s_i^h have unit mean. Their variances are denoted by σ_k^2 and σ_h^2, respectively. For simplicity, we assume that at the initial distribution, s_i^k and s_i^h are uncorrelated, that is, their covariance $\sigma_{k,h} = 0$ at time 0.

Lemma 1 *Household i's income share is a convex combination of its physical and human capital shares:*

$$s_i^y = \phi s_i^k + (1 - \phi)\, s_i^h, \tag{38}$$

where

$$\phi \equiv \frac{\alpha\,(1-\beta)\,(1-u) + \beta\,(1-\alpha)\,u}{(1-\beta)\,(1-u) + (1-\alpha)\,u} \in (0,1)\,. \tag{39}$$

If $\alpha = \beta$, then $\phi = \alpha = \beta$. If $\alpha \neq \beta$, then ϕ is positively related to the fraction of factors of production allocated to the sector which is more physical capital intensive. That is, $\partial\phi/\partial\,(1-u) > 0$ for $\alpha > \beta$ and $\partial\phi/\partial u > 0$ for $\alpha < \beta$.

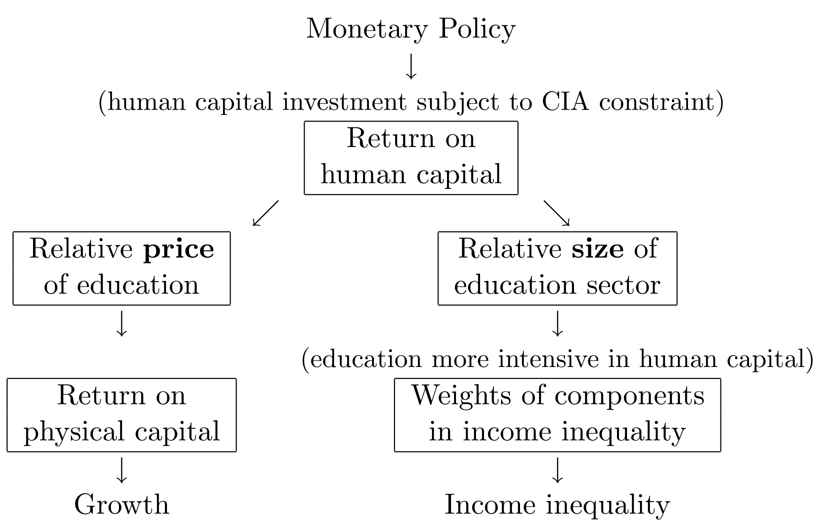
Proof. See the Appendix. ■

The result in Lemma 1 is quite intuitive. In fact, the weight ϕ in (38) is the share of national income accruing to physical capital. Naturally, expanding the relative size of the sector that is more intensive in physical capital will increase the demand for that factor of production, bid up its rental rate, thereby raises its share in total income.

4 Inflation, Growth, and Inequality

The mechanism of our model can be depicted as follows. Monetary policy affects inflation and hence the return on human capital as human capital investment is subject to the CIA constraint. This, in turn, affects growth by changing the relative *price* of education services and the return on physical capital. At the same time, the change in the return on human capital causes a change in the relative *size* of the education sector, which is more intensive in human capital. This then exerts an influence on income inequality by altering the weights attached to the two types of heterogeneities.

Illustration of the Mechanism



4.1 Monetary Frictions and Growth

We are interested in the implications of money growth (and inflation) on long-run economic growth and income inequality. To this end we look at the model's balanced growth path (BGP), along which consumption, physical and human capital stock, real balance, and sectoral output all grow at constant rates while sectoral allocation of factor inputs, the relative price of education, and inflation remain constant. That is, along the BGP,

$$\frac{\dot{p}}{p} = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{m}}{m} = \frac{\dot{\pi}}{\pi} = 0. \tag{40}$$

(Recall that c, k, and m are all expressed relative to the stock of human capital.) Furthermore, each household's physical capital, human capital, and real balance all grow at constant rates. The growth rates, however, are not restricted to be identical across households.

One can show that the BGP also features the constancy of individual households' shares of physical and human capital, and therefore the constancy of their income shares. That is, each individual household's capital stock grows at the same rate as the aggregate stock. Its income also grows at the same rate as the aggregate income. We can therefore talk meaningfully about the "long-run" income distribution since the distribution has a well defined steady state which obtains when the aggregate economy rests at its BGP.

Proposition 1 *Along the BGP, individual households' shares of physical and human capital are constant, i.e.,* $\dot{s}_i^k = \dot{s}_i^h = 0$ *for all* i*. So are their income shares, i.e.,* $\dot{s}_i^y = 0$ *for all* i*.*

Proof. (1) Note that $\int \left(\dot{H}_i/H_i\right) s_i^h = 1$ always holds. Along the BGP, $\dot{H}_i/H_i$ is constant over time for all i (it might be different across households).

Applying mathematical induction, it is easy to show that s_i^h is constant for all i. That is, $\dot{s}_i^h/s_i^h = \dot{H}_i/H_i - \dot{H}/H = 0$.

(2) $\dot{s}_i^h = 0$ implies that e_i/H_i, and in turn $\dot{e}_i/e_i$, is independent of i. Then using the time derivatives of the individual and aggregate CIA constraints, we obtain

$$\frac{\dot{M}_i}{M_i} - \frac{\dot{M}}{M} = \left(\frac{\dot{C}}{C} - \frac{\dot{p}}{p} - \frac{\dot{Y}^e}{Y^e}\right)\left(\frac{C_i}{M_i} - \frac{C}{M}\right) = 0.$$

Thus $\dot{M}_i/M_i = \dot{M}/M = \theta - \pi$. Given the results that $\dot{s}_i^h = 0$ and $\dot{M}_i/M_i = \dot{M}/M$, we can write $\dot{s}_i^k$ as

$$\dot{s}_i^k = \left(\frac{\dot{K}_i}{K_i} - \frac{\dot{K}}{K}\right) s_i^k = \frac{w-m}{k} s_i^h - \frac{w-m}{k} s_i^k.$$

The aggregate transversality condition, $\lim_{t\to\infty} \lambda K e^{-\rho t} = 0$, requires that $\dot{\lambda}/\lambda + \dot{K}/K - \rho < 0$, or $r > \dot{K}/K$. From the aggregate budget constraint, $\dot{K}/K = r + (w-m)/k$, we have $(w-m)/k < 0$. Hence $\dot{s}_i^k$ is linear in s_i^k with a positive constant coefficient. To be consistent with a long-run equilibrium with nondegenerate distribution of capital, we must have $\dot{s}_i^k = 0$ for all i for all time.

(3) It is obvious from (38) that s_i^y is constant over time for all i. ■

To see clearly how the presence of monetary frictions affects the economy's long-run performance, it would be worthwhile to first review the mechanics of growth in the corresponding real environment, whose BGP can be characterized by the following system:

$$\frac{\dot{p}}{p} = r(p) - \frac{w(p)}{p} = 0, \tag{41}$$

$$\frac{\dot{c}}{c} = \frac{1}{1-\gamma}\left[r(p) - \rho\right] - \psi = 0, \tag{42}$$

$$\frac{\dot{k}}{k} = \left[y^f(p,k) - c\right]\frac{1}{k} - \psi = 0, \tag{43}$$

where the aggregate growth rate (common for consumption, physical and human capital, and outputs) is simply

$$\psi = \frac{\dot{H}}{H} = y^e(p,k). \tag{44}$$

Equation (41) solely determines the relative price of education, p, by the equality $r(p) = w(p)/p$. Depicted in Figure 1a, $r(p)$ is downward sloping while $w(p)/p$ is upward sloping, which is precisely the manifestation of the *Stolper-Samuelson Theorem*. The value of p is uniquely pinned down by the intersection of $r(p)$ and $w(p)/p$ curves. Equation (42) then yields the aggregate growth rate, $\psi(p) = [r(p) - \rho]/(1-\gamma)$. The larger the relative price p, the lower the rental rate $r(p)$, and the lower the growth rate ψ, as the lowered return on physical capital discourages its accumulation and suppresses growth. Equation (44), in turn, gives the value of k, the ratio of physical and human capital. The condition $\psi(p) = y^e(p,k)$ is depicted in Figure 1b, where for a given p, the horizontal line represents $\psi(p)$ while the curve $y^e(p,k)$ is downward sloping in k, reflecting the *Rybczynski effect*. Finally, equation (43) determines the value of c.

Now, going back to our model with monetary frictions, we notice that equations (42)-(44) remain valid for the characterization of BGP, whereas the equation that determines p, i.e., (41), has to be modified to incorporate the distortion associated with positive nominal interest rates as follows:

$$r(p) - \frac{w(p)}{p} \frac{1}{1 + r(p) - \psi(p) + \theta} = 0, \tag{45}$$

where we have used the fact that along the BGP,

$$\pi = \theta - \psi(p) \tag{46}$$

(see (37)). Whenever $\pi > -r$, or equivalently, $\theta > -(r - \psi)$, the (net) nominal interest rate is strictly positive ($R > 1$), which reduces the return on human capital. This will raise the relative price of education. In Figure 1a, the dashed curve, $w(p)/(pR)$, is everywhere below $w(p)/p$, resulting in a higher value of p than in the real model. The intuition is that the CIA constraint on education purchase makes investment in human capital more costly and pushes up its price. The increase in p then raises the value of k as it lowers the aggregate growth rate ψ and shifts the curve $y^e(p, k)$ up, as illustrated in Figure 1b. That is, human capital becomes relatively more scarce.

We summarize these findings in the following proposition.

Proposition 2 *Suppose $\alpha > \beta$. Compared to the real model, the economy with monetary frictions has a higher relative price of education p and lower aggregate growth rate ψ, as long as the gross nominal interest rate R is greater than one.*

Proof. See the Appendix. ■

The above proposition indicates that the *presence* of monetary frictions distorts sectoral allocations and exerts an adverse effect on growth. It is also interesting to see how *changes* in the extent of these frictions alter the extent of distortions. In particular, we study how changes in the money growth rate, θ, affect the economy's BGP. It is not difficult to show that faster growth of money supply will lead to higher inflation and lower the aggregate growth rate.

Proposition 3 *Suppose $\alpha > \beta$. An increase in the money growth rate θ raises the BGP inflation rate π and the relative price of education p, and reduces the aggregate growth rate ψ.*

Proof. See the Appendix. ■

The reasoning behind Proposition 3 can again be illustrated by Figure 1. Note that the gross nominal interest rate

$$R = 1 + r(p) + \pi = 1 + \frac{\rho}{1-\gamma} - \frac{\gamma}{1-\gamma} r(p) + \theta.$$

An increase in θ raises R for all p and therefore shifts the dashed curve downward in Figure 1a, resulting in a higher value for p. Interestingly, this implies that R must be higher with higher θ, signifying a larger monetary friction.[8] The increase in p also leads to a lower aggregate growth ψ and a higher ratio of physical to human capital k, as the severer monetary friction makes human capital more scarce (see Figure 1b). Finally the inflation rate $\pi = \theta - \psi$ goes up.

[8] To see this, note that along the BGP $r(p) = w(p)/(pR)$ holds. Hence R has to be higher whenever p is higher.

Money is therefore not superneutral in our model in that it affects adversely the aggregate growth rate. Essentially, the higher inflation (and the higher nominal interest rate) associated with an increase in money growth leads households to economize their real balances and to cut education expenditures, which makes human capital more expensive and retards growth. This also implies that money growth also affects the cross-sector allocation of resources in our model. As we pointed out before, the sectoral allocation of factors of production is key to understanding the behavior of income inequality. The dependence of income inequality on sectoral allocations gives rise to an added dimension of the non-superneutrality of money, whereby changes in money growth alter the distribution of income across agents. The next subsection investigates the effects on income inequality of changes in θ.

4.2 Sectoral Allocations and Inequality

The determination of the sectoral allocation is illustrated in Figure 1c. The fraction of factor inputs allocated to the education sector, u, is pinned down by the intersection of ψ and $y^e = uA^e \left(K^e/L^e\right)^{\beta}$, the latter of which is linear in u for a given value of p (and hence K^e/L^e). The effect of the presence of monetary frictions shows up as a decline in the aggregate growth rate ψ and a steeper y^e line (the dashed lines), resulting in a lower value of u, as compared to the real model (the solid lines). This effect gets stronger as the money growth rate θ increases, since the reduction in the aggregate growth and the rise of the relative price of education are pushed by even greater extent. We therefore have $du/d\theta < 0$. As a result, the share of income that goes to physical capital, ϕ, increases with θ as ϕ is decreasing in u as Lemma 1 indicates. It turns out that how changes in θ affect income inequality σ_y^2 depends on the comparison of the two types of heterogeneities in our model.

Proposition 4 *An increase in θ enlarges (reduces) income inequality when physical-capital heterogeneity dominates (is dominated by) human-capital heterogeneity in the sense that $\sigma_k^2 > (<) \frac{1-\phi}{\phi}\sigma_h^2$.*

Proof. With $\sigma_{k,h} = 0$, equation (38) implies

$$\sigma_y^2 = \phi^2 \sigma_k^2 + (1-\phi)^2 \sigma_h^2. \tag{47}$$

Since σ_k^2 and σ_h^2 are fixed along the BGP, we have

$$\frac{d\sigma_y^2}{d\theta} = \left[2\phi\sigma_k^2 - 2(1-\phi)\sigma_h^2\right] \frac{d\phi}{du}\frac{du}{d\theta}.$$

The result follows. ■

As indicated by (47), one can decompose income inequality σ_y^2 into a component associated with physical-capital heterogeneity and the other associated with human-capital heterogeneity. Regardless of the cross-sector difference of factor intensities and the comparison between σ_k^2 and σ_h^2, a higher money growth rate leads to an expansion of the goods producing sector relative to the education sector, which in turn raises the income share of physical capital when the goods producing sector is more intensive in that factor of production. It hence raises the contribution of σ_k^2 to the overall income inequality while reducing the contribution of σ_h^2. The overall effect depends on the relative size of the two types of heterogeneity as spelt out in Proposition 4. In particular, it is positive when the heterogeneity in physical capital is the dominant force.

We regard the case of $\alpha > \beta$ as the empirical plausible one. Hence our model predicts a negative impact of money growth on aggregate growth, along with a positive effect on inflation. Variations in money growth therefore trace out a *negative* relation between inflation and growth. In the light of Proposition 4, variations in money growth also trace out a *positive* relation between inflation and inequality and a *negative* relation between growth and inequality when physical-capital heterogeneity dominates. The latter two relations will reverse signs when it is human-capital heterogeneity that dominates. The results are summarized in Table 1.

Table 1: Growth-Inflation-Inequality Relations ($\alpha > \beta$)

	$\sigma_k^2 > \frac{1-\phi}{\phi}\sigma_h^2$	$\sigma_k^2 < \frac{1-\phi}{\phi}\sigma_h^2$	$\sigma_k^2 = \frac{1-\phi}{\phi}\sigma_h^2$
Inflation-Growth	Negative	Negative	Negative
Inflation-Inequality	Positive	Negative	Unrelated
Growth-Inequality	Negative	Positive	Unrelated

4.3 The Growth, Inflation, and Inequality Relations: Some Existing Evidence

Although our study is intended to be theoretical, it is useful to partially review the available empirical evidence concerning the relation among growth, inflation, and income inequality. Regarding the growth-inflation relation, early empirical work indicates that high-inflation countries tend to grow slower than low-inflation countries. Fischer (1991) reports that the slowgrowth countries have an average inflation rate slightly above 30 percent, while the fast-growth countries average only 12 percent inflation. Gylfason and Herbertsson (2001) list 17 studies for which all but one find a significant decrease in the growth rate from increasing the inflation rate from 5 to 50%. It is also found that the negative effect is marginally stronger at low inflation rates and marginally weaker as the inflation rate rises. This negative and highly nonlinear effect is strongly supported in Sarel (1996) , Judson and Orphanides (1996) , Ghosh and Phillips (1998) , Khan and Senhadji (2001) , and Gillman, Harris, and Matyas (2003) . More recent studies find a "threshold" rate of inflation, above which the effect on growth is strongly significant and negative, but below which the effect is insignificant and positive. This threshold level has been found through testing to be at 1% inflation rate for industrialized countries and at 11% for developing countries (Khan and Senhadji 2001). Using instrumental variables, however, the negative nonlinear inflation-growth effect has been reinstated at all *positive* inflation rate levels for both developed and developing country samples (Ghosh and Phillips 1998, Gillman, Harris, and Matyas 2003).

Most studies on inflation and inequality indicate a positive relationship between the two. The cross-country regression in Romer and Romer (1998) shows that a one-percentage-point rise in average inflation is associated with a rise in the Gini coefficient of 0.2 points, and that the null hypothesis of no relationship is rejected. They also report that for the sample consisting of OECD countries as of 1973, there is a quantitatively large and statistically significant positive association between inequality and average inflation. The evidence for a strong positive correlation between inequality and inflation for democratic countries is presented in Beetsma (1992). Al-Marhubi (1997) finds a positive correlation between inequality and average inflation similar to the one reported in Romer and Romer. He also finds that this result is robust to controlling for political stability, central bank independence, and openness. In Easterly and Fischer (2000), direct measures of improvement in the well-being of the poor are negatively correlated with inflation in pooled cross-country regressions. Looking at a sample of 51 industrialized and developing countries over the time period from 1966 to 1990, Albanesi (2002) computes the OLS estimates of the relation between the inflation tax (defined as the net inflation rate divided by the gross inflation rate) and inequality and finds that the estimated slope coefficient is 0.4561 for the full sample. This corresponds to a 2% rise in the inflation tax rate associated with a one standard deviation (7 points) increase in the Gini coefficient.

The empirical evidence on the relationship between growth and income inequality has been inconclusive. Alesina and Rodrik (1994) , Persson and Tabellini (1994) , Perotti (1996) , and others find a negative relationship between the two. More recent studies, for example, Li and Zou (1998) , Forbes (2000) , and Lundberg and Squire (2003) , however, obtain a positive, or at least more ambiguous relationship. Barro (2000) finds that the relationship between growth and inequality depends on the wealth level of each country: it tends to be negative for poorer countries, and be positive for richer countries.

Under the assumption that the goods producing sector is relatively more intensive in physical capital and that education purchase is subject to monetary frictions, our model delivers results consistent with the empirical evidence on the growth-inflation relation. Our model would also be consistent with the evidence on the inflation-inequality relation so long as heterogeneity in physical wealth dominates that in agents' skills, i.e., heterogeneity in human capital.

5 Factor Intensities and the (Non-)Superneutrality of Money

Thus far our results rely on the assumption that there exist cross-sector factor intensity differentials, in particular that the education sector is relatively intensive in human capital. To highlight the role played by such differentials it would be useful to discuss the case where they are absent. Suppose the factor intensities are identical across the two sectors, i.e., $\alpha = \beta$. It is easy to show that in this case the two sectors have the same capital-labor ratio. That is, $K^f/L^f = K^e/L^e = K/H = k$. Both the rental and wage rates are now functions of k (instead of p) alone:

$$w = (1-\alpha) A^f k^{\alpha}, \qquad r = \alpha A^f k^{\alpha-1}.$$

The equality of factor intensities also renders the relative price of education equal to the cross-sector productivity ratio:

$$p = \frac{A^f}{A^e}.$$

Setting r equal to $w/(pR)$ (analog of equation (45)) thus gives the BGP value of k. As in Figure 2a, k is determined by the intersection of the downward sloping $r(k)$ curve and the upward-sloping $w(k)/(pR)$ curve.[9] Finally, the fraction of factor inputs allocated to the education sector, u, is pinned down by the condition

$$\psi = y^e = uA^e k^\alpha,$$

as in Figure 2b.

Figure 2 also demonstrates what happens to k, ψ, and u as the money growth rate θ increases. The curve $w(k)/(pR)$ shifts down, resulting in a higher k and a lower r, which in turn reduces the aggregate growth rate ψ. Moreover, the increase in k makes the y^e line steeper, leading to a lower u. These results are similar to those presented for the case where $\alpha > \beta$, except that p is unaffected by changes in θ. An important difference arises regarding the coefficient ϕ in the expression for the income inequality (47). In the current situation, the income share of physical capital, ϕ, is simply equal to the common capital intensity:

$$\phi = \alpha = \beta.$$

In other words, although changes in the money growth still shift factor inputs away from the education sector $(du/d\theta < 0)$, it doesn't exert any impact on the distribution of income. These results are summarized in the following proposition.

[9] Note that here R is a function of k : $R = 1 + r(k) + \pi = 1 + \frac{\rho}{1-\gamma} - \frac{\gamma}{1-\gamma} r(k) + \theta.$

Proposition 5 *If $\alpha = \beta$, then an increase in money growth negatively affects the aggregate growth rate, but is inequality-neutral.*

Therefore sectoral allocations matter for income inequality only when the two production sectors are different in a nontrivial way.

6 Conclusions

In this paper we have developed a monetary, two-sector, heterogeneous-agent endogenous growth model with both physical and human capital. We find that monetary frictions on human capital investment and cross-sector factor intensity differentials play important roles in generating the non-superneutrality of money with respect to growth, the relative price of sectoral outputs, cross-sector allocations of resources, and the inequality of income across households.

The framework developed here provides a convenient starting point for various extensions in future research. First, one can introduce variable aggregate labor supply into the model and investigates its interaction with sectoral allocations of factor inputs in the mechanism through which monetary policy affects growth, inflation, and inequality. Second, in addition to the balanced growth analysis conducted in the present paper, one can analyze the model's transitional dynamics. Finally, it will be also interesting to study whether fiscal policies, such as education subsidies, can mitigate the distortions caused by monetary frictions and the behavior of growth, inflation, and income inequality.

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Appendix:

Proof of Lemma 1.

It is easy to see that $\phi = \frac{rK}{Y}$, which is the share of total income accruing to physical capital. Using (23), (25), and the two production functions, we can write ϕ as in (39). When $\alpha \neq \beta$, $\partial\phi/\partial u = \frac{(\beta-\alpha)(1-\alpha)(1-\beta)}{[(1-\beta)(1-u)+(1-\alpha)u]^2}$. It is negative when $\alpha > \beta$ and positive when $\alpha < \beta$.

Proof of Proposition 2.

The relative price of education is determined by $r(p) = w(p)/(pR)$. Differentiating this equation with respect to the nominal interest rate R gives

$$\frac{dp}{dR} = \frac{r}{(w/p)' - r'R} > 0$$

since $w(p)/p$ is increasing in p and $r(p)$ is decreasing in p. Given that $R = 1$ in the real environment, the price in the model with monetary frictions is higher as long as $R > 1$. Differentiating equation (42) with respect to p yields $d\psi/dp = r'(p)/(1-\gamma) < 0$. Therefore $d\psi/dr = (d\psi/dp)/(dp/dR) < 0$.

Proof of Proposition 3.

Differentiating equation (45) with respect to θ and using the result that $d\psi/dp = r'(p)/(1-\gamma)$, we obtain

$$\frac{dp}{d\theta} = \frac{w/p}{(w/p)' R + (w/p)\gamma r'/(1-\gamma) - r'R^2} > 0.$$

Thus $d\psi/d\theta < 0$. Equation (46) implies that $d\pi/d\theta = 1 - d\psi/d\theta$, and

$$\frac{d\pi}{d\theta} = \frac{(w/p)' R - (w/p) r'/(1-\gamma) - r'R^2}{(w/p)' R + (w/p)\gamma r'/(1-\gamma) - r'R^2} > 0.$$

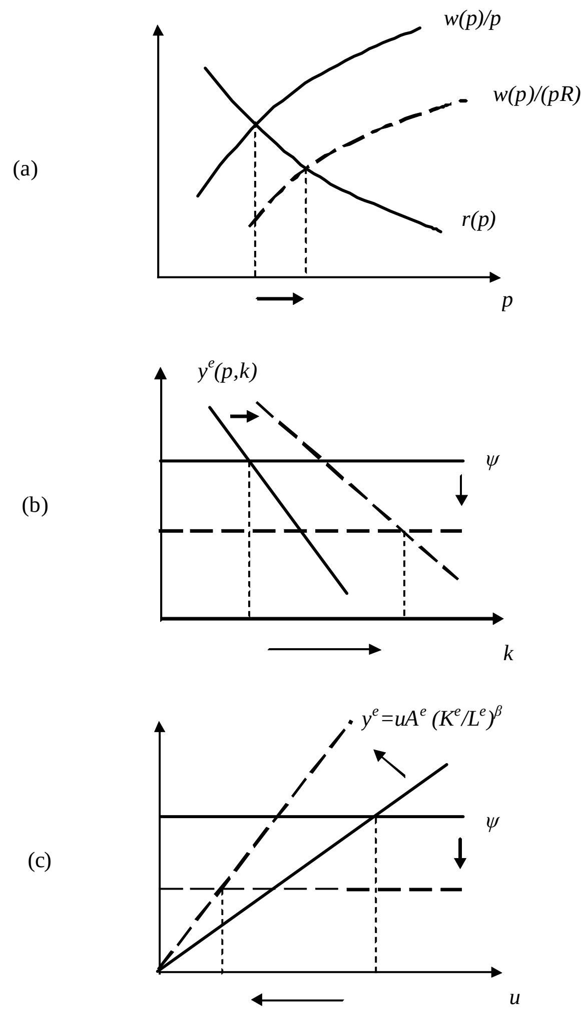
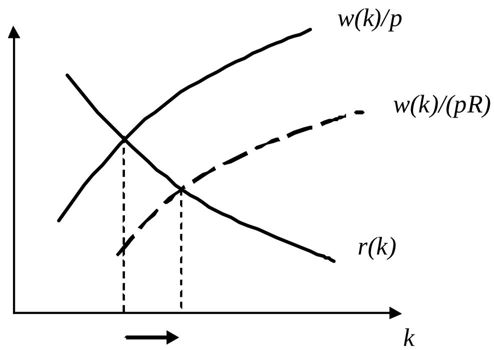


Figure 1: The case of $\alpha > \beta$.

(a)



(b)

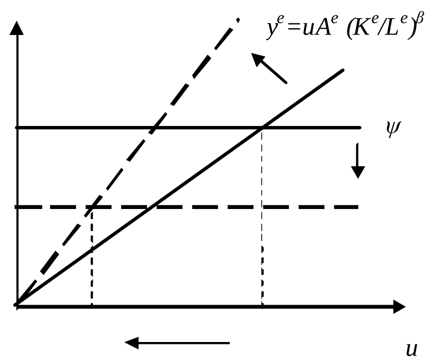


Figure 2: The case of $\alpha = \beta$.