Migration and Business Cycle Dynamics

Christie Smith and Christoph Thoenissen

August 2018

JEL classification: F41, F44, F22, E32

www.rbnz.govt.nz/research/discusspapers/

Discussion Paper Series

ISSN 1177-7567
Abstract

Shocks to net migration matter for the business cycles. Using an estimated dynamic stochastic general equilibrium (DSGE) model of a small open economy and a structural vector autoregression, we find that migration shocks account for a considerable proportion of the variability of per capita GDP. Migration shocks matter for the capital investment and consumption components of per capita GDP, but they are not the most important driver. Migration shocks are also important for residential investment and real house prices, but other shocks play a larger role in driving housing market volatility. In the DSGE model, the level of human capital possessed by migrants relative to that of locals materially affects the business cycle impact of migration. The impact of migration shocks is larger when migrants have substantially different levels of human capital relative to locals. When the average migrant has higher levels of human capital than locals, as seems to be common in most OECD economies, a migration shock has an expansionary effect on per capita GDP and its components.
Non-technical summary

We examine the business cycle effects that arise from an expansion of the population due to migration. We identify the contribution that migration shocks make to cyclical fluctuations, and illustrate their dynamic impact. The analysis presented here is conducted in per capita terms.

We develop a dynamic stochastic general equilibrium (DSGE) model to explore the consequences of migration. Households consume foreign- and domestically-produced goods, and housing. Domestically produced goods are produced using effective labour and physical capital, where effective labour is a composite of physical labour and human capital. The level of human capital is particularly important for the dynamics of the migration shock as it amplifies the labour supply impact of a migration impulse.

The model also embodies other frictions. Physical capital, for example, has a variable utilisation rate and adjusting the stock of capital is costly. Variable utilisation and capital adjustment costs are important for the dynamics of shocks. Variable utilisation provides an extra margin of adjustment in response to a shock, while capital adjustment costs serve to slow the propagation of shocks to investment.

The model has three stocks of capital: physical capital; housing; and human capital. A migration inflow erodes the per capita stock of physical and housing capital, but the effect on the stock of human capital is ambiguous because migrants may have more or less human capital than domestic residents. The stock of migrant human capital relative to local resident human capital has a material impact on the dynamics of the migration impulse and on the contribution that migrant shocks make to business cycle fluctuations.

Using the estimated DSGE model, we show that migration shocks account for a considerable portion of the variability of per capita gross domestic product (GDP). While migration shocks matter for the capital investment and consumption components of per capita GDP, there are other drivers of cyclical fluctuations in these expenditure components that are even more important. Migration shocks also matter for residential investment and real house prices, but once again other shocks play a larger role in driving volatility.

In the DSGE model, the level of human capital possessed by migrants relative to that of locals materially affects the business cycle impact of migration. The impact of migration shocks is larger when migrants have substantially different – larger or smaller – levels of human capital relative to locals. When the average migrant has higher levels of human capital than locals, as seems to be common in most OECD economies, a migration shock has an expansionary effect on per capita GDP and its components, which also accords with the evidence from a restricted structural vector autoregression.
1 Introduction

In recent years, migration flows have been large. Very large. Large in absolute numbers of migrants and large relative to non-migrant populations in destination countries. According to the United Nations, there has been a 17 percent increase in the world-wide stock of migrants between 2010 and 2017.\footnote{www.un.org/en/development/desa/population/migration/data/index.shtml, accessed 9 May 2018.} Over the same period the stock of migrants rose by 18 percent in Western Europe, by 13 percent in the United States, 16 percent in Canada and by 18 percent in Oceania.\footnote{See Peri (2016) for more cross-country detail, though for a slightly earlier period.} In the world as a whole, population growth has been 8.5 percent during this period but in high-income countries it has been a paltry 3.8 percent. Thus, it is clear that migrant stocks are growing much more quickly than overall populations.

The economic causes and consequences of migration are complex and multi-dimensional, affecting both origin and destination countries. Kerr and Kerr (2011) and Nathan (2014) provide surveys that discuss various facets of migration, while Constant and Zimmermann (2013) and Chiswick and Miller (2015) provide handbooks on the topic. A particular focus of the literature has been on the effect of migration on the labour market (Borjas, 1999a; Dustmann et al., 2005; Borjas, 2014; Burstein et al., 2017). Much of this analysis has a strong microeconometric focus,\footnote{See, for example, the discussion paper series of the Centre for Research and Analysis of Migration.} sometimes based on partial equilibrium models or models that exploit cross-country or regional variation. The macroeconomic consequences of migration, and in particular the general equilibrium \textit{business cycle} consequences, are less well understood and have not been researched in much depth in the international literature. One notable exception is the work by Mandelman and Zlate (2012), which focuses on international risk sharing via migrants’ remittances.

In this paper, we focus on the role of skilled migrants as a driver of the business cycle in destination countries. How does skilled migration affect the per capita level of gross domestic product (GDP) and its components? How does skilled migration affect the real exchange rate, and finally, do shocks to skilled migration drive the business cycle?

To address these questions, we develop and estimate a dynamic stochastic general equilibrium model for a small open economy that experiences net migration flows. We fit this model to the New Zealand economy, because of the availability of excellent migration data. All arrivals and departures in New Zealand are subject to reporting requirements and virtually all migrants

\begin{thebibliography}{99}
\bibitem{A} \textit{A} and \textit{B} (2006) \textit{A journal} 1(2) 1-10.
\end{thebibliography}
arrive or depart by air, which provides a natural bottleneck for data collection. Migration flows into New Zealand have also been substantial in recent years, providing much-needed variation for econometric analysis. For example, net migration has increased working-age population in New Zealand by 1 percent in each of the three years from 2015-2017, and continues at a fast pace in 2018.

The key difference between migration flows and natural population increase is that migrants have pre-existing accumulations of financial and human capital, while babies enter the world with no capital balances. In our analysis we model migration allowing for human capital, but for simplicity we do not model stocks of financial capital.\(^4\) (We discuss this difference more in section 3.) Migration flows also affect the productive capacity of the economy immediately and cannot be perfectly anticipated in advance, though migration flows do exhibit substantial persistence.

Our analysis enables us to determine the contribution of migration shocks to the business cycle. We illustrate that the skill level of migrants relative to locals materially influences the dynamic impact of migration on the host economy. Borjas (1999a) notes that “the labour market impact of immigration hinges crucially on how the skills of migrants compare to those of natives in the host country”. We show that relative skill levels also matter for macroeconomic aggregates such as consumption and investment, in addition to labour market variables such as hours worked and wages. Migration shocks account for more of the volatility of the business cycle if migrants’ level of human capital differs from that of locals. If migrants have a higher level of human capital than locals, the effects of migration are expansionary on a per capita basis and migration shocks can account for a large fraction of the volatility of GDP and its components. When migrants have the same human capital as locals, migration shocks account for only a small fraction of the overall volatility of GDP. While still expansionary on a per capita basis, this kind of migration causes much less volatility for the host economy.

The literature on the business cycle effects of migration can be traced back to Jerome (1926), who explored the implications of immigration into the United States in the early twentieth century. However, the modern literature on the macroeconomic effects of migration, using time series and structural macroeconomic models, is relatively sparse. Our work is related to Weiske (2017a,b), who look at the macroeconomic effects of migration and population growth in the United States (US). Using constructed working-age net migration data for the United States in a vector autoregression, Weiske

\(^4\) On average, migrants entering New Zealand in 2004 and 2005 (excluding refugees) had about NZD36,000 worth of gross assets DOL (2009, table 6.4).
(2017a) finds that the short-run effects of migration are consistent with standard growth theory, ie real wages fall and investment increases. However, Weiske also finds that migration shocks make only a modest contribution to US business cycle dynamics. The latter result is not entirely surprising, since data from the Department of Homeland Security and the US Census Bureau suggest that the per annum migration rate for the United States has been below 1 percent since 1915 and, with two exceptions, has been below 0.4 percent since 1925.\(^5\)

Localised waves of migrants have been used in the literature as exogenous shocks to understand the effect of migration on wages, unemployment rates and house prices. Card (1990), for example, uses the 1980 Mariel boatlift of refugees from Cuba to Florida to examine the effects of migration on labour markets. The Mariel immigrants, relatively unskilled, increased the population of Miami by seven percent. Yet, according to Card, the influx had little effect on wages and unemployment rates overall. Borjas (2017), however, argues that the wage effects were substantial for the high school dropouts that were directly competing with the influx of unskilled migrants. The analytical framework developed below focuses on the aggregate effects of skilled migrant flows, not on the effect on specific types of labour grouped by skill. We will see, however, that relative skill-levels are particularly important for the macroeconomic consequences of migration shocks.

For some countries, the effects of migration shocks are more substantial. Furlanetto and Robstad (2016), for example, use Norwegian data and find that positive migration shocks are expansionary and are a major driver of the dynamics of unemployment, though they are unimportant for house prices.\(^6\) Barrell et al. (2010) examine a particular facet of migration, namely the migration that occurred following the accession of ten Eastern European countries into the European Union, highlighting large flows into Ireland and the United Kingdom. In a Bundesbank working paper, Stähler (2017) examines the macro impact of refugees in Germany. In Stähler’s model, refugees from the rest-of-the-world are absorbed only gradually into the labour force. Refugees initially increase output, via a demand channel, but the later dynamics depend on whether refugees accumulate more or less qualifications than locals.

In New Zealand, much of the literature on migration focuses on the hous-

---

\(^5\) See [https://www.dhs.gov/immigration-statistics/yearbook/2016/table1](https://www.dhs.gov/immigration-statistics/yearbook/2016/table1) and the Haver population series A111POPG10. These percentages are indicative since the immigration series are for fiscal years, and do not align perfectly with the Census numbers.

\(^6\) Likewise, Saiz (2003) finds that the Mariel boatlift had little impact on house prices in Miami, though rents increased by substantially more than other metropolitan areas.
ing market. Using a structural vector autoregression, Coleman and Landon-Lane (2007) find that migration has an extremely large impact on house prices, unlike the result reported for Norway by Furlanetto and Robstad (2016). Stillman and Maré (2008) apply microeconometric techniques to New Zealand census and house sales data to examine the impact of population and migration on house prices at a local, disaggregated level; they find no impact of foreign-born migrants on local house prices, though returning New Zealanders seem to have a statistically significant impact. In contrast, McDonald (2013) investigates the composition of New Zealand migration and finds that constituent parts of net migration have different macro consequences: migrant arrivals are found to have more substantial impact on house prices than migrant departures and the citizenship of migrants also appears to have implications for the domestic (New Zealand) impact of migration. In a similar vein, Vehbi (2016) finds that the age-composition of migrants matters, with (presumably wealthier) 30-49 year old migrants having more substantial effects on consumption, house prices, rents, and residential investment than 17-29 year old migrants.

Having briefly described the literature we now turn to the specification of the structural model that we use to investigate the impact of migration flows on the business cycle.

2 A model of migration in a small open economy

We analyse the effects of migration shocks on business cycle dynamics using a dynamic stochastic general equilibrium (DSGE) model of a small open economy. The standard small open economy model is augmented with two features that have non-trivial implications for the economy’s dynamic response to migration shocks. First, we allow for human capital accumulation, such that migration can affect not just the physical capital stock per head, but also the stock of human capital per capita. Importantly, the two forms of capital need not be affected by a migration shock in the same way. Second, we introduce a residential housing sector into the model. This addition allows us to analyse the effect of migration on residential real estate prices, and also sectoral labour flows. In other words, does migration cause labour to flow from the production of goods into the production of houses? We briefly discuss these two modelling choices in relation to the macroeconomic environment in New Zealand.

New Zealand’s Immigration Act 2009 provides the current framework for
migration into New Zealand. This legislation is augmented with regulations that specify application requirements for different visa categories. Visas are available for entrepreneurs, investors, skilled migrants, refugees, Pacific Islanders, and others. Of most note, in the context of our analysis, is the use of points-based criteria to rank applicants for many visa categories. Comparatively little use is made of visa ballots,\textsuperscript{7} such as those used to allocate ‘Diversity Immigrant Visas’ (green cards) in the United States. In the ten and a half years from fiscal year 2007/8, roughly 463 thousand migrants have had visa applications approved by New Zealand immigration authorities.\textsuperscript{8} Some 263 thousand successful applicants (circa 57 percent of successful applicants) entered New Zealand as ‘Skilled Migrants’ or via investor, entrepreneur or other skill-related categories. A further 163 thousand migrants (35 percent) were approved for family-related reasons, around 15 thousand visas (3.2 percent) were granted for refugees, 16 thousand visas were approved for people from the Pacific (3.5 percent); and a little over 5 thousand people were provided visas for various other reasons (primarily by ministerial direction). While the measurement of human capital is clearly fraught, the importance of skilled, investor, and entrepreneurial migrants provides some support for the view that the ‘average’ migrant might have more human capital than domestic residents.

As mentioned above, we also explicitly model the housing sector. We incorporate housing into our analysis because housing is an important component of the capital stock, and demand for houses is directly and immediately affected by an increase in population – from migrants and residents alike. Residential investment is also one of the most volatile components of gross domestic product, contributing to business cycle fluctuations. Furthermore, construction is an important sector of the New Zealand labour market. According to the Quarterly Employment Survey, the proportion of full-time equivalents employed in construction has increased from below 5 percent in the early 1990s to around 9 percent in the most recent data in 2017. The links between house values and consumption (and therefore aggregate demand) also receives continued emphasis in the monetary policy statements of the Reserve Bank of New Zealand, in part reflecting the fact that a substantial proportion of New Zealanders’ domestic wealth is tied up in home ownership.

\textsuperscript{7} There are exceptions to this generalisation: Gibson et al. (2018) discuss the effects of migration on Tongan migrants who enter into New Zealand via a lottery.

2.1 Households

In our model, households maximise expected utility defined over consumption, housing services, labour effort, and skill accumulation. The period utility function is

\[ U_t = \left( j^c_t \ln c_t + j_t \ln h_t - \frac{\phi_0}{1 + \eta} (n_t + s_t)^{1+\eta} \right) \] (1)

where \( c_t \) is consumption per capita, \( h_t \) are housing services per capita, \( j^c_t \) and \( j_t \) are shocks that affects the utility agents derive from consumption and housing services, respectively. \( n_t \) denotes working hours, and \( s_t \) is on-the-job training hours per capita. The final consumption good, \( c_t \), consists of a domestically produced good, \( c^d_t \), and an imported good, \( c^f_t \). More precisely, the final good is defined as a constant elasticity of substitution (CES) aggregate:

\[ c_t = \left[ c^d_t \left( \frac{c^d_t}{c^f_t} \right)^{\theta - 1} + (1 - \theta) \left( \frac{c^f_t}{c^d_t} \right)^{\theta - 1} \right]^{\frac{1}{\theta}} \] (2)

Here \( \theta \) denotes the elasticity of substitution between the two types of goods and \( \theta \) is the share of the domestically produced good in final consumption. The price index of the final good, \( P_t \), is chosen to be the numeraire. Consequently, all other prices are expressed relative to the home final good. For example, the relative price of domestically produced goods, \( p^d_t \), denotes the ratio \( \frac{P^d_t}{P_t} \). Households maximise expected utility subject to the flow budget constraint:

\[ c_t + p^f_t b_t + q^H_t h_t + p^l_t l_t = (1 + r_{t-1}) p^f_t \frac{N_{t-1}}{N_t} b_{t-1} + q^H_t (1 - \delta_h) \frac{N_{t-1}}{N_t} h_{t-1} \]
\[- \quad + w_t n_t \frac{N_{t-1}}{N_t} d_{t-1} + (p^l_t + R^l_t) \frac{N_{t-1}}{N_t} l_{t-1} + \pi_t \] (3)

The size of the working-age population at time \( t \) is denoted by \( N_t \). All variables in the model are expressed on a per capita basis, which implies that all carried-over stocks, such as housing, bonds, human capital and land in equation (3) are deflated by the term \( \frac{N_{t-1}}{N_t} \), which is the inverse of the gross growth rate of working-age population. Households consume goods, \( c_t \), buy bonds that pay out in units of foreign-produced goods, \( b_t \), buy housing services, \( h_t \) at price \( q^H_t \), and buy land, \( l_t \) at price \( p^l_t \). Households finance these expenditures through wage income, \( w_t n_t d_t \) (reflecting both hours \( n_t \) and human capital \( d_t \)); the return they receive from the bonds purchased in the previous period, \( (1 + r_{t-1}) b_{t-1} \); from the rental returns to their land holdings, \( R^l_t l_{t-1} \); from selling the un-depreciated housing services purchased
last period, \((1 - \delta_h)h_{t-1}\); and through dividend income, \(
abla_t\) that accrues to households as owners of the production sector. The stock of human capital, denoted \(d_t\), evolves according to the following law of motion:

\[
d_t = \left(\frac{N_{t-1}}{N_t} d_{t-1} s_t\right)^{\phi_s} N_{t}^{2\phi_s - 1} + (1 - \delta^d) \frac{N_{t-1}}{N_t} d_{t-1}
\]

(4)

where \(\left(d_{t-1} s_t\right)^{\phi_s}\) denotes the production technology that turns effective time investment into human capital and \(\delta^d\) denotes the depreciation rate of human capital. In modelling the accumulation of human capital we largely follow Kim and Lee (2007). Setting the parameter \(\phi_s < 1\) ensures that growth is exogenous. In our case with exogenous population growth we have to set \(\phi_s = 1/2\) to rule out a scale effect from population growth.

2.2 Household’s first order conditions

Equations (5) - (11) are the optimality conditions for consumption, hours worked, hours spent training, the accumulation of human capital, bonds, housing, and land. The marginal utility of consumption at time \(t\) in these equations is denoted \(\mu_t\) and the multiplier on the accumulation constraint for human capital is denoted \(\lambda_t\).

\[
j^c_t/c_t - \mu_t = 0 \quad (5)
\]

\[
-\phi_0(n_t + s_t)^n + \mu_t w_t \frac{N_{t-1}}{N_t} d_{t-1} = 0 \quad (6)
\]

\[
-\phi_0(n_t + s_t)^n + \lambda_t \phi_s \frac{N_{t-1} d_{t-1} s_t}{s_t} = 0 \quad (7)
\]

\[
-\lambda_t + \mu_t w_t n_t + \beta E_t \lambda_{t+1} \left[ \phi_s \frac{N_{t+1}}{N_t} d_{t+1} \left(1 - \delta^d\right) \frac{N_{t}}{N_{t+1}} \right] = 0 \quad (8)
\]

\[
-\mu_t + \beta E_t \mu_{t+1} \frac{p^f_{t+1}}{p^l_t} \frac{N_{t}}{N_{t+1}} (1 + r_t) = 0 \quad (9)
\]

\[
-q^H_t + \frac{1}{(h_t \mu_t)} + \beta E_t \frac{N_{t}}{N_{t+1}} \frac{\mu_{t+1}}{\mu_t} \left(1 - \delta_h\right) q^H_{t+1} = 0 \quad (10)
\]

\[
-p^l_t + \beta E_t \frac{N_{t}}{N_{t+1}} \frac{\mu_{t+1}}{\mu_t} (p^l_{t+1} + R^l_{t+1}) = 0 \quad (11)
\]

2.3 Firms

Households supply firms with effective labour, defined as \(n_t d_{t-1} \frac{N_{t-1}}{N_t} = e n_t\), which is remunerated with the real wage \(w_t\). Note that the opportunity cost
of investing in human capital is borne exclusively by the household and not the firm. Households divide total effective labour, $en_t$, between the goods producing sector, supplying $en_t^g$ units of labour, and the construction sector, supplying $en_t^H$ units of labour.

$$en_t = en_t^g + en_t^H$$

(12)

**Goods sector**

Goods-producing firms produce a tradable good $y_t$ whose price in terms of the numeraire good is $p_t^h$. Firms maximise cash-flow defined as the difference between the value of output and expenditure on wages and investment, $x_t$:

$$\pi_t^y = p_t^h y_t - w_t en_t^g - x_t$$

(13)

subject to a production technology that combines effective labour and utilised capital:

$$y_t = a_t \left( u_t \frac{N_{t-1}}{N_t} k_{t-1} \right)^{\alpha} (en_t^g)^{1-\alpha}.$$  

(14)

The usual law of motion of the capital stock is defined as:

$$k_t = (1 - \delta(u_t))k_{t-1} \frac{N_{t-1}}{N_t} + a_t^i t(x_t/x_{t-1}).$$

(15)

where the depreciation rate $\delta()$ is a function of the utilisation rate, $u_t$. The function $t(x_t/x_{t-1})$ represents investment adjustment costs, as per Christiano et al. (2005), and $a_t^i$ denotes a shock to the marginal efficiency of investment (MEI). The standard optimality condition for capital and investment are:

$$q_t = E_t \beta \frac{N_t}{N_{t+1}} \frac{\mu_{t+1}}{\mu_t} \left( p_t^h \frac{\partial y_{t+1}}{\partial k_t} + q_{t+1}(1 - \delta(u_{t+1})) \right)$$

(16)

$$1/a_t^i = q_t \frac{\partial t(x_t, x_{t-1})}{\partial x_t} + \beta E_t \left( \frac{\mu_{t+1}}{\mu_t} q_{t+1} \frac{\partial t(x_{t+1}, x_t)}{\partial x_t} \right)$$

(17)

$$\alpha p_t^h \frac{\partial y_t}{\partial u_t} = q_t \delta'(u_t) k_t$$

(18)

**Construction sector**

Our housing and construction sector is based on Iacoviello (2005). Housing stock is built using effective labour, land and home-produced intermediate goods, $m_t$. Profits in the construction sector at time $t$ are defined as $\pi_t^H$, with

$$\pi_t^H = q_t^H H_t - w_t en_t^H - R_t^H l_{t-1} - p_t^h m_t$$

(19)
where \( q_t^H \) denotes the price of newly built housing stock. Labour mobility across sectors ensures that builders face the same wage costs as do goods producing firms, \( w_t \). The rental rate of land faced by the construction sector is denoted by \( R_t^l \). Profits are maximised subject to the following production technology for new housing:

\[
H_t = a_t^H \left( \frac{N_{t-1}}{N_t} \right)^{\xi_l} \left[ e^{n_t^H (1-\xi_l)} w_t^m m_t^m \right]^{\xi_m} \tag{20}
\]

The production of houses is, like the production of goods, subject to an AR(1) technology shock, \( a_t^H \). The construction firm maximises profits by choosing effective labour, land and intermediate inputs optimally:

\[
(1 - \xi - \xi_m)q_t^H H_t \frac{H_t}{e^{n_t^H}} = w_t \tag{21}
\]

\[
\xi q_t^H \frac{H_t}{l_{t-1}} = R_t^l \tag{22}
\]

\[
\xi_m q_t^H \frac{H_t}{m_t} = p_t^h \tag{23}
\]

Every period, households sell their un-depreciated housing stock and purchase new homes with the proceeds. Market clearing implies that the supply of new houses equals the net increase in the housing stock.

\[
H_t = h_t - (1 - \delta_h) \frac{N_{t-1}}{N_t} h_{t-1} \tag{24}
\]

The total amount of land in the economy is fixed, but as the population grows the supply of land per household diminishes. A temporary increase in migration, or indeed just natural population growth, would imply an ever decreasing amount of land per household. From a modelling perspective, the steady state around which we are linearising the model would therefore not be deterministic. We get around this problem by assuming that land is re-zoned for building purposes as the population grows. As the supply of building land grows along with the population, the supply of land per household remains constant:

\[
l_t = 1. \tag{25}
\]

2.4 Current account

Having described the household and production sectors above, this section presents the final equations needed to close the model. Market clearing in
the goods market is described by equation (26):

\[ y_t - m_t = v \left( p^h_t \right)^{-\theta} (c_t + x_t) + e x^h_t. \]  \quad (26)

The home produced good is used in the production of the domestically consumed final good and the domestically used investment good, and is also exported and used in construction. Export demand from abroad is assumed to be of the form:

\[ e x^h_t = v^* \left( \frac{r e r_t}{p_t^h} \right)^{\theta^*} y^*_t \]  \quad (27)

with \( y^*_t \) denoting total foreign demand for the domestic good. Substituting the market clearing conditions from the goods and labour markets into the household budget constraints yields the economy-wide current account equation:

\[ y_t = c_t + x_t + m_t + p^f_t b_t - p^f_t (1 + r_{t-1}) \frac{N_{t-1}}{N_t} b_{t-1} \]  \quad (28)

Finally, we close the model by introducing a debt elastic interest rate premium that allows for small deviations of the domestic real interest rate from the world rate when the domestic net foreign asset position deviates from its steady state level. This eliminates the unit-root in bond holdings:

\[ 1 + r_t = (1 + r_t^*) e^{-\phi_b (b_t - \bar{b})} \]  \quad (29)

### 2.5 Driving processes

The model economy is driven by seven shocks all of which take the shape of an AR(1) process:

\[ a_t = \rho_a a_{t-1} + \epsilon_{at} \]  \quad (30)

\[ a^H_t = \rho_h a^H_{t-1} + \epsilon_{ht} \]  \quad (31)

\[ j_t = \rho_j j_{t-1} + \epsilon_{jt} \]  \quad (32)

\[ j^c_t = \rho_{jc} j^c_{t-1} + \epsilon_{jct} \]  \quad (33)

\[ a^i_t = \rho_i a^i_{t-1} + \epsilon_{it} \]  \quad (34)

\[ y^f_t = \rho_y y^f_{t-1} + \epsilon_{yft} \]  \quad (35)

\[ v_t = \rho_v v_{t-1} + \epsilon_{vt} \]  \quad (36)

Equations (30) and (31) are total factor productivity processes in goods production and construction, respectively. Equations (32) and (33) represent preference shocks for housing and consumption, while (34) denotes the MEI.
shock process. World output and net migration follow the processes denoted in (35) and (36). Specifically, the migration process is defined as $v_t \equiv \ln \left( \frac{N_t}{N_{t-1}} \right)$.

Modelling migration as an exogenous process is a simplifying assumption, with some empirical support in our reduced form analysis, depending on how the model is specified. The literature also provides a degree of support for this assumption. Mitchell et al. (2011) find that simple autoregressive models can provide more accurate forecasts of migration in the United Kingdom than models that include economic or policy factors – in part because policy factors are hard to forecast.\footnote{Conversely, theory emphasizes that migration should be endogenous to domestic and foreign conditions, see for example Borjas (1999a). Alternative empirical methodologies do uncover endogenous effects at some frequencies: Mayda (2010), for example, conducts a panel data analysis based on annual data from 14 developed countries and finds that ‘pull’ factors in destination economies, such as relative income levels, do affect migration flows, though ‘push’ factors have only small effects.}

3 Migration versus population growth

What is the key difference between migration and population growth? In the model, the main effect of both migration and population growth is to dilute existing stocks of capital, housing, human capital, and net foreign assets on a per capita basis. What differentiates migration from population growth is that for human capital, at the very least, migration need not reduce the per capita level of the relevant stock. Indeed, migration may even raise the average human capital in the economy.

To illustrate the effect of migrants arriving with human capital, consider the log-linearised evolution of $d_t$ over time when migrants arrive with no human capital:

$$
\hat{d}_t = \phi_s \delta^d \left[ \hat{d}_{t-1} - v_t + \hat{s}_t \right] + (1 - \delta^d) \left[ \hat{d}_{t-1} - v_t \right] \tag{37}
$$

Unskilled migration reduces the per capita stock of human capital in the economy. Skilled migration in the model would imply that the evolution of human capital per head is not affected by migration. We thus amend the equation above as:

$$
\hat{d}_t = \phi_s \delta^d \left[ \hat{d}_{t-1} - (1 - \chi)v_t + \hat{s}_t \right] + (1 - \delta^d) \left[ \hat{d}_{t-1} - (1 - \chi)v_t \right] \tag{38}
$$

where $\chi$ is strictly positive and takes the value of 1 when migrants possess the same level of human capital as natives, or greater than 1 when migrants have a higher average level of human capital.
Our way of modelling migration, as an exogenous increase in the working-age population that dilutes aggregate stocks is appropriate for the specific question of the paper, i.e. to study the impact of migration on aggregate domestic variables that apply to both locals and migrants. In contrast, different modelling assumptions would have been required for migration if the question was about the impact of immigrants on the labour market outcomes of local workers, or on different types of local workers differentiated by skill level.

4 Estimation strategy

The principal aim of our model is to shed light on the short-run macroeconomic effects of migration shocks and assess their contribution to the dynamics of the business cycle. To this end, we implement a Bayesian estimation procedure. We estimate the model using macroeconomic data for New Zealand, an economy that has experienced both large and volatile migration flows in recent decades. In addition, New Zealand is one of the very few countries for which reliable data on working-age net migration is available.

We focus our analysis on ‘permanent and long-term’ (PLT) arrivals and departures. PLT arrivals are people arriving for a stay of 12 months or more, including New Zealanders returning after an absence overseas of 12 months or more. Conversely, PLT departures are New Zealanders departing for 12 months or more and migrants leaving after a stay of 12 months or more in New Zealand. Net migration figures in New Zealand are often decomposed into net migration between New Zealand and Australia and net migration relative to the rest of the world. Australian and New Zealand citizens largely have freedom of movement between the two countries, including the right to work. In Figure 1, net migration between New Zealand and Australia is summarised by the blue bars and that between New Zealand and the rest of the world by the red bars. Over the sample, there was negative net migration between New Zealand and Australia, offset by positive migration between the rest of the world and New Zealand. Since about 2014, net migration to Australia has dried up, while net migration into New Zealand from the rest of the world has increased. As a result, annual net migration has risen to about 1.5 percent of the total resident population, while working-age migration has increased by a slightly larger percentage. In the four years 2014-2017, working age population increased by over 5.5 percent from net migration alone.

10 These numbers are based on the destination country and country of origin, rather than country of citizenship.
4.1 Data

We use real per capita GDP, real per capital private consumption, real gross fixed capital formation (investment), real house prices, working age net migration and a trade-weighted aggregate of world gross domestic product, all as defined in table 2 as our observables to estimate the model. The national account and migration data are sourced from Statistics New Zealand, while the house price data are from Quotable Value New Zealand. The trade-weighted world GDP data are compiled by the Reserve Bank of New Zealand. The data sample runs from 1992Q1-2017Q2.

GDP, residential investment, gross fixed capital formation (investment), and private consumption which are sourced from the national accounts. We also use a trade-weighted measure of world GDP and working age net migration. All series are seasonally adjusted. The national accounts and migration data are transformed into per capita terms by dividing by seasonally-adjusted working age (15-65 year old) population. We take the natural logarithm of trending series and then apply the local linear projections of Hamilton (2017) to compute the trends, and hence cycles, of the data series. This de-trending method is particularly straightforward to implement and consists of regressing the representative data series $x_{t+h}$ against a constant and the data $x_t$, $x_{t-1}$, $x_{t-2}$, and $x_{t-3}$, where $h = 8$ quarters. This filter is one-sided and thus avoids the so-called ‘end-point’ problem commonly associated with the Hodrick-Prescott filter. The filter has the added advantage that, given that four lags are used, it simultaneously strips out seasonality. Cycles derived from seasonally adjusted data and unadjusted data are virtually indistinguishable. Furthermore, the detrended series also has a mean of zero provided that a constant is included in the local linear projection. (See Hamilton 2017 for a thorough discussion of the virtues of this detrending method.) As the filter is not yet widely used, and as our migration data are not well-known, we illustrate the detrended data in figure 2. As can be see in the figure, the cycles obtained from a seasonally adjusted series and from an unadjusted migration series are virtually identical.

Over the course of our sample working age population has increased rapidly, from around 2.66 million people in 1992 to 3.84 million in 2017Q2, making it difficult to translate percentage changes into the number of migrants entering the country. The largest quarterly migrant impulse in the raw data in percentage terms corresponded to an increase in working age population of 0.4 percent in a single quarter, in 2015Q4. In this quarter (in unofficially seasonally adjusted terms) a little more than 15,100 working age PLT migrants entered the country in raw terms.

---

11 The migration series is not logged as it takes both positive and negative values.
The standard deviation of the \textit{detrended} migration series is 0.00125 in quarter-on-quarter percent terms. Thus, in an ordinary year a one standard deviation increase in population from migration corresponds to roughly a 1/2 percent of working age population. The largest detrended seasonally adjusted migration inflow in a quarter, 0.0031 percent, occurred in 2003Q1, and was nearly \(2\frac{1}{2}\) times as large as the standard deviation of the detrended series. Approximately 10,500 working age migrants entered New Zealand in that particular quarter in the raw data.

The rest-of-world gross domestic product series is a trade-weighted average of the GDPs from 17 countries. We have backcast the series 2 quarters using an earlier vintage of this trade-weighted GDP, based on slightly fewer countries. Working age net migration is computed from Statistics New Zealand’s permanent and long-term arrivals and departures data. The working-age data are assembled from age cohort data. We seasonally adjust the per capita working-age net migration data using a default implementation of X12.\footnote{The executable file for X12 is available from the United States Census Bureau. We use an X12 implementation embedded in IRIS, \url{https://github.com/IRIS-Solutions-Team}.} Table 1 defines the raw data, while table 2 describes the transformations applied to the raw data.

Table 3 reports the standard deviations, the standard deviation of variable \(i\) relative to that of GDP and first-order autocorrelation to the observables. New Zealand GDP per capita is considerably more volatile than our measure of World GDP. Residential investment is 5.67 times as volatile as GDP and more volatile than gross fixed capital formation (investment). Real house prices are 3.8 times as volatile as GDP. Unlike most other developed economies, New Zealand consumption is somewhat more volatile than GDP. Net migration per capita in New Zealand is volatile by OECD standards, but is still only about 5\% as volatile as real GDP.

\subsection*{4.2 Calibration and priors}

Columns 3 - 5 of Table 4 report the priors, means and the standard deviations of the parameters to be estimated. Most of our priors are fairly standard, see for instance Kamber et al. (2015). We do however, differ from the literature along several dimensions. Specifically, we attach a very tight prior to the share of capital, \(\alpha\), with a prior mean of 0.33, as is standard in the real business cycle literature. Likewise, the AR(1) coefficients for world GDP and net migration, \(\rho_y\) and \(\rho_v\) respectively, have a prior that corresponds to estimates of these coefficients from single equation methods, as do the standard errors of the these two shocks, \(\epsilon_y\) and \(\epsilon_v\). In each case, we estimate the parameter,
but choose a relatively small standard error for our prior. These tight priors are implemented to prevent biases in the domestic equations from contaminating our estimates of these foreign impulses via the systems estimation of the model.

Preliminary efforts to estimate the ratio of human capital for migrants relative to domestic residents, $\chi$ proved problematic, so we calibrate this parameter and later report a sensitivity analysis to illustrate how the dynamics of the model are affected by this parameter. The bottom half of Table 4 reports the calibrated parameters. Most of these are standard and only two parameters merit a special mention: the ratio of residential investment to consumption, which we set at 0.12 to match New Zealand data, and the above mentioned parameter $\chi$, which we set at 1.85. The latter value is the relative level of human capital of migrants into New Zealand as reported in Boubtane et al. (2016). We carry out sensitivity analysis around our choice of its parameter value in section 6.

4.3 Estimation results

Columns 6 - 8 of Table 4 report the posterior mean and lower and upper limits of 90% Bayesian confidence intervals from the posterior distribution. The share of capital in the production of goods has a posterior mean of 0.33 and the share of land in the housing sector one of 0.61. Capital depreciates 2.7 percent per quarter. The inverse of the labour supply elasticity, $\eta$, has a posterior mean of 3.7. The trade elasticity or the intra-temporal elasticity of substitution between home and foreign produced goods is estimated at 2.55, which implies that home and foreign-produced goods are highly substitutable for one another. The openness parameter, $\gamma$, also has a tight prior and corresponds to the ratio of exports plus imports to GDP. For New Zealand, this value is around 0.33. Parameters $acu$ and $ac$ are the capacity utilisation elasticity and the investment adjustment cost parameter, respectively. $\phi^b$ measures the bond holding costs. The data suggests a low mean value 0.2 of one percent.

Total factor productivity (TFP) in goods production and housing is persistent, with estimated AR(1) coefficients of 0.76 and 0.72, respectively. The corresponding standard deviations of the innovations are 0.03 and 0.04, respectively. The shocks to preferences for housing and consumption have AR(1) coefficients of 0.86 and 0.83, respectively. Whereas these two shocks have a similar persistence, the housing preference shock is more volatile than the consumption shock. The investment specific technology shock has a low autocorrelation coefficient and large standard deviation. The magnitude of this shock process is similar to estimates from Kamber et al. (2015). Working
age migration per capita, estimated with a tight prior, is persistent with an associated AR(1) coefficient of 0.89 and a standard deviation of the migration impulse of 0.001.

5 A migration shock

An increase in migration is a rise in a country’s population and its labour supply. As a result, a positive migration shock, initially at least, reduces the per capita value of stocks such as capital, housing and bond holdings. As our calibration assumes that migrants have a higher stock of human capital than locals, the per capita stock of human capital rises in response to a migration shock. Much of the transitional dynamics of the model economy are therefore driven by the reversion of these stocks to their steady-state values following a shock to migration.

Another key driver of the model’s dynamics following a migration shock is the response of the real exchange rate or (near-synonymously) the terms of trade. Figures 3-4, which shows the impulse responses of the model using the mean of the estimated parameters, suggest that the terms of trade, defined as the price of foreign to home-produced goods, appreciate following an unexpected increase in migration. The reason the terms of trade appreciate is that a migration shock raises absorption of home-produced goods. The estimation results suggest that agents have a significant degree of home-bias in both consumption and investment expenditure (the smaller is the openness parameter γ, the greater is home bias), which raises demand for domestically
Table 2: Data transformations

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log per capita income</td>
<td>$y = HAM(\log(GDP/pop^{WA}))$</td>
</tr>
<tr>
<td>Log per capita residential investment</td>
<td>$H = HAM(\log(I_{res}/pop^{WA}))$</td>
</tr>
<tr>
<td>Log per capita gross fixed capital formation</td>
<td>$x = HAM(\log(X/pop^{WA}))$</td>
</tr>
<tr>
<td>Log per capita private consumption</td>
<td>$c = HAM(\log(C/pop^{WA}))$</td>
</tr>
<tr>
<td>Log real house prices</td>
<td>$q^h = HAM(\log(P^h \cdot 1000/CPI))$</td>
</tr>
<tr>
<td>Detrended migration per capita</td>
<td>$v = HAM(M/pop^{WA})$</td>
</tr>
<tr>
<td>Log trade-weighted foreign GDP</td>
<td>$y^* = HAM(\log(GDP^*))$</td>
</tr>
</tbody>
</table>

$HAM()$ represents the Hamilton filter used to detrend all series. $\log()$ is the natural logarithm. Migration is not logged because it can assume negative values. A Matlab file implementing these transformations is available upon request.

Table 3: Observables and model moments

<table>
<thead>
<tr>
<th></th>
<th>Std Dev ($\sigma$)</th>
<th>$\sigma_i/\sigma_y$</th>
<th>Corr($Y_t, Y_{t-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP per capita</td>
<td>0.0264</td>
<td>1</td>
<td>0.8453</td>
</tr>
<tr>
<td>Residential Investment per capita</td>
<td>0.1496</td>
<td>5.67</td>
<td>0.8694</td>
</tr>
<tr>
<td>Investment per capita</td>
<td>0.1134</td>
<td>4.30</td>
<td>0.8297</td>
</tr>
<tr>
<td>Consumption per capita</td>
<td>0.0275</td>
<td>1.04</td>
<td>0.8408</td>
</tr>
<tr>
<td>Real House Prices</td>
<td>0.1006</td>
<td>3.81</td>
<td>0.8715</td>
</tr>
<tr>
<td>World GDP</td>
<td>0.0164</td>
<td>0.62</td>
<td>0.8864</td>
</tr>
<tr>
<td>Migration per capita</td>
<td>0.0013</td>
<td>0.05</td>
<td>0.8904</td>
</tr>
</tbody>
</table>

Note: All data, except for net migration per capita, are in logs and all are de-trended using the Hamilton filter.
## Table 4: Estimated parameters values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Share of capital</td>
<td>N 0.330 0.010</td>
<td>0.330 0.314 0.346</td>
</tr>
<tr>
<td>σh</td>
<td>Share of land in housing</td>
<td>N 0.700 0.050</td>
<td>0.614 0.561 0.667</td>
</tr>
<tr>
<td>δ</td>
<td>Depreciation rate capital</td>
<td>N 2.500 0.500</td>
<td>2.748 1.944 3.538</td>
</tr>
<tr>
<td>η</td>
<td>Frisch elasticity</td>
<td>Γ 2.000 0.750</td>
<td>3.733 2.211 5.251</td>
</tr>
<tr>
<td>θ</td>
<td>Intratemp. subst. elasticity</td>
<td>N 1.000 0.250</td>
<td>2.550 2.498 2.590</td>
</tr>
<tr>
<td>γ</td>
<td>Openness</td>
<td>β 0.300 0.010</td>
<td>0.337 0.321 0.353</td>
</tr>
<tr>
<td>a cu</td>
<td>Capacity-U curvature</td>
<td>β 0.500 0.150</td>
<td>0.669 0.479 0.865</td>
</tr>
<tr>
<td>ac</td>
<td>Investment adjustment costs</td>
<td>N 4.000 1.500</td>
<td>6.313 4.433 8.131</td>
</tr>
<tr>
<td>100·φb</td>
<td>Bond adjustment costs</td>
<td>Γ⁻¹ 1.000 5.000</td>
<td>0.205 0.152 0.256</td>
</tr>
<tr>
<td>ρa</td>
<td>Persistence tech.</td>
<td>β 0.500 0.200</td>
<td>0.762 0.710 0.814</td>
</tr>
<tr>
<td>ρah</td>
<td>Persistence housing tech.</td>
<td>β 0.500 0.200</td>
<td>0.718 0.613 0.826</td>
</tr>
<tr>
<td>ρy</td>
<td>Persistence foreign demand.</td>
<td>β 0.886 0.010</td>
<td>0.887 0.871 0.903</td>
</tr>
<tr>
<td>ρj</td>
<td>Persistence housing pref.</td>
<td>β 0.500 0.200</td>
<td>0.860 0.806 0.917</td>
</tr>
<tr>
<td>ρjc</td>
<td>Persistence consumption pref.</td>
<td>β 0.500 0.200</td>
<td>0.830 0.780 0.879</td>
</tr>
<tr>
<td>ρi</td>
<td>Persistence investment-specific</td>
<td>β 0.500 0.150</td>
<td>0.272 0.145 0.397</td>
</tr>
<tr>
<td>ρm</td>
<td>Persistence migration</td>
<td>β 0.890 0.010</td>
<td>0.890 0.874 0.906</td>
</tr>
<tr>
<td>ϵa</td>
<td>Std dev. tech.</td>
<td>Γ⁻¹ 0.004 1.500</td>
<td>0.030 0.026 0.034</td>
</tr>
<tr>
<td>ϵh</td>
<td>Std dev. housing tech.</td>
<td>Γ⁻¹ 0.005 1.500</td>
<td>0.038 0.032 0.043</td>
</tr>
<tr>
<td>ϵyf</td>
<td>Std dev. foreign demand</td>
<td>Γ⁻¹ 0.007 1.500</td>
<td>0.007 0.006 0.008</td>
</tr>
<tr>
<td>ϵj</td>
<td>Std dev. housing pref.</td>
<td>Γ⁻¹ 0.005 0.500</td>
<td>0.535 0.335 0.728</td>
</tr>
<tr>
<td>ϵi</td>
<td>Std dev. investment-specific</td>
<td>Γ⁻¹ 0.005 1.500</td>
<td>0.366 0.244 0.483</td>
</tr>
<tr>
<td>ϵjc</td>
<td>Std dev. consumption pref.</td>
<td>Γ⁻¹ 0.004 1.500</td>
<td>0.034 0.030 0.039</td>
</tr>
<tr>
<td>ϵv</td>
<td>Std dev. migration</td>
<td>Γ⁻¹ 0.001 1.500</td>
<td>0.001 0.001 0.001</td>
</tr>
</tbody>
</table>

Calibrated

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>χ</td>
<td>Relative human cap of migrants</td>
</tr>
<tr>
<td>100·δh</td>
<td>Depreciation rate housing</td>
</tr>
<tr>
<td>β</td>
<td>Discount rate</td>
</tr>
<tr>
<td>δh</td>
<td>Depreciation rate human cap.</td>
</tr>
<tr>
<td>φs</td>
<td>Skill accumulation</td>
</tr>
<tr>
<td>j</td>
<td>Steady-state j</td>
</tr>
<tr>
<td>n + s</td>
<td>Hours worked + training</td>
</tr>
<tr>
<td>Ξm</td>
<td>Share of traded goods in</td>
</tr>
<tr>
<td>H/c</td>
<td>H - C ratio</td>
</tr>
</tbody>
</table>
produced goods by more than the demand for imports, and hence leads to an appreciation of the terms of trade. An appreciation of the terms of trade raises the return to domestic factors of production and increases the purchasing power of domestic consumers. The real appreciation, caused by the positive migration shock, thus has a positive wealth effect on consumption.

An increase in migration lowers the per capita physical capital stock. This reduction in capital per capita, along with the appreciation in the terms of trade, has the effect of raising the marginal product of capital. Thus owners of installed capital unambiguously benefit from an increase in migration. The increased return on capital stock raises investment. At the same time, an increase in migration raises the utilisation rate of capital. As Brunow et al. (2015, p. 1030) note, a constant returns to scale technology implies that per capita income declines when labour supply increases are not accompanied by corresponding increases in capital. In our model, however, changes in capacity utilisation partially offset the movements in capital per capita that arise with migration inflows and outflows.

Boubtane et al. (2016) estimate that the relative stock of human capital for migrants into New Zealand is 1.85 times that of the average domestic resident for the 1986-2006 period. Only the United States, with an estimated

Table 5: Variance decomposition at the posterior mean

<table>
<thead>
<tr>
<th>Observables</th>
<th>$\epsilon_a$</th>
<th>$\epsilon_h$</th>
<th>$\epsilon_{yf}$</th>
<th>$\epsilon_j$</th>
<th>$\epsilon_i$</th>
<th>$\epsilon_{pc}$</th>
<th>$\epsilon_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.36</td>
<td>0.04</td>
<td>0.00</td>
<td>0.35</td>
<td>0.04</td>
<td>0.02</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>[0.22, 0.49]</td>
<td>[0.02, 0.06]</td>
<td>[0.00, 0.00]</td>
<td>[0.12, 0.56]</td>
<td>[0.01, 0.07]</td>
<td>[0.01, 0.02]</td>
<td>[0.11, 0.27]</td>
</tr>
<tr>
<td>Investment</td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.70</td>
<td>0.00</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>[0.05, 0.18]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.01]</td>
<td>[0.55, 0.85]</td>
<td>[0.00, 0.00]</td>
<td>[0.07, 0.27]</td>
</tr>
<tr>
<td>Residential</td>
<td>0.00</td>
<td>0.46</td>
<td>0.00</td>
<td>0.50</td>
<td>0.00</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>investment</td>
<td>[0.00, 0.00]</td>
<td>[0.23, 0.72]</td>
<td>[0.00, 0.00]</td>
<td>[0.25, 0.76]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.01]</td>
<td>[0.01, 0.04]</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.07</td>
<td>0.56</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>[0.18, 0.29]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.00]</td>
<td>[0.25, 0.76]</td>
<td>[0.00, 0.04]</td>
<td>[0.04, 0.10]</td>
<td>[0.48, 0.62]</td>
</tr>
<tr>
<td>Real house prices</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.88</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>[0.01, 0.08]</td>
<td>[0.00, 0.01]</td>
<td>[0.00, 0.00]</td>
<td>[0.79, 0.98]</td>
<td>[0.00, 0.02]</td>
<td>[0.00, 0.03]</td>
<td>[0.01, 0.07]</td>
</tr>
</tbody>
</table>
ratio of 0.97, has a ratio below 1.\textsuperscript{13} The remaining countries examined by Boubtane et al. have ratios from 1.01–2.87 (Greece and Ireland respectively). Poot and Stillman (2016) find that migrants into New Zealand have 12.81 years of education on average, whereas New Zealand born individuals average a little less at 12.44 years of education. In 2006, 34 percent of migrants had bachelor degrees, while only 18 percent of New Zealand-born residents had such degrees.\textsuperscript{14} The mapping between different measures of schooling and human capital is difficult to resolve, so later we explore the sensitivity of our results to this key ratio.

As the empirical evidence suggests that migrants have a higher stock of human capital than New Zealand locals, we observe an increase in the per capita human capital stock following an increase in migration. As the transitional dynamics are characterised by a reversion to the pre-migration mean, the representative household reduces investment in skill acquisition. Less time spent training, means more time spent on hours worked. As a result, effective hours per capita increase following a rise in migration. The combination of a lower capital stock and an increased supply of effective labour, pushes down the wage rate. On impact, this effect is offset by the appreciation of the terms of trade. After a couple of quarters, the wage rate falls, reverting back to the initial steady state in the medium run. The increase in effective hours plus the increase in capacity utilisation allow output per head to rise in response to a positive migration shock.

In the housing market, the per capita stock of housing is reduced by a sudden increase in migration. Given that migrants have the same preferences over housing and consumption as locals, there will be more households trying to purchase the existing housing stock than before. As a result, the demand for new houses as well as the price of housing rises along with the return on land. The increase in demand for new housing stock, or rather the increase in relative price of housing, stimulates construction activity by shifting mobile resources such as labour into the construction sector. Building houses requires land, labour and intermediate goods. Although total effective hours per worker increase because skilled migration raises effective hours per worker, there is some reallocation of labour effort from the goods into the construction sector in response to increase in relative prices. Effective hours in the construction sector increase by more than in the goods producing sector. To ensure that the post-migration steady-state is the same as the pre-migration steady state, we assume that the supply of building land

\textsuperscript{13} Borjas (1994, 1999b) confirms that the skill level of US immigrants is less than locals.
is allowed to grow with the population.\footnote{In the model, the per capita housing stock falls in response to an unexpected increase in migration not because locals generously share their housing stock with new migrants, but because on aggregate the housing stock is sold and repurchased in every period - a consequence of having only one-period assets and liabilities. Given that the supply of houses is fixed on impact, house prices must rise and stocks purchased per household will fall when there is an increase in migration.}

GDP is the sum of goods production and construction denoted by the solid (blue) line in the top left panel of figure 3. In the estimated model, goods production initially grows faster than overall GDP, though construction in the housing market overtakes around the three year mark. In summary, an increase in skilled migrants is expansionary for a small open economy. Even though the wage rate falls, per capita consumption, investment and GDP rises. Migration raises the return to stocks of physical capital and land and can temporarily reduce the return to human capital if migrants bring with them higher stocks of human capital. Our business cycle results contrast with the cross-country panel data analysis of Brunow et al. (2015), who find that decadal averages of per capita GDP are unrelated to decadal movements in net migration.

5.1 Does migration drive the business cycle?

Having analysed the dynamics of a migration shock in the model, we now consider whether migration is an important driver of the business cycle. Table 5 presents the variance decomposition at the posterior mean of our estimated model, with the lower and upper limit of the 90 percent Bayesian confidence intervals in square brackets underneath.

Over our sample, the median contribution of the migration shock is 19 percent of the variance of observed GDP per capita. The rest of the variance is accounted for, in roughly equal parts, by the TFP shock and the preference for housing shock. Recall that GDP consists of output of goods as well as housing. Migration is thus one of the main drivers of the variance of New Zealand GDP. For per capita consumption, migration is the third most important driver accounting for on average 12 percent of the variance, behind the consumption preference shock and the productivity shock. For investment, the migration shock accounts for on average 17 percent, which makes it the second most important driver behind the MEI (marginal efficiency of investment) shock. The role of migration shocks for the volatility of the housing market variables is modest, between 4 percent for real house prices and 3 percent for residential investment. The variance of residential investment is split relatively even between the housing sector productivity
shock and the demand for housing shock, with a further 3% accounted for by migration. The variance of house prices is largely accounted for by the housing demand shock, which contributes 88% to the variability of real house prices. Migration accounts for 4%, which is more than is accounted for by the housing supply shock.

Given the relatively low degree of trade openness and the ability of the terms of trade to insulate the economy against foreign shocks, it is not surprising that the shock to world GDP has virtually no effect on the variances of our observables.

6 Sensitivity analysis: The relative human capital of migrants

One of the key assumptions of our DSGE model is that, on average, migrants have higher human capital levels than locals. As our data is not informative about this parameter, we calibrated this parameter $\chi$ to a baseline value of 1.85, which is the value estimated by Boubtane et al. (2016) for New Zealand for the period from 1986 to 2006. We justified this claim by noting that skilled and entrepreneurial migrants are a large proportion of total migration into New Zealand. Here we re-estimate the model under the assumption that migrants’ human capital stock does not differ from that of locals, and explore the contribution that migration shocks then make to the variability of our observables.

Tables 6 and 7 reports the parameter estimates and the variance decomposition for a model where $\chi$ has been set to 1. Cancelling out the effects of migration on the stock of human capital does not significantly alter the model’s parameter estimates, see Table 6, but does significantly reduces the contribution of the migration shock to the variance of our observables. For per capita GDP, the contribution falls from around 19 percent to 0.2 percent, for residential investment the figure drops from 3 percent to 0 percent. For consumption per capita the migration shock’s contribution of the total variance falls from 12% to just 3% and for house prices from 4% down to 1%. Our results thus imply that migration has less of an effect on the business cycle when migrants are closer to the local population in terms of their human capital. Our business cycle results thus cohere with an observation by Dustmann et al. (2005, p. F324), namely that “labour market effects of immigration depend most importantly on the structure of the receiving economy, as well as the skill mix of immigrants, relative to the resident population.” Figure 5 illustrates how the contribution of migration shocks varies with the
parameter $\chi$. The relationship is U-shaped, with minima for $\chi \in (0.5, 1)$, i.e., where migrants have lower or equivalent levels of human capital relative to locals.

7 A SVAR look at the data

Having investigated the business cycle effects of migration via a DSGE model, we now look at the data using a structural vector autoregression (SVAR). As SVARs embody fewer restrictions, we do not necessarily expect to find the exact same dynamics as in the DSGE model. Instead, our focus is on the qualitative effects of a migration shock on the variables in our data set. In this section, we ask whether or not a migration shock in an SVAR is expansionary for the components of GDP, whether it raises residential investment and house prices, and whether it causes the real exchange rate to appreciate. In other words, are the qualitative dynamics of an SVAR comparable with those generated by the estimated DSGE model?

We develop an SVAR from the same observable variables that were used to estimate the DSGE model, but augmented with the real wage and the real exchange rate, both logged and detrended as previously described. We specify the VAR as follows

$$A_0 y_t = A(L) y_{t-1} + u_t.$$  \hfill (39)

where $A_0$ is a $k + 1 \times k + 1$ matrix; $y_t$ is a $k + 1$ column vector of variables, including a 1 to account for constants, and $A(L) \equiv A_1 L + A_2 L^2 + \ldots + A_p L^p$ denotes a lag polynomial where $L$ is the lag operator, such that $L y_t = y_{t-1}$. The vector $u_t$ represents the mean-zero, serially uncorrelated exogenous shocks with diagonal variance-covariance matrix $\Sigma_u$. The reduced form errors thus have a variance covariance matrix $A_0^{-1} \Sigma_u (A_0^{-1})'$. 

Migration shocks are identified through a recursive identification scheme. We treat world GDP as an exogenous variable, ordered first in the causal ordering, followed by per capita migration.\footnote{The migration process is equivalent to what one would obtain from a VARX model with contemporaneous and lagged world GDP included as exogenous regressors.} Migration shocks are assumed to be uncorrelated with contemporaneous shocks to domestic variables. While we employ a Cholesky decomposition to identify the world GDP and migration shocks, we disregard the exact ordering of the subsequent domestic shocks, since they are not of material importance to our migration analysis. The lack of contemporaneous correlation seems a reasonable identifying assumption given that obtaining a visa or going through the logistics of leaving a job and moving from one country can be a lengthy process.

16
### Table 6: Estimated parameters values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Prior</th>
<th>Posterior</th>
<th>Mean</th>
<th>Std Dev</th>
<th>(5%)</th>
<th>(95%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Share of capital</td>
<td>$N$</td>
<td></td>
<td>0.330</td>
<td>0.010</td>
<td>0.330</td>
<td>0.313</td>
</tr>
<tr>
<td>$\alpha_h$</td>
<td>Share of land in housing</td>
<td>$N$</td>
<td></td>
<td>0.700</td>
<td>0.050</td>
<td>0.612</td>
<td>0.560</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate capital</td>
<td>$N$</td>
<td></td>
<td>2.500</td>
<td>0.500</td>
<td>2.791</td>
<td>2.011</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Frisch elasticity</td>
<td>$\Gamma$</td>
<td></td>
<td>2.000</td>
<td>0.750</td>
<td>3.778</td>
<td>2.223</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Intratemp. subst. elasticity</td>
<td>$N$</td>
<td></td>
<td>1.000</td>
<td>0.250</td>
<td>2.551</td>
<td>2.501</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Openness</td>
<td>$\beta$</td>
<td></td>
<td>0.300</td>
<td>0.010</td>
<td>0.338</td>
<td>0.322</td>
</tr>
<tr>
<td>$a_{cu}$</td>
<td>Capacity-U curvature</td>
<td>$\beta$</td>
<td></td>
<td>0.500</td>
<td>0.150</td>
<td>0.667</td>
<td>0.477</td>
</tr>
<tr>
<td>$ac$</td>
<td>Investment adjustment costs</td>
<td>$N$</td>
<td></td>
<td>4.000</td>
<td>1.500</td>
<td>6.348</td>
<td>4.525</td>
</tr>
<tr>
<td>$100 \cdot \phi^b$</td>
<td>Bond adjustment costs</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>1.000</td>
<td>5.000</td>
<td>0.204</td>
<td>0.152</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence tech.</td>
<td>$\beta$</td>
<td></td>
<td>0.500</td>
<td>0.200</td>
<td>0.756</td>
<td>0.703</td>
</tr>
<tr>
<td>$\rho_{ah}$</td>
<td>Persistence housing tech.</td>
<td>$\beta$</td>
<td></td>
<td>0.500</td>
<td>0.200</td>
<td>0.714</td>
<td>0.609</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Persistence foreign demand</td>
<td>$\beta$</td>
<td></td>
<td>0.886</td>
<td>0.010</td>
<td>0.887</td>
<td>0.871</td>
</tr>
<tr>
<td>$\rho_j$</td>
<td>Persistence housing pref.</td>
<td>$\beta$</td>
<td></td>
<td>0.500</td>
<td>0.200</td>
<td>0.861</td>
<td>0.806</td>
</tr>
<tr>
<td>$\rho_{jc}$</td>
<td>Persistence consumption pref.</td>
<td>$\beta$</td>
<td></td>
<td>0.500</td>
<td>0.200</td>
<td>0.833</td>
<td>0.785</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Persistence investment-specific</td>
<td>$\beta$</td>
<td></td>
<td>0.500</td>
<td>0.150</td>
<td>0.272</td>
<td>0.145</td>
</tr>
<tr>
<td>$\rho_v$</td>
<td>Persistence migration</td>
<td>$\beta$</td>
<td></td>
<td>0.890</td>
<td>0.010</td>
<td>0.890</td>
<td>0.874</td>
</tr>
<tr>
<td>$\epsilon_a$</td>
<td>Std dev. tech.</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>0.004</td>
<td>1.500</td>
<td>0.030</td>
<td>0.026</td>
</tr>
<tr>
<td>$\epsilon_{ah}$</td>
<td>Std dev. housing tech.</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>0.005</td>
<td>1.500</td>
<td>0.037</td>
<td>0.032</td>
</tr>
<tr>
<td>$\epsilon_{gf}$</td>
<td>Std dev. foreign demand</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>0.007</td>
<td>1.500</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td>$\epsilon_j$</td>
<td>Std dev. housing pref.</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>0.005</td>
<td>0.500</td>
<td>0.533</td>
<td>0.337</td>
</tr>
<tr>
<td>$\epsilon_{jc}$</td>
<td>Std dev. investment-specific</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>0.005</td>
<td>1.500</td>
<td>0.367</td>
<td>0.247</td>
</tr>
<tr>
<td>$\epsilon_{ce}$</td>
<td>Std dev. consumption pref.</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>0.004</td>
<td>1.500</td>
<td>0.034</td>
<td>0.030</td>
</tr>
<tr>
<td>$\epsilon_{vc}$</td>
<td>Std dev. migration</td>
<td>$\Gamma^{-1}$</td>
<td></td>
<td>0.001</td>
<td>1.500</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Calibrated

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$</td>
<td>Relative human cap of migrants</td>
<td>1</td>
</tr>
<tr>
<td>100 $\cdot \delta_h$</td>
<td>Depreciation rate housing</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>$1/1.01$</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>Depreciation rate human cap.</td>
<td>0.01</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Skill accumulation</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tilde{j}$</td>
<td>Steady-state $j$</td>
<td>0.7</td>
</tr>
<tr>
<td>$n + s$</td>
<td>Hours worked + training</td>
<td>1/3</td>
</tr>
<tr>
<td>$\xi_m$</td>
<td>Share of traded goods in housing</td>
<td>0.1</td>
</tr>
<tr>
<td>$H/c$</td>
<td>H - C ratio</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Table 7: Variance decomposition at the posterior mean

<table>
<thead>
<tr>
<th>Observables</th>
<th>Shocks</th>
<th>ϵ_a</th>
<th>ϵ_b</th>
<th>ϵ_yf</th>
<th>ϵ_i</th>
<th>ϵ_j</th>
<th>ϵ_v</th>
<th>ϵ_v'</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td></td>
<td>0.46</td>
<td>0.06</td>
<td>0.00</td>
<td>0.35</td>
<td>0.10</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.28, 0.62]</td>
<td>[0.03, 0.09]</td>
<td>[0.00, 0.00]</td>
<td>[0.13, 0.56]</td>
<td>[0.03, 0.16]</td>
<td>[0.02, 0.04]</td>
<td>[0.00, 0.00]</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.88</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.03, 0.14]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.01]</td>
<td>[0.81, 0.95]</td>
<td>[0.00, 0.00]</td>
<td>[0.01, 0.05]</td>
</tr>
<tr>
<td>Residential investment</td>
<td></td>
<td>0.00</td>
<td>0.56</td>
<td>0.00</td>
<td>0.42</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.00, 0.00]</td>
<td>[0.32, 0.81]</td>
<td>[0.00, 0.00]</td>
<td>[0.16, 0.66]</td>
<td>[0.00, 0.01]</td>
<td>[0.00, 0.02]</td>
<td>[0.00, 0.01]</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.11</td>
<td>0.66</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.15, 0.25]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.00]</td>
<td>[0.00, 0.02]</td>
<td>[0.06, 0.15]</td>
<td>[0.59, 0.73]</td>
<td>[0.02, 0.04]</td>
</tr>
<tr>
<td>Real house prices</td>
<td></td>
<td>0.06</td>
<td>0.01</td>
<td>0.00</td>
<td>0.87</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.01, 0.10]</td>
<td>[0.00, 0.01]</td>
<td>[0.00, 0.00]</td>
<td>[0.77, 0.98]</td>
<td>[0.00, 0.05]</td>
<td>[0.00, 0.06]</td>
<td>[0.00, 0.01]</td>
</tr>
</tbody>
</table>

The table reports the theoretical variance decomposition at the posterior mean in percent for the baseline model with migrant human capital in excess of local \( \chi = 1 \). The numbers in brackets are the 5% and 95% confidence intervals. All observables are defined as per data transformations.

We use a multivariate Bayesian information criterion (MBIC) to determine the lag structure of the model. Unsurprisingly, given the comparatively strong penalty on the number of parameters and hence the preference for parsimony, the MBIC implies that the reduced form VAR has a single lag.\(^\text{17}\)

In our DSGE model we assume that the migration impulse is exogenous to the domestic economy. This assumption may seem a little implausible, since the propensity to migrate should reflect the relative costs and benefits in home and foreign countries (see for example Clark et al. 2007, Mayda 2010, and Hatton 1995). To explore whether domestic pull factors are material explanators of our migration series we conduct a simple Granger-causality test. We focus on the migration equation alone, and consider simple autoregressive processes against single equations that have lags of domestic variables as additional regressors.

When the migration equation from a VAR(1) is compared to an AR(1) process for migration, a likelihood ratio test cannot reject the restrictions embedded in the AR(1). The dynamics of the migration equation are predominantly affected by its own lags. This block exogeneity assumption is not material to our qualitative results. The impulse responses are virtually

\(^\text{17}\) The Hannan-Quinn information criterion also implies a VAR model with one lag, while the Akaike information criterion implies that four lags should be included.
identical if one specifies the migration equation as an autoregressive process or as an unrestricted equation from a VAR(1), with feedback from lagged domestic variables. Specifying migration as an exogenous process does, however, have the benefit of making our SVAR model broadly comparable with the reduced form of the DSGE model.\footnote{Given that the exogenous shocks in the DSGE model are AR(1) processes the reduced form of the DSGE model corresponds to a VAR(2).}

In Figure 6 we report the impulse response shock to migration. The independent and identically distributed migration shock has the same standard deviation as in the DSGE model (0.00059), which of course is propagated via the AR(1) process used to model the migration series. In the long-run this shock corresponds to a cumulative impulse of roughly 0.0053 percent of working age population. This is larger than the simple standard deviation of the working age migration series, but reflects the fact that migration impulses exhibit a strong degree of autocorrelation. The migration shock is associated with a statistically significant increase in consumption, investment, residential investment, and house prices. GDP per capita also increases, but is not significantly different from zero. The VAR impulse response also confirms one of the key transmission mechanisms of a migration shock, namely the appreciation of the real exchange rate associated with a migration shock.

\section{Conclusion}

Migration shocks matter for the business cycle. Using a dynamic stochastic general equilibrium model of a small open economy estimated on data for New Zealand, we find that migration shocks account for a considerable proportion of the variability of per capita GDP. For the components of per capita GDP, migration shocks matter, but are not the key drivers. Even for residential investment and real house prices, migration shocks are important, but by no means the key driver of the variation in these variables.

An unexpected positive migration shock is found to be expansionary in terms of per capita real GDP and its components and is associated with an initial appreciation of the terms of trade. As expected, migration benefits the owners of fixed assets such as capital or housing: the returns on these assets rise with an influx of migrants. The return on human capital is also affected by the relative human capital of migrants versus locals. If, as in our case, migrants have an initially higher level of human capital than locals, the real wage, or the return on effective labour falls.

The relative level of human capital of migrants also affects the extent to which migration shocks contribute to the volatility of per capita GDP.
We conduct a sensitivity analysis on the relative level of human capital. We find that the impact of migration shocks for the business cycle is much diminished if new migrants and locals have similar levels of human capital. When we assume that migrants have the same level of human capital as locals, migration shocks make only a minor contribution to the variances of per capita GDP and other macro variables.

Figure 1: Net working-age migration flows into and out of New Zealand

Note: The solid black line denotes non-seasonally adjusted working-age net migration into New Zealand. This figure is split into net migration into New Zealand from Australia (blue bars; predominantly an out-flow) and net migration into New Zealand from all countries other than Australia (red bars).
Figure 2: Net working-age migration flows de-trended via local linear projections

Note: The dash-dot (blue) line is the net migration working age impulse relative to the size of total working age population on a quarterly basis, seasonally adjusted but not de-trended. The solid and dotted (red) lines are the cycles derived from applying the Hamilton local linear projection to the dash-dot series and to an equivalent series that has not been seasonally adjusted. These cyclically adjusted series have means of zero by construction. The latter two lines illustrate that the Hamilton local projection can be used to eliminate both trends and seasonality.
Figure 3: A migration shock (Panel A)

Note: An increase in migration in a small open economy. In the ‘output’ panel, the blue line denotes GDP, while the black line denotes the log-deviation of home-produced traded goods. In the ‘Hours’ panel, the blue line denotes total effective hours supplied by households, the black line denotes effective hours devoted to goods production, while the dashed-dotted line denotes effective hours in construction. In the panel GDP growth, the blue line denotes growth of total GDP, while the black line denotes growth of GDP per capita.
Figure 4: A migration shock (Panel B)

Note: An increase in migration in a small open economy.
Figure 5: The role of migration shocks as a function of $\chi$

Note: This figure reports the variance contributions of the migration shock for different values of the relative human capital parameter $\chi$, migrant capital to local, with all other parameters held constant at the mean posterior values.
Figure 6: A migration shock in a VAR

Notes: An increase in migration in a small open economy. Migration shock in the VAR(1) is identified by a Cholesky decomposition where the filtered world GDP series is ordered first followed by the migration per capita series which is ordered second. All data series are logged and de-trended using the Hamilton (2017) filter, as in Bayesian estimation. The dashed lines are 68% confidence intervals. A decline in the real exchange rate series denotes a real appreciation.
References


1 Steady State

\[
\begin{align*}
\mu &= 1/c \\
0 &= \phi_0(n + s)^y + (1/c)wd \\
0 &= -\phi_0(n + s)^y + \lambda \phi_s \frac{(ds)\phi_s}{s} \\
\lambda &= (1/c)wn + \beta \lambda \left[ \phi_s \frac{(sd)\phi_s}{d} + (1 - \delta^d) \right] \\
1 &= \frac{(ds)\phi_s}{d} + (1 - \delta^d) \\
1 &= \beta(1 + r) \\
\frac{q^H h}{c} &= \frac{j}{(1 - \beta(1 - \delta^d))} \\
p^l &= \beta \left( p^l + R^l \right)
\end{align*}
\]

\[
\begin{align*}
n &= \frac{(1 - \beta(\delta_d\phi_s - 1) + 1)}{\phi_s\delta_d} \\
n + s &= 1/3 \\
n &= \frac{(n + s)^2}{n + s + 1} \\
s &= (n + s) - s \\
y &= \frac{1/\beta - (1 - \delta)}{\alpha} \\
c &= \frac{y}{k} - \delta \\
k &= \frac{(y/k)^{1/(\alpha - 1)}}{n} \\
n^y &= \frac{yk}{k c h \delta_h q^H} \\
n^h &= \frac{1}{k c h \delta_h q^H} \\
n &= 1 + \frac{n^y}{n^h}
\end{align*}
\]
2 MCMC convergence

This appendix illustrates diagnostics to assess whether the Markov chains have converged to the posterior distributions of interest. Two Markov chains were iterated 2,000,000 times to estimate the posterior distribution of the vector of parameter. As noted by Brooks and Roberts (1998) and others, no diagnostic can guarantee convergence. Nevertheless, these figures provide some reassurance that the chains have indeed converged to their stationary distributions. The figures below illustrate the deciles for each chain, computed recursively as the chains are iterated forward (following a 100,000 burn-in period to reduce bias from initial conditions). Convergence implies that like-deciles from the two chain should asymptote to the same values, which should remain constant as the sample is extended. The variability evident in some deciles for some parameters seems fairly modest and is unlikely to be of economic significance.