Model for Analysis and Simulations (MAS)*

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Abstract

This document presents a first version of the Model for Analysis and Simulations (MAS), a new general equilibrium model for the Chilean economy. The document describes the structure and the main components of the model, and analyzes some impulse-response functions. It also presents a preliminary evaluation of the model comparing some moments in the model with the corresponding ones in the data.

1 Introduction

The paper presents a first version of the Model for Analysis and Simulations (MAS), a new dynamic stochastic general equilibrium model for the Chilean economy.

The document presents broadly the basic structure of the model. It carefully describes the equations that characterize the dynamic behavior of each variable, discusses its main properties by examining the impulse-response functions of several variables to some exogenous shocks, and presents a preliminary evaluation by comparing some moments in the model with the corresponding one in the data.

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Dynamic stochastic general equilibrium models (DSGE) with nominal stickiness have become a popular tool for monetary policy analysis in recent years.\footnote{See, for instance, Goodfriend and King (1997), Rotemberg and Woodford (1997), Clarida, Gali and Getler (1999), Altig et al. (2003, 2004), Benigno and Benigno (2003), Christiano, Eichenbaum and Evans (2005), Gali and Monacelli (2005), Schmitt-Grohé and Uribe (2001).} The main advantage of this type of models is that their structural interpretation allows to overcome the Lucas’ (1976) critique. Although this is clearly an advantage over traditional models for policy analysis, the fact that DSGE models have been perceived to perform poorly in terms of quantitative predictions has been one of the reasons why policymakers have neglected using this type of models. In fact, many central banks around the world still rely on macroeconometric models that lack microfoundations for projections and analysis.\footnote{The current macromodel used by the Chilean central bank (MEP) is a semi-structural model that combines elements of micro-founded dynamic models, with reduced form equations with no structural interpretation. For a complete description of the MEP, see CBC (2003).}

Over the past few years, key contributions on estimation techniques have given a new status to DSGE models as useful tools for forecasting and policy analysis.\footnote{The recent contributions in estimating monetary general equilibrium models are Ireland (2004), Smet and Wouters (2003a,b), Del Negro et al. (2004). Additionally, Schorfheide (2000) and Del Negro and Schorfheide (2004) propose methodologies to evaluate the fit of the general equilibrium models. A good review is provided by Ruge-Murcia (2003).} Moreover, this research agenda has given new insights to the relative empirical importance of different frictions and shocks in explaining macroeconomic data. These new insights, in turn, have contributed to further development of DSGE models.

The model in this paper is a dynamic stochastic microfounded general equilibrium model with nominal and real rigidities, that is closely related to the New Open Economy Macroeconomic models. The main characteristics of this version of the model are the following: prices and wages are sticky, there are adjustment costs in investment, the exchange rate pass-through to import prices is imperfect in the short run, and there is habit in consumption. On the supply side the model includes three main productive sectors: tradables, non-tradables and a commodity export sector.

In this version, parameters are chosen so as to mimic some steady state features of...
the Chilean economy. We study the propagation of external fluctuations to the domestic economy by analyzing the impulse-response functions of a selected set of variables to three exogenous shocks: commodity export price shock, an oil price shock, and an external interest rate shock. We focus on those three shocks because empirical evidence has highlighted that external shocks explain a significative part of the Chilean business cycles. ⁴

The impulse-response functions of the benchmark model are compared with three alternative specifications that remove three rigidities one-by-one: sticky wages, investment adjustment cost, and habit formation in consumption. From this exercise we conclude that both investment and consumption inertia are relevant to get a *hump shaped* response of GDP to these external shocks, as traditional observed in the data. This inertia, in turn, is reflected in a similar pattern for employment (hours worked). Sticky wages are important to obtain a muted response of real wages to different shocks, but they induce larger fluctuations in employment.

As a preliminary way of evaluating the model, we compare the cross-correlations between output and of the main variables generated with the model to those in the data. To compute the cross-correlations of the variables with output we use an estimated variance-covariance matrix of all exogenous shocks. The results confirm the relevance of the three rigidities analyzed to get more a plausible dynamics for the main variables. However, this exercise also shows some pitfalls of the model. First, the real exchange rate in all models considered is significatively pro-cyclical while in the data the opposite is true. Second, inflation precedes GDP with significative negative correlation, a feature that is not captured by the benchmark model and its alternatives. Third, the models miss an internal propagation mechanism that causes the aggregate demand to accelerate after an increase in GDP.

Section 2 presents a scheme of the model economy, and the main equations that characterize it. In this section, prices and quantities are aggregated and we also look at the market clearing conditions and the required elements to define the general equilibrium

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of the economy. In section 3 we discuss the parameters chosen to calibrate the model. As we said, in this version of the model, parameters are chosen to match the main aggregate features of the Chilean economy. Section 4 describes the effect of different exogenous shocks by analyzing the impulse-response functions. Section 5 provides a preliminary empirical evaluation comparing the cross correlation of the model with the data. Finally, section 6 concludes.

2 Model Economy

2.1 Basic Structure

The model economy is meant to characterize the main features of the Chilean economy, a small open economy that exports commodities linked to natural resources.

Figure 1 presents a diagram with the structure of the model economy. There are three basic agents: households, government and foreign agents. There are also three basic domestic inputs: labor, capital and natural resources. Labor is directly provided by households. Capital is provided by a representative firm that is owned by households. The stock of natural resources is determined exogenously and its owners are foreign investors and the government.

There are four types of firms. A first group of firms produce differentiated varieties of intermediate non-tradable goods, using labor, capital, and a foreign input (oil). A second group produce differentiated varieties of intermediate tradable goods, using also labor, capital, and a foreign input. This type of firms sell their production both domestically and abroad. A third type of firms are importers that distribute domestically different varieties of foreign intermediate goods. Each of the firms in these three sectors has monopoly power over the variety that produces or distributes. For the same reason profits are non-zero in all three sectors.

There is a fourth firm that produces commodities that are not consumed domestically but exported. This firm has no market power. It takes the international price of the commodity as given (e.g. the law of one price holds) and produces utilizing capital and
natural resources.

The imported input that is used in the production of domestic intermediate goods is sold directly from foreign producers—there is no retailing for this input. We think of this input to represent oil.

 Tradable and non-tradable domestic intermediate goods, and foreign intermediate goods are assembled in different ways to produce three types of final goods: one that is accumulated to increase the capital stock; one that is consumed by households; and one that is consumed by the government. The final good consumed by households also includes oil.

 Households supply a differentiated labor service and receive the corresponding wage compensations. Each household has a monopoly power over the type of labor service it provides. They are the owners of firms producing intermediate goods, capital and of import retailing firms. Therefore, they receive income corresponding to the monopoly rents generated in those sectors, and capital income.

 The government receives taxes from households and a share of the rents generated in the primary sector (commodity export sector). The remaining share of those rents goes abroad as dividends for the foreign investor owners.

 We assume the population \( N_t \) grows at a rate \( n \). The model exhibits a balanced growth path. Let \( \Omega_t \) be the labor augmenting technology. We assume that in steady state labor productivity grows at a rate \( g_y \); \( \Omega_t = (1 + g_y) \Omega_{t-1} \). We denote by \( G \) the gross rate of growth in steady state,

\[
G = (1 + n) (1 + g_y)
\]

 Finally, we also assume that in the steady state, the inflation rate is exogenously determined by the monetary authority and it is non-zero.

### 2.2 Households

The domestic economy is inhabited by a continuum of households indexed by \( j \in [0, 1] \). Each household is composed by \( N_t \) members. The expected present value of the utility
of household \( j \) is given by:

\[
U_t (j) = E_t \sum_{i=0}^{\infty} \beta^i \bar{U} \left( C_{t+i}(j), \frac{\tilde{M}_{t+i}(j)}{P_{C,t+i}}, l_{t+i}(j) \right) N_{t+i},
\]

where \( l_t (j) \) is the number of hours worked by a member of household \( j \), \( \tilde{C}_t (j) \) is total consumption per capita, and where \( \tilde{M}_t (j) \) corresponds to total nominal balances per capita held by household \( j \) at the beginning of period \( t \). Function \( \bar{U} (\cdot) \) corresponds to the utility of a particular member of household \( j \).

Each household member provides labor to two sectors in the economy (tradables and non-tradables). The total number of hours worked by members of household \( j \) is given by:

\[
L_t (j) = L_{N,t} (j) + L_{H,t} (j),
\]

where \( L_{N,t} (j) = N_t l_{N,t} (j) \) is the total number of hours worked in the non-tradable sector and \( L_{H,t} (j) = N_t l_{H,t} (j) \) the total number of hours worked in the tradable sector.

### 2.2.1 Asset market structure and budget constraint

Domestic households have access to three different types of assets: money \( M_t (j) \), foreign bonds \( B^*_t (j) \), and domestic public bonds \( B_t (j) \). There are no adjustment costs in the portfolio composition. However, each time a domestic household borrows from abroad it must pay a premium over the international price of external bonds. This premium is introduced in the model to obtain a well defined steady state for the economy.\(^5\)

The budget constraint for household \( j \), expressed in terms of units of the final con-

\(^5\)See Schmitt-Grohe and Uribe (2003) for different ways to get steady state independent of initial conditions for small open economy models.
sumption good, is given by

\[
N_t \left[ \frac{\tilde{M}_t(j)}{P_{C,t}} + \frac{\tilde{B}_t(j)}{P_{C,t}(1 + i_t)} + \frac{e_t \bar{B}_t^*(j)}{P_{C,t}(1 + i_t^*)} \Theta \left( \frac{e_t B_t^*}{P_{X,t} X_t} \right) \right] \leq \left[ \frac{\tilde{M}_{t-1}(j)}{P_{C,t}} + \frac{\tilde{B}_{t-1}(j)}{P_{C,t}} + \frac{e_t \bar{B}_{t-1}^*(j)}{P_{C,t}} \right] + N_t \left[ (1 - \tau_L) \frac{W_t(j)}{P_{C,t}} l_t(j) + \frac{\tilde{l}_t(j)}{P_{C,t}} - \tilde{C}_t(j) + \tilde{\tau}_t + \tilde{d}_t(j) \right].
\]

(3)

where \( P_{C,t} \) is the consumption baske deflator, \( \tilde{\Pi}_t(j) \) are after tax profits received from different firms, \( e_t \) is the nominal exchange rate, \( W_t(j) \) is the nominal wage set by household \( j \), \( \tilde{\tau}_t \) are per capita real lump sum net transfers from the government. The term \[ \Theta \left( \frac{e_t B_t^*}{P_{X,t} X_t} \right) \] corresponds to the premium domestic households have to pay each time they borrow from abroad, where \( B_t^* = N_t \int_0^1 \tilde{B}_t^*(j) dj \) is the aggregate net foreign asset position of the economy and where \( P_{X,t} X_t \) is the nominal value of exports.\(^6\) The term \( \tilde{d}_t(j) \) corresponds to the net payments receive by household \( j \) from participating in a contingent asset market that insures households against fluctuations in employment due to sticky wages. Assuming the existence of such a market ensures that consumption of all households is the same, independently of the labor income they receive each period.

Notice that we assume the premium depends on the aggregate net foreign asset position of the economy. This implies that households take \( \Theta(\cdot) \) as given when deciding their optimal portfolios. In other words, households do no internalize the effect on the premium of changes in their foreign asset positions. We impose the following restrictions on this function: \( \Theta(\cdot) = 1 \), and \( \Theta(\cdot) = 1 \) only when \( \frac{e_t B_t^*}{P_{X,t} X_t} = \frac{e B^*}{P_{X} X} \); \( \Theta(\cdot) \) is a twice differentiable function, and decreasing in the neighborhood around \( \frac{e B^*}{P_{X} X} \). Here \( B^* \) corresponds to the steady state real net foreign position, while \( P_{X} X \) is the steady state value of exports.

\(^6\)Since the economy is growing in steady state the net asset position is also growing in the long run. Therefore, in order to have a stationary risk premium it is necessary that this premium be a function of the ratio of the net asset position and some variable that grows at the same rate in steady state. We choose export since that could represent a form of international collateral (see Caballero and Krishnamurthy, 2001)
2.2.2 Consumption and saving decisions

Households choose consumption and the composition of their portfolios by maximizing (1) subject to (3). The first order conditions for the problem that household $j$ must solve imply the following relations:

$$
\tilde{U}_{m,t} = \tilde{U}_{C,t} \frac{i_t}{1 + i_t},
\tag{4}
$$

$$
\tilde{U}_{C,t} = (1 + i_t) \beta E_t \left( \tilde{U}_{C,t+1} \frac{P_{C,t}}{P_{C,t+1}} \right),
\tag{5}
$$

$$
\tilde{U}_{C,t} = (1 + i^*_t) \Theta (\cdot) E_t \left( \tilde{U}_{C,t+1} \frac{e_{t+1}}{e_t} \frac{P_{C,t}}{P_{C,t+1}} \right).
\tag{6}
$$

Here $\tilde{U}_{C,t}$ and $\tilde{U}_{m,t}$ denote the derivatives of $\tilde{U} \left( \tilde{C}_t, \frac{\tilde{M}_t}{P_{C,t}}, l_t \right)$ with respect to $\tilde{C}_t$ and $\tilde{M}_t / P_{C,t}$. Notice that since households’ decisions regarding consumption and money balances are symmetric we have omitted index $j$ in derivatives of $\tilde{U} (\cdot)$.

Moreover, if we assume that preferences are homothetic and complete domestic financial markets, then the income or wealth distribution has no effect on the marginal decision of households.

Equation (4) represents the demand for real balances. Equations (5) and (6) correspond to the first order conditions for domestic and external bonds, respectively. Combining these two equations we obtain an expression for the uncovered interest rate parity:

$$
(1 + i_t) E_t \left( \tilde{U}_{C,t+1} \frac{P_{C,t}}{P_{C,t+1}} \right) = (1 + i^*_t) \Theta (\cdot) E_t \left( \tilde{U}_{C,t+1} \frac{e_{t+1}}{e_t} \frac{P_{C,t}}{P_{C,t+1}} \right).
\tag{7}
$$

2.2.3 Labor supply decisions and wage setting

Each household $j$ is a monopolistic supplier of a differentiated labor service. There is a set of perfect competitive labor service assemblers that hire labor from each household and combine it into an aggregate labor services unit, $l_t$, that is then used by the intermediate goods producers. The labor service unit is defined as,

$$
l_t = \left( \int_0^1 l_t(j) \frac{e_{t-1}}{e_t} \, dj \right)^{\frac{e_{t+1}}{e_t}}.
\tag{8}
$$

\footnote{For convenience we have omitted the transversality condition, which states that the present value of the household’s total assets at the infinite future cannot be less than zero. This condition implies that $\beta(1 + n)(1 + g_y)^{(1-1/\sigma)} < 1$.}

\footnote{As we will see below, households differ in the amount of labor (hours) they supply in the market. Therefore, we need to assume that the utility function is separable in labor.}
The optimal composition of this labor services unit is obtained by minimizing its cost, given the different wages set by different households. In particular, the demand for the labor service provided by household \( j \) is:

\[
l_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_L} l_t,
\]

where \( W_t(j) \) is the wage rate set by household \( j \) and \( W_t \) is an aggregate wage index defined as

\[
W_t = \left( \int_0^1 W_t(j)^{1-\epsilon_L} dj \right)^{\frac{1}{1-\epsilon_L}}.
\]

Following Erceg, Henderson and Levin (2000) we assume that wage setting is subject to nominal rigidity à la Calvo (1983). In each period, each type of household faces a constant probability \((1 - \phi_L)\) of being able to re-optimize its nominal wage. We assume there is an updating rule for all those households that cannot re-optimize their wages. In particular, if a household cannot re-optimize during \( i \) periods between \( t \) and \( t + i \), then its wage at time \( t + i \) is given by

\[
W_{t+i}(j) = \Gamma_i W_t W_t(j)
\]

where \( \Gamma_i W_t \) describes an adjustment rule for wages that is defined as:

\[
\Gamma_i W_t = \sum_{j=1}^i (1 + \pi_{C,t+j-1})^{\kappa_W} (1 + \pi_{t+j})^{1-\kappa_W} (1 + g_y)
\]

This “passive” adjustment rule implies that workers who do not optimally reset their wages update them by considering a geometric weighted average of past CPI inflation and the inflation target set by the authority, \( \pi_t \). The presence of \((1 + g_y)\) in the expression above is included in order to avoid large real wage dispersion along the steady state growth path. Once a household has decided a wage, it must supply any quantity of labor service demanded at that wage.

A particular household \( j \) that is able to re-optimize its wages at \( t \) solves the following problem:

\[
\max_{W_t(j)} = E_t \left\{ \sum_{i=0}^{\infty} \phi_L A_{t+i} \left( \frac{W_t(j) \Gamma_i W_{t+i}}{P_{C,t+i}} + \frac{\bar{U}_{l,t+i}}{\bar{U}_{C,t+i}} \right) l_{t+i}(j) \right\}
\]
subject to the demand (9) and the updating rule for the nominal wage (11). Variable $\Lambda_{t,t+i}$ is the relevant discount factor between periods $t$ and $t+i$, and is given by $\Lambda_{t,t+i} = \beta \frac{U_{C,t+i}}{U_{C,t}}$. $\tau_L$ is a labor income tax which we assume constant.

The first order condition for this problem implies the following optimal wage:

$$W_{t}^{*} = \frac{\epsilon_L}{\epsilon_L - 1} \frac{E_t \left\{ \sum_{i=0}^{\infty} \phi_i^L \Lambda_{t,t+i} \left( -\frac{U_{t,i}}{U_{C,t+i}} \right) I_{t+i}(j) \right\}}{E_t \left\{ \sum_{i=0}^{\infty} \phi_i^L \Lambda_{t,t+i} \left[ \left( 1 - \tau_L \right) \Gamma W_{t} \right] P_{C,t+i} \right\} I_{t+i}(j)}$$

(12)

When wages are fully flexible ($\phi_L = 0$) the optimal wage is just a markup over the marginal rate of substitution between labor and consumption,

$$(1 - \tau_L) \frac{W_{t}^{*}}{P_{C,t}} = -\frac{\epsilon_L}{\epsilon_L - 1} \frac{U_{t,t}}{U_{C,t}}.$$

(13)

2.3 Commodity export sector ($S$)

There is a unique firm that produces and exports a commodity good (good $S$). All production is exported. The international demand for this good is completely elastic at a given price $P_{S,t}^*$. The commodity good is produced using capital and an input $K$ whose endowment is determined exogenously, and whose growth rate is equal to $G - 1$.\(^9\) We think of this factor as representing natural resources (in the case of Chile, copper).\(^10\) The technology in this sector is given by:

$$Y_{S,t} = F^S(K_t, K_{S,t}),$$

(14)

where $K_{S,t} = u_{S,t} K_{S,t}$ is the effective capital utilized by the firm. Capital is specific to this firm and cannot be used by any other firm in the economy. Variable $u_{S,t}$ represents the utilization rate of capital, and $K_{S,t}$ is the stock of capital available in $t$. We assume that $F^S(.)$ is a homogeneous function of degree one. However, since $K_t$ is exogenous there are diminishing returns to $K_{S,t}$ at a given date $t$.

\(^9\)We need to impose such a restriction in order to have balance growth in sector $S$ consistent with the growth of the rest of the economy.

\(^10\)Copper exports account for about 40% of Chilean exports.
The capital good utilized in sector $S$ is a bundle composed by domestic intermediate tradable goods, $Y_{H,t}(I_S)$, and foreign intermediate goods, $Y_{F,t}(I_S)$:

$$I_{S,t} = \left( \frac{\gamma_{I_S}}{\theta_S} (Y_{H,t}(I_S))^{\frac{\theta_S-1}{\theta_S}} + (1 - \gamma_{I_S})^{\frac{1}{\theta_S}} (Y_{F,t}(I_S))^{\frac{\theta_S-1}{\theta_S}} \right)^{\frac{\theta_S}{\theta_S-1}}. \tag{15}$$

The firm decides the optimal composition in (15) by minimizing the cost of investment, $P_{I_S,t}I_{S,t} = P_{H,t}Y_{H,t}(I_S) + P_{F,t}Y_{F,t}(I_S)$, subject to a certain level of investment. The solution to this problem defines the demand for intermediate domestic tradable and foreign goods:

$$Y_{H,t}(I_S) = \gamma_{I_S} \left( \frac{P_{H,t}}{P_{I_S,t}} \right)^{-\theta_S} I_{S,t}, \tag{16}$$

$$Y_{F,t}(I_S) = (1 - \gamma_{I_S}) \left( \frac{P_{F,t}}{P_{I_S,t}} \right)^{-\theta_S} I_{S,t}, \tag{17}$$

where $P_{I_S,t}$ is the price deflator of investment in sector $S$, while $P_{H,t}$ and $P_{F,t}$ are the corresponding price indices of domestic and foreign intermediate tradable goods. The price index $P_{I_S,t}$ is obtained by replacing (16) and (17) into (15) and is given by $P_{I_S,t} = \left( \gamma_{I_S} P_{H,t}^{1-\theta_S} + (1 - \gamma_{I_S}) P_{F,t}^{1-\theta_S} \right)^{\frac{1}{1-\theta_S}}$.

The firm decides how much capital to accumulate and the rate at which capital is used each period, taking the price of the commodity, $P_{S,t}$, has given in order to maximize the discounted expected value of its profits. The law of one price holds for this good. Therefore, its price in domestic currency is just $P_{S,t} = e_t P^*_S$, where $P^*_S$ is the exogenous international price for this good.

Let $\Pi_t(S) = (1 - \tau_S) P_{S,t} F^S(K_t, K_{S,t}) + \tau_S P_{I_S,t} K_{S,t} \delta_S - P_{I_S,t} I_{S,t}$ be the period $t$ profit for the firm. $\tau_S$ is the tax for capital income in this sector. Then the firm must solve the following problem:

$$\max_{\{I_{S,t+i}\}_{i=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \Lambda_{t+i} \frac{\Pi_{t+i}(S)}{N_{t+i} P_{C,t+i}} \right\} \tag{18}$$

subject to the law of motion for capital,

$$K_{S,t+1} = (1 - \delta_S (u_{S,t})) K_{S,t} + \Phi_S (I_{S,t}/I_{S,t-1}) I_{S,t}, \tag{19}$$

11 Non-tradable intermediate goods are a negligible component of investment in the copper sector.

12 $\Pi_{t+i}(S)/N_{t+i}$ are the per capita profits which are the relevant to be discounted with $\Lambda_{t,t+i}$. 

11
Here $\delta_S(u_{S,t})$ is the rate of depreciation, which depends on the utilization rate of capital. In particular, we assume that $\delta_S(\cdot)$ is an increasing convex function. The function $\Phi_S(I_{S,t}/I_{S,t-1})$ represents the technology that transforms current and past gross investment, $I_{S,t}$, into installed physical capital to be used in the following period (Christiano, Eichenbaum and Evans, 2005). This function satisfies: $\Phi_S(G) = 1$, $\Phi_S'(G) = 0$, and $\Phi_S''(G) < 0$.

The evolution of investment and the capital stock in this sector are characterized by the following system of difference equations:

$$\frac{P_{I_{S,t}}}{P_{C,t}} = \frac{Q_{S,t}}{P_{C,t}} \left( \Phi_S \left( \frac{I_{S,t}}{I_{S,t-1}} \right) + \Phi_S' \left( \frac{I_{S,t}}{I_{S,t-1}} \right) - \frac{\Lambda_{t,t+1} Q_{S,t+1}}{1 + n} \frac{\Phi_S' \left( \frac{I_{S,t+1}}{I_{S,t}} \right)}{\frac{I_{S,t+1}}{I_{S,t}} - 1} \right),$$

$$Q_{S,t} = E_t \left[ \frac{\Lambda_{t,t+1} Q_{S,t+1}}{1 + n} \left( 1 - \tau_S \right) \frac{P_{S,t+1}}{P_{C,t+1}} F_{K_{S,t+1}} u_{S,t+1} + \tau_S \frac{P_{S,t+1}}{P_{C,t+1}} \delta_S \frac{Q_{S,t+1}}{P_{C,t+1}} (1 - \delta_S (u_{S,t+1})) \right],$$

where $F_{K_{S,t+1}}$ is the partial derivative of $F^S(K, K_S)$ with respect to $K_S$ at time $t + 1$.

Equation (20) relates investment with the shadow price of capital, $Q_{S,t}$, in sector $S$. Equation (21) defines the law of motion of the variable $Q_{S,t}$.

The first order condition for the optimal utilization rate of capital is the following:

$$(1 - \tau_S) \frac{P_{I_{S,t}}}{P_{C,t}} F_{K_{S,t}} = \frac{Q_{S,t}}{P_{C,t}} \delta_S' (u_{S,t}) K_{S,t}$$

2.4 Final consumption good assembly ($C$)

A unique final consumption good is assembled by a set of firms that use a constant return to scale technology. The production function utilized by those firms is the following:

$$C_t = \left( \alpha_C \psi_C^{-1} (C_{Z,t}) \psi_C^{-1} + (1 - \alpha_C) \psi_C^{-1} (C_{O,t}) \psi_C^{-1} \right) \psi_C^{-1},$$

where $C_{Z,t}$ corresponds to a basket of goods that we call core consumption, and $C_{O,t}$ is consumption of oil (energy). Parameter $\psi_C$ corresponds to the elasticity of substitution.
between core consumption and consumption of oil, and parameter $\alpha_C$ defines the share of core consumption. Core consumption is given by the following basket:

$$C_{Z,t} = \left( \gamma_C \left( Y_{T,t} (C) \right)^{\frac{\nu_C - 1}{\nu_C}} + (1 - \gamma_C) \left( Y_{N,t} (C) \right)^{\frac{\nu_C - 1}{\nu_C}} \right)^{\frac{\nu_C}{\nu_C - 1}},$$

where $Y_{T,t} (C)$ is a basket of intermediate tradable goods, and $Y_{N,t} (C)$ is a basket of intermediate non-tradable goods. The basket of intermediate tradable goods combines a basket of domestic intermediate tradable goods, $Y_{H,t} (C)$, and a basket of foreign intermediate tradable goods, $Y_{F,t} (C)$, as follows:

$$Y_{T,t} (C) = \left( \omega \left( Y_{H,t} (C) \right)^{\frac{\nu_C - 1}{\nu_C}} + (1 - \omega) \left( Y_{F,t} (C) \right)^{\frac{\nu_C - 1}{\nu_C}} \right)^{\frac{\nu_C}{\nu_C - 1}}. \quad (25)$$

Each assembler of the final consumption good chooses the optimal combination of oil, intermediate tradable and intermediate non-tradable goods in order to minimize the cost of the final consumption good faced by the households, $P_{C,t}C_t = P_{CZ,t}C_{Z,t} + (1 + \tau_C)P_{O,t}C_{O,t}$ subject to (23), (24), and given the cost of core consumption, $P_{CZ,t}C_{Z,t} = P_{TC,t}Y_{T,t} (C) + (1 + \tau_C)P_{N,t}Y_{N,t} (C)$, and the cost of the tradable basket, $P_{TC,t}Y_{T,t} (C) = (1 + \tau_C) (P_{H,t}Y_{H,t} (C) + P_{F,t}Y_{F,t} (C))$. Parameter $\tau_C$ corresponds to a tax on final consumption. From this minimization problem we obtain the following:

$$C_{O,t} = (1 - \alpha_C) \left( 1 + \tau_C \left( \frac{P_{O,t}}{P_{C,t}} \right) \right)^{-\psi_C} C_t \quad (26)$$

$$Y_{N,t} (C) = (1 - \gamma_C) \alpha_C \left( 1 + \tau_C \left( \frac{P_{N,t}}{P_{CZ,t}} \right) \right)^{-\eta_C} \left( \frac{P_{CZ,t}}{P_{C,t}} \right)^{-\psi_C} C_t \quad (27)$$

$$Y_{H,t} (C) = \omega_C \gamma_C \alpha_C \left( 1 + \tau_C \left( \frac{P_{H,t}}{P_{TC,t}} \right) \right)^{-\theta_C} \left( \frac{P_{TC,t}}{P_{CZ,t}} \right)^{-\eta_C} \left( \frac{P_{CZ,t}}{P_{C,t}} \right)^{-\psi_C} C_t, \quad (28)$$

$$Y_{F,t} (C) = (1 - \omega) \gamma_C \alpha_C \left( 1 + \tau_C \left( \frac{P_{F,t}}{P_{TC,t}} \right) \right)^{-\theta_C} \left( \frac{P_{TC,t}}{P_{CZ,t}} \right)^{-\eta_C} \left( \frac{P_{CZ,t}}{P_{C,t}} \right)^{-\psi_C} C_t. \quad (29)$$

Equations (26), (27), (28) and (29) define the demands for each type of intermediate goods and for oil that originate in households’ consumption. The price index of consumption goods is given by:

$$P_{C,t} = \left( \gamma_C P_{CZ,t}^{1-\psi_C} + (1 - \gamma_C) ((1 + \tau_C)P_{O,t})^{1-\psi_C} \right)^{\frac{1}{1-\psi_C}}, \quad (30)$$

This tax is included in the model in order to match certain steady state features of the Chilean economy. This is a value added tax and for that reason is only effectively paid by the final consumers.
The price index for core consumption is:

\[ P_{C,t} = \left( \gamma_C P_{T,t}^{1-\eta_C} + (1 - \gamma_C) (1 + \tau_C) P_{N,t}^{1-\eta_C} \right)^{\frac{1}{1-\eta_C}}, \]  

(31)

where the price index of the intermediate tradable basket utilized in consumption is given by

\[ P_{T,t} = (1 + \tau_C) \left( \omega_C P_{H,t}^{1-\theta_C} + (1 - \omega_C) P_{F,t}^{1-\theta_C} \right)^{\frac{1}{1-\eta_C}}. \]

### 2.5 Government consumption good assembly \((G)\)

The government in our model economy consumes a final good that is different from the final good consumed by households. We differentiate government consumption from household consumption to capture the effects of aggregate demand composition on relative prices (i.e., real exchange rate).\(^{15}\) The consumption good of the government is composed as follows:

\[ G_t = \left( \gamma_G Y_{T,t} (G) \right)^{\frac{\eta_G-1}{\eta_G}} + (1 - \gamma_G) \left( \gamma G Y_{N,t} (G) \right)^{\frac{\eta_G-1}{\eta_G}}, \]  

(32)

where \( Y_{T,t} (G) \) corresponds to a basket of intermediate tradable goods, and \( Y_{N,t} (G) \) to a basket of intermediate non-tradable goods. The basket of intermediate tradable goods combines a basket of domestic intermediate tradable goods, \( Y_{H,t} (G) \), with a basket of intermediate foreign goods, \( Y_{F,t} (G) \), using the following technology:

\[ Y_{T,t} (G) = \left( \frac{1}{\omega_G} Y_{H,t} (G) \right)^{\frac{\eta_G-1}{\eta_G}} + (1 - \omega_G) \left( \frac{1}{\eta_G} Y_{F,t} (G) \right)^{\frac{\eta_G-1}{\eta_G}}. \]  

(33)

The government chooses the quantities of tradable and non-tradable goods it consumes by minimizing its expenditure, \( P_{G,t} G_t = P_{T,t} Y_{T,t} (G) + P_{N,t} Y_{N,t} (G) \), subject to (32). From this minimization we obtain the following demands for each type of goods:

\[ Y_{N,t} (G) = (1 - \gamma_G) \left( \frac{P_{N,t}}{P_{G,t}} \right)^{-\eta_G} G_t, \]  

(34)

\(^{15}\)Edwards (1989), Froot and Rogoff (1991), and De Gregorio and Wolf (1994) provide international evidence of effects of government spending on the real exchange rate. Moreover, Arrau et al. (1992) and Arellano and Larrain (1994) highlight the significance of government spending expansion on real exchange rate appreciation for the Chilean case.
\[ Y_{H,t}(G) = \omega_G \gamma_G \left( \frac{P_{H,t}}{P_{G,t}} \right)^{-\theta_G} \left( \frac{P_{G,t}}{P_{T,G,t}} \right)^{-\eta_G} G_t, \quad (35) \]

\[ Y_{F,t}(G) = (1 - \omega_G) \gamma_G \left( \frac{P_{F,t}}{P_{G,t}} \right)^{-\theta_G} \left( \frac{P_{T,G,t}}{P_{G,t}} \right)^{-\eta_G} G_t. \quad (36) \]

where the government expenditure deflator is given by

\[ P_{G,t} = \left( \gamma_G P_{T,G,t}^{1-\eta_G} + (1 - \gamma_G) P_{N,t}^{1-\eta_G} \right)^{\frac{1}{1-\eta_G}}, \quad (37) \]

and where the price index of the tradable goods utilized in the assembly of government consumption is given by

\[ P_{T,G,t} = \left( \omega_G P_{H,t}^{1-\eta_T} + (1 - \omega_G) P_{F,t}^{1-\eta_T} \right)^{\frac{1}{1-\eta_T}}. \]

### 2.6 Capital goods assembly \((I_D)\)

The capital good utilized by firms producing intermediate goods is assembled by a single firm that combines bundles of intermediate tradable and non-tradable goods utilizing a constant return to scale technology:

\[ I_{D,t} = \left( \frac{1}{\gamma_{I_D} (Y_{T,t}(I_D))^{\frac{\eta_I}{\eta_I}} + (1 - \gamma_{I_D}) \frac{1}{\eta_I} (Y_{N,t}(I_D))^{\frac{\eta_I}{\eta_I}}} \right) \frac{1}{\gamma_{I_D} + (1 - \gamma_{I_D}) \frac{1}{\eta_I}}. \]

\[ (38) \]

where the basket of intermediate tradable goods is given by:

\[ Y_{T,t}(I_D) = \left( \omega_{I_D} (Y_{H,t}(I_D))^{\frac{\theta_{I_D}}{\theta_{I_D}}} + (1 - \omega_{I_D}) \frac{1}{\theta_{I_D}} (Y_{F,t}(I_D))^{\frac{\theta_{I_D}}{\theta_{I_D}}} \right)^{\frac{1}{\theta_{I_D}}}. \]

\[ (39) \]

Demands for each type of goods are obtained by minimizing the cost of investment, \(P_{I,t,I_D,t} = P_{T,I,D,t} Y_{T,t}(I_D) + P_{N,t} Y_{N,t}(I_D)\). They are given by

\[ Y_{N,t}(I_D) = (1 - \gamma_{I_D}) \left( \frac{P_{N,t}}{P_{I_D,t}} \right)^{-\eta_I} I_{D,t}. \]

\[ (40) \]

\[ Y_{H,t}(I_D) = \omega_{I_D} \gamma_{I_D} \left( \frac{P_{H,t}}{P_{T,I,D,t}} \right)^{-\theta_I} \left( \frac{P_{T,I,D,t}}{P_{I_D,t}} \right)^{-\eta_I} I_{D,t}. \]

\[ (41) \]

\[ Y_{F,t}(I_D) = (1 - \omega_{I_D}) \gamma_{I_D} \left( \frac{P_{F,t}}{P_{T,I,D,t}} \right)^{-\theta_I} \left( \frac{P_{T,I,D,t}}{P_{I_D,t}} \right)^{-\eta_I} I_{D,t}. \]

\[ (42) \]

The investment deflator is given by

\[ P_{I,t} = \left( \gamma_{I_D} P_{T,I,D,t}^{1-\eta_I} + (1 - \gamma_{I_D}) P_{N,t}^{1-\eta_I} \right)^{\frac{1}{1-\eta_I}}, \]

\[ (43) \]
where the price index of the basket of intermediate tradable goods utilized in the assembly of capital goods is given by

\[ P_{T,I,D,t} = \left( \omega_{I,D} P_{I,D,t}^{1-\theta_{I}} + (1 - \omega_{I,D}) P_{F,t}^{1-\theta_{I}} \right)^{\frac{1}{1-\theta_{I}}} \]

A representative firm in this sector decides how much capital to accumulate and the rate at which capital could be used by intermediate goods producers. Let \( \Pi_t(I_D) = (1 - \tau_D) R^K_t u_{D,t} \bar{K}_{D,t} + \tau_D P_{I,D,t} \delta_D \bar{K}_{D,t} - P_{I,D,t} I_{D,t} \) be the period \( t \) profit for the representative firm, where \( R^K_t \) is the rental price of capital goods to intermediate firms and \( \tau_D \) is a capital income tax. The firm decides how much to invest and the utilization rate of capital \( u_{D,t} \) by solving the following problem:

\[
\max_{\{I_{D,t+1}\}_{t=0}^{\infty}} E_t \left\{ \sum_{i=0}^{\infty} \Lambda_{t+i} \frac{\Pi_{t+i}(I_D)}{N_{t+i} P_{C,t+i}} \right\} \tag{44}
\]

subject to the law of motion of the capital stock,

\[
\bar{K}_{D,t+1} = (1 - \delta_D(u_{D,t})) \bar{K}_{D,t} + \Phi_D(I_{D,t}/I_{D,t-1}) I_{D,t} \tag{45}
\]

As in sector \( S \), the depreciation rate of capital, \( \delta_D(u_{D,t}) \), depends on the utilization rate of capital. We assume that function \( \delta_D(\cdot) \) is an increasing convex function, which is such that in steady state \( u_D = 1 \).

The function \( \Phi_D(I_{D,t}, I_{D,t-1}) \) summarizes the technology that transform current and past new capital goods into installed capital stock to be used the next period. Again, we assume this function to have the following properties \( \Phi_D(\mathcal{G}) = 1, \Phi'_D(\mathcal{G}) = 0, \) and \( \Phi''_D(\mathcal{G}) < 0 \).

Investment and the evolution of the capital stock can be characterized by the following system of difference equations:

\[
\frac{P_{I_{D,t}}}{P_{C,t}} = E_t \left[ \frac{\Lambda_{t,t+1} Q_{D,t+1}}{1 + n} \left( \frac{A_{I_{D,t}}}{A_{I_{D,t-1}}} \right) \right] \tag{46}
\]

\[
\frac{Q_{D,t}}{P_{C,t}} = E_t \left[ \frac{\Lambda_{t,t+1} Q_{D,t+1}}{1 + n} \left( \frac{1 - \tau_D}{1 - \delta_D} \right) + \frac{\tau_D P_{I_{D,t+1}} \delta_D}{P_{C,t+1}} \right] \tag{47}
\]
Equation (46) relates investment with the shadow real price of capital in sector $I_D$, $Q_{D,t}/P_{C,t}$. Equation (47) defines the law of motion of $Q_{D,t}/P_{C,t}$. In particular, this equation shows how this variable depends on the expected rental price of capital relative to the price of consumption, $\frac{R^k_{t+1}}{P_{C,t+1}}$, and the expected evolution of its own value.

The first order condition for the optimal utilization rate of capital is the following:

\[(1 - \tau_D) \frac{R^K_t}{P_{C,t}} K_{D,t} = Q_{D,t} \frac{\delta_{D}'}{P_{C,t}} (u_{D,t}) K_{D,t} \] (48)

2.7 Intermediate goods assembly

Aggregate bundles of intermediate non-tradable, and domestic and foreign intermediate tradable goods are defined over a continuum of varieties, $z_N$, $z_H$ and $z_F$, as follows:

\[Y_{a,t} = \left( \int_0^1 (Y_{a,t}(z_a))^{\frac{1}{1-\epsilon_a}} d z_a \right)^{\frac{1}{1-\epsilon_a}} \quad \text{for} \quad a = N, H, F \] (49)

Variable $Y_{a,t}(z_a)$ corresponds to the quantities of varieties $z_a$ utilized in the assembly of the corresponding aggregate quantities $Y_{a,t}$. Parameter $\epsilon_a$ defines the price elasticity of demand for each type of variety.

Demands for the three types of varieties are obtained by minimizing the cost of each one of the three aggregate baskets. From this minimization problem we obtain:

\[Y_{H,t}(z_H) = \left( \frac{P_{H,t}(z_H)}{P_{H,t}} \right)^{-\epsilon_H} Y_{H,t}, \] (50)

\[Y_{N,t}(z_N) = \left( \frac{P_{N,t}(z_N)}{P_{N,t}} \right)^{-\epsilon_N} Y_{N,t}, \] (51)

\[Y_{F,t}(z_F) = \left( \frac{P_{F,t}(z_F)}{P_{F,t}} \right)^{-\epsilon_F} Y_{F,t}. \] (52)

where $Y_{H,t} = Y_{H,t}(C) + Y_{H,t}(I_S) + Y_{H,t}(G) + Y_{H,t}(I_D)$, $Y_{N,t} = Y_{N,t}(C) + Y_{N,t}(G) + Y_{N,t}(I_D)$, and $Y_{F,t} = Y_{F,t}(C) + Y_{F,t}(I_S) + Y_{F,t}(G) + Y_{F,t}(I_D)$.\(^{16}\) The respective aggregate price indices are defined as

\[P_{a,t} = \left( \int (P_{a,t}(z_a))^{1-\epsilon_a} d z_a \right)^{\frac{1}{1-\epsilon_a}} \quad \text{for} \quad a = N, H, F. \] (53)

\(^{16}\)The foreign demand for domestic tradable intermediate goods, $Y_{H,t}(F)$, is defined below.
2.8 Domestic intermediate sectors

Sectors $H$ and $N$ are characterized by a set of domestic firms producing differentiated varieties of tradable goods. These varieties are then utilized in the assembly of either final consumption goods, capital goods, or government consumption goods. Each firm produces a unique variety and has monopoly power over it. Price adjustment is infrequent, as specified below.

2.8.1 Technology

Each firm in sectors $H$ and $N$ produces with the following technology:

$$F^d (K_{d,t} (z_d), L_{d,t} (z_d), O_{d,t} (z_d)) = A_{d,t} \left[ v^{\frac{1}{\sigma_d}} \left( K_{d,t}^{\eta_d} (z_d) (\Omega_t L_{d,t} (z_d))^{1-\eta_d} \right)^{\frac{\sigma_d-1}{\sigma_d}} \right. + (1 - v)^{\frac{1}{\sigma_d}} \left( O_{d,t} (z_d) \right) \left. \right]^\frac{\sigma_d}{\sigma_d-1}, \text{ for } d = H, N$$

where $L_{d,t} (z_d) = N_t l_{d,t} (z_d)$ corresponds to the total quantity of labor utilized by a firm producing variety $z_d$, $K_{d,t} (z_d)$ is the quantity of capital that the firm utilizes, and $O_{d,t} (z_d)$ corresponds to the quantity of imported inputs utilized. $A_{d,t}$ is a sector specific technology shock.

As was said, each firm has monopoly power over the variety it produces. In turn, prices are infrequently adjusted. Once a firm has set its price it must satisfy the quantity demanded at the given price. Each period, the firm decides how much imported input to use and capital and labor to hire by minimizing total cost of production conditional on the quantity it must produce. The problem for a generic firm $z_d$ in period $t$ is:

$$\min \left\{ L_{D,t} (z_d), K_{t} (z_d), O_{D,t} (z_D) \right\} W_t L_{d,t} (z_d) + R^K_t K_{d,t} (z_d) + P_{O,t} O_{d,t} (z_d)$$

subject to the technological restriction (54). From the first order condition we obtain:

$$\frac{R^K_t}{W_t} = \frac{F^d_{K,t} (z_d)}{\Omega_t (z_d) F^d_{L,t} (z_d)}$$

$$\frac{R^K_t}{P_{O,t}} = \frac{F^d_{K,t} (z_d)}{F^d_{O,t} (z_d)}$$
where $F_{d,t}(z_d)$ is the derivative of $F^d$ with respect to input $f$ for firm $z_d$ evaluated at time $t$.

Since the technology has constant returns to scale and there are no adjustment costs for inputs, the marginal cost does not depend on the scale of production, i.e. marginal costs are the same for all firms in sector $d$. We can express marginal cost as follows:

$$MC_{d,t} = \frac{W_t L_{d,t}(z_d) + R^K_t K_{d,t}(z_d) + P_{O,t} O_{d,t}(z_d)}{A_{d,t} F^d(K_{d,t}(z_d), L_{d,t}(z_d), O_{d,t}(z_d))}.$$ (58)

### 2.8.2 Pricing policy

Following Calvo (1983) we assume that firms adjust their prices infrequently, at the moment of receiving a signal. Each period the probability of receiving a signal and adjusting prices is $1 - \phi_d$ for all firms, independently of their history. If the firm does not receive a signal, then it follows a simple updating rule defined by function $\Gamma_{d,t}$. Therefore, if a firm $z_d$ receives a signal in period $t$ then it will adjust the price of its variety, $P_{d,t}(z_d)$, so as to maximize the following expression:

$$\max_{P_{d,t}(z_d)} \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} (\phi_d)^i \Lambda_{t,t+i} \frac{\Gamma_{d,t}^i P_{d,t}(z_d) - MC_{d,t+i}}{N_{i+1} P_{C,t+i}} Y_{d,t+i}(z_d) \right\},$$

subject to the restriction imposed by the technology and considering the demand it faces for its variety (50 or 51). The passive adjustment rule is given by

$$\Gamma_{D,t}^i = \prod_{j=1}^{i} \left( 1 + \pi_{d,t+j-1} \right)^{1-\xi_d} \left( 1 + \pi_{t+j} \right)^{1-\xi_d}$$ (60)

where $1 + \pi_{d,t} = (P_{d,t}/P_{d,t-1})$, and where $\pi_{t+j}$ corresponds to the inflation target set by the authority. Parameter $\xi_d$ captures the degree of “indexation” in the economy. The larger this parameter, the larger the weight of past inflation in defining new prices—therefore current inflation. This will imply a Phillips curve with a larger backward-looking component.

The optimal price of a firm that is able to set its price at time $t$, $P_{op_{d,t}}(z_d)$, is obtained by solving (59) subject to either (50) or (51);

$$P_{op_{d,t}} = \frac{\epsilon_d}{\epsilon_d - 1} \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} (\phi_d)^i \Lambda_{t,t+i} \frac{MC_{d,t+i}}{N_{i+1} P_{C,t+i}} Y_{d,t+i}(P_{d,t+i})^\epsilon_d \right\}.$$ (61)
Notice that since marginal costs are independent of the scale of production the optimal resetting price is the same for all firms receiving a signal. Therefore, we suppress the index in the optimal price for firm $z_d$. Without nominal rigidities ($\phi_d = 0$) we have that the optimal price for each domestic tradable variety is a mark-up over the marginal cost:

$$P_{d,t}^{\text{flex}} = \frac{\epsilon_d}{\epsilon_d - 1} MC_{d,t}^{\text{flex}},$$

(62)

where the mark-up $\frac{\epsilon_d}{\epsilon_d - 1}$ depends on the price elasticity of demand faced by the firm.

Profits for a generic firm $z_d$ in sector $d$ are $\Pi (d, z_d) = P_{d,t} (z_d) Y_{d,t+i} (z_d) - W_{t} L_{d,t} (z_d) - R_{t} K_{d,t} (z_d) - P_{O,t} O_{d,t} (z_d)$.

2.9 Foreign intermediate goods retailing sector ($F$)

There is a group of domestic firms that import foreign intermediate goods and sell them domestically. Each firm imports a unique foreign variety and has monopoly power over domestic retailing of that variety.\footnote{We think that each firm has a brand representation.} A retailing firm adjusts the domestic price of the foreign variety infrequently, whenever it receives a signal. The signal arrives with probability $1 - \phi_F$ each period. Therefore, when a generic firm $z_F$ receives a signal, it chooses a new retailing price by maximizing the following expression:

$$\max_{P_{F,t} (z_F)} E_t \left\{ \sum_{i=0}^{\infty} (\phi_F)^i \Lambda_{t+i} P_{F,t+i} (z_F) - e_{t+i} (1 + \tau_F) F_{F,t+i} (z_F) - e_{t+i} (1 + \tau_F) P_{F,t+i} (z_F) - N_{t+i} P_{C,t+i} (z_F) - e_{t+i} (1 + \tau_F) P_{O,t} O_{d,t} (z_d) \right\},$$

(63)

subject to the domestic demand for variety $z_F$ (52). $\tau_F$ is import tax on this type of foreign goods. The first-order condition in this case is the following:

$$P_{F,t}^{\text{op}} = \frac{\epsilon_F}{\epsilon_F - 1} E_t \left\{ \sum_{i=0}^{\infty} (\phi_F)^i \Lambda_{t+i} e_{t+i} (1 + \tau_F) F_{F,t+i} (z_F) - e_{t+i} (1 + \tau_F) P_{F,t+i} (z_F) - N_{t+i} P_{C,t+i} (z_F) - e_{t+i} (1 + \tau_F) P_{F,t+i} (z_F) \right\}. \quad (64)$$

Under flexible prices the optimal price charged by an intermediary $z_F$ is just a mark-up over the price of variety $z_F$ abroad:

$$P_{F,t}^{\text{flex}} (z_F) = \frac{\epsilon_F}{\epsilon_F - 1} e_{t} (1 + \tau_F) P_{F,t}^{*}. \quad (65)$$
2.10 Government

We assume the government to be composed by the fiscal and monetary authorities. The government’s budget, therefore, corresponds to the consolidated budget of both authorities.

2.10.1 Budget constraint

The only asset owned by the government is its share in the commodity exporting company $(S)$. Government liabilities are public bonds held by the private sector, and money. The consolidated budget constraint of the government, expressed in terms of units of the final consumption good, is given by:

$$
\frac{P_{G,t}}{P_{C,t}} G_t - Tax_t - \frac{\Pi_t (S, G)}{P_{C,t}} =
N_t \int \frac{\tilde{M}_t(j) - \tilde{M}_{t-1}(j)}{P_{C,t}} dj + N_t \int \frac{\tilde{B}_t(j)(1 + i_t) - \tilde{B}_{t-1}(j)}{P_{C,t}} dj
$$

(66)

where $\Pi_t (S, G)$ corresponds to the share of the government in the profits of the commodity exporting company. $Tax_t$ are real net tax revenues of the government:

$$
Tax_t = -N_t \tilde{\tau}_t + \frac{\tau_C}{1 + \tau_C} N_t \int \tilde{C}_t(j) dj + \tau_S \left( \frac{P_{S,t}}{P_{C,t}} F^S(K_t, K_{S,t}) - \delta_S \frac{P_{I_{S,t}}}{P_{C,t}} K_{S,t} \right)
$$

$$
- \delta_D \frac{P_{I_{D,t}}}{P_{C,t}} K_{D,t} + \tau_D \left( \frac{Z_t}{P_{C,t}} u_{D,t} K_{D,t} - \delta_D \frac{P_{I_{D,t}}}{P_{C,t}} K_{D,t} \right)
$$

$$
+ N_t \int \tau_L \frac{W_t(j)}{P_{C,t}} l_t(j) dj + \tau_F c_t P_{F,t} Y_{F,t} + \tau_O c_t P_{O,t} O_t
$$

In the expression above, $Y_{F,t}$ is the total amount of imported goods and $O_t$ is the total amount of imported oil (or energy goods/inputs). $\tau_O$ is a tax on imported oil.

2.10.2 Fiscal policy

Fiscal policy is defined as a stochastic process for real public expenditure:

$$
G_t = G(1 - \rho_G) G^{\rho_G} (G_{t-1})^{\rho_G} (\Omega_t N_t)^{1 - \rho_G} \exp(\varepsilon_{gt}),
$$

(67)

where $E_{t-1} \varepsilon_{gt} = 0$. We assume that net transfers $\tilde{\tau}_t$ are adjusted continuously so as to satisfy the budget constraint (66), in response to either a public expenditure shock or a change in the interest rate induced by the monetary policy discussed below.
2.10.3 Monetary policy

We assume that the monetary policy could be modelled as a Taylor rule:

\[ 1 + i_t = (1 + \dot{i})^{1-\phi_i} E_t \left[ (1 + \dot{i}_{t-1})^{\phi_i} \left( \frac{P_{C,t+1}}{P_{C,t}} \right)^{(1-\varphi_i)\varphi_y} \left( \frac{Y_t}{\bar{Y}_t} \right)^{(1-\varphi_i)\varphi_y} \right] \exp(\nu_{m,t}) \]  

where \( \bar{Y}_t \) can be specified as flexible prices GDP or the balanced growth path GDP, and where \( \nu_{m,t} \) is a monetary policy shock that represents deviations from the policy rule.

2.11 Foreign agents

Foreign agents demand the commodity good \( S \) and intermediate tradable goods produced domestically. The demand for the commodity good is completely elastic at the price \( P_{S,t}^{*} \).

Foreign demand for domestically produced tradable intermediate goods depends on the relative price of this type of goods and foreign aggregate demand. In particular, we assume that foreign demand for goods \( H \), \( Y_{H,t}(F) \), is given by

\[ Y_{H,t}(F) = \zeta^* \left( \frac{P_{H,t}^*}{P_{F,t}^*} \right)^{-\eta^*} C_t^* \]

where \( \zeta^* \) corresponds to the share of domestic intermediate goods in the consumption basket of foreign agents, and where \( \eta^* \) is the price elasticity of the demand function.

We assume that the law of one price holds for exports of intermediate goods. Then, we have that the demand for domestic intermediate tradable goods can be written as

\[ Y_{H,t}(F) = \zeta^* \left( \frac{P_{H,t}^*}{P_{F,t}^*} \right)^{-\eta^*} RER_t^{\eta^*} C_t^* , \]

where \( RER_t \) is the real exchange rate. This variable is defined simply as the relative price of the foreign consumption basket, \( P_{F,t}^* \), relative to the price of the domestic consumption basket,

\[ RER_t \equiv \frac{e_t P_{F,t}^*}{P_{C,t}^*} . \]
2.12 General Equilibrium

2.12.1 Factor markets equilibrium

Total demand for capital corresponds to the sum of the demands for capital in sectors $H$ and $N$. Capital stock supply at any moment $t$ is given by (45). The equilibrium condition is, therefore,

$$u_{D,t} \tilde{K}_{D,t} = K_{H,t} + K_{N,t}$$  \hspace{1cm} (71)$$

Total labor demand corresponds to the sum of labor demands from sectors $H$ and $N$. Labor supply is given by the aggregation of (2) over the continuum of households. The equilibrium condition is

$$L_t = L_{H,t} + L_{N,t}.$$  \hspace{1cm} (72)$$

2.12.2 Aggregate resources constraint and current account

The aggregate resources constraint is obtained by aggregating the budget constraint over the continuum of households. Combining the aggregate budget constraint of the private sector and the budget constraint of the government (66) we obtain the following expression for the aggregate resource constraint of the economy:

$$\frac{P_{G,t}}{P_{C,t}} G_t + \frac{e_t P_{F,t}^* B^*_{t-1}}{P_{C,t}} \left(1 + i^*_t\right) \Theta \left(B^*_t\right) = \frac{e_t P_{F,t}^* B^*_{t-1}}{P_{C,t}} + \frac{W_t}{P_{C,t}} L_t + \frac{\Pi_t}{P_{C,t}} - C_t,$$  \hspace{1cm} (73)$$

where $\Pi_t = \Pi_t(S, G) + \Pi_t(H) + \Pi_t(N) + \Pi_t(F) + \Pi_t(ID)$ corresponds to total aggregate profits received by domestic agents (households and government) from different firms in the economy. We assume that the government owns a share $\kappa$ of the commodity exporting firm. The rest is owned by foreign investors.$^{18}$

Substituting the corresponding expressions for profits into (73) and rearranging terms we obtain the following expression for the current account of the economy:

$$e_t P_{F,t}^* \frac{B^*_t}{P_{C,t}} \left(1 + i^*_t\right) \frac{B^*_{t-1}}{P_{F,t}^*} \left(RER_t B^*_t / P^*_t\right) - \frac{e_t P_{F,t}^* B^*_{t-1}}{P_{C,t}} \left(\frac{P_{F,t}^*}{P_{F,t}^* - 1}\right) + \frac{\Pi_t(S, F)}{P_{C,t}} X_t - \frac{P_{M,t}}{P_{C,t}} M_t$$  \hspace{1cm} (74)$$

$^{18}$ $\Pi_t(S, G)$ is defined as $(P_{S,t} Y_{S,t} - P_{I_{S,t} I_{S,t}} - (1 - \kappa) \Pi_t(S)$, where $\Pi_t(S)$ is given by the expression described in section 2.3 that includes capital income taxes. In other words, the profits of the government in the commodity exporting sector consider the collected taxes on the profits of foreign investors.
where $\Pi_t(S, F) = (1 - \kappa) \Pi_t(S)$ is the share of foreign investors in the profits of the firm exporting the commodity. If $\Pi_t(S, F) > 0$, then there are remittances by foreign investors, while if $\Pi_t(S, F) < 0$ then there is a net foreign investment flow into this sector.

Total value of exports and imports is given by

$$P_{X,t}X_t = e_t P_{S,t}^* Y_{S,t} + P_{H,t} Y_{H,t}(F)$$

$$P_{M,t} M_t = e_t P_{F,t}^* Y_{F,t} + e_t P_{O,t}^* (O_t)$$

where $Y_{F,t} = Y_{F,t}(C) + Y_{F,t}(G) + Y_{F,t}(IS) + Y_{F,t}(ID)$ and $O_t = C_{O,t} + O_{H,t} + O_{N,t}$.

Notice that we do not have a direct measure of real exports and imports since each is composed by different types of goods. Therefore, we compute “real” exports and imports by measuring them with reference to a particular base year:

$$X_t = e_0 P_{S,0}^* Y_{S,0} + P_{H,0} Y_{H,0}(F), \quad M_t = e_0 P_{F,0}^* Y_{F,0} + e_0 P_{O,0}^* O_t.$$  \hspace{1cm} (75)

The corresponding implicit deflator is defined as $P_{X,t} = \frac{P_{X,t} X_t}{X_t}$, and $P_{M,t} = \frac{P_{M,t} M_t}{M_t}$.

### 2.1.2.3 Gross domestic product

Using the expression for the current account above we can define the following relationship

$$P_t Y_t = P_{C,t} C_t + P_{I,t} I_t + P_{G,t} G_t + P_{X,t} X_t - P_{M,t} M_t$$  \hspace{1cm} (76)

where $P_t Y_t = P_{S,t} Y_{S,t} + P_{H,t} Y_{H,t} + P_{N,t} Y_{N,t} - e_t P_{O,t}^* (O_{H,t} + O_{N,t}) + \left(P_{F,t} - e_t P_{F,t}^* \right) Y_{F,t} + \tau_C P_{C,t} C_t/(1 + \tau_C) + \tau_F e_t P_{F,t}^* Y_{F,t} + \tau_O e_t P_{O,t}^* O_t$ corresponds to nominal GDP, measured as the sum of value added in each domestic sector plus consumption and import taxes collected by the government. To obtain a measure of real GDP it is necessary to define a base year. Let $t = 0$ be that particular year. Normalizing $P_0 = 1$ we have that real

---

\footnote{Strictly speaking, $Y_{F,t}$ should include a multiplicative term not necessarily different from one outside of the steady state. This term appears due to the distortions of prices of the imported varieties. However, this term is omitted since it is zero in a first-order log-linear approximation.}
GDP (GDP measured at constant prices of the base year) is given by

\[ Y_t = P_{S,0}Y_{S,t} + P_{H,0}Y_{H,t} + P_{N,0}Y_{N,t} - e_0 P_{O,0}^* (O_{H,t} + O_{N,t}) + (P_{F,0} - e_0 P_{F,0}^*) Y_{F,t} \]

\[ + \tau_C C_{t,0}/(1 + \tau_C) + \tau_F e_0 P_{F,0}^* Y_{F,t} + \tau_O e_0 P_{O,0}^* O_{F,t} \]

Finally, GDP implicit deflator is defined by \[ P_t^d = \frac{P_t Y_t}{Y_t}. \]

### 2.12.4 Exogenous process

The exogenous variables evolve according to the following stochastic process:

\[
\begin{align*}
\frac{P_{S,t}^*}{P_{F,t}^*} & = \left( \frac{P_{S}^*}{P_{F}^*} \right)^{1 - \rho_S} \left( \frac{P_{S,t-1}^*}{P_{F,t-1}^*} \right)^{\rho_S} e^{\varepsilon_{st}} \\
G_t & = G^{(1 - \rho_G)} G^{\rho_G} (G_{t-1})^{\rho_G} (\Omega_{t} N_{t})^{1 - \rho_G} \exp(\varepsilon_{gt}) \\
A_{N,t} & = (A_N)^{1 - \rho_N} A_{N,t-1}^{\rho_N} e^{\varepsilon_{Nt}} \\
A_{H,t} & = (A_H)^{1 - \rho_H} A_{H,t-1}^{\rho_H} e^{\varepsilon_{Ht}} \\
(1 + \pi_{F,t}^*) & = (1 + \pi_{F,t-1}^*)^{1 - \rho_{\pi}^*} \left(1 + \pi_{F,t-1}^*\right)^{\rho_{\pi}^*} \exp(\varepsilon_{\pi,t}) \\
\frac{P_{O,t}^*}{P_{F,t}^*} & = \left( \frac{P_{O}^*}{P_{F}^*} \right)^{1 - \rho_O} \left( \frac{P_{O,t-1}^*}{P_{F,t-1}^*} \right)^{\rho_O} e^{\varepsilon_{ot}} \\
C_t^* & = (C^*)^{1 - \rho_C} (C_{t-1}^*)^{\rho_C} e^{\varepsilon_{ct}} \\
(1 + i_t^*) & = (1 + i_{t-1}^*)^{1 - \rho_i^*} \left(1 + i_{t-1}^*\right)^{\rho_i^*} e^{\varepsilon_{it}} \\
\exp(\nu_{m,t}) & = \exp(\rho_m \nu_{m,t-1} + \varepsilon_{m,t}) 
\end{align*}
\]

It is assumed that \( E_{t-1} \varepsilon_{vt} = 0 \) for \( v = s, g, N, H, \pi^*, o, c, i, m \).

### 3 Parameterization and solution

To solve the model we first solve for the non-stochastic steady state by using numerical methods. Then we solve the log-linearized decision rules from the behavioral equations of the model. We use the \( QZ \) factorization described in Uhlig (1997).
Table 2 presents the chosen values for the structural parameters of the model. Many of these parameters were taken directly from the existing DSGE literature. Others were chosen so as to match long-run features of the Chilean economy.

We assume an annual long-run labor productivity growth of 3.5%. The working age population grows 1% per year. The annual inflation rate is 3%, which is consistent with the target value defined by the Central Bank of Chile in 1999. These values imply an annual long run nominal interest rate of about 7.5%.

For the benchmark case we consider a log utility with habit formation in consumption:

\[
\tilde{U} = \frac{\mu}{\mu - 1} \log \left( a(\tilde{C}_t(j) - bH_t)^{\frac{\mu - 1}{\mu}} + (1 - a)(\tilde{M}_t(j)/P_c^t)^{\frac{\mu - 1}{\mu}} \right) - \psi l_t(j)^{1+\sigma_L} \frac{1}{1 + \sigma_L}
\]

where \( H_t = \tilde{C}_{t-1} \) is the external habit in consumption. Parameter \( b \) controls the level of inertia in consumption. For the base parameterization, \( b \) is set equal to 0.8. We also analyze a case where there is no habit in consumption (\( b = 0 \)). \( \sigma_L \) is the inverse of the labor supply elasticity, which is set equal to 1. Notice that a unitary labor supply elasticity is lower than traditional values used in the U.S. real business cycle models. However, this value is in line with microeconomic estimates for Chile.\(^{20}\) The remaining parameters of the preferences are related with money demand.\(^{21}\)

To calibrate the consumption bundle we use the shares deducted from the official CPI basket. Oil and energy consumption account for approximately 4% of the total basket. Amongst the components of core consumption, 50% correspond to non-tradable goods, 25% is domestic tradable goods, and the remaining 25% are imported goods. The elasticity of substitution between non-tradable and tradable consumption goods is 1.0, while the elasticity of substitution between domestic tradable and imported consumption goods is a little higher (1.5).

When we include labor wage stickiness, we assume that wages are reset optimally every two years on average. Given the Calvo setting, this implies that roughly 87.5% of

\(^{20}\)See Mizala and Romaguera (1999).

\(^{21}\)Parameters \( a \) and \( \mu \) are chosen to match money demand semi-elasticity and long run consumption base money velocity. The calibrated value of \( a \) is very close to 1, which allow us to abstract from the effects of real balance fluctuations in the marginal value of consumption.
the workers set their wages considering past consumption price inflation.\textsuperscript{22}

We calibrate the share of the commodity exporting sector in the total of exports to 40%, which resembles the share of copper exports in total exports for the Chilean economy. The foreign debt as percentage of GDP is calibrated to 30%, which is also consistent with Chilean data.\textsuperscript{23} Government spending is calibrated to 11% of GDP.

For the commodity exporting sector, we assume that the share of natural resources on production is about 35%. The annual depreciation rate of the capital stock is set to 7.2%. We suppose that the elasticity of substitution between domestic tradable and imported goods for investment in this sector is low (we assume a value of 0.2).

We assume that government consumes only non-tradable goods. This assumption defines the parameters of the government’s consumption bundle. For the production function of the capital good utilized by intermediate producers, we assume that the elasticity of substitution between non-tradable and tradable goods is 0.5, while the one between domestic tradable and imported goods is fixed at 0.5. The annual depreciation rate of this type of capital is 5.8%.\textsuperscript{24}

Producers of domestic tradable and non-tradable intermediate goods have access to a constant return technology. We assume that the non-tradable intermediate goods technology is relatively more labor-intensive than the technology utilized by tradable goods producers. In particular, we set $\eta_H$ equal to 0.40 while $\eta_N$ is equal to 0.30. We also assume a low share of imported inputs (oil) in both domestic sectors ($\upsilon = 0.02$) and a low degree of substitution of this input with the labor and capital ($\theta_H = \theta_N = 0.3$).

Price stickiness parameters are set such that intermediate goods producers and retailers of imported goods reset optimally their prices every four quarters on average, while the indexation to past inflation is 50%, and the remaining 50% is based on the inflation target.

\textsuperscript{22}see Jadresic, 2002, footnote 9.
\textsuperscript{23}The total external debt of the Chilean economy is currently 50% of its annual GDP. However, the central bank holds reserves of about 20% of GDP. Therefore, the net position of the country is about 30% of annual GDP.
\textsuperscript{24}The depreciation rate is lower than for the capital in the commodity export sector because of the composition of the investment goods.
Finally, the foreign demand elasticity for domestic tradable goods is set to 0.5, and the upward sloping supply of external funds (function $\Theta(.)$) is assumed to be very elastic: 

$$\Theta'(.) (eB^*/P_X X)/\Theta(.) = 0.01.$$  

This choice of parameters generates several aggregate steady state statistics that can be compared to the Chilean data (see Table 3). The labor share in the model reaches a value of 46.4% that looks low compared to the U.S., but is in line with the Chilean data. Private consumption represents 60% of GDP, which is only 3% below the empirical counterpart (however, non-durable consumption is 59% of GDP in the Chilean data). Both exports (26.5%) and investment (25%) as percentage of GDP in the model are similar to the Chilean values. Net exports as percentage of GDP are 2.6% higher than in the historical data over the last 15 years. Oil (energy) imported goods are 13.5% of the total of imports in the model while in the data oil accounts for 10% of total imports.

Our model has a non-tradable good sector that is a little smaller than the non-tradable sector observed in the Chilean economy. Also, the non-commodity tradable sector is slightly overestimated, and the level of imports is lower than in the data. Overall, however, the figures from the model are very close to their empirical counterparts.

4 Impulse response analysis

In this section we analyze the impulse-response functions of the model to three types of exogenous fluctuations: (i) shock to the US$ price of the exported commodity good; (ii) shock to the US$ price of imported input (oil); and (iii) shock to the foreign nominal interest rate. For each shock we compute the responses of a selected set of variables under four different specifications of the model. The benchmark case corresponds to the full blown model described in section 2. The other three cases are (a) no habit formation in consumption; (b) flexible wages; and (c) no adjustment cost in investment. For this last case we consider a specification with quadratic adjustment in capital (a Tobin’s Q.

$^{25}$Since we introduce this function only to generate well defined dynamics around the steady state (and to realistically model the risk premium), we want to mimic the result that would be obtained in the case of purely exogenous international interest rate for a small open economy.
model) and a constant utilization rate for capital.

4.1 Exported commodity good price shock

Figures 2 and 3 show the responses of a selected set of variables to a 10% innovation in the price of the exported commodity good. Since the government owns a stake in the firm that exports this commodity, the shock implies an increase in fiscal revenues. Moreover, given that government spending is fixed, lump-sum taxes are decreased in order to keep the government’s budget balanced. Therefore, private disposable income and private consumption rise after the shock.\(^{26}\) As expected, when the model does not consider habit formation in consumption this variable jumps up on impact. For the other cases we observe the typical hump-shaped response to the shock.

The expansion in aggregate demand has a positive impact on total GDP. Investment increases for two reasons. On the one hand, the expansion in output raises the marginal productivity of capital, which stimulates capital accumulation. On the other, the shock produces a real appreciation of the exchange rate. Since capital goods are mainly assembled from imported intermediate goods, this real appreciation makes investment cheaper. Notice that to obtain a hump-shaped response of output it is necessary to include investment adjustment costs in the model. Habit formation in consumption is not enough by itself to get such a pattern.

Notice also that the shock produces what has been termed Dutch disease. The real appreciation of the exchange rate comes together with a reduction in the quantity of non-commodity exports (domestic intermediate tradable goods). Exports of the commodity good increase (not shown in the figure) but the negative effect of the shock on non-commodity exports dominates. Therefore, total exports fall. On the other hand, both the expansion in investment and consumption raises the demand for imports.

Except for the model with flexible wages, labor (hours) and real wages increase after the shock. The rise in real wages is explained by both a reduction in the labor supply

\(^{26}\)The Ricardian equivalence holds in our model. Therefore the precise timing pattern of the government deficits and surpluses does not matter for household’s decisions.
and an increase in the labor demand after the shock. The expansion in the labor demand is associated with the increase in the capital stock—that raises the marginal product of labor. The contraction in labor supply is due to the increase in consumption that reduces the incentive to work (income effect). When wages are sticky, a certain fraction of the workers—those that can not adjust their wages—must supply any quantity of labor demanded at the given wage rate. Therefore, in this case the income effect is diminished and the response of labor to the shock is increased. When wages are flexible a more significative part of the labor market adjustment to the expansion in labor demand is absorbed by wages, and the response of labor is dampened. In fact, when wages are flexible the equilibrium number of hours worked falls initially after the shock. Consistently with the previous observation, the response of output to the shock is diminished when the model considers flexible wages.

Despite the increase in real wages, the shock causes a reduction in inflation. Although the increase in the real wage causes inflation in both domestic intermediate sectors to rise (not shown), there is a nominal appreciation of the exchange rate that pushes down foreign goods inflation. This last effect dominates the response of CPI inflation to the shock.

4.2 Imported input (oil) price shock

Figures 4 and 5 display the responses to a 10% innovation in the price of the imported input (US$ oil price). The shock has a negative income effect on households through two channels. First, profits from domestic intermediate goods producers fall because they utilize oil as an input—for which the demand is relatively inelastic. Second, oil consumption is more expensive in terms of domestic production. Also, this shock induces an increase in the real interest rate which produces a inter-temporal substitution effect on consumption. Both effects tend to reduce aggregate consumption. The fall in consumption lowers GDP and this, in turn, depresses investment. Notice that the effect of the shock on output gets considerably larger when habit formation or adjustment cost
for investment is removed. Otherwise, the effect on output is rather small.\footnote{Empirical evidence for Chile shows that this type of shocks have low impact on output (Franken, et al. (2004)).}

Exports initially fall in response to the shock. This is consistent with the real appreciation of the exchange rate that occurs on impact. However, after a few quarters the real exchange rate depreciates and exports grow above their steady-state level. To understand why there is a real depreciation of the currency after some periods, it is important to remember that our parameterization implies a very low degree of substitution of oil in the consumption bundle. Therefore, the demand for domestically produced varieties falls in line with the reduction in oil consumption. As a consequence the price of domestic intermediate tradable goods relative to the international price level tends to decrease, moving up the real exchange rate.\footnote{We do not include the potential effect on foreign inflation after the increase in the imported input (oil) price.}

In general, this shock implies a reduction in the real wage because of both a contraction in labor demand and an increase in the labor supply. The latter can be explained by the income effect associated with the fall in consumption. As discussed before, under flexible wages this income effect is bigger and the expansion in labor supply tempers the fall in employment with a larger reduction in real wages. Initially, CPI inflation rises both as a direct consequence of the oil price shock, and because of the rise in inflation of domestically produced goods. However, since this shock is transitory, the decrease of the oil price back to its steady state dominates over the more persistent response of core inflation (not shown in the figure). This results in a negative effect on CPI inflation after one quarter.

### 4.3 Foreign interest shock

Figures 6 and 7 present the responses to a shock on the foreign interest rate equivalent to an increase of 100 basis points (annualized).

Since households are net debtors in the international capital markets, this shock implies a contraction on households’ disposable income in the short run. At the same
time, the interest rate differential between domestic and foreign interest rates becomes negative after the shock. The nominal exchange rate depreciates on impact and, given the stickiness of domestic prices, there is a real depreciation of the domestic currency. Imported goods get more expensive in terms of domestic produced goods. For both reasons, current consumption has to decrease in response to the shock.

The rise of the domestic interest rate and the perspectives of less income in the short run reduces the incentive to invest. Moreover, the observed nominal devaluation makes investment goods relatively more expensive because they are more intensive in imported intermediate goods than the final consumption good. In equilibrium, we observe a contraction in the marginal productivity of capital and a reduction in investment.

The real depreciation of the exchange rate generates a rise in exports. This expansion in exports is quantitatively strong and occurs on impact, which may be at odds with the data.\textsuperscript{29} Imports, in turn, fall because of the contraction in consumption and investment. The fall in import is reinforced through the expenditure switching effect associated with the real depreciation of the currency.

Since aggregate demand falls, labor demand shrinks. However, the contraction in consumption drives up the labor supply. These two effects unambiguously move real wages down. Except for the case with flexible wages, the reduction in labor demand is larger than the expansion in labor supply. This, in turn, induces a contraction in employment (hours worked). In the case of flexible wages, the fall in consumption generates a more significative income effect in the labor supply, which implies an expansion of employment in the short run.

The response of inflation to this shock is positive. First, the observed nominal devaluation increases inflation of imported goods. Second, CPI inflation jumps up on impact due to the rise of oil price measured in domestic currency.

Despite labor share and real wage fall in response to the shock, the change in relative prices makes the relevant marginal cost for domestic firms producing intermediate goods

\textsuperscript{29}Although we have no robust evidence on the impact of the exchange rate on exports, we think that the response of exports to movements in the real exchange rate should follow an hump-shape pattern. We are currently working on an adjustment mechanism to include such a feature.
to rise. On the one hand, the domestic currency price of oil increases. Given the low degree of substitution between this input and the other factors in production, this price increase drives costs up. On the other hand, when the wages are sticky they are more rigid than domestic goods prices. Therefore wages measured in terms of the domestic goods prices on impact fall by less than when measured in terms of CPI. The increment in the marginal cost for domestic firms means that core inflation also rises in response to the shock.

In the case of flexible wages, there is a greater reduction in nominal wages, which drives real wages down further. This effect generates a deeper reduction in cost for domestic firms. This translates into a negative core inflation in the short run (not shown in the figure).

5 An Empirical Evaluation of the Model

We proceed to empirically evaluate the model following the RBC literature. We calculate cross correlations of the variables with GDP at various leads and lags. The results are compared to the empirical ones reported by Restrepo and Soto (2004).

To compute the moments of the variables we consider simultaneously all the shocks of the model. To compute the covariance matrix of the innovation to those shocks, we first estimate an autoregressive process for each of the exogenous variables. We use quarterly data of oil price, foreign inflation, foreign output (weighted average of output from the main trade partners), copper price, and foreign interest rate (Libor). The underlying identification assumption for the innovations is that the Chilean economy is small enough to consider all those variables as exogenous. This assumption seems quite plausible. The other four shocks in the model (i.e. government expenditure shock, monetary policy shock, and productivity shocks in the tradable and non-tradable sectors) are not easily identified from the data. As a first approach, we take as exogenous government expenditure. This assumption is clearly not realistic. However, at this point there are no good empirical works trying to identify exogenous expenditure shocks for Chile. Moreover, in the model we are actually assuming the government decides its expenditure profile in a
completely exogenous way. Productivity shocks in each sector are taken to be proportional to total factor productivity, as estimated in CBC (2003). Finally, we use a series of monetary policy shocks identified from an estimated interest rate rule by Caputo and Liendo (2005).30

We detrend each series by using an HP filter and estimate an AR(1) process for each one of them.31 The covariance matrix of the residuals is shown in Table 1.32

Table 1: Covariance matrix exogenous processes

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_s$</th>
<th>$\epsilon_g$</th>
<th>$\epsilon_N$</th>
<th>$\epsilon_H$</th>
<th>$\epsilon_o$</th>
<th>$\epsilon_i^*$</th>
<th>$\epsilon_i^*$</th>
<th>$\epsilon_m$</th>
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<tr>
<td>$\epsilon_s$</td>
<td>73.17</td>
<td>0.36</td>
<td>0.00</td>
<td>0.00</td>
<td>3.33</td>
<td>30.74</td>
<td>0.97</td>
<td>1.11</td>
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<tr>
<td>$\epsilon_g$</td>
<td>0.36</td>
<td>0.17</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.16</td>
<td>0.31</td>
<td>0.00</td>
<td>-0.02</td>
</tr>
<tr>
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<td>1.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\epsilon_H$</td>
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<td>1.05</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$\epsilon_i^*$</td>
<td>30.74</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>2.15</td>
<td>118.73</td>
<td>1.12</td>
<td>0.02</td>
</tr>
<tr>
<td>$\epsilon_i^*$</td>
<td>0.97</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>1.12</td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>$\epsilon_m$</td>
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<td>-0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>0.02</td>
<td>0.08</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Figures 8 to 11 display the correlations with current output of a selected group of variables at various leads and lags. Figure 8 corresponds to the model with flexible wages. The pattern exhibited by output in the model is quite similar to the one in the data. Both consumption and investment tend to lead output in the model whereas in the data these two variables seem to move after output. As a consequence net exports also appear to be leading output in the model. The correlation pattern of employment is similar to the one in the data, but the magnitudes of such correlations are lower in the model. In the case of real wages, the model delivers a much stronger contemporaneous correlation than in the data, although the correlation pattern at various leads and lags

30 We thank Rodrigo Caputo for providing the series with the identified monetary policy shocks.
31 Results do not change significantly if we instead estimate a VAR(1) with all the variables.
32 The covariance of the productivity shock and the other shocks is not well defined since we do not have independent measures of TFP for each sector. We just assumed that productivity shocks are uncorrelated to other shocks in the model.
is, in general, similar.

Notice that the model is unable to generate a contemporaneous negative correlation between the real exchange rate and output, as observed in the data. Moreover, the contemporaneous correlation between these two variables in the model is not only positive but also quite strong. The model also fails to deliver a negative correlation between output and inflation contemporaneously. More generally, the correlation between output and inflation at various lead and lags is very small in the model when compared with the data.

Figure 9 shows the case of the model without adjustment cost for investment. In this case, output becomes less persistent, making its autocorrelation at various leads and lags much weaker than in the data. Also, the correlation between current output and future investment (two leads) becomes negative, which is also at odds with the data. The correlation pattern of consumption follows the data closely. In this case, consumption continues to be as persistent as in the previous case. However, since output is now much less persistent, the correlation pattern of the model resembles now much more the data.

The contemporaneous correlation between employment and output is too strong in the model, while the pattern exhibited by real wages diverts from the one in the data. Again, the model fails to generate a negative correlation between the real exchange rate and output.

Figure 10 presents the correlations for the case of the model without habit in consumption. In this case, the correlation between current output and future consumption becomes much weaker (even negative) while the correlation pattern of investment follows more closely the data than the previous two cases. Again, the contemporaneous correlation between output and employment is too strong, and the model fails to replicate the pattern depicted by real wages. Although the contemporaneous correlation between inflation and output in the model resembles the data closely and the sign of the correlations at various leads and lags is the same as in the data, the magnitude of such correlations is much smaller in the model.

Finally, figure 11 displays the benchmark model as described in section 2. In this case
the correlation patterns of almost all variables resemble very much like in the flexible wages model. However, in this case we observe a much stronger correlation between employment and output in the model, the correlation pattern between past real wages and output gets closer to the data, and so does the correlation pattern between inflation and output.

6 Conclusions and directions for further work

This paper presents the Model for Analysis and Simulations (MAS), a new DSGE model for the Central Bank of Chile. This version of the model includes the following features: prices and wages are sticky, there are adjustment costs in investment, the exchange rate pass-through to import prices is imperfect in the short run, and there is habit formation in consumption. On the supply side, this version of the model includes three main productive sectors: tradables, non-tradables and a commodity export sector.

The paper presents an analysis of the impulse-response functions of a selected group of variables to three exogenous shocks, namely a shock to the price of an export commodity good, an oil price shock, and a foreign interest rate shock. The model is empirically evaluated by comparing the cross correlations between output and some variables at various leads and lags, with the corresponding moments in the data.

In general, the model is able to replicate the signs of the correlations between output and a set of variables at various leads and lags. However, the model exhibits correlations of current output with past investment and consumption that are stronger than in the data. At the same time, the correlation of output with future values of those variables in the data seems to be stronger than in the model.

A major weakness of the model is that it fails to replicate the negative correlation between output and the real exchange rate observed for the Chilean economy. The model also has to be improved to better capture the precise pattern correlation of output and inflation at different leads and lags.

For a small open economy like Chile, exogenous international capital inflows have played an important role in determining business cycle fluctuations. In this version of
the model, all fluctuations in international capital flows are reflected in the uncovered interest parity condition (UIP). However, it is well known that in macro data the UIP hypothesis fails. Moreover, in the model exogenous fluctuations in the risk premia are observationally equivalent to changes in the foreign interest rate. For the baseline parameterization we used the empirical fluctuation in the Libo rate as a proxy of the variance of both the exogenous risk premium and the foreign interest rate. It is clear that by taking the Libo rate as our proxy we are leaving outside an important determinant of capital inflows, and of the business cycle. Given the correlation pattern of the real exchange rate and output in response to this type of shock, improving this building block of the model could be a valuable step forward.

Respect to the strong correlation between current output and future consumption and investment, the data suggest a sort of propagation mechanism from output to those two variables not incorporated in the model. For the case of investment, balance sheet effects could play a role to introduce persistency and a feedback effect from output to investment. For the case of private consumption, the existence of “hand to mouth” consumers could be an important element to be considered in subsequent versions of the model.
References


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<td>$n$</td>
<td>0.25%</td>
<td>Population growth rate [quarterly]</td>
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<tr>
<td>$g$</td>
<td>$g.y$</td>
<td>0.86%</td>
<td>Labor productivity growth [quarterly]</td>
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<td>$\pi_C$</td>
<td>$\pi_c$</td>
<td>0.74%</td>
<td>Inflation rate [quarterly]</td>
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**Homes preferences and wage setting**

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<td>0.9975</td>
<td>utility discount factor</td>
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<tr>
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<td>$\mu$</td>
<td>0.2</td>
<td>elasticity of money demand to $i/(1 + i)$</td>
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<tr>
<td>$\sigma$</td>
<td>$\sigma$</td>
<td>1.0</td>
<td>elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>$\sigma_L$</td>
<td>1.0</td>
<td>inverse of labor supply elasticity</td>
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<tr>
<td>$\epsilon_L$</td>
<td>$\epsilon_L$</td>
<td>26</td>
<td>elasticity of substitution among labor services varieties</td>
</tr>
<tr>
<td>$\phi_L$</td>
<td>$\phi_L$</td>
<td>0.875</td>
<td>probability of reoptimizing wages</td>
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<tr>
<td>$\kappa_W$</td>
<td>$\kappa_W$</td>
<td>1.0</td>
<td>wage indexation on inflation</td>
</tr>
<tr>
<td>$\tau_L$</td>
<td>$\tau_L$</td>
<td>10%</td>
<td>labor income tax</td>
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</table>

**Commodity export sector**

<table>
<thead>
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<th>Parameter</th>
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<th>Value</th>
<th>Description</th>
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</thead>
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<tr>
<td>$\alpha_S$</td>
<td>$\alpha_S$</td>
<td>0.65</td>
<td>capital share in commodity export sector</td>
</tr>
<tr>
<td>$\theta_S$</td>
<td>$\theta_S$</td>
<td>0.20</td>
<td>elasticity of substitution between NNRR and $K_S$ in $Y_S$</td>
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<tr>
<td>$\gamma_{I_S}$</td>
<td>$\gamma_{I_S}$</td>
<td>0.2</td>
<td>domestic tradable goods share in $I_S$</td>
</tr>
<tr>
<td>$\eta_{I_S}$</td>
<td>$\eta_{I_S}$</td>
<td>0.3</td>
<td>elasticity of substitution between $Y_H(I_S)$ and $Y_F(I_S)$</td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>$\delta_S$</td>
<td>1.85%</td>
<td>depreciation rate of $K_S$ (quarterly)</td>
</tr>
<tr>
<td>$\Phi_S^{(1)}$</td>
<td>$\Phi_S^{(1)}$</td>
<td>2.0</td>
<td>second derivative investment adj. cost</td>
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<tr>
<td>$\delta_S^{(1)}/\delta_S^{(1)}$</td>
<td>$\delta_S^{(1)}/\delta_S^{(1)}$</td>
<td>20.0</td>
<td>marginal cost of utilization elasticity</td>
</tr>
<tr>
<td>$\tau_S$</td>
<td>$\tau_S$</td>
<td>10%</td>
<td>capital income tax in sector $S$</td>
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**Consumption goods assembly**

<table>
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<th>Description</th>
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<td>$\alpha_C$</td>
<td>$\alpha_C$</td>
<td>0.96</td>
<td>Core consumption share</td>
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<tr>
<td>$\psi_C$</td>
<td>$\psi_C$</td>
<td>0.05</td>
<td>elasticity of substitution between oil and core consumption</td>
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<tr>
<td>$\eta_C$</td>
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<td>elasticity of substitution between $Y_T(C)$ and $Y_N(C)$</td>
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<tr>
<td>$\gamma_C$</td>
<td>$\gamma_C$</td>
<td>0.5</td>
<td>tradable goods share in $C$</td>
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<tr>
<td>$\theta_C$</td>
<td>$\theta_C$</td>
<td>1.5</td>
<td>elasticity of substitution between $Y_H(C)$ and $Y_F(C)$</td>
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<tr>
<td>$\omega_C$</td>
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<td>0.5</td>
<td>domestic tradable goods share in $Y_T(C)$</td>
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<td>$\tau_C$</td>
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<td>15%</td>
<td>consumption tax</td>
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**Retail assembly**

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<td>elasticity of substitution between $z_H$ varieties in $H$</td>
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<tr>
<td>$\epsilon_F$</td>
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<td>elasticity of substitution between $z_F$ varieties in $F$</td>
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<td>$\epsilon_N$</td>
<td>$\epsilon_N$</td>
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<td>elasticity of substitution between $z_N$ varieties in $N$</td>
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**Government consumption assembly**

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<td>$\eta_G$</td>
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<td>$\gamma_G$</td>
<td>$\gamma_G$</td>
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<td>Tradable goods share in $G$</td>
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<tr>
<td>$\theta_G$</td>
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<td>elasticity of substitution between $Y_H(G)$ and $Y_F(G)$</td>
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<td>$\omega_G$</td>
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<td>Value</td>
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<td><strong>Capital goods assembly</strong></td>
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<td>$\tau_D$</td>
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<td>$\phi_N$</td>
<td>phi_N</td>
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<td>change of price probability in sector $H$</td>
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<td>change of price probability in imported goods</td>
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<td>$\chi$</td>
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<td>Share of commodity sector owned by the government</td>
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<tr>
<td>$\rho$</td>
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Table 2 (cont.)

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<th>Description</th>
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<td>( \rho_S )</td>
<td>( \text{rho}_pF_s )</td>
<td>0.8</td>
<td>AR(1) coefficient in ( \hat{p}_{S,t} )</td>
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<tr>
<td>( \rho_G )</td>
<td>( \text{rho}_g )</td>
<td>0.8</td>
<td>AR(1) coefficient in ( \hat{g}_t )</td>
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<td>( \rho_N )</td>
<td>( \text{rho}_a_n )</td>
<td>0.5</td>
<td>AR(1) coefficient in ( \hat{a}_{N,t} )</td>
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<tr>
<td>( \rho_H )</td>
<td>( \text{rho}_a_h )</td>
<td>0.5</td>
<td>AR(1) coefficient in ( \hat{a}_{H,t} )</td>
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<tr>
<td>( \rho_{\pi^*} )</td>
<td>( \text{rho}_pF_f )</td>
<td>0.0</td>
<td>AR(1) coefficient in ( \hat{p}_{F,t} )</td>
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<tr>
<td>( \rho_i^* )</td>
<td>( \text{rho}_iF )</td>
<td>0.7</td>
<td>AR(1) coefficient in ( \hat{i}_{t}^* )</td>
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<tr>
<td>( \rho_m )</td>
<td>( \text{rho}_m )</td>
<td>0.0</td>
<td>AR(1) coefficient for the monetary shock ( \nu^m_t )</td>
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Other SS variables fixed as parameters

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<tr>
<td>( \frac{P_sY_s}{P_X X} )</td>
<td>( P_sY_s/P_X X )</td>
<td>40%</td>
<td>Commodity export share of total exports</td>
</tr>
<tr>
<td>( \frac{eB^*}{P_Y ('annual')} )</td>
<td>( eBF/(PyY) )</td>
<td>-30%</td>
<td>Foreign debt as GDP percentage</td>
</tr>
<tr>
<td>( \frac{P_G G}{P_Y} )</td>
<td>( P_G G/PyY )</td>
<td>11%</td>
<td>Government consumption as GDP percentage</td>
</tr>
<tr>
<td>( v )</td>
<td>( v )</td>
<td>8</td>
<td>Transaction velocity in consumption</td>
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Table 3: Principal Statistics of the Steady State

<table>
<thead>
<tr>
<th>Variable</th>
<th>SS Value</th>
<th>Data(^a)</th>
<th>Description</th>
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<tbody>
<tr>
<td>(WL_{PY})</td>
<td>46.4%</td>
<td>53%</td>
<td>Labor share of GDP</td>
</tr>
<tr>
<td>(P_{C}C_{PY})</td>
<td>60.3%</td>
<td>63% (^b)</td>
<td>Consumption share of GDP</td>
</tr>
<tr>
<td>(P_{I}I_{PY})</td>
<td>25.1%</td>
<td>25%</td>
<td>Investment share of GDP</td>
</tr>
<tr>
<td>(P_{X}X_{PY})</td>
<td>26.5%</td>
<td>30%</td>
<td>Exports share of GDP</td>
</tr>
<tr>
<td>(P_{X}X - P_{M}M_{PY})</td>
<td>3.7%</td>
<td>1%</td>
<td>Net exports to GDP</td>
</tr>
<tr>
<td>(P_{O}(O_H + O_X + O_t(C))_{PM/M})</td>
<td>13.5%</td>
<td>10.3% (^c)</td>
<td>Oil share of imports</td>
</tr>
<tr>
<td>(H(S,F)_{PY})</td>
<td>3.7%</td>
<td>2.5% (^d)</td>
<td>Dividends paid to foreign investors to GDP</td>
</tr>
<tr>
<td>(P_{S}Y_{S})</td>
<td>10.6%</td>
<td>8%</td>
<td>Commodity export sector share of GDP</td>
</tr>
<tr>
<td>(P_{H}Y_{H} - P_{O}O_{H} + (P_{F} - cP_{F}^*)Y_F)</td>
<td>34.0%</td>
<td>31.6%</td>
<td>Non-commodity tradable goods share of GDP</td>
</tr>
<tr>
<td>(P_{N}Y_{N} - P_{O}O_{N})</td>
<td>46.4%</td>
<td>50.4%</td>
<td>Non-tradable share of GDP</td>
</tr>
</tbody>
</table>

\(^a\)Most data values are based on Restrepo and Soto (2004).

\(^b\)This figure includes durable consumption. Non-durable consumption is 50% of GDP.

\(^c\)1996-2003 average imports of oil and related as a percentage of goods and services imports.

\(^d\)1996-2003 net profit average of direct investment as a percentage of GDP.
Figure 1: Basic structure of MAS
Figure 2: Responses to a commodity export price shock

Commodity export price

Consumption

Investment

GDP

Exports

Imports

- benchmark
- flex. wages
- no inv adj cost
- no habit
Figure 3: Responses to a commodity export price shock (cont.)
Figure 4: Responses to a oil price shock

- Oil price
- Consumption
- Investment
- GDP
- Exports
- Imports

Legend:
- benchmark
- flex. wages
- no inv adj cost
- no habit
Figure 5: Responses to an oil price shock (cont.)

- RER
- Domestic interest rate
- Real wage
- Hours worked
- Inflation rate
- Labor share

Legend:
- Benchmark
- Flexible wages
- No interest adjustment cost
- No habit
Figure 6: Responses to a foreign interest rate shock

- **Foreign int rate**
  - benchmark
  - flex. wages
  - no inv adj cost
  - no habit

- **Consumption**
- **Investment**
- **GDP**
- **Exports**
- **Imports**
Figure 7: Responses to a foreign interest rate shock (cont.)

- **RER**
  - Benchmark
  - Flex. wages
  - No inv adj cost
  - No habit

- **Domestic int rate**

- **Real wage**

- **Hours worked**

- **Inflation rate**

- **Labor share**
Figure 8: Cross correlation - model with flexible wages
Figure 9: Cross correlation - model without investment adjustment cost

- GDP
- Investment
- Net exports to GDP ratio
- Employment
- Real Wages
- Real Exchange Rate
- Consumption
- Inflation

Model vs Data comparison for various economic indicators.
Figure 10: Cross correlation - model without habit in consumption
Figure 11: Cross correlation - Benchmark model