RESERVE BANK
OF NEW ZEALAND

The Forecasting and Policy System: the core model

Research paper No. 43

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All errors, omissions, and views expressed in this paper are the sole responsibility of the authors.

Contents

Abstract

The Reserve Bank of New Zealand's new *Forecasting and Policy System* has been designed for use in both policy analysis and quarterly economic projections. The core model, which lies at the heart of FPS, contains several important design features that make it notably different from previous models developed at the Reserve Bank. First, the model has been designed from a top-down perspective, with the focus on its aggregate properties. Its parameters have been calibrated rather than relying solely on econometric estimates. Second, solution paths converge to a well-defined steady state which is consistent with a balanced growth equilibrium. Stock-flow accounting is enforced. Finally, the model's dynamic properties are the product of a structure for adjustment costs, explicit modelling of expectations, and the endogenous behaviour of the fiscal and monetary authorities.

The behavioural core of the model is derived from the Blanchard-Buiter-Weil model of overlapping generations in an one-good small open economy. The optimising behaviour of households and firms together with the decisions taken by government determine desired asset stock positions. Expenditure flows are adjusted to achieve and maintain those desired positions. The monetary authority adjusts monetary conditions on the basis of the deviation of projected inflation outcomes from its inflation target.

Preface

The Reserve Bank has published several macroeconomic models of the New Zealand economy over the past quarter century. Throughout, the stated aim has been to provide a framework for medium-term projections and to aid the analysis of policy issues. With each new model, estimation techniques have been improved and the theoretical content of individual equations strengthened. However, this equation-by-equation method of construction has meant that relatively little attention has been paid to the properties of the whole model.

The model described in this document represents a regime-shift in our approach to constructing macroeconomic models. Our over-riding emphasis with FPS has been to ensure that the core model as a whole embodies properties that are sensible theoretically and empirically. As part of this process, its parameters have been calibrated from a wide range of information, rather than relying solely on econometric estimates. Although this practice is somewhat controversial within the economics profession, we believe the end results presented in this paper will be seen as justifying our approach.

Our new modelling approach represents a break from past Reserve Bank models in two further respects. First, we do not rely on one model to accomplish all of our objectives for forecasting and policy analysis; instead, we develop a system of models, each designed to accomplish specific tasks. The full system consists of the core model discussed in this document and a series of ‘indicator’ and ‘satellite’ models for short-term forecasting and detailed analysis. Second, we base the core model on a general-equilibrium one-good framework with a well-defined steady-state, stock-flow identities, and budget constraints. In particular, the model embodies overlapping generations of consumers, profit-maximising firms, endogenous monetary and fiscal policy, forward-looking behaviour, costly adjustment, ‘time-to-build’ constraints, and an asymmetric inflation process.

The model was constructed through the collaboration of the staff of the Reserve Bank, QED Solutions, and the Bank of Canada. This combination of local knowledge and international model-building expertise has been crucial to our success in meeting deadlines within budgeted cost. Further, this collaboration has built expertise at the Reserve Bank.

The project has also benefited from advances in computing hardware and software. The model is coded in Troll and uses the ‘Stacked Time’ algorithm to solve out the forward-looking solutions.[1] Stacked-Time generally solves faster than the Fair-Taylor method and is more robust to nonlinearities.[2] Practically, it reduces size restrictions on the model and allows for fast projection rounds. Moreover, faster simulation times

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1 See Hollinger and Spivakovsky (1996), Hollinger (1996).

2 See Armstrong *et al* (1995).

make computationally-intensive stochastic simulation analyses feasible. These techniques are increasingly required for modern policy analysis.

The model documented in this paper represents our 'work-in-progress' after an initial 18 month development schedule. As part of the development process, the model was used in 'shadow projections' following the December 1996 and March 1997 projection rounds. These trials confirmed that the model was sufficiently well-developed to use for the preparation of quarterly projections. Accordingly, the model was used to prepare the projections reported in the June 1997 *Monetary Policy Statement*.

Nevertheless, much remains to be done to complete the full FPS system. Significant further development of the indicator and satellite models will occur over the next year. Further adjustments to the calibration of the core model can be expected as we gain experience from successive projection rounds, policy simulation experiments, and new econometric evidence. Further ahead, the structure of the core model may also be the focus of development work.

Ultimately, FPS will be a success if it comes to be treated by Reserve Bank staff as a reservoir of knowledge about the systematic behaviour of the New Zealand economy. This in turn will be measured by the extent to which FPS continues to be used in both projections and policy analysis.

Eric Hansen
Manager, Forecasts and Model Research

August 1997.

1. Introduction

This document describes the core macroeconomic model of the Reserve Bank's new *Forecasting and Policy System* (FPS)[3]. It contains an overview of the goals and scope of the System, followed by a technical description of the characteristics of the core model.[4]

The decision to develop a new macroeconomic model was motivated by the demands of monetary policy in the 1990s. To achieve its inflation target, the Reserve Bank requires sound economic analysis outlining the current state of the economy and its expected evolution in the future. It also requires a thorough understanding of the risks facing policy. The goal of the FPS project was to develop a framework that would allow a unified approach to the preparation of quarterly economic projections and research on alternative policy choices. The framework was therefore named the *Forecasting and Policy System*.

A formal macroeconomic model sits at the core of FPS. This design reflects the view that there are unique advantages to models. These advantages come from the discipline that a model imposes on economic analysis. When examining policy issues, this discipline may come, for example, from accounting and behavioural constraints on agents' behaviour, or the distinction drawn between short run and long run. For projections, this discipline becomes an aid for analysts by making the consequences of their sectoral views on the larger economy more transparent.

An extensive review of formal macroeconomic models used throughout the world determined that several key features would be required for the new model. These included: a top-down approach, calibration of the model's parameters, a steady state, stock-flow accounting, a well-defined supply side, a realistic specification of the expectations process, tractable solution paths, and endogenous monetary and fiscal policy.

The emphasis on the model as a system is reflected in the layout of this report. The model has been built from a top-down perspective rather than a more traditional 'bottom-up' approach. Accordingly, the following chapters stress the whole rather than its parts.

In Chapter 2 a broad perspective on the model's design is presented. The intended uses of the model and the tensions that these often-conflicting demands create are discussed. Key design features that distinguish the FPS model from previous models at the Reserve Bank are then outlined. Chapter 3 contains details of the economic theory embodied in the model. The aim of this chapter is to provide a description of the economic structure, emphasising the overall behaviour of agents and their

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3 For an overview of the Forecasting and Policy System, see Black *et al*, (1997).

4 This document does not, however, deal with how the model will be used to produce quarterly economic projections or implications for the conduct of monetary policy. These issues warrant a separate treatment.

interactions in markets. In Chapter 4, this same framework is used to describe the steady state. The steady state is characterised in terms of ratios to a numeraire price or normalised output, with trend growth removed. The model's dynamic properties are illustrated using a set of simulation experiments in Chapter 5. These simulations are also useful for illustrating the features discussed in Chapter 2 and the theoretical structure outlined in Chapter 3. We finish in Chapter 6 with some concluding remarks. Appendices list the model's equations and mnemonics.

2. The design of FPS and the core model

The Forecasting and Policy System represents a new approach to modelling at the Bank. Likewise, the core model at the heart of FPS is substantially different from previous Bank models in its features, theory, and behaviour. This Chapter discusses the design of FPS and the core model. Section 2.1 begins with a discussion of the FPS concept. It describes the different demands policy analysis and projections place on a model, and how the System was designed to handle both tasks. Once these demands were identified, it was clear that several features would be required in the core model. These are discussed in section 2.2.

2.1 The FPS concept

Policy experiments and forecasting place quite different demands on macroeconomic models. The Bank's experiences with economic modelling and those of other institutions show a tension between models that are rich enough to relate closely to the data and those that are tractable enough to be practically useful for analysis of alternative policy choices. Typically, models that forecast well in the short term are often seriously misleading when used for policy analysis.

One might think that a solution would be to use one model for projections and another for policy analysis. However, in addition to the resource costs of maintaining two large macroeconomic models, there are two basic problems with such an approach. First, the same long-run general equilibrium considerations that are essential for policy analysis are also critical for determining the inflation outcomes central to the forecasting process. Second, in order to analyse alternative policy choices effectively, it is important to capture all the short-run dynamic adjustment processes in the economy. The solution chosen is a single, core macroeconomic model that has the necessary structure to support policy analysis, and that can be used as part of a larger system to support economic projections.

2.1.1 Using a model for policy analysis

The basic requirement for a policy model is that it should produce sensible policy conclusions. Yet all too frequently the economic models used for policy analysis are incomplete, with the resulting policy conclusions being highly misleading. In part, this problem arises because of the way these models have been built: when 'assembled' from a number of individually-estimated equations, the partial-equilibrium properties of individual behavioural equations are emphasised and the properties of the system ignored. This focus on individual equations may divert attention from important economic and accounting restrictions on the general equilibrium solution. All this can lead, quite unwittingly, to model properties that are inconsistent or perverse.

There has also been an historical tendency for models to grow more and more complex, as more equations are added in order to gain detail. Added complexity can have a high cost: the original economic logic of the model is obscured or damaged, the model becomes slower to run and harder to maintain, and the results become less transparent and harder to understand.

Ideally, a model for policy analysis would not suffer from these compromises. For policy experiments, a model should help clarify the key issues. To do so, it must embody the key behavioural relationships of the economy in a transparent framework. The model should be able to run different scenarios quickly and easily. This is particularly important when using more computationally-intensive techniques, such as stochastic simulation analysis, that are essential to approximate the type of uncertain world within which monetary policy must operate.

Fortunately, many of the policy questions of interest are not critically dependent on the current state of the economy. This allows us to reduce some of the demands on a policy model. In FPS, policy questions may be tackled by using the core general equilibrium model by itself. Normally such analysis would be done using the model's steady-state solution as a control solution for the analysis. The outputs of the analysis are in the form of 'shock-minus-control' values of the model's response to shocks around the steady state. This is the methodology we adopt for describing the model's properties (see Chapter 5). Using the core model in this way maximises clarity, providing an aggregate, general equilibrium answer to the policy question being considered.

2.1.2 Using a model for quarterly projections

In contrast, for quarterly projections the starting point matters considerably. Developments in the short run have important implications for inflation outcomes. Because of this, it is important to incorporate detailed information available from current sources about the short run. This involves judgements about the current state of the economy, the nature of the disequilibria, and how persistent the disequilibria are expected to be.

Projections should also help to clarify the kinds of medium-term policy issues that come from adjustment towards some long-run equilibrium. In other words, projections require a framework capable of incorporating rich short-run dynamics, together with medium-term dynamics that are consistent with a sensible long-run scenario.

Macroeconomic models tend to be quite aggregated. However, simply making an existing model more detailed does not necessarily produce a successful projection model, since added complication leads to higher maintenance costs and more difficulty in interpreting outputs. It certainly risks making the model unsuitable for policy analysis.

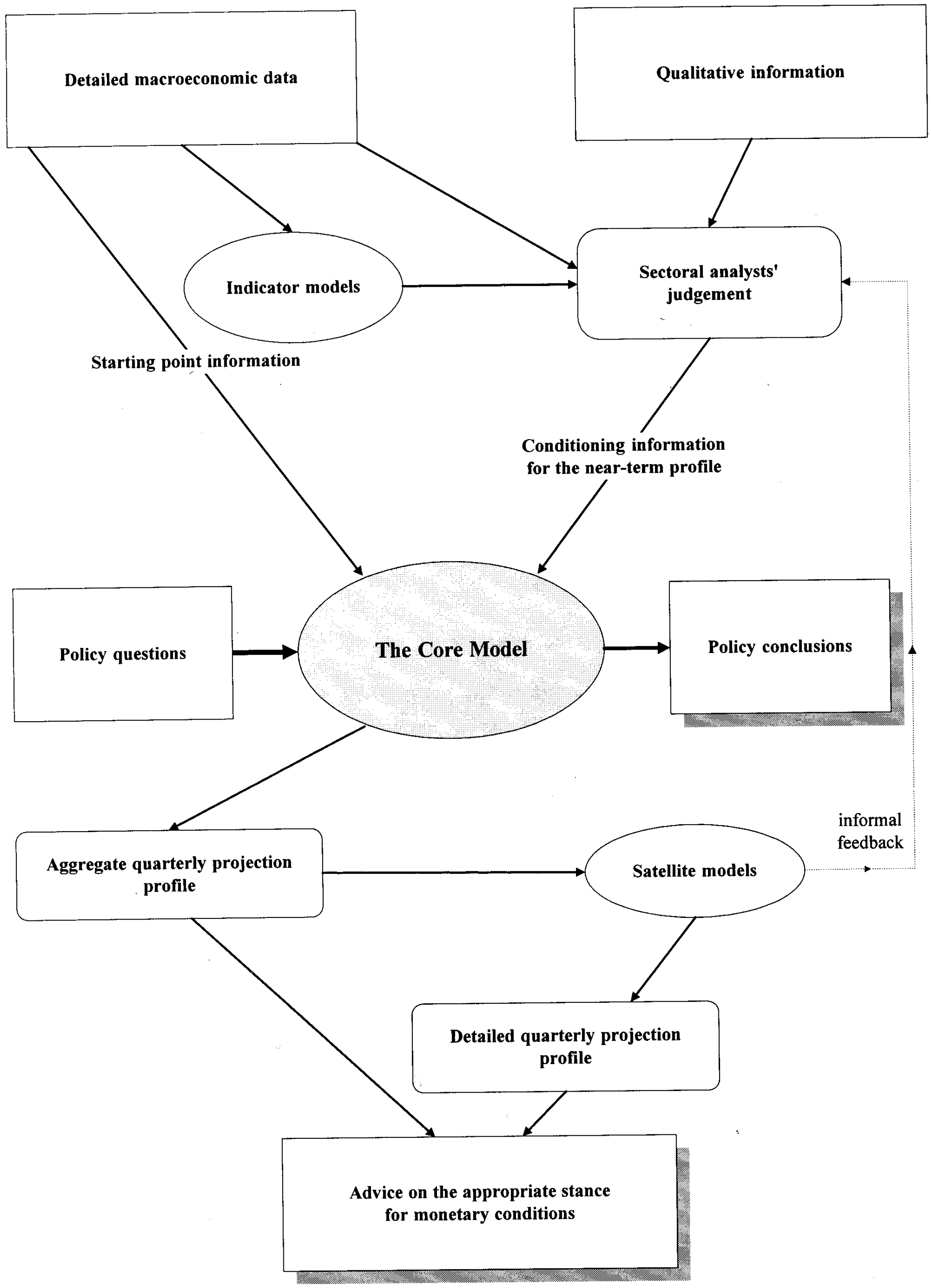
Instead, the FPS 'projection environment' consists of the core model plus two types of small models. The core model incorporates the behavioural relationships among the key macroeconomic concepts that are most central to monetary policy issues. *Indicator models* are pure time-series models designed to capture the high frequency information in the data. *Satellite models* decompose the highly aggregated concepts from the core model into more detailed sectoral information.

By using indicator models, the very short-run properties of data are able to influence the projected path for monetary conditions without making these properties permanent in the core model. This is important because these short-run properties often change substantially through time, particularly in New Zealand where significant structural change has been so pervasive. Indicator models can be updated quickly without having to change the core model. Second, indicator models themselves can be used quickly and easily between projections to see if new data are suggesting that the view on the current state of economy needs to be revised. In their own right, indicator models are useful analytical tools for examining the information content of the data.

Using satellite models to decompose the aggregate concepts used in the core model ensures that the advantages of the core model's general equilibrium solutions are retained, while allowing simulation times to be very fast. To the extent possible, the satellite models embody the same design philosophy as the core model; dynamic adjustment occurs around a well-defined equilibrium path. Detailed results from the satellite models are more readily assessed by sectoral analysts at the level at which they have specific information. This allows them to see the implication of their aggregate views at a detailed level, possibly leading to a revision of those views. In this way, the projection environment becomes a tool where different views can be tested and re-examined, leading to better quality projections.

A graphical representation of the full Forecasting and Policy System is represented in Figure 2.1. While it may appear complicated, the essence of this configuration is that it essentially allows the analyst to use as much of a model as is needed.

Figure 2.1
The Forecasting and Policy System



2.2 Key design features of the core model

As the FPS concept took form, it became clear that the core model would require a number of key design features, regardless of the theoretical paradigm chosen. These design features follow logically from the demands of policy analysis and projections described in the previous section.

2.2.1 A top-down approach

Many old-style macroeconomic models were built from the bottom up; simply as collections of a large number of independently-estimated behavioural equations. Most of the effort was put into the properties of the individual equations and very little, if any, into the properties of the system as a whole. Consequently, the solution paths from these models were only sensible for the first few quarters of the simulation period. Eventually, the lack of focus on the general equilibrium nature of the system became evident as the solution paths in the medium term became increasingly less plausible. Accounting identities and important constraints on agents' behaviour were frequently ignored.

Models developed this way tended to produce seriously misleading policy conclusions and poor medium-term forecasts. Too often, more behavioural detail was added, hoping that this would 'fix' problems. In this way, the original theoretical paradigm became more obscured by *ad hoc* measures, rendering the model less adaptable, harder to use, harder to understand, and, consequently, less relevant for the policy analyst. New estimation techniques were often held out as a means of improving the performance of these models. However, although new estimation techniques may have improved the behaviour of individual equations or sectors, they still failed to address the fundamental point that it is the general-equilibrium properties of the model as a *system* that matter.

By contrast, a top-down approach places priority on the overall properties of the model. The principal aim of this approach is to ensure the model maintains fidelity to the theoretical paradigm, while at the same time producing solution paths with sensible aggregate dynamic properties.

2.2.2 Calibration rather than direct estimation

When estimating equations for models, a natural tendency is to aim for goodness of in-sample fit. This often occurs without the researcher realising that this may imply undesirable properties for the system as a whole. Models built this way usually overfit the data, leading to breakdown over time.

Moreover, the New Zealand context is particularly unfavourable for classical estimation techniques. Data series have short spans and observations are often low frequencies, mitigating the usefulness of systems estimators. They also contain large structural breaks and are subject to frequent and large revisions.

Rather than estimate single equations, we calibrate the model. Calibration draws on theory and a wide range of statistical techniques to choose parameter values for the model that result in sensible aggregate properties. In this sense, calibration is complementary to the top-down approach, making the emphasis on the system responses a practical proposition.

2.2.3 A well-defined steady state

Some models do not possess a steady state. In many cases, such models will not solve beyond some limiting forward horizon. When they will, their solution paths can drift without defined limits, since they are not constrained in some way to converge upon a stable, long-run equilibrium. Since we never reach 'the long run', some might argue that a steady state is merely a theoretical nicety. Yet many policy issues require us to talk about the long run. More fundamentally, since economic theory tells us that the behaviour of economic agents will be guided by their desire to achieve some long-run position, a failure to converge to a steady-state equilibrium indicates that the model is either incomplete or flawed in some way, and will be misleading over horizons of concern to policymakers.

Solution paths in the core model converge to a well-defined steady state, consistent with a balanced-growth equilibrium. The steady state serves several purposes. At a practical level, convergence of dynamic paths to a steady state serves to eliminate multiple equilibria and dynamic instabilities. While multiple equilibria are a feature of some theoretical macroeconomic models, they are not helpful in a model used every day for analysis in a policy institution.

For policy experiments and projections, important differences are obtained between temporary and permanent disturbances to the economy. When the model embodies a steady state, this distinction between permanent and temporary disturbances is brought into focus. Permanent disturbances are those that affect the steady-state solution. The process of adjustment towards a new steady state creates quite different problems for monetary policy than arise in dealing with temporary disturbances. A well-defined steady state is also important for projections; a forecast path produced by the model should not converge to a value that is implausible or internally inconsistent. In short, being forced to look at the long run is an important discipline on the way we think about policy at all horizons.

2.2.4 Stock-flow accounting

Consistent stock-flow accounting is another important dimension of the core model.[5] First, it ensures consistency within the steady state. In the FPS model, an important aspect underlying the behaviour of expenditures is the adjustment of flows to achieve and maintain long-run desired asset stocks. For example, profit-maximising firms aim for a desired level of capital stock. To achieve this goal, adjustments in investment flows may be required each quarter.

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5 In addition to stock-flow accounting, which maintains 'internal' consistency, it is also important that the model be compatible with national accounts concepts, which is a key element in maintaining 'external' consistency. The model database is not described here and will be dealt with in separate documentation.

Second, together with budget constraints, the stock-flow accounting has important implications for the sustainability of agents' actions. Models that do not impose these constraints allow 'free lunches', where an agent's present expenditure choice has no repercussions on future expenditure choices. This is particularly damaging for a policy model. By contrast, the stock-flow accounting in the core model places important behavioural constraints on agents' dynamic behaviour. Households, firms, and government cannot spend without any regard to their debt burden. The monetary authority cannot stimulate higher consumption on a sustained basis since households will not ignore forever the consequent decline in their wealth.

2.2.5 A well-defined supply side

In the core model, the productive capacity of the economy depends on the evolution of technical progress, the level of productive capital maintained by firms, and the labour services that households are willing to supply. For policy experiments, the supply side is an important element in the model's reaction to real shocks. It also plays an important role in the inflation process, by allowing the concept of the output gap to be articulated.

2.2.6 Realistic and tractable dynamic properties

Given its role for both policy exercises and quarterly projections, an important requirement of the core model is that it produce dynamic paths that are both realistic (that is, that they are plausible) and tractable (that are able to be clearly understood and easily analysed.) As a means to this end, dynamic adjustment in the core model has several conceptually distinct sources: 'intrinsic' dynamics, 'expectational' dynamics, and the effects of fiscal and monetary policy. These components allow for the realistic and tractable dynamic solution paths essential for a policy model. Short-run paths can be more easily understood, since, notionally, they can be 'devolved' into the effects coming from these three distinct sources.

Intrinsic dynamics

The microfoundations literature presents us with many justifications for slow adjustment in agents' behaviour towards their desired positions, such as menu costs and overlapping contracts. Underlying all of these theories is a notion that, for some reason, adjustment is often costly. This is what we identify as intrinsic dynamics.

In the core model, much use has been made of polynomial adjustment cost structures. These afford considerable control over the model's dynamics, allowing a good match to observed behaviour. In different markets, the speeds of adjustment can be expected to be quite different - asset prices, for example, tend to adjust more quickly than real variables such as employment and output. By calibrating each adjustment cost form differently, the model is able to capture these differing speeds of adjustment.

Expectational dynamics

The way in which private agents form their expectations will have important implications for dynamic adjustment. When exogenous disturbances move the economy away from equilibrium, and policy actions are introduced to offset the disturbance, agents' expectations are not immediately re-anchored at equilibrium. Therefore, other real variables in the economy must follow disequilibrium paths until expectations are re-anchored. These expectational dynamics have very important consequences for both fiscal and monetary policy.

In the FPS model, we explicitly model expectations that people hold about the future, such as expected inflation. This means that private agents will look to the actions of policy agents, such as the monetary authority. The monetary authority can bring about changes in expectations by changing its policy. Since this means that private agents' decisions are no longer invariant to policy decisions, the use of forward-looking expectations goes some way towards addressing the Lucas critique. It avoids the problems associated with formal macroeconometric models with estimated decision rules that are invariant to policy actions.[6] However, it does not make the model immune to the critique. A valuable next step would be to specify how agents learn about the new policy rules, although as yet there is no generally-accepted theory of learning in macroeconomics.[7] To address this issue completely, one would need to know the true structure of the economy and agents' decisions rules.[8]

Many economists believe that the use of fully model-consistent expectations in policy analysis assumes too great a level of understanding of the economy and too great a level of information. Nevertheless, the inclusion of some element of forward-looking structure in expectations is now widely accepted. The solution we use is to model expectations as some weighted combination of the model-consistent forecast and some other function of the recent data.

Expectations are therefore not 'rational' in the strict Muth (1961) sense. This 'mixed' representation of expectations formation can be thought of as an approximation to an aggregate economy made up either of a mixture of agents with different levels and costs of information, or of identical agents all unsure of the true structure of the economy and the nature of its shocks.

This representation has important implications for monetary policy. It avoids the unrealistic properties of some New Classical models. Models where expectations adjust instantaneously are uninteresting from a monetary policy point of view, since they are not consistent with a broad range of stylised facts that emerge from the data. Since expectations do not adjust instantaneously, the monetary authority cannot benefit from announcement effects. In this model, the monetary authority has to take actions to re-anchor inflation expectations at the policy target.[9]

Endogenous monetary and fiscal policy

For use in addressing policy questions, an obvious requirement for a model is that it clearly express the roles of monetary and fiscal policy and how they affect private

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6 It is in reference to these kinds of models that Lucas and Sargent (1981) would remark famously, "econometric models are of *no* value in guiding policy."

7 See Marcet and Sargent (1987a, 1987b, 1988a, 1988b), and Laxton, Ricketts and Rose (1994).

8 That is, one would need to uncover the 'deep parameters' of constraints, technologies, and agents' preferences. See Lucas and Sargent (1981).

9 The structure of the model does, however, allow us to experiment with credibility effects. This is an avenue for future research.

agents. To this end, the behaviour of the two policy authorities is characterised by reaction functions. There are several reasons for making policy endogenous.

First, there are technical advantages to expressing the role of policy in terms of policy targets. In the case of the fiscal authority, policy targets avoid time-inconsistency problems that can arise in forward-looking models. That is, long-run targets precommit the authority so that it cannot 'choose' fiscal policies that are, in the language of dynamic game theory, subgame perfect.[10] In the case of the monetary authority, a monetary policy target provides a nominal anchor to the model.

Second, if the monetary policy objective is some form of nominal closure such as an inflation target, then the Lucas critique makes it clear that policy measures cannot be added exogenously onto a model economy. In this case policy is necessarily dynamic, for it is as much a policy choice not to react as to act in the first place. Because of this, we specify fully-endogenous reaction functions to characterise monetary and fiscal policy.

Hence, just as it solves for equilibrium flows that achieve desired stocks, the model also solves for conditions that work to achieve policy targets. This represents a new approach to modelling at the Bank. That is, instead of asking 'What happens to inflation if we set particular monetary conditions?', the model poses the question 'What monetary conditions are required to get inflation to its target?'

For the purposes of a monetary policy model, fiscal policy can be represented relatively simply, by numerical targets such as a debt ratio. With a fiscal reaction function, the policymaker is then precommitted, and private agents can use this fact to form their expectations about the fiscal authority's future actions. The fiscal reaction function does not generally require leads and can be written simply in terms of current deviations from target.

On the other hand, it is very important the monetary reaction function be forward-looking. As is well-known, monetary transmission works with a lag. Consequently, if the monetary authority is trying to stabilise inflation then it needs to be forward-looking.[11] For this reason, the monetary reaction function targets future inflation. If, however, the monetary authority was to target contemporary or past inflation, then it would find itself perpetually 'behind the game.' Neither inflation nor the real economy would ever be stabilised.[12]

Specifying a forward-looking policy rule is appropriate, since the monetary policy framework in New Zealand requires the policymaker to think in terms of the conditions required to achieve and maintain an inflation target. This maintenance requires the policymaker to think in terms of future outcomes when assessing what policy actions are appropriate now. Increasingly, the Bank has to deal with policy issues where the reactions of private agents to fiscal policy measures are at the core of

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10 See Kydland (1975, 1977).

11 See McCallum (1994a).

12 For a discussion of forward- versus backward-looking monetary policy rules, see Clark, Laxton and Rose (1995).

the policy issue. The use of endogenous reaction functions allows the general equilibrium interactions of fiscal and monetary policy to be made more explicit.

3. The economic structure of the core model

This chapter outlines the economic theory embodied in the core model. The model describes the behaviour of a set of stylised agents, whose interactions lead to flows of income and expenditures, stock accumulations, and changes in prices. These interactions take place in the context of a small open economy. These agents provide a natural framework for describing the system. This begins with the theory of the household in section 3.1, followed by the firm in section 3.2, government in section 3.3, and the external sector in section 3.4. Section 3.5 describes the labour market. With the description of the real side of the economy complete, the nominal economy is described in terms of the monetary authority, interest rates and the exchange rate (section 3.6), and prices and inflation (section 3.7). Section 3.8 contains a note on growth.

The stock-flow logic outlined in section 2.2.4 is an important feature of the model. Much of the real economy can be described by the theory determining how (domestic) agents' desired levels of assets determine their expenditure choices. The stock-flow logic is part of an accounting framework that achieves internal consistency in asset stocks, expenditure and income flows, taxation, and relative prices. At the same time, the components of dynamics described in section 2.2.6 are particularly important to the evolution of the economy. Intrinsic dynamics, expectational dynamics, and the behaviour of the monetary and fiscal authorities all interact to determine the dynamic adjustment path of the economy.

Although the theory is presented sequentially by type of agent, it is worth noting that no agent's particular decision is taken independently of other agents' actions. This general equilibrium perspective will be more thoroughly illustrated in Chapters 4 and 5.

A note on the structure of the core model's equations

In the core model, a distinction is made between the 'equilibrium' path and the 'dynamic' path. The equilibrium path is specified in equations that describe the long-run solution for the model economy. These equations, for the most part, abstract from the short-run adjustment behaviour and nominal effects. The dynamic equations specify how intrinsic dynamics, expectational dynamics and monetary and fiscal policy affect the convergence path to the long-run equilibrium. The dynamic responses in these equations have been calibrated to match the stylised facts for the New Zealand economy. Once all desired asset stocks are achieved, expectations realised, and no disturbances are influencing agents behaviour, actual values will have converged to equilibrium values.

As a result of its two layers, the model allows for a clear distinction between short-run and long-run influences. This contributes notably to fulfilling the requirement of tractable solution paths. Consequently, the model is 'two-tiered', in the sense that for every variable there is usually an equilibrium equation and a dynamic equation. For clarity and brevity, in general only the equilibrium equations are discussed in the text that follows. Moreover, not all of the equilibrium equations are presented, and those that appear are often stylised representations of the equations in the model code (see Appendix A). Therefore, this Chapter does not provide an account of the full richness of the model's dynamics. Instead, these properties are illustrated via simulation experiments in Chapter 5.

3.1 The household

Households are represented by two groups of consumers in the FPS model: 'rule-of-thumb' consumers and 'forward-looking' consumers. Forward-looking consumers save, on average, and hold all the assets of the household sector. Rule-of-thumb consumers simply spend all their disposable income each period, neither providing savings nor holding assets.

The theoretical core of the model lies in the optimisation problem for forward-looking consumers. The structure is based on the overlapping-generations framework of Yaari (1965), Blanchard (1985), Weil (1989), and Buiter (1988), but in a discrete-time form as in Frenkel and Razin (1992) and in Black, Laxton, Rose and Tetlow (1994).[13] In this model, the forward-looking consumer maximises the expected present value of lifetime utility subject to a budget constraint and a fixed probability of death.

The population size and age structure is determined by the simplest possible demographic assumptions. We assume that new consumers enter according to a fixed birth rate, and that existing consumers exit the economy according to a fixed probability of death. For the supply of labour, we assume that each consumer offers a unit of labour services each period.[14] Thus, labour is supplied inelastically with respect to the real wage.

In the next two sections, the equilibrium behaviour of individual households is presented. Section 3.1.3 shows how these results are aggregated.

3.1.1 The optimisation problem for forward-looking consumers

The basic problem for forward-looking consumers can be stated as follows: at time *t*, an individual consumer who was born at time *a* chooses a path for consumption to maximise the present value of expected lifetime utility, subject to a budget constraint.

Utility

More formally, consumers maximise an expected discounted sum of period-by-period utility that is given by a constant relative risk aversion utility function,

(3.1) $$U = \mathrm{E}\left(\sum_{s=0}^{\infty} \delta^s \frac{cfl_{a,t+s}^{\;1-\sigma^{-1}}}{1-\sigma^{-1}} \varepsilon_s \right)$$

where *cfl* is consumption by forward-looking consumers, σ is the elasticity of intertemporal substitution, δ is the discount factor, E denotes a mathematical

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13 FPS shares the same basic theoretical framework as the Bank of Canada's Quarterly Projection Model. However, whereas QPM's steady-state model is static, the underlying equilibrium structure in FPS is dynamic. We largely abstract from this dynamic dimension of the equilibrium structure in the discussion that follows.

14 We also assume that all workers are equally-and fully-productive from birth till death.

expectation and ε is a random variable that takes the value one if the consumer is alive and zero if not.

Left in this form, however, the problem has no deterministic solution, since the expectation is undefined. One way around this problem is to assume fixed lifetimes.[15] However, in this model, we assume that consumers do not know how long they will live, but instead face a constant probability of death each period, γ. This allows us to define ε by a Poisson process at rate $1-\gamma$, so that the probability that the consumer will be alive *s* periods hence (i.e., that ε_s is one) is $(1-\gamma)^s$. We can then rewrite equation (3.1) as

(3.2) $$\sum_{s=0}^{\infty}\left((1-\gamma)\delta\right)^s \frac{cfl_{a,t+s}^{\;1-\sigma^{-1}}}{1-\sigma^{-1}},$$

where the expectation operator is now implicit in variables with leads. In other words, expected utility is evaluated on the basis of the probability of survival. This allows the expression to be stated in terms of its *certainty equivalent* form. The effective rate at which consumers discount the future is then equal to $(1-\gamma)\delta$.

The budget constraint

Consider now the consumer's budget constraint. In each period, consumers supply labour for which they receive labour income. After paying taxes net of any transfers from the government, they are left with disposable income, *ydfl*. Consumers also receive income from financial assets, *fa*, at the rate *rcon*.[16] In addition, there is a transfer we call *risk*.[17] This income can be spent on consumption or used to accumulate financial assets, so that the consumer's budget constraint takes the form

(3.3) $$pc_t cfl_{a,t} + fa_{a,t} = \frac{(1+rcon_{t-1})}{1-\gamma} fa_{a,t-1} + ydfl_{a,t} + risk_{a,t}$$

where *pc* is the relative price of consumption goods.

Equation (3.3) shows an interesting twist on the usual budget constraint. The household interest rate is divided by the probability of survival, so that the return to individuals is higher than the market rate. This reflects an assumption about the existence of insurance companies that receive all agents' financial assets when they die.[18] In return for turning their assets over upon death, all consumers receive an additional return on their financial assets. The sum of all these additional returns is

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15 See, for example, Diamond (1965).

16 We assume that each consumer starts with no assets at birth. The level of financial assets depends on the age of the consumer.

17 The *risk* component reflects the practical reality that we want to apply this model to a world with various assets that bear quite different rates of return. We use one notional rate of return for consumer assets, *rcon*, in the formal problem, and put any differences, asset by asset, into a 'risk' term that is treated as a lump-sum transfer to households. Configured this way, the term simply ensures that income accounting identities hold, i.e., that all payments are recorded as income somewhere in the system.

18 We assume that the insurance company charges an actuarilly-fair premium, and that the redistribution is costless. This could be thought of as a state-run venture.

simply equal to the value of the assets held by consumers who died in that period. It follows that the *effective* rate of interest for surviving individuals is $(1+rcon)/(1-\gamma)-1$. This is an important feature to which we will return later.

Utility maximisation

The consumer's problem is to maximise expected utility, given by equation (3.2), subject to the budget constraint in equation (3.3). The first-order condition from this problem is that

(3.4) $$cfl_{a,t+1} = \left[\delta(1+rcon)\frac{pc}{pc_{t+1}}\right]^{\sigma} cfl_{a,t}$$

which neatly states the problem independently of γ.[19]

Wealth

It is convenient at this stage to explain the concepts of *human wealth* and *total wealth*. Human wealth, *hwfl*, is defined as the expected value of labour income and risk income:

(3.5) $$hwfl_{a,t} = \sum_{s=0}^{\infty}(1-\gamma)^s \frac{ydfl_{a,t+s} + risk_{a,t+s}}{\prod_{i=0}^{s-1}\left(1+rcon_{t+i}\right)}$$

while total wealth, *twfl*, is defined as the expected value of the individual's consumption stream:

(3.6) $$twfl_{a,t} = \sum_{s=0}^{\infty}(1-\gamma)^s \frac{pc_{t+s}cfl_{a,t+s}}{\prod_{i=0}^{s-1}\left(1+rcon_{t+i}\right)}$$

Total wealth can be also thought of, perhaps more intuitively, as human wealth plus the discounted value of income from financial assets.

These equations can be usefully written in Euler equation form. Respectively,

(3.7) $$hwfl_{a,t} = ydfl_{a,t} + risk_{a,t} + \frac{1-\gamma}{1+rcon_{t+1}}hwfl_{a,t+1}$$

and

(3.8) $$twfl_{a,t} = pc_t c_{a,t} + \frac{1-\gamma}{1+rcon_{t+1}}twfl_{a,t+1}$$

Then, from the budget constraint, total wealth, human wealth and financial assets are related by

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[19] The probability of death influences the effective discount factor and the effective rate of interest in the same manner. Since the marginal rates of substitution between the levels of consumption at t and t+1 equal the market discount factor, it follows that the discrepancy between the effective (risk-adjusted) rate of interest available to individual consumers and the market (risk-free) rate applicable to the model society does not distort the intertemporal allocation of consumption.

(3.9) $$twfl_{a,t} = \frac{(1+rcon_{t-1})}{1-\gamma} fa_{a,t-1} + hwfl_{a,t}$$

The standard first-order conditions lead to a law of motion for consumption,

(3.10) $$pc_t cfl_{a,t} = mpcw_{a,t} twfl_{a,t}$$

where the marginal propensity to consume out of wealth is given by

(3.11) $$mpcw_{a,t}^{-1} = (1-\gamma)\delta^{\sigma}\left(\frac{pc}{pc_{t+1}}(1+rcon_t)\right)^{\sigma-1} mpcw_{a,t+1}^{-1} + 1$$

Since we assume that the utility function exhibits constant relative risk aversion, with elasticity of intertemporal substitution σ, the coefficient of relative risk aversion is $1/\sigma$. For risk-neutral agents $\sigma = 1$. The marginal propensity is then simply one minus the effective discount rate. The more agents discount the future - either because they expect to die sooner or they simply prefer present to future consumption - the more they consume out of their total wealth in each period. When σ is less than one, the propensity to consume is now also a function of the sequence of future interest rates and the path for consumption prices. The lower is the value for σ, the larger is the interest rate needed to induce consumers to save more and thereby tilt the time profile for their consumption path upwards. Equivalently, a lower value for σ means that, given a level of interest rates, individual savings will be lower.

3.1.2 Rule-of-thumb consumers

Rule-of-thumb consumers receive labour income each period and, after taxes and transfers from the government, are left with disposable income. They consume all of that disposable income each period. Formally, for a consumer born at time *a*, consumption at time *t*, *crt*, is given by:

(3.12) $$pc_t crt_{a,t} = ydrt_{a,t}$$

where *ydrt* is disposable income for rule-of-thumb consumers.

3.1.3 Aggregate equilibrium and behaviour

The aggregation problem

Two basic problems arise when we try to move from individual to aggregate behaviour. First, the existence of overlapping generations with differing propensities to consume and differing levels of wealth means that we cannot aggregate a consumption function.[20] The assumption that the probability of death, γ, is constant overcomes this problem. As a consumer's expected lifetime is given by $1/\gamma$, this expectation is now constant throughout his or her life. Hence agents are of different ages, with differing levels of wealth, but all have the same horizon and propensities to consume.

Second, an implication of equations (3.9) to (3.11) is that agents aim to consume everything out of their expected lifetime incomes. Since lifetimes are not known with

[20] See Modigliani (1966).

certainty, some individuals will die holding assets. To facilitate aggregation, some mechanism is required to deal with these assets; they cannot simply be ignored. This is where the existence of the insurance company comes in. Residual income passes to the insurance company that distributes it costlessly to those still alive who hold financial assets. Stated more formally, individual consumption depends on total individual wealth, with propensity $1-(1-\gamma)\delta$ (assuming risk neutrality for the moment), where the discount rate for labour income is $rcon\cdot(1-\gamma)$. Aggregate financial wealth, however, accumulates at the net rate *rcon*. The difference is a transfer, via the assumed life insurance company, from those who die to those who remain alive. It is not therefore an addition to aggregate wealth. In effect, uncertainty about the time of death is netted out in the aggregate by the existence of the insurance company. [21]

With these assumptions in place we can derive an aggregate consumption function. First, let $n_{a,t}$ denote the size of the cohort born at time *a* that is still alive at time *t*. Then aggregate quantities are related to individual quantities according to

(3.13) $$x_t = \sum_{a=-\infty}^{t-1} n_{a,t} x_{a,t}$$

which embodies an assumption that agents do not consume in the first period of life (or, alternatively, all births are at the end of the period).

Having assumed a fixed probability of death for the individual, the aggregate number of deaths, by age cohort, is fixed and known with certainty.[22] Then, the following holds precisely for the size of the cohort born at time *a*:

(3.14) $$n_{a,t} = (1-\gamma) n_{a,t-1}$$

and the total population itself evolves according to

(3.15) $$n_t = (1-\gamma) n_{t-1} + \beta (1-\gamma) n_{t-1}$$

where β is the birth rate.[23]

Aggregate equilibrium for forward-looking consumers

Forward-looking consumers' budget constraints aggregate to:

(3.16) $$pc_t cfl_t + fa_t = (1+rcon_{t-1})\, fa_{t-1} + ydfl_t + risk_t$$

given the assumption that financial wealth is zero at birth.

We assume that both disposable income and risk are identical for consumers of all ages. In this case, human wealth is independent of age and evolves according to

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21 Equivalently, we can assume the existence of annuities. This is to ensure that agents can insure against uncertainty and make their choice of a utility-maximising savings rate. (See Yaari, 1965.) Another, equivalent, way of looking at it is that this ensures that there are no unintended bequests.

22 We are implicitly assuming large numbers of cohorts.

23 It is assumed that the people who are to die in a period do so before they have any children.

(3.17) $$hwfl_t = ydfl_t + risk_t + \frac{1-\gamma}{1+rcon_{t+1}} hwfl_{t+1}$$

Aggregate total, human and financial wealth are related by

(3.18) $$twfl_t = (1+rcon_{t-1})\, fa_{t-1} + hwfl_t$$

and, from the first-order condition,

(3.19) $$pc_t cfl_t = mpcw_t twfl_t$$

where the marginal propensity to consume out of wealth is given by

(3.20) $$mpcw_t^{-1} = (1-\gamma)\delta^{\sigma}\left(\frac{pc_t}{pc_{t+1}}(1+rcon_t)\right)^{\sigma-1} mpcw_{t+1}^{-1} + 1$$

Completing aggregate equilibrium

Aggregation for rule-of-thumb consumers is trivial. Equation (3.12) simply becomes

(3.21) $$pc_t crt_t = ydrt_t$$

and aggregate consumption follows

(3.22) $$c_t = cfl_t + crt_t.$$

We use a parameter, λ, to set the proportion of rule of thumb consumers - λ splits disposable income into two streams for the two types of consumers.

Aggregate equilibrium behaviour

In the limiting case where agents do not expect to die, agents discount the future according to the parameter δ. Given a constant probability of death, the effective discounting is $(1-\gamma)\delta$, so alternatively, even if δ is equal to one, agents will discount the future so long as they have expectations of finite lifetimes. This makes the effective discount rate higher, the higher the probability of death. This results in an important property called 'overdiscounting,' wherein households discount the future more heavily than would a social planner or an infinitely-lived household. This property has been found to be very useful in reconciling the predictions of forward-looking models of consumer behaviour with the data.

Under various limiting assumptions, this set-up produces well-known results. The general framework is consistent with the Permanent Income Hypothesis of Friedman (1957).[24] By letting the probability of death and the birth rate go to zero, we obtain the infinite horizon case, expressed as a representative agent problem. Then the equation of motion for consumption reduces to Hall (1978).[25]

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[24] However, note that the "main drawback of this approach is that it captures the finite horizon aspect of life but not the change in behaviour over life, the 'life-cycle' aspect of life." (Blanchard, 1985, p.224) Extensions to the Blanchard-Weil-Buiter framework to capture life-cycle effects have recently been outlined in Faruqee, Laxton and Symansky (1996).

[25] For further comments on the distinction between the overlapping generations model and the infinite horizon case, see Buiter (1988).

Equivalently, one interpretation of the overlapping-generations framework is to think of families or cohorts, rather than individuals. In this case, γ describes the probability that current members of the family do not have a bequest motive. Hence if γ and β (the birth rate) were set to zero, Ricardian equivalence would hold.

3.1.4 Dynamic behaviour

The dynamic equations take this underlying theoretical structure and add a number of elements. At this level we add polynomial adjustment costs (see "The structure of adjustment costs", box) and a direct effect of monetary policy. Additionally, we add an income-cycle effect that makes forward-looking consumer behaviour more sensitive to income than predicted by the formal theory. This, of course, provides another marginal channel of influence for monetary policy, since monetary actions affect the aggregate income cycle through other channels.

3.2 The firm

The firm is the second optimising agent in the core model. The formal introduction of a supply side requires us to go beyond the simple endowment economy of the Blanchard-Weil-Buiter framework. The firm is modelled in standard fashion, but, as with the characterisation of the consumer, some extensions are made that help capture important features of the New Zealand economy.

The structure of adjustment costs

This box contains a description of the general framework used to characterise the intrinsic dynamics in the core model, arising from adjustment costs.[26] Adjustment costs influence consumption by forward-looking households, capital formation, import shares, exports, relative prices and unemployment.

The essential idea behind the adjustment cost structure is that the adjustment path is chosen to balance two types of cost: the loss from not being at the 'optimal' value (that is, the cost of being in disequilibrium), and the costs of adjusting to that optimal value. These adjustment costs include a cost of change (a first difference) and a cost of changing the rate of adjustment (a second difference). The latter can be termed the 'speed' at which changes are put into effect. Generally, for analytical tractability, all elements of the general cost function are assumed to be quadratic. In such a case, the net effect is to introduce inertia into the process of adjustment, with a quasi-linear structure and with two leads and lags in the Euler equation.

Let y be the variable to be chosen, and y^* its 'desired' or 'equilibrium' value. The general intrinsic adjustment model is derived from choosing $y_{t+\tau}$, all $t > 0$, to minimise the following cost function:

(A1) $$E\left[\sum_{\tau=0}^{\infty}\left(y_{t+\tau}-y^{*}_{t+\tau}\right)^{2}+\sum_{i=0}^{n}\phi_{i}\left(A_{i}(L)y_{t+\tau}\right)^{2}\right]$$

[26] See also Pesaran (1991) and Tinsley (1993).

where $A_i(L)$ is a polynomial in the lag operator (see below) and *n* is the order of the adjustment process. The first term of this equation penalises being away from the desired value at time $t+\tau$. The second term is chosen so as to penalise changes in *y*. Having high weights in ϕ will lead to gradual adjustment.

In FPS, two different forms for A_i are used, depending on whether the adjustment is for a stock or not. If the adjustment is for a flow or relative price, then A_i is chosen so that changes in the *i*th (equally-weighted) moving average of *y* are penalised. This change is given by

(A2) $$\sum_{j=0}^{i}\left(y_{t+\tau-j}\right)-\sum_{j=0}^{i}\left(y_{t+\tau-1-j}\right)=y_{t+\tau}-y_{t+\tau-1}$$

so that A_i is given by

(A3) $$A_i(L)=1-L^{i+1}$$

For stocks, A_i is chosen so that changes in the *i*th difference of *y* are penalised. This leads to the definition

(A4) $$A_i(L)=(1-L)^i$$

Having these two different styles of adjustment costs facilitates the calibration of the model. For flows, the data suggest that past flows are good indicators of the current flow. This is captured in the model by placing high weights on changes in moving averages which penalises movements in *y* compared to what it was in the past. For stocks, the problem is slightly different. In this case, it was found that when the first form of adjustment is used, the flows resulting from changes in the stock are very volatile. The second form, which penalises the differences, provides a similar path for the level of the stock, but one with smoother changes in flows.

In the core model, each application is customised to include special features to capture the interactions among variables in 'disequilibrium' cycles. These terms give us more scope to calibrate the model to reflect the stylised facts of economic cycles. One can think of these terms as reflecting information such as would be attained from estimated VARs linking the disequilibrium movements of the variables. In a sense, this has the effect of making the adjustment parameters sensitive to economic variables, as in Smyth (1984), Ouliaris and Corbae (1985) and Rose and Selody (1985), for example. However, in our analysis, this information is overlaid on a quadratic structure with fixed parameters, whereas these authors make the parameters of a first-order process functions of economic variables.

In FPS, investment and capital are modelled from the perspective of a representative firm. This firm acts to maximise profits subject to the usual constraints. It is assumed that these firms are perfectly competitive, with free entry and exit to markets. Firms produce goods, pay wages for labour services, and purchase capital. We further assume an open economy where real interest rates are determined by foreign capital markets in the long run. There are only two physical inputs to production: labour and capital. The production technology is Cobb-Douglas, with constant returns to scale and a constant elasticity of output with respect to capital, α.

The solution to the firm's profit maximisation problem determines the level of capital stock, output, and the real wage. In the event that total factor productivity changes, the firm adjusts the level of the capital stock, and consequently output changes by more than just the change in productivity. There are, of course, other ways in which the firm may be modelled. However, this way is simple and leads to a model of the firm that is compatible with balanced growth.

This framework is extended in two directions. First, firms face adjustment costs for capital. That is, when firms adjust their level of investment they incur an additional cost over and above the cost of the investment goods themselves. The behavioural implication of this is that the firm tends to adjust its desired capital stock more slowly than it would in the absence of such costs.[27] Second, firms face a time-to-build constraint. This means that investment today becomes effective for producing output only a number of periods into the future. It has the important implication that an investment boom precedes the consequent increase in productive capacity.

3.2.1 The representative firm

The firm's maximisation problem

Firms maximise a discounted cash flow stream subject to a production function and two accumulation constraints. The profit each period is the return to capital, which is taxed at the corporate tax rate, plus depreciation allowances, less investment expenditures. The firm's profit each period is therefore given by

(3.23) $$\Pi_t = (1 - tk_t)\cdot[pfc_t y_t - w_t l_t] + depr \cdot tk \cdot kt_{t-1} - pi_t i_t,$$

where *tk* is the effective tax rate on net capital income, *w* represents the real wage, *l* the level of labour, *depr* the depreciation rate, *kt* the level of capital stock for tax purposes, *i* is investment, *pi* is its relative price, and *pfc* is price at factor cost.

Production and capital

The production function for the firm is given by a standard Cobb-Douglas function in labour and capital:

(3.24) $$y_t = tfp_t \cdot kp_{t-1}{}^{\alpha} \cdot l_t{}^{(1-\alpha)},$$

where *tfp* represents total factor productivity, *kp* is a measure of productive capital stock, and α is the exponent on capital. An implication of the conditions of a competitive equilibrium is that this parameter also represents the equilibrium share of income that will accrue to capital.

Equilibrium capital stock comes from the first-order condition relating the marginal product of capital to the cost of capital:

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[27] This is an important step towards matching the stylised facts of the business cycle. In typical RBC-type models, small changes in interest rates produce large revisions in desired capital and therefore large swings in investment. Leeper and Sims (1994) find that introducing adjustment costs allows their business cycle model to better replicate observed investment patterns.

(3.25) $$\alpha \cdot pfc_{t+1} y_{t+1} \cdot (1 - tk_t) = (1 - rk_t) \cdot pk_t - (1 - depr) \cdot pk_{t+1},$$

The return on capital will depend on capital's share of income, the level of firm taxation, *tk*, and the price of outputs at factor cost. We put an element of forward-looking behaviour in the firm's equilibrium path by making the return to capital a function of expected output and expected output prices. We do this to parallel the general idea in the Blanchard-Weil-Buiter framework of household choice.

The cost of capital is positively related to interest rates and depreciation, multiplied by the shadow price of physical capital, *pk*.[28] In turn, this price is related to the price of investment goods, and the tax treatment of capital.[29] When all prices are one and the rate of capital taxation is zero, the expressions (3.24) and (3.25) collapse to a more easily readable form:

(3.26) $$kp_t = \frac{\alpha \cdot y_{t+1}}{rk_t + depr}$$

Investment

Given desired capital stock, the investment flows required follow from the perpetual inventory identity. In the steady state, a constant flow of investment is therefore required to replace depreciation and build the stock at the balanced growth rate.

3.2.2 Extensions to the representative firm

Time-to-build

In practice, the accumulation identity for productive capital stock is modified slightly:

(3.27) $$kp_t = (1 - depr) \cdot kp_{t-1} + \mathrm{A}(\mathrm{L}) \cdot i_t$$

where $\mathrm{A}(\mathrm{L})$ is an *n*-dimensional polynomial in the lag operator. The interpretation of the accumulation equation for productive capital, *kp*, is that investment creates productive capacity only with an additional delay. If the coefficients in A are 'hump-shaped', as they are in FPS, then investment comes on line slowly at first, and then faster, until after about eight quarters it is fully on line. This time-to-build constraint has important implications for the way monetary policy responds to supply shocks. Since supply conditions do not adjust immediately following such shocks, the required monetary response is less than it would otherwise have been.

Tax and depreciation

The interactions of tax and depreciation have important effects on the cost of capital that should be captured in a general equilibrium model. However, the treatment of capital for tax purposes is made difficult by complex revaluation effects of specific types and vintages of capital. We employ some simplifications to make the problem tractable. First, whilst in reality different types of capital have different depreciation rates for tax purposes, in this model all capital is treated in the same way. Second, tax

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28 The shadow price follows from the usual Lagrangian specification. In this way the cost of capital includes the opportunity cost of deferred investment.

29 Note that in this expression, in order to get contemporaneous capital and cost of capital terms, the depreciation variable has to enter with a corresponding lead.

credit systems are based on historical cost accounting procedures, so that it is the size of the nominal expenditure that matters for tax purposes. This implies non-superneutralities arising from the interaction of inflation and the cost of capital, but we deliberately exclude these.

3.2.3 Dynamic behaviour

Since the model is written such that investment flows are solved from the capital cumulation identity, the dynamic adjustment structure is added to the actual capital stock equation. Polynomial adjustment costs, the effects of monetary policy, and expectational dynamic effects all influence the actual level of capital. The dynamic path for actual investment flows reflects this. As with the specification of all dynamic adjustment processes in the model, the actual level of capital converges to the desired level.

3.3 Government in FPS

Government has the power to collect taxes, raise debt, make transfer payments, and purchase output. As with households and firms, the structure of the model requires clear objectives for government in the long run. However, whereas households and firms have explicit maximising objectives, we directly impose fiscal policy choices for government debt and spending.

Government faces a binding intertemporal budget constraint. We use this to solve for the labour income tax rate that supports the fiscal choices. Government is non-neutral in the model, and the interactions of debt, spending and taxes create powerful effects through the rest of the model economy via the overlapping generations structure.

3.3.1 Policy choice and the fiscal reaction function

Government's behaviour is encapsulated by the intertemporal budget constraint that balances revenues against expenditures. We model government as a relatively simple entity that buys output and transfers resources to households. It finances these activities by borrowing or taxing.

Revenue sources in the model include new debt issue (borrowing), direct taxes on households and firms, and indirect taxes (including import tariffs).[30] Expenditures include the cost of servicing debt, government purchases, and transfers to households. The budget constraint therefore takes the form

(3.28) $$gb_t + t_t^{iy} + t_t^k + t_t^l = \left(1 + rgb_{t-1}\right) \cdot gb_{t-1} + pg_t \cdot g_t + gtr_t ,$$

where *gb* is the stock of government debt; *t* represents tax revenues from indirect taxes, capital, and labour income, respectively; *rgb* is the interest rate on government debt; *g* represents government expenditures; *pg* is their relative price; and *gtr* represents transfers to the household.[31]

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30 We ignore the potential for seignorage and the inflation tax on nominal balances as a source of revenue.

31 This specification could also include transfers to firms (subsidies), but there are none at present.

This budget constraint is essential to the workings of the stock-flow accounting mechanism in the model. However, the budget constraint by itself is not sufficient to ensure long-run equilibrium. We impose fiscal policy choices for long-run debt, transfers to households and government spending; all expressed as ratios to output. Taxes on firms' profits and the six indirect tax rates are also exogenous.[32,33] The labour income tax rate is then set to ensure that the target debt ratio is achieved. In this way, the intertemporal budget constraint functions as a fiscal reaction function.

Conceivably, any of the variables could be made the fiscal instrument - this is a reaction function that can be customised to do whatever is wanted for any particular application. There is only an absolute restriction that the intertemporal budget constraint must be respected in any formulation - that is, we must get to the target debt and expenditure ratios. That said, in general we find it clearer to characterise government in terms of choices for debt and spending, leaving taxes to adjust over time.[34] Moreover, this configuration is a good match to the current institutional environment, where clear, stable fiscal objectives for spending and debt are required under the 1994 *Fiscal Responsibility Act*. Similarly, if taxes are to be the fiscal instrument, then there is no technical reason why labour income taxes should be endogenous. However, it is the direct income tax that is most fully integrated into the choice theory. Thus we are best able to capture the general equilibrium consequences of the budget constraint when we use this as the residual tax.

In real life, government has an important role in receiving and redistributing income. In some models, where an emphasis is placed on the distributional aspects of fiscal policy, welfare functions are used to derive optimal levels of debt, expenditures and taxes. We choose to take announced fiscal policy targets in the current institutional framework as given, and let the model derive the general equilibrium consequences for the real and nominal sides of the economy. Moreover, while the strength of the Blanchard-Buiter-Weil framework is the clarity it imposes on intertemporal issues, its comparative weakness is in distributional issues. For these two reasons the behavioural element of government is restricted to that implied by the workings of the intertemporal budget constraint.

3.3.2 Government's dynamic behaviour

We characterise the dynamic paths for government purchases and transfers by simple partial adjustment structures where the actual value converges to its steady-state value over time.

In keeping with the usual core model structure, the equilibrium labour income tax path and the actual path are modelled separately. While the equilibrium labour income tax rate must support the steady-state debt target, its dynamic adjustment path must bring debt to that target level, contingent on all the other fiscal settings. The dynamic

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32 The model tax structure conceptually includes direct taxes on households' income and firms' profits, and indirect taxes on consumption, investment and government expenditures. A further layer is provided by indirect taxes on imported goods.

33 Note that this implies that the effective overall indirect tax rate will change if the relative shares of consumption, investment and government expenditures or their relative prices change.

34 For the purposes of closing the model, there are a number of potential rules. Debt, for example, could be made the fiscal instrument. However, in the non-Ricardian-equivalent environment of FPS, this would mean that there would be no unique steady state.

equation also differs by the introduction of autoregressive terms. This is to simulate the notion of policy inertia. Note that this *does not* mean that the intertemporal budget constraint is violated; it simply means that the government does not attempt to adjust taxes too quickly to try to achieve its target fiscal positions at all times. We then introduce extra terms which draw a distinction between tax rate changes that come about in reaction to other changes in the economy and tax changes that reflect a permanent change in fiscal policy. Other tax rates are specified exogenously.

3.3.3 Government's effects on other agents

Government and households

There are important points regarding the relationship between government and consumers. Since labour supply is exogenous, consumers behave as though the labour income tax is a lump sum tax, even though it is specified as a rate on labour income. However, even though taxes are non-distortionary, Ricardian equivalence does not hold in the model. Consider a temporary decrease in taxes at time *t* associated with a permanent increase in debt and taxes at a later date. The effect on forward-looking consumers at time *t* is given by the effect on human wealth. Note that consumers have a positive probability of death, whereas government is infinitely-lived and has the power to tax the unborn. Consumers therefore discount at a higher rate than government. Since human wealth is the present discounted value of non-interest income minus taxes, discounted at the rate $(1+rcon)\cdot(1-\gamma)$, a decrease in taxes today will temporarily increase disposable income and hence consumption. Only in the limiting case where consumers expect to live forever and there is no population growth will Ricardian equivalence hold.[35]

In the pure Blanchard framework, government debt displaces foreign assets in households' wealth. The displacement is one for one when the interest rate equals the rate of pure time preference. It will be less than one when (as is the case in the model) the interest rate is less than the rate of pure time preference. These results are quite different to the infinite horizon case, where the level of government debt has no effect on the steady-state level of financial assets.

Government and firms

Government directly affects firms by taxing their incomes. Taxes also play an important role in the cost of capital via those indirect taxes which affect the relative price of investment goods. Government also affects expected returns in the short run through its influence on aggregate demand.

Government and the monetary authority

In terms of the monetary policy problem, fiscal policy affects aggregate demand in three ways. First, the sequence of labour income taxes will usually affect human wealth and thus consumption. Second, the structure of indirect taxes (such as tariffs) has important effects on relative prices, which will affect the levels of consumption

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35 The inclusion of birth in the model is of high practical significance. The addition of new individuals or cohorts to the economy with a given death rate raises the 'turnover' rate of individuals, making the model less Ricardian equivalent. This extension by Weil (1989) is an important step towards matching observed behaviour.

and investment. Finally, the level of government spending affects aggregate demand directly.

3.4 The external sector

Unlike domestic agents, foreign agents are not modelled behaviourally. They are represented via a set of exogenous conditions determined in world markets, namely the prices for goods and capital. We assume that they are unaffected by the actions of domestic agents.

3.4.1 Foreign debt

In section 3.1, we saw how the household model pins down a value for financial asset equilibrium. By imposing the asset identity

(3.29) $$fa_t = k_t + gb_t + nfa_t$$

we solve for a level of net foreign assets, *nfa*, that satisfies the domestic financial asset decision. The net foreign asset position reconciles the government's debt level, firms' equilibrium level of capital stock, and households' desired financial asset position.

Several combinations of steady-state consumption and holdings of net foreign assets potentially exist in this framework. They depend on the basic specification of household choice. If the equilibrium real interest rate, which is determined in the rest of the world, equals the pure rate of time preference, then domestic agents would neither borrow from nor lend to the external sector. They would be exactly as impatient as the agents in the rest of the world. If the interest rate is greater than the pure rate of time preference, then the ratio of net foreign assets to output is positive; the country is a net creditor. If the interest rate is lower than the pure rate of time preference, then the country is a net borrower and the ratio of net foreign assets to output is negative.

An increase in the real interest rate determined in the rest of the world therefore increases the level of foreign assets. This is analogous to agents in the rest of the world becoming less patient relative to domestic agents. An increase in the probability of death, γ, is analogous to domestic agents becoming less patient, implying lower domestic accumulation of assets.

3.4.2 External trade

The trade balance and current account

Having determined the level of net foreign assets, the same stock-flow logic that applies to the rest of the model now determines external trade. Through an accumulation equation for net foreign assets, we have a level for the equilibrium trade balance, *xbal*, consistent with this choice for net foreign assets,

(3.30) $$nfa_t = \left(1 + rnfa_{t-1}\right) \cdot nfa_{t-1} + xbal_t$$

where *rnfa* is the interest rate on net foreign assets. In effect, the equilibrium real exchange rate gets determined to support this equilibrium. For example, if higher net exports are required in equilibrium, then the equilibrium real exchange rate will depreciate to support this higher net export position.

Given an equilibrium solution for net foreign assets, the current account is then the sum of the trade balance and interest payments on net foreign assets, or alternatively, the rate of change of net foreign assets.

Imports

The model is configured under the assumption that a share of consumption, investment and government expenditures comes from foreign markets, and that this exhausts imports. The levels of consumption, investment and government imports are determined endogenously as proportions of total consumption, investment and government expenditures. The proportions of imports in these expenditures are made functions of their respective import prices (including tariff effects), relative to the price of domestically-sourced equivalents. Hence a permanent depreciation of the real exchange rate or a rise in the relative world prices of these goods will lower the proportions of consumption, investment and government expenditures that come from abroad.

Exports

The equilibrium level of exports is made a function of the domestic currency prices of exports in the rest of the world. Either a permanent increase in rest of world prices or a permanent depreciation of the real exchange rate will increase exports.

3.4.3 Net export dynamics

In keeping with the core model structure, net export dynamics are a function of costs of adjustment, disequilibrium in relative prices (which will capture temporary exchange rate effects), and disequilibrium in domestic demand.

3.5 Income and the labour market

The previous four sections describe the expenditure flows that follow from asset equilibrium. In this section, we complete the description of the real economy by accounting for income flows. Part of this has already been dealt with: households receive income from assets (as defined in section 3.1.1), and firms receive income from production (as described in section 3.2.1). It remains to describe income accruing to households from the labour market.

At the same time the labour market completes the representation of the real economy, it leads us to the nominal economy. Workers care about the purchasing power of their real wages, and the determination of nominal wages has important dynamic effects on the overall inflation process.

3.5.1 Real wage equilibrium

Equilibrium labour supply is exogenous in the model.[36] In order to quantify labour supply, we normalise units such that each person supplies labour services at the rate of one unit per year, when employed. We then simply specify an explicit terminal condition for the equilibrium unemployment rate, u, and an implicit terminal

[36] Since the utility function does not include consumption as an argument, there is no labour-leisure trade-off in the FPS model.

condition for the participation rate. In the model code, the labour input is therefore given by (1-u). The growth rate of labour input is then determined by the population growth rate.

Given labour supply, the equilibrium real wage, w, is derived from the standard condition for the marginal product of labour,

(3.31) $$w_t = (1-\alpha)\cdot pfc_t \cdot \frac{y_t}{1-u_t}$$

The real wage here is measured in the units of the numeraire price, the price of domestically produced and consumed goods at factor cost - that is, the money wage is deflated by the numeraire price. This explains the presence of the price at factor cost, *pfc*, in the equation; it converts the units to factor cost as required for the firm's first order condition. We also include two other measures of real wages in the model. The producer real wage, *wp*, is deflated by the relative price of output at factor cost. The consumer real wage, *wc*, is deflated by the relative price of consumption goods at factor cost.

3.5.2 Household income

Labour income, *ylab*, is determined by aggregating wages of those employed:

(3.32) $$ylab_t = w_t \cdot (1-u_t)$$

We then take into account personal direct taxes to define disposable income. We split the result between the rule-of-thumb consumers and the forward-looking consumers, using the proportion λ. Disposable income also includes government transfers; proportionately more transfers are directed to rule-of-thumb consumers.

3.5.3 Nominal wages

The introduction of a measure for the consumer real wage allows us to approach the dynamics of nominal wage determination. We introduce a structure that captures the essential features of a bargaining environment, as well as providing the basis for an explicit account of how wage costs influence the inflation process.

The core of this structure is embedded in an equation that models nominal wage growth arising from various pressures in the labour market. Some of these arise from the state of the business cycle. For example, upward pressure can come from shortages in labour, as accounted for by a labour supply gap. A gap term on total factor productivity and its equilibrium value allows for a wage effect of firms producing more or less than the normal level of output with the existing stock of capital. We allow this to have an effect on wages without a simultaneous effect on unemployment. This term can be thought of as capturing a cyclical 'hours' effect on wages.

Other pressures may arise from disturbances to the economy. Suppliers of labour care about their wage in terms its purchasing power over consumption goods, and this plays a role in the determination of nominal wage growth. Various terms create the bargaining metaphor: workers are mindful of the erosion of their purchasing power, and this leads them to target a consumer real wage that may diverge from the firm's profit maximising real wage. For example, changes in indirect tax rates or import prices will generate a temporary compensating upward movement in nominal wage demands. The extent to which these pressures translate into changes in nominal wage inflation depends on the state of the business cycle.

Bargaining from the worker's perspective is asymmetric - there is no resistance to rises in the nominal wage, but they are sticky downwards. There is also inertia in the bargaining environment, since only a fraction of labour contracts come up for negotiation each quarter. Eventually, however, nominal wage inflation, adjusted for productivity growth, must converge on numeraire price inflation for equilibrium to hold in the nominal economy. Note that there is also a level condition that ensures that real wages converge to the marginal product of labour.

3.6 The monetary authority, interest rates, and the exchange rate

In this section, we turn to the last of the five agents, the monetary authority. The role of the monetary authority in FPS is to anchor the nominal side of the economy. We specify a monetary policy target for the inflation rate and characterise the actions of the monetary authority by a forward-looking reaction function.

We begin by describing the equilibrium structure of interest rates. In the long run, real interest rates are exogenous. However, the monetary authority has the ability to move short-term nominal interest rates temporarily. This allows it to alter the price of consuming and investing today versus tomorrow. Further, because prices are sticky in the short run, movements in nominal rates are translated into changes in real interest rates that affect the real exchange rate through the Uncovered Interest Parity condition. The monetary authority is able to influence aggregate demand through these two channels, thereby achieving the demand conditions required to reach its inflation rate target.

We draw a distinction between inflation and inflation expectations (see section 3.7). This structure implies that a necessary condition for monetary equilibrium is that inflation expectations are anchored at the policy target.

3.6.1 The equilibrium structure of interest rates

Before describing the monetary authority, we need to provide a structure that determines the equilibrium values of interest rates. The stylised facts from financial markets strongly suggest that interest rates are systematically different across asset types. For example, the equity premium leads us to expect a higher interest rate for investment finance than on sovereign debt.[37] The framework that addresses these issues is very simple, but provides a full equilibrium structure of real and nominal rates into which we can introduce a monetary authority.

To begin with, we assume a small, open economy with well-integrated capital markets. We therefore model an equilibrium short-term real interest rate as an exogenous world real interest rate plus an exogenous sovereign risk premium. In modelling long-term (10-year) interest rates in FPS, we exploit the measure of the

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[37] See Mehra and Prescott (1985)

hypothetical 5-year rate that comes from the expectations theory of the term structure of interest rates. The model computes a hypothetical 5-year rate in the usual manner, from the compounded sequence of short rates, plus an exogenous term premium. The model's long rate is assumed to have the same equilibrium value as this 5-year rate, but we add dynamic terms to reflect the fact that long rates tend to be more closely correlated with short rates in the cycle than would be suggested by the expectations theory using model-consistent expectations.[38] Equilibrium real rates for households, government, firms and the rate on net foreign assets are similarly modelled as the ten-year rate plus the appropriate exogenous risk premia.

Then it remains to determine nominal rates. Given an *ex ante* equilibrium inflation rate, equilibrium nominal short and long rates can be solved for sequentially. An identity for the slope of the yield curve then completes the equilibrium structure.

3.6.2 The monetary authority

Whereas the objectives of the other agents anchor the real side of the economy, the role of the monetary authority is to anchor the nominal side. To do this we have to specify a nominal anchor. In FPS, we approximate the 0 to 3 per cent target range specified in the current *Policy Targets Agreement* (PTA) by assuming an inflation rate target of 1.5 per cent annual CPI inflation. In the steady state, all relative prices converge to fixed values and all domestic prices therefore grow at the target rate of inflation.

Although specifying the target is straightforward, there are many different ways to characterise the way that the monetary authority acts to achieve this target. In FPS, the behaviour of the monetary authority is characterised by a forward-looking reaction function that adjusts nominal short-term interest rates when projected inflation deviates from the policy target:

(3.33) $$rs_t - rl_t = rs_t^* - rl_t^* + \sum_{i=1}^{j} \theta_i \left(\pi_{t+i}^e - \pi_{t+i}^T \right),$$

where *rs* and *rl* are short and long nominal interest rates, respectively; rs^* and rl^* are their equilibrium equivalents, π^e is expected inflation and π^T is the policy target.

There are several points worth noting about this specification. First, while it is the slope of the yield curve that appears on the left hand side, we emphasise that the short-term nominal interest rate is the policy instrument. Using the slope of the yield curve offers a parsimonious method of introducing the full information of the term structure into agents' decision-making. Since long rates can move in response to changes in inflation expectations, it also has the advantage of making the distinction between non-monetary shocks and monetary actions. Second, we specify the dynamic path for the slope of the yield curve around the equilibrium slope and this combined with an equilibrium condition on short-term interest rates ensure that the real interest rate converges to equilibrium. Third, the number of leads, *j*, and the weights on them, θ_i,

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[38] We use the 5-year horizon to avoid doubling the number of leads that would be implied if we were to model the 10-year horizon formally. This simulation speeds simulation times considerably.

are a calibration choice. The function has positive weights on leads six to eight (quarters). They are set this way to proximate the policy horizon that the Bank focuses on, reflecting the fact that monetary policy must be forward looking if inflation is to be stabilised at the target rate. Finally, the variable targeted in the reaction function is the annual rate of change in a consumer price index, conceptually ex-interest and indirect consumption taxes, reflecting the outline of the PTA.

3.6.3 Exchange rates

The equilibrium exchange rate is determined by the trade flows required to service the equilibrium net foreign asset position.

The dynamic path for the real exchange rate is determined by a version of the uncovered interest parity condition. Often the biasedness of forward exchange rates as predictors of future spot rates is taken as proof that UIP does not hold, and given the large empirical literature rejecting unbiased forward rates, it would appear brave to include this as a key behavioural relationship in a formal model.[39] However, the results are perfectly consistent with a system in which the monetary authority uses the link from real interest rates to the real exchange rate to target some nominal variable.[40]

However, the exchange rate does not act as a pure 'jumper' because its expected value has some inertia from a lag as well as some weight on the equilibrium value. Note also that the UIP condition does not bind precisely in the real exchange rate equation.

(3.34) $$v_t^e = \mu_1 \cdot v_{t+1} + \mu_2 \cdot v_{t-1} + \left(1 - \mu_1 - \mu_2\right) \cdot v_{t+1}^*$$

(3.35) $$v_t = \eta_1 \cdot v_{t-1} + \eta_2 \cdot v_t^e \cdot \frac{\left(1 + r_t\right)}{\left(1 + rrow_t + rp_t\right)} + \left(1 - \eta_1 + \eta_2\right) \cdot v_t^*$$

Equation (3.34) describes the expectation of the real exchange rate as a function of own lags and leads, as well as a term, v^*, representing its fundamental long-run value. (In these equations, the exchange rate is measured as the price of domestic currency in units of foreign currency. A rise in v is therefore an appreciation.) The actual dynamic equation for v is presented in equation (3.35), where *rrow* represents real interest rates in the rest of the world and *rp* is a sovereign risk premium. The nominal exchange rate, *s*, is solved for using the ratio of foreign and domestic price levels.

3.6.4 The transmission mechanism

Some have suggested classifying transmission channels of monetary policy into the effects from intertemporal substitution, the exchange rate, cash flows, wealth and assets, and credit and liquidity.[41] Most of these channels are reflected in FPS. However, the highly simultaneous nature of the model does not permit us to provide an explicit decomposition of the effects. The monetary authority succeeds in influencing the real economy by being able to move nominal short-term interest rates. This results in changes to the intertemporal prices of consumption and investment, so changing the level of demand. Further, because prices are sticky, nominal rate

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39 See, for example, the surveys by Hodrick (1987) and Meese (1989).

40 See McCallum (1994b).

41 See, for example, Mishkin (1995) and articles referred to therein.

movements result in real rate movements to which the real exchange rate responds. The real exchange rate movements influence the net export position. Both exchange rate and interest rate movements affect household wealth, which has consequent effects on consumption demand. Movements in interest rates also change the service cost of government debt, which leads to income tax effects that alter consumption demand.

Inflation responds to a change in the output gap, both directly and through an indirect effect on wages. Further, changes in the exchange rate have direct (and quantitatively important) effects on import prices.

Formally, there is no explicit credit channel in the model. Similarly, there is no direct link from nominal rates to the formation of inflation expectations. Agents do not attempt to learn about the actual inflation target from monetary policy actions. However, credibility effects are an interesting and important dimension of monetary policy and a topic for future research. There is also no explicit role for money in the model; we assume that the monetary authority can alter the price directly without operating through the quantity. It would be possible to write the reaction function in terms of the quantity of money and invert a money demand equation to solve for the short-term nominal interest rate. However, we do not feel at this stage that this would add anything to the analytical usefulness of the model as a projection or policy analysis tool.

Since there are lags between movements in the policy instrument and the subsequent responses of inflation, monetary policy cannot stabilise inflation immediately after a disturbance. This is not remedied by simply raising the weights in the monetary reaction function - at the extreme, high weights on a short policy horizon will lead to instrument instability. The framework makes it clear that more attractive policy outcomes are obtained by identifying shocks and their implications early, and then reacting to them promptly.

3.7 Relative prices and inflation

While many aspects of the real economy in the FPS model are simple, a relatively complex structure exists for the nominal economy. This reflects the use of the model as a tool for analysing monetary policy. The foundation of this structure is provided by a system of relative prices that differentiates ‘goods’ according to their uses. We impose inflation dynamics on top of this. The complete structure characterises inflation as coming potentially from many sources, which has the implication that no single monetary response will always be appropriate. Similarly, since relative prices can and often do change to an important degree, the definition of inflation matters, so care has been taken to build in several measures of inflation to make these differences clear.

3.7.1 Relative prices

Formally, FPS is a one-good model on the supply side. The same substance manages to feed people and build factories at the same time. Also, consumers, firms and government treat imported goods as equivalent to domestic goods, except for price. While this allows for considerable advantages of simplicity and transparency, the one-good framework suffers in that it provides no rationale for trade. Further, without a distinction between traded and non-traded goods, real exchange rate determination is not as theoretically motivated as one might like.

Instead of attempting to build a multiple-good model, we have instead imposed a relative price structure on the FPS model. This framework differentiates uses of the one good according to the price the good holds for that purpose. We use the familiar expenditure components from the real economy to define these categories. For example, 'consumption' goods will typically have a different price from 'investment' goods. We also split each of these categories into 'domestic' and 'imported' components. The split affords channels for important exchange rate and foreign price affects.

The relative price block is disciplined by a formal accounting structure. Since this is a *relative* price system, we need a numeraire price, which in FPS is the price of domestically-produced and consumed goods at factor cost. Nominal values have to add up in the same way that real values are constrained to add up. For example, the relative price of imports has to be consistent with the relative prices of the consumption, investment and government good components of imports. Second, indirect taxes are an important source of differences in relative prices, and they too have to be accounted for. Import tariffs are levied on the nominal value of imported goods, and indirect expenditure taxes are then applied to the overall nominal value of expenditures.

Although purchasing power parity is an attractive theoretical proposition about equilibrium levels, the data for New Zealand simply fail to support it. Consequently, the relationship between foreign and domestic prices in the model is characterised by what might usefully be thought of as *relative* purchasing power parity. In equilibrium, a permanent wedge can exist between the domestic dollar price of imported goods and the world price converted into domestic currency using the exchange rate. However, given the steady-state wedge, purchasing power parity holds on the margin, along equilibrium adjustment paths. We also add a dynamic structure that allows for temporary divergence from the equilibrium values. This approach has the attractive feature that it allows foreign price movements to be transmitted to domestic prices, in accordance with conventional theory, while recognising that permanent price wedges can exist. These price wedges reflect such real factors as different transportation and distribution costs.

3.7.2 The sources and dynamics of inflation

With a system for relative prices in place, we can now turn to the question of how to characterise inflation. Obviously, the model has to be able to address how and why the inflation rate changes over time. It is important to remember that all agents - households, firms, government and foreigners - can contribute to inflationary pressures. There are four main sources of inflation in the FPS model.

First, we model a small open economy that depends on imports for many of its goods. The economy is a price taker on world markets. There are, therefore, significant direct effects from foreign prices and fluctuations in the exchange rate. Second, price pressures arising from demand conditions are modelled using the concept of the output gap. Deviations of aggregate demand from productive capacity are entered directly into the inflation process. Since the output gap incorporates both supply and demand, supply shocks are just as important as demand shocks in the inflation process. Third, changes to the costs of production inputs can accelerate inflation, even if there are no demand pressures. These costs include wages and indirect taxes. Finally, forward-looking agents will adjust their forecasts of inflation when they see shocks, so forward-looking inflation expectations are an important part of the inflation process.

While all four channels described above enter the inflation process directly, there are important links between them. The structure of the model emphasises that different shocks flow through to inflation in different ways, and so present different problems to the monetary authority.

3.7.3 The Phillips curve

The Phillips curve formally embodies these ideas. In the FPS model, the equation takes the basic form

(3.36) $$\pi_t = (1-\alpha)\mathrm{B}_1(\mathrm{L})\cdot\pi_t + \alpha\cdot\pi_t^e + \mathrm{B}_2(\mathrm{L})\cdot\left(y_t^d - y_t^p\right) + \mathrm{B}_3(\mathrm{L})\cdot\left(y_t^d - y_t^p\right)^+$$

where π represents inflation, π^e expected inflation, y_t^d measures the demand for output, and y_t^p potential output and α is a coefficient. $\mathrm{B}(\mathrm{L})$ denotes a polynomial in the back-shift operator, while $(\ )^+$ is an annihilation operator which filters out negative values of the output gap.

This is rather different from the simplest accelerationist Phillips curve, where α is zero. Instead, in this specification, inflation is modelled as only partially autoregressive. This inertia could come from many sources, but the structure is not meant to approximate a particular theoretical story. No one theory adequately explains all the observed persistence in inflation. Instead, these coefficients are calibrated to produce a system with overall properties that match stylised facts about sluggishness in prices observed in the data.

Inflation expectations

Inflation expectations themselves are characterised as partially autoregressive and partially model-consistent:

(3.37) $$\pi_t^e = (1-\gamma)\mathrm{B}(\mathrm{L})\cdot\pi_t + \gamma\cdot\mathrm{F}(\mathrm{L})\cdot\pi_{t+1}$$

where γ is a coefficient and $\mathrm{F}(\mathrm{L})$ is a polynomial in the forward-shift operator.[42]

Models with backward-looking reaction functions and purely backward-looking expectations often fail to converge and are capable of producing seriously misleading policy conclusions. On the other hand, models where expectations are purely model-consistent are uninteresting from a monetary policy point of view. In a world where expectations adjust instantaneously to the model-consistent solution, the monetary authority has little work to do, except to announce the target.

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[42] A mixed model with forward- and backward-looking components is explored in Buiter and Miller (1985).

The term for inflation expectations enters with an homogeneity restriction on expectations and lagged inflation, often referred to as the natural rate restriction. This restriction implies that there is no permanent trade-off between the level of inflation and the level of output. In any steady state, with inflation at the target rate, inflation expectations at the target rate, and all other disequilibrium influences at zero, this equation collapses to the condition that the output gap must be zero. This is regardless of the level of the inflation rate. Moreover, since inflation expectations are partially forward-looking, any attempt by a policymaker to push output above potential on a sustained basis will lead to accelerating inflation without bound. Note that the Phillips curve cannot determine the level of inflation, only its dynamic properties. Looking at it from the other side, if we suppose that the real economy is in equilibrium, then the equation says, essentially, that inflation will be whatever it is expected to be. This is why we say that the fundamental role of monetary policy in FPS is to ensure that expectations of inflation are anchored to the target rate. That is what providing a nominal anchor means in the model.

The output gap

The last two terms in equation (3.37) introduce the output gap. The gap is defined not as the gap between output and its equilibrium value, but as actual output less potential. Potential output differs from equilibrium in that the currently productive stock of capital is used instead of the equilibrium stock. Its use reflects the view that inflation depends on a difference between actual output, reflecting the level of demand, and the level of output that is capable of being produced now, on a sustainable basis.

Asymmetry

A key feature of the inflation process is revealed in the last term, an extra effect from positive values of the output gap. This term makes the Phillips curve asymmetric. The implication of asymmetry for the monetary authority is profound; an asymmetric Phillips curve implies that it takes more action from the monetary authority to lower inflation than to raise it. Once a positive shock to inflation is understood, the output losses are minimised with a prompt monetary policy response.[43]

Extensions to the basic model

In general, inflation is ‘pulled’ along by the output gap or inflation expectations. Nonetheless, we extend the Phillips curve to include terms which accommodate more traditional, ‘cost-push’ ideas of inflation. The specification of wage terms in the Phillips curve allows for nominal wage inflation (adjusted for productivity) to affect price inflation directly and for price inflation to help re-equilibrate real wages. This wage-price feedback process between the Phillips curve and nominal wage setting is very important for the dynamics of the model. It creates an important source of stickiness with which the monetary authority has to contend.

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[43] For more on the evidence for and policy implications of asymmetry in the output-inflation process, see Clark, Laxton and Rose (1996), Laxton, Meridith and Rose (1995), Laxton, Rose and Tetlow (1993), Laxton, Ricketts and Rose (1994), and Razzak (1997).

We also add the capability to include the effects of changes to indirect tax rates. These can be thought of as ‘piggy-back’ effects where firms take the opportunity to hide behind tax increases to try to introduce additional price increases.

Shocks to foreign prices may have an effect on domestic prices independent of the effect through the exchange rate. Similarly, given a positive terms of trade shock, inflation pressure over and above direct pass-through effects may arise as firms move up their supply curves in order to meet increased demand.

3.7.4 From Phillips curve to CPI inflation

The previous section provided a relatively detailed analysis of the link from inflation pressures into realised inflation. However, we know that there is no one ‘inflation rate’ - it all depends on what is measured. The Phillips curve maps these pressures into a measure of inflation based on the numeraire price. Since we want the monetary authority to target a level of underlying CPI inflation, we have to extend this measure of inflation.

The problem here is that the CPI is a basket of 319 goods, whereas FPS is a one-good model. We cannot therefore ‘assemble’ a measure of CPI inflation from its component parts by identity. We get part of the way by deriving a measure of consumer price inflation based on the relative price of consumption goods. From there we model the CPI ex interest and GST based on empirical work on the relationship between the model deflator and the official series. The exchange rate was found to be important in this relationship.

At this level, ‘net’ and ‘gross’ measures of inflation are introduced. This allows us to consider the question of whether agents ‘see through’ the levels effects of changes in indirect tax rates, or whether they affect inflation expectations.

3.8 Growth

The model is written so that variables are divided by the current level of trend output.[44] Thus, all variables are strictly stationary, which allows a steady state to be defined in terms of fixed values. Trend real growth and inflation of nominal variables are later reintroduced so as to produce output data in levels.

In this way, the model is consistent with a balanced growth equilibrium. From Cobb-Douglas production technology, this growth satisfies the basic conditions of a neoclassical growth model.[45] In keeping with this model, the overall trend growth measure is defined from its components, trend population growth and trend labour productivity growth, as $\dot{y} = \dot{n} + \dot{q}$. $\dot{q}$ can be thought of as measuring technical

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[44] Nominal variables are also divided by the deflator for domestically-produced and consumed goods at factor cost, so that every price is a relative price.

[45] A production function of the form $Y = F(K, L)$ is said to be *neoclassical* if (i) for $K > 0$ and $F > 0$, $F(\cdot)$ exhibits positive first and negative second derivatives in each argument; (ii) $F(\cdot)$ exhibits constant returns to scale; and (iii) the marginal product of capital (labour) approaches infinity as capital (labour) goes to zero, and approaches zero as capital (labour) goes to infinity (the Inada conditions). For further details see Barro and Sala-i-Martin (1995).

progress in efficiency units of labour, rather than total factor productivity. Technological progress is introduced through a scalar term on the production function, and this reflects total factor productivity, following standard practice in calculating measures such as the Solow residual.

Note, however, that although the basic growth model is neoclassical, the capital-labour ratio is not constant with respect to productivity growth (because of the assumption of exogenous labour supply). No matter how technological progress arises, in equilibrium all benefits accrue to labour through the real wage.

Trend population growth and trend productivity growth are both exogenous. The constraint $\dot{y} < r$ applies, preventing Ponzi schemes. There are no variations in these trend growth components over time, although actual productivity growth will vary through the cycle.

4. The steady state

The steady state solution should be thought of as the outcome for the economy if all disequilibria were resolved, all expectations were realised, and no exogenous disturbances were influencing the behaviour of economic agents.

The steady-state solution has three key functions: it provides the dynamic simulation algorithm with an end point, it serves as an external consistency check on the exogenous end-point values, and it provides a stable, internally-consistent anchor for dynamic behaviour.

In a forward-looking model, a set of terminal conditions is a requirement for a solution to the two-point boundary problem. Since a variable's value at date *t* is a function of past and future realisations, the solution algorithm requires a set of end-point conditions in order for it to be able to solve for the dynamic paths. Some techniques impose derivative conditions at a terminal period, such that a variable reaches a certain rate of change. Alternatively, terminal conditions can be imposed as levels conditions. Following the latter approach, the steady state of the core model provides terminal conditions for the solution algorithm in level terms.[46]

The structure of the steady-state model and a subset of exogenously imposed steady-state values together determine the steady-state solution. Comparing this solution to the actual data serves as a check on the consistency of those exogenously specified long-run values.

Finally, and most importantly, the steady-state solution provides the key anchor for the dynamic structure of the model. Along the dynamic adjustment path, flows respond to the desired equilibrium position to achieve and subsequently maintain the relevant stock positions required to support that equilibrium.

Although often referred to as if it were constant, the steady-state actually represents a balanced growth *path* given by the underlying neo-classical growth structure of the core model. To derive a stationary representation of this growth path, we define variables in terms of ratios to normalised output, relative to a numeraire price, or in rates of change.

Calculating a set of exogenous steady-state values consistent with the data has been a difficult task. Several issues have created problems:

- In some cases, we required data that do not exist. An example is the capital stock.
- Many series that must be stationary for a true steady state to exist exhibit strong trends over history.
- In most cases, New Zealand data series are short, making it difficult to tell whether observations are within the range of a 'normal' cycle or part of a trend.

[46] This approach also helps to speed convergence.

- Our recent history of reform means that there are many breaks in data series. A frequent question is whether a given series is converging to a new level or simply reflecting adjustments that require historically high or low levels in the short term.

Our aim has been to specify a steady-state solution that is consistent both with regard to the logic of the model and the Bank's view of the economy. It is worth emphasising that as the economy develops and new information comes to hand, the parameterisation of the steady state can be expected to change. However, the same logic outlined here will apply to any future revisions.

We begin this Chapter with an overview of the steady state, summarising the model economy in equilibrium (section 4.1). In sections 4.2.1 to 4.2.8, we discuss the steady-state choices using the same presentational structure as in the previous chapter. We stress, however, that while it is convenient to approach the steady-state choices in the same way as we describe the theory, steady-state choices for one sector will not be independent of choices for another sector. Finally, we summarise the numerical steady state in section 4.3.

4.1 An overview of the steady state

A large part of the steady-state story is embodied in the ratios to output of four key stocks: capital, government bonds, financial assets, and net foreign assets. Given the depreciation rate, the ratio of capital to output determines the required investment flows. The ratios of government debt to output and government spending to output determine the required level of taxation. Given labour income net of taxes, households' choice for the ratio of financial assets to output determines simultaneously the sustainable flow of consumption and the ratio of net foreign assets to output. The net export position is then determined by the service cost (benefit) of net foreign assets.

Given the equilibrium choices for the cost of capital and the rate of return on capital, the solution to the firm's optimisation problem is a capital-to-output ratio of approximately 1.7. This is considerably higher than our estimate of the current ratio of 1.5. The level of the current ratio primarily reflects that, recently, capital growth has been outstripped by the very rapid growth in real output associated with restructuring. However, the steady-state ratio appears quite reasonable when a slightly longer historical period is considered. Maintaining a capital-to-output ratio of 1.7 requires that investment flows represent 18 per cent of output. This is above the current share of investment in output. Part of the increase in the share of investment in output arises because of the equilibrium choice for the rate of depreciation of 8.5 per cent. This depreciation rate reflects the view that computers and other machinery, which depreciate more quickly than do buildings, will become a larger share of the capital stock than at present.

The steady-state choice for government debt is 20 per cent of output. Although this is somewhat below its current level, it is consistent with announced fiscal objectives. Combined with our equilibrium choice for the government expenditure share of output of 17.5 per cent, 11 per cent for transfers, and the choices for indirect tax rates, this debt ratio implies a reduction in the average labour income tax rate from its current level of 23 per cent to roughly 17 per cent.

Given an expectation of labour income net of taxes, households are able to determine their human wealth. Given human wealth, supporting a desired consumption flow requires a certain level of financial assets. The financial asset-to-output ratio has been calibrated to approximately 0.7, so that combined with human wealth, the sustainable share of consumption in output is 66 per cent. This is lower than the current share. The implicit view, then, is that the recent share of consumption in output is unsustainable in the long-run without a very dramatic increase in real financial assets.

Together, the capital stock, the level of debt, and desired financial assets imply a net foreign asset-to-output ratio of -1.05, lower than our estimate of the current position of -0.95. Given the terms of trade and the interest rate that must be paid on foreign liabilities, servicing this stock of foreign debt requires a real net export flow of -2 per cent of output. The equilibrium terms of trade are such that this net export volume yields a positive nominal net export flow sufficient to meet the service cost of the (negative) net foreign asset position.

4.2 The parameterisation of the steady state

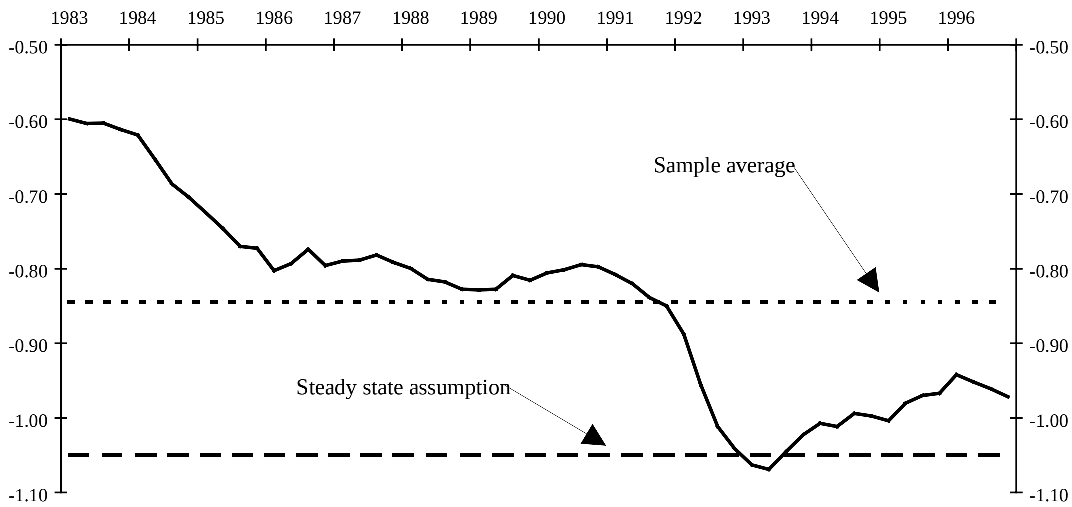
4.2.1 Households

Forward-looking households aim to maximise the utility from their expected lifetime consumption. They must take a decision about how much to consume now out of their current income and how much to save. This will depend on how much they value the future. Given a level of interest rates, the rate at which households discount the future proximately determines the level of wealth that households desire to hold.

So how do we determine the discount rate? To derive the long-run solution, the level of financial assets is set exogenously and one component of the discount rate is solved for endogenously. In the Blanchard-Weil-Buiter models, the discount rate equals the households' discount factor, δ, times the probability of survival, $(1-\gamma)$. By choosing γ and setting an equilibrium value for financial assets the model solves for δ. The probability of survival is set at 0.98. This choice implies an effective working lifetime of roughly 50 years. By taking a view on the steady state value for financial assets, we are taking a view on how 'impatient' our model economy is.

The steady-state solution of -1.05 for the NFA position implied by the levels of capital, government debt and financial assets suggests some further declines from its current level of -0.95 (see Figure 4.1).

Figure 4.1
Ratio of net foreign assets to GDP



The flow of net exports required to service this NFA position depends on the terms of trade faced by the domestic economy and the interest rate that must be paid on these liabilities. Consequently, households' desired financial asset position helps determine both the level of savings and the current account position in the steady state. If households are more impatient, more production has to be exported in equilibrium owing to the lower level of domestically-owned assets. In an open economy, the real exchange rate is the key price that must adjust to achieve the required net export flow. However, for the control solution, we make the real exchange rate exogenous so that exports are forced to adjust.[47] The required level for exports will depend on both the level of imports and the terms of trade faced by the domestic economy.

In addition to the effective discount rate outlined above, an additional parameter needs to be chosen to fully specify the utility function for maximising agents - the elasticity of intertemporal substitution, σ. The inverse, $1/\sigma$, is the coefficient of relative risk aversion. In line with Mehra and Prescott's (1985) survey, a value of 0.66 was chosen for this parameter.[48] While there is a trend towards lower values from recent research, 0.66 still implies a relatively high degree of risk aversion.[49]

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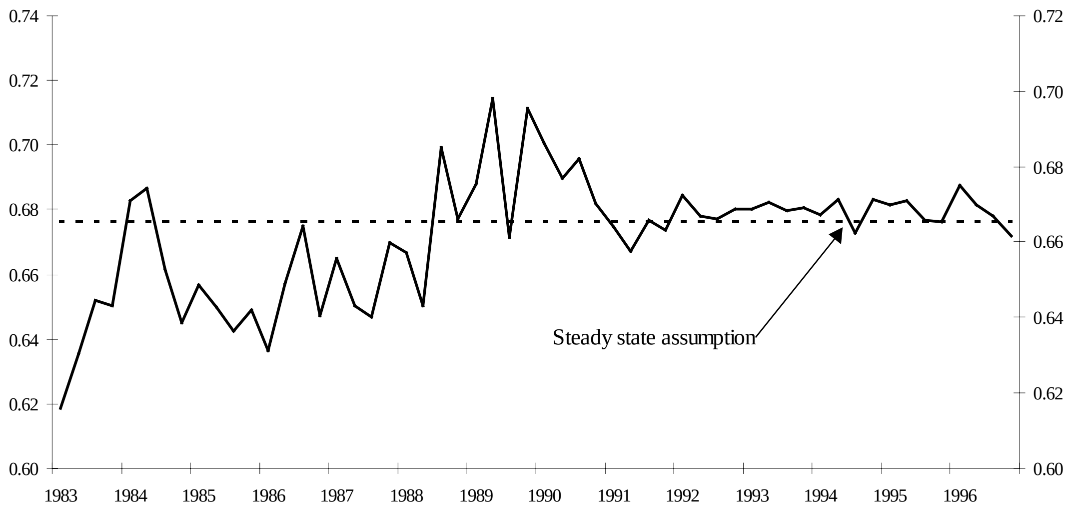
47 The NFA ratio and the real exchange rate are exogenous 'in control', but are both *endogenous* in simulation. Hence, a permanent shock may result in a new steady state with different values for NFA and the real exchange rate. This can be seen in the shock responses in Chapter 5. The ability to invert the model in this way for solving out the control solution is a convenient feature of the TROLL operating software.

48 Estimating σ is a difficult task. Attempts using Hayashi and Sim's (1983) modified IV estimator as used in Patterson and Peseran (1992) did not produce sensible results.

49 Hall (1988) goes so far as to argue that there is no evidence for a positive value σ.

In this way, the specification of household choice proximately determines the amount of wealth held by households. However, since wealth is unobservable, we pay particular attention to the resulting solution for the consumption flow. We see in Figure 4.2 that steady-state consumption is below recent values. However, the view is that the recent historical proportions are unsustainable. The available data suggest that these consumption levels have been achieved by running down financial assets relative to output. This trend is not expected to continue.

Figure 4.2
Ratio of consumption to GDP



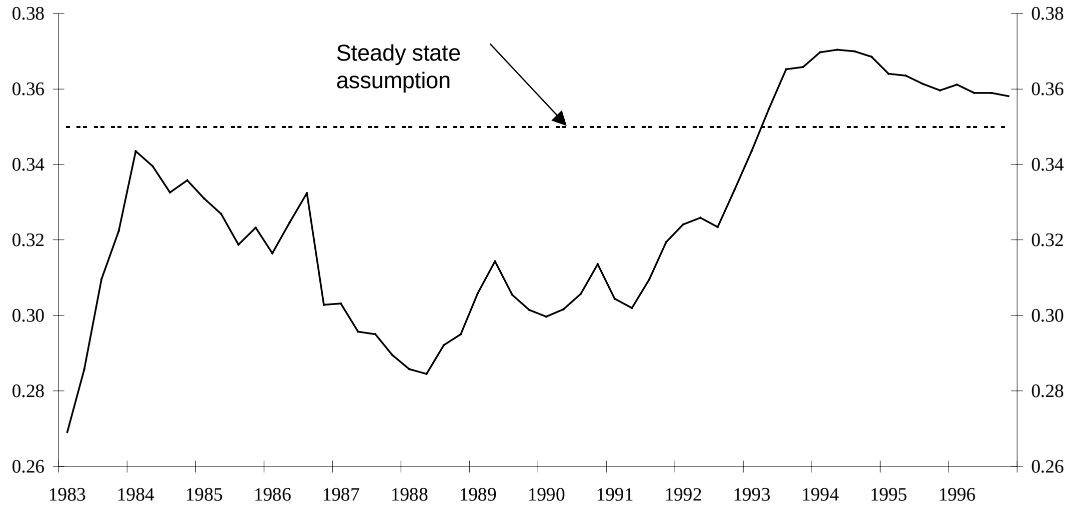
4.2.2 Firms

Specifying the steady state conditions for the firm requires a view on labour input and the factors determining the cost of capital and the return on capital.

The supply of labour is exogenous in equilibrium. The long-run labour input depends on population growth, the participation rate, and the equilibrium rate of unemployment *u_eq*. The equilibrium rate of unemployment has been set at 5 per cent, a rate broadly consistent with those used in other macroeconomic models for industrialised countries with similar labour market structures. It is worth noting that the particular rate chosen has *no* implications for the achievable target rate of inflation; any rate between 0 and 99 per cent could be chosen and the model would still converge to the target inflation rate.

The equilibrium capital stock is the solution to the first-order condition relating the marginal product and marginal cost of capital. The return on capital is largely a function of α, the exponent on capital in the production function. A rise in α increases returns and causes a rise in desired capital stock. The equilibrium share is estimated from national income accounts. As shown in Figure 4.3, the equilibrium share of income accruing to capital is set at 0.35.

Figure 4.3
Capital’s share of income



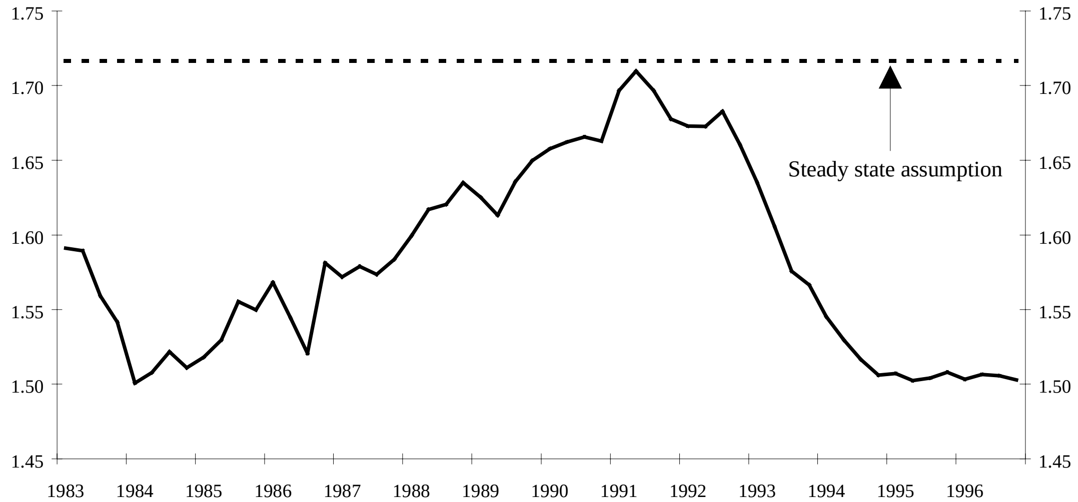
It might be argued that there is an increasing trend in this series, and that α could be set higher. However, the consequence of this would be a larger capital-to-output ratio and a larger investment share of output.

The cost of capital is a function of the depreciation rate, the price of investment, tax rates, and equilibrium interest rates. The steady-state value for the depreciation rate is set at 8.5 per cent. We consider this to be towards the lower side of estimates, but a higher value would also demand a higher investment flow to maintain the capital stock. Underlying the choice for the depreciation rate is an increased penetration of computer equipment in the capital stock, to some extent reflecting expected further declines in computer prices. These declining computer prices contribute to a decline in the price of investment goods that will increase the desired capital stock (see Figure 4.13).

Increased tax rates work to lower the desired capital stock by reducing the return on capital. The tax rate on profits, *tk*, is exogenous. The tax on capital income is set at 17 per cent, which is an approximate average of its rate over the last few years (see Figure 4.9).

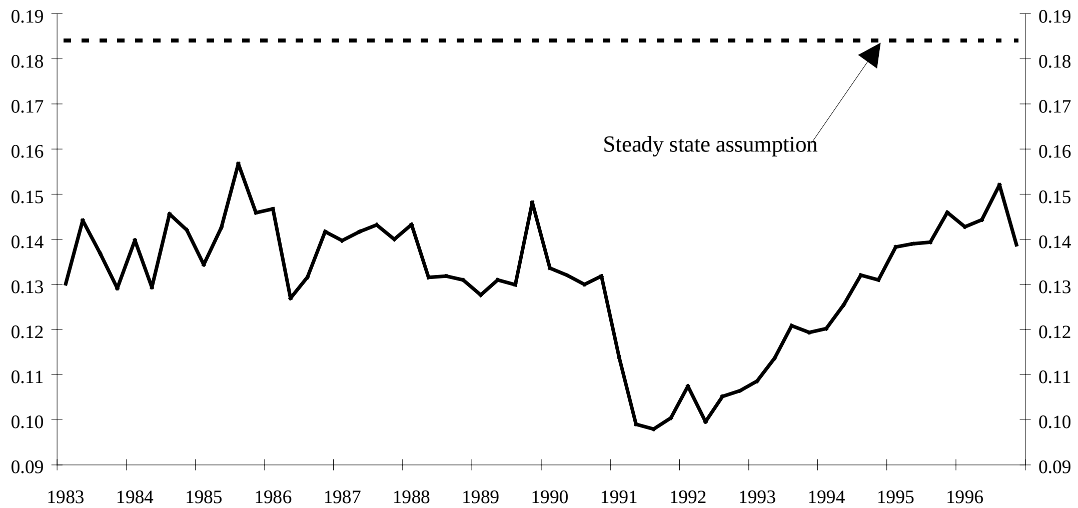
Finally, firms face an exogenous risk premium on their finance. Given the choices for other components of the cost of capital, the risk premium has been used to achieve a total cost of capital broadly consistent with conventional estimates for comparable small open economies. An equity premium of 5.75 per cent yields a cost of capital of approximately 22 per cent.

Figure 4.4
Ratio of capital stock to GDP



These steady-state assumptions lead to a desired level of capital of roughly 170 per cent of output. Even this relatively conservative parameterisation of the marginal conditions yields a steady-state value that is notably higher than the estimate of the current level.[50] As seen in Figure 4.5, the consequence is that the flow of investment required to sustain this equilibrium level of capital is considerably above recent historical experience. The view is that the structural reforms of the last decade have considerably improved the long-term productivity prospects for the New Zealand economy and firms will increase capitalisation to take full advantage.

Figure 4.5
Ratio of investment to GDP



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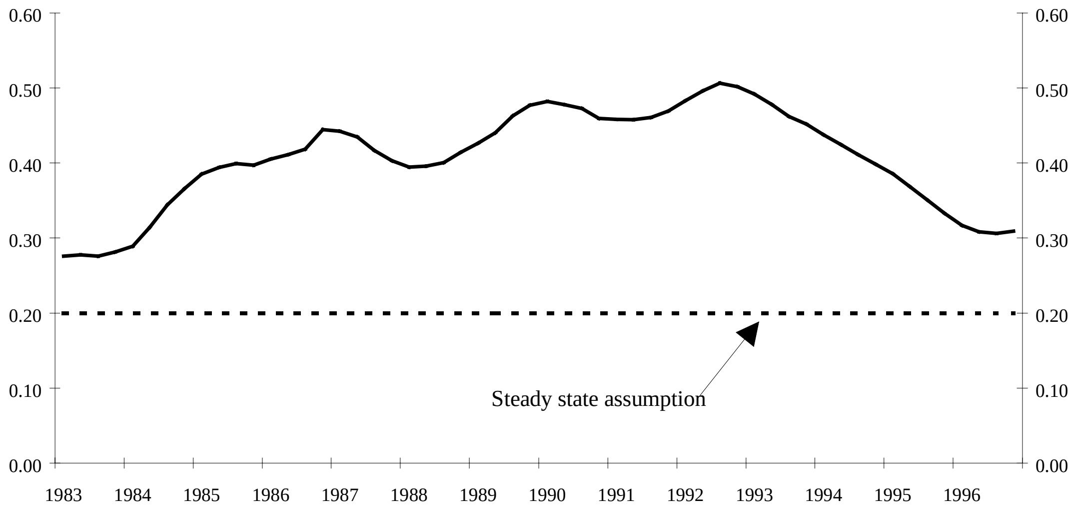
50 All estimates of the capital stock are based on the perpetual inventory method, consistent with the core model definition of private investment.

4.2.3 Government

As with households and firms, the steady state for fiscal policy is summarised by a stock-flow equilibrium. Unlike private agents, however, the government's long-term asset position and expenditure flows have been imposed exogenously rather than modelled as an endogenous solution to a maximisation problem. Reconciliation of these choices is achieved through the government budget constraint. The behavioural element of government is the steady-state solution for labour income taxes.

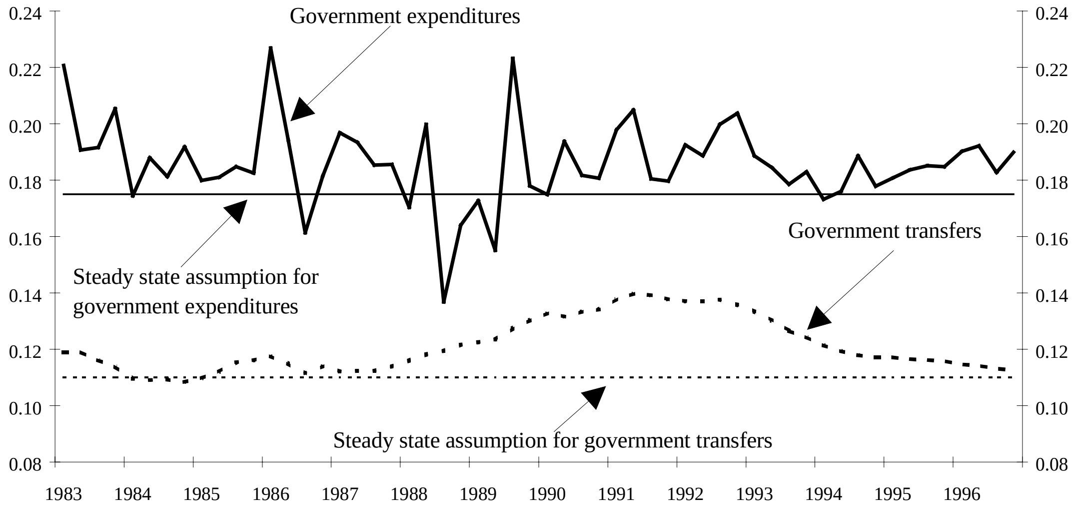
Based on announced fiscal plans, the government debt target is set at 20 per cent of output. This is lower than the last fifteen years, but well within reach given the rate of government debt repayment.

Figure 4.6
Ratio of government net debt to GDP



The choice of government expenditure is also based on previously announced plans, with a value of 17.5 per cent of output. On the income side, we set government transfers to households at 11 per cent of output.

Figure 4.7
Ratio of government spending to GDP



Given these policy choices, it remains to specify the rest of the revenue structure. To do this we have to calculate a set of effective tax rates and choose their steady-state values.

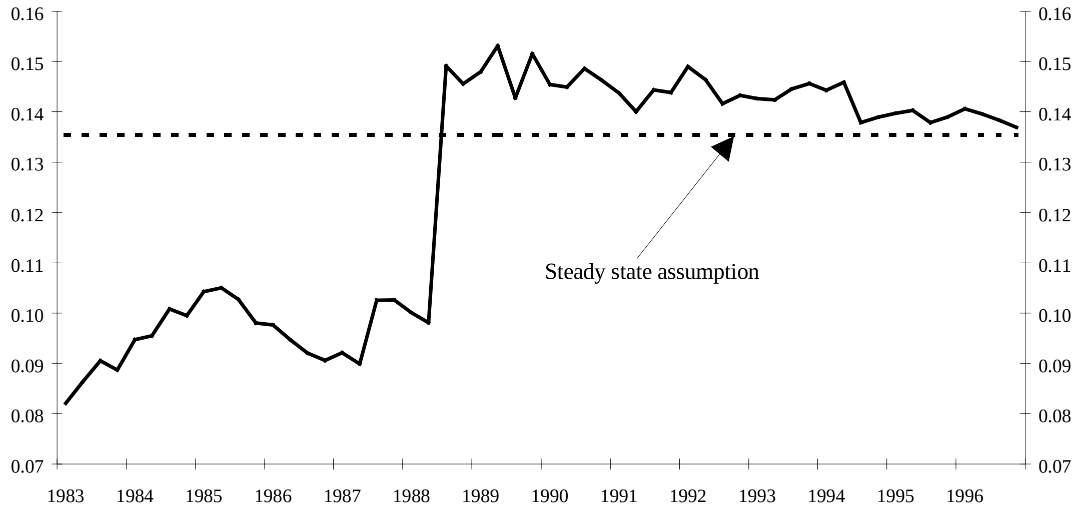
Conceptually, taxes take two forms: direct, applying to income flows; or indirect, applying to expenditure flows. Indirect taxes are applied in two stages, first on goods as they enter the economy, and second as they are sold to end users:

Table 4.1
The structure of taxes in the core model

	Direct	**Indirect**	
Households	tax on labour income	tax on consumption goods	duty on imported consumption goods
Firms	tax on capital income	tax on investment goods	duty on imported investment goods
Government		tax on government expenditures	duty on imported government expenditures

All of the indirect rates are exogenous. In general, they have been chosen on the basis of current values or recent averages. Some reduction in effective tariff rates has been built in, in accordance with announced policy. This leads to an assumption about the average overall indirect tax rate which is close to current values:

Figure 4.8
Average indirect tax rate

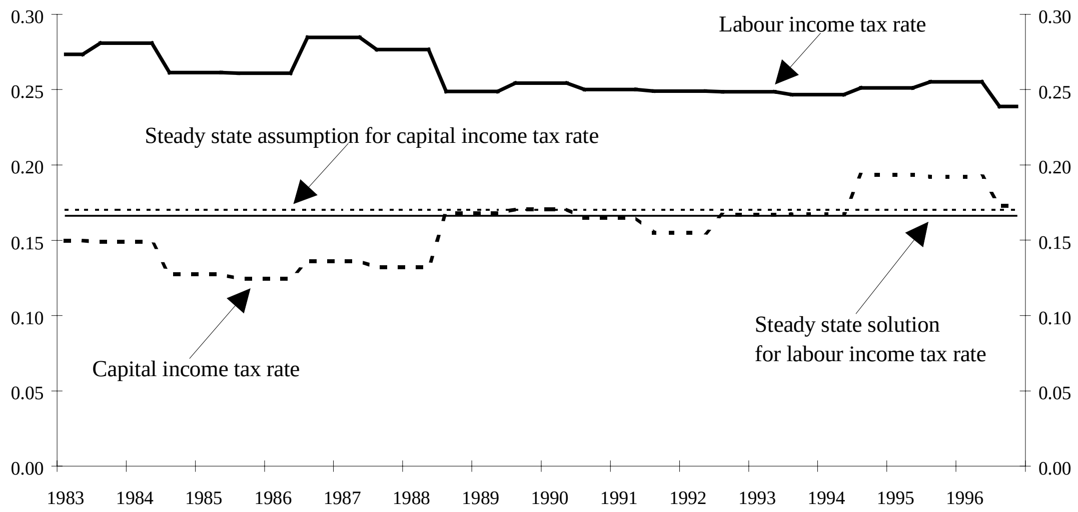


The large increase in effective indirect tax rates in the late 1980s is seen easily in Figure 4.8. Given the policy choices outlined earlier and the revenue structure, the government budget constraint solves for a steady-state labour income tax rate of 16.6 per cent. This rate is considerably lower than present values (Figure 4.9).

4.2.4 The foreign sector, debt, and external trade

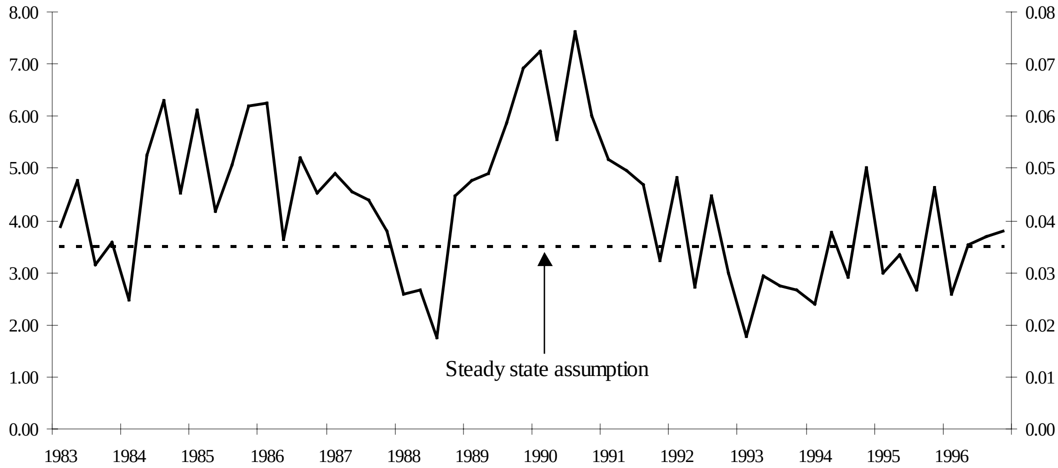
In contrast to the domestic economy, the steady state of the foreign sector is characterised by relative prices rather than by stock ratios. Under our small open economy assumption, the New Zealand market simply takes as given the prices of goods and finance set in world markets.

Figure 4.9
Effective direct tax rates



The world real interest rate for the model is calculated as a weighted average of U.S. and Australian real interest rates and is shown in Figure 4.10. The steady-state value is set at 3.5 per cent.

Figure 4.10
World interest rates



The equilibrium domestic short-term rate equals this rate plus a country risk premium of 100 basis points. The long-term domestic interest rate includes an additional term premium of 50 basis points. In essence, any amount of debt may be sold in world markets at this price.

Similarly, conditions for trade are summarised by the world prices of export, consumption, investment, and government goods. Given these prices, domestic agents choose the import shares of consumption, investment, and government expenditures, denoted by *cm_c*, *im_i* and *gm_g* respectively.[51] The selection of import shares is difficult, since there is no National Accounts record of imports by expenditure class. Using available data and some simple assumptions regarding relative shares, estimates of imports by expenditure class were constructed (Figure 4.11). Values for the shares of imports in consumption, investment and government expenditures of 37, 45 and 14 per cent respectively were chosen based on these constructed series.

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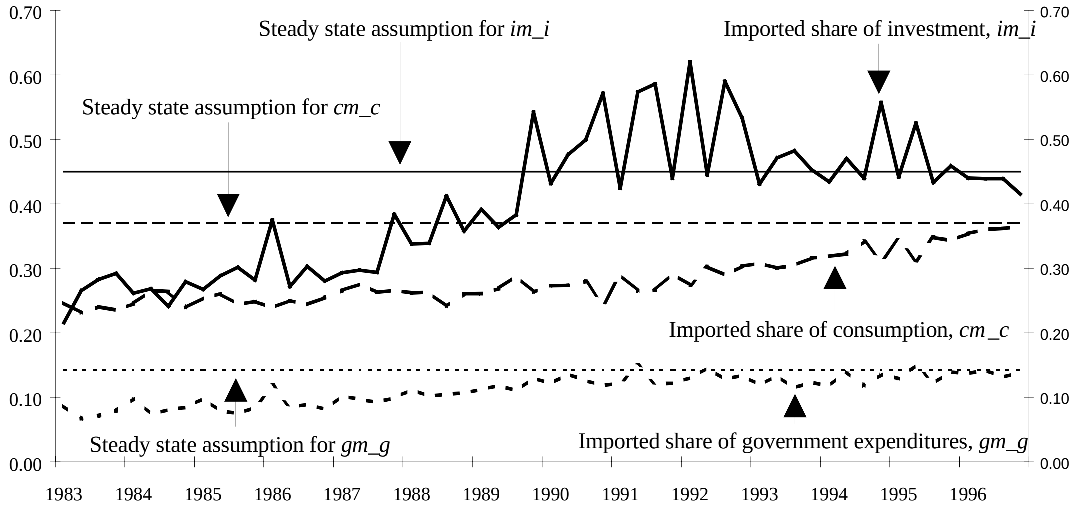
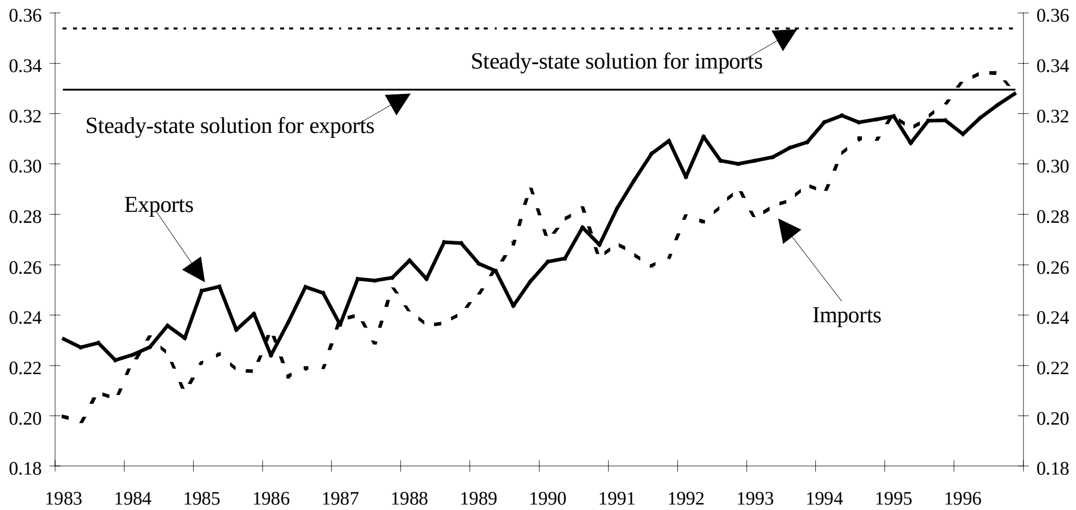
[51] Note that these shares are endogenous in simulation and only exogenous for setting the control solution.

Figure 4.11
Import shares of expenditures



Given the net foreign assets position and the terms of trade, the import shares effectively determine the levels of imports and exports in the steady state. Although the solutions for the expenditure shares of exports and imports that are graphed in Figure 4.12 are slightly higher than recent history, they are quite reasonable given expected equilibrium relative prices. The gap between the export and import shares is the level of real net exports required to service the net foreign asset position. In the steady state, the net export volume, *x-m*, is negative. However, under our assumptions for relative prices (see subsection 4.2.7), the nominal trade balance is positive. The latter is a necessary requirement for a stable ratio of net foreign assets to output. With a lower NFA position, the trade balance would have to be higher still.

Figure 4.12
Ratios of exports and imports to GDP



4.2.5 The labour market

Although very important for determining dynamic properties, the labour market is specified very simply in the steady state. The labour input is summarised by the steady-state unemployment rate, the participation rate, and population growth (see section 4.2.8). The participation rate is assumed to be constant and, consequently, labour force and population growth are identical. In the two-factor production technology, total national income is split between capital and labour, with labour's share of output (1-α) determining equilibrium real wages.

4.2.6 The monetary authority, interest rates, and the exchange rate

The role of the monetary authority is represented by one parameter in the steady-state solution. We specify the nominal anchor as an inflation rate target of 1.5 per cent.

The equilibrium interest rate structure, which started with the specification of a world real interest rate and a country risk premium for New Zealand (section 4.2.4), is completed by the choice of exogenous risk premia for government debt (0 per cent), net foreign assets (0 per cent), capital (5.75 per cent), and the portfolio return to households, *rcon* (2 per cent).

Although the steady-state real exchange rate is set at one, permanent wedges between domestic and foreign prices are allowed to exist in equilibrium. Pure purchasing power parity is not imposed.

4.2.7 Relative prices

All domestic prices in the model are specified relative to the deflator for domestically-produced and consumed goods at factor cost, *pd*. The GDP deflator is denoted by *py*. There are six key exogenous relative prices that we use for the control solution: the price of consumption relative to the GDP deflator, *pc_py_eq*, the price of investment, *pi_py_eq*, the price of government expenditures, *pg_py_eq*, the price of exports, *px_py_eq*, the deflator for domestically-produced investment goods, *pid_eq*, and the deflator for domestically-produced government goods, *pgd_eq*.

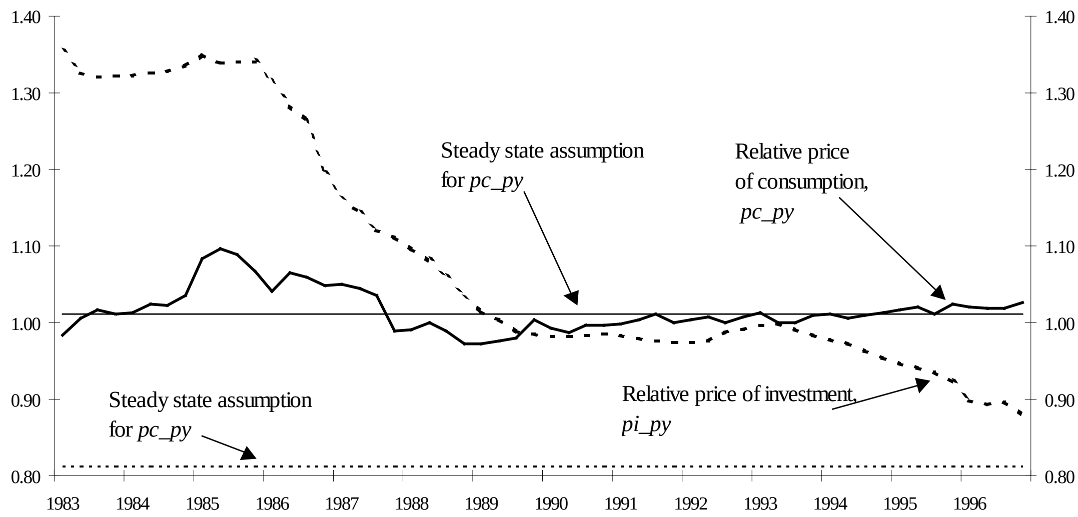
The two exogenous domestic prices, *pid_eq* and *pgd_eq*, determine a value for the deflator for domestically-produced consumption goods, *pcd_eq*. Weighted by real volumes, the three components of the deflator for domestically-produced and consumed goods must sum to one, the deflator *pd_eq* relative to itself.

Given the three components of *pd_eq*, the exogenously-specified prices relative to the GDP deflator, relative foreign prices, and indirect taxes, the price block solves for the model prices relative to *pd_eq*. Calibration constants are used to allow for the fact that the law-of-one-price, notionally imposed by the price block, is not well supported by the data.

The data on prices of expenditures relative to the GDP deflator were examined to help determine equilibrium values for exogenously-specified relative prices. Typically, relative prices are not stationary and selecting steady-state values requires considerable judgement. The rebasing of SNA data to a 1991/92 base year has removed some of the relative price distortions in the data, but not all. For example, there are still noticeable trends in relative prices from ongoing reductions in the prices of computers. This is particularly noticeable in the case of the relative price of investment.

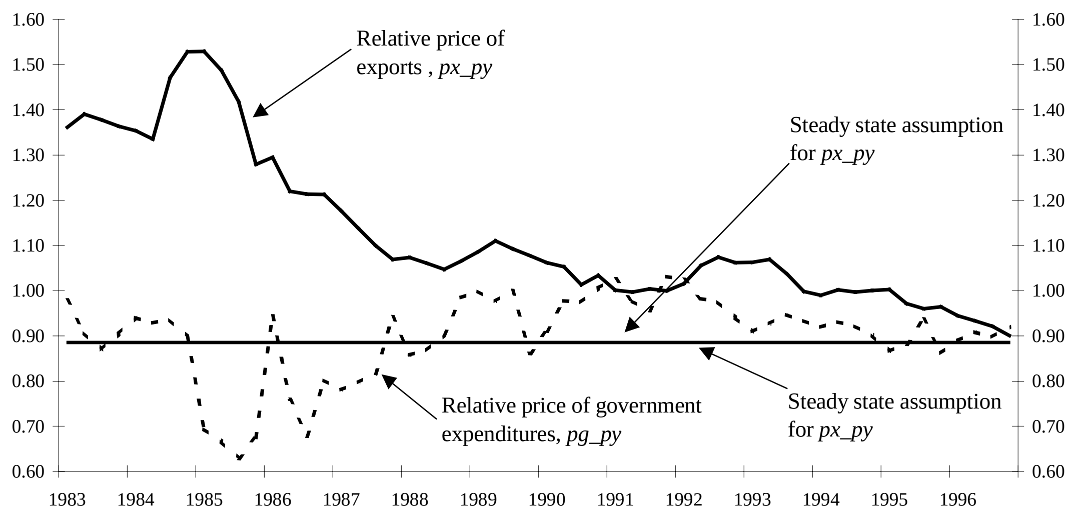
On the grounds that the historical downward trend in the relative price of investment goods can be expected to continue for some time to come, a steady state value of 0.81 was chosen for *pi_py_eq*. For consumption, we took the view that an approximate average of recent historical behaviour is appropriate; we set *pc_py_eq* equal to 1.01.

Figure 4.13
Relative prices of consumption and investment



The price of government expenditures is set to 0.88, close to its recent average. Some further declines are expected in export prices and a value for *px_py* of 0.89 was chosen. The price of imports relative to the GDP deflator is determined residually by the model's identities.

Figure 4.14
Relative prices of government expenditures and exports



The choice of relative prices cannot be made exclusively by eye-balling time series. A view is taken with a mind to the consequences for the resulting equilibrium expenditure flows. All else equal, an alternative set of relative prices would result in a different equilibrium for expenditure flows with these flows supporting a different set of equilibrium stock ratios.

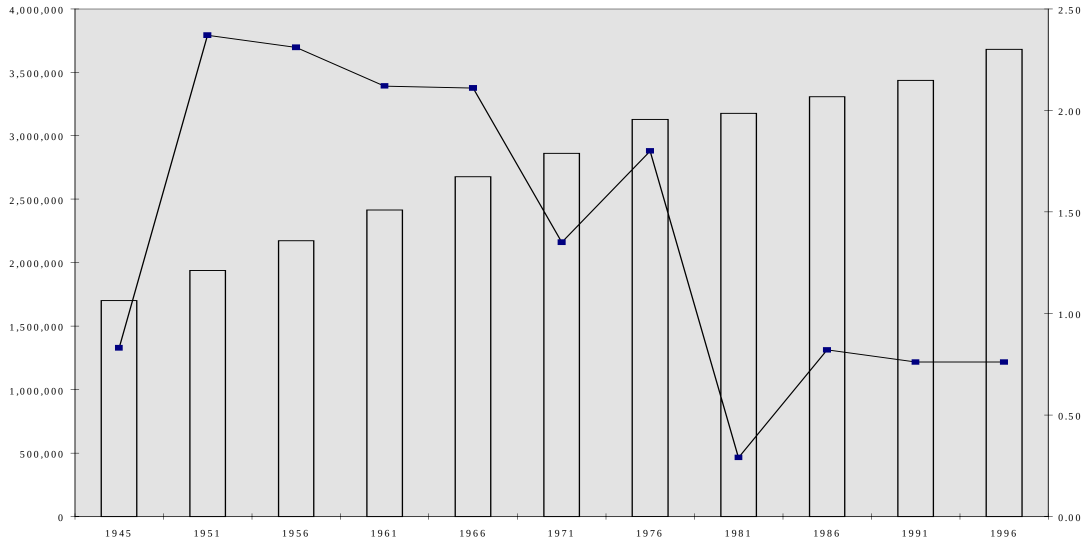
4.2.8 Growth

The equilibrium rate of population growth plays three roles in the model. First, without the introduction of new generations the economy would logically collapse. Second, births increase the degree of non-Ricardian equivalence. Third, population growth provides the first factor making up overall trend growth. With an exogenous unemployment rate and constant participation rate, the growth in population equals the growth in the labour input.

Some judgement is required in selecting this population growth rate. The census data show that the population growth rate has varied substantially through the post-war years:

Figure 4.16

New Zealand population
(level (lhs), annual growth rate (rhs))



A variety of population projections from the Demography Division of Statistics New Zealand were examined, considering fertility, life expectancy, net migration and participation rates.[52] Net migration is especially important in their outlook. A medium-term outlook of a 1 per cent per annum growth rate in the labour force is their average scenario and has been used as the population growth rate in the model.

Our stance on productivity growth is the subject of considerable on-going scrutiny. In international comparisons, New Zealand's labour productivity growth rate fell between the 1950s and 1990s (see, for example, Summers and Heston, 1991, 1995). On the basis of international evidence, an equilibrium growth rate of labour-augmenting technical progress of 1.5 per cent has been chosen.

Population growth of 1 per cent combined with labour productivity growth of 1.5 per cent, yields an annual trend growth rate of 2.5 per cent. This is consistent with most views about long-run growth in leading industrialised economies.

4.3 The numerical steady state

Table 4.2 contains the solution of the model for the main macro variables. All quantities are expressed in real base period units, normalised on output.

52 See Statistics New Zealand (1996).

Table 4.2
The numerical steady state of the core model

Growth			
Labour force growth rate	*ndot*	exogenous.	0.010
Labour productivity growth rate	*qdot*	exogenous	0.015
Preferences			
Discount parameter	δ	endog. for control	
Probability of death	γ	exogenous.	0.02
Elasticity of intertemporal substitution	σ	exogenous	0.66
Proportion of rule-of-thumb consumers	λ	exogenous	0.30
Production			
Total factor productivity	*tfp_eq*	endog. for control	0.832439
Capital's share of income	α	exogenous	0.35
Cost of capital	*cc_eq*	endogenous	0.224728
Depreciation	*depr*	exogenous	0.085
Income and asset accumulation			
Real wage	*wa_eq*	endogenous	0.684211
Savings rate	*(1-mpcw_eq)*	endogenous	0.925531
Financial assets	*fa_eq*	endogenous	0.734516
Government debt	*gb_eq*	exogenous	0.20
Capital	*k_eq*	endogenous	1.726287
Net Foreign Assets	*nfa_eq*	exog. for control	-1.05
Expenditures			
Consumption	*c_eq*	endogenous	0.665219
Investment	*i_eq*	endogenous	0.184142
Government	*g_eq*	exogenous	0.175
Exports	*x_eq*	endogenous	0.329658
Imports	*m_eq*	endogenous	0.354020
Prices (relative to GDP deflator)			
Consumption	*pc_py_eq*	exog. for control	1.011804.
Investment	*pi_py_eq*	exog. for control	0.812372
Government	*pg_py_eq*	exog. for control	0.883301
Exports	*px_py_eq*	exog. for control	0.887168
Imports	*pm_py_eq*	endogenous	0.762383
Prices (relative to absorption deflator)			
Consumption	*pc_eq*	endogenous	1.143208
Domestic consumption	*pcd_eq*	endogenous	1.03
Imported consumption	*pcm_eq*	endogenous	0.910201
Investment	*pi_eq*	endogenous	0.917875
Domestic investments	*pid_eq*	exog. for control	0.935093
Imported investment	*pim_eq*	endogenous	0.710647

Government expenditures	*pg_eq*	endogenous	0.998015
Domestic government expenditures	*pgd_eq*	exog. for control	0.96
Imported government expenditures	*pgm_eq*	endogenous	0.871712
Exports	*px_eq*	endogenous	1.002385
Imports	*pm_eq*	endogenous	0.860772
International prices			
Foreign consumption goods	*pcrow_eq*	endog. for control	0.52
Foreign investment goods	*pirow_eq*	endog. for control	0.41
Foreign government goods	*pgrow_eq*		0.65
Foreign export goods	*pxrow_eq*	endog. for control	1.01
Trade			
Real net exports	*netx_eq*	endogenous	-0.024361
Trade balance	*xbal_eq*	endogenous	0.025714
Real exchange rate	*v_eq*	exog. for control	1.0
Import penetration			
Consumption	*cm_c*	exog. for control	0.370
Investment	*im_i*	exog. for control	0.450
Government	*gm_g*	exog. for control	0.143
Fiscal			
Debt-to-output target	*gbtar_eq*	exogenous	0.20
Transfers rate	*gtr_eq*	exogenous	0.11
Labour income tax rate	*td_eq*	exogenous	0.167825
Capital income tax rate	*tk_eq*	exogenous	0.17
Consumption expenditure tax rate	*tic_eq*	exogenous	0.170
Investment expenditure tax rate	*tii_eq*	exogenous	0.090
Government expenditure tax rate	*tig_eq*	exogenous	0.015
Duty rate	*ticm/tiim/tigm_eq*	exogenous	0.025
Interest rates			
World interest rate	*rrow_eq*	exogenous	0.035
Country risk premium	*rp_eq*	exogenous	0.010
Risk premium on long bonds	*rt5_eq*	endogenous	0.005
Risk premium on government debt	*rgb_rl_eq*	exogenous	0.000
Risk premium on capital	*rk_rl_eq*	exogenous	0.0575
Risk premium on net foreign assets	*rnfa_r_eq*	exogenous	0.000
Risk premium for consumers	*rcon_r_eq*	exogenous	0.020

5. Dynamic model properties

In this chapter, we illustrate the key dynamic properties of the model using simulation experiments. This is an effective means to understand how the theory outlined in Chapter 3 fits together in a general equilibrium framework. Concomitantly, it emphasises the properties of the system, rather than the specification of individual equations.

We begin, therefore, in section 5.1 by briefly describing the calibration of the dynamic properties. We regard calibration as a method that allows us to incorporate a wide range of evidence from the data and the international economic literature, and it is particularly useful where clear econometric evidence is difficult to come by. Section 5.2 follows with a presentation of simulation experiments that illustrate the properties of the model economy, highlighting many of the key features of the model's design.

5.1 Calibrating dynamic properties

As noted in section 2.2.2, the core model is 'calibrated' rather than directly estimated.[53] This method has been chosen for two basic reasons: first, it is complementary to the top-down approach described in section 2.2.1, and second, it avoids some of the problems of direct econometric estimation.[54]

One approach to parameterising the model would entail casting it into a well-specified probabalistic framework and estimating all parameters by a systems estimator. However, even though the core model is compact by historical standards, it is too large to be estimated as a system. Even for the estimation of small systems, New Zealand data series are often too short. Moreover, they are subject to large and frequent revisions. Past experience suggests that a model relying exclusively on estimated parameters will face large changes in properties when re-estimated with updated data. This problem is further compounded by structural breaks in the time series from extensive structural and institutional reform in New Zealand in the last decade.[55]

A wide range of evidence has been used to calibrate the model. Simple things, such as respecting the character of the New Zealand data, in terms of ratios or shares, will have an impact on model properties.

Further data analysis, commonly associated with Real Business Cycle modelling, has also been used. This includes various forms of statistical analysis, such as mean-variance analysis. These techniques have the advantage that they do not require long data spans. Such analysis shows, for example, that a stylised fact of the New Zealand

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[53] For a review of calibration procedures, see Kim and Pagan (1995).

[54] One of the main advantages of classical econometric techniques is that they provide a set of metrics by which to evaluate such notions as goodness of fit. Recent work has sought to introduce such measures to calibration theory. See, for example, Canova (1994), Gallant (1996), Gregory and Smith (1990), Hoover (1995), and Watson (1993).

[55] For a discussion of the reform process, see Silverstone *et al* (1996) and Evans *et al* (1996).

business cycle is that imports are more volatile than exports, and that this relative volatility has become stronger in the 1990s.

Econometric evidence has also been considered. For example, various long-run price elasticities were estimated using the Phillips-Hansen (1990) procedure.[56] A small system was estimated by Generalised Method of Moments, yielding estimates for the Phillips curve.[57]

However, all of these estimates are considered as evidence towards a stylised picture of the New Zealand business cycle, rather than being taken at face value.[58] On occasion, a technique might produce evidence that would result in model properties that are inconsistent with priors or other evidence. When this occurs, it seems only prudent to go with the weight of evidence, even if this means overriding results from a technique that has yielded plausible evidence elsewhere. As an example, the box (overleaf) discusses the use of vector autoregressions.

The model's parameterisation can also be tested by how well the model is able to replicate the historical experience. This technique has also been used to inform the calibration. However, given the degree of structural change in New Zealand, it has to be remembered that the model should reflect the present, rather than the past.

In the following section, we present responses to a number of simulation experiments. These experiments have also been used to inform the choice of parameter values. Parameters that produce plausible demand shock responses, for example, may not produce sensible responses to an exchange rate shock. In this way, the model structure itself can be used as a 'filter' for all types of evidence - by repeated simulation of the model under well-defined experiments, the properties of the system are emphasised and the evidence assessed.

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56 See Phillips and Hanson (1990), Phillips (1993).

57 See Razzak (1997).

58 Examples of New Zealand business cycle research include Featherstone (1994), Hall *et al* (1996), Kim (1994), Kim *et al* (1994), Kim *et al* (1995), McNelis and Schmidt-Hebbel (1993), Razzak (1994a), Razzak (1994b) and Wells and Evans (1985).

FPS Calibration and vector autoregression models

Builders of macroeconomic models in other countries have found that vector autoregression (VAR) models can provide useful information for calibrating model properties. Although VARs have insufficient theoretical structure to examine the long-run impact of shocks, they are well suited to capturing short-run persistence and cross correlations in the data. Thus, VARs often provide a useful guide to an economy's short-term response to shocks. For this reason, during the calibration of the model's dynamic properties, we examined the results of a variety of VAR models.

We were particularly interested in the short-term impact of an exogenous change to monetary policy, in the form of a shock to interest rates.[59] During the course of our work, we estimated a large number of VAR models, ranging from small freely-estimated models with no theoretical restrictions, to more sophisticated structural VARs that imposed restrictions on the VAR's structure.[60] Most of these models generated poor impulse-response properties, even when we used procedures that have worked successfully in other countries.

The absence of successful results is not entirely surprising. The economic and institutional structure of New Zealand, including the monetary policy framework, has changed considerably over the last two decades. Indeed, it would be disheartening if such wide-ranging reforms did not alter substantially the time series properties of New Zealand data.

It is disappointing that plausible impulse responses were not always found to condition the calibration of FPS. However, the poor results are also a salutary reminder about the pitfalls of relying exclusively on econometric analysis to build a macroeconomic model. In the face of these difficulties, we studied empirical evidence for New Zealand and other small open countries (such as Australia, Canada, and the United Kingdom) to ensure that the FPS responses are within reasonable bounds.

We are also conducting further research into VAR modelling with New Zealand data. Ultimately, we would like to determine which aspects of the New Zealand economy and data properties are causing the poor performance of VARs for New Zealand.

5.2 Simulation experiments

The simulation experiments presented here trace out the model's solution path to shocks that impact on the model economy. These shock responses work well for illuminating the properties of the model, and the particular shocks that follow were chosen to illustrate specific features.

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59 See, for example, Gerlach and Smets (1995).

60 For different approaches to Structural VARs, see Blanchard and Quah (1993) and Shapiro and Watson (1988).

The structure of dynamic and equilibrium equations affords a clear distinction between permanent and temporary shocks. An important feature is that the model identifies the new long run equilibrium implied by a permanent shock. Such shocks illustrate that short-run dynamic effects arise from the adjustment required to attain the new steady state. This includes effects on inflation to which the monetary authority must react.

All of these shocks are unanticipated. Implicitly, however, full information is assumed – when the shock hits, the nature of the shock is known and understood by all agents. Partial-information and learning effects can be expected to make the dynamic responses quite different in certain shocks, and are an issue for further research.

The model's solution paths to these shocks are presented in shock-minus-control graphs. The experiments assume that the economy is initially at the steady state, and the graphs trace out the paths of variables in terms of their differences from the original equilibrium. These responses as either per cent or percentage point deviations. The units on the horizontal axis are years.

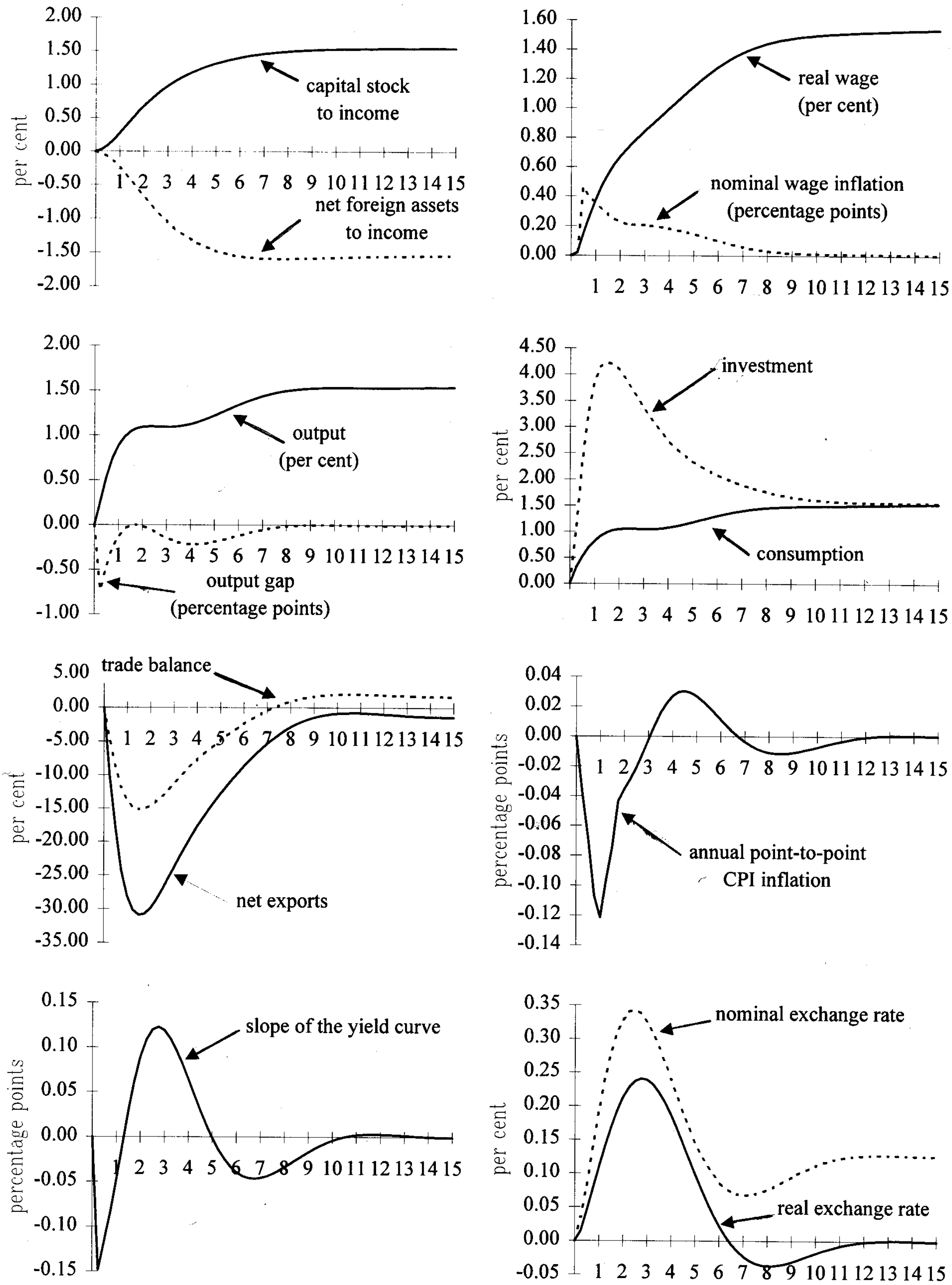
5.2.1 A permanent increase in total factor productivity

An important feature of the core model is that it has a fully-articulated supply side. Therefore, the model is not restricted to considering demand-side experiments only. To show this feature, we use an unanticipated, permanent increase to equilibrium total factor productivity, with no increase in productivity in the rest of the world. The shock has a conventional interpretation as a positive supply shock. Because the shock is permanent, it will affect the long-run equilibrium. When considering permanent shocks, it often facilitates understanding to examine the long-run implications before tracing out the dynamic adjustment required to achieve the new long-run position. That is the approach that will be used in the descriptions of the simulation experiments that follow.

An increase in total factor productivity raises the equilibrium desired capital stock because of the resulting increase in the marginal product of capital (see Figure 5.1) To support this new stock level, firms must increase their equilibrium investment expenditures. This added capital means that in the long run, potential output increases by more than just the 1 per cent increase to productivity. As predicted by the model's neoclassical theory of income distribution, potential output increases by about 1.6 per cent above control. In the long run, the real wage rises by the same amount and some of the resulting increase in income is saved. The increase in equilibrium household financial assets is equal to the increase in output, leaving the ratio of net foreign assets to output unchanged.

Along the dynamic adjustment path to the new equilibrium, investment must rise above its new long-run position to build the required capital stock. Consumers also respond to their improved wealth position by increasing consumption. However, the demand for goods and services does not increase as rapidly as productive capacity and a small excess supply gap opens up. The monetary authority responds to the projected decline in inflation below target by decreasing short-term interest rates to stimulate demand. Despite the easing, annual CPI inflation dips slightly below the target. The fiscal authority also responds to the new supply conditions as the increase in the level of output allows for an increase in debt to maintain the target debt-to-income ratio. The easing in the net direct tax rate helps stimulate consumption, helping to reduce the extent of the excess supply gap that opens up.

Figure 5.1
A permanent increase in domestic productivity



Through the bargaining process, nominal wage inflation picks up as workers start to receive some of their increased marginal product. The decline in the price level arising from the deflation also helps increase real wages up to their new equilibrium level. The increase in disposable income allows households to increase both consumption and their holdings of financial assets. Although households' financial assets increase, domestic agents do not hold all of the increase in domestic assets. The level of net foreign assets declines as foreigners purchase some of the claims on the rise in capital and government debt. The boom in investment and the increase in consumption demand serve to stimulate imports and the appreciation of the exchange rate in the medium term reduces exports. The resulting trade deficits eventually achieve the new lower level of net foreign assets. Once the equilibrium level of net foreign assets is achieved, net exports return to their previous *share* of output.

Although monetary and fiscal policy responses are required to maintain their respective targets, their roles in this shock are relatively minor. The real side of the economy responds relatively quickly to the initial disequilibrium arising from the innovation to productivity. The economy has essentially converged on the new equilibrium ten years after the initial disturbance, with the bulk of adjustment completed after five years.

5.2.2 A decrease in the government's debt-to-income target

In this experiment, government reduces its debt-to-output target by 10 percentage points. This simulation experiment is useful for illustrating the dynamic adjustment required to achieve a new long-run equilibrium, the response of the supply side, the stock-flow relationships, the non-neutrality of fiscal policy due to overdiscounting, and the central role of the real exchange rate in an open economy. In particular, it draws attention to the interaction of monetary and fiscal policy.

In the long run, lower government debt implies lower labour income taxes. The decline in government debt also means that there are fewer domestic assets available for households' financial wealth portfolios (over-discounting of future tax liabilities means that households view government debt as net wealth). Since households' wealth was initially in equilibrium, they must replace the government debt in their wealth portfolios. With no change in capital stock, this implies that they must purchase assets from the external sector.

All else equal, the new equilibrium net foreign asset-to-output ratio would increase by exactly the same amount as the fall in the government debt-to-income ratio. The decline in net foreign liabilities means a lower debt-servicing burden, so the domestic economy will not need to export as much output. Consequently, the real exchange rate appreciates and the previously-exported goods are now available for domestic use. Some are consumed, as consumption's share of output is higher in the new equilibrium, and some are invested because of the real exchange rate's effect on the cost of capital. The appreciation in the exchange rate reduces the price of imported investment goods, thereby lowering the cost of capital. This induces firms to increase their desired capital stock, increasing the long-run productive capacity of the economy.

In summary, the new long-run has a lower labour income tax rate, an improved net foreign asset position, a lower net export position supported by a stronger real exchange rate, a higher level of both consumption and investment, and more productive capacity.

The dynamic adjustment looks quite different from the long-run equilibrium. To achieve the reduction in outstanding debt, tax rates are raised on labour income. With less disposable income, households reduce consumption and demand falls below supply. The forward-looking monetary authority responds to the projected decline in inflation below its target by reducing short-term interest rates. The exchange rate depreciates in response to the smaller differential between domestic and foreign interest rates. The exchange rate effect more than offsets the interest rate effect, and a temporary increase in the user cost of capital results in an initial decline in investment that contributes to the excess supply. However, the exchange rate depreciation also stimulates a temporary export boom, and that, combined with a significant decline in imports, results in a large improvement in the current account. The improved current account flows cumulate into a less negative net foreign asset position, moving the economy towards its new long-run position.

The easing in monetary conditions eventually raises demand above supply some two years after the initial change in the fiscal target. The stimulus works primarily through the trade sector. Excess demand over a three year period re-anchors inflation expectations at the mid-point of the target range. Once sufficient foreign assets have been accumulated and the long-run decline in the cost of capital arises, both consumption and investment increase to their new levels. The increase in investment also builds the capital stock to its new level and productive capacity reaches its new equilibrium. Once short-term interest rates re-equilibrate, the exchange rate converges to its new equilibrium level. Net exports then lock in at their new lower level.

To summarise, the long-run effect of the shock has been to raise households' ability to consume, since less output has to be diverted to service foreign liabilities. However, in the short run a temporary sacrifice is required by households in the form of higher income taxes. Although this is a fiscal policy shock, monetary policy plays a key role in the transition from short run to long run by temporarily easing monetary conditions.

5.2.3 A shift in the composition of taxes

Changes in the composition of taxes can be used to show how the overlapping-generations framework upon which the model is founded allows for intergenerational effects from fiscal policy. This experiment involves an unanticipated, permanent two percentage point increase in the indirect tax on consumption goods.[61]

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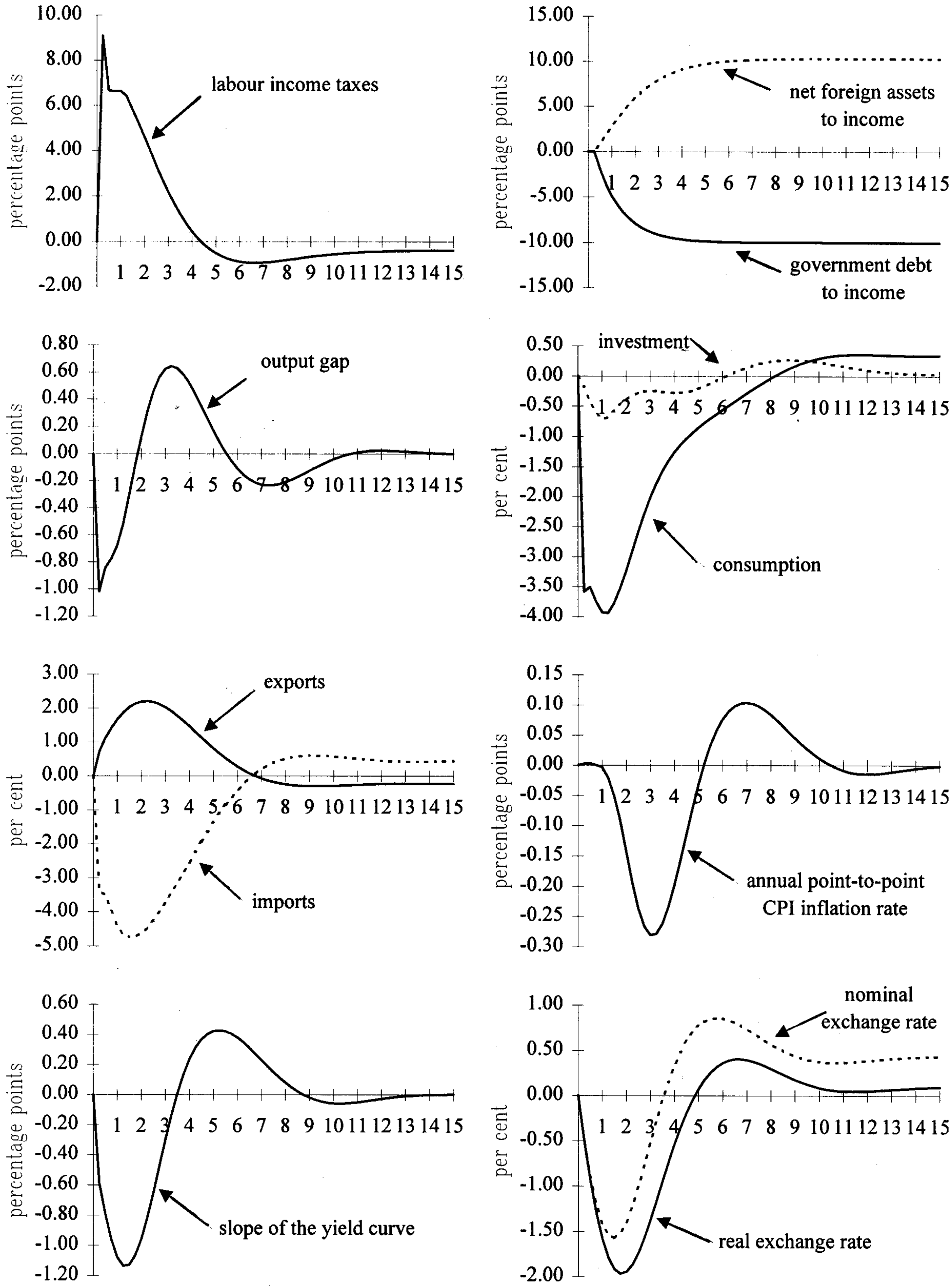
[61] Note that conceptually, the indirect tax on consumption goods includes GST, excise taxes, and other miscellaneous indirect taxes that are paid by households (such as gaming duties and motor vehicle charges). It does not include import tariffs, which are handled elsewhere.

Figure 5.2
A permanent decrease in the government's debt-to-income target



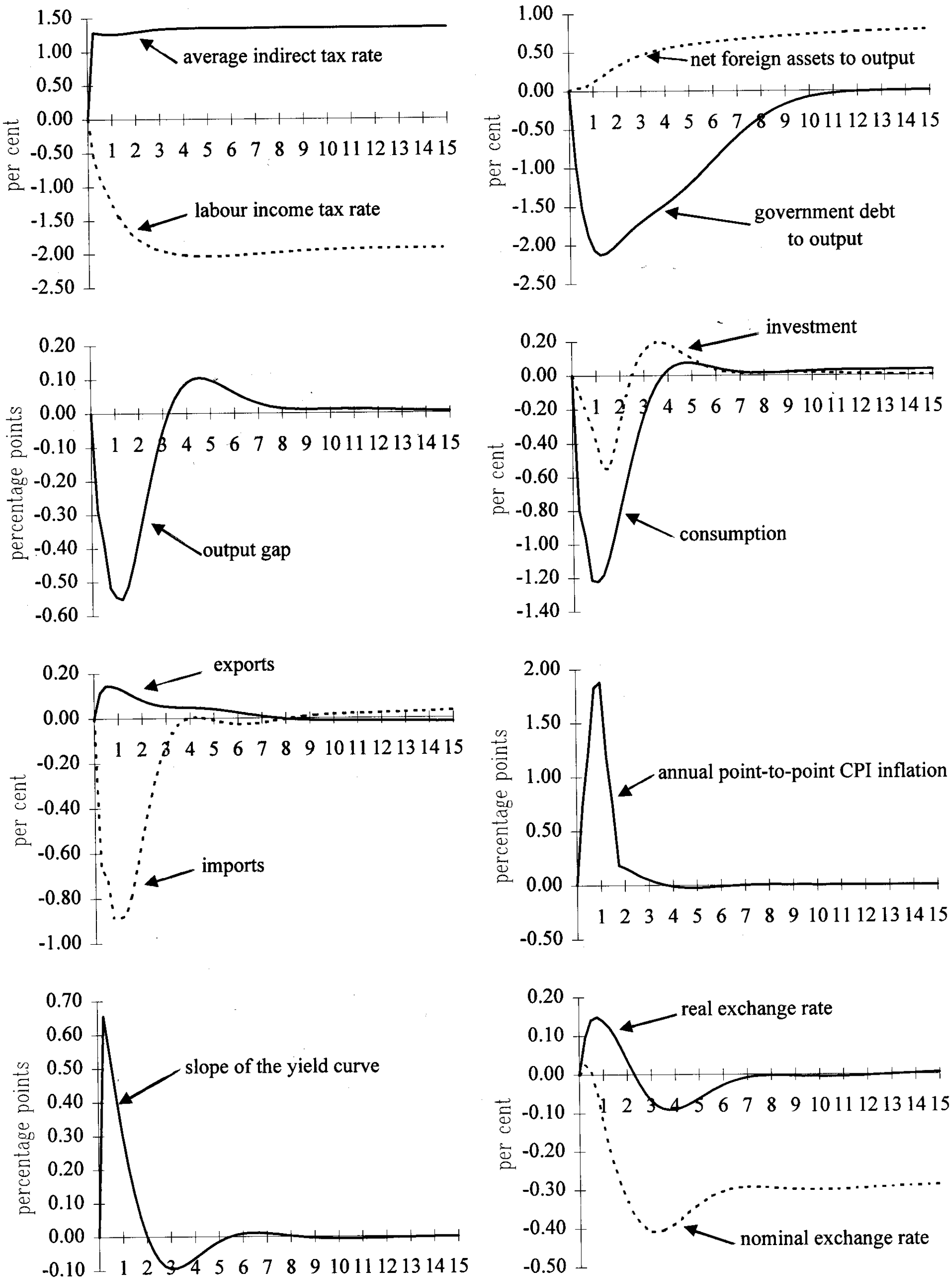
In the model, all tax rates except the rate of direct tax on labour income are exogenous, as is the ratio of government spending to output. Consequently, the equilibrium government budget constraint forces the labour income tax rate to adjust so that the long-run policy goal for government debt is supportable. Therefore, a permanent increase in indirect taxes allows the equilibrium rate of direct tax on labour income to fall. The fall in the equilibrium income tax rate means that human wealth increases, encouraging households to increase their savings. Households' equilibrium financial asset holdings rise. As a result, fewer goods need to be exported in the new equilibrium because the increased domestic savings improves the net foreign asset position. The equilibrium real exchange rate appreciates to support this lower net export position. Because the appreciation in the real exchange rate reduces the cost of capital through the price of imported investment goods, firms increase their desired capital-to-output ratio and potential output increases marginally in the long run. The increase in domestic wealth also means that households are able to support a higher level of consumption in equilibrium.

Although consumption increases in the long run, an important intergenerational effect of this tax composition change makes households initially worse off on average. The burden of the increase in the consumption tax in this experiment falls disproportionately on older generations. The purchasing power of their accumulated financial assets has fallen with the increase in the consumption tax. For younger generations, with little in the way of financial assets, the reduction in the purchasing power of those assets is relatively minor and they benefit from the reduced labour income tax.

When the shock hits, the dominant dynamic effect is the loss of purchasing power facing the older generation, so that aggregate consumption falls. Investment demand also initially falls owing to lower expected returns and the monetary policy response. Monetary policy tightens to prevent the price level effect of the increase in consumption taxes from becoming entrenched in inflation expectations. Because of the weakness in domestic demand, however, the tightening is short-lived and is quickly reversed. The real exchange rate is ill-affected by this initial tightening; the dominant change is a depreciation that helps to support demand over much of the medium term. Weak domestic demand for imports and an increase in exports generates the increase in net foreign assets that eventually allows domestic households to achieve their new desired financial asset position.

As time goes by, the younger generations accumulate more financial assets and consumption demand recovers. Investment demand also strengthens as expected returns recover and monetary conditions ease. Policy is able to ease rather quickly because of the weakness in demand conditions generated by the disturbance and because of the fact that only a portion of indirect tax effects feed into inflation expectations. After ten years, the new equilibrium has essentially been achieved.

Figure 5.3
A shift in the composition of taxes



5.2.4 Changes to the inflation target

The core model incorporates an asymmetric Phillips curve. The asymmetry is such that more inflation is generated by excess demand than deflation is generated by an identical amount of excess supply. The following simulation experiments use changes in the targeted rate of inflation to illustrate the degree of the asymmetry.[62] The simulation path in Figure 5.4 traced out by the solid line reflects an increase of one percentage point in the target inflation rate. The dashed line traces out the simulation path under a one percentage point decline.

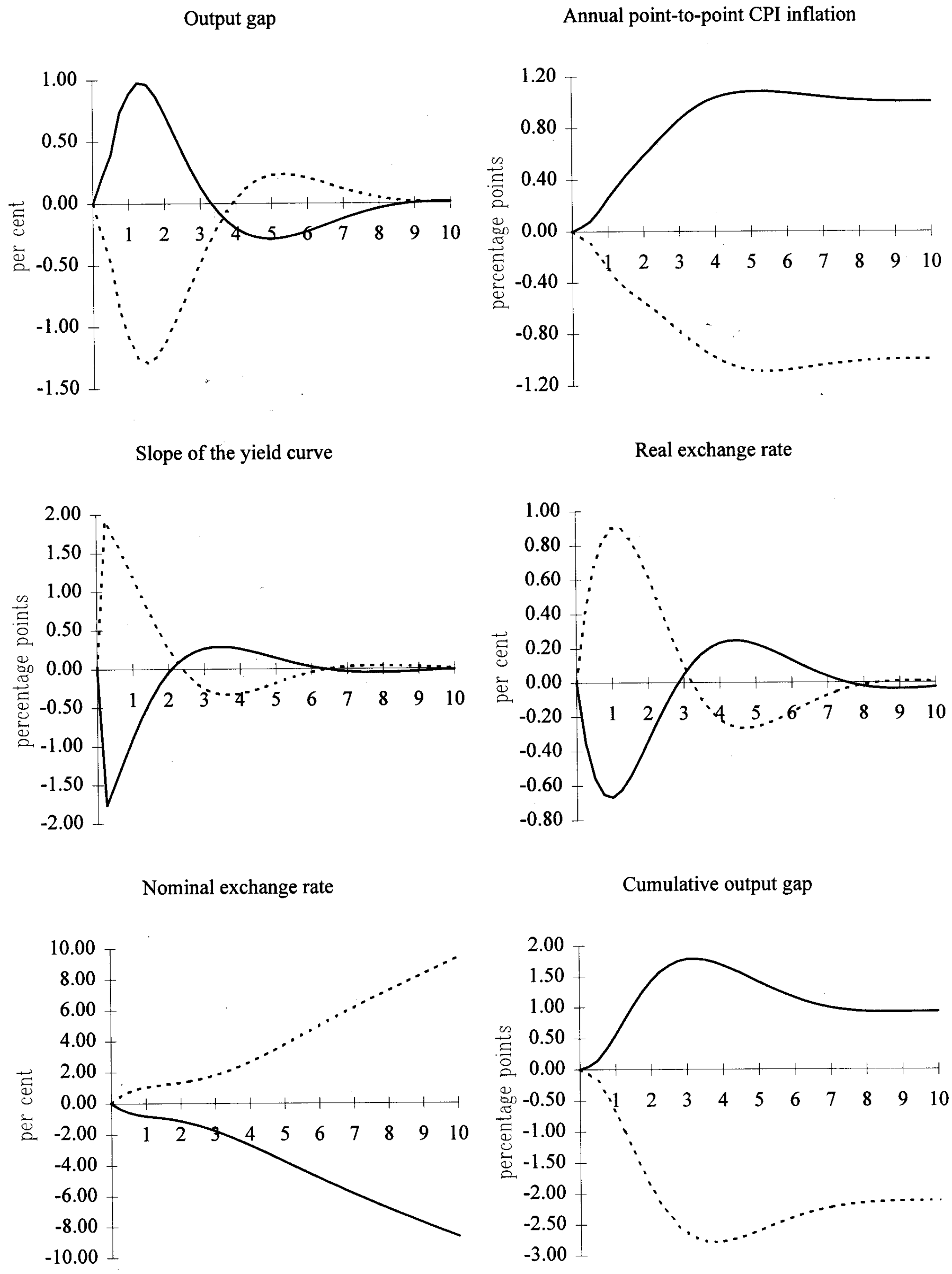
To increase inflation permanently by one percentage point, the monetary authority must shift inflation expectations up. To do so, it reduces nominal short-term interest rates. This serves to stimulate demand for both consumption and investment goods. Further, the decline in the nominal rate also produces a decline in the real short-term interest rate that causes the exchange rate to depreciate. The lower real exchange rate stimulates net exports. Aggregate demand for goods and services exceeds the economy's productive capacity for roughly three years. These demand pressures cause inflation and, consequently, inflation expectations to rise.

After two years, the stance of monetary policy reverses. Demand pressures ease and a small excess supply gap opens up to prevent inflation from accelerating more than one percentage point above its previous level. After six years, policy returns to control as inflation expectations are anchored one percentage point higher. The net gain in output, illustrated by the cumulative output gap, is roughly one per cent of one year's output.

The simulation responses for a decline in the inflation target are qualitatively similar to the above experiment except that the direction of the changes are reversed. However, there are important quantitative differences. As noted above, more excess supply is required to anchor inflation expectation one percentage point lower than excess demand is required to shift them up by one percentage point. Consequently, the required policy tightening is both stronger and longer lasting when the inflation target is reduced by one percentage point. The cumulative output loss under the deflation experiment is roughly twice the cumulative output gain under the inflation experiment. The degree of asymmetry has been calibrated, using these two experiments, to be roughly two-to-one.

62 Note that the core model is neutral and super-neutral with respect to the rate of inflation. There are no permanent real effects arising from changes to the target rate of inflation.

Figure 5.4
Changes to the inflation target



5.2.5 Monetary reaction to an increase in demand

As outlined in the previous experiment, the inflation process in the core model is fundamentally driven by relative demand and supply conditions. Further, inflation accelerates more rapidly when demand exceeds supply than it decelerates when supply exceeds demand. An implication of this asymmetry is that delaying the policy response to an inflationary disturbance serves only to increase the amount of output that must be forgone to re-anchor inflation expectations at the target rate.[63]

To illustrate this point, we compare two responses to an unanticipated autonomous increase in the demand for goods and services. The shock is configured as an increase in consumption and investment expenditures lasting four quarters. The results differ because of the response of monetary policy. In the first case, monetary reaction is unrestrained (the solid line). In the second case, monetary reaction is delayed four quarters (the dashed line). Both cases are presented in Figure 5.5.

When the monetary authority's response is delayed, the nominal short-term interest rate does not change, relative to the long-term interest rate, for four quarters after the shock has hit. The main result of delaying the monetary response is a rapidly growing inflation problem. As inflation gathers momentum, inflation expectations accelerate. Once the monetary authority responds, a more severe tightening is required to return inflation to the target rate. The required contraction in output is substantially larger and longer-lasting in the delayed case. In other words, following a shock which raises demand above its sustainable level, a cumulative output loss is necessary for inflation to be returned to its target rate. This is true even when there is no delay in the policy response. However, if inflation pressures are allowed to build for longer, a larger cumulative output loss is incurred.

5.2.6 An increase in nominal wage demands

The wage-price nexus in the core model has prices leading wages in response to shocks originating outside the labour market. However, accelerating costs can be an important source of inflationary pressure to which monetary policy must be vigilant. Therefore, the core model has the appropriate channels to allow for traditional 'cost-plus' influences originating in the labour market. The equilibrium level of real wages in the core model is determined by the marginal product of labour. A permanent increase in real wages requires a permanent improvement in labour productivity. Temporary accelerations in nominal wage inflation cannot, therefore, succeed in changing the level of real wages permanently.

To illustrate this we use an unanticipated one percentage point increase to the annual rate of nominal wage growth for one quarter. The shock can be thought of as a temporary increase in labour's bargaining power that results in firms accepting a nominal wage agreement that allows real wages to exceed the marginal product of labour.

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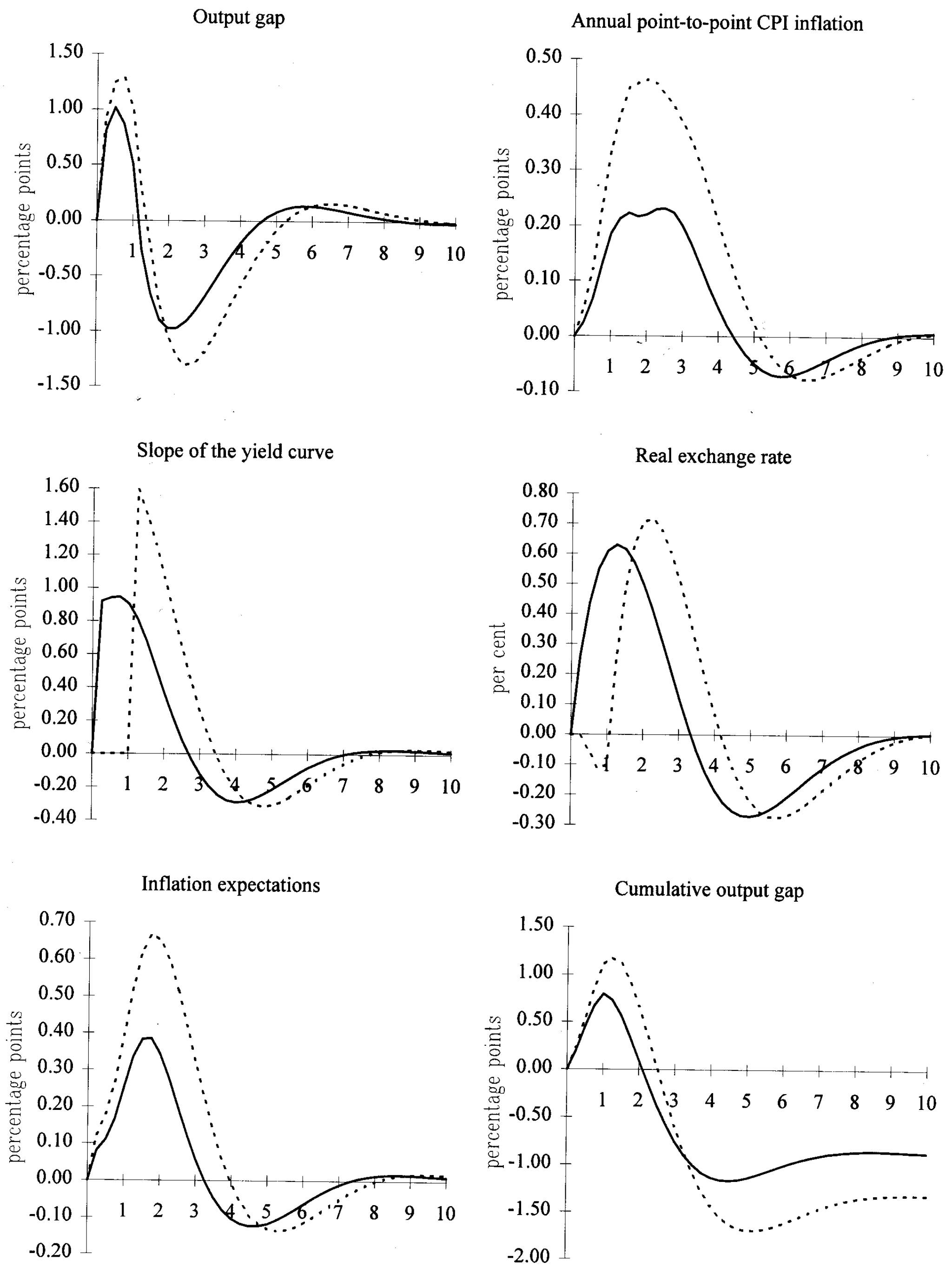
[63] Note that this excludes the *gains* from maintaining price stability.

Figure 5.5
Two responses to an autonomous demand shock



When the shock occurs and real wages rise above their long-run equilibrium, firms try to pass on these higher costs by raising prices. This generates inflationary pressures from the cost side.[64] Further, the temporary increase in real wages raises disposable incomes and stimulates consumption demand. Increased consumption creates a positive output gap, adding additional inflationary pressures. The forward-looking monetary authority projects that inflation will rise above target and therefore tightens monetary conditions immediately. The tightening in monetary conditions eventually generates excess supply as demand for consumption, investment and net exports all slow. Subsequently, inflation is re-anchored at the policy target.

The real wage is restored to its equilibrium level by a combination of lower nominal wage inflation and the acceleration in price inflation. The initial increase in nominal wages causes an increase in unemployment and unemployment rises further as demand conditions weaken. As the real wage, demand conditions, and inflation re-equilibriates, so does unemployment. The only long-term consequence is some upward drift in the price level.

5.2.7 A temporary improvement in investor confidence in New Zealand assets

This section describes an experiment where there is a temporary, unanticipated appreciation of the real exchange rate. One interpretation of this experiment is that it represents a temporary rise in confidence in assets denominated in New Zealand dollars. Although this increase in the real exchange rate has important implications for the real side of the economy and subsequent inflationary consequences, it also has direct price implications.

Although uncovered interest parity is a key determinant of exchange rate behaviour in the core model, the exchange rate does not behave as a pure jumper variable. The exchange rate process has been modelled to capture the type of persistent behaviour we observe in the data. Consequently, the monetary authority cannot exploit the UIP condition to simply unwind the appreciation by reducing the short-term nominal interest rate. The improvement in investor confidence transmits into an appreciation in the real exchange rate that is temporary but persistent.

Upon impact, exchange rate appreciation starts to exert downward pressure on domestic prices through two channels. First, the exchange rate appreciation reduces import prices. Second, the real appreciation weakens export demand and stimulates import demand, causing demand for domestically-produced goods to ease and a negative output gap to open up. In response, the monetary authority immediately reduces short-term interest rates. (It is important to note that there is no direct response to the exchange rate itself in the reaction function. The monetary reaction arises from the consequences of the shock for domestic inflation.) Consumption, and (to a lesser degree) investment, respond and eventually demand for domestically produced goods strengthens sufficiently to generate the excess demand conditions required to re-anchor inflation expectations.

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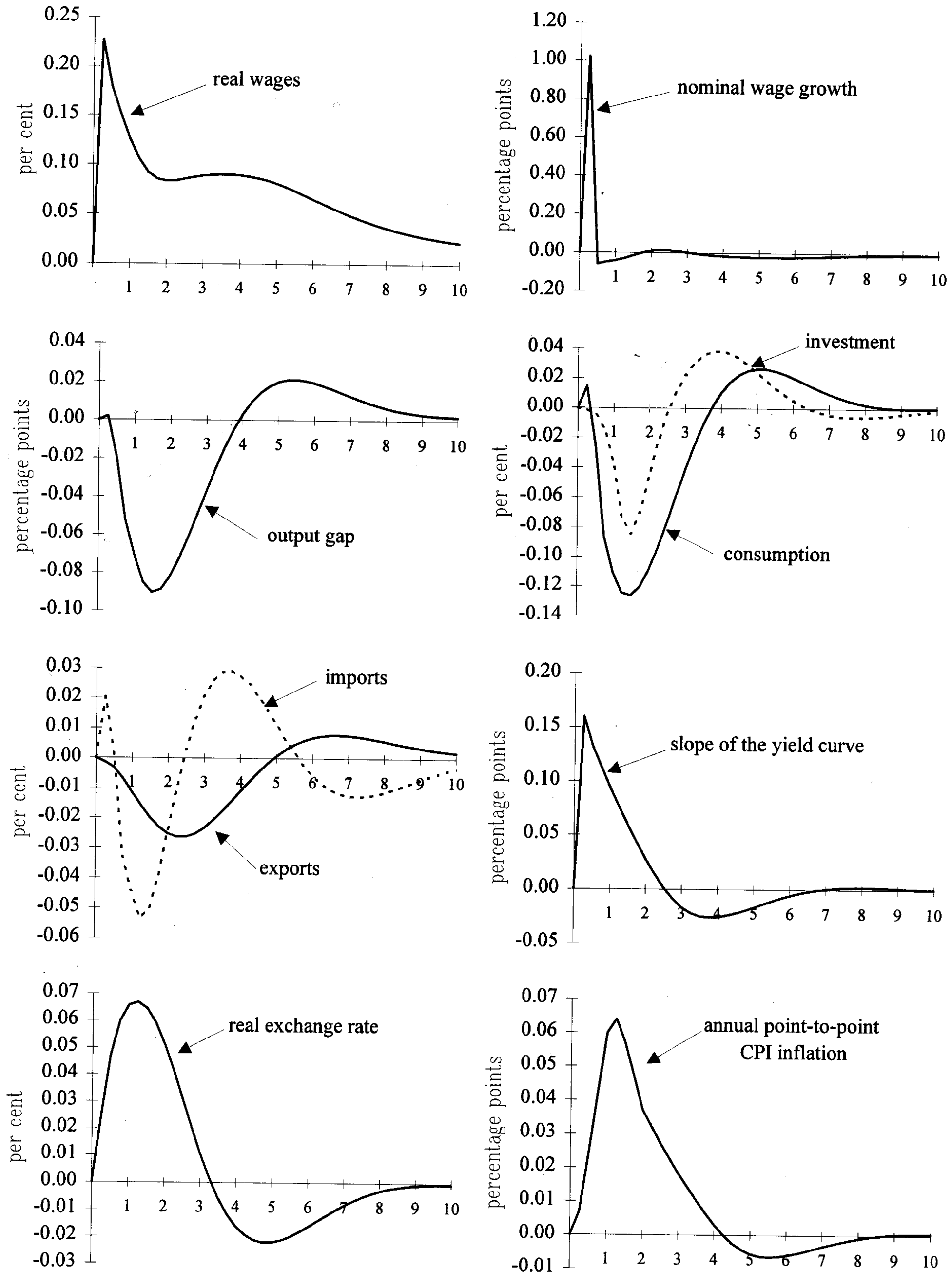
[64] Note that there is no inertia in wage inflation itself. In this version of the model, any persistence in nominal wage inflation comes about only as a result of persistence in inflation and inflation expectations feeding back into wage demands.

Figure 5.6
An increase in nominal wage demands



5.2.8 An improvement in New Zealand's terms of trade

Here we present two simulation experiments, both involving improvements in the terms of trade faced by the domestic economy. The key difference is that in one experiment the improvement is permanent; in the other, the improvement is temporary. Whereas the economy shifts to a new equilibrium following the permanent improvement, there are no long-run effects on the real economy following the temporary one. We employ two variations on a terms-of-trade shock here to make this point clear. The terms of trade improvements embody a 0.5 per cent rise in world export prices and a 0.5 per cent fall in world import prices. The temporary improvement lasts for eight quarters.

Focusing on the permanent shock first (the solid line), the most notable long run effect of the improvement in the terms of trade is the increase in productive capacity. Firms benefit from permanently-increased returns to capital and a permanent reduction in the cost of capital. Consequently, their desired capital-to-output ratio increases. The increase in capital intensity also increases the return accruing to labour and households' human wealth increases. Households save some of this increase and their desired financial asset position rises. However, households do not hold all the increase in domestic capital and bonds, so that the equilibrium net foreign asset-to-output ratio declines slightly. Despite the slight decrease in the NFA ratio, the terms-of-trade effect is sufficiently large that fewer net exports are required in equilibrium to service foreign liabilities. As a result, the equilibrium real exchange rate appreciates. A larger share of domestic production becomes available for domestic uses; the shares of consumption and investment in output are higher in the new equilibrium. Initially, consumption and investment demand both respond strongly to start the adjustment to the new equilibrium. Households start to consume some of their increased human wealth and firms start investing to achieve their new desired capital level. However, the productive capacity of the economy does not respond as rapidly because the time-to-build feature means that the increase in investment becomes productive only slowly. As a result, demand pressures build that quickly offset the direct deflationary pressure from declining import prices. The initial easing in policy is short-lived as the forward-looking monetary authority recognises that the demand effects will dominate. The real exchange rate immediately starts to appreciate and the tightening in monetary conditions is sufficient to ease demand pressures primarily through consumption and net exports. The worsening trade balance allows the required resources to be imported and the net foreign asset position declines to its new equilibrium level. Inflation is re-anchored and the economy converges on its new equilibrium with a higher level of output and larger shares going to domestic uses, consumption and investment.

Figure 5.7
A temporary real exchange rate appreciation

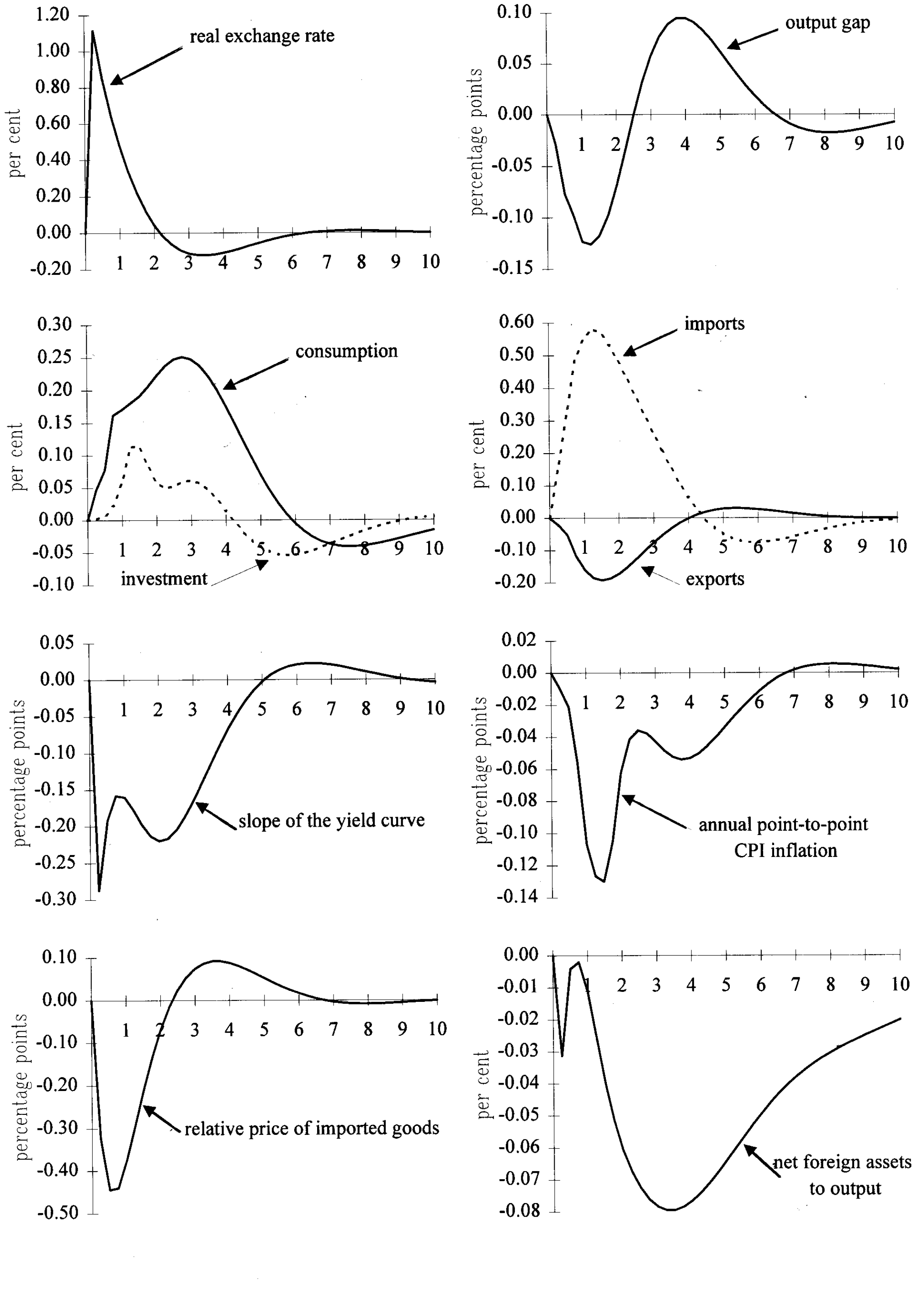
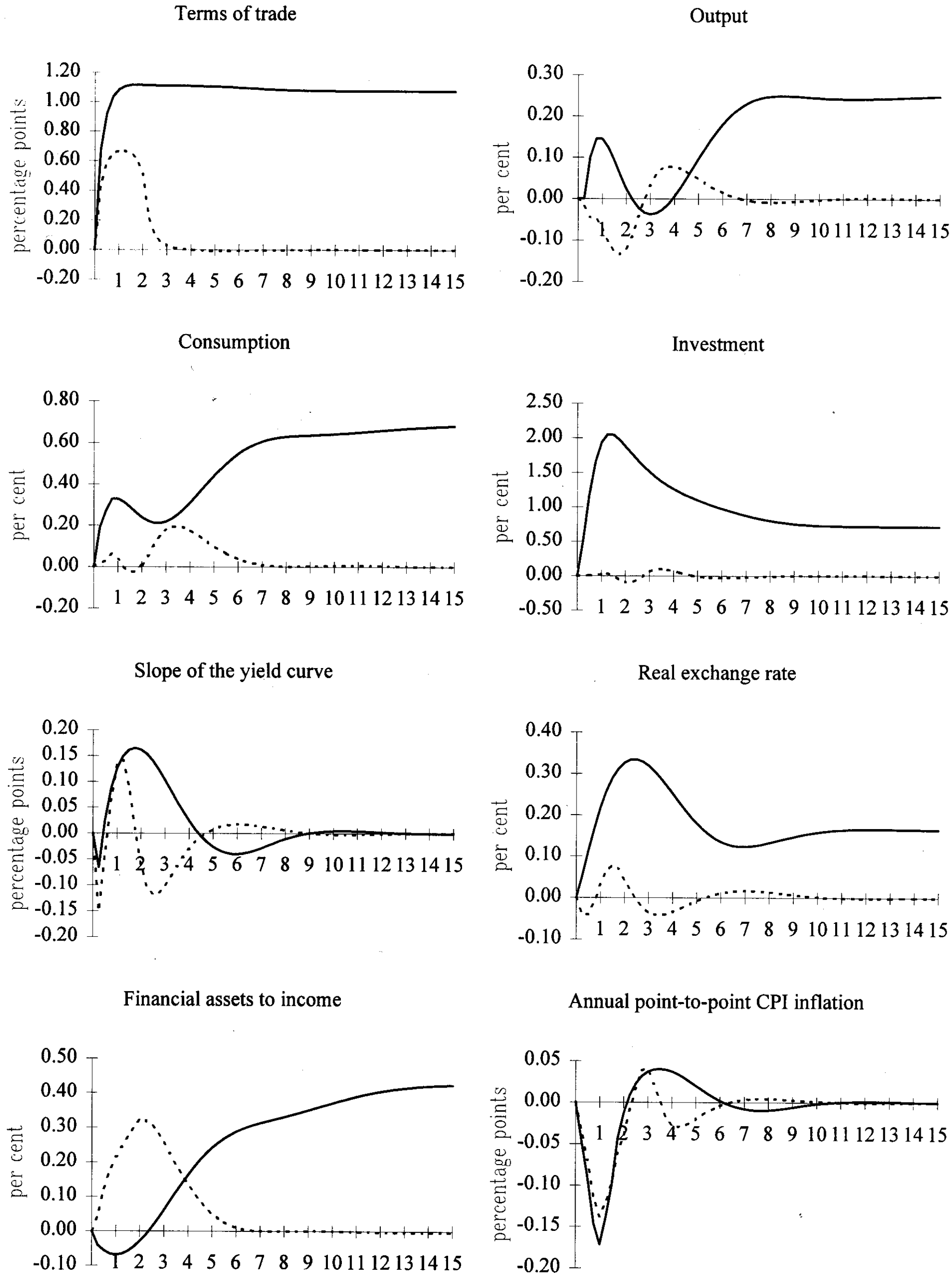


Figure 5.8
Permanent and temporary improvements in the terms of trade



The adjustment to the temporary improvement (the dotted line) is quite different. Aggregate demand initially falls as there are no long-run influences driving the dynamic adjustment of consumption and investment. Net exports decline because lower import prices increase import penetration. The initial easing in monetary policy, though longer-lived and larger than under the permanent improvement, is also reversed fairly quickly. However, the reason is somewhat different; under the temporary change, the monetary authority sees that the direct price effect will be reversed. Policy then eases again to ensure sufficient demand pressures arise to re-anchor inflation expectations at the target. The monetary authority is assisted by the demand pressures that arise because of the initial improvement in the terms of trade. It results in a more positive current account, which cumulates into a net foreign asset position that is above equilibrium. Households respond by increasing consumption. Eventually the positive output gap re-anchors inflation expectations at the target.

This is a very useful place to end the discussion of model properties. The significant differences that arise under permanent and temporary disturbances illustrated by these two simulation experiments are a useful reminder of just how important the inputs into the model are. The interpretation of current developments as either permanent or temporary disturbances implies significantly different reactions from the monetary authority. Under the permanent terms-of-trade improvement the dominant job of the monetary authority is to reduce demand pressures. Under the temporary improvement it is primarily trying to support demand. Models are tools and the quality of the policy advice produced depends not only on the quality of the tool but the underlying analysis embodied in the inputs used by the model.

This discussion also illustrates the importance of the assumption we have made that the shocks, while not anticipated, are known and understood from the start, once they have occurred. If private agents and the monetary authority had to learn about the shock and its duration, the dynamics and the monetary control problem could be very different. Discussion of the conduct of monetary policy under uncertainties like this is a very important topic for future research using the model.

6. Concluding remarks

This document has described the core macroeconomic model that lies at the heart of the Reserve Bank's new *Forecasting and Policy System*. One aim has been to show how the objectives of the project motivated the design of a complete system with a modern macroeconomic model at its core. Another has been to describe the underlying theory embodied in the core model and illustrate its behavioural properties.

The design of FPS has been very much a product of history. The twin goals of projection and policy analysis, with their different demands, led to the concept of a macroeconomic model at the centre of a system. Past experience also provided other lessons that informed the design of the core model. Before even considering the economic structure of the model, it was clear that it would have to have several features that would make it different from previous models at the Bank. The roles of steady state, the stock-flow accounting, the supply side, expectations, and endogenous policy reaction will hopefully now be clear. The top-down approach, together with calibration, has made building a model with these features feasible in the New Zealand environment.

The theoretical content of the core model places it among the more advanced macroeconomic models used by policy makers world-wide. Rather than explain every one of the model's equations, the discussion has focused on the fundamentals underlying the optimising behaviour of the key agents in the model. The outline of the parameterisation of the core model's steady state illustrated the discipline that it imposes on views of how the economy is likely to evolve in the future. The dynamic simulation paths produced under various shock experiments illustrated how the theory embodied in the structure and the calibration choices come together to determine the model's properties. At all stages, in keeping with the way the model was built, the emphasis has been on the properties and behaviour of the model as a whole, rather than on individual elements.

The model has already proven useful in the production of quarterly economic projections. After being tested in two 'shadow' projections in December 1996 and March 1997, it was used as the central tool for the preparation of the economic projection reported in the June 1997 *Monetary Policy Statement*. The next year will see the model's role in this area expand as it is used more to evaluate the real risks that monetary policy makers face operating in an uncertain world. Further, work is under way to use the model in a stochastic environment so that policy analysis conducted with the model will also take into account some of the very real uncertainties that face policy makers.

While the design of FPS has been a product of history, we have also learned many lessons in the process of building the core model. For example, the steady state of the model has been an important aid to consistency, and proven to be informative in its own right on policy issues. When calibrating the model, we learned that empirical evidence must be interpreted with great care. It is important to realise that parameter estimates derived from microeconomic research or from small system techniques, even if very sound, may not map well into a highly-aggregated macroeconomic system. Concomitantly, it is very clear that it is difficult to make judgements about the properties of the model on the basis of individual equations or parameters. It is for this reason that specific equation coefficient values have not been mentioned and simulation experiments were used to illustrate the model's properties. Another important lesson relates to the preparation of a model database. Considerable attention has been directed towards deriving data series in accordance with their economic meaning in the model, rather than their National Accounts interpretation. Improvements in data consistency have mapped into clear improvements in the behaviour of the model.

However, none of this should be taken to imply that there is no more to be learnt, or that the model will not need to be developed further. While the model has been designed to be economically quite 'complete', it is still highly stylised and quite aggregated. Questions naturally arise as to whether the model is 'complete enough'. For example, since terms of trade shocks are very important for a small open economy like New Zealand, should the core model be extended to a multiple-good framework? Initially the satellite models are intended to facilitate extra analysis of these types of issues. However, only time will really tell whether these issues can be handled adequately in the satellite structure or need to be part of the general-equilibrium framework of the core model.

There are numerous other directions for future research that may prove quite fruitful. Endogenous labour supply would extend the model to allow for analysis of labour-leisure trade-off issues and the distortionary effects of income taxes. One might conjecture that there are important linkages from fiscal and monetary policy to the country risk premium, and that the premium should therefore be determined behaviourally within the core model. Extensions to allow for non-superneutralities and endogenous growth would extend the model's ability to examine issues of optimal policy. In all cases, decisions about extending model will be based on the resource costs involved and the expected benefits in terms of either projection accuracy or the enhancement of policy analysis.

Models are by construction abstractions. No macroeconomic model will ever be able to incorporate all the behavioural aspects of the real world and, consequently, no model will ever be able to project the future perfectly, even if there were no unanticipated shocks. In other words, a model should not be expected to produce 'The Truth.' However, a properly designed model that captures the salient behavioural features of the economy can be a very useful tool for examining many of the policy issues of concern to a monetary authority. The design of FPS and its core model has been focused on precisely this.

All models are wrong, but some are useful
George Box

Appendices

Appendix A: Equations for the core model

In the equations that follow, variables with a suffix *_eq* are equilibrium values. The equilibrium values will converge on the steady-state (*ss*) values chosen for the steady state calibration. However, the *_eq* values differ, in principle, from the *ss* values when the model is shocked. The model's theory provides predictions about 'equilibrium' adjustment in the absence of adjustment costs. This is the basic source of the *_eq* structure. Dynamic equations overlay the *_eq* structure. Some of these contain adjustment cost terms, which end in *adj*. For brevity, these are not expanded out.

Variables with the form v(i) indicate the model solution for the variable v *i* periods ahead (ie a lead of *i* periods). Variables with the form v(-i) indicate the value of the variable v *i* periods back (ie a lag of *i* periods). * denotes multiplication and ** exponentiation. Variables with the form *x_y* are usually ratios (x over y). For relative interest rates, we use the same notation for an absolute difference.

Real levels are deflated by an index of the trend level of real output. Nominal variables are also divided by the price of domestically-produced and consumed goods at factor cost (*pd*). In other words, every price in the core model is a relative price. Thus, all variables in the core model are strictly stationary in the sense that their steady state values are particular numbers. Trend real growth and inflation of nominal values are reintroduced separately in a procedure that transforms and adds to the output from the core model.

The core model is written with all units in quarterly units. Thus, for example, all expenditure flows are at quarterly rates. Similarly, all growth rates, inflation rates and interest rates are quarter-over-quarter, unless otherwise stated.

The standard expenditure add-up identities are included, but there are two twists. The first is the conversion to create stationarity. Thus *y* is not constant dollar output. If Y is a constant dollar measure, it is rescaled according to y(t)=Y(t)/(Y0*(1+ydot)**t), where Y0 is the value in a base period (ie we also normalise the units of y to be 0.25). The same rescaling is carried out for *c*, *i*, etc. Thus they are proportional *shares* of *y*. Also, the prices are not the System of National Account deflators. For example, *py* must be interpreted as PY/PD, where the upper case names are the SNA prices.

The rescaling is particularly important for cumulation equations. For example, the normal investment equation, in real form, would be K(t)=(1-depr)*K(t-1)+I(t). Dividing by Y0(1+ydot)**t, and using the core model's measurement convention, that k(t)=K(t)/(Y0(1+ydot)**t), etc, the reason for the growth corrections in the *i* and *i_eq* equations should be immediately clear. In general, in accumulation equations, if there is a lag (lead) there will be a 'divide' ('multiply') rescaling of that term. In some cases, where it does not really matter, such as in the *c* adjustment equation, we leave these terms out to preserve readability. In accumulation equations, this detail is not optional.

Finally, the equations are listed with a label, followed by a colon. Labels are arbitrary in a simultaneous system - the entire *n* equations of a nonsingular system determine *n* unknowns. In TROLL, furthermore equations do not have to be written in a form already normalised on a particular variable. In fact, the label can be a variable that does not actually appear in the equation. Therefore, these labels indicate at best that the equation *proximately* determines the variable in question. They should not be used as if the model were simply a set of reduced-form equations.

Growth accounting

```
ndot:       ndot = ndot_eq
qdot:       qdot = qdot_eq
ydot:       ydot = (1+ndot)*(1+qdot) - 1
ydot_eq:    ydot_eq = (1+ndot_eq)*(1+qdot_eq) - 1
```

Expenditure accounts

Output

```
y:          y = c + i + g + x - m
py:         py*y = pc*c + pi*i + pg*g + px*x - pm*m
py_eq:      y_eq = c_eq + i_eq + g_eq + x_eq - m_eq
pfc:        py*y = (1+tiy)*pfc*y
pfc_eq:     py_eq*y_eq = (1+tiy_eq)*pfc_eq*y_eq
```

Consumption

```
c:          c = crt + cfl
crt:        crt = ydrt/pc
cfl:        cfl = (cfl_eq + cv1*((ydfl(-2)/pc(-2))/(ydfl_eq(-2)/pc_eq(-2))-1)
            - cv2*(rsl(-2)-rsl_eq(-2))*cfl_eq(-2)
            + cv3*(nfa/pc-nfa_eq/pc_eq) - cfladj)

c_eq:       c_eq = cfl_eq + crt_eq
crt_eq:     pc_eq*crt_eq = ydrt_eq
cfl_eq:     pc_eq*cfl_eq = mpcw_eq*twfl_eq + zeta*(fa_eq-fa_ss)
mpcw_eq:    1/mpcw_eq = (1-gamma)*delta**sigma *(pc_eq/pc_eq(+1)
            *(1+rcon_eq))**(sigma-1)/mpcw_eq(1) + 1
twfl_eq:    twfl_eq = hwfl_eq + (1+rcon_eq(-1))*fa_eq(-1)/(1+ydot_eq)
hwfl_eq:    hwfl_eq = ydfl_eq + risk_eq + (1-gamma) * (1+qdot_eq)
            *hwfl_eq(1)/(1+rcon_eq(1))
fa_eq:      fa_eq + pc_eq*cfl_eq = ydfl_eq + risk_eq + (1+rcon_eq(-1))
            *fa_eq(-1)/(1+ydot_eq)
fa_ss:      fa_ss = fa_ss(1)
```

Investment

```
i:          k = (1-depr)*k(-1)/(1+ydot) + i
i_eq:       k_eq = (1-depr_eq)*k_eq(-1)/(1+ydot_eq) + i_eq
kp_eq:      kp_eq = (1-ip1-ip2-ip3-ip4-ip5-ip6-ip7)*k_eq
```

```
                 +ip1*k_eq(-1)/(1+ydot_eq)+ip2*k_eq(-2)/(1+ydot_eq)**2
                 +ip3*k_eq(-3)/(1+ydot_eq)**3+ip4*k_eq(-4)/(1+ydot_eq)**4
                 +ip5*k_eq(-5)/(1+ydot_eq)**5+ip6*k_eq(-6)/(1+ydot_eq)**6
                 +ip7*k_eq(-7)/(1+ydot_eq)**7
kp:              kp = (1-ip1-ip2-ip3-ip4-ip5-ip6-ip7)*k
                 +ip1*k(-1)/(1+ydot)+ip2*k(-2)/(1+ydot)**2
                 +ip3*k(-3)/(1+ydot)**3+ip4*k(-4)/(1+ydot)**4
                 +ip5*k(-5)/(1+ydot)**5+ip6*k(-6)/(1+ydot)**6
                 +ip7*k(-7)/(1+ydot)**7
```

Government expenditures

```
g:               g = g1*g(-1) + (1-g1)*g_eq
g_eq:            g_eq = g2*g_eq(-1) + (1-g2)*g_y_eq*y_eq
gtr:             gtr = gtr1*gtr(-1) + (1-gtr1)*gtr_eq
gtr_eq:          gtr_eq = gtr2*gtr_eq(-1) + (1-gtr2)*gtr_y_eq*y_eq
```

External trade, imports, exports, and net exports

```
m:               m = cm + im + gm
m_eq:            m_eq = cm_eq + im_eq + gm_eq
cm:              cm = cm_c*c
cm_eq:           cm_eq = cm_c_eq*c_eq
cm_c:            cm_c = cm_c_eq - cmv1*((1+ticm)*pcm(-1)/pcd(-1) -(1+ticm_eq)
                 *pcm_eq(-1)/pcd_eq(-1)) - cm_cadj
cm_c_eq:         cm_c_eq = cm_c0 - cms2*(1+ticm_eq)*pcm_eq/pcd_eq
im:              im = im_i*i
im_eq:           im_eq = im_i_eq*i_eq
im_i:            im_i = im_i_eq - imv1*((1+tiim)*pim(-1)/pid(-1)
                 -(1+tiim_eq)*pim_eq(-1)/pid_eq(-1)) - im_iadj
im_i_eq:         im_i_eq = im_i0 - ims2*(1+tiim_eq)*pim_eq/pid_eq
gm:              gm = gm_g*g
gm_eq:           gm_eq = gm_g_eq*g_eq
gm_g:            gm_g = gm_g_eq - gmv1*((1+tigm)*pgm(-1)/pgd(-1)
                 - (1+tigm_eq)*pgm_eq(-1)/pgd_eq(-1)) - gm_gadj
gm_g_eq:         gm_g_eq = gm_g0 - gms2*(1+tigm_eq)*pgm_eq/pgd_eq,

x:               x = x_eq + xv1*(px(-2)-px_eq(-2))*x_eq(-2) - xadj
x_eq:            x_eq/y_eq = x_y0 + x2*px_eq

xbal:            xbal = px*x - pm*m
xbal_eq:         xbal_eq = px_eq*x_eq - pm_eq*m_eq
netx:            netx = x - m
netx_eq:         netx_eq = x_eq - m_eq
```

Income accounts

Wages and labour income

```
wa:              wa = wa(-1)*(1+wdot)/((1+pdot)*(1+qdot))
wa_eq:           wa_eq = (1-alpha)*pfc_eq*y_eq/(1-u_eq)
wp_eq:           wp_eq =  wa_eq/pfc_eq
```

```
wp:          wp = wa/pfc
wc_eq:       wc_eq = wa_eq/pc_eq
wc:          wc = wa/pc
wctar:       wctar = MAX((1-wcf0)*wc_eq+wcf0*wc_eq(1)
             + (wc1*((1-wcf0)*wc_eq+wcf0*wc_eq(1))
             +(1-wc1)*(wcl1*wctar(-1) + wcl2*wctar(-2)
             + wcl3*wctar(-3) + (1-wcl1-wcl2-wcl3)*wctar(-4))))
wdot_eq:     wdot_eq = (1+pdot_eq)*(1+qdot_eq) - 1
wdot:        1+wdot = (1+qdot)*(1+wpe1
             *(wp2*pdote(-2)+wp3*pdote(-3)+wp4*pdote(-4)
             +wp5*pdote(-5)+wp6*pdote(-6)
             +(1-wp2-wp3-wp4-wp5-wp6)*pdote(-7))
             +(1-wpe1)*(wp2*pcdote(-2)+wp3*pcdote(-3)+wp4*pcdote(-4)
             +wp5*pcdote(-5)+wp6*pcdote(-6)+(1-wp2-wp3-wp4-wp5-
             wp6)*pcdote(-7)))
             + wd1*MAX(wctar(-1)/wc(-1)-1,0)
             + wd2*(wp_eq(-1)/wp(-1)-1)
             + wd3*(u_eq(-1)-u(-1))+ wd4*MAX(u_eq(-1)-u(-1),0)
             + wd7*(u_eq(-2)-u(-2))+ wd8*MAX(u_eq(-2)-u(-2),0)
             + wd5*MAX(pc(-1)/(1+tic(-1))/pc(-5)*(1+tic(-5))-1,0)
             + wd6*MAX((tic_eq-tic_eq(-4))/(1+tic_eq(-4)),0)
             + wd0*(tfp(-2)/tfp_eq(-2)-1)

ylab:        ylab = wa*(1-u)
ylab_eq:     ylab_eq =  wa_eq*(1-u_eq)
```

Disposable income

```
ydrt:        ydrt = ( (1-td)*(ylab+yd1*gtr) + (1-yd1)*gtr)*lamda
ydrt_eq:     ydrt_eq = ((1-td_eq)*(ylab_eq+yd1*gtr_eq) + (1-yd1)*gtr_eq)*lamda
ydfl:        ydfl = ( (1-td)*(ylab+yd2*gtr) + (1-yd2)*gtr)*(1-lamda)
ydfl_eq:     ydfl_eq = ((1-td_eq)*(ylab_eq+yd2*gtr_eq) + (1-yd2)*gtr_eq)*(1-
             lamda)
```

Risk income

```
risk_eq:     risk_eq = ((rk_eq(-1)-rcon_eq(-1))*pka(-1)*k_eq(-1)
             + (rgb_eq(-1)-rcon_eq(-1))*gb_eq(-1)
             + (rnfa_eq(-1)-rcon_eq(-1))*nfa_eq(-1) )/(1+ydot_eq)
             + pka*k_eq - pi_eq*i_eq - (1-depr_eq)*pk_eq*kp_eq(-1)/(1+ydot_eq)
             -((1+rk_eq(-1))*(pka(-1)*k_eq(-1)-pk_eq(-1)*kp_eq(-1))
             - tk_eq*depr_eq*pka(-1)*k_eq(-1) )/(1+ydot_eq)
risk:        risk = ri1*risk(-1) + (1-ri1)*(risk_eq)
```

Stocks

Capital

```
k:           k = k_eq + kv1*(y(-4)/y_eq(-4)-1)
             - kv2*(rsl(-4)-rsl_eq(-4))*k_eq(-4) - kadj
k_eq:        cc_eq = alpha*pfc_eq(1)*y_eq(1)*(1+ydot_eq)/kp_eq
cc_eq:       cc_eq*(1-tk_eq) = (1+rk_eq)*pk_eq - (1-depr_eq)*pk_eq(1)
```

```
dt_eq:      dt_eq = (dt_eq(1)*(1-depr_eq) + pka*depr_eq*tk_eq)/(1+rk_eq)
pk_eq:      (1-ip1-ip2-ip3-ip4-ip5-ip6-ip7)*pk_eq
            + ip1*pk_eq(1)/(1+rk_eq) + ip2*pk_eq(2)/(1+rk_eq)/(1+rk_eq(1))
            + ip3*pk_eq(3)/(1+rk_eq)/(1+rk_eq(1))/(1+rk_eq(2))
            + ip4*pk_eq(4)/(1+rk_eq)/(1+rk_eq(1))/(1+rk_eq(2))/(1+rk_eq(3))
            +ip5*pk_eq(5)/(1+rk_eq)/(1+rk_eq(1))/(1+rk_eq(2))/(1+rk_eq(3))/
            (1+rk_eq(4)) + ip6*pk_eq(6)/(1+rk_eq)/(1+rk_eq(1))/(1+rk_eq(2))/
            (1+rk_eq(3))/(1+rk_eq(4))/(1+rk_eq(5)) + ip7*pk_eq(7)/(1+rk_eq)/
            (1+rk_eq(1))/(1+rk_eq(2))/(1+rk_eq(3))/(1+rk_eq(4))/(1+rk_eq(5))/
            (1+rk_eq(6)) = ((1-pk1)*pi_eq+pk1*pi_ss) - dt_eq
            + ke1*(i_eq-i_ss)
i_ss:       i_ss  = i_ss(1)
pi_ss:      pi_ss = pi_ss(1)
```

Government bonds and taxes

```
gbtar_eq:   gbtar_eq = gbtar_y_eq*y_eq
gbtar:      gbtar = gbtar_eq
gb_eq:      gb_eq = gb_eq(-1) + td1*(gb_eq-gbtar) + td2*(gb_eq-gb_eq(-1))
gb:         gb + td*(ylab+(yd1*lamda+yd2*(1-lamda))*gtr) + tiy*pfc*y
            + tk*(pfc*y - ylab - depr*pka(-1)*k(-1)/(1+ydot))
            = (1+rgb(-1))*gb(-1)/(1+ydot) + pg*g + gtr
td_eq:      gb_eq + tiy_eq*pfc_eq*y_eq
            + td_eq*(ylab_eq+(yd1*lamda+yd2*(1-lamda))*gtr_eq)
            + tk_eq*(pfc_eq*y_eq-ylab_eq-depr_eq*pka(-1)
            *k_eq(-1)/(1+ydot_eq))
            = (1+rgb_eq(-1))*gb_eq(-1)/(1+ydot_eq) + pg_eq*g_eq +gtr_eq
td:         td = (tdl1*td(-1)+tdl2*td(-2)+tdl3*td(-3)+tdl4*td(-4)
            + (1-tdl1-tdl2-tdl3-tdl4)*(td_eq +td3*(gb(-1)-gb_eq(-1))
            - td4*(gbtar_y_eq-gbtar_y_eq(-1))))
tiy_eq:     tiy_eq = (tic_eq*pc_eq*c_eq/(1+tic_eq) + ticm_eq*pcm_eq*cm_eq
            + tii_eq*pi_eq*i_eq/(1+tii_eq) + tiim_eq*pim_eq*im_eq
            + tig_eq*pg_eq*g_eq/(1+tig_eq)
            + tigm_eq*pgm_eq*gm_eq)/(pfc_eq*y_eq)
tiy:        tiy = (tic*pc*c/(1+tic) + ticm*pcm*cm + tii*pi*i/(1+tii)
            + tiim*pim*im + tig*pg*g/(1+tig) + tigm*pgm*gm)/(pfc*y),
```

Net foreign assets and the asset identities

```
pka:        pka = (1-pk0)*pka(-1) + pk0*pi_ss
nfa:        nfa = (1+rnfa(-1))*nfa(-1)/(1+ydot) + xbal
nfa_eq:     nfa_eq = (1+rnfa_eq(-1))*nfa_eq(-1)/(1+ydot_eq) + xbal_eq
fa:         fa = pka*k + gb + nfa
z_eq:       fa_eq = pka*k_eq + gb_eq + nfa_eq
```

Production and the labour market

```
tfp:        y = 0.25*tfp*(kp(-1)/(1+ydot))**alpha*(1-u)**(1-alpha)
y_eq:       y_eq = 0.25*tfp_eq*(kp_eq(-1)/(1+ydot_eq))**alpha
            *(1-u_eq)**(1-alpha)
yp:         yp = 0.25*tfp_eq*(kp(-1)/(1+ydot_eq))**alpha*(1-u_eq)**(1-alpha)
```

ip: kp = (1-depr)*kp(-1)/(1+ydot) + ip
ip_eq: kp_eq = (1-depr_eq)*kp_eq(-1)/(1+ydot_eq) + ip_eq

u: u = u_eq - uv2*(y(-2)/yp(-2)-1) -uv3*(y(-3)/yp(-3)-1)
+ uv1*(wp(-1)/wp_eq(-1)-1) - uadj

The monetary authority, interest rates, and exchange rates

Interest rates

rn: rsl = rsl_eq + rsl1*(tpdot(1)-pdottar(1))
+ rsl2*(tpdot(2)-pdottar(2))+rsl3*(tpdot(3)-pdottar(3))
+ rsl4*(tpdot(4)-pdottar(4))+rsl5*(tpdot(5)-pdottar(5))
+ rsl6*(tpdot(6)-pdottar(6))+rsl7*(tpdot(7)-pdottar(7))
+ rsl8*(tpdot(8)-pdottar(8))+rsl9*(tpdot(9)-pdottar(9))
+ rsl10*(tpdot(10)-pdottar(10))+ rsl11*(tpdot(11)-pdottar(11))
+ rsl12*(tpdot(12)-pdottar(12)) + rns1*(rn - rn(-1))

pdottar: pdottar = pdottar_eq
pdot_eq: pdot_eq = pdottar

rn_eq: 1+rn_eq = (1+r_eq)*(1+pdot_eq(1))
rn5_eq: 1+rn5_eq = (1+rn5_eq(1)) * ((1+rt5_eq)/(1+rt5_eq(1)))*
((1+rn_eq)/(1+rn_eq(20)))**(1/20)
rn5: 1+rn5 = (1+rn5(1)) * ((1+rt5_eq)/(1+rt5_eq(1)))*
((1+rn)/(1+rn(20)))**(1/20)
rnl_eq: rnl_eq = rn5_eq
rnl: rnl = rl1*(1+rn)*(1+rt5_eq) + rl2*(1+rn5)
+ (1-rl1-rl2)*(1+rnl_eq) - 1
r: 1+rn = (1+r)*(1+pdot(1))
r5_eq: 1+r5_eq = (1+r5_eq(1)) * ((1+rt5_eq)/(1+rt5_eq(1)))*
((1+r_eq)/(1+r_eq(20)))**(1/20)
r5: 1+r5 = (1+r5(1))*((1+rt5_eq)/(1+rt5_eq(1)))*
((1+r)/(1+r(20)))**(1/20)
rl_eq: rl_eq = r5_eq
rl: 1+rl = rl1*(1+r)*(1+rt5_eq) + rl2*(1+r5) + (1-rl1-rl2)*(1+rl_eq)
rsl: 1+rsl = (1+rn)/(1+rnl)
rsl_eq: 1+rsl_eq = (1+rn_eq)/(1+rnl_eq)
r_eq: r_eq = rrow_eq + rp_eq
rk_eq: rk_eq = rl_eq + rk_rl_eq
rgb: rgb = rl + rgb_rl
rgb_eq: rgb_eq = rl_eq + rgb_rl_eq
rnfa: rnfa = rl + rnfa_rl
rnfa_eq: rnfa_eq = rl_eq + rnfa_rl_eq
rcon_eq: rcon_eq = rl_eq + rcon_rl_eq
rp: rp = rp_eq
rgb_rl: rgb_rl = rgb_rl_eq
rnfa_rl: rnfa_rl = rnfa_rl_eq

Exchange rates

```
ze:         ze = zf1*z(1) +zl1*z(-1) + (1-zf1-zl1)*z_eq(1)
z:          z = z1*z(-1) + z2*ze*(1+rrow+rp)/(1+r) + (1-z1-z2)*z_eq
```

Inflation expectations and the Phillips curve

```
pdote:      pdote = (1-(pde0+pde1+pde2+pde3+pde4+pde5+pde6+pde7+pde8))*
            (pdl1*pdot(-1)+pdl2*pdot(-2)+pdl3*pdot(-3)
            +(1-pdl1-pdl2-pdl3)*pdot(-4))
            + pde1*pdot(1) + pde2*pdot(2) + pde3*pdot(3)
            + pde4*pdot(4) + pde5*pdot(5) + pde6*pdot(6)
            + pde7*pdot(7) + pde8*pdot(8) + pde0*pdottare
pcdote:     pcdote = (1-(pde0+pde1+pde2+pde3+pde4+pde5+pde6+pde7+pde8))*
            (pdl1*ncpidot(-1)+pdl2*ncpidot(-2) + pdl3*ncpidot(-3)
            +(1-pdl1-pdl2-pdl3)*ncpidot(-4))
            + pde1*ncpidot(1) + pde2*ncpidot(2) + pde3*ncpidot(3)
            + pde4*ncpidot(4) + pde5*ncpidot(5) + pde6*ncpidot(6)
            + pde7*ncpidot(7) + pde8*ncpidot(8) + pde0*pdottare
            + pde9*((tiy_eq-tiy_eq(-4))/(1+tiy_eq(-4)))
pdot:       pdot = pdf1*pdote + (1-pdf1)*(pda1*pdot(-1)
            +pda2*pdot(-2)+pda3*pdot(-3)+(1-pda1-pda2-pda3)*pdot(-4))
            + pda6*(pm/pm(-1)-1) + pda7*(px/px(-1)-1)
            + pd0*(y(0)/yp(0)-1) + pd1*(y(-1)/yp(-1)-1)
            + pd2*(y(-2)/yp(-2)-1)+ pd3*MAX(y(-1)/yp(-1)-1,0)
            + pda4*((1+wdot)-(1+qdot)*(1+pdottare))
            + pda5*((1+wdot(-1))-(1+qdot(-1))*(1+pdottare(-1)))
            - pda8*(wa_eq/wa_eq(-1)-1)
            + pda9*((tiy-tiy(-1))/(1+tiy(-1)))
pdottare:   pdottare = (ptl1*pdottare(-1) + ptl2*pdottare(-2)
            + (1-ptl1-ptl2)*(ptl3*(pdot(6)+pdot(8))/2+(1-ptl3)*pdottar_eq))
pcdot:      (1+pcdot) = (1+pdot)*pc/pc(-1)
pcdot4:     (1+pcdot4)**4 = (1+pcdot)*(1+pcdot(-1))*(1+pcdot(-2))
            *(1+pcdot(-3))
npcdot:     (1+npcdot) = (1+pdot)*(pc/(1+tic))/(pc(-1)/(1+tic(-1)))
npcdot4:    (1+npcdot4)**4 = (1+npcdot)*(1+npcdot(-1))*(1+npcdot(-2))
            *(1+npcdot(-3))
cpidot:     cpidot = cp0*pcdot+cp1*pcdot(-1)+cp2*pcdot(-2)
            +(1-cp1-cp2-cp0)*pcdot(-3) -cp3*(z(0)/z(-3)-1)
ncpidot:    ncpidot = cp0*npcdot+cp1*npcdot(-1)+cp2*npcdot(-2)
            +(1-cp1-cp2-cp0)*npcdot(-3) -cp3*(z(0)/z(-3)-1)
cpidot4:    cpidot4 = (((1+cpidot)*(1+cpidot(-1))*(1+cpidot(-2))
            *(1+cpidot(-3))**0.25)-1)
ncpidot4:   ncpidot4 = (((1+ncpidot)*(1+ncpidot(-1))*(1+ncpidot(-2))
            *(1+ncpidot(-3))**0.25)-1)
tpdot:      tpdot = pt0*cpidot4 + pt1*ncpidot4 + (1-pt0-pt1)*(pdot+pdot(-1)
            +pdot(-2)+pdot(-3))/4
```

Relative prices

pc:	pc*c = (1+tic)*(pcd*(c-cm) + (1+ticm)*pcm*cm)
pc_eq:	pc_eq*c_eq = (1+tic_eq)*(pcd_eq*(c_eq-cm_eq)
	+ (1+ticm_eq)*pcm_eq*cm_eq)
pcm:	pcm = pcm_eq + pcmv1*(z-z_eq)*pcrow_eq
	+ pcmv2*(pcrow-pcrow_eq)*z_eq - pcmadj
pcm_eq:	cm_eq = (1-pcm1)*pcm_eq(-1) + pcm1*(pcrow_eq*z_eq+pcm0)
pcd:	pcd = pcd_eq + pcdv1*(y/yp-1) - pcdadj
pi:	pi*i = (1+tii)*(pid*(i-im) + (1+tiim)*pim*im)
pi_eq:	pi_eq*i_eq = (1+tii_eq)*(pid_eq*(i_eq-im_eq)+
	(1+tiim_eq)*pim_eq*im_eq)
pim:	pim = pim_eq + pimv1*pirow_eq*(z(-1)-z_eq(-1))
	+ pimv2*(pirow-pirow_eq)*z_eq - pimadj
pim_eq:	pim_eq = (1-pim1)*pim_eq(-1) + pim1*(pirow_eq*z_eq+pim0)
pg:	pg*g = (1+tig)*(pgd*(g-gm) + (1+tigm)*pgm*gm)
pg_eq:	pg_eq*g_eq = (1+tig_eq)*(pgd_eq*(g_eq-gm_eq)
	+ (1+tigm_eq)*pgm_eq*gm_eq)
pgm:	pgm = pgm_eq + pgmv1*pgrow_eq*(z(-1)-z_eq(-1))
	+ pgmv2*(pgrow-pgrow_eq)*z_eq - pgmadj
pgm_eq:	pgm_eq = (1-pgm1)*pgm_eq(-1)+ pgm1*(pgrow_eq*z_eq+pgm0)
pgd:	pgd = pgd_eq + pgdv1*(y/yp-1) - pgdadj
px:	px = px_eq + pxv1*pxrow_eq*(z-z_eq) + pxv2*(x/x(-1)-1)
	+ pxv3*(pxrow-pxrow_eq)*z_eq - pxadj
px_eq:	px_eq = (1-px1)*px_eq(-1) + px1*(pxrow_eq*z_eq+px0)
pm:	pm*m = pcm*cm + pim*im + pgm*gm
pm_eq:	pm_eq*m_eq = pcm_eq*cm_eq + pim_eq*im_eq + pgm_eq*gm_eq
pid_eq:	pcd_eq*(c_eq-cm_eq) + pid_eq*(i_eq-im_eq) + pgd_eq*(g_eq-gm_eq)
	= c_eq-cm_eq + i_eq-im_eq + g_eq-gm_eq
pid:	pcd*(c-cm) + pid*(i-im) + pgd*(g-gm) = c-cm + i-im + g-gm
pgd_eq:	pgd_eq = pid_eq*pg_pi0
pcd_eq:	pcd_eq = pcd_eq(-1)*(1+0.1*(1-0.75)*(z_ss/z_ss(-1)-1))
	+0.75*(pcd_eq(-1)-pcd_eq(-2))
z_ss:	z_ss = z_ss(1)
y_ss:	y_ss = y_ss(1)

Appendix B: Mnemonics

alpha, α	Capital's share of production
c	Consumption
c_eq	Equilibrium consumption
cc_eq	Equilibrium user cost of capital
cfl	Consumption by forward-looking consumers
cfl_eq	Equilibrium consumption by forward-looking consumers
cm	Imports of consumption goods
cm_c	Proportion of consumption goods imported
cm_c_eq	Equilibrium proportion of consumption goods imported
cm_c0	Level calibration term in the *cm* share equation
cm_eq	Equilibrium imports of consumption goods
cpidot	Inflation rate for the CPI
cpidot4	Annual inflation rate for the CPI
crt	Consumption by rule-of-thumb consumers
crt_eq	Equilibrium consumption by rule-of-thumb consumers
delta, δ	Household discount rate
depr	Depreciation rate on capital
depr_eq	Equilibrium depreciation rate on capital
dt_eq	Equilibrium capital value of tax depreciation
fa	Real financial assets
fa_eq	Equilibrium real financial assets
fa_ss	Steady-state real financial assets
g	Government expenditures
gamma, γ	Probability of death
g_eq	Equilibrium government expenditures
g_y_eq	Equilibrium target ratio of government expenditure to output
gb	Real government assets
gb_eq	Equilibrium government assets
gbtar	Government debt target
gbtar_eq	Equilibrium government debt target
gbtar_y_eq	Equilibrium target ratio of government bonds to output
gm	Imports of government goods
gm_eq	Equilibrium imports of government goods
gm_g	Proportion of government goods imported
gm_g_eq	Equilibrium proportion of government goods imported
gm_g0	Level calibration term in the *gm* share equation
gtr	Government transfers
gtr_eq	Equilibrium government transfers
gtr_y_eq	Equilibrium target ratio of government transfers to output
hwfl_eq	Equilibrium human wealth

i	Investment
i_eq	Equilibrium investment
i_ss	Steady-state investment
im	Imports of investment goods
im_eq	Equilibrium imports of investment goods
im_i	Proportion of investment goods imported
im_i_eq	Equilibrium proportion of investment goods imported
im_i0	Level calibration term in the *im* share equation
ip	Investment added to productive capital
ip_eq	Equilibrium investment added to productive capital
k	Capital stock, inclusive of investments not yet productive
k_eq	Equilibrium capital stock, inclusive of investments not yet productive
kp	Production capital
kp_eq	Equilibrium production capital
lamda, λ	Proportion of rule-of-thumb consumers
m	Imports
m_eq	Equilibrium imports
mpcw_eq	Equilibrium marginal propensity to consume out of wealth
ncpidot	Inflation rate for the CPI, net of indirect tax
ncpidot4	Annual inflation rate for the CPI, net of indirect tax
ndot	Population growth rate
ndot_eq	Equilibrium population growth rate
netx	Net exports
netx_eq	Equilibrium net exports
nfa	Real net foreign asset ratio
nfa_eq	Equilibrium real net foreign asset ratio
nfa_ss	Steady-state real net foreign asset ratio
npcdot	Inflation rate for the price of consumption, net of indirect tax
npcdot4	Annual inflation rate for the price of consumption, net of indirect tax
pc	Relative price of consumption
pc_eq	Equilibrium relative price of consumption
pc_py_eq	Equilibrium relative price of consumption relative to the price of output
pcd	Relative price of domestic consumption goods
pcd_eq	Equilibrium relative price of domestic consumption goods
pcdot	Inflation rate for the price of consumption
pcdot4	Annual inflation rate for the price of consumption
pcdote	Expected inflation rate for the price of consumption
pcm	Relative price of imported consumption goods
pcm_eq	Equilibrium relative price of imported consumption goods
pcm0	Level calibration term in the *pcm_eq* equation
pcrow	Relative price of consumption goods in the rest of the world

pcrow_eq	Equilibrium relative price of consumption goods in the rest of the world
pdot	Inflation rate for the price of domestically-produced and consumed goods at factor cost
pdot_eq	Equilibrium inflation rate
pdote	Expected inflation rate
pdottar	Target inflation rate
pdottar_eq	Equilibrium target inflation rate
pdottare	Expected target inflation rate
pfc	Relative price of output at factor cost
pfc_eq	Equilibrium relative price of output at factor cost
pg	Relative price of government goods
pg_eq	Equilibrium relative price of government goods
pg_pi0	Level calibration term in the *pgd_eq* equation
pg_py_eq	Equilibrium price of government goods relative to the price of output
pgd	Relative price of domestic government goods
pgd_eq	Equilibrium relative price of domestic government goods
pgm	Relative price of imported government goods
pgm_eq	Equilibrium relative price of imported government goods
pgm0	Level calibration term in *pgm_eq* equation
pgrow	Relative price of government goods in the rest of the world
pgrow_eq	Equilibrium relative price of government goods in the rest of the world
pi	Relative price of investment goods
pi_eq	Equilibrium relative price of investment goods
pi_py_eq	Equilibrium relative price of investment relative to the price of output
pi_ss	Steady-state relative price of investment goods
pid	Relative price of domestic investment goods
pid_eq	Equilibrium relative price of domestic investment goods
pim	Relative price of imported investment goods
pim_eq	Equilibrium relative price of imported investment goods
pim0	Level calibration term in *pim_eq* equation
pirow	Relative price of investment goods in the rest of the world
pirow_eq	Equilibrium relative price of investment goods in the rest of the world
pk_eq	Equilibrium relative price of capital stock
pka	Relative price of capital stock
pm	Relative price of imported goods
pm_eq	Equilibrium relative price of imported goods
pm_py_eq	Equilibrium relative price of imported goods relative to the price of output
prowdot	Inflation rate in the rest of the world
prowdot_eq	Equilibrium inflation rate in the rest of the world
px	Relative price of export goods
px_eq	Equilibrium relative price of export goods
px_py_eq	Equilibrium relative price of export goods relative to the price of output

px0	Level calibration term in the *px_eq* equation
pxrow	Relative price of export goods in the rest of the world
pxrow_eq	Equilibrium relative price of export goods in the rest of the world
py	Relative price of output
py_eq	Equilibrium relative price of output
qdot	Trend growth in labour-augmenting technical progress
qdot_eq	Equilibrium growth rate of labour-augmenting technical progress
r	1-quarter real interest rate
r_eq	Equilibrium 1-quarter real interest rate
r5	5-year real interest rate
r5_eq	Equilibrium 5-year real interest rate
rcon_eq	Equilibrium real interest rate for consumers
rcon_rl_eq	Equilibrium real risk premium for consumers
rgb	Real interest rate on government bonds
rgb_eq	Equilibrium real interest rate on government bonds
rgb_rl	Real risk premium on government bonds
rgb_rl_eq	Equilibrium real risk premium on government bonds
risk	Transfer to individuals from asset holders
risk_eq	Equilibrium transfer to individuals from asset holders
rk_eq	Equilibrium real interest rate on capital
rk_rl_eq	Equilibrium real risk premium on capital
rl	10-year real interest rate
rl_eq	Equilibrium 10-year real interest rate
rn	1-quarter nominal interest rate
rn_eq	Equilibrium 1-quarter nominal interest rate
rn5	5-year nominal interest rate
rn5_eq	Equilibrium 5-year nominal interest rate
rnfa	Real interest rate on net foreign assets
rnfa_eq	Equilibrium real return on net foreign assets
rnfa_rl	Real risk premium on net foreign assets
rnfa_rl_eq	Equilibrium real risk premium on net foreign assets
rnl	10-year nominal interest rate
rnl_eq	Equilibrium 10-year nominal interest rate
rp	Country real risk premium
rp_eq	Equilibrium country real risk premium
rrow	Real interest rate in the rest of the world
rrow_eq	Equilibrium real interest rate in the rest of the world
rsl	Slope of the yield curve
rsl_eq	Equilibrium slope of the yield curve
rt5_eq	Equilibrium 5-year term premium
s	Nominal (trade-weighted) exchange rate, measured as the price of domestic currency in units of foreign currency (positive movement is appreciation)

sigma, σ	Consumers' coefficient of intertemporal substitution
td	Net direct (labour income) tax rate
td_eq	Equilibrium net direct tax rate
tfp	Total factor productivity
tfp_eq	Equilibrium total factor productivity
tic	Indirect tax rate on consumption goods
tic_eq	Equilibrium indirect tax rate on consumption goods
ticm	Tariff rate on imported consumption goods
ticm_eq	Equilibrium tariff rate on imported consumption goods
tig	Indirect tax rate on government goods
tig_eq	Equilibrium indirect tax rate on government goods
tigm	Tariff rate on imported government goods
tigm_eq	Equilibrium tariff rate on government goods
tii	Indirect tax rate on investment goods
tii_eq	Equilibrium indirect tax rate on investment goods
tiim	Tariff rate on imported investment goods
tiim_eq	Equilibrium tariff rate on investment goods
tiy	Average indirect tax rate
tiy_eq	Equilibrium average indirect tax rate
tk	Tax rate on profits
tk_eq	Equilibrium tax rate on profits
tpdot	Weighted average of gross and net inflation rates, used in reaction function
twfl_eq	Equilibrium total wealth
u	Rate of unemployment
u_eq	Equilibrium rate of unemployment
ugap	Unemployment gap
v	Real (trade-weighted) exchange rate, measured as the price of domestic currency in units of foreign currency (positive movement is appreciation)
wa	Real wage in absorption units
wa_eq	Equilibrium real wage in absorption units
wc	Consumer real wage
wc_eq	Equilibrium consumer real wage
wctar	Consumers' target real wage
wdot	Rate of change of nominal wages
wdot_eq	Equilibrium rate of change of nominal wages
wp	Producer real wage
wp_eq	Equilibrium producer real wage
x	Exports
x_eq	Equilibrium exports
x_y0	Level in the *x_eq* equation
xbal	Trade balance in absorption units
xbal_eq	Equilibrium trade balance in aborption units

y	Output
y_eq	Equilibrium output
y_ss	Steady-state output
ydfl	Real disposable income for forward-looking consumers
ydfl_eq	Equilibrium real disposable income for forward-looking consumers
ydot	Trend output growth rate
ydot_eq	Equilibrium trend output growth rate
ydrt	Real disposable income for rule-of-thumb consumers
ydrt_eq	Equilibrium real disposable income for rule-of-thumb consumers
ylab	Real labour income
ylab_eq	Equilibrium real labour income
yp	Potential output
ypgap	Output gap
z	Real (trade-weighted) exchange rate, measured as the price of foreign currency in units of domestic currency
ze	Expected real exchange rate
zeta, ζ	Coefficient on *fa_ss* gap in *c_eq* for forward-looking consumers
z_eq	Real equilibrium exchange rate
z_ss	Steady-state real exchange rate
z_sv	Calibration choice for steady-state real price of foreign exchange

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