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Bond premia, monetary policy and exchange rate dynamics*

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Abstract

I propose a model in which uncertainty about the value of ex-post payoffs drives a wedge - a short-term bond premium - between an observed short-term benchmark interest rate and the unobserved discount rate, that is used to allocate consumption over time. The model helps to explain disconnect between exchange rates and interest rate fundamentals; disconnect between measures of risk that price bonds and measures of risk that price currencies; and why exchange rates are "too smooth" relative to the volatile discount rates implied by equity premia. In the model with risk, the exchange rate response to monetary policy is observationally similar, whether monetary policy moves the discount rate or the bond market premium. Between policy changes, interest rate stabilisation (i) isolates the currency from variation in the discount rate; and (ii) shifts the expression of bond risk from bond yields to the currency premium. Those tradeoffs provide a risk-based interpretation of the monetary policy trilemma.

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Non-technical summary

The risk-free interest rate means different things to different people. One notion of the risk-free rate is the return on a bond that has no risk of default, such as a short-term government bill. Another is the unobserved discount rate, that is used to allocate consumption over time. To observe the latter, we would need a bond that paid off in units of consumption. Observed interest rates should compensate the holder of the bond for delaying consumption, for expected losses, in units of consumption value, and for consumption risk. Consumption risk premia compensate the holder for consumption volatility. An asset that performs poorly in bad times should pay a yield above the risk-free rate, because holding such an asset makes consumption more volatile.

This paper sets out an exchange rate model that incorporates (i) a wedge between observed short-term interest rates and the underlying discount rate - a short-term bond premium, and (ii) monetary policy. The model helps to explain three empirical regularities: the disconnect between exchange rates and interest rate fundamentals; the disconnect between measures of risk that price bonds and measures of risk that price currencies; and why exchange rates are less volatile than discount rates implied by equity premiums. The intuition is, respectively: currencies do not respond to the bond premium component of interest rates when the premium compensates the holder for risk; bond risk can be reflected in the bond yield or in the currency, but not in both; and monetary policy stabilisation of the observed interest rate isolates the currency from variation in the underlying discount rate.

In the model with risk, monetary policy can move the underlying discount rate or the short-term bond market premium. Under standard assumptions, the exchange rate response to a monetary tightening looks the same in either case (the currency initially appreciates sharply (Dornbusch, 1976), and then depreciates gradually to offset the higher *risk-adjusted* interest payoff, period by period). However, if exchange rates do not fully adjust without capital flows, then adjustment may involve the building and shedding of risk, with implications for international spillovers.

In the model with risk, monetary policy affects the exchange rate in two other ways. Stabilising interest rates between monetary policy decisions (i) isolates the currency from variation in the discount rate, which we think is very volatile; and (ii) shifts the expression of bond risk from bond yields to the currency premium. These tradeoffs provide a risk-based interpretation of the monetary policy trilemma.

1 Introduction

The risk-free rate means different things to different people. A short-term government interest rate, that is liquid and has low credit default risk, is often used as a proxy for the 'risk-free' interest rate in empirical models. In theory, the risk-free rate is usually defined by the inter-temporal marginal rate of substitution (IMRS), or discount rate, that is used to allocate consumption over time and to price risk (Cochrane, 2001). To observe the discount rate, we would need a bond that paid off in units of consumption utility.

A growing body of literature differentiates between observed short-term rates and the unobserved discount rate (Cochrane 2016 provides a recent discussion). Empirically, observed short-term rates and the IMRS match neither in terms of variance nor covariance. Equity premiums imply that discount rates are very volatile, unless risk aversion is implausibly high (Brandt et al. 2006, Cochrane and Hansen 1992, Hansen and Jagannathan 1991 and Mehra and Prescott 1985), while observed short-term rates reflect a high degree of monetary policy smoothing. Canzoneri et al. (2007) show that discount rates constructed from consumption data and common specifications of preferences are negatively correlated with ex-post Fed funds rate, implying that the wedge between the two - which I will call the short-term bond premium - is positively correlated with the stance of monetary policy. Similarly, Nagel (2014) shows that short-term liquidity spreads are positively correlated with the stance of monetary policy. In our models, the discount rate is generally a function of expected consumption growth and other utility function variables. In contrast, the short-term policy rate is driven by inflation and other Taylor rule variables. There is no obvious reason to equate the two. In the finance literature, there is ample evidence that short-term premiums are empirically important.[1]

In this paper, I propose an exchange rate asset price model that allows for (i) uncertainty about the value of ex-post payoffs, that drives a wedge between the observed policy rate and the IMRS, ex-ante - a short-term bond premium, and (ii) monetary policy intervention in short-term bond markets. The paper

[1] For example, although government default risk is usually low relative to other rates, Della Corte et al. (2015) find relative default risk to be significant in explaining exchange rate behaviour. Krishnamurthy and Vissing-Jorgensen (2012), Amihud et al. (2005) and Duffie (1996) show that short-term safety premia reflected in US Treasuries interest rates are empirically important. Lustig and Verdelhan (2007) show that high interest rate currencies tend to perform poorly in bad times, and argue that investors demand higher yields on bonds denominated in high interest currencies because holding them makes consumption more volatile.

builds on the application of risk corrections to exchange rate models employed by Lustig and Verdelhan (2007) and Backus et al. (2001).[2] I consider three variants of the model. In the first, there is full risk sharing and no role for monetary policy. In that model, the bond premium helps to explain the forward premium puzzle.[3] The currency does not depreciate to offset a higher interest return when the higher return is compensation for risk.

The second and third variants of the model incorporate monetary policy. The second is the traditional view, whereby there is no short-term bond premium and monetary policy moves the discount rate, which I will refer to as the 'risk-free' rate. The third variant features both monetary policy control of the observed short-term rate and short-term bond risk.

The presence of a short-term bond premium raises the question: Does monetary policy intervention in short-term money markets move the risk-free rate, or the bond premium, or both? The traditional answer is the risk-free rate, but there are several reasons to think that policy moves the premium. Canzoneri et al. (2007) argue that a rise in the policy rate may slow activity and expected consumption growth, so *reduce* the risk-free rate to explain the negative empirical correlation. That mechanism implies that a policy tightening increases the short-term premium. Similarly, Nagel (2014) shows that short-term liquidity spreads are positively correlated with the stance of monetary policy. Finally, recent papers link intervention in longer-term bond markets to movements in the term premium (Bernanke 2013, and references therein). It is reasonable to expect that purchases/sales of securities in short-term markets and long-term markets may have similar effects.

If monetary policy works through the short-term bond premium, what are the implications? When prices adjust without flows (Fama, 1965), the exchange rate response to monetary policy is the same, whether policy moves the short-term bond premium or the risk-free interest rate. In response to a monetary policy tightening, the home currency initially appreciates sharply and then depreciates slowly. The initial appreciation eliminates all future excess *risk-adjusted* returns on home bonds, and the subsequent depreciation offset the higher *risk-adjusted* interest return, period by period. When there are limits to capital-free arbitrage (Shleifer and Vishny, 1997), the adjustment

[2] The paper augments Munro (2014) by including a role for monetary policy.

[3] A large literature (eg Della Corte et al. (2015), Sarno et al. (2012), Lustig and Verdelhan (2007), Brennan and Xia 2006, Duarte and Stockman (2005), Obstfeld and Rogoff (2002), Backus et al. (2001), Fama (1984)) has argued that failure to account for risk may be the cause of the weak empirical link between currency movements and relative returns. However, that literature has mostly assumed that short-term T-bills or eurodollar deposits are risk free.

mechanism may involve the building and shedding of risk.

In a model with short-term bond risk, monetary policy is associated with two other exchange rate dynamics. Both are related to interest rate stabilisation between policy changes. Both are tradeoffs between the expression of volatility in bond yields or in the currency, and together they provide a risk-based interpretation of the monetary policy trilemma.[4]

The first relates to the expression of risk. Taking the risk-free rate as given, when monetary policy stabilises the observed interest rate, it prevents variation in short-term bond risk from being expressed in the bond market. When the shadow price of risk is not reflected in the bond market premium, then it must be reflected in the currency premium. Interest rate stabilisation therefore shifts variation in the underlying price of short-term bond risk from bond yields to the exchange rate. That tradeoff provides an interpretation of the disconnect between measures of risk that price currencies and measures of risk that price bonds (Sarno et al. (2012) or equities (Burnside, 2012)). Intuitively, bond risk must be expressed somewhere. When risk is symmetrically priced, it is reflected in the interest rate, but not in the currency. When a risk is asymmetrically priced, it is reflected in the currency.

The second relationship between interest rate stabilisation and the exchange rate relates to the expression of variation in the risk-free rate, and provides an interpretation of why exchange rates are 'too smooth' (Brandt et al., 2006). Unless risk aversion is implausibly high, risk-free rates implied by equity prices are very volatile (Hansen and Jagannathan, 1991). Brandt et al. (2006) show that, if exchange rates reflect relative risk-free rates, then either exchange rates should be considerably more volatile than they are, or home and foreign risk-free rates must be correlated, implying a higher degree of risk-sharing than is typically estimated.[5]

The model proposed here provides another interpretation. When policy intervention stabilises the observed short-term interest rate, but the underlying risk-free rate is volatile, then the wedge between the two - the short-term bond premium - must be volatile and negatively correlated with the risk-free rate. That volatility in the bond premium reflects policy intervention to

[4] In a financially open economy, the trilemma is usually stated as a tradeoff between independent monetary policy (stabilising the interest rate) and exchange rate stability.

[5] Full risk-sharing is rejected empirically (Backus and Smith, 1993). Kose et al. (2003) show that cross-country consumption correlations did not increase in the 1990s, despite financial integration. More recently, employing a different empirical approach, Flood and Matsumoto (2009) find that consumption growth rates have converged, suggesting that international risk sharing has improved during a period of globalization.

stabilise the observed rate, rather than a change in the market price of risk. With the shadow price of risk unchanged, policy intervention must generate a currency premium that is positively correlated with the risk-free rate. The effect of a rise in the foreign risk-free rate is offset by a rise in the foreign currency premium. The former appreciates the foreign currency, but the latter depreciates it, leaving the exchange rate stable relative to the volatile risk-free rate.

Brandt et al. (2006) interpret the puzzle as evidence for a relatively high degree of risk sharing. In contrast, the interpretation of the exchange rate volatility puzzle here is similar to that of Chien et al. (2015) in that it involves a deviation from complete markets. In Chien et al. (2015), non-participation in financial markets drives down the cross-country correlation in aggregate consumption, implying a low degree of risk-sharing. Here the deviation from full risk sharing is associated with monetary policy intervention in short-term money markets. When consumption growth is proportional to the risk-free rate, the same mechanism provides an explanation for disconnect between relative consumption growth exchange rates (Backus and Smith, 1993).

The next section restates the standard exchange rate asset price model, allowing for (i) uncertainty about the value of ex-post short-term bond payoffs that generates a short-term premium, ex ante and (ii) policy intervention in short-term bond markets. Section 3.1 considers the case with no policy intervention. Section 3.2 considers the traditional model with monetary policy, but no bond market premium. Section 3.3 considers a model with both monetary policy and short-term bond risk. Section 4 discusses approaches to extending the model to general equilibrium 5 concludes.

2 A model with short-term bond risk

This section restates the standard exchange rate asset price model, incorporating short-term bond risk. The risk-free rate is defined as the IMRS, which is the rate used to discount the future and to price risk. A contracted interest rate compensates the holder for delaying consumption (the risk-free rate), for expected losses, and for the risk that the asset performs poorly when consumption utility rises. In the absence of policy intervention, the wedge between the foreign and home contracted rates and the relative risk-free rates - the bond premium - compensates for the relative risk of holding the foreign bond. The premium includes relative default risk (Della Corte et al., 2015), relative term premia, relative liquidity risk (Nagel 2014, Duffie 1996, Amihud and Mendelson 1991) and currency revaluation risk (Lustig and Verdelhan, 2007).

In this section, the home and foreign investors' first-order conditions for home and foreign bonds are used to derive two equations. The exchange rate is defined by uncovered interest parity (UIP) that equates the home investor's expected returns on home and foreign short-term bonds:

$$q_t \quad = \quad -r_t^d - \lambda_t + E_t q_{t+1} \tag{1}$$

where q_t is the logarithm of the real exchange rate, defined as the value of the foreign currency in units of home currency, r_t^d is the home-foreign real short-term interest differential and E_t indicates expectations at time, t. The expected foreign currency 'excess return' $\lambda_t = E_t \Delta q_{t+1} - r_t^d$ can be expressed in terms of expected losses and risk corrections. Equation (1) can be read: the home currency is expected to depreciate ($E_t \Delta q_{t+1} > 0$), to offset a higher home *risk-adjusted* return ($r_t^d + \lambda_t$).

A second equation expresses the observed interest rate differential as relative risk-free returns, net of the foreign bond premium:

$$r_t^d \quad = \quad (r_t^f - r_t^{f*}) - \lambda_t^R \tag{2}$$

where r_t^f and r_t^{f*} are the unobserved home and foreign risk-free interest rates respectively, and λ_t^R is the foreign (relative to home) short-term bond market premium. Uncertainty about the value of ex-post bond payoffs generates bond premia in ex-ante yields.

Variants of the model with and without risk and monetary policy are considered. The model encompasses the standard exchange rate asset price model (Engel and West 2010, Engel and West 2005, Dornbusch 1976), but allows for a broader set of interest rate-exchange rate dynamics.

2.1 One-period risk-free bonds

The risk-free rate, r_t^f, is defined by the home investor's IMRS: his willingness to give up a unit of consumption today to consume $(1 + r_t^f)$ units of consumption next period. The risk-free rate is defined in terms of expectations and is not known with certainty at time, t. Following Cochrane (2001), the 'stochastic discount factor' (SDF), M_{t+1}, of the home investor is:

$$M_{t+1} = E_t \beta U_{C,t+1} / U_{C,t} = \frac{1}{1+r_t^f} \tag{3}$$

where β is the home investor's subjective discount factor, $U_{C,t}$ is the marginal utility of consumption, and E_t indicates expectations at time t. The results do not depend on a particular specification of the SDF, so there is no reason to specify a utility function, but simply to postulate that it exists. The risk-free rate is lower when people save more because they are patient (β), they are averse to varying consumption across time (inter-temporal substitution), they are averse to varying consumption across states (risk aversion), or they expect consumption growth to be volatile (precautionary savings).

The SDF and gross asset returns are assumed to be conditionally log-normal. Taking the logarithm of equation (3),

$$logM_{t+1} \quad = \quad -r_t^f$$

Similarly, the foreign real, risk-free interest rate, r_t^{f*}, is defined by the foreign investor's SDF:

$$logM_{t+1}^* = -r_t^{f*} \tag{4}$$

where M_{t+1}^* is the foreign investor's SDF, β^* is her subjective discount factor.

2.2 The bond market premium, λ_t^R

The home investor's first order condition (pricing equation) for the home bond is usually stated as:

$$1 \quad = \quad E_t[M_{t+1}(1 + r_t)] \tag{5}$$

where $1 + r_t$ is the expected payoff at time $t + 1$. The pricing equation equates the dis-utility of giving up one unit of consumption today to the expected, discounted benefit from consuming $(1 + r_t)$ next period. The log pricing equation (see A for detailed derivations) is:

$$logE_t(1 + r_t) \quad = \quad r_t^f - cov_t(m_{t+1}, r_t), \tag{6}$$

where $m_t = log(M_t)$. When the ex-post payoff $1 + r_t$ is assumed to be observed, with certainty, at time, t, the final covariance term is zero (for example, Colacito and Croce (2013) (appendix C) and Lustig and Verdelhan (2007) (appendix B)).

This paper departs from the standard approach by allowing for uncertainty about the value of ex-post payoffs. Specifically, the first-order condition is restated as:

$$1 \quad = \quad E_t[M_{t+1}(1 + r_t^c)Z_{t+1}] \tag{7}$$

where r_t^c is the ex-ante contracted interest rate, and Z_{t+1} reflects factors that may affect the value of the payoff at $t+1$. Such factors may include expected losses from default, from inflation, or from selling the bond before maturity, uncertainty regarding the evolution of the risk-free rate over the holding period and expected covariances of those factors with consumption utility. The log pricing equation is then:

$$logE_t(1+r_t^c) \sim r_t^c \;=\; r_t^f - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1}), \qquad (8)$$

where $z_{t+1} = log(Z_{t+1})$. The second term on the right hand side of (8) captures expected losses and the final term on the right hand side is a risk correction. The risk correction increases the yield on assets with payoffs that are expected to be positively correlated with consumption growth (negatively correlated with consumption utility growth). Holding such assets makes consumption more volatile (Cochrane, 2001). Appendix A presents examples in which default risk, a term premium and liquidity risk drive a wedge between the contracted rate and the risk-free rate.[6] Uncertainty about the ex-post payoff generates a premium in the ex-ante contracted yield.

Similarly, the contracted return in the foreign bond market can be expressed as the foreign investor's risk-free rate, r_t^{f*}, plus expected losses and a risk correction that is priced according to her SDF, M_t^*:

$$logE_t(1+r_t^{c*}) \sim r_t^{c*} \;=\; r_t^{f*} - logE_t(Z_{t+1}^*) - cov_t(m_{t+1}^*, z_{t+1}^*) \qquad (9)$$

where $m_t^* = log(M_t^*)$ and $z_{t+1}^* = log(Z_{t+1}^*)$. Combining (8) and (9), the observed short-term home-foreign interest differential can be expressed as the relative risk-free return plus a relative bond premium, λ_t^R:

$$\begin{aligned} r_t^d &= r_t^c - r_t^{c*} \\ &= (r_t^f - r_t^{f*}) - \lambda_t^R \ \text{ where,} \\ \lambda_t^R &= logE_t(Z_{t+1}) - logE_t(Z_{t+1}^*) + cov_t(m_{t+1}, z_{t+1}) - cov_t(m_{t+1}^*, z_{t+1}^*) \qquad (10) \end{aligned}$$

The bond premium drives a wedge between the observed, contracted rate on the bond and the unobserved risk-free rate. The premium reflects uncertainty about ex-post payoffs in local currency terms.

[6] UIP is typically viewed in terms of buying and holding short-term bonds, such as t-bills or eurodollar deposits, to maturity. That doesn't mean that liquidity risk doesn't matter. If other holders of the securities value them for liquidity, then a liquidity premium is priced into the yield ex-ante.

2.3 Uncovered interest parity and currency revaluation risk

I assume, for convenience and without loss of generality, that the currency is priced by the home investor (eg. the foreign country is small and has a floating exchange rate). The home investor's first order condition for the foreign short-term bond is:

$$Q_t = E_t[M_{t+1}(1 + r_t^{c*})Z^*_{t+1}Q_{t+1}], \quad (11)$$

where Q_t is the real exchange rate (value of the foreign currency in units of home currency). Equation (11) equates the cost of buying one unit of the foreign bond this period, Q_t, to the expected, discounted return on the foreign bond at $t + 1$, in home currency terms. The log pricing equation is:

$$\begin{aligned} r_t^{c*} &= r_t^f - logE_t(Z^*_{t+1}) - E_t\Delta q_{t+1} - \tfrac{1}{2}var_t(\Delta q_{t+1}) \\ &\quad -cov_t(m_{t+1}, z^*_{t+1}) - cov_t(m_{t+1}, \Delta q_{t+1}), \end{aligned} \quad (12)$$

where $q_t = log(Q_t)$. From the perspective of the home investor, the foreign bond premium reflects expected losses, including expected depreciation of the foreign currency, $E_t\Delta q_{t+1} + \frac{1}{2}var_t(\Delta q_{t+1})$, and risk corrections that reduce the yield on bonds that are expected to perform well when the marginal utility of consumption rises (eg safe-haven currencies). Holding such bonds makes consumption less volatile (Lustig and Verdelhan, 2007).

Combining the home investor's pricing equation for the home short-term bond (8) with the home investor's pricing equation for foreign bonds (12), gives the UIP condition that equates the expected return on the home bond to the expected return on the foreign bond:

$$\begin{aligned} q_t &= -r_t^d - \lambda_t + E_t(q_{t+1}) \text{ where,} \\ \lambda_t &= logE_tZ_{t+1} - logE_tZ^*_{t+1} - \tfrac{1}{2}var_t(\Delta q_{t+1}) \\ &\quad +cov_t(m_{t+1}, z_{t+1}) - cov_t(m_{t+1}, z^*_{t+1}) - cov_t(m_{t+1}, \Delta q_{t+1}) \end{aligned} \quad (13)$$

The foreign currency 'excess return',λ_t, reflects the home investor's pricing of home and foreign bond risk.

Equation 13 encompasses the standard asset price model of the exchange rate. If the home risk-adjusted interest rate rises, or is expected to rise, relative to the foreign risk-adjusted rate, the home currency should sharply appreciate à la Dornbusch (1976) and then slowly depreciate to its equilibrium value, over the period of higher home *risk-adjusted* returns. The initial appreciation eliminates all future excess *risk-adjusted* returns, and the subsequent depreciation offsets the higher interest *risk-adjusted* payoffs each period, so there is no excess return to holding the home or the foreign asset.

2.4 The currency premium, λ_t^{FX}

It is useful to divide the currency 'excess return', λ_t, into two parts: the bond market premium, λ_t^R, and a 'currency premium', λ_t^{FX}. Accordingly, the currency premium is defined as the currency 'excess return' (13), net of the bond premium (10):

$$\begin{aligned} \lambda_t^{FX} &= \lambda_t - \lambda_t^R \\ &= -cov_t(m_{t+1} - m_{t+1}^*, z_{t+1}^*) - cov_t(m_{t+1}, \Delta q_{t+1}) - \tfrac{1}{2} var_t(\Delta q_{t+1}) \end{aligned} \quad (14)$$

Defined this way, the currency risk premium reflects the difference between the home and foreign investors' pricing of foreign bond risk.[7] As in Lustig and Verdelhan (2007), Backus et al. (2001) and Chien et al. (2015), the currency premium reflects asymmetric pricing of risk. The last two terms on the right-hand side of (14) are standard and reflect asymmetric pricing of currency revaluation risk (Lustig and Verdelhan 2007, Obstfeld and Rogoff 1996).

The first term on the right-hand side of (14) reflects asymmetric pricing of foreign bond risks, reflected in z_{t+1}^* such as relative liquidity risk and relative credit risk, that are the focus of this paper. When the home and foreign investor have access to a full set of state-contingent bonds, the ratio of their marginal utility growth is the expected change in the exchange rate, $M_{t+1}\Delta Q_{t+1} = M_{t+1}^*$. Therefore, their pricing of the foreign bond yields the same contracted interest rate, conditional on Z_{t+1}^*.

$$1 = E_t M_{t+1} \frac{Q_{t+1}}{Q_t}(1 + r_t^{c*})Z_{t+1}^* = E_t M_{t+1}^*(1 + r_t^{c*})Z_{t+1}^*$$

Combining (9) and (12), and substituting in the risk sharing condition $r_t^f - r_t^{f*} = m_{t+1}^* - m_{t+1} = E_t \Delta q_{t+1}$,

$$\begin{aligned} 0 &= r_t^f - r_t^{f*} - E_t\Delta q_{t+1} - \tfrac{1}{2} var_t(\Delta q_{t+1}) \\ &\quad - cov_t(m_{t+1} - m_{t+1}^*, z_{t+1}^*) - cov_t(m_{t+1}, \Delta q_{t+1}) \\ &= cov_t(m_{t+1}^* - m_{t+1}, z_{t+1}^*) + cov_t(m_{t+1}, \Delta q_{t+1}) + \tfrac{1}{2} var_t(\Delta q_{t+1}) \end{aligned}$$

When markets are complete, the currency premium λ_t^{FX} is zero. All risks are priced symmetrically, and are reflected in both the bond market premium λ_t^R and in the currency excess return λ_t. The source of market completeness of

[7] If market pricing were a weighted average of the home and foreign investors' pricing, the currency premium would reflect different pricing of home bonds as well as foreign bonds.

interest in this paper, is the effect of monetary policy intervention in short-term bond markets that, for a given expectation of future consumption growth, drives a wedge between the shadow price of risk, λ_t, and the short-term bond premium, λ_t^R.

Defining the foreign currency premium as the foreign currency excess return net of the relative foreign bond market premium, the exchange rate (1) can be expressed as its expected future value, the observed interest differential and an excess return that includes the relative bond premium and a currency premium:

$$q_t = -\underbrace{((r_t^f - r_t^{f*}) - \lambda_t^R)}_{\text{Observed interest differential}} - \underbrace{(\lambda_t^R + \lambda_t^{FX})}_{\text{`Excess return'}\lambda_t} + E_t q_{t+1}$$

or as its expected future value, the risk-free interest differential and the foreign currency premium:

$$q_t = -\underbrace{(r_t^f - r_t^{f*})}_{\text{Risk-adjusted differential, } r_t^d} - \underbrace{\lambda_t^{FX}}_{\text{Currency premium, } (\lambda_t - \lambda_t^R)} + E_t q_{t+1} \quad (15)$$

Foreign bond risk can be reflected in the bond market premium (the risk is priced symmetrically) or reflected in the currency premium (the risk is priced asymmetrically), but not in both. That decomposition is consistent with the finding that measures of risk that price bonds do little to price currencies, but that term premia have modest predictive power for currency excess returns.[8]

Asymmetries in the pricing of risk reflect incomplete risk sharing and may arise because of market segmentation (Chien et al., 2015), or monetary policy intervention in the short-term bond markets. The case of interest for this paper is policy intervention in short-term bond markets.

[8] See Chen and Tsang (2013) and Sarno et al. (2012) for analysis of the predictive power of term structure factors for currencies and currency excess returns. Since term structure models assume the short-term observed rate to be risk-free, term structure factors likely capture a subset of the risks reflected in z_{t+1}. That is especially the case if the observed rates are smooth compared to potentially volatile risk-free rates and risk premia. Burnside (2012) makes a similar observation regarding equities: measures of risk that price equities have weak power for pricing currencies, and conversely, non-traditional measures of risk that price currencies (eg dollar factor, high-to-low carry, volatility and skewness) do little to price equities.

3 Model dynamics

This section and the following two sections present three variants of the model. This section considers the complete markets case, whereby risk is priced symmetrically, and there is no role for monetary policy. The other two variants reflect two views of monetary policy. The second makes the traditional assumption that a monetary tightening raises the risk-free interest rate (Dornbusch, 1976). That model assumes the policy rate is equal to the inter-temporal rate of substitution. The third variant considers the case when monetary policy moves the short-term bond premium (Nagel 2014 and Canzoneri et al. 2007). In that case, monetary policy intervention creates asymmetries in the pricing of risk.

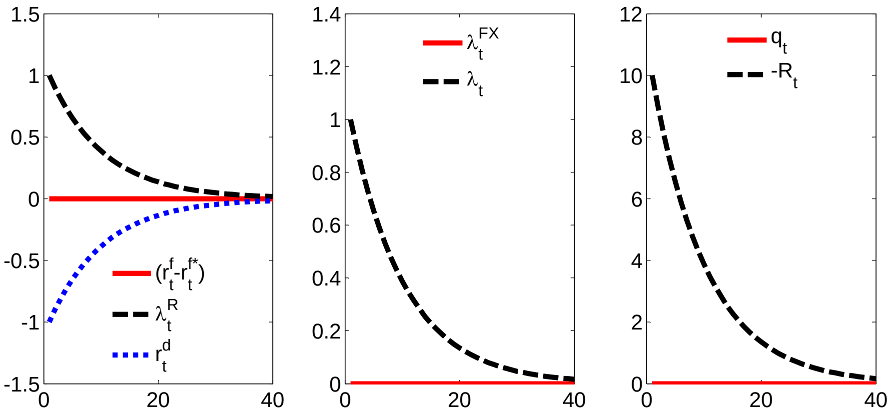
The model dynamics are illustrated using impulse response functions. Innovations in the driving factors are assumed to follow AR(1) processes with AR(1) coefficients of 0.9 and unit variance shocks. In all cases, the exchange rate is assumed to be floating so that the currency reflects the market price of risk. In contrast, monetary policy intervention in short-term bond markets may drive the bond market premium away from the market price of risk.

3.1 Model A: No monetary policy

The complete markets case is a useful benchmark case when thinking about the role of monetary policy intervention in short-term bond markets. In the absence of monetary policy, and with risk priced symmetrically, there is only one driving factor: the relative bond market premium. Home and foreign risk-free rates are equal ($r_t^f = r_t^{f*}$) and the home and foreign investor price risk using the same SDF. The interest rate differential reflects only the relative short-term bond premium. The currency 'excess return' is equal to the relative bond market premium, so that the currency premium, $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ is absent. The relative bond market premium reflects the relative risk characteristics of the foreign and the home bond, such as relative sovereign credit risk and relative liquidity. Relative liquidity may reflect factors such as relative market size, contractual differences between the two bonds, and idiosyncratic supply and demand in the two bond markets. Foreign bond risk is fully reflected in both the relative bond market premium and in the currency 'excess return'.

Figure 1 shows the response to a rise in foreign bond risk, $\lambda_t = \lambda_t^R$. A rise in foreign bond risk is reflected in both the foreign bond market (λ_t^R rises, left hand panel of Figure 1) and in the currency market (λ_t rises, centre panel). The higher foreign bond premium increases the observed foreign interest rate, so reduces the observed home-foreign interest rate differential, r_t^d (left hand panel), and increases the relative expected path of foreign returns $-R_t = E_t \sum_{k=0}^{\infty} \lambda_{t+k}^R$ (right hand panel). The currency premium is zero because risk is priced symmetrically.

Figure 1
Response to a rise in foreign bond risk (priced symmetrically)



Notes: The vertical axis is in percent per period. The horizonal axis is in units of time. Variables: $r_t^f - r_t^{f*}$ is the home-foreign risk-free interest rate differential; λ_t^R is the foreign bond premium relative to the home premium; $r_t^d = r_t^f - r_t^{f*} - \lambda_t^R$ is the home-foreign observed interest rate differential; λ_t is the foreign currency 'excess return'; $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ is the currency premium; $R_t = E_t \sum_{k=0}^{\infty} r_{t+k}^d$ is the expected relative interest rate path; q_t is the real exchange rate (the value of the foreign currency in units of home currency).

The forward premium puzzle: empirical evidence

This variant of the model delivers the forward premium puzzle. Since the bond premium compensates the holder for risk, the foreign currency does not depreciate to offset the higher foreign bond premium. The foreign currency exhibits 'excess returns' equal to the higher foreign bond payoff and the exchange rate is disconnected from observed relative interest returns.

Tests of UIP usually regress the change in the exchange rate on the observed short-term interest differential (the Fama equation):

$$\Delta q_{t+1} = c + \beta r_t^d + \varepsilon_{t+1} \tag{16}$$

If the observed interest rate differential, r_t^d, is uncorrelated with ε_{t+1}, then we expect to estimate $\beta = 1$. Empirically, however, estimates of β are almost always less than one, and often less than zero. For example, for a range of currency pairs, Engel (2016)[9] estimates -1.4 $< \hat{\beta} <$ 0.6 (real terms) and $-2.7 < \hat{\beta} < 0.6$ (nominal terms). Fama (1984) estimates $-1.6 < \hat{\beta} < 0.3$ (nominal), and Sarno et al. (2012) estimate $-5.5 < \hat{\beta} < 0.2$ (nominal). The estimates from all of those studies span $\beta = 0$, the value predicted by the variant of the model with no monetary policy, in which short-term bond risk is priced symmetrically and is the only source of interest rate volatility. High interest rate currencies have expected 'excess returns' on their short-term bonds (Engel (2016), Fama (1984) and references therein).[10]

Subtracting q_{t+1} from both sides of equation (1) and rearranging:

$$\Delta q_{t+1} = r_t^d + \underbrace{\lambda_t + [q_{t+1} - E_t(q_{t+1})]}_{\epsilon_{t+1}}, \tag{17}$$

If r_t^d is correlated with ϵ_{t+1}, then the estimated value, $\hat{\beta}$, in equation (16) is:

$$\hat{\beta} \;=\; \frac{cov(r_t^d, \Delta q_{t+1})}{var(r_t^d)} = \frac{cov(r_t^d, (r_t^d + \epsilon_{t+1}))}{var(r_t^d)} = 1 + \frac{cov(r_t^d, \epsilon_{t+1})}{var(r_t^d)} \tag{18}$$

Assuming rational expectations, the expectational error in square brackets in (17) should be uncorrelated with variables observed at t. However, the risks reflected in λ_t are known at time, t. Moreover, in the two-equation model (1) and (2), risk can not be treated as exogenous because λ_t^R and λ_t reflect common premia (equations 10 and 13). For example, both reflect $logE_t(Z_{t+1}^*) - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1})$, and with risk sharing, $\lambda_t^R = \lambda_t$. Common premia in λ_t and λ_t^R mean that $r_t^d = (r_t^f - r_t^{f*}) - \lambda_t^R)$ and λ_t are correlated. Estimation of equation (16) is biased relative to the coefficient of unity expected in a risk-free model. That bias is the result of a measurement problem: we do not observe the risk-free rate. The estimated value of β is:

$$\hat{\beta} \;=\; 1 + \frac{cov(r_t^d, \lambda_t)}{var(r_t^d)} \tag{19}$$

In this variant of the model, we expect to estimate $\beta = 0$ in equation 16.In equation (15), the observed interest differential reflects only the bond premium which is completely offset by the currency excess return. In equation 19, $\frac{cov(r_t^d, \lambda_t)}{var(r_t^d)} = -1$, so that we expect to estimate $\beta = 0$. There is complete disconnect between the exchange rate and the interest differential.

[9] Engel estimates a parameter $\beta_q = 1 - \beta$. See Engel (2016), Table 3.

[10] See Engel (2016), Engel (2014), Engel (1996), and Flood and Rose (1996) for reviews.

What empirical evidence is there that the short-term premium plays a role in estimates of $\beta < 1$? It is useful to break the covariance term $cov(r_t^d, \lambda_t)$ into two components:

$$\hat{\beta} \;=\; 1 + \frac{cov(r_t^d, \lambda_t^R)}{var(r_t^d)} + \frac{cov(r_t^d, \lambda_t^{FX})}{var(r_t^d)} \tag{20}$$

If the unobserved components $(r_t^f - r_t^{f*})$, λ_t^R and λ_t^{FX} are independent then the only non-zero term in equation (20) is $\frac{cov(r_t^d, \lambda_t^R)}{var(r_t^d)} = -\frac{var(\lambda_t^R)}{var(r_t^d)}$.[11] With risk sharing, only that term exists, and it is equal to -1. More generally, both terms may be relevant.

To inform on whether a short-term premium is relevant, we would like to know how measures of short-term bond risk covary with observed interest differentials and with currency excess returns. Papers that relate currency excess returns directly to measures of risk include Della Corte et al. (2015) and Kiley (2013). Della Corte et al. (2015) find relative sovereign risk, proxied by relative sovereign CDS spreads to be a significant predictor of currency excess returns, with R^2 statistics of typically about 0.25-0.30 for regressions of currency returns on relative CDS spreads. They also find relative sovereign risk to be a significant predictor of higher moments of returns. Their results are consistent with the idea that relative risk is empirically important for currency movements. Kiley (2013) uses an interest rate with different risk characteristics as an instrument for the risk-free rate and finds greater support for UIP, conditional on monetary policy events. His results also support the idea that accounting for the risk premium component of relative interest returns is important in understanding currency movements.

Canzoneri et al. (2007) construct measures of the US real risk-free rate using consumption data and common specifications of preferences. They show that model-implied risk-free rates tend to be negatively correlated with the ex-post Federal Funds rate (first three columns of Table 1). Their reported moments can be used to calculate the first covariance term in equation (20), as reported in Table 1. Assuming no change in foreign variables, Canzoneri et al. (2007)'s moments imply that $\frac{cov(r_t^d, \lambda_t^R)}{var(r_t^d)}$ contributes -5.0 to -0.8 to the estimation bias (final column of Table 1).

That bias doesn't directly translate into $\hat{\beta}$ because of the final term in equation (20). That term can be broken into two pieces: $\frac{cov((r_t^f - r_t^{f*}), \lambda_t^{FX})}{var(r_t^d)} - \frac{cov(\lambda_t^R, \lambda_t^{FX})}{var(r_t^d)}$.

[11] Munro (2015) reports exchange rate decompositions assuming that innovations in level variables are independent.

A large literature shows that, in the absence of a bond premium, it is very difficult to generate the first of those terms in a magnitude large enough to account for the forward premium puzzle. However, there is reason to expect that monetary policy stabilisation creates a tradeoff between the risks reflected in the bond premium and the currency premium (section 3.3) so that $cov(\lambda_t^R, \lambda_t^{FX}) = -1$, contributing +1 to $\hat{\beta}$. Taking that into account, a conservative interpretation of the reported covariances in Table 1 would be $-3.0 < \beta < 1.2$, with an average value of $\beta = 0.06$, which is well below one and near the full risk sharing implied value of 0.

A similar, though qualitative result, is supported by the empirical evidence presented in Nagel (2014). The importance of the covariance between the observed interest rate and the bond premium, $cov(r_t^d, \lambda_t^R)$, Nagel shows that a short-term liquidity premium in US treasuries is positively correlated with the ex-post US policy interest rate. The liquidity premium is the spread between the yields on t-bills and repo securities of the same maturity, and both backed by US government paper. In contrast to repos, tbills can be sold in a liquid market at any time, so pay a yield below repos, reflecting their liquidity value. That evidence also implies that $cov(r_t^d, \lambda_t^R) < 0$.

What do we know about the covariance between the observed interest rate and the currency premium − the term $cov(r_t^d, \lambda_t^{FX})$ in (20)? In the absence of a short-term bond premium, theoretical models that seek to generate covariance between the observed interest differential and the currency excess return require both asymmetrical exposure to some kind of shock and unusual forms of preferences, such as recursive Epstein Zin preferences, to generate enough covariance between relative risk-free rates and the currency excess return Colacito and Croce (2013).

In two-country term structure models, there is a tension between fitting yield curves and fitting currency movements (for example, Sarno et al. 2012). At face value, the result that yield curve factors − level, slope and curvature − do little to explain currency excess returns could be taken to imply that bond risk accounts for a modest component of the currency excess return. The relevance of those papers for the covariances in (20) is, however, limited to the term-premium component of the short-term bond premium derived here. The reason is that the decomposition of longer-term rates into the rollover of a short-term rate and a term premium is, conceptually very different to decomposing the short-term rate into a risk-free rate and a short-term premium. For example, there are reasons to think that the risk-free rate and short-term premium may both be volatile. However, that volatility is not apparent in observed interest rates, so is not captured in yield curve factors.

In summary, reported covariances between measures of a variety of types of short-term bond risk support the idea that a short-term bond premium can severely bias tests of UIP in a direction and magnitude consistent with the forward premium puzzle.

While a volatile bond premium can deliver $\hat{\beta} = 0$, Engel (2016), shows that high interest rate currencies, not only have expected excess returns on their short-term bonds, but also are strong in level terms, even stronger than the path implied by $-R_t$ in Figure 1. A rise in a symmetrically priced foreign bond market premium does not deliver both. In Figure 1, the exchange rate is weak relative to the path implied by $-R_t$. Engel argues that the model needs two driving factors: a volatile 'liquidity return' that delivers the forward premium puzzle, and a persistent monetary policy shock that dominates the level. The next two sections consider the role of monetary policy.

3.2 Model 2: Monetary policy, no short-term bond premium

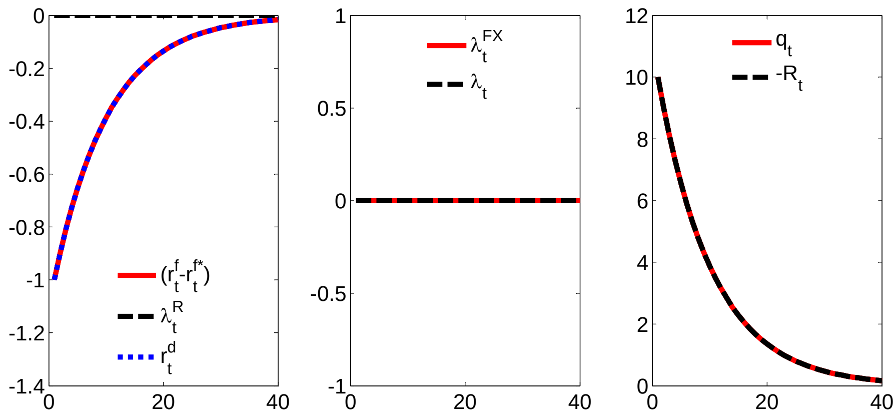
This variant of the model is the traditional view of monetary policy that assumes that the risk-free interest rates is set by the monetary authority. In this variant of the model, the exchange rate is driven by two factors: the 'risk-free' rate, and an exogenous currency 'excess return'. In this variant of the model, there is no short-term bond premium, so the the currency premium is equal to the currency 'excess return', which can be interpreted in terms of relative currency revaluation risk (the last two terms on the right-hand side of (14), as in, for example, Lustig and Verdelhan (2007)).

Response to monetary policy (via the risk-free rate)

The exchange rate response to a rise in the foreign 'risk-free' policy rate is illustrated in Figure 2. When the foreign risk-free interest rate, r_t^{f*}, rises, the risk-free home-foreign interest differential $r_t^f - r_t^{f*}$ and the observed interest differential r_t^d fall (left-hand panel). The foreign currency immediately appreciates (q_t rises, red line in right hand panel) to reflect the higher expected future path of the foreign interest rate, $R_t = E_t \sum_{k=0}^{\infty} r_{t+k}^d \sim \frac{1}{1-0.9} \Delta r_t^d$ (right hand panel). The initial appreciation eliminates all future excess returns relative to the long-run equilibrium value of the currency. The foreign currency subsequently depreciates to offset the higher foreign return, period by period.

The magnitude of the initial appreciation reflects both the magnitude of the rise in r_t^{f*} and its expected persistence.

Figure 2
Response to a foreign monetary tightening (via the risk-free rate)



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

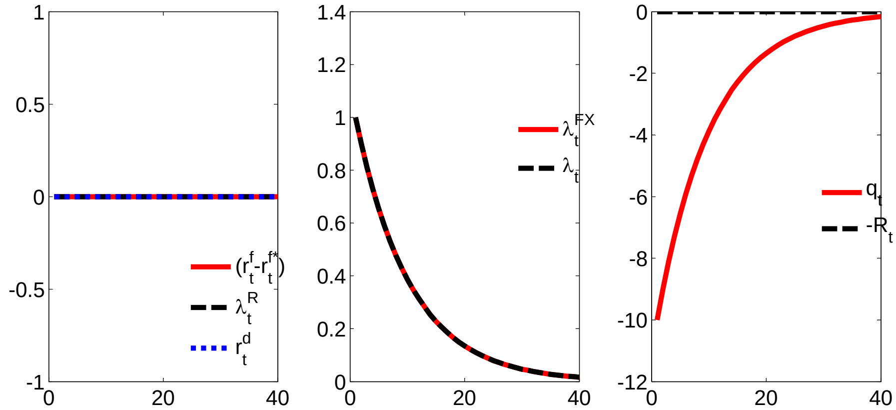
Response to currency risk

The response to a rise in the currency excess return is shown in Figure 3. In the absence of a short-term bond premium, the currency excess return only reflects asymmetries in the pricing of currency revaluation risk (the last two terms on the right-hand side of (14)) as in Lustig and Verdelhan (2007), Backus et al. (2001) and Chien et al. (2015).

The forward premium puzzle literature, tells us that, empirically, the currency excess return is correlated with the observed interest differential (Fama, 1984). In a model without a short-term bond premium, that covariance needs to be generated endogenously through asymmetries between currency area.

This variant of the model, a high interest rate currency is a strong currency (Engel, 2016), but it does not deliver the forward premium puzzle. Empirically, it is very difficult to generate enough covariance between asymmetric risk and the interest rate differential to explain the forward premium puzzle. For example, in a model with asymmetric exposure to shocks, Colacito and Croce (2013) employ recursive Epstein-Zin preferences, with an elasticity of inter-temporal substitution that is greater than one, to generate large

Figure 3
Response to a rise in the foreign currency excess return



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

enough covariances to account for empirical regularities. In contrast, a short-term bond premium can deliver the forward premium puzzle for common specifications of preferences (Table 1).

3.3 Model 3: Monetary policy and short-term bond risk

The view that monetary policy intervention in short-term markets moves the premium component of returns, while unconventional, is consistent with evidence from three perspectives. First, the equity premium literature implies that, unless risk aversion is implausibly high, risk-free rates are very volatile (Hansen and Jagannathan, 1991). In contrast, observed short-term government rates are influenced by a high degree of monetary policy smoothing. Second, the positive correlation between short-term bond premia and the stance of monetary policy reported in Canzoneri et al. (2007) and Nagel (2014) are consistent with a monetary tightening raising the short-term premium. Third, purchases of long-term bonds appear to compress risk premia rather than to lower expectations of future short-term interest rates (Bernanke (2013), and references therein). Purchases and sales of securities in short term markets may be expected to have similar effects.

In this variant of the model, there are three driving factors. The risk-free rate is assumed to be determined by expectations about future consumption, or other factors affecting utility, that are exogenous in this partial equilibrium setting. The observed interest rate is determined by monetary policy. In the absence of policy intervention in FX markets, the currency excess return reflects the market price of risk. In the presence of policy intervention in bond markets, bond market premia may not reflect the market price of risk.

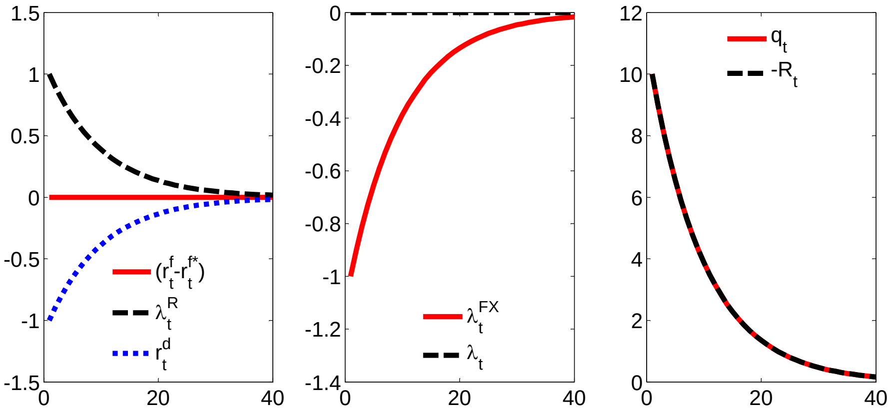
Response to monetary policy (via the bond premium)

The response to a foreign monetary tightening is shown in Figure 4. Monetary intervention in the foreign short-term bond market raises the observed foreign interest rate, reducing the home-foreign observed interest differential, r_t^d (left-hand panel). Taking the foreign risk-free rate as initially unchanged,[12] the foreign bond market premium, λ_t^R, rises (left hand panel of Figure 4). The rise in the foreign bond market premium reflects policy intervention in short-term bond markets, rather than a change in the riskiness of the foreign bond. The market price of risk, reflected in λ_t, is unchanged (centre panel). Therefore policy intervention creates asymmetry in the pricing of bond risk, giving rise to a currency premium ($\lambda_t^{FX} = \lambda_t - \lambda_t^R$). For no-arbitrage conditions to hold in the currency market, the foreign currency premium falls (red line in centre panel), appreciating the foreign currency (q_t rises, red line in the right hand panel). In contrast to the response to symmetrically-priced bond risk in Figure 1, where the high interest rate currency was weak relative to the expected interest rate path $-R_t$, here the high interest rate currency is strong, in line with the expected relative interest rate path (Engel, 2016).

The response to the conventional (via the risk-free rate) and unconventional (via the premium) views of monetary policy are the same. In both cases, adjustment can occur as a price response to a higher *risk-adjusted* foreign return. However, in a model with risk, adjustment can also occur through the price of risk. When the home investor is offered a higher foreign return, but his assessment of foreign economic fundamentals, reflected in r_t^{f*}, and foreign relative bond risk, λ_t, are unchanged, then adjustment is needed to equalise expected returns on the home and foreign bonds. In the standard model, we assume that prices adjust without flows (Fama, 1965). In the absence of capital-free arbitrage (Shleifer and Vishny, 1997), the adjustment mechanism

[12] Canzoneri et al. (2007) argue that, the foreign risk-free rate falls, implying a rise in the foreign bond premium of more than the rise in the observed rate. In a very flexible model, after an immediate fall in consumption, expected consumption growth and the risk-free rate may rise. The exchange rate response is qualitatively the same in those cases. The higher foreign interest rate is reflected either in a higher foreign risk-free rate or in a lower currency premium.

Figure 4
Response to a foreign monetary tightening (via the bond premium)



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

may involve a building or shedding of risk with implications for the nature of international spillovers.

In this model, a high interest rate currency is strong in level terms (Engel, 2016), but cannot account for the forward premium puzzle. To deliver the forward premium puzzle, the bond market premium needs to be a combination of a volatile, symmetrically priced premium (Figure 1 that delivers the forward premium puzzle), and a persistent, asymmetrically priced premium, driven by monetary policy (Figure 4) that dominates the exchange rate level. That is the proposed solution of Engel (2016). Engel's model is motivated by liquidity in the utility function. In contrast, this paper is motivated by a more generalised model of short-term bond risk that is independent of the specification of preferences. A summary of Engel's model in the framework of this paper is presented in C.

Exchange rates are "too smooth": a monetary policy explanation

In a model with a short-term bond premium, two other dynamics are relevant to monetary policy. Both are associated with monetary policy interest rate stabilisation between policy changes. This section considers the response to changes in the risk free rate, when the policy rate is held constant. The next section considers the response to changes in the market price of risk when the policy rate is held constant.

Brandt et al. (2006) show that, although floating exchange rates are volatile, risk-free rates implied by asset prices are much more volatile (Hansen and Jagannathan, 1991). If exchange rates reflect relative risk-free interest rates, then either home and foreign risk-free rates are correlated, implying a higher degree of risk sharing than standard estimates (see footnote 5), or exchange rates are "too smooth". The combination of a volatile underlying risk-free rate and monetary policy stabilisation of the observed interest rate provides an additional explanation.

The response to a change in the foreign risk-free rate is shown in Figure 5. With the observed rate held steady by policy intervention in the short-term bond market, a rise in the foreign risk-free rate compresses the foreign bond premium, λ_t^R (Figure 5, left panel). A volatile foreign risk-free rate is associated with a volatile foreign bond market premium and the two are negatively correlated (the home-foreign risk-free differential and the foreign bond premium are positively correlated, Figure 5, left panel).

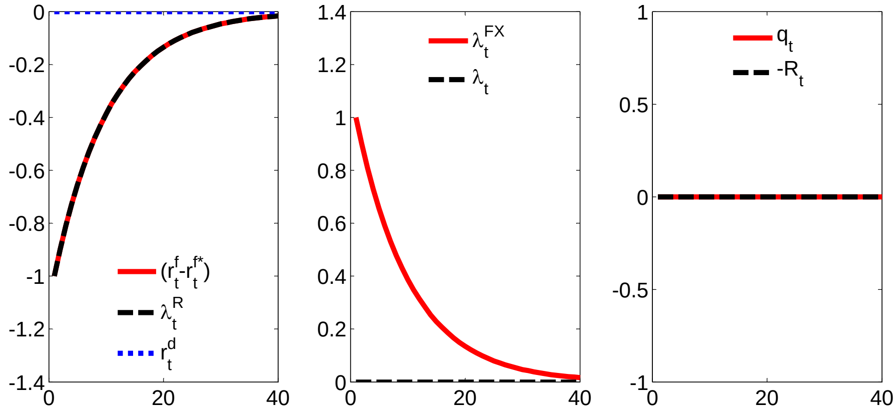
The compressed foreign bond premium reflects monetary policy intervention to stabilise the observed rate, rather than a change in the riskiness of the foreign bond. The home investor's pricing of foreign bond risk, λ_t, is unchanged (centre panel). For no-arbitrage to hold in the currency market, the foreign currency premium $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ must rise (centre panel of Figure 5).

The opposing effects of the higher foreign risk-free rate and the higher foreign currency premium leave the exchange rate flat relative to the path implied by the risk-free rate. That response is in contrast to the traditional Dornbusch (1976) response to a rise in the risk-free rate illustrated in Figure 2.

The isolation of the exchange rate from fluctuations in risk-free rates is independent of the degree of risk-sharing. In contrast, Brandt et al. (2006) et al interpret the puzzle as evidence for correlation between home and foreign risk-free rates, that implies greater risk-sharing than typically estimated; and Chien et al. (2015) provide a segmented markets explanation that implies a low degree of risk sharing.

Isolating the exchange rate from variations in relative risk-free rates is, in a model with power utility, the same as isolating the exchange rate from variation in relative consumption growth. Therefore this variant of the model also provides an interpretation of the Backus and Smith (1993) puzzle or consumption−real exchange rate anomaly, that correlations between consumption and real exchange rates are zero or negative. Here, the correlation is zero.

Figure 5
Response to a rise in the foreign risk-free rate, with a stable policy rate



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

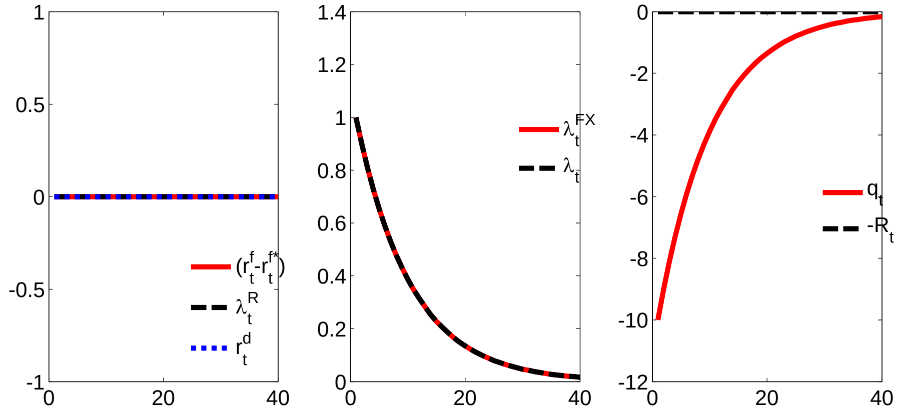
The trilemma: a tradeoff in the expression of risk

The second dynamic related to monetary policy stabilisation is associated with a change in the market price of risk. While monetary policy stabilisation of the observed rate isolates the exchange rate from variation in relative risk-free rates, it shifts variation in the underlying riskiness of the short-term bond to the exchange rate.

Figure 6 shows the response to a rise in foreign bond risk, while the policy rate is held steady. Taking the underlying risk-free rate and policy rate as constant, the foreign bond market premium ,λ_t^R, must also be unchanged (left hand panel). The rise in foreign bond risk is reflected in the currency 'excess return' , λ_t, but not in the bond market. Therefore the currency premium, $\lambda_t^{FX} = \lambda_t - \lambda_t^R$ rises, depreciating the foreign currency (right hand panel). When policy intervention prevents foreign bond risk from being expressed in the foreign bond market premium, it appears in the currency premium.

That tradeoff between interest rate volatility and exchange rate volatility provides a risk-based interpretation of the monetary policy trilemma. In a financially open economy, policy can stabilise the interest rate (independent monetary policy) or the exchange rate, but not both. In a modern macroeconomic model, the trilemma is understood in terms of uncovered interest rate parity: taking the foreign interest rate as given, policy can stabilise the home interest rate, or the exchange rate, but not both. Arbitrage in large

Figure 6
Response to a rise in foreign bond risk, with stable policy rate



The vertical axis is in percent per period. The horizonal axis is measured in time periods. See footnote to Figure 1 for variable definitions.

fixed-income and foreign exchange markets (UIP) determines the other. In this variant of the model, the monetary policy tradeoff between exchange rate volatility and interest rate volatility is a tradeoff between relative bond risk being reflected in bond yields or in the currency.

In summary, this variant of the model, delivers the empirical regularities that a high interest rate currency is strong in level terms and "too smooth" relative to volatile risk free rates, but does not deliver the forward premium puzzle. To deliver the forward premium puzzle, the measured interest rate (eg. treasury bill or eurodollar deposit) needs to reflect short-term bond risk as well as the policy rate, and the risk component needs to dominate the variance of the measured interest rate. That is consistent with the fact that, policy rates exhibit very little volatility, between policy changes, compared to other short-term interest rates.

4 General equilibrium

For tractability, the effects of including a short-term bond premium have discussed so far in a partial equilibrium environment. A short-term premium can be included in a general equilibrium New Keynesian model in several ways. The simplest is to break the equality between the policy interest rate and the IMRS, identifying the former from inflation (and other Taylor-type rule variables), and the latter from consumption data (and other utility function variables) conditional on the specification of utility. The difference between is the short-term bond premium.

A second approach is to generate a premium endogenously. Ideally, a premium would be generated from the characteristics of the borrower or the bond contract, such as ability to sell the bond or to use it as collateral, that in turn affects the ability of the model agent(s) to smooth consumption. An example is Woodford (2016), based on Stein (2012). That could include terms such as expected losses from inflation, or from rigidities in the ability to sell the bond, before maturity, to smooth consumption ($E_t(Z_{t+1})$ terms in equation 8). A less micro-founded, but more tractable approach is to include liquidity in utility (for example Engel (2016), based on Nagel (2014)).

Those approaches can be implemented in a linear model. Risk arises more naturally in a non-linear model (for example Colacito and Croce 2013), where endogenously generated premia also include covariance terms ($cov_t(m_{t+1}, z_{t+1})$ terms in equation 8). The drawback is the complexity of a New Keynesian open economy model with higher order solution methods.

5 Conclusion

This paper builds on a growing literature that departs from the assumption that observed short-term interest rates are risk-free (Cochrane 2016, Nagel 2014, Canzoneri et al. 2007, Duffie 1996 and Amihud and Mendelson 1991). Departing from that assumption, to incorporate short-term bond risk, has wide-ranging effects and provides several avenues for further research.

Short-term bond risk provides an interpretation of several empirical regularities. The currency doesn't depreciate to offset a short-term premium, because the premium compensates the holder for risk (the forward premium puzzle). Bond risk can be expressed in bond yields or in the currency, but not in both, leading to disconnect between measures of risk that price bonds and that price currencies (see Sarno et al. (2012) and, for equities, (Burnside, 2012)). When risks are symmetrically priced in the bond markets and the currency market, they are reflected in the relative bond premium and the currency excess return, but not in the currency. When risk is asymmetrically priced in the bond markets and the currency market, the wedge between the two - the currency premium - is reflected in the currency.

In the model with short-term bond risk, the exchange rate response to a change in monetary policy looks the same, whether it works through the risk-free rate or through the bond premium, but the mechanism may involve the building and shedding of risk. In a model with short-term bond risk, monetary policy stabilisation of observed rates, between policy changes, has two additional effects. Interest rate stabilisation isolates the exchange rate from variation in the, potentially volatile, risk-free rate. That provides an interpretation of why exchange rates are "too smooth" relative to the volatile risk-free rates implied by equity premia (Chien et al. 2015, Brandt et al. 2006), and of the empirical disconnect between exchange rate movements and relative consumption growth (Backus and Smith, 1993). Second, by preventing the expression of bond risk in bond yields, interest rate stabilisation shifts the expression of bond risk to the currency. Those tradeoffs provide a risk-based interpretation of the monetary policy trilemma.

The proposed framework provides a means of identifying bond and currency premia as latent variables in macroeconomic models, by breaking the equality between the rate of inter-temporal substitution and the policy interest rate. There are at least two implications for general equilibrium models. One is that, when the inter-temporal rate of substitution is identified from consumption data and the specification of preferences, rather than equated to observed short-term interest rates, agents' discounting of future payoffs may be very different. A more volatile discount rate may have important implications for forward-looking variables such as investment.

A second implication relates to the role of capital flows. In the standard macroeconomic model, the exchange rate response to policy assumes capital-free arbitrage (prices adjust without the need for capital flows, Fama 1965). In the absence of capital-free arbitrage (Shleifer and Vishny, 1997), adjustment in a model with risk may involve the building and shedding of risk, which has implications for the nature of international spillovers.

Table 1
The short-term premium and bias in tests of UIP

The Fama equation: $\Delta q_{t+1} = c + \beta r_t^d + \varepsilon_{t+1}$

	Reported moments			Implied bias
Model	σ^{r_t}	$\sigma^{r_t^f}$	$corr(r_t, r_t^f)$	$cov(r_t^d, \lambda_t^R)/var(r_t^d)$
	(a)	(b)	(c)	$-(a^2 - a * b * c)/a^2$
CRRA	2.39	1.66	-0.37	-1.3
Fuhrer	2.39	31.25	-0.07	-1.9
Abel	2.39	26.55	-0.36	-5.0
CC	2.39	1.64	-0.37	-1.3
CEE	2.39	7.39	-0.09	-1.3
Abel(iid)	2.39	2.32	0.17	-0.8

Source: First three columns from Canzoneri et al. (2007) and references therein. Notes: CRRA (coefficient of relative risk aversion): standard, additively separable preferences, CC: Campbell and Cochrane, CEE: Christiano, Eichenbaum and Evans. Apart from the CRRA utility function, all involve some form of habit in consumption. The home bond spread is the data (observed Federal Funds rate, r_t) net of the model risk-free rate, r_t^f. That is, $\lambda_t^{Rh} = r_t - r_t^f$. Recall that λ_t^R is defined as the *foreign* bond premium net of the home bond premium. The covariance between the data and the implied bond premium is calculated as follows: $cov(r_t, \lambda_t^{Rh}) = cov(r_t, (r_t - r_t^f)) = var(r_t) - cov(r_t, r_t^f) = var(r_t) - corr(r_t, r_t^f)\sigma^{r_t}\sigma^{r_t^f}$. Assuming that foreign variables are unchanged, the implied estimation bias is $-cov(r_t, \lambda_t^{Rh})/var(r_t)$.

References

Aiyagari, S. R. and M. Gertler (1991). Asset returns with transactions costs and uninsured individual risk. *Journal of Monetary Economics 27*(3), 311–331.

Amihud, Y. and H. Mendelson (1991). Liquidity, maturity, and the yields on us treasury securities. *Journal of Finance 46*, 1411–1425.

Amihud, Y., H. Mendelson, and L. Pedersen (2005). Liquidity and asset prices. *Foundations and Trends in Finance 1*(4), 269–364.

Backus, D., S. Foresi, and C. Telmer (2001). Affine term structure models and the forward premium anomaly. *The Journal of Finance 56*(1), pp. 279–304.

Backus, D. K. and G. W. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics 35*(3-4), 297–316.

Bernanke, B. S. (2013). Long-term interest rates. Remarks at the Annual Monetary/Macroeconomics Conference: The Past and Future of Monetary Policy, Sponsored by Federal Reserve Bank of San Francisco, San Francisco, California, 1 March 2013.

Brandt, M. W., J. H. Cochrane, and P. Santa-Clara (2006). International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics 53*(4), 671–698.

Brennan, M. J. and Y. Xia (2006). International Capital Markets and Foreign Exchange Risk. *Review of Financial Studies 19*(3), 753–795.

Burnside, C. (2012). Carry trades and risk. In I. W. M. Jessica James and L. Sarno (Eds.), *Handbook of Exchange Rates*, Handbook of Monetary Economics, pp. 283–312. John Wiley & Sons.

Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2007). Euler equations and money market interest rates: A challenge for monetary policy models. *Journal of Monetary Economics 54*(7), 1863–1881.

Chen, Y. and K. Tsang (2013). What does the yield curve tell us about exchange rate predictability? *The Review of Economics and Statistics 95*(1), 185–205.

Chien, Y., H. Lustig, and K. Naknoi (2015). Why are exchange rates so smooth? a segmented asset markets explanation. Working Paper 2015-39, Federal Reserve Bank of St Louis.

Cochrane, J. (2001). *Asset Pricing*. Princeton University Press.

Cochrane, J. H. (2016). Macro-finance. NBER Working Papers 22485, National Bureau of Economic Research.

Cochrane, J. H. and L. P. Hansen (1992, June). Asset Pricing Explorations for Macroeconomics. In *NBER Macroeconomics Annual 1992, Volume 7*, NBER Chapters, pp. 115–182. National Bureau of Economic Research, Inc.

Colacito, R. and M. M. Croce (2013, December). International Asset Pricing with Recursive Preferences. *Journal of Finance 68*(6), 2651–2686.

Della Corte, P., L. Sarno, M. Schmeling, and C. Wagner (2015). Exchange rates and sovereign risk. Technical report, mimeo, http://ssrn.com/abstract=2354935.

Dornbusch, R. (1976). Expectations and exchange rate dynamics. *Journal of Political Economy 84*(6), 1161–76.

Duarte, M. and A. C. Stockman (2005). Rational speculation and exchange rates. *Journal of Monetary Economics 52*(1), 3–29.

Duffie, D. (1996). Special repo rates. *The Journal of Finance 51*(2), 493–526.

Engel, C. (1996). The forward discount anomaly and the risk premium: A survey of recent evidence. *Journal of Empirical Finance* (3), 123–192.

Engel, C. (2014). Exchange rates and interest parity. *Handbook of International Economics, vol. 4 (Elsevier)*.

Engel, C. (2016). Exchange rates, interest rates and the risk premium. *American Economic Review* (2), 436–474.

Engel, C. and K. West (2005). Exchange rates and fundamentals. *Journal of Political Economy 113*(3), 485–517.

Engel, C. and K. West (2010). Global interest rates, currency returns, and the real value of the dollar. *American Economic Review 100*(2), 562–567.

Fama, E. F. (1965). The behavior of stock market prices. *Journal of Business 38*, 34–105.

Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics 14*(3), 319–338.

Feldhtter, P. and D. Lando (2008). Decomposing swap spreads. *Journal of Financial Economics 88*(2), 375–405.

Flood, Robert, N. M. and A. Matsumoto (2009). International risk sharing during the globalisation era. Working Paper 09/209, International Monetary Fund.

Flood, R. P. and A. K. Rose (1996). Fixes: Of the Forward Discount Puzzle. *The Review of Economics and Statistics 78*(4), 748–52.

Hansen, L. P. and R. Jagannathan (1991). Implications of Security Market Data for Models of Dynamic Economies. *Journal of Political Economy 99*(2), 225–62.

Kiley, M. T. (2013). Exchange rates, monetary policy statements, and uncovered interest parity: before and after the zero lower bound. Finance and Economics Discussion Series 2013-17, Board of Governors of the Federal Reserve System.

Kose, M. A., E. Prasad, and M. Terrones (2003). How does globalization affect the synchronization of business cycles? *American Economic Review 93*(2), 57–62.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy* (2).

Lustig, H. and A. Verdelhan (2007). The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review 97*(1), 89–117.

Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*(2), 145–161.

Munro, A. (2014). Exchange rates, expectations and risk: UIP unbound. Working Paper 73/2014, Centre for Applied Macroeconomic Analysis (CAMA).

Munro, A. (2015). Exchange rates, expected returns and risk: what can we learn from Asia-Pacific currencies? *BIS Papers 82*.

Nagel, S. (2014). The liquidity premium of near-money assets. Working Paper 20265, National Bureau of Economic Research (NBER).

Obstfeld, M. and K. Rogoff (1996). *Foundations of International Macroeconomics*. Cambridge, MA: MIT Press.

Obstfeld, M. and K. Rogoff (2002). *Risk and Exchange Rates*. Cambridge: Cambridge University Press.

Sarno, L., P. Schneider, and C. Wagner (2012). Properties of foreign exchange risk premiums. *Journal of Financial Economics 105*(2), 279–310.

Shleifer, A. and R. W. Vishny (1997). The limits of arbitrage. *Journal of Finance 52*(1), 35–55.

Stein, J. C. (2012). Monetary policy as financial-stability regulation. *Quarterly Journal of Economics 127*(1), 57–95.

Vayanos, D. (1998). Transaction Costs and Asset Prices: A Dynamic Equilibrium Model. *Review of Financial Studies 11*(1), 1–58.

Wong, A., D. Leung, and C. Ng (2016). Risk-adjusted covered interest parity: theory and evidence. Working Paper 16/2016, Hong Kong Institute for Monetary Research.

Woodford, M. (2016, May). Quantitative Easing and Financial Stability. NBER Working Papers 22285, National Bureau of Economic Research, Inc.

Appendices

A Derivations

The risk-free rate

The home investor's real stochastic discount factor (SDF), M_{t+1}, between period t and $t+1$ is defined as:

$$M_{t+1} \quad = \quad \beta E_t \frac{U'_{C,t+1}}{U'_{C,t}}$$

where β is the subjective discount factor, and $U'_{C,t}$ is the marginal utility of consumption. Appealing to no-arbitrage and the Fundamental Theorem of Asset Pricing, there is no need to specify the form of the SDF, but simply to postulate that it exists. The home gross risk-free interest rate $1 + r_t^f$ is defined by:

$$\frac{1}{1+r_t^f} \quad = \quad M_{t+1} \tag{A.1}$$

Following Lustig and Verdelhan (2007), I assume the stochastic discount factor and asset returns to be conditionally log-normal.[13] Define $x_{t+1} = log(X_{t+1})$. If x_{t+1} is normally distributed, then $X_{t+1} = e^{(x_{t+1})}$ is log-normally distributed and $E_t(X_{t+1}) = e^{(E_t(x_{t+1}) + \frac{1}{2}\sigma^2_{x,t})}$. Taking logs, (A.1) becomes:

$$-r_t^f \quad = \quad logM_{t+1} = E_t m_{t+1} + \tfrac{1}{2} var_t(m_{t+1}) \tag{A.2}$$

Risk corrections

The home investor's pricing equation (Euler equation) for the home short-term bond is:

$$1 \quad = \quad E_t\big(\, M_{t+1}(1+r_t^c) Z_{t+1} \,\big)$$

where r_t^c is the ex-ante observed, contracted rate on the short-term bond (eg tbill), and Z_t, captures uncertainty about the ex-post payoff at time $t+1$.

[13] Alternatively, one can use the definition of covariance, $cov(M, X) = E(MX) - E(M)E(X)$, and take a log approximation.

Taking logs,

$$
\begin{aligned}
0 &= logE_t\big(M_{t+1}(1+r_t^c)Z_{t+1}\big) \\
&= log\big(e^{(E_t m_{t+1}+E_t r_t^c+E_t z_{t+1}+\frac{1}{2}var_t(m_{t+1}+r_t^c+z_{t+1}))}\big) \\
&= E_t m_{t+1}+\tfrac{1}{2}var_t(m_{t+1})+E_t r_t^c+\tfrac{1}{2}var_t(r_t^c)+E_t z_{t+1}+\tfrac{1}{2}var_t(z_{t+1}) \\
&\quad +cov_t(m_{t+1},r_t^c)+cov_t(m_{t+1},z_{t+1}) \qquad \text{(A.3)}
\end{aligned}
$$

Since the contracted rate, r_t^c is known with certainty ex-ante, the term $cov_t(m_{t+1}, r_t^c)$ is zero, and $logE_t(1+r_t^c) \sim r_t^c$. Only systematic risk (covariances with m_{t+k}) is priced, since idiosyncratic risk can be diversified away.

Combining (A.2) and (A.3) the home investor's pricing equation for the home short-term bond is:

$$r_t^c \;=\; r_t^f - logE_t(Z_{t+1}) - cov_t(m_{t+1}, z_{t+1}) \qquad \text{(A.4)}$$

The equivalent log pricing equation for the foreign bond, from the foreign investor's perspective is:

$$r_t^{c*} = r_t^{f*} - logE_t(Z_{t+1}^*) - cov_t(m_{t+1}^*, z_{t+1}^*) \qquad \text{(A.5)}$$

where r_t^{c*} is the coupon rate on the foreign bond, paid at time $t+1$ in foreign currency, Z_{t+1}^* captures uncertainty about payoffs on the foreign bond, and $r_t^{f*} = -log(E_t M_{t+1}^*)$ is the foreign risk-free rate. Subtracting (A.5) from (A.4) gives equation 2), with the relative bond market premium defined in 10.

Examples in which ex-post payoffs are not known with certainty follow in the next three sections.

Default premium

Consider a 1-period bond that is contracted at a gross rate $(1 + r_t^c)$, payable at $t + 1$, but that defaults with a non-zero probability. The pricing equation is

$$1 = E_t[M_{t+1}(1 + r_t^c)(1 - d_{t+1})]$$

where $d_t \in$ [0,1] captures both the probability of default and loss in the event of default. The log pricing equation is:

$$
\begin{aligned}
0 &= log[E_t\big(M_{t+1}(1+r_t^c)(1-d_{t+1})\big)] \\
&= log\big(e^{(E_t m_{t+1}+E_t r_t^c-E_t d_{t+1}+\frac{1}{2}var_t(m_{t+1}+r_t^c-d_{t+1}))}\big) \\
&= -r_t^f + r_t^c + logE_t(1-d_{t+1}) - cov_t(m_{t+1}, d_{t+1})) \\
r_t^c &= r_t^f - logE_t(1-d_{t+1}) + cov_t(m_{t+1}, d_{t+1})
\end{aligned}
$$

The contracted rate is known ex-ante, so $logE_t(1+r_t^c) \sim r_t^c$ and $cov_t(r_t^c, m_{t+k}) = 0$. Defining the default premium as the difference between the contracted rate and the risk-free rate,

$$\begin{aligned}\text{default premium} &\equiv r_t^c - r_t^f \\ &= -log(E_t(1-d_{t+1})) + cov_t(m_{t+1}, d_{t+1}),\end{aligned}$$

The default premium reflects the expected loss and a risk correction that increases the contracted yield if losses from default are expected to be higher when the marginal utility of consumption is expected to rise. The default premium is part of the bond premium, λ_t^R (equation 2).

Inflation premium

Written in real terms, inflation risk is part of the default premium. Written in terms of the nominal interest rate:

$$1 = E_t[M_{t+1}\frac{(1+i_t^c)}{(1+E_t\pi_{t+1})}(1-d_{t+1})]$$

where, d_{t+1} now excludes the effects of inflation.

$$\begin{aligned}0 &= log[E_t\big(M_{t+1}\frac{(1+i_t^c)}{(1+E_t\pi_{t+1})}(1-d_{t+1})\big)] \\ &= log\big(e^{(E_t m_{t+1}+E_t i_t^c - E_t\pi_{t+1} - E_t d_{t+1} + \frac{1}{2}var_t(m_{t+1}+i_t^c-\pi_{t+1}-d_{t+1}))}\big) \\ &= -r_t^f + i_t^c - log(E_t(1+\pi_{t+1}) + cov_t(m_{t+1}, \pi_{t+1})) \\ &\quad +logE_t(1-d_{t+1}) - cov_t(m_{t+1}, d_{t+1}) \\ r_t^c &= i_t^c - E_t\pi_{t+1} = r_t^f \underbrace{+\tfrac{1}{2}var_t(\pi_{t+1}) - cov_t(m_{t+1}, \pi_{t+1})}_{\text{inflation risk}} \\ &\quad -logE_t(1-d_{t+1}) + cov_t(m_{t+1}, d_{t+1})\end{aligned}$$

The inflation premium is part of the bond premium, λ_t^R (equation 2).

Term premium

While the assumption that the term premium is small is relevant for overnight securities, for the monthly or quarterly returns that are typically used in empirical exchange rate analysis, term premia may be material. Consider a two-period fixed-rate bond that pays a certain $(1+r_{2,t}^c)^2$ at $t+2$. The pricing equation, $1 = E_t[M_{t+1}M_{t+2}(1+r_{2,t}^c)^2]$, equates the cost of buying the bond today with the expected value of the payoff at $t+2$. The log of the pricing equation is:

$$\begin{aligned}
0 &= logE_t[M_{t+1}M_{t+2}(1+r^c_{2,t})^2] \\
&= log\big(e^{(E_t m_{t+1}+E_t m_{t+2}+2E_t r^c_{2,t}+\frac{1}{2}var_t(m_{t+1}+m_{t+2}+2r^c_{2,t}))}\big) \\
&= -E_t r^f_t - E_t r^f_{t+1} + 2E_t r^c_{2,t} + cov_t(m_{t+1}, m_{t+2}) \\
2r^c_{2,t} &= E_t r^f_t + E_t r^f_{t+1} - cov_t(m_{t+1}, m_{t+2}) \qquad \text{(A.6)}
\end{aligned}$$

Since the contracted rate $r^c_{2,t}$ is known with certainty at time t, $cov_t(r^c_{2,t}, m_{t+k}) = 0$. However, the state of the economy, and so the marginal utility of consumption in subsequent periods is not known with certainty, so $cov_t(m_{t+1}, m_{t+2}) \neq 0$. Defining the term premium as the holding-period return on the 2-period bond net of the return on rolling over a 1-period risk-free bond,

$$\begin{aligned}
\text{term premium} &\equiv 2r^c_{2,t} - r^f_t - r^f_{t+1} \\
&= -cov_t(m_{t+1}, m_{t+2})
\end{aligned}$$

The term premium compensates the holder for uncertainty about the marginal utility of consumption in the future. To generate a positive term premium, the stochastic discount factor must have negative serial correlation: $cov_t(m_{t+1}, m_{t+2}) < 0$. Negative serial correlation in the risk-free rate means that holding a multi-period bond with a fixed nominal payoff makes consumption more volatile. If the payoff $r^c_{2,t}$ helps to smooth consumption today, the negative serial correlation between m_{t+1} and m_{t+2} means that it is unlikely to help to smooth consumption next period. The term premium is part of the bond premium, λ^R_t (equation 2).

Liquidity premium

The price of a short-term bond can deviate considerably from its hold-to-maturity value because of collateral value, demand and supply, and short-term safety factors.

Consider holding the 2-period bond described above, but with a non-zero probability that the bond will be sold, at $t+1$, to smooth consumption, subject to a liquidation cost. The pricing equation is

$$1 = E_t[M_{t+1}M_{t+2}(1+r^s_{2,t})^2(1-d_{t+1})]$$

where $(1+r^s_{2,t})$ is the gross yield on the bond that may need to be sold, and d_{t+1} captures both the probability that the bond will be sold at $t+1$ and the expected discount if the bond is sold, relative its hold-to-maturity value. The log of the pricing equation is:

$$\begin{aligned} 2r^s_{2,t} &= E_t r^f_t + E_t r^f_{t+1} - logE_t(1-d_{t+1}) - cov_t(m_{t+1}, m_{t+2}) \\ &\quad + cov_t(m_{t+1}, d_{t+1}) + cov_t(m_{t+2}, d_{t+1}) \end{aligned}$$

The observed, contracted rate on the bond reflects expected risk-free returns, expected losses from selling the bond before maturity, a term premium, and a risk correction that increases the yield on the bond if losses are expected to be greater when the marginal utility of consumption is expected to rise.

Defining the liquidity premium as the yield on the bond that is sold at a discount at $t+1$, net of the yield on the 'liquid' bond (A.6),

$$\begin{aligned} \text{liquidity premium} &\equiv r^s_{2,t} - r^c_{2,t} \\ &= \tfrac{1}{2}(logE_t(1-d_{t+1}) + cov_t(m_{t+1}, d_{t+1})) \end{aligned}$$

the liquidity premium captures the expected loss from selling the bond, $logE_t(1-d_{t+1})$, and a risk correction. If investors are more likely to liquidate bonds to smooth consumption when the marginal utility of consumption rises, the expected loss from selling illiquid bonds is likely to be positively correlated with m_{t+1}. The expected discount d_{t+1} can also be interpreted as a transaction cost associated with selling the bond or 'specialness' (Krishnamurthy and Vissing-Jorgensen (2012), Vayanos (1998) and Aiyagari and Gertler (1991)). If a bond is expected to sell at a premium in bad times $E_t d_{t+1} < 0$, for example when the market wants to hold high quality assets and collateral − a 'flight to quality' response − then the yield on the bond will be lower, reflecting its expected liquidity value. Feldhtter and Lando (2008), Duffie (1996) and Amihud and Mendelson (1991) estimate short-term safety factors, to be substantial for US Treasuries. (Nagel, 2014) links liquidity premia to monetary policy. The liquidity premium is part of the bond premium, λ^R_t (equation 2).

Uncovered interest parity and currency revaluation risk

The home (global) investor's pricing equation for the foreign short-term bond is:

$$Q_t = E_t M_{t+1}(1 + r^{c*}_t) Z^*_{t+1} Q_{t+1},$$

where Q_t is the real exchange rate (a rise is a depreciation of the home currency). Taking logs,

$$\begin{aligned}
0 &= logE_t\big(M_{t+1}(1+r_t^{c*})Z_{t+1}^*\frac{Q_{t+1}}{Q_t}\big) \\
&= log\Big(e^{(E_t m_{t+1}+E_t r_t^{c*}+E_t z_{t+1}^*+E_t\Delta q_{t+1}+\frac{1}{2}var_t(m_{t+1}+r_t^{c*}+z_{t+1}^*+\Delta q_{t+1}))}\Big) \\
&= E_t m_{t+1}+\tfrac{1}{2}var_t(m_{t+1})+E_t r_t^{c*}+\tfrac{1}{2}var_t(r_t^{c*})+E_t z_{t+1}^*+\tfrac{1}{2}var_t(z_{t+1}^*) \\
&\quad +E_t\Delta q_{t+1}+cov_t(m_{t+1},z_t^*)+cov_t(m_{t+1},\Delta q_{t+1})+\tfrac{1}{2}var_t(\Delta q_{t+1}) \\
q_t &= -r_t^f+r_t^{c*}+logE_t(Z_{t+1}^*)+E_t q_{t+1}+\tfrac{1}{2}var_t(\Delta q_{t+1}) \\
&\quad +cov_t(m_{t+1},z_t^*)+cov_t(m_{t+1},\Delta q_{t+1}) \qquad \text{(A.7)}
\end{aligned}$$

Subtracting, (A.4) from (A.7) gives the UIP condition (equation 1) and the currency 'excess return' in (13):

$$\begin{aligned}
\lambda_t &= (logE_t Z_{t+1}-logE_t Z_{t+1}^*+cov_t(m_{t+1},(z_{t+1}-z_{t+1}^*))-cov_t(m_{t+1},\Delta q_{t+1}) \\
&\quad -\tfrac{1}{2}var_t(\Delta q_{t+1})
\end{aligned}$$

that equates the expected return on a home bond to expected return on a foreign bond, from the home investor's perspective.

The currency premium

The currency premium is defined as:

$$\begin{aligned}
\lambda_t^{FX} &= \lambda_t-\lambda_t^R \\
&= (logE_t(Z_{t+1})-logE_t(Z_{t+1}^*) \\
&\quad +cov_t(m_{t+1},z_{t+1})-cov_t(m_{t+1},z_{t+1}^*)-cov_t(m_{t+1},\Delta q_{t+1}) \\
&\quad -\tfrac{1}{2}var_t(\Delta q_{t+1})-(logE_t(Z_{t+1})-logE_t(Z_{t+1}^*) \\
&\quad +cov_t(m_{t+1},z_{t+1})-cov_t(m_{t+1}^*,z_{t+1}^*)) \\
&= cov_t(m_{t+1}^*,z_{t+1}^*)-cov_t(m_{t+1},z_{t+1}^*)-cov_t(m_{t+1},\Delta q_{t+1})-\tfrac{1}{2}var_t(\Delta q_{t+1}) \\
&= cov_t\big((m_{t+1}^*-m_{t+1}),z_{t+1}^*\big)-cov_t(m_{t+1},\Delta q_{t+1})-\tfrac{1}{2}var_t(\Delta q_{t+1})
\end{aligned}$$

When the home and foreign investor have access to a full set of state-contingent bonds, the ratio of their marginal utility growth is the expected change in the exchange rate, $M_{t+1}\Delta Q_{t+1}=M_{t+1}^*$. Therefore, their pricing of the foreign bond yields the same interest rate.

$$1 = E_t M_{t+1}\frac{Q_{t+1}}{Q_t}(1+r_t^{c*})Z_{t+1}^* = E_t M_{t+1}^*(1+r_t^{c*})Z_{t+1}^*$$

Combining (9) and (12), and substituting in the risk sharing condition $r_t^f - r_t^{f*} = m_{t+1}^* - m_{t+1} = E_t \Delta q_{t+1}$,

$$\begin{aligned}
0 &= r_t^f - r_t^{f*} - E_t \Delta q_{t+1} - cov_t(m_{t+1} - m_{t+1}^*, z_{t+1}^*) - cov_t(m_{t+1}, \Delta q_{t+1}) \\
&\quad - \tfrac{1}{2} var_t(\Delta q_{t+1}) \\
0 &= cov_t(m_{t+1}^* - m_{t+1}, z_{t+1}^*) + cov_t(m_{t+1}, \Delta q_{t+1}) + \tfrac{1}{2} var_t(\Delta q_{t+1})
\end{aligned}$$

In he complete markets case, risks are priced symmetrically and there is no currency premium $\lambda_t^{FX} = 0$.

B Fama conditions

(Fama, 1984) defines the forward exchange rate as the certainty-equivalent of the future spot rate:

$$\begin{aligned}
F_t &\equiv S_t \frac{1 + i_t^c}{1 + i_t^{c*}} \\
\text{in logs, } f_t &= s_t + i_t^c - i_t^{c*} \qquad \text{(B.1)}
\end{aligned}$$

where F_t is the forward nominal exchange rate, S_t is the nominal spot exchange rate, $f_t = log(F_t)$, and $s_t = log(S_t)$. Fama decomposes the forward exchange rate into an expected future spot rate and a premium, ρ_t:

$$f_t = E(s_{t+1}) + \rho_t$$

The log forward discount $f_t - s_t$ can be expressed as an expected depreciation and a premium:

$$f_t - s_t = \underbrace{E(s_{t+1}) - s_t}_{\text{expected depreciation}} + \rho_t$$

Using (B.1) to substitute for $f_t - s_t$,

$$i_t^c - i_t^{c*} = \underbrace{E(s_{t+1}) - s_t}_{\text{expected depreciation}} + \rho_t$$

Defining the real interest differential r_t^d as $i_t^c - E_t(\pi_{t+1}) - (i_t^{c*} - E_t(\pi_{t+1}^*))$, and subtracting the expected home-foreign inflation differential from both sides,

$$\begin{aligned}
i_t^c - E_t(\pi_{t+1}) - (i_t^{c*} - E_t(\pi_{t+1})) &= E_t(s_{t+1}) + E_t(p_{t+1}^*) - E_t(p_{t+1}) \\
&\quad - (s_t + p_t^* - p_t) + \rho_t \\
r_t^d &= E_t(\Delta q_{t+1}) + \rho_t
\end{aligned}$$

Comparing B.2 and equation (1), we can see that Fama's Premium ρ_t is equal to $-\lambda_t$.

In Fama's two regressions,

$$\begin{aligned} f_t - s_{t+1} &= \alpha_1 + \beta_1(f_t - s_t) + \epsilon_{1,t+1}, \text{ and} \\ s_{t+1} - s_t &= \alpha_2 + \beta_2(f_t - s_t) + \epsilon_{2,t+1} \end{aligned}$$

β_1 picks up the component of $f_t - s_t = i_t^d$ that shows up reliably in $f_t - s_{t+1}$, and β_2 picks up the component of i_t^d that shows up reliably in $s_{t+1} - s_t$. The left-hand side variable in the first equation is:

$$\begin{aligned} f_t - s_{t+1} &= \underbrace{f_t - E_t(s_{t+1})}_{-\lambda_t} - \underbrace{(s_{t+1} - E_t(s_{t+1}))}_{\text{expectational error}, \varepsilon_{1,t+1}} \\ &= -\lambda_t^R - \lambda_t^{FX} + \varepsilon_{t+1} \end{aligned}$$

Writing the explanatory variable $f_t - s_t$ as $i_t^d = (r_t^f - r_t^{f*}) - \lambda_t^R + E_t(\pi_{t+1}) - E_t(\pi^*_{t+1})$, and abstracting from covariances between variables, the component of i_t^d captured in β_1 is the bond premium λ_t^R which has a negative sign in both i_t^d and in $f_t - s_{t+1}$. The model proposed here predicts that β_1 has a value $\frac{var(\lambda_t^R)}{var(i_t^d)}$.

The left-hand side variable of Fama's second equation is:

$$\begin{aligned} s_{t+1} - s_t &= \underbrace{s_{t+1} - E_t(s_{t+1})}_{\text{expectational error}, \varepsilon_{t+1}} + \underbrace{E_t(s_{t+1}) - s_t}_{i_t^d + \lambda_t} \\ &= i_t^d + \lambda_t^R + \lambda_t^{FX} + \varepsilon_{t+1} \end{aligned}$$

The component of i_t^d captured in β_2 comes from the first two terms. Abstracting from covariances between the premium and expected depreciation, the model predicts that β_2 has a value $1 - \frac{var(\lambda_t^R)}{var(i_t^d)}$. When variation in i_t^d is all from the relative bond premium component, then $\beta_2 = 0$. For $\beta_2 < 0$, $var(\lambda_t^R) > var(i_t^d)$, implying covariance between the two components or between i_t^d and λ_t^{FX}.

Fama also shows that

$$\begin{aligned} \beta_1 - \beta_2 &= \frac{var(\rho_t) - var(E_t(s_{t+1} - s_t))}{var(i_t^d)} \\ \text{since,} \quad & \beta_1 = 1 - \beta_2 \\ 1 - 2\beta_2 &= \frac{var(\rho_t) - var(E_t(s_{t+1} - s_t))}{var(i_t^d)} \end{aligned}$$

For $\beta_2 < 0$, the right hand side must be positive, implying that the variance of the premium component of i_t^d, which is λ_t^R, must be greater than the variance of the expected depreciation $(r_t^d + \lambda_t)$.

C Comparison with Engel 2016 model

In this appendix, the model of Engel (2016) is restated in the framework derived in this paper as a basis for understanding similarities and differences. The two models are similar in spirit, but different in their motivation/derivation and different in some assumptions that affect model dynamics. In Engel's model, a liquidity premium arises directly in a linear model, motivated by a cash in advance constraint, and implemented through liquidity in the utility function. In the more generalised framework in this paper, bond premia arise, from expected gains/losses and from covariance of those gains/losses with consumption utility growth. The borrower or contract features that give rise to premia are not modeled explicitly, but premia can potentially be identified as latent variables.

Household optimisation problem

Abstracting from deposits, the representative household/investor derives utility from consumption, C_t, and liquidity, L_t.

$$\begin{aligned}
&\max_{C_t,B_t} \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, L_{t+k}) \qquad\qquad 0 < \beta < 1 \\
&s.t. \qquad P_tC_t + B_t + S_tB_t^* \;=\; Y_t + (1 - i_{t-1})B_{t-1} + S_t(1 - i_{t-1}^*)B_{t-1}^* \\
&\qquad\qquad\qquad\qquad\quad L_t \;=\; \kappa(\epsilon_t)\frac{B_t}{P_t}
\end{aligned}$$

where B_t is holdings of the home currency bond , B_t^* is holdings of the foreign currency bond, S_t is the nominal exchange rate, and i_t, i_t^* are home and foreign nominal policy interest rates. $\kappa(\epsilon_t)$ reflects the effect of monetary policy intervention on home bond liquidity. The real stochastic discount factor (SDF), M_t, defines the inter-temporal rate of substitution between consumption in period t and period $t + 1$. The inverse of the SDF is the gross real risk-free interest rate $1 + r_t^f$:

$$\text{define the real SDF :} \quad M_{t+1} = \beta E_t(\frac{U_{C,t+1}}{U_{C,t}}) = \frac{1}{1 + r_t^f}$$

Define $X_t = \frac{U_{L,t+1}}{U_{C,t}}$ as the marginal rate of substitution between consumption and liquidity, or the shadow value of liquidity, measured in units of consumption. First order conditions for the home investor are:

$$B_t: \quad 1-\kappa(\epsilon_t)X_t \quad = E_t\big[M_{t+1}\frac{(1+i_t)}{1+E_t(\pi_{t+1})}\big] \tag{C.1}$$

$$B_t^*: \quad 1 \quad = E_t\big[M_{t+1}\frac{(1+i_t^*)}{1+E_t(\pi_{t+1})}\frac{S_{t+1}}{S_t}\big] \tag{C.2}$$

Foreign assets are assumed not to provide liquidity services for the home investor and vice versa. Assuming a symmetrical foreign economy, the first order conditions of the foreign investor are:

$$B_t: \quad 1 \quad = E_t\big[M_{t+1}^*\frac{(1+i_t)}{1+E_t(\pi_{t+1}^*)}\frac{S_t}{S_{t+1}}\big] \tag{C.3}$$

$$B_t^*: \quad 1-\kappa(\epsilon_t^*)X_t^* \quad = E_t\big[M_{t+1}^*\frac{(1+i_t^*)}{1+E_t(\pi_{t+1}^*)}\big] \tag{C.4}$$

Define the home and foreign real policy interest rates, respectively, as $r_t = i_t - E_t\pi_{t+1}$ and $r_t^* = i_t^* - E_t(\pi_{t+1}^*)$. The respective risk-free real rates are $r_t^f = -m_t$ and $r_t^{f*} = -m_t^*$. Define the real exchange rate as $q_t = s_t - p_t + p_t^*$. For ease of notation, define $x_t + \eta_t \equiv log(1-\kappa(\epsilon_t)X_t)^{-1} - log(1-\bar{\kappa}\bar{X})^{-1}$ and $x_t^* + \eta_t^* \equiv log(1-\kappa(\epsilon_t^*)X_t^*)^{-1} - log(1-\bar{\kappa}\bar{X^*})^{-1}$, where $x_t,\ x_t^*$ capture the linearised components of $X_t,\ X_t^*$ respectively, and $\eta_t,\ \eta_t^*$ capture the linearised components of monetary intervention, respectively $\kappa(\epsilon_t),\ \kappa(\epsilon_t^*)$. First order approximations of (C.1) - (C.4), written in real terms, are:

$$r_t = r_t^f - x_t - \eta_t \tag{C.5}$$

$$r_t^* = r_t^{f*} - x_t^* - \eta_t^* \tag{C.6}$$

$$r_t^* = r_t^f - E_t(\Delta q_{t+1}) \tag{C.7}$$

$$r_t = r_t^{f*} + E_t(\Delta q_{t+1}) \tag{C.8}$$

In (C.5) and (C.6), the real interest rate is the sum of the risk-free rate and a bond market premium. The former reflects expected consumption utility growth. The premium reflects the shadow price of liquidity, x_t, and the stance of monetary policy, η_t. If monetary policy is set according to a Taylor-type rule, then there is a mapping between η_t, η_t^* and monetary policy settings, reflected in r_t, r_t^*.[14] In the absence of monetary policy, $\eta_t = \eta_t^* = 0$.

[14] Although monetary policy sets the nominal interest rate, the rules can be written in real terms, reflecting the Taylor principle: the real interest rate needs to rise in response to a rise in inflation.

In both models, monetary policy intervention in the local bond market is a source of market incompleteness. In Engel's model, asymmetric valuation of liquidity (only local investors value local currency liquidity) generates a second source of deviation from complete markets.[15] That

Uncovered interest parity

Combining (C.5) and (C.7) gives the UIP condition from the home investor's perspective:

$$q_t = -(r_t - r_t^*) - x_t - \eta_t + E_t(q_{t+1}) \tag{C.9}$$

Combining (C.6)and (C.8) gives the UIP condition from the foreign investor's perspective:

$$q_t = -(r_t - r_t^*) + x_t^* + \eta_t^* + E_t(q_{t+1}) \tag{C.10}$$

Engel (2016) assumes that the home and foreign investors' pricing of the bonds is equally weighted in the foreign exchange market. Averaging (C.9) and (C.10),

$$q_t = -(r_t - r_t^*) - \underbrace{\tfrac{1}{2}(x_t - x_t^* + \eta_t - \eta_t^*)}_{\lambda_t} + E_t(q_{t+1}) \tag{C.11}$$

Combining (C.5) and (C.6), the observed interest differential is

$$r_t - r_t^* = (r_t^f - r_t^{f*}) - \underbrace{(x_t + \eta_t - x_t^* - \eta_t^*)}_{\lambda_t^R}$$

The currency premium in this model is:

$$\begin{aligned} \lambda_t^{FX} &= \lambda_t - \lambda_t^R \\ &= -\tfrac{1}{2}(x_t - x_t^* + \eta_t - \eta_t^*) \end{aligned}$$

[15] In the presence of a liquid spot and forward currency market, an agent needing home liquidity should be able to borrow foreign currency, convert it at the spot rate and enter into a forward exchange rate contract to obtain local currency liquidity. Occasionally, for example following the Lehman bankruptcy, that option is risky and material deviations from covered interest parity are observed. See Wong et al. (2016) for a risk-based discussion.

The forward premium puzzle

In Engel's model, λ_t and λ_t^R have a common component $\frac{1}{2}(x_t - x_t^* + \eta_t - \eta_t^*)$. That symmetrically priced liquidity component is not offset by currency movement because it is reflected in both λ_t and in λ_t^R. The common component nets to zero in the UIP equation (C.11). Intuitively, the currency does not depreciate to offset that component because all participants view it as compensation for risk.

If $var(x_t)$, $var(x_t^*)$ are large compared to $var(\eta_t)$, $var(\eta_t^*)$, as Engel proposes, and x_t, x_t^* are not highly correlated, then $\hat{\beta}$ estimated in $\Delta q_{t+1} = c + \beta r_t^d + \varepsilon_{t+1}$ is biased downwards from one. If the variance of r_t^d mainly reflects relative liquidity premia, then $\hat{\beta} \to \frac{1}{2}$.

If both home and foreign price home and foreign liquidity, then x_t and x_t^* would all be fully reflected in both λ_t and in λ_t^R. In that case, a volatile x_t, x_t^* would deliver $\hat{\beta} = 0$.

Complete markets:

When markets are complete, the home and foreign investors' pricing of the home and foreign bonds yield the same interest rates since $M_t \Delta Q_{t+1} = M_t^*$. In that case, combining (C.5) with (C.8) and (C.7) with (C.6)

$$r_t^f - x_t - \eta_t = r_t^{f^*} + E_t \Delta q_{t+1} \quad \text{and} \quad r_t^{f*} + x_t^* + \eta_t^* = r_t^f - E_t \Delta q_{t+1}$$

Substituting in the risk-sharing condition $r_t^f - r_t^{f*} = E_t \Delta q_{t+1}$ gives

$$x_t + \eta_t = 0 \quad \text{and} \quad x_t^* + \eta_t^* = 0$$

In the complete markets case, Engel's model delivers $\beta = 1$. In contrast, in the model proposed in this paper, the complete markets limit delivers the forward premium puzzle $\beta = 0$ because both home and foreign investors value liquidity in both markets.

Exchange rate level

Equation (C.11) can be written as:

$$q_t = -(r_t^f - r_t^{f*}) + \underbrace{\frac{1}{2}(x_t - x_t^* + \eta_t - \eta_t^*) + E_t(q_{t+1})}_{-\lambda_t^{FX}}$$

Applying Engel's assumption that monetary policy is persistent relative to the shadow price of risk, suppose that η_t, η_t^* are persistent AR(1) processes with AR(1) coefficients ρ and ρ^*, while x_t, x_t^* are much less persistent. Substituting forward,

$$q_t \sim -\sum_{k=0}^{\infty}(r_t^f - r_t^{f*}) + \tfrac{1}{2}(\frac{\eta_t}{1-\rho} - \frac{\eta_t^*}{1-\rho^*}) + E_t(\bar{q})$$

The sum of the forward relative interest rate path, R_t, is:

$$R_t = \sum_{k=0}^{\infty}(r_t^d) \sim \sum_{k=0}^{\infty}(r_t^f - r_t^{f*}) - (\frac{\eta_t}{1-\rho} - \frac{\eta_t^*}{1-\rho^*}) + E_t(\bar{q})$$

In level terms, and abstracting from over-and under-adjustment in general equilibrium discussed in Engel (2016), a currency with a high risk-free rate follows the expected path $-R_t$. A currency with a low currency premium is strong relative to it's long-run value, but weak relative to the expected path of $-R_t$ because it reflects only half of the monetary policy component of the interest rate path.

In contrast, if investors were to price only the shadow value of liquidity and monetary policy were to affect only the local bond market interest rate directly, then a volatile price of risk would deliver $\beta = 0$ and a persistent policy shock would be fully reflected in q_t as well as in $-R_t$. In that case the two models would be very similar, with the exception of the liquidity in utility assumption.

The model proposed here and the model in Engel (2016) are very similar in spirit, but different in derivation and the assumptions discussed above. In both models, exchange rates are driven by three factors: the underlying risk-free rate (IMRS), the market price of risk, and the effects of policy intervention. To meet both empirical regularities discussed in Engel (2016), the observed interest rate needs to be highly influenced by a monetary policy shock to deliver the exchange rate level, and also to reflect relative short-term risk characteristics, to deliver the forward premium puzzle.