A structural model for policy analysis and forecasting: NZSIM

Gunes Kamber, Chris McDonald, Nick Sander, Konstantinos Theodoridis

November 2015

JEL classification: C51, E3, E52

www.rbnz.govt.nz/research/discusspapers/

Discussion Paper Series

ISSN 1177-7567
Abstract

We describe the underlying structure of the new forecasting and policy model used at the Reserve Bank of New Zealand. This paper outlines the dynamic stochastic general equilibrium part of the model, which is deliberately kept small so that it is easily understood and applied in the forecasting context. We also discuss the key transmission channels in the model, estimate the model’s parameters and evaluate its ability to explain New Zealand data.
1 Introduction

In this paper we introduce the main elements of the Reserve Bank of New Zealand’s economic forecasting and policy analysis model (NZSIM). This model is one of the tools used to create the forecasts published in the Reserve Bank’s Monetary Policy Statements. It also underlies some of the internal and externally published policy analysis conducted at the RBNZ. We summarise key features of the framework and outline the approach we have taken in designing it.

Our model has been designed to articulate the structure of the monetary transmission mechanism and to incorporate interactions between the wide range of variables we use in the process of policy decision-making. The model gives a credible account of key relationships present in the data and replicates the salient features of the business cycle. Moreover it is flexible enough to easily examine the consequences of various economic developments and counterfactual scenarios while keeping the whole structure sufficiently tractable, easy to understand and to communicate. Over time, we use our model to express and store our institutional research, experience and knowledge.

The design of the NZSIM framework builds upon the lessons and experience from the development and use of macroeconomic models at the Bank. The central element of our modelling strategy is a small open economy dynamic stochastic general equilibrium (DSGE) model with optimising agents. This model provides theoretical underpinnings to match and understand the key correlations in New Zealand economic data. The DSGE centre is deliberately kept small relative to previous models used at the RBNZ. This parsimonious structure is more transparent and easier to communicate. However, this simplicity and tractability is not entirely costless since restricting the coverage of the model’s structure reduces the insights it can add in articulating the drivers of the New Zealand business cycle.

Our approach is to expand our DSGE centre by incorporating additional simple auxiliary relationships. These relationships are based on empirical or theoretical research and reflect our institutional experience. These auxiliary equations enable us to represent our institutional views on a large number of important economic channels clearly and concisely. Our criteria for deciding how a feature should be introduced into our framework (either with full theoretical detail or based on empirical research) is to consider the complexity intro-
duced and empirical gain achieved from various modelling methods. The theoretically consistent centre allows us to view the auxiliary relationships as essentially correlated exogenous drivers and maintains our ability to produce credible counterfactuals. In section 6, we illustrate, with an example, the details of our approach for selecting and assessing an additional feature to incorporate into our framework.

An alternative approach would be to include into the DSGE model all the economic mechanisms of interest to the RBNZ. However, our experience suggests that the many interactions in large models can be difficult to model. Complex models may obscure the key insights that policy makers seek from a policy model with sometimes little empirical improvement. Therefore we have adopted an approach to only include features that improve the empirical fit.

Naturally our approach contains risks. Our auxiliary reduced-form relationships are affected by our DSGE centre and in some cases influence that centre in turn. Adding such additional features into the framework may, if one is not careful, produce unexpected interactions with the remainder of the system, contradicting existing features of the model or violating the general equilibrium accounting conditions or constraints. We adopt and outline a systematic approach to ensure that the theoretical and general equilibrium implications of each feature we are including are acceptable and as desired and that the feature improves the fit of the model.

While including auxiliary relationships relaxes the trade-off between model transparency and comprehensiveness, it does not remove all constraints on model size. A well-designed policy model is more than simply producing forecasts for everything. A variety of forecasting models already exist at the RBNZ to predict the evolution a large number of economic variables. Often, the models that produce the best forecasts – being purely statistical – are silent on what economic fundamentals are driving their forecasts. Unsurprisingly, policymakers want to know and understand the drivers of any model output and the underlying assumptions and relationships producing it. Our central policy framework complements these models by articulating the underlying economic forces generating the

---

3 Alternative approaches exist to incorporate missing channels or to allow the analysis of off-model variables in conjunction with the output from a central model. We are not the first to face this constraint and several methods to manage this problem are discussed in the literature. Burgess et al. (2013) suggest using multiple models - each simple and tractable - to obtain comprehensive coverage of economic channels. Caldana et al. (2011) (and others) suggest augmenting (via correlated shocks) a DSGE model with simple factor relationships to connect new variables and transmission channels quickly into the DSGE. In addition, when the implications of monetary policy for off-model variables are of interest to policymakers, Schorfheide et al. (2010) propose a method where DSGE variables are connected to large numbers of off-model variables as if the DSGE variables were dynamic factors.

4 As Kapetanos et al. (2007) argue: “The conceptual [structural] model is unlikely to be able to match an existing data set without becoming impossibly complex and thereby losing its main attraction of having a readily comprehensible structure.”
forecasts. Once identified, the frameworks’ perspectives on the state of the economy and the policy implications can be discussed.

Given the importance we place on the parsimony of the model it becomes very important to prioritize what features of the business cycle we wish the model to capture. In our view, there is no constant set of “most important” features. Every business cycle is different in terms of the relative prominence of different drivers and our framework may leave out a driver that later becomes important. As such, we intend our framework to evolve over time as our needs change and as the economic profession progresses in its understanding of business cycles. While we allow the framework to evolve, we intend to retain the DSGE structure at the centre of the framework. This paper therefore focuses on the central DSGE structure of our framework and provides some discussion and examples on how the model is used and how we approach the inclusion of additional features.

The central model is a small open economy New Keynesian model. As such, it builds on the already well-established literature on structural models of New Zealand, especially on its predecessor at the Reserve Bank, KITT. Section 2 presents a detailed description of the structure of the model, showing how the model dynamics are determined by economic agents’ decision rules, budget constraints and market clearing conditions.

Section 3 focuses on the empirical properties of the model. First, using Bayesian econometric techniques, the parameters of the model and the dynamics of the exogenous drivers are estimated by matching the model to a number of macroeconomic aggregates. Second, using the estimated parameter values we evaluate the model’s ability to account for the business cycle dynamics in New Zealand. Third, we discuss the transmission channels of the estimated model by examining model-implied behaviour of macroeconomic variables following unexpected movements in interest rates and exchange rates, respectively.

Section 4 uses an example to illustrate how knowledge of the model’s transmission channels can be used to produce alternative scenarios. While technical tools exist to adjust the behaviour of model variables, the endogenous nature of the modelling framework can provide many alternative explanations for a given movement in a single variable. To reflect the desired scenario it is important to select the appropriate shock or combination of shocks to generate the desired change in model predictions. Our example demonstrates that depending on the particular lever used, the same adjustment to the path of interest rates can have different qualitative impacts on other model variables.

5 Such as Benigno and Thoenissen (2003), Gali and Monacelli (2005), Lubik and Schorfheide (2005), Adolfson et al. (2007), Justiniano and Preston (2010a) and Justiniano and Preston (2010b).
6 See for example Albertini et al. (2012), Kam et al. (2009), Lees et al. (2011), Lubik and Schorfheide (2007) and Matheson (2010).
Section 5 presents and discusses an extension to the baseline model. Give the general equilibrium structure, a model extension can have non-trivial impacts on the model’s behaviour. Being mindful of this, our adjustment process requires an investigation of these impacts and an assessment of the net benefits of systematically including additional features. To demonstrate this process, we consider the theoretical appeal and empirical gain from allowing household’s and firm’s expectations of inflation to be driven by an alternative partially backward looking process.

2 Technical description of the model

This section provides the details of NZSIM, including a description of the optimisation problems faced by the agents in this economy. The first order conditions of these problems describe the evolution of the model’s variables. The baseline version of this model assumes that agents in this economy have full information about the structure and the state of the economy.

The economy is comprised of three optimising agents: households, domestic firms and import distributing firms. Households sell their labour to domestic firms and consume output produced domestically or imported from abroad. Domestic firms employ labour (provided by households) to produce domestic goods. We assume they hold a fixed amount of capital, which simplifies the model structure considerably and allows us to explicitly link domestic inflation to the output gap. General equilibrium models with typical investment adjustment costs perform poorly when explaining investment dynamics without heavily relying on investment specific shocks (Justiniano et al., 2010, Kamber et al., 2012). Therefore, we keep the model structure simple by omitting capital. The import distributing (tradable) firms purchase imports and domestic output to produce tradable goods. The domestic output can be thought of as representing the distribution costs faced by import retailers.

We model firms as competing in monopolistically competitive environments with price adjustment costs `a la Rotemberg (1982). The adjustment costs restrict the adjustment of prices in response to economic shocks, resulting in inefficient fluctuations in output. Including these adjustment costs in the model suggests powerful effects of monetary policy on real economic variables and, in addition, suggests that appropriately set monetary policy can improve welfare.
2.1 Households

2.1.1 Utility function

In each period, households consume \((c_t)\), save via a financial intermediary issuing a bond \((B_t)\) and pay a lump-sum tax to the government \((T_t)\). They are required to send a transfer overseas \((\text{Trans}_t)\) and supply labour \((n_t)\). We assume that each household has an identical utility function and supplies differentiated labour. Investment is not modelled as a choice variable of the household. Capital letters denote nominal variables and lower case letters real variables. The model includes a stochastic trend in total factor productivity \((A_t)\). The utility function is defined as

\[
U(c_{t+s}(j), n_{t+s}(j)) = E_t \sum_{s=0}^{\infty} \beta^s \omega^s_{t+s} \left\{ \frac{1}{1-\sigma} \left( c_{t+s}(j) - \chi c_{t+s-1} - \frac{\omega^{s}_{t+s} z_{t+s}(j)}{1+\sigma} \right)^{1-\sigma} \right\}
\]

where \(n_t\) is labour, \(\omega^s_t\) represents stochastic changes in the household’s discount factor, \(\omega^n_t\) is a shock to the household’s disutility of labour and \(z_t\) is a preference shifter that evolves according to

\[
z_{t+s}(j) = z_{t+s-1}(j) \left( c_{t+s}(j) - \chi c_{t+s-1} \right)^v \left( \frac{A_{t+s}}{A_{t+s-1}} \right)^{1-v} v \in [0, 1]
\]

Our utility function specification nests the one proposed by Jaimovich and Rebelo (2009), where the parameter \(v\) controls the importance of the wealth effect in households’ labour supply curve. When \(v\) is set to 0, preferences become non-separable as in Greenwood et al. (1988) (henceforth referred to as GHH). When \(v\) is set to one, preferences are of the same class as King et al. (1988). We also allow households to form habits not only in terms of consumption (as in Christiano et al., 2005; Smets and Wouters, 2007; Adolfson et al., 2007) but also in terms of working hours. As is explained by Schmitt-Grohé and Uribe (2008) and Kamber (2010), habit formation in hours worked adds persistence to the labour market even when wages are fully flexible. This device can be viewed as an approximation to capture search and matching type frictions in the labour market.

The household budget constraint is

\[
R_{t-1}^h \frac{B_{t-1}(j)}{P_t} + \left( 1 - \Psi_W \left( \frac{W_t(j)}{W_{t-1}(j)} \right) \right) \frac{W_t(j)}{P_t} n_t(j) + \frac{T_t(j)}{P_t} + \frac{\text{Trans}_t(j)}{P_t} = \frac{B_t(j)}{P_t} + \frac{C_t(j)}{P_t} + \frac{T_t(j)}{P_t}
\]

where \(W_t\) is the nominal wage, \(R_{t}^h\) is the (gross) nominal interest rate and \(P_t\) is the
aggregate price level. $\Psi_W \left( \frac{W_t}{W_{t-1}} \right)$ captures the cost of resetting wages and $\Xi_t$ denotes the profits made by monopolistic firms.

Aggregate consumption is a function of domestic ($C_{D,t}$) and tradable ($C_{T,t}$) goods. It is described by

$$C_t(j) = \left( 1 - \theta_C \right) \frac{1}{\eta} C_{D,t}(j)^{\frac{\eta-1}{\eta}} + \theta_C C_{T,t}(j)^{\frac{\eta-1}{\eta}}$$

where $\eta$ is the elasticity of substitution between domestic and tradable goods and $\theta_C$ measures trade openness. Optimal consumption and debt are derived by maximising the utility function subject to the budget constraint.

### 2.1.2 Labour supply

Households are monopolistic suppliers of their own differentiated labour (indexed by $j$). They set their nominal wage $W_t(j)$ and supply any amount of labour demanded at that wage. They face a quadratic adjustment cost, measured in terms of their total nominal wage bill. This prevents agents from being able to change nominal wages rapidly. The specification of the adjustment cost function,

$$\Psi_W \left( \frac{W_t(j)}{W_{t-1}(j)} \right) = \frac{\psi_W}{2} \left( \frac{W_t(j)}{W_{t-1}(j) (\Delta A \pi)^{1-\gamma}} \right)^2 - 1$$

The inclusion of the steady state values of TFP ($\Delta A$) and inflation ($\pi$), ensures that $\Psi_W (\cdot) = 0$ in the steady state.

There exists a ‘labour packager’ that costlessly transforms heterogenous household labour into a homogenous labour bundle, which is hired by firms. The labour packager, which operates under perfect competition, combines labour from all households to create the labour bundle. Aggregate labour is given by

$$n_t = \left[ \int_0^1 \left( n_t(j) \right)^{\frac{1}{\lambda_{w,t}}} dj \right]^{\lambda_{w,t}}$$

The demand for labour of type $j$ is given by

$$n_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{\frac{1}{\lambda_{w,t}-1}} n_t$$

The aggregate wage is given by

$$W_t = \left[ \int_0^1 \left( W_t(j) \right)^{1-\gamma} dj \right]^{1-\lambda_{w,t}}$$

Households decide how much labour to supply by maximising the utility function subject to the budget constraint and the per capita demand for labour equation.
2.2 Financial intermediary

Households’ save by buying bonds from perfectly competitive financial intermediaries (indexed by \( f \)). Each financial intermediary issues a one period bond \( B_t(f) \) at the beginning of period \( t \) and repays at a gross rate of \( R^f_t \) in period \( t + 1 \). The intermediaries utilize the household’s savings by investing in a portfolio of domestic (\( B^d_t(f) \)) and foreign bonds (\( B^f_t(f) \)). The prices of these bonds are subject to risk premiums that depend on the overall debt in the economy expressed as ratio to GDP. Each intermediary’s objective is to

\[
\max_{B^d_t(f), B^f_t(f)} E_t \left[ R_t B^d_t(f) \Psi_{B^d} \left( \frac{B_t}{P_t}, \omega_t^R \right) + R_t S_t \frac{B^f_t(f)}{S_{t+1} P_t} \Psi_{B^f} \left( \frac{B_t}{P_t}, \omega_t^R, \omega_t^S, \frac{R_{t-1} - R_t}{\Delta S_t}, \Delta S_t \right) - R^f_t B_t(f) \right] 
\]

subject to

\[
\frac{B_t(f)}{P_t} = \frac{B^d_t(f)}{P_t} + \frac{B^f_t(f)}{S_t P_t}
\]

where the optimization problem is specified in domestic currency terms and both \( R_t \) and \( R^f_t \) represent the gross interest rate earned on domestic and foreign bonds respectively. \( S_t \) is the nominal exchange rate defined so that a rise corresponds to an appreciation. The real exchange rate is therefore \( Q_t = \frac{S_t P_t}{P_t^*} \).

The risk premia are given by

\[
\Psi_{B^d} \left( b_t, \omega_t^R \right) = \exp \left\{ -\psi_B \left( \frac{b_t}{y} - \frac{b}{y} \right) \right\} \\
\Psi_{B^f} \left( b_t, \omega_t^R, \omega_t^S, \frac{R_{t-1}}{\pi_{t+1}}, \frac{Q_t}{Q_{t+1}} \pi_{t+1} \right) = \exp \left\{ -\psi_{UIP} \left( \frac{R_{t-1}}{\pi_{t+1} - \pi_{t+1}} - 1 \right) + \omega_t^R + \omega_t^S \right\}
\]

where \( b_t = B_t/P_t \) represents the real economy wide quantity of bonds issued to households.

The first risk premium function \( \Psi_{B^d} \left( b_t, \omega_t^R \right) \) is one of the methods suggested by Schmitt-Grohé and Uribe (2003) to close the model. This device ensures that the net foreign asset position does not become explosive (see Adolfson et al., 2007; Harrison and Oomen, 2010; Justiniano and Preston, 2010b; Christiano et al., 2011; Mumtaz and Theodoridis, 2012). Following the literature, the elasticity parameter \( \psi_B \) is set to a very small number minimising the contribution of this term in the overall dynamics of the model.

The second risk premium function \( \Psi_{B^f} \left( b_t, \omega_t^R, \omega_t^S, \frac{R_{t-1}}{\pi_{t+1}}, \frac{Q_t}{Q_{t+1}} \pi_{t+1} \right) \) is not a standard modelling feature and it has been added to capture persistent deviations of the exchange from the UIP condition. Our UIP modification, similar to that used for the KITT model (Lees
This modification is motivated by the observation that standard UIP implies a purely forward-looking exchange rate. With this structure, the response of the exchange rate is in general too large to economic shocks, causing the exchange rate in the model to be more volatile than in the data. In our modification, the parameter \( \psi_{uip} \) controls for the smoothness of exchange rate movements, by making the contemporaneous exchange rate depend on the past values of exchange rates and interest rates. In the empirical section, we evaluate whether the New Zealand data support non-zero values of this parameter.

2.3 Domestic producers

There is a continuum of monopolistically competitive firms producing domestic goods (indexed by \( i \)). Domestic firms employ labour \( (n_t(i)) \) and are subject to both permanent \((A_t)\) and temporary \((\omega_t^D)\) TFP shocks. The production function is given by

\[
y_t(i) = \omega_t^D A_t n_t(i)^\alpha - F_D A_t
\]

where \( F_D \) is a fixed cost associated with the production process.\(^9\)

Firms maximise their discounted future stream of profits taking into account both production and menu costs,

\[
\max_{P_{D,t}(i)} E_t \left[ \sum_{s=0}^{\infty} \Lambda_{t+s} \frac{P_{D,t+s}}{r_{D,t+s}} \left( \frac{P_{D,t+s}(i)}{r_{D,t+s}} - m_{CD,t+s} - \Psi_{PD} \left( \frac{P_{D,t+s}(i)}{r_{D,t+s}} \right) y_{t+s}(i) \right) - F_D A_{t+s} \right]
\]

where \( \Lambda_{t+s} = \beta^s \omega_{t+s}^D \lambda_{t+s} \omega_t^D A_t \) is the stochastic discount factor with \( \lambda_t \) representing the marginal utility of consumption, \( P_{D,t} \) is the domestic price level and \( m_{CD,t} \) is the (real) marginal cost of domestic firms.\(^10\)

The price adjustment cost for firm \( i \) is measured in terms of output and is given by

\[
\Psi_{PD} \left( \frac{P_{D,t}(i)}{P_{D,t-1}(i)} \right) = \psi_{D} \left( \frac{P_{D,t}(i)}{P_{D,t-1}(i)} (\pi_D)^{1-\gamma_D} (\frac{P_{D,t-1}}{r_{D,t-2}})^{\gamma_D} - 1 \right)^2 y_t
\]

Firms sell differentiated goods to the final output aggregator with firm \( i \) facing demand

\[
y_t(i) = \left( \frac{P_{D,t}(i)}{P_{D,t}} \right)^{\lambda_{D,t}} y_t
\]

where

\[
\lambda_{D,t} = (1 - \rho_{\lambda_D}) \lambda_D + \rho_{\lambda_D} \lambda_{D,t-1} + \sigma_{\lambda_D} \omega_t^D
\]

\(^9\)The fixed cost is set such that in steady state firms make zero economic profits

\(^{10}\)The marginal utility of consumption is obtained from the first order conditions of the households’ utility maximisation problem.
is a stochastic process that describes the evolution of domestic firms’ markups.

The combined quantity of output is given by the CES aggregator

\[ y_t = \left[ \int_0^1 y_t(i) \frac{1}{\lambda_D,t} \, di \right]^{\lambda_D,t} \]

### 2.4 Imports

The tradable and domestic sectors are modelled in similar ways. The tradable aggregation firm combines differentiated tradable goods which are produced from imports. Imports are differentiated by intermediate producers (retailers) that also incur some (domestic) distribution costs. Let \( y_{T,t}(\kappa) \) be the tradable output produced by retailer \( \kappa \). The final import good is a CES composite of individual retail goods

\[ y_{T,t} = \left[ \int_0^1 y_{T,t}(\kappa) \frac{1}{\lambda_T,t} \, d\kappa \right]^{\lambda_T,t} \]

Cost minimisation implies that each retailer faces an isoelastic demand for its output, given by

\[ y_{T,t}(\kappa) = \left( \frac{P_{T,t}(\kappa)}{P_{T,t}} \right)^{\frac{\lambda_T,t}{1-\gamma_T}} y_{T,t}(\kappa) \]

where \( P_{T,t} \) is the tradable price level. Each retailer faces a quadratic cost of adjusting prices measured in terms of the differentiated import good

\[
\Psi_{P_T} \left( \frac{P_{T,t}(\kappa)}{P_{T,t-1}(\kappa)} \right) = \frac{\psi_T}{2} \left( \frac{P_{T,t}(\kappa)}{P_{T,t-1}(\kappa)} \right)^{1-\gamma_T} \left( \frac{P_{T,t-1}(\kappa)}{P_{T,t-2}} \right)^{\gamma_T} - 1 \right)^2 y_{T,t}(\kappa)
\]

and solves the following maximisation problem

\[
\max_{P_{T,t}(\kappa)} E_t \left[ \sum_{s=0}^{\infty} \Lambda_{t+s} \frac{P_{T,t+s}(\kappa)}{P_{T,t+s}} \left( \left( \frac{P_{T,t+s}(\kappa)}{P_{T,t+s-1}(\kappa)} - m_{CT,t+s} - \Psi_{P_T} \left( \frac{P_{T,t+s}(\kappa)}{P_{T,t+s-1}(\kappa)} \right) \right) y_{T,t+s}(\kappa) - \varsigma T A_{t+s} \right) \right]
\]

where \( m_{CT,t} \) is the marginal cost faced by the tradable firm. Note that \( \varsigma T \) is a parameter that represents a tax whose value is set to ensure that the tradable firm makes zero profits in the steady state.

Each firm’s production function is given by

\[
y_{T,t}(\kappa) = \left[ \frac{1}{\tau} m_t(\kappa) \frac{\sigma_T-1}{\sigma_T} + (1 - \tau) \frac{1}{\gamma_T} \left( y_{T,t}(\kappa) \right)^{\frac{\sigma_T-1}{\sigma_T}} \right]^{\frac{\sigma_T}{\gamma_T}} \]

where \( m_t \) are imports, \( y_{D,t}^T \) represents the amount of domestic production used for the distribution of imports and \( \tau \) determines the share of imports in the production of tradable goods.
2.5 Exports

There is a continuum of exporting firms indexed by \( h \) on the unit interval. Each firm \( h \) buys a homogenous domestic good from a domestic retailer for the price \( P_{D,t} \) and differentiates it. They then sell these differentiated goods to the foreign economy, facing the following demand schedule,

\[
C_{X,t} (h) = \left( \frac{P^*_{X,t} (h)}{P^*_{X,t}} \right)^{\lambda_{X,t} - 1} C_{X,t}
\]

where \( P^*_{X,t} \) is the price of the export goods in foreign currency and \( C_{X,t} \) is the quantity of exports sold.

Similar to Gali and Monacelli (2005), Justiniano and Preston (2010b) and Adolfson et al. (2007), we assume that the foreign demand for exports is given by

\[
C_{X,t} = \varpi \left( \frac{P^*_{X,t} P^*_{t}}{P^*_{t}} \right)^{-v_f} y^*_t
\]

where \( y^*_t \) and \( P^*_{t} \) denote the foreign demand and foreign price level respectively.

Each exporter faces a quadratic cost of adjusting their prices as set in the export market currency. The cost for firm \( h \) is proportional to the volume of exports, with the cost per unit given by

\[
\Psi_{P_X} \left( \frac{P^*_{X,t} (h)}{P^*_{X,t-1} (h)} \right) = \frac{\psi_X}{2} \left( \frac{P^*_{X,t} (h)}{P^*_{X,t-1} (h)} \pi_X (\frac{P^*_{X,t-1}}{P^*_{X,t-2}})^{1-\gamma_X} - 1 \right)^2
\]

and the firm then selects its price in order to maximise lifetime profits

\[
\max_{P^*_{X,t} (h)} E_t \left[ \sum_{s=0}^{\infty} \Lambda_{t+s} P^*_{X,t+s} \left( \left( \frac{P^*_{X,t+s} (h)}{P^*_{X,t+s-1} (h)} \right) - mc_{X,t+s} - \Psi_{P_X} \left( \frac{P^*_{X,t+s} (h)}{P^*_{X,t+s-1} (h)} \right) \right) C_{X,t+s} (h) - \varsigma X A_{t+s} \right]
\]

where \( mc_{X,t} \) is the marginal cost faced by the export firm.

2.6 Net foreign asset position

The evolution of foreign debt \( \left( B^f_t \right) \) is given by

\[
B^f_t = R^f_{t-1} \Psi_{B^f} (\cdot) B^f_{t-1} + P^*_{X,t} C_{X,t} - P^*_{M,t} m_t
\]

where \( P^*_{M,t} \) is the world price of imports in foreign currency.

2.7 Policy

In the model, the central bank sets the nominal interest rate. At this stage, we assume the interest rate decisions of the central bank are guided by a policy rule rather than by
optimal policy, which would seek to maximise a model-based welfare criterion. The policy rule (in log deviations from steady state) is given by

\[ \hat{r}_t = \rho r \hat{r}_{t-1} + (1 - \rho r) [\phi_x E_t \hat{\pi}_{t+1} + \phi_y \hat{y}_t + \phi_{\Delta y} \Delta \hat{y}_t + \omega_t] \]

where \( y_t \) is the output gap, \( \pi_t \) is CPI inflation and hats indicate log deviations from steady state.

This specification is a generalised Taylor rule. The parameter \( \rho_r \) governs the degree of interest rate smoothing. The monetary authority is assumed to respond to inflation expectations at time \( t+1 \) and the long run response to inflation is measured by the parameter \( \phi_x \). The policy rule also allows the interest rate to respond to movements in the level and growth rate of the output gap. Our definition of the output gap is derived from the model. It is defined as the difference between actual output and the level of output that would prevail in the absence of nominal rigidities. Finally, \( \omega_t \) is a monetary policy shock representing deviations from the monetary policy rule.

The particular choice of the policy rule is motivated by our desire to have an empirical rule that describes the RBNZ’s underlying policy rule. The RBNZ is tasked with ensuring price stability in New Zealand. In particular, the Policy Targets Agreement between the Minister of Finance and the Governor of the RBNZ requires the Reserve Bank to “keep future CPI inflation outcomes between 1 per cent and 3 per cent on average over the medium term, with a focus on keeping future average inflation near the 2 per cent target midpoint” and “in pursuing its price stability objective, the Bank shall implement monetary policy in a sustainable, consistent and transparent manner, have regard to the efficiency and soundness of the financial system, and seek to avoid unnecessary instability in output, interest rates and the exchange rate.”

Given the price stability objective of the RBNZ, an option would be to specify the interest rate rule as a function of inflation expectations alone. However, the model-based expectations may not fully capture the RBNZ’s forecasts of inflation at that point in time. Instead by allowing for additional terms in the policy rule, we aim to capture the effects of overall economic movements (both directly and via households’ and firms’ inflation expectations) on the RBNZ’s interest rate discussions. Moreover, a more generic rule may also reduce the effect of model misspecification in estimating the model’s structural parameters.

Fiscal policy is represented very simply in the model; the government collects \((\varsigma^X \text{ and } \varsigma^T)\), issues new debt to the financial intermediary \((B_t^d)\) and finances government spending \((g_t)\).

\[11 \text{ The full PTA can be accessed from http://www.rbnz.govt.nz/monetary_policy/policy_targets_agreement/}. \]
Lump sum taxes \( (T_t) \) adjust to ensure that the government’s budget is always balanced (i.e. \( B^d_t = 0 \)).

2.8 Log-linear model

The dynamics of the model are fully characterised by the first order conditions from the optimisation problems presented in this section together with the market clearing conditions. The presence of permanent TFP shocks introduces a stochastic trend in real variables. We first write the model in terms of stationary variables and solve for the deterministic steady state. We then log-linearise the model around this steady state. The resulting log-linear equations are summarised in Table 1. These equations are used in the following empirical sections.

3 Empirical analysis

In this section, the model’s parameters are estimated on New Zealand economic data. This section discusses the Bayesian estimation procedure, the data and priors used in estimation, the empirical fit and a selection of the resulting impulse responses.

3.1 Data and estimation

We use Bayesian techniques to estimate the model’s parameters.\(^\text{12}\) Our estimation uses quarterly data for New Zealand from the period 1993Q2 to 2013Q1, which covers most of New Zealand’s inflation targeting era. Over this sample the monetary regime has remained largely unchanged and has focused on price stability. Although the monetary policy mandate has emphasised price stability since 1989, actual price stability was not achieved until 1992. We select 1993Q2 as the starting point of our sample as we believe that by this point the RBNZ had credibly convinced the public of its commitment to price stability.

The data series that we observe are: output growth, consumption growth, export growth, the nominal interest rate, nominal wage inflation, headline inflation, domestic inflation, nominal exchange rate growth and terms of trade growth.\(^\text{13}\) Output is defined as real gross domestic product (GDP). Consumption and exports are in real terms and come from the national accounts. Our interest rate measure is the New Zealand 90-day bank bill rate. Nominal wage inflation is constructed as the percent change in average hourly

---

\(^{12}\)We do not describe the details of the estimation approach here. Further details are available in An and Schorfheide (2007) and Del Negro and Schorfheide (2011).

\(^{13}\)All growth and inflation rates are represented as quarterly percent changes.
earnings (ordinary time, private sector) from the Quarterly Employment Survey. Headline and domestic inflation are percent change in headline consumer price index and in non-tradable goods prices, both excluding the goods and services tax (GST) changes in 2010. The nominal exchange rate is the New Zealand dollar trade weighted index and, lastly, the terms of trade is the ratio of export to import prices from the Overseas Trade Index (OTI). Output, consumption and exports data are transformed to per capita terms using working age population from the Household Labour Force Survey. All the data are seasonally adjusted, except the exchange rate and the interest rate. Interest rate and exchange rate data are sourced from the RBNZ. All other data are produced by Statistics New Zealand.

The measurement equations that relate the model variables to the observable variables are outlined in table 2. Figure 1 plots all the data that we use in the estimation. Note that our observable set of variables does not include a measure of demand for New Zealand’s exports. This is motivated by the fact that it is hard to construct a measure reflecting the total external demand faced by the New Zealand economy. In particular, our model places a tight constraint on the relationship between foreign demand and total exports. However, typical measures of foreign demand, such as trade weighted aggregates of trading partner’s GDP, are only weakly correlated with export data. The same observation applies to measures of foreign interest rates and prices. This is one possible reason why estimated small open economy models typically predict a low share of foreign shocks in the variance decompositions for domestic variables (Justiniano and Preston, 2010a). Our strategy consists of treating foreign variables as latent variables and inferring their dynamics using information from domestic variables, the exchange rate and the terms of trade.

We estimate the posterior mode, by numerically maximising the log posterior density function (a combination of prior information and the likelihood of the data). Then, we estimate the full posterior distribution using the Metropolis Hastings algorithm. In the remaining part of this section, we discuss the calibration and choice of priors for the estimated parameters.

Before estimating, we calibrate some parameters to be consistent with the mean values in the data. We set the discount factor \( \beta \) to 0.99, which gives an annual steady state real interest rate around 4 per cent. We assume households work for 20 percent of their allocated time in the steady state, which approximately corresponds to the ratio of average weekly hours worked to total hours in a week. The share of tradable goods in the consumption basket \( \theta_C \) is assumed to be the share of tradable goods in the CPI, which is approximately 0.44 in New Zealand. The share of imported goods in the production of
tradable goods ($\tau$) is set to 0.8 to match the average share of total imports over GDP. Following Schmitt-Grohé and Uribe (2003), we ensure the stationarity of the model by setting the elasticity of the risk premium to the level of debt ($\Psi_B$) to 0.001.

The model allows for a rich structure with many frictions. However, we do not use all features in our baseline version. We set the intertemporal elasticity of substitution parameter to 1 and the preference shifter to 0, collapsing the consumer preferences to the GHH utility function. Further, we turn off habit formation in hours worked. We relax these assumptions and discuss the results in subsection 3.2.

**Priors**

The third column of table 3 details the priors used in estimation of the model’s parameters. We discuss a subset of these priors below.

We assume the consumption habit parameter comes from a beta distribution, with a mean of 0.4 and a standard deviation 0.05. Our prior mean is lower than similar estimated models for other economies. This low value is motivated by the fact that the persistence of consumption growth in New Zealand is much lower than in many other countries. Several authors who estimated DSGE models on NZ data (such as Justiniano and Preston, 2010a) have found low values for habit persistence. Obviously, our choice of prior has some effect on the posterior distribution. We have experimented by setting the mean of the prior for the habit persistence to a higher value. That left us, however, with unrealistically long delays in the transmission of transitory nominal shocks.

On the price setting side, we estimate the inverse of the price adjustment cost parameters. The priors for the inverse of these parameters are from a gamma distribution, with means of 1/200 and standard deviations of 1/800. It is not straightforward to provide an economic interpretation of the size of this parameter. But, given the other calibrated parameters, the equivalent of this parameter in a Calvo (1983) type price setting framework would be 0.8, implying that prices are changed on average every 5 quarters.

We have three parameters governing the demand elasticities in the model: the consumer’s elasticity of substitution between domestic and tradable goods ($\eta$), tradable firms’ elasticity of substitution between domestic and imports ($\eta_T$) and the export demand elasticity.

---

14 The shares of tradable output in consumption and consumption share of GDP imply that an import share of 0.8 is appropriate to deliver New Zealand’s observed import share of GDP.

15 The standard deviation for the prior on the tradable adjustment cost parameter is slightly larger than the other parameters (1/600). This prior for tradable adjustment costs was chosen because the volatility of tradable inflation is considerably smaller than the volatility of import prices, implying larger nominal rigidities.
There is no direct evidence on what these elasticities should be at the macroeconomic level in New Zealand. We impose diffuse priors for $\eta$, $\eta_T$ and $\eta_X$ each centered on a mean of 1.5. These demand elasticity priors come from a gamma distribution.

We centre our prior for the parameter governing the interest rate response to inflation movements to 2. We assume that the priors for the other parameters in the interest rate equation are normally distributed with a mean of 0.15 and standard deviation 0.1. Finally, the prior on the interest rate smoothing parameter $\rho_r$ is beta distributed with mean 0.75 and standard deviation 0.05.

All the exogenous disturbances follow independent AR(1) processes. The prior means for the persistence of shocks are set to 0.5. The priors for standard deviations of the shocks are from the inverse gamma distribution with a mean of 0.1 with the exception of the permanent TFP shock. Our prior lowers the standard deviation of this shock so that trend movements in GDP, consumption and wages are smooth.

**Posterior inference**

The last two columns in table 3 present the posterior mean and 90% probability intervals for the estimated parameters. We have also plotted the posterior distributions along with the prior distributions in figure 2. We test whether the data is informative for our parameter estimates using the methodology of Koop et al. (2011). Table 5 shows the results from this test. Most of our structural parameters are identified by the data, though most of the shock persistence and volatility parameters are not well identified.

We find that the largest nominal rigidities are in the tradable goods sector. While the adjustment cost parameter for the tradable sector is considerably higher than all other sectors, in terms of Calvo equivalents, tradable prices change approximately as often as wages (once every 3 years). This rigidity in tradable goods prices reconciles the fact that import prices are volatile (as a result of exchange rate movements) but tradable prices respond only moderately. Domestic and export prices appear to be adjusted on average every 0.75 to 1.25 years.

According to these estimates, substitution between imports and domestic production appears to largely occur at the production level with the import substitution elasticity ($\eta_T$) estimated to be 1.05. At the retail level, tradable and domestic goods appear to be complements ($\eta = 0.81$).

In terms of the policy rule, our parameter estimates suggest that interest rates have a substantial degree of persistence. The posterior mean of the interest rate smoothing
In the remainder of this section, we focus on the empirical performance of the model.

3.2 Empirical fit

We consider two metrics regarding the empirical fit of the model – second order moments, and marginal likelihood comparisons.

Second order moments

An intuitive method to evaluate a model’s empirical fit is to compare second order moments (the cross-correlations and standard deviations) of the data to those implied by the model. Table 6 presents the standard deviations of the observed variables and figure 3 displays their cross-correlations with up to eight lags. Model based moments are computed by taking 1000 draws from the posterior parameter estimates and for each draw simulating 100 artificial datasets composed of 84 observations – the length of our sample – for each variable. In these results, we report the median and 90 per cent probability intervals of both the model and data.

There is a reasonable overlap between the empirical and model-implied confidence intervals for the standard deviations of the observable data. Although the model somewhat overestimates the volatilities of some variables (particularly consumption growth), overall the model is able to replicate the second moments of the data well. In addition, the estimated model is able to reproduce the rankings of the volatilities. The model-implied standard deviations for all the inflation variables are lower than output, and wage inflation is more volatile than CPI and non-tradable inflation. Furthermore, the volatilities of the interest rate, export growth and the exchange rate are all higher than the volatility of output, which is in line with the data.

The model performs similarly to a BVAR in replicating the cross-correlation in the data. The model is not able to capture all the cross correlations in the data, but it does capture most of the reduced form lead-lag dynamics in the data.
Marginal likelihoods

An alternative method of evaluating the model fit is to formally evaluate the marginal likelihood of the model given the observed data. Unless used to compare models, marginal likelihoods hold little information, so we examine which of the model’s features are important by removing a selection of features one at a time and re-estimating the model. We consider the importance of features included within the baseline such as removing habit persistence in consumption ($\chi_c$), the UIP smoother ($\psi_{uip}$), wage adjustment costs, domestic goods price adjustment costs and the tradable goods price adjustment costs. In addition, we consider adding features that are excluded from the baseline such as estimating habit persistence in hours worked ($\chi_n$) and allowing the preference shifter ($v$) to be different from 0. In table 7 we present the posterior modes for each parameter when turning on and off different features of our model. We also show the marginal likelihoods for each model including the baseline with larger numbers indicating improved empirical fit. For comparison, the log of the marginal likelihood of the baseline model is -854.

We find that habit persistence in consumption is important to fit the data well. When we set $\chi_c = 0$, we find that the log of the marginal likelihood falls to -869. This suggests that consumption and GDP tend to respond to shocks in a hump-shaped or gradual manner. This is consistent with previous studies of consumption and GDP responses in New Zealand, see for example Buckle et al. (2007), Haug and Smith (2012) and Bloor and Matheson (2008). In a similar manner, the smoothing term in the UIP equation helps to fit the data. When we set $\psi_{uip} = 0$ the log marginal likelihood falls to -859. This likely occurs because the pure UIP equation struggles to capture the persistence of movements in the exchange rate.

 Tradable price rigidities dramatically improve the marginal likelihood of the model relative to rigidities in domestic goods prices or wages. This reflects the extra degree of volatility in the real marginal cost of tradable goods relative to these other sectors. Without tradable rigidities, the price of tradable goods would be extremely volatile. Any change in the exchange rate would have an immediate one-to-one effect on their prices. Hampton (2001) shows, however, that a 10 percent depreciation in the NZD exchange rate results in only a 0.5 percent rise in New Zealand tradable goods prices in the short run. This small pass-through is consistent with strong nominal rigidities in our model. For the domestic or non-tradable firms, marginal cost is determined by the output gap and the real wage. These are far less volatile and more persistent. As such, the model does not need large nominal rigidities in the domestic goods sector.
Moving to extensions to our baseline, we consider more flexible specifications of the utility function. Both allowing for habit persistence in hours worked and allowing $v$ to be different from zero increase the log marginal likelihood of the model. These extensions can be seen as an approximation to more involved interactions between consumption and labour supply decisions or labour market rigidities. As such, these results suggest that a more elaborate treatment of flows in the labour market – such as search and matching frictions – might provide more insights into the interaction between labour market outcomes and macroeconomic dynamics.

We also consider adjusting the expectations formation process to allow more persistent expectations formation. This also improves the model’s empirical fit. For brevity we examine in detail the impact of only one of these extensions – persistent inflation expectations – in section 5.

3.3 Impulse response analysis

Impulse responses are an essential tool to clarify the transmission mechanisms of the model – they describe how specific shocks will affect the model variables. Here we discuss two key impulse responses: a monetary policy shock and an exchange rate shock.

Monetary policy shock

The first shock that we consider is a monetary policy shock that temporarily raises the interest rate by 1 percent (figure 4). This shock represents an event where the central bank in the model deviates from its policy rule. In this case, economic conditions do not warrant the interest rate increase. In the charts, the solid blue line is from the posterior mode and the confidence bands show the 5th and 95th percentiles of 1000 draws from the MCMC chain. The interest rate and the inflation responses have been annualised and the exchange rate is defined so that an increase represents an appreciation.

Monetary policy achieves its targets in this model through three channels: i) the demand channel, ii) the exchange rate channel and iii) the expectations channel. Interest rates can have a powerful impact on aggregate demand in the economy through their impact on household spending decisions. In NZSIM, households select how much to consume and save based on their expectation of the entire path of interest rates. The equations presented in subsection 2 not only detail how households optimise between consumption in the current and subsequent periods, but across all future periods. Therefore, monetary policy

\[^{16}\text{In NZSIM, the term structure of interest rates reflects market participants' beliefs about short term rates.}\]
can have a powerful effect on households’ consumption, both through affecting current interest rates and influencing households’ expectations of interest rates in the future. This channel is reflected in figure 4 by the hump-shaped fall in consumption when households face higher interest rates. Because households all choose to increase their savings in the current period and there is no investment in NZSIM, domestic demand falls which lowers domestic GDP.

In addition to the demand channel, monetary policy has a considerable impact on the exchange rate. The modified UIP equation predicts large and persistent movements in the exchange rate when domestic monetary policy is adjusted relative to foreign monetary policy. The exchange rate affects inflation directly by lowering the cost of production in the tradable sector. Tradable firms face lower imported input costs and pass on some of this to households. In turn lower tradable inflation results in lower CPI inflation. In addition, the exchange rate affects economic activity through export demand and two substitution effects: a production substitution effect in the tradable sector toward imports as an input into production, and a consumption substitution effect as households choose to consume more of the tradable good. Both the lower exports and the substitution effects result in lower demand for domestic production which, as production is reduced, lowers marginal costs in the domestic sector and therefore inflation.

The expectations channel amplifies the demand and exchange rate channels. As mentioned above, households’ consumption decisions are determined by households’ expectations of future interest rates. In addition, the modified UIP arbitrage condition provides a role for future interest rate differentials to affect the current value of the exchange rate. If the central bank is able to affect these expectations, households will adjust their consumption and speculators will move the exchange rate without any need for the central bank to adjust interest rates in the current period. In NZSIM, the central bank commits to (on average) follow the policy rule specified in subsection 2.7. The particular type of interest rate rule – via these expectations – can have a strong impact on the path of inflation and output.

**Exchange rate shock**

Here we consider the impact of a 1 percent appreciation of the NZD TWI that cannot be explained by interest rate differentials (detailed in figure 5). While this shock is modelled as a change in the risk premium required to hold foreign bonds, this shock can also be thought of as representing changes in international investors’ preferences for investing in
The higher TWI reduces the NZD price of both import and exports. Lower imports prices reduce the cost of production for the import distributing firms. This decline in their marginal cost is partially passed on to households who increase their demand for tradable output and in doing so are able to increase their total consumption. While total consumption increases, the expenditure switching toward imports is sufficiently large that consumption of domestic goods falls. This contributes to a reduction in GDP. In addition, the high exchange rate reduces export receipts and as firms raise prices to contain their losses, export volumes fall. This reduction in exports leads to a further reduction in GDP. Lower GDP leads to lower domestic marginal costs and reduces domestic inflation. As tradable and non-tradable inflation are lower, the central bank reduces interest rates which slowly brings the exchange rate back to its pre-shock level.

4 Alternative policy scenarios

Although our model captures many of the key aspects of the New Zealand economy, it is not expected to fully summarise all the information available to the Reserve Bank. A key design goal of the NZSIM framework is to ensure that expert judgment or information from other models can be incorporated effectively. As discussed, the model contains many structural shocks that can act as levers to adjust the model’s output. For example, if we have off-model information about the future evolution of a particular variable, that information can be fed into the forecast by selecting a series of future shocks. In this section we provide an example – via a conditional forecast exercise – of how model output can be adjusted and how the adjustment should be interpreted. We show that, when adjusting model output, the choice of the shock or the set of shocks is crucial because the qualitative nature of each shock determines the flow-on effects onto other variables.

Consider a scenario where instead of rising gradually to neutral levels, we want to keep interest rates constant for 8 quarters. Such a scenario may have several causes. One possible reason could be that the central bank underestimates the strength of the economy and mistakenly keeps rates below where appropriate. Alternatively, if instead the central bank had advance warning that the outlook had deteriorated, it might then decide to hold rates constant. Figures [6] and [7] represent the economic impacts of these scenarios respectively.

\footnote{Using a FAVAR approach that exploits information contained in more than 300 macroeconomic data series, Karagedikli et al. (2013) investigates the effects of an exchange rate shock whose definition is very close to ours. Our estimates of the macroeconomic effects of an exchange rate shock are qualitatively and quantitatively very similar to their results.}
In the first scenario (figure 6), we impose a constant interest rate forecast using monetary policy shocks only. The monetary policy shocks here characterize setting the interest rate at lower than appropriate levels (the appropriate interest rate is shown in our ‘baseline’ scenario). The interest rate path starts diverging after mid-2013, as the baseline scenario has the interest rate increasing from then. Accordingly, the dynamics of other variables are different after that point. Consistent with the impulse responses in section 3, the persistently lower interest rates induce households to reduce their savings and increase consumption. This yields higher output and imports. To increase production, domestic firms increase their demand for labour, generating higher wages. This raises the marginal cost of production and results in higher prices. Overall, the model predicts higher output in the near term and a persistent rise in inflation.

In the second scenario we are agnostic about the specific non-monetary factors that lead to constant interest rates. Instead we apply the methodology of Waggoner and Zha (1999) and Benes et al. (2008) to select the statistically most likely – given the estimated shocks’ persistence and standard deviations – combination of non-monetary shocks to deliver the target interest rate track. We interpret this as a general deterioration in economic conditions that, consistent with the estimated policy rule, would result in the central bank leaving policy unchanged. The forecasts under this scenario are displayed in figure 7. Because we do not allow monetary policy shocks to explain the lower interest rates, inflationary pressures must considerably ease to justify the central bank maintaining low interest rates.

This example shows how the same interest rate track can be consistent with completely different economic outlooks. While the information in the scenario may be imposed on a specific economic variable, the model has several degrees of freedom to generate this adjustment. Therefore, any change to model output has to be accompanied with an economic story. Indeed, our design of the framework provides intuitive connections between the model’s fundamental drivers and most economic stories.

We use these tools to approximate economic channels missing from our framework. These tools allow us to flexibly capture missing channels when needed, e.g. in response to particular economic events that occur only infrequently. As each business cycle has its own particular drivers and these drivers are not necessarily predictable in advance, it is likely that our model may miss a subsequently necessary channel. If it transpires that a particular missing channel is repeatedly important, we will consider incorporating auxiliary relationships to systematically capture this missing channel. In the next section we present, with an example, our procedure for incorporating new channels into the framework.
5 Extensions – Adaptive inflation expectations

In this section we present an extension to the baseline model and we outline the approach we have taken to implement it. Our example focuses on the dynamics of inflation expectations. As outlined in the previous sections, the inflation dynamics are driven by forward looking Phillips curves and therefore inflation expectations are one of the key determinants of inflation dynamics in the model. Because of the importance of expectations in determining inflation, the Reserve Bank uses a range of survey measures of inflation expectations. In particular, the survey measure of firms’ two-year ahead inflation expectations has been a reliable indicator of inflationary pressure over the last two decades (See figure 8). However, one of the key features of this measure of inflation expectations is its persistence – more so than predicted by our model – and its slow movement over the business cycle (Figure 9). This is despite our baseline model allowing features that would increase the persistence of inflation and – in a model consistent manner – inflation expectations.

All else constant, persistent inflation expectations make the process of inflation stabilization more difficult. For example, if expectations are elevated and intrinsically persistent, then negative output gaps are required for an extended period to bring inflation to its target. If our baseline model assumes an insufficiently persistent expectations process, monetary policy might not be sufficiently responsive to stabilize inflation at the desired rate.

There exist several structural explanations for why we might not be capturing the persistence of inflation expectations – such as bounded rationality, learning dynamics or imperfect credibility. However, introducing such mechanisms into our model would add considerable complexity. Our strategy is to achieve a similar outcome by assuming a flexible linear inflation expectations formation process that also nests our baseline specification. Using the same dataset in figure 1, we re-estimate the model with this additional feature. We then compare the two models’ empirical fit to the data. In addition, we compare whether the implied inflation expectations in this variant are closer to the survey measures.

The particular expectations formation process we adopt is

\[
\pi_t^e = \rho_{pe1} \pi_{t-1}^e + (1 - \rho_{pe1})[\rho_{pe2} E_t \pi_{t+1} + (1 - \rho_{pe2}) \pi_{t-1} ]
\]

\(\rho_{pe1}\) is the weight on past inflation expectations and \(\rho_{pe2}\) is the relative weight on rational expectations and last period’s observed inflation. When \(\rho_{pe1} = 0\) and \(\rho_{pe2} = 1\) expectations
are rational, as in the baseline model. Expectations of domestic, tradable, and wage inflation all follow this process with sector-specific expectations variables, but the same values for $\rho_{1}^{pe}$ and $\rho_{2}^{pe}$.

Our priors for $\rho_{1}^{pe}$ and $\rho_{2}^{pe}$ are beta distributions with a mean of 0.5 and a standard deviation of 0.1. We find that the posterior mode for $\rho_{1}^{pe}$ is around 0.9 and for $\rho_{2}^{pe}$ is around 0.5. This produces markedly more persistent model-implied inflation expectations - as shown in figure 9. Indeed, the marginal likelihood of the adaptive expectations version of the model is higher than the baseline (-804 versus -854), suggesting that allowing persistence in inflation expectations considerably improves the in-sample fit of the model. One caveat, however, is that the estimate of the weight on rational versus backward looking expectations ($\rho_{2}^{pe}$) is not well identified.

Figure 10 compares the impulse responses of the baseline model and the adaptive expectations extension to a monetary policy shock. Real variables and interest rates show similar dynamics, with price variables – particularly domestic inflation – being markedly more persistent. This increase in persistence occurs because the adaptive expectations change the Phillips curve. These changes emphasise past expectations of future marginal costs rather than simply future marginal costs. This makes the responses of prices and wages more sluggish.

Overall, we see this specification as a useful first step towards improving the modeling of inflation expectations in New Zealand and in small open economy models. This specification is now an integral part of our policy framework.

6 Conclusion

We have outlined the underlying structure of NZSIM and our process for settling on its design. The model is a parsimonious, structural small open economy model driven by a number of economic shocks. We augmented its structure with simplistic relationships to widen its coverage while keeping its size manageable. We apply rigorous assessment criteria to ensure that our extensions improve the model’s empirical performance and behave in a theoretically desirable manner.

---

\[^{19}\text{Hagedorn (2011) uses a similar approach. He finds that adaptive expectations reduce the strength of the expectations channel when implementing a disinflationary policy. As such, the central bank in the model has to be more proactive with interest rates in order to achieve its inflation target. As described in Milani (2012), there are many alternative formal ways of introducing departures from rational expectation hypothesis into structural dynamic macroeconomic models.}\]

\[^{20}\text{We adjust the priors for } \Psi_{D}, \Psi_{T} \text{ and } \Psi_{W} \text{ to a mean of 100 and standard deviation of 20. With adaptive expectations adding persistence to inflation, price adjustment costs do not need to be as high.}\]
Our model is able to match the salient features of New Zealand macroeconomic data relatively successfully. The model produces similar volatilities and correlations to those in the data. No model is able to include all the factors that might be relevant for constructing a macroeconomic forecast. Our model, however, is rich enough to implement alternative economic and policy scenarios. We have illustrated an example of this by introducing off-model information into the model forecast.

The model presented here is a snapshot of current macroeconomic modelling at the RBNZ. We continue to consider alternative models and additional extensions when creating forecasts and alternative policy scenarios.

References


Figure 1: Data used to estimate the model

Note: This figure details the data used in the estimation of NZSIM. For details on how to obtain this data refer to subsection 3.1.
Figure 2: Prior and posterior distribution of the estimated parameters

Note: This figure shows the prior (grey distribution), the estimate of the posterior mode (green) and the posterior distribution (black) from 500,000 draws from a Markov chain Monte Carlo procedure.
Note: This figure shows the prior (grey distribution), the estimate of the posterior mode (green) and the posterior distribution (black) from 500,000 draws from a Markov chain Monte Carlo procedure.
Figure 3: Correlations

Note: This figure shows the autocorrelations and cross correlations of NZSIM, a BVAR and the data. Diagonal plots show autocorrelations of variables and the off-diagonal plots show the correlations of the variables labelled in the rows against the k-th lag of the variables labelled in the columns.
Figure 4: Monetary policy shock

Note: This figure shows the impulse responses from a 1% annualised monetary policy shock. The bands come from 1000 draws randomly selected from the estimated MCMC draws. The thick blue line comes from a single model with parameters from the estimate of the posterior mean.

Figure 5: Exchange rate shock

Note: This figure shows the impulse responses from a 1% exchange rate shock. The bands come from 1000 draws randomly selected from the estimated MCMC draws. The thick blue line comes from a single model with parameters from the estimate of the posterior mean.
Figure 6: Alternative interest rate scenario - monetary policy shock

Note: This figure compares the model based forecast to an alternative scenario where interest rates are held constant. Refer to section 4 for details.

Figure 7: Alternative interest rate scenario - multiple shocks

Note: This figure compares the model based forecast to an alternative scenario where interest rates are held constant. Refer to section 4 for details.
Figure 8: CPI inflation and the RBNZ 2 year ahead inflation expectation survey measure (in annual percent change)

Figure 9: CPI inflation expectations (2 year ahead, annual percent change)

Note: This figure compares two-year ahead inflation expectations in the baseline model to those from the extended model in section 5. The 90% confidence bands represent uncertainty around Kalman filter estimates of the model based expectations.
Figure 10: Responses to a 1 percent monetary policy shock

Note: This figure shows the impulse responses to a 1% expansionary monetary policy shock for two models: the baseline model and the extended model detailed in section 5.
8 Tables

Table 1: Linearized model equations

The equations shown below represent the linearized model equations for various economic variables. The text includes various economic terms and equations, which are typically used in macroeconomic models to analyze the effects of different shocks on the economy.

The consumption equation, for example, is shown as:
\[ c_t = \frac{y_t}{1+r} (c_{t-1} - \Delta \tilde{y}_t) + \frac{1}{1+r} \left[ \frac{\omega_{t+1}^{a} + \alpha (1+\chi) \Delta E_t \tilde{y}_{t+1}^{a} - (1-\chi) (1+\omega_{t+1}^{a} + \alpha (1+\chi) \Delta E_t \tilde{y}_{t+1}^{a})/\sigma_{t+1}^{a}}{\sigma_{t+1}^{a}} \right] (\tilde{r}_t^{a} - E_t \tilde{r}_{t+1}^{a}) + \frac{(1-\sigma_{t+1}^{a}) (1-\omega_{t+1}^{a} + \alpha (1+\chi) \Delta E_t \tilde{y}_{t+1}^{a})/\sigma_{t+1}^{a}}{\sigma_{t+1}^{a}} \tilde{c}_t^{a} \]

The wage Phillips curve is represented as:
\[ \pi_t^{w} = \frac{\gamma}{1+\gamma} \pi_{t-1}^{w} + \frac{\delta}{1+\gamma} \pi_{t+1}^{w} + \frac{1}{1+\gamma} \left[ \frac{1}{\gamma - (1+\gamma)(\lambda^{w} + \hat{\lambda})} \right] (\tilde{m}_t - \tilde{y}_t + \hat{\lambda}) \]

Imports demand is given by:
\[ \tilde{m}_t = -\eta q_{t+1} - \eta \pi_t (1+\tau) (\tilde{p}_{m,t} - \tilde{p}_{D,t}) \]

Domestic production is defined as:
\[ \tilde{y}_{t+1} = \frac{\lambda}{1+\gamma} \tilde{y}_t + \frac{1}{1+\gamma} \left[ \frac{1}{\gamma - (1+\gamma)(\lambda^{w} + \hat{\lambda})} \right] \tilde{m}_t - \tilde{y}_t + \hat{\lambda} \]

Marginal cost of tradable is:
\[ \tau_{t+1} = \frac{\gamma}{1+\gamma} \pi_{t-1}^{w} + \frac{\delta}{1+\gamma} \pi_{t+1}^{w} + \frac{1}{1+\gamma} \left[ \frac{1}{\gamma - (1+\gamma)(\lambda^{w} + \hat{\lambda})} \right] (\tilde{m}_t - \tilde{y}_t + \hat{\lambda}) \]

Exports demand is:
\[ \tilde{e}_{t+1} = \tilde{e}_{t} - \tilde{e}_{t-1} - \tilde{e}_{t+1} = \tilde{q}_{t} - \tilde{q}_{t+1} - \psi_{t} \tilde{r}_{t} - \tilde{r}_{t} - \tilde{q}_{t+1} - \tilde{r}_{t+1} - \tilde{q}_{t} - \tilde{r}_{t} + \psi_{t+1} + \omega_t \]

Balance of payments is defined as:
\[ \hat{b}_t = \frac{\lambda}{1+\gamma} \hat{b}_t + \frac{1}{1+\gamma} \tilde{y}_t \]

Utility function is:
\[ \hat{y}_t = \frac{\gamma}{1+\gamma} \hat{y}_t + \frac{1}{1+\gamma} \tilde{y}_t - \frac{1}{1+\gamma} \tilde{m}_t \]

U/P is represented as:
\[ \hat{r}_t = E_t \pi_{t+1}^{a} - \hat{r}_t - \hat{r}_{t-1} = \hat{q}_t - \hat{q}_{t+1} - \psi_{t} \tilde{r}_{t} - \tilde{r}_{t} - \hat{r}_{t+1} - \hat{r}_{t-1} + \psi_{t} \tilde{r}_{t} - \tilde{r}_{t} - \hat{r}_{t+1} - \hat{r}_{t-1} + \omega_t \]

Central Bank policy rule is:
\[ \hat{r}_t = \rho_t \hat{r}_{t-1} + (1 - \rho_t) \left[ \phi_{t} E_t \pi_{t+1}^{a} + \phi_{t} \tilde{y}_t + \phi_{t} \Delta \tilde{y}_t + \omega_t \right] \]

All variables are log-deviations from steady state. Real variables are in per capita terms. $r$ = nominal interest rate, $\pi^D$ = domestic inflation, $\pi^w$ = nominal wage inflation, $\pi^T$ = tradable inflation, $\omega$ = real wage, $q$ = real exchange rate, $p_D$ = relative domestic prices, $p_T$ = relative tradable prices, $b$ = foreign debt, $c$ = consumption, $y$ = GDP, $m$ = imports, $g$ = government, $\pi^{rs}$ = marginal rate of substitution, $c_X$ = exports.

$\rho_t$ = labour, $\hat{a}$ = total factor productivity, $p_X$ = export prices, $P_M$ = wholesale import prices, $\gamma$ = foreign GDP, $\omega^D$ = temporary productivity changes, $X$ = mark-up shocks, $\omega$ = other shocks.$E_t$ preceding variables represent expectations taken at time $t$. 

36
Table 2: Measurement equations

<table>
<thead>
<tr>
<th>Observable</th>
<th>Model variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output growth</td>
<td>$y_t - y_{t-1} + \Delta a_t$</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>$c_t - c_{t-1} + \Delta a_t$</td>
</tr>
<tr>
<td>Export growth</td>
<td>$x_t - x_{t-1} + \Delta a_t$</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$4 \times (r_t + r + \pi + \Delta a_t (\sigma - 1) + 100(1/\beta - 1))$</td>
</tr>
<tr>
<td>Wage inflation</td>
<td>$\pi^w_t + \pi^w + \pi + \Delta a_t$</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>$\pi_t + \pi$</td>
</tr>
<tr>
<td>Domestic inflation</td>
<td>$\pi_{D,t} + \pi_D$</td>
</tr>
<tr>
<td>Change in the exchange rate</td>
<td>$q_t - q_{t-1} - \pi_t + \pi^*_t$</td>
</tr>
<tr>
<td>Growth in the terms of trade</td>
<td>$\pi^<em>_t - \pi^</em>_{m,t} + \bar{tot}$</td>
</tr>
</tbody>
</table>
Table 3: Prior and Posterior distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Prior</th>
<th>Posterior mean</th>
<th>Prob. interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity Growth</td>
<td>$\Delta tfp$</td>
<td>$\Gamma(0.40, 0.10)$</td>
<td>0.29</td>
<td>[0.23, 0.36]</td>
</tr>
<tr>
<td>Consumption Habit Parameter</td>
<td>$\chi_c$</td>
<td>$\beta(0.40, 0.05)$</td>
<td>0.50</td>
<td>[0.44, 0.56]</td>
</tr>
<tr>
<td>Domestic Sector Indexation Parameter</td>
<td>$\gamma_D$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.37</td>
<td>[0.23, 0.51]</td>
</tr>
<tr>
<td>Tradable Sector Indexation Parameter</td>
<td>$\gamma_T$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.25</td>
<td>[0.14, 0.35]</td>
</tr>
<tr>
<td>Wage Indexation Parameter</td>
<td>$\gamma_W$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.25</td>
<td>[0.15, 0.36]</td>
</tr>
<tr>
<td>Export Sector Indexation Parameter</td>
<td>$\gamma_X$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.48</td>
<td>[0.33, 0.64]</td>
</tr>
<tr>
<td>Tradable Price Elasticity of Demand</td>
<td>$\eta$</td>
<td>$\Gamma(1.50, 1.00)$</td>
<td>0.81</td>
<td>[0.28, 1.33]</td>
</tr>
<tr>
<td>Import Price Elasticity of Demand</td>
<td>$\eta_T$</td>
<td>$\Gamma(1.50, 1.00)$</td>
<td>1.05</td>
<td>[0.51, 1.58]</td>
</tr>
<tr>
<td>Export Price Elasticity of Demand</td>
<td>$\eta_X$</td>
<td>$\Gamma(1.50, 1.00)$</td>
<td>0.30</td>
<td>[0.11, 0.49]</td>
</tr>
<tr>
<td>Central Bank Output Growth Parameter</td>
<td>$\phi_{\Delta y}$</td>
<td>$N(0.15, 0.10)$</td>
<td>0.14</td>
<td>[-0.02, 0.29]</td>
</tr>
<tr>
<td>Central Bank Inflation Parameter</td>
<td>$\phi_y$</td>
<td>$\Gamma(2.00, 0.10)$</td>
<td>1.97</td>
<td>[1.80, 2.13]</td>
</tr>
<tr>
<td>Central Bank Output Parameter</td>
<td>$\phi_y$</td>
<td>$N(0.15, 0.10)$</td>
<td>0.20</td>
<td>[0.09, 0.32]</td>
</tr>
<tr>
<td>Domestic Price Adjustment Cost</td>
<td>$\frac{1}{\sigma_{\nu}}$</td>
<td>$\Gamma(\frac{3}{200}, \frac{500}{200})$</td>
<td>1.77</td>
<td>[1.77, 1.77]</td>
</tr>
<tr>
<td>Tradable Sector Price Adjustment Cost</td>
<td>$\frac{1}{\sigma_{\nu}^*}$</td>
<td>$\Gamma(\frac{1}{200}, \frac{500}{200})$</td>
<td>2.76</td>
<td>[1.07, 2.11]</td>
</tr>
<tr>
<td>Wage Adjustment Cost</td>
<td>$\frac{1}{\sigma_{\nu}^*}$</td>
<td>$\Gamma(\frac{1}{200}, \frac{500}{200})$</td>
<td>2.76</td>
<td>[1.07, 2.11]</td>
</tr>
<tr>
<td>Export Price Adjustment Cost</td>
<td>$\frac{1}{\sigma_{\nu}^*}$</td>
<td>$\Gamma(\frac{1}{200}, \frac{500}{200})$</td>
<td>2.76</td>
<td>[1.07, 2.11]</td>
</tr>
<tr>
<td>Persistence of Preference Shock</td>
<td>$\phi_p$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.48</td>
<td>[0.33, 0.63]</td>
</tr>
<tr>
<td>Persistence of Government Spending</td>
<td>$\phi_g$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.88</td>
<td>[0.84, 0.93]</td>
</tr>
<tr>
<td>Persistence of Import Price Shock</td>
<td>$\phi_{im}$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.50</td>
<td>[0.34, 0.67]</td>
</tr>
<tr>
<td>Persistence of Domestic cost-push shock</td>
<td>$\phi_D$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.35</td>
<td>[0.22, 0.48]</td>
</tr>
<tr>
<td>Persistence of Tradable Cost-Push Shock</td>
<td>$\phi_T$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.32</td>
<td>[0.18, 0.45]</td>
</tr>
<tr>
<td>Persistence of Export Price Shock</td>
<td>$\phi_X$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.51</td>
<td>[0.35, 0.67]</td>
</tr>
<tr>
<td>Persistence of Monetary Policy Shock</td>
<td>$\phi_r$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.50</td>
<td>[0.37, 0.63]</td>
</tr>
<tr>
<td>Interest Rate Smoothing</td>
<td>$\rho_r$</td>
<td>$\beta(0.75, 0.05)$</td>
<td>0.84</td>
<td>[0.80, 0.88]</td>
</tr>
<tr>
<td>Persistence of Exchange Rate Shock</td>
<td>$\phi_q$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.71</td>
<td>[0.59, 0.83]</td>
</tr>
<tr>
<td>Persistence of Total Factor Productivity Shock</td>
<td>$\phi_{tfp}$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.51</td>
<td>[0.34, 0.67]</td>
</tr>
<tr>
<td>Persistence of Wage Cost-Push Shock</td>
<td>$\phi_{w}$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.27</td>
<td>[0.16, 0.38]</td>
</tr>
<tr>
<td>Persistence of Foreign Demand Shock</td>
<td>$\phi_{w^*}$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.91</td>
<td>[0.88, 0.94]</td>
</tr>
<tr>
<td>Inverse Frisch Elasticity</td>
<td>$\sigma_n$</td>
<td>$\Gamma(2.00, 1.00)$</td>
<td>1.16</td>
<td>[0.64, 1.66]</td>
</tr>
<tr>
<td>Standard Deviation of Preference Shock</td>
<td>$\sigma_c$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>3.89</td>
<td>[2.03, 5.76]</td>
</tr>
<tr>
<td>Standard Deviation of Government Spending</td>
<td>$\sigma_g$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>3.55</td>
<td>[2.43, 4.64]</td>
</tr>
<tr>
<td>Standard Deviation of Import Price Shock</td>
<td>$\sigma_{ip}$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>0.75</td>
<td>[0.40, 1.11]</td>
</tr>
<tr>
<td>Standard Deviation of Domestic cost-push shock</td>
<td>$\sigma_D$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>0.12</td>
<td>[0.09, 0.16]</td>
</tr>
<tr>
<td>Standard Deviation of Tradable Cost-push shock</td>
<td>$\sigma_T$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>0.31</td>
<td>[0.20, 0.41]</td>
</tr>
<tr>
<td>Standard Deviation of Export Price Shock</td>
<td>$\sigma_X$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>0.49</td>
<td>[0.22, 0.76]</td>
</tr>
<tr>
<td>Standard Deviation of Monetary Policy shock</td>
<td>$\sigma_r$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>0.12</td>
<td>[0.08, 0.15]</td>
</tr>
<tr>
<td>Standard Deviation of Exchange Rate Shock</td>
<td>$\sigma_q$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>0.73</td>
<td>[0.37, 1.08]</td>
</tr>
<tr>
<td>Standard Deviation of TFP Shock</td>
<td>$\sigma_{tfp}$</td>
<td>$\Gamma^{-1}(0.03, 2.00)$</td>
<td>0.13</td>
<td>[0.05, 0.20]</td>
</tr>
<tr>
<td>Standard Deviation of Wage cost-push Shock</td>
<td>$\sigma_{w}$</td>
<td>$\Gamma^{-1}(0.01, 2.00)$</td>
<td>0.25</td>
<td>[0.17, 0.33]</td>
</tr>
<tr>
<td>Standard Deviation of Foreign Demand Shock</td>
<td>$\sigma_{w^*}$</td>
<td>$\Gamma^{-1}(0.10, 2.00)$</td>
<td>1.64</td>
<td>[1.12, 2.14]</td>
</tr>
<tr>
<td>UIP Smoother</td>
<td>$\psi_{up}$</td>
<td>$\beta(0.50, 0.10)$</td>
<td>0.53</td>
<td>[0.38, 0.69]</td>
</tr>
</tbody>
</table>

Where $\beta(\mu, \sigma)$ is the beta distribution, $\Gamma(\mu, \sigma)$ is the gamma distribution, $N(\mu, \sigma)$ is the normal distribution, $\Gamma^{-1}(\alpha, \theta)$ is the inverse gamma distribution ($\alpha$ and $\theta$ are scaling parameters), and $U(lb, ub)$ is the uniform distribution. The probability interval is a 90% interval.

* The Calvo equivalents of the posterior mode of these Rotemberg parameters are: 0.71, 0.92, 0.92 and 0.82 respectively.
Table 5: Koop et al. (2011) test of parameter identification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Convergence rates (samples)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 (100)</td>
</tr>
<tr>
<td>( \Delta tfp )</td>
<td>2.24</td>
</tr>
<tr>
<td>( \chi_c )</td>
<td>1.15</td>
</tr>
<tr>
<td>( \sigma_n )</td>
<td>1.45</td>
</tr>
<tr>
<td>( \eta )</td>
<td>1.56</td>
</tr>
<tr>
<td>( \eta_T )</td>
<td>1.47</td>
</tr>
<tr>
<td>( \eta_X )</td>
<td>0.91</td>
</tr>
<tr>
<td>( \gamma_W )</td>
<td>1.46</td>
</tr>
<tr>
<td>( \gamma_D )</td>
<td>1.22</td>
</tr>
<tr>
<td>( \gamma_T )</td>
<td>1.48</td>
</tr>
<tr>
<td>( \gamma_X )</td>
<td>1.36</td>
</tr>
<tr>
<td>( \psi_W )</td>
<td>1.59</td>
</tr>
<tr>
<td>( \psi_D )</td>
<td>1.30</td>
</tr>
<tr>
<td>( \psi_T )</td>
<td>2.50</td>
</tr>
<tr>
<td>( \psi_X )</td>
<td>1.20</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>1.58</td>
</tr>
<tr>
<td>( \phi_{\pi} )</td>
<td>0.57</td>
</tr>
<tr>
<td>( \phi_y )</td>
<td>1.86</td>
</tr>
<tr>
<td>( \phi_{\Delta y} )</td>
<td>1.33</td>
</tr>
<tr>
<td>( \psi_{uip} )</td>
<td>1.31</td>
</tr>
<tr>
<td>( \vartheta_c )</td>
<td>1.48</td>
</tr>
<tr>
<td>( \vartheta_D )</td>
<td>1.06</td>
</tr>
<tr>
<td>( \vartheta_T )</td>
<td>1.56</td>
</tr>
<tr>
<td>( \vartheta_w )</td>
<td>1.33</td>
</tr>
<tr>
<td>( \vartheta_r )</td>
<td>1.65</td>
</tr>
<tr>
<td>( \vartheta_q )</td>
<td>0.94</td>
</tr>
<tr>
<td>( \vartheta_p^* )</td>
<td>1.52</td>
</tr>
<tr>
<td>( \vartheta_X )</td>
<td>1.30</td>
</tr>
<tr>
<td>( \vartheta_{p,n} )</td>
<td>1.76</td>
</tr>
<tr>
<td>( \vartheta_{tfp} )</td>
<td>1.16</td>
</tr>
<tr>
<td>( \vartheta_g )</td>
<td>3.38</td>
</tr>
<tr>
<td>( \epsilon_c )</td>
<td>0.89</td>
</tr>
<tr>
<td>( \epsilon_D )</td>
<td>1.01</td>
</tr>
<tr>
<td>( \epsilon_T )</td>
<td>1.05</td>
</tr>
<tr>
<td>( \epsilon_W )</td>
<td>0.90</td>
</tr>
<tr>
<td>( \epsilon_r )</td>
<td>1.05</td>
</tr>
<tr>
<td>( \epsilon_q )</td>
<td>0.77</td>
</tr>
<tr>
<td>( \epsilon_y^* )</td>
<td>1.07</td>
</tr>
<tr>
<td>( \epsilon_X )</td>
<td>1.65</td>
</tr>
<tr>
<td>( \epsilon_{p,m}^* )</td>
<td>0.86</td>
</tr>
<tr>
<td>( \epsilon_{\Delta tfp} )</td>
<td>1.20</td>
</tr>
<tr>
<td>( \epsilon_g )</td>
<td>0.82</td>
</tr>
</tbody>
</table>

This test uses the law of large numbers to assess whether the data is informative for posterior estimates of parameters. If the (actual) data is informative for estimates near the posterior mode, then the posterior variances of the parameters should tend to 0 at a rate of \( \frac{1}{T} \). Simulated data of varying sample sizes are generated using the posterior mode estimates and then the model is re-estimated on this simulated data. The rate of convergence of the posterior variances of the estimates from the simulated data can then be compared to \( \frac{1}{T} \). Convergence rates near \( \frac{1}{T} \) suggest that the parameter estimates are identified. The table shows the convergence ratios for samples of 100, 200, 500, 2500 and 7500.
Table 6: Standard Deviations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>Model [5 95]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rates</td>
<td>2.14</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td>[1.75, 6.29]</td>
<td></td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>0.34</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>[0.23, 0.5]</td>
<td></td>
</tr>
<tr>
<td>Domestic Inflation</td>
<td>0.31</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>[0.26, 0.65]</td>
<td></td>
</tr>
<tr>
<td>Wage Inflation</td>
<td>0.52</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>[0.31, 0.71]</td>
<td></td>
</tr>
<tr>
<td>Exchange Rate Growth</td>
<td>3.73</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td>[2, 5.08]</td>
<td></td>
</tr>
<tr>
<td>GDP Growth</td>
<td>0.76</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>[1.17, 2.86]</td>
<td></td>
</tr>
<tr>
<td>Consumption Growth</td>
<td>0.82</td>
<td>1.64</td>
</tr>
<tr>
<td></td>
<td>[1.11, 2.81]</td>
<td></td>
</tr>
<tr>
<td>Export Growth</td>
<td>2.26</td>
<td>3.59</td>
</tr>
<tr>
<td></td>
<td>[2.37, 6.62]</td>
<td></td>
</tr>
<tr>
<td>Terms of Trade</td>
<td>2.31</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td>[1.18, 3.16]</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Comparing the importance of the model’s features

<table>
<thead>
<tr>
<th></th>
<th>Base</th>
<th>$\chi_c = 0$</th>
<th>$\alpha_p = 0$</th>
<th>$\gamma_W = 0$</th>
<th>$\gamma_T = 0$</th>
<th>$\gamma_D = 0$</th>
<th>$\chi_n$</th>
<th>$v$</th>
<th>Adaptive Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode of the estimated parameters</td>
<td>Δtfp</td>
<td>0.28</td>
<td>0.27</td>
<td>0.28</td>
<td>0.4</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>$\psi_W$</td>
<td>299.84</td>
<td>235.32</td>
<td>305.83</td>
<td>0.01</td>
<td>207.72</td>
<td>296.95</td>
<td>311.91</td>
<td>299.14</td>
<td>205.54</td>
</tr>
<tr>
<td>$\psi_D$</td>
<td>248.26</td>
<td>253.55</td>
<td>243.44</td>
<td>2475.25</td>
<td>276.52</td>
<td>249.45</td>
<td>252.12</td>
<td>307.85</td>
<td></td>
</tr>
<tr>
<td>$\psi_T$</td>
<td>3366.86</td>
<td>4277.7</td>
<td>3746.62</td>
<td>3007.67</td>
<td>0.01</td>
<td>3267.71</td>
<td>3423.89</td>
<td>4189.32</td>
<td>403.33</td>
</tr>
<tr>
<td>$\psi_X$</td>
<td>371.09</td>
<td>420.41</td>
<td>248.74</td>
<td>304.21</td>
<td>201.37</td>
<td>359.87</td>
<td>359.85</td>
<td>379.65</td>
<td>261.92</td>
</tr>
<tr>
<td>$\phi_{tr}$</td>
<td>1.96</td>
<td>1.96</td>
<td>1.97</td>
<td>1.98</td>
<td>1.98</td>
<td>1.96</td>
<td>1.97</td>
<td>1.97</td>
<td>1.93</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.21</td>
<td>0.14</td>
<td>0.23</td>
<td>0.15</td>
<td>0.26</td>
<td>0.26</td>
<td>0.21</td>
<td>0.09</td>
<td>0.25</td>
</tr>
<tr>
<td>$\phi_\Delta$</td>
<td>0.12</td>
<td>0.1</td>
<td>0.13</td>
<td>0.16</td>
<td>0.33</td>
<td>0.33</td>
<td>0.21</td>
<td>0.11</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_T$</td>
<td>0.85</td>
<td>0.79</td>
<td>0.85</td>
<td>0.83</td>
<td>0.74</td>
<td>0.86</td>
<td>0.85</td>
<td>0.83</td>
<td>0.81</td>
</tr>
<tr>
<td>$\psi_{up}$</td>
<td>0.52</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.54</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.51</td>
<td>0.35</td>
<td>0.53</td>
<td>0.52</td>
<td>0.47</td>
<td>0.56</td>
<td>0.51</td>
<td>0.46</td>
<td>0.47</td>
</tr>
<tr>
<td>$\rho_\Delta$</td>
<td>0.76</td>
<td>0.72</td>
<td>0.87</td>
<td>0.72</td>
<td>0.8</td>
<td>0.78</td>
<td>0.76</td>
<td>0.71</td>
<td>0.75</td>
</tr>
<tr>
<td>$\rho_D$</td>
<td>0.48</td>
<td>0.88</td>
<td>0.45</td>
<td>0.63</td>
<td>0.58</td>
<td>0.48</td>
<td>0.5</td>
<td>0.75</td>
<td>0.4</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>0.35</td>
<td>0.36</td>
<td>0.35</td>
<td>0.49</td>
<td>0.27</td>
<td>0.5</td>
<td>0.36</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
<td>$\rho_T$</td>
<td>0.3</td>
<td>0.27</td>
<td>0.3</td>
<td>0.31</td>
<td>0.5</td>
<td>0.3</td>
<td>0.31</td>
<td>0.3</td>
<td>0.21</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>0.26</td>
<td>0.25</td>
<td>0.26</td>
<td>0.5</td>
<td>0.23</td>
<td>0.25</td>
<td>0.27</td>
<td>0.27</td>
<td>0.23</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.48</td>
<td>0.49</td>
<td>0.46</td>
<td>0.53</td>
<td>0.91</td>
<td>0.46</td>
<td>0.48</td>
<td>0.54</td>
<td>0.88</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>0.73</td>
<td>0.71</td>
<td>0.83</td>
<td>0.7</td>
<td>0.54</td>
<td>0.72</td>
<td>0.73</td>
<td>0.71</td>
<td>0.54</td>
</tr>
<tr>
<td>$\rho_{\Delta_p}$</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.98</td>
<td>0.91</td>
<td>0.92</td>
<td>0.92</td>
<td>0.49</td>
</tr>
<tr>
<td>$\rho_{\Delta TP}$</td>
<td>0.5</td>
<td>0.47</td>
<td>0.5</td>
<td>0.29</td>
<td>0.48</td>
<td>0.32</td>
<td>0.49</td>
<td>0.47</td>
<td>0.52</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.86</td>
<td>0.84</td>
<td>0.86</td>
<td>0.87</td>
<td>0.9</td>
<td>0.84</td>
<td>0.85</td>
<td>0.82</td>
<td>0.89</td>
</tr>
</tbody>
</table>