Structural VARs, deterministic and stochastic trends: Does detrending matter?

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Structural VARs, deterministic and stochastic trends: 
Does detrending matter?*

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Abstract

We highlight how detrending within Structural Vector Autoregressions (SVAR) is directly linked to the shock identification. Consequences of trend misspecification are investigated using a prototypical Real Business Cycle model as the Data Generating Process. Decomposing the different sources of biases in the estimated impulse response functions, we find the biases arising directly from trend misspecification are not trivial when compared to other widely studied misspecifications. Misspecifying the trend can also distort impulse response functions of even the correctly detrended variable within the SVAR system. A possible solution hinted by our analysis is that increasing the lag order when estimating the SVAR may mitigate some of the biases associated with trend misspecification.

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1 Introduction

While trends are ubiquitous in macroeconomic time series, dealing with them is often not straightforward. Within Structural Vector Autoregression (SVAR) frameworks, assumptions regarding the trend play an important role in the identification of structural shocks in a system. Contributions by Pagan and Pesaran (2008) and Fisher et al. (2014) highlight how choices pertaining to the handling of trends impact upon the identification of whether shocks in a system are transitory or permanent. We extend upon this strand of the literature by investigating the implications of misspecification in the detrending procedure. In particular, the main contribution of our work quantifies the severity of the biases with regards to trend misspecification in SVARs. We isolate the biases induced by trend misspecification by introducing a simple decomposition of the estimated SVAR biases. Thereafter, we propose a possible practical solution for empirical work in order to mitigate the biases induced by trend misspecification.

A common approach for dealing with trends in empirical macroeconomics is to employ a filter. Two common filters are deterministic detrending through a linear, and perhaps a quadratic, trend term or to first difference. A decision whether to difference or deterministically detrend variables is a researcher’s prior stand on how the variables respond to different identified shocks and whether shocks are transitory or permanent. There is thus an inherent link between identification and detrending. To illustrate the point, some methods of identification in an SVAR framework range from placing restrictions on contemporaneous relationships, long-run relationships, imposing directional responses of variables to particular shocks or some combination of those just mentioned. Each has implications for inferences when studying the response of the economy to different structural shocks. For instance, a common means of orthogonalising the shocks in an SVAR system is to impose short-run, often zero, restrictions on the contemporaneous response of variables to particular shocks. Suppose the orthogonalisation is on a variable like the difference of real output, with differencing occurring prior to estimation in order to detrend the variable. Without any other restrictions, all shocks will impose a long-run impact on the level of real output. We define this type of shock with a long-run impact on the level of at least one variable in the system as a permanent shock. While shocks having a permanent long-run impact is consistent with some shocks discussed within the macroeconomic literature (i.e. productivity shocks), they are also inconsistent with a large class of shocks (e.g. demand shocks, monetary shocks, etc). Despite these implications, it is unfortunate the subtle link between identification and detrending does not appear fully appreciated within the empirical literature.

1 The impact of a shock, $\zeta$, of size 1, at time $t$ to a variable $w$ at time $t+i$ is $\frac{\partial w_{t+i}}{\partial \zeta}$. A shock is transitory on $w$ if $\lim_{i \to \infty} \frac{\partial w_{t+i}}{\partial \zeta} = 0$. Otherwise, it is permanent. If $w$ was first differenced, then $\lim_{i \to \infty} \frac{\partial w_{t+i}}{\partial \zeta} = \sum_{j=0}^{\infty} \frac{\partial \Delta w_{t+j}}{\partial \zeta}$. Unless a prior restriction is imposed, this sum will in general not equal to zero, implying a permanent effect on $w$.

2 For example, Peersman (2005) and Canova et al. (2007) feature monetary policy shocks which are permanent on output due to the first differencing of output before the VAR estimation. A more recent development in empirical macroeconomics is the use of large datasets to extract factors within the FAVAR framework (see e.g. Bernanke et al. (2005). FAVAR practitioners typically routinely first difference all their trending data as a matter of practicality, but often with a failure to recognise the subtlety of inducing permanent shocks in all these variables. Both Del Negro and Schorfheide (2004) and Inoue and Ross (2011) also feature trend stationary output in their DSGE models, but difference output in the SVAR section of their work. This is an inconsistency as the DSGE
In this paper, we use Monte Carlo experiments to study the issue of trend misspecification in SVARs. The Data Generating Process (DGP) are standard Real Business Cycle (RBC) models in the spirit of Hansen (1985). The RBC model under consideration differs only with regards to the trend specification induced by the underlying technology shock process. Output specified in the DGP will then either evolve according to a stochastic or deterministic trend depending on the properties of the underlying technology shock in the DGP. We then estimate bi-variable SVARs using artificial data from the DGP, with a misspecified trend. Given the link between identification and trend specification, this trend misspecification will also have an impact on the shock identification of the SVAR. Our question is then how serious are such trend misspecifications for subsequent inference as we characterise the sources of these biases.

The properties of whether output is a deterministic or stochastic trend remains a point of contention in empirical macroeconomics. For likely the same reasons, prominent SVAR studies mainly feature these two forms of trend specifications, making our choice to contrast deterministic and stochastic trends a natural one (see, e.g. Morley et al., 2003; Perron and Wada, 2009). Given our focus, we do not discuss in depth SVAR specifications with trend properties which are inconsistent with workhorse DSGE/RBC models, such as using levels.

The empirical literature is laden with instances where an SVAR is likely to contain biases when generated from RBC/DSGE model (see e.g. Christiano et al., 2003; Ravenna, 2007; Chari et al., 2008; Carlstrom et al., 2009). Therefore, one expects even if the trend is properly specified, the SVAR will contain some degree of bias. It follows that any analysis will require isolating the bias which is directly induced by the misspecified trend. To deal with this, a feature of our analysis is a decomposition of the biases in order to distinguish between biases which are and are not a consequence of the trend misspecification. We find biases due to trend misspecification are non-trivial, relative to other widely studied biases. This emphasises incorrect assumptions about trend processes can cause considerable biases in the estimated impulse response functions, further highlighting the importance about the choice of detrending within an SVAR framework. Researchers using SVARs for empirical work should at least be mindful of the implicit assumptions within their models. As an SVAR is a system of interdependent equations, the corresponding impulse response functions of the correctly detrended variable, like hours worked in our study, are also distorted if the trend in output is misspecified. That is, there can be significant spillover from trend misspecification of the trending variable to the correctly detrended variable within the system.

Misspecifying the trend in an SVAR induces a VARMA structure. This in principle suggests a long lag order may be able to obviate some of the biases which are a consequence of trend misspecification. While this claim has merit, we conduct Monte Carlo experiments which suggest model features transitory shocks (around a trend) to output, but the corresponding SVAR has permanent shocks to output.

3For example, Blanchard and Perotti (2002), Boivin and Giannoni (2006) and Rossi and Zubairy (2011) all feature some form of deterministic detrending for output. At the other end of the spectrum, Blanchard and Quah (1989), King et al. (1991) and Gali (1999) all utilise identifying assumptions of productivity being the source of permanent variation to output.

4Sims et al. (1990) advocate using levels for SVAR estimation but levels are inconsistent with the way data is being treated in workhorse RBC/DSGE models unless one is explicit about the cointegration relationships. Similarly, we do not discuss in depth the Hodrick and Prescott (1997) filter, apart from in the supplementary appendix, as the filter induces an inconsistency between the data and that which is generated by RBC/DSGE models (see Fukac and Pagan, 2010 for a discussion on this point).
common lag order selection based on information criteria are likely to choose a lag order which is too short. Our practical advice is that practitioners may well be better off just choosing rule of thumb lag orders in SVARs, which are put forward to capture sufficiently long and variable dynamics (e.g. 4 lags for quarterly data), rather than relying on information criteria if the objective is to obviate biases from trend misspecification.

The remainder of the paper is as follows. Section 2 describes the theoretical model and the identification of the model within a SVAR setting. Section 3 explicitly links the theoretical RBC model with features of the SVAR to motivate the design of the simulation study. The simulation setup and the decomposition of biases to study the consequences of trend misspecification are then discussed in Section 4. The results are presented in Section 5 before some concluding remarks in Section 6.

2 Theoretical Model and Identification

We study an RBC model similar to that used by Hansen [1985]. The parsimony of the model structure appeals with fewer identifying restrictions for the SVAR. The following subsections present the theoretical RBC model used as the DGP and a discussion of the SVAR identification.

2.1 The Theoretical Model

Under this framework, the households’ problem is given by

\[
\max_{\{C_t, H_t, K_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \frac{(H_t/B_t)^{1+\eta}}{1+\eta} \right\}
\]

subject to

\[
\begin{align*}
C_t + I_t &= R_t K_t + W_t H_t \quad (1) \\
I_t &= K_{t+1} - (1-\delta) K_t \quad (2) \\
\ln(B_{t+1}) &= \rho_B \ln(B_t) + \epsilon_{B,t+1} \quad (3)
\end{align*}
\]

where \( \beta \in (0,1) \) is a discount factor, \( \eta > 0 \) is an inverse Frisch elasticity of labour supply, \( \delta \in (0,1) \) is a depreciation rate of capital, \( \rho_B \in (0,1) \) is a measure of persistence and \( \epsilon_{B,t+1} \sim \mathcal{N}(0,\sigma_B^2) \) is an exogenous Gaussian shock process.

Households in this economy optimise their expected discounted lifetime utility by choosing each period consumption \( (C_t) \), hours worked \( (H_t) \) and next-period capital holdings \( (K_{t+1}) \) subject to their budget constraint \( (1) \), a capital accumulation equation \( (2) \) and a stationary AR(1) exogenous process \( (B_t) \). The shock process, \( \epsilon_{B,t} \), presented in Equation \( (3) \) can be interpreted as either a shock to labour supply, preference, or demand of households. In this paper, we refer to this innovation as a non-technology shock. Furthermore, as the process is stationary, the shock has a transitory impact upon variables in the system. Households earn income by supplying capital and labour services to firms. Income is either consumed or invested. Let \( I_t \) be investment, \( R_t \) be the rental rate of capital and \( W_t \) be the wage rate at period \( t \).
We can write the problem for firms as

$$\max_{\{K_t, H_t\}} \{Y_t - R_t K_t - W_t H_t\}$$

subject to

$$Y_t = K_t^\alpha (Z_t H_t)^{1-\alpha}$$

where $\alpha \in (0, 1)$ is a capital share and $Z_t$ is a technology innovation to productivity.

The firms maximise their profit subject to the labour-augmenting Cobb-Douglas production function. Here revenue is obtained by selling goods to households, denoted by $Y_t$, while costs are incurred from renting households’ capital and labour services.

**Technology Shock Process**

The aim of the theoretical model is to generate the aforementioned RBC model with a technology shock which can be either transitory or permanent. We therefore consider two specifications of the technology shock to achieve this. The permanent and transitory technology shock process entail a stochastic and deterministic trend process respectively. The stochastic trend specification can be represented as

$$\tilde{Z}_{t+1} = \frac{Z_{t+1}}{Z_t}$$
$$\ln(\tilde{Z}_{t+1}) = (1 - \rho_z) \ln(\gamma) + \rho_z \ln(\tilde{Z}_t) + \epsilon_{z,t}$$

where $\gamma$ is the average growth rate, $\rho_z$ is persistence in the growth rate of the technology shock and $\epsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2)$ is a Gaussian shock. Under this specification, the technology innovation has a permanent impact on the level of $Y_t, C_t, K_t, W_t$ and $Z_t$ causing these variables to inherit unit roots. Therefore, one can obtain stationary variables with the following transformation; $\tilde{Y}_t = \frac{Y_t}{Z_t}$, $\tilde{C}_t = \frac{C_t}{Z_t}$, $\tilde{K}_{t+1} = \frac{K_{t+1}}{Z_t}$ and $\tilde{W}_t = \frac{W_t}{Z_t}$. Note that $K_{t+1}$ is detrended by $Z_t$ as it is determined within period $t$. Hereinafter, we refer to this model as RBC-rw.

On the other hand, the deterministically trending process assumes that the technology innovation grows at a constant rate $\gamma$. The process can be represented as

$$Z_t = \gamma^t \tilde{Z}_t$$
$$\ln(\tilde{Z}_{t+1}) = \rho_z \ln(\tilde{Z}_t) + \epsilon_{z,t+1}$$
$$\ln(Z_t) = t \ln(\gamma) + \rho_z \ln(\tilde{Z}_{t-1}) + \epsilon_{z,t}.$$  

We obtain the stationary equilibrium condition by detrending the variables with the deterministic trend $\gamma$. The transformed variables are then given by $\tilde{Y}_t = \frac{Y_t}{Z_t}$, $\tilde{C}_t = \frac{C_t}{Z_t}$, $\tilde{K}_{t+1} = \frac{K_{t+1}}{Z_t}$ and $\tilde{W}_t = \frac{W_t}{Z_t}$. Unlike the RBC-rw specification, the technology shock considered here only has a transitory impact to all variables in the system. The variables are thus trend stationary. We refer this model as RBC-dt.
Data Generating Process

Given this framework, we have \( \tilde{X}_t = (\tilde{K}_t, \tilde{Z}_t, B_t)' \) as unobserved state variables, \( \tilde{Q}_t = (\tilde{Y}_t, H_t)' \) as observable variables and \( \epsilon_t = (\epsilon_{z,t}, \epsilon_{B,t})' \) as exogenous shocks where \( t = 1, 2, \ldots, T \). By implementing the first-order approximation, a stable Rational Expectation Equilibrium solution to the log-linearised system of an RBC model has the following linear state-space representation,

\[
\begin{align*}
\tilde{x}_{t+1} &= R\tilde{x}_t + S\epsilon_{t+1} \\
\tilde{q}_t &= M\tilde{x}_t 
\end{align*}
\]

where \( \tilde{x}_t \) and \( \tilde{q}_t \) are column vectors of log-deviation state variables and observable variables from the steady state values, \( R \), \( S \) and \( M \) are matrices of reduced-form parameters and \( \epsilon_t \epsilon_t' = \Sigma_\epsilon \) is a diagonal covariance matrix.

Given \( \tilde{x}_0 \), we can simulate data of output and hours worked \( Q = \{Y_t, H_t\}_{t=0}^T \). Hours worked is always integrated of order zero, \( H_t \sim I(0) \). The time series properties for output however depends on the underlying trend process. If the trend process is a deterministic trend as in the case of RBC-dt, output is trend stationary and integrated of order zero, \( Y_t \sim I(0) \). On the other hand, output is integrated of order one, \( Y_t \sim I(1) \), when the underlying trend process is a stochastic trend as in the case of RBC-rw. The characteristics of the structural shocks in the system also differ across model specifications. While the non-technology shock always has transitory impacts, the technology shock has transitory impact only under RBC-dt. In the alternative RBC-rw case, technology shocks have a permanent impact upon the level of output.

The values of structural parameters used in the DGPs are summarised in Table 1. These parameter values are standard in the literature (see, e.g. Lindé, 2009).

2.2 Structural VAR Identification

Let \( \tilde{Q}_t = (\tilde{Y}_t, \tilde{H}_t)' \) be a column vector of (demeaned) observable variables where these “hat” variables represent time series generated from one of the DGPs described in the previous section. Define \( \tilde{q}_t \) as a vector of transformed observable variables, either by first differencing or linear detrending. The VAR in the transformed variable is

\[
\Phi(L)\tilde{q}_t = u_t = A\nu_t
\]

where \( \Phi(L) \) is a lag polynomial, \( I - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p \) of finite lag order \( p \). \( A \) is a matrix of contemporaneous impact multipliers. \( u_t \) and \( \nu_t \) are the reduced-form and structural innovations with covariance matrix \( \Sigma_u \) and \( \Sigma_\nu \) respectively. We define the structural shocks in this system as a technology \((\nu_t^T)\) and a non-technology \((\nu_t^{NT})\) shock. \( \Sigma_\nu \) is diagonal by assumption, embedding the idea that the structural shocks in the model are orthogonal. The econometrician can estimate \( \Phi(L) \) consistently using least squares. However, the identification issue arises because there is one free parameter in this bi-variate system due to assuming the structural innovations are orthogonal (i.e. \( \Sigma_\nu \) is diagonal whereas \( \Sigma_u \) is not).
Therefore, while the reduced-form VAR in Equation (6) can be easily estimated by least squares, the econometrician can only estimate the SVAR in Equation (7) by invoking some identifying restrictions. A common way to view this problem is the econometrician can compute impulse response functions with knowledge of both \( \Phi(L) \), which is estimated by least squares, and \( A \), which comes about after making some identifying assumptions. As the following will make clear, identifying strategies are linked to the trend assumption which the econometrician makes.

**Long-Run Restrictions**

Suppose the econometrician believes that output exhibits a unit root solely due to a long-run impact from a technology shock while hours worked is an I(0) process. Blanchard and Quah (1989) offer an identification strategy to impose this long-run restriction in an SVAR framework. As output is assumed to be an I(1) process, the series enters the SVAR in first difference to achieve stationarity. To impose this restriction, first let the vector \( \hat{q}_t = (\Delta \hat{y}_t, \hat{h}_t)' \), where \( \hat{y}_t \) and \( \hat{h}_t \) are logged output and hours worked respectively. We can rewrite the VAR in Vector Moving Average (VMA) form as follows. From Equation (7), we have

\[
\hat{q}_t = \Phi^{-1}(L)A\nu_t \quad (8)
\]

\[
= \Psi(L)\nu_t \quad (9)
\]

where \( \Psi(L) = \Phi^{-1}(L)A \). Expanding (9) and substituting in the elements of the vector \( \hat{q}_t \) and \( \nu_t \), we obtain

\[
\begin{pmatrix}
\Delta \hat{y}_t \\
\hat{h}_t
\end{pmatrix}
= \Psi(L)
\begin{pmatrix}
\nu^T_t \\
\nu^{NT}_t
\end{pmatrix}
= \begin{pmatrix}
\psi_{11}(L) & \psi_{12}(L) \\
\psi_{21}(L) & \psi_{22}(L)
\end{pmatrix}
\begin{pmatrix}
\nu^T_t \\
\nu^{NT}_t
\end{pmatrix}
= \sum_{i=0}^{\infty} \psi_{11}^i \nu^T_t + \sum_{i=0}^{\infty} \psi_{12}^i \nu^{NT}_t
\]

By assuming the non-technology shock has no long-run impact on output, the required restriction is then \( \psi_{12}(1) = \sum_{i=0}^{\infty} \psi_{12}^i = 0 \). After imposing this restriction on \( \Psi(L) \), it is straightforward to recover \( A \).

**Short-Run Restrictions**

If the econometrician believes that both output and hours worked are I(0) processes, all structural shocks in the system are assumed to be transitory, with output transitory around a deterministic trend. Output is linearly detrended before estimation and thus \( \hat{q}_t = (\tilde{y}_t - \lambda_t, \tilde{h}_t)' \) where \( \lambda \) represents the coefficient on the deterministic trend. Regardless of the identification procedure here, shocks will always be transitory on the variables in \( \hat{q}_t \), namely detrended...
output and hours worked. In order to invoke one identifying restriction, we place one zero restriction directly on the contemporaneous matrix, $A$. A natural way to identify a technology shock is to assume that a non-technology shock has no contemporaneous impact on output. This will almost by construction allow a large share of the forecast error variance to be explained by the technology shock. Our identification also directly appeals to the intellectual foundations of RBC models, where technology shocks are the dominant drivers of the business cycle.

This restriction amounts to identification with a Cholesky decomposition of the covariance matrix, ordering output first. By assumption, output will not respond contemporaneously to non-technology shocks. Given a restriction is imposed on the short-run dynamics in the model, namely the impact response to non-technology shocks, we refer to this as imposing short-run restrictions. From Equation $7$, the proposed identification of this system is then given by

$$
\begin{pmatrix}
\hat{y}_t - \lambda_t \\
\hat{h}_t
\end{pmatrix} = \Phi^{-1}(L) \begin{pmatrix}
A_{11} & 0 \\
A_{21} & A_{22}
\end{pmatrix} \begin{pmatrix}
\nu_t^T \\
\nu_{NT}^T
\end{pmatrix}.
$$

3 Link Between RBC and SVAR

Before discussing the simulation setup, it is worth discussing the link between the theoretical RBC model, which we use as the DGP, and the SVAR identification. The use of long-run restrictions allows for permanent shocks given output is differenced prior to estimation. One can reconcile features of the long-run restrictions with the RBC-rw. The non-technology shock in the theoretical RBC-rw model does not have any long-run impact on output, which is used as the restriction to identify both shocks.

With the Cholesky decomposition, all shocks are assumed to be transitory. All the variables are thus assumed to be fluctuating either around a deterministic trend or an unconditional mean. Therefore, the features of the theoretical RBC-dt model, where all shocks are transitory, are consistent with using short-run restrictions. However, there is still a misspecification with the Cholesky decomposition since output is restricted not to react contemporaneously to a non-technology shock upon impact. While this is not consistent with the theoretical RBC-dt model, we view this restriction as providing the linearly detrended SVAR with the “best” shot at matching the theoretical model. We stress at this stage that, despite the theoretical inconsistency, the Cholesky decomposition is largely able to recover the properties of the transitory technology shock in our Monte Carlo simulation. The biases induced by the Cholesky decomposition are small relative to the other sources of biases in our experiments when the deterministic trend is properly specified. At the same time, the bias decomposition exercise described in the next section will make comparisons relative to

\footnote{Note this does not in anyway imply short-run restrictions necessarily identify transitory shocks. The shocks here are transitory because of the detrending, though this impacts upon the choice to impose an identifying restriction on the short-run dynamics of the model. Another popular identification procedure is to impose impact sign restrictions on variables (e.g. Uhlig 2005). Such restrictions do not tie down the long-run properties of the shocks. Therefore, using sign restrictions on differenced data without additional restrictions must entail these are also by construction permanent shocks.}

\footnote{Carlstrom et al. (2009) study a similar issue by investigating the consequence of using Cholesky identification when it is not consistent with a theoretical New Keynesian DSGE model.}
correctly specifying the trend. It should also be noted that imposing a Cholesky decomposition for identification is a plausible strategy from an empirical perspective within the SVAR literature.

Another discrepancy is that the theoretical models under both trend specifications have a state variable, capital ($K_t$), which is omitted in the VAR estimation, implying an infinite order VAR is the correct specification to map the theoretical model to the SVAR (see e.g. Kapetanios et al., 2007; Ravenna, 2007; Chari et al., 2008; Poskitt and Yao, 2012). A finite order VAR will thus be biased from lag length truncation. We isolate the lag truncation by decomposing the components of the biases which are a direct consequence of detrending, and the component which is not. The latter includes issues that plague SVARs even if the detrending methods are sound, and is relevant to the prior discussion on lag length truncation and the imperfect ability of the Cholesky decomposition to recover the theoretical model’s impulses. Therefore, our experimental setup goes a step further by isolating these components for the analysis through our decomposition. This tool allows a comparison of the components which are due to trend misspecification. The decomposition is described within the next section.

4 Simulation Setup

As previously discussed, the long-run and short-run restrictions should correctly recover the correct shock properties for the RBC-rw and RBC-dt respectively. We label these cases as having the correct trend specification. The purpose of this paper is to determine the magnitude of the distortion in the estimated impulse response functions which trend misspecification in an SVAR induces. Table 2 summarises the experimental design of our Monte Carlo exercise. Under our first trend misspecification study, output is generated I(0) by an RBC model with a deterministic trend (RBC-dt). The econometrician then wrongly first differences the series and identifies the SVAR using long-run restrictions. In the second misspecification, output is generated I(1) by an RBC model with a stochastic trend (RBC-rw). However, the econometrician wrongly detrends the series and implements short-run restrictions to identify the SVAR. Hours worked is correctly specified in levels, consistent with both DGPs. As mentioned earlier, responses to the non-technology shock are misspecified under the short-run, zero contemporaneous, restriction. The responses to this shock are thus not a fair comparison to study trend misspecification. Our analysis is therefore based on the estimated impulse response functions to a technology shock.

4.1 Bias Decomposition

The total bias can be the defined as the average of the difference between the estimated impulse response functions derived from a misspecified SVAR and the true ones deduced from the corresponding RBC model. For example, in Trend Misspecification 1, the total bias in the estimated impulse response functions is

$$\text{Total Bias} = \frac{1}{N} \sum_{i=1}^{N} \left[ IRF^{(i)}(rw, LR) - IRF(\text{RBC-dt}) \right]$$

(10)
where the first term is the estimated impulse response functions from an SVAR assuming a random-walk process and imposing long-run restrictions as a shock identifying strategy and the second term is true responses from an RBC model with a deterministic trend process and \( N = 10,000 \) is the number of simulations.

To study the consequences of trend misspecification, we quantify the size of these components by linearly decomposing the total bias in Trend Misspecification 1 expressed in Equation (10) as follows.

\[
\text{Total Bias} = \frac{1}{N} \sum_{i=1}^{N} \left[ \hat{IRF}^{(i)}(rw, LR) - \hat{IRF}^{(i)}(dt, SR) \right] + \frac{1}{N} \sum_{i=1}^{N} \left[ \hat{IRF}^{(i)}(dt, SR) - IRF(RBC-dt) \right].
\]

The decomposition in Equation (11) distinguishes between two sources of biases. Biases as a consequence of detrending are termed \textit{detrending bias}, while biases which are not a consequence of detrending are termed \textit{non-detrending bias}. The non-detrending bias occurs due to the imperfect ability of the SVAR to mimic the theoretical impulses for a variety of reasons not linked to detrending. Recall these include issues ranging from lag length truncation to a degree of identification bias given the inability to perfectly pin down the matrix of impact multipliers, \( A \). It should be clear that if the SVAR is correctly detrended, the detrending bias disappears. Hence, the non-detrending bias in Trend Misspecification 1 equals the bias incurred in Correct Trend Specification 2, when the data is correctly linearly detrended. Similar decomposition of the total bias can also be done for Trend Misspecification 2. Likewise, the non-detrending bias in this case measures the bias which occurs with Correct Trend Specification 1. The decomposition enables us to isolate the \textit{non-detrending bias}, which allows our analysis to cleanly draw out the implications of trend misspecification. If we further expand Equation (11),

\[
\text{Total Bias} = \frac{1}{N} \sum_{i=1}^{N} \left[ \hat{IRF}^{(i)}(rw, LR) - \hat{IRF}^{(i)}(dt, LR) \right] + \frac{1}{N} \sum_{i=1}^{N} \left[ \hat{IRF}^{(i)}(dt, LR) - \hat{IRF}^{(i)}(dt, SR) \right] + \frac{1}{N} \sum_{i=1}^{N} \left[ \hat{IRF}^{(i)}(dt, SR) - IRF(RBC-dt) \right].
\]

The detrending bias is now further decomposed into two components, namely the \textit{direct detrending bias} and the \textit{indirect detrending bias}. The direct detrending bias is the average difference between the estimated impulse response functions of the correct and incorrect detrending procedure as implied by a corresponding RBC model, keeping constant the
identifying restrictions assumed by the econometrician. This is isolated from the subsequent identification problem which occurs because of the wrong detrending procedure in the first step. We term this the indirect detrending bias. This bias is due to imposing incorrect identifying restrictions as a consequence of detrending and captured by the second component. This bias component is computed as the average difference between the estimated impulse response functions with different identifying restrictions, given a detrending procedure consistent to that of the corresponding RBC model.

Intuitively, from Equation (8), the responses of transformed variables $\hat{q}_t$ to structural innovations $\nu_t$ are influenced by both the estimated reduced-form coefficients $\Phi(L)$ and the matrix $A$. Specifically, an incorrect assumption about the trend process in output leads to trend misspecification and in turn causes estimation bias in the reduced-form coefficients, $\Phi(L)$. The bias in the coefficient estimates is also exacerbated by the nonlinear mapping involved with imposing identifying restrictions in the matrix $A$ as it is also a function of the estimated coefficients. Furthermore, the incorrect trend assumption also leads to an error in identifying structural shocks and hence affects the way an econometrician imposes restrictions on the matrix $A$ (i.e. imposing long-run or short-run restrictions). Another way of thinking about this bias is as follows. If detrending did not lead to a change in the matrix of impact multipliers of the shocks, namely $A$, then it is straightforward to see that the second component of the total bias would be eliminated since detrending had no marginal impact in distorting the identification of shocks. In sum, the decomposition offers a framework for thinking about how detrending within the SVAR framework induces different biases in the impulse response functions.

5 Results and Discussion

We first present the results from the baseline Monte Carlo study to first establish the role of trend misspecification. Thereafter, we discuss practical implications and how practitioners may be able to mitigate these biases.

5.1 Monte Carlo Simulation

Figures 1 and 2 present the findings from our Monte Carlo study. The true responses derived from the DGPs are presented by a dashed line with circles. Red area represents estimated impulse response functions of the SVAR from 10,000 data sets of 200 observations. The dynamics of the true responses can be described as follows. The impulse response function converges back to zero in the long-run due to the transitory nature of the shock in the deterministic trend case (denoted RBC-dt) under its true DGP. This is shown with the dashed line with circles in Figures 1. Analogously, the stochastic trend case is depicted in Figure 2 (denoted RBC-rw). The DGP impulse response function increases and reaches a new steady state level of output instead of converging back to zero. Hours worked, on the other hand, has a transitory response under both specifications. The stochastic trend DGP reveals a hump-shape behaviour of hours worked rather than a sharp increase and decay as per the response of output depicted under a deterministic trend specification. The hump shape response in hours is due to households’
expectation of higher productivity in the future. Households are thus motivated to substitute some of their time toward current leisure and do not initially increase their labour supply as much as in the case of a deterministic trend specification. Later on, hours worked is adjusted to cope with a new long-run level of output. Under both specifications, output and hours worked increase in response to a positive technology shock. Note that we have chosen to only consider hours which are stationary, as standard theoretical models will find it challenging to produce hours which are nonstationary, and even for those that do, their relative fit to the data is mixed (see Chang et al., 2007). Nonstationary hours, which entails a set of specification issues like detrending for hours (e.g. Gali, 1999), or specifying separate trend and cyclical components of hours (e.g. Basistha, 2009), do produce empirical responses of hours decreasing in response to positive technology shocks. This is an admittedly important empirical debate which we do not delve deeper given the focus of our paper will not allow us to be able to adequately address the issue.

Mishandling the trend in output unsurprisingly affects the behaviour of estimated impulse response functions as shown in Figures 1 and 2. Figure 1a plots the estimated impulse response functions from a misspecified SVAR using long-run restrictions along with the true responses derived from RBC-dt. As the econometrician incorrectly first differences output and imposes long-run restrictions, the estimated impulse response functions of output to a positive technology shock converges to a new steady state instead of exhibiting the transitory behaviour implied by the underlying DGP. Furthermore, on average, the estimated responses are biased upward except upon impact. The bias decomposition presented in Figure 1b suggests that the main source of bias is the direct detrending bias. The error in estimating impulse response functions is thus mainly due to mishandling the trend. Comparatively, the indirect detrending bias contributes a smaller fraction to the total bias and only distorts the short-run dynamics. In sum, biases in output stemming from incorrect detrending are non-trivial compared to other sources of biases.

We consider our other misspecification. According to Figure 2a, wrongly imposing short-run restrictions will cause the estimated impulse response functions of output to decay over time even though the true response exhibits a permanent response as implied by RBC-rw. As the zero long-run impact of the output response to technology shock is imposed by the detrending, and then identification, procedure, we should expect a downward bias in the estimated impulse response functions. The downward bias should also get larger at longer horizons because detrending imposes a long-run effect on the system. Figure 2b shows this is indeed the case. The direct detrending bias is the dominant source of error at all horizons and is particularly noteworthy at longer horizons. Our simulations also show that the reversion of the output level to zero can be slow to take effect, perhaps reflecting the permanent shock in the underlying DGP. However, this is insufficient to prevent the obvious downward bias due to mishandling the trend in output.

While one would expect the output response to be severely biased given it is the variable with the misspecified trend, any implications to the correctly detrended variable may not seem obvious at first glance. However, we do find significant spillover of biases to the correctly detrended variable, hours worked, induced by misspecifying the trend in output. That is, the trend misspecification does not only affect the estimated impulse response functions of output to a technology shock. The estimated impulse response functions of hours worked to a technology
shock are also distorted as the system is interdependent. In both misspecifications we consider, the shapes of the impulse response function of hours worked are fairly well preserved. Even so, they both have a tendency to exhibit upward bias in the initial periods after the technology shock. The response in Trend Misspecification 2 though, has a tendency to follow up upward biases with downward biases about 10 periods after the shock. Unlike the estimated impulse response functions of output, the dominant source of biases for hours worked is generally split between the direct and indirect detrending bias. Even so, this serves to reinforce our point that mishandling the trend can lead to severely biased impulse response functions.

We also consider increasing the relative size of the technology shock to the non-technology shock given there is evidence indicating that the SVAR will be better identified when the shock signal is much stronger with a larger relative sized shock (see, e.g. Erceg et al., 2005; Paustian 2007; Chari et al., 2008). In particular, we set the magnitude of the standard deviation of technology shocks to be twice that of the non-technology shock. As depicted in Figure 3, increasing the relative size of the technology shock reduces the non-detrending bias, compared to Figures 1b and 2b. This is a direct consequence of the larger shock signal better identifying the technology shock. However, even so, the detrending bias remains prominent. In principle, it is possible that the non-detrending bias may dominate the detrending bias (both direct and indirect) in larger model structures. However, what our exercise reveals is that the biases induced by trend misspecification do not disappear even under a different parameterisation of the model.$^7$

If anything, a similar degree of detrending bias remains under both parameterisations.

The results from our Monte Carlo study should be sufficient to raise warning flags. Issues like lag length truncation and identification of SVARs receive much attention given their role in allowing the researcher to interpret the data within the context of their chosen model. Even so, our Monte Carlo simulations reveal that trend misspecification is non-trivial, and could even potentially be a greater source of bias compared to these widely studied biases. Spillovers of biases to correctly detrended variables within the system are also non-trivial. Our results reiterate a reminder that researchers should be careful to link their detrending procedures with the identification of the structural model they have in mind.

5.2 Implications for Inference

We explore the effect that trend misspecification has on inference. Statistical inferences are often drawn based on the confidence intervals or confidence sets. Paradoxically, inferences can still be equally valid with or without the statistical bias as long as the biased estimator produces confidence intervals which still accommodate the true model parameters, producing what can be termed unbiased inference. A good example is by Christiano et al. (2003), who show despite the bias of SVARs in mimicking the dynamics of RBC/DSGE models, the inference one draws are not drastically different once sampling uncertainty is incorporated into the analysis. Therefore, while we have established the possibility of trend misspecification severely biasing the estimated

$^7$We discuss the size of the relative variance because there are implications associated with shock identification. In addition, for many parameters in our benchmark case like the discount rate and the capital share are often calibrated, and thus pose less issues fixing those particular parameters. The only other parameter, which may be a source of disagreement is the persistence of the shocks. However, we find the persistence of the shocks only alters the timing, but not the profile of the biases. We address this more explicitly in the supplementary appendix.
impulse response functions, it seems natural to investigate whether this has an effect on inference.

To conduct this exercise, we generate 68% confidence intervals for the impulse response functions on each run of the Monte Carlo simulation in order to investigate the coverage rate of the impulse response functions relative to the true DGP. The choice of 68% is motivated as per the typical choice of VAR practitioners. Our approach of studying the coverage rates of misspecified models relative to the true DGP has strong parallels with Christiano et al. (2003) and Wiriyawit (2014).

In a repeated sampling exercise, if the estimators are unbiased and the measure of sampling uncertainty sound, we can expect coverage rates will roughly coincide with the nominated confidence interval, 68% in our exercise (i.e. the model has good size properties). Of course, the models here are misspecified relative to the DGP derived from the RBC model on some dimension. That is, even correctly detrending the data will give rise to some sources of biases, like the truncation and identification biases as discussed earlier. Therefore, we can only study the marginal change in the quality of inference for the same DGP moving from one with a properly specified trend to one with a misspecified trend.

Table 3 presents the coverage rates of impulse response functions at selected horizons. In the case of having RBC-rw as the DGP, it is clear that trend misspecification deteriorates the quality of inference at most horizons of interest. The coverage rates under trend misspecification are mostly below 50% which is poor from the perspective of using 68% confidence interval. The lower coverage rates given trend misspecification imply that the 68% confidence interval contains the true DGP responses less often than in the case when output is correctly first differenced. The coverage rates for output in the case of C1 tends to be very large as output is first differenced, and thus cumulated. Through differencing, these output responses are estimates of sums and therefore are susceptible to large variances. The coverage rates for M2 for hours is close to zero at short horizons and is noticeably worse than when the trend is properly specified (C1).

In the case of having RBC-dt as the DGP, it is not immediately clear that trend misspecification strictly deteriorates the quality of inference. We first focus on coverage rates for the response of output. The coverage rates for the output response when the econometrician correctly specifies a trend stationary process vary from 2% to 68%. The corresponding coverage rates for output response when incorrectly using long-run restrictions are however above 70%. The excessively high coverage rate even though the econometrician incorrectly differencing output in this case is once again, due to a consequence of cumulative impulse response functions. This thus explains the excessively high coverage rate yielding less than meaningful inferences under this trend misspecification case. Coverage rates for the response of hours worked, on the other hand, have different pattern. At a horizon 20 periods after the shock, correctly linearly detrending and then applying the Cholesky decomposition yields coverage rates for the response of hours worked at the horizons of interest of around 60%, close to nominal coverage. Coverage rates under misspecification at short horizons appear to be close to nominal coverage, but fall to zero at a horizon of 20 quarters. Therefore,

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8For each data set generated by the DGP, we first estimate the VAR impulses. Thereafter, we bootstrap using the bias corrected bootstrap recommended by Kilian (1998). We then apply the relevant detrending and identification strategy to generate the distribution of the impulse response functions. We then count the proportion of simulations where the 68% confidence interval contains the true DGP responses to obtain the coverage rate of the corresponding impulse response function.
it appears in the case of a deterministic trend in output, trend misspecification only seems to have an effect on the coverage rates at long horizons, but misspecifying the trend would produce good inferences at short horizon. This is however, most likely due to the identification strategy involved. The long-run restrictions does not impose zero restrictions on any of impact multipliers, unlike the Cholesky decomposition. Therefore, with the virtue of the identification strategy being less restrictive at shorter horizons has allowed the misspecified model to produce better inference at shorter horizons. This is also noticeably that the bias at horizon zero in Figures 1b and 3a are close to zero for the same reason, but become larger at longer horizons. However, once we move out to longer horizons, the correctly detrended model achieves close to nominal coverage.

With the exception of the feature of the long-run restrictions imposing less restrictive impact restrictions, which aids the inference within the model with the misspecified trend in the case of a deterministic trend in output at very short horizons, the overall conclusions are similar to the analysis of the biases. Considering a role for sampling uncertainty still results in the same biases carrying over when the trend is misspecified.

5.3 VARMA Structure

We focus attention to possible solutions in this section. To identify the source of trending bias, recall our true VAR structure

$$\Phi(L)(y_t - y^T_t) = u_t$$

where the true model log variables $y_t$ is measured as a log deviation from the correct trend component $y^T_t$ and $u_t$ is the (reduced form) innovation.

Suppose that an econometrician makes a wrong assumption on the trend component, resulting in an incorrect detrending methodology imposed on SVAR model. We can thus write the VAR structure as

$$\Phi(L)(y_t - y^*_t + y^*_t - y^T_t) = u_t,$$

$$\Phi(L)(y_t - y^*_t) = u_t - \Phi(L)(y^*_t - y^T_t)$$

where $y^*_t$ is trend component assumed by the econometrician and can either be a unit root or a deterministic trend. If the assumed trend coincides with the true one, then $y^*_t = y^T_t$. The final term $\Phi(L)(y^*_t - y^T_t)$ thus drops out and the VAR is estimated correctly. Otherwise, as $y^*_t$ is not identical to $y^T_t$, the above expression becomes a VARMA structure and the last term is the source of direct detrending bias. It is also straightforward to see the source of the indirect detrending bias. In particular, the composite error term estimated by the VAR will now be $u_t - \Phi(L)(y^*_t - y^T_t)$. The additional term $\Phi(L)(y^*_t - y^T_t)$ is the source of the indirect detrending bias because the identification of the SVAR will involve decomposing $u_t - \Phi(L)(y^*_t - y^T_t)$ instead of just $u_t$.

The VARMA structure presents a means of alleviating the bias induced by incorrectly detrending. Under the Wold representation, a VARMA is just a reparameterisation of a VAR of infinite lag order. Therefore, a simple practical solution may be to just increase the lag order of the VAR. To see if this indeed holds true, we study the problem first in population.
We isolate the direct detrending bias from indirect detrending bias by imposing a theoretical restriction from our DGP to an estimated SVAR. This methodology is implemented with guidance from the approach used by Kapetanios et al. (2007) and Ravenna (2007). The theoretical identification is then imposed by setting the first row of impact matrix $A$ corresponding to the impact of a technology shock is set to be consistent with the structural shock defined in RBC. By doing so, we eliminate the indirect detrending bias and can focus only on the direct detrending bias. We then repeat the simulation exercises presented in Section 4 imposing this new identification scheme in population ($N = 20,000$). We also vary the lag length from $p = 2$ to $p = 100$ imposed in an estimated SVAR model.

The findings are presented in Figure 4. Once output is incorrectly detrended, we can observe larger bias due to an emerging of a direct detrending bias in impulse response estimates, as shown in Figure 4a. However, by including more lags in an estimated SVAR, it is clear that the direct detrending bias gets smaller. Particularly, short lag SVAR still shows permanent property of estimated technology shock to output at longer horizon as an econometrician incorrectly first difference output. Longer lag SVARs, on the other hand, can better capture the transitory property of the true technology shock to both output and hours worked. The hours worked responses of both specifications further underscores this point. If we increase the lag order of the VAR, the hours worked response does become much closer to the true DGP even if the trend is misspecified.

However, the preceding discussion may only remain a theoretical point. As Figure 4 illustrates, to be able to capture the true impulse responses given trend misspecification, the SVAR may require about 100 lags, which is not practical in empirical work. Moreover, this is also conditioned on one possessing perfect knowledge of the identification of the shocks in the system. Given we make the point that the identification is explicitly linked to the detrending procedure, it is not certain such a lag order can do an adequate job even if it is feasible for empirical work.

The natural follow up question is therefore how knowledge of a VARMA structure is useful for practical purposes. Given the experiment which isolates the identification problem appears quite promising in population, we apply similar ideas to finite samples. We simulate data from our DGPs, misspecify the trend relative to the DGP and then choose the VAR lag order through three common lag order procedures. These lag order procedures are the common information criteria, namely the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and a likelihood ratio test of the lag order. The likelihood ratio test starts by tests for the statistical significance of the $p^{th}$ lag of a VAR($p$) against a VAR($p-1$). This procedure is repeated until the null is rejected. Note that as the true lag order implied by our DGP is theoretically a VAR($\infty$), the true lag order will never be within the set of models evaluated by the information criteria.

The results of our lag length testing procedure is presented in Figure 5. The AIC and BIC will almost certainly choose a lag order that is too low. The likelihood ratio tests, on the other hand, may seem increasing the lag order appears to worsen inference for output in the misspecification of the RBC-rw DGP, as per Figure 4b, this is not true. Because the DGP is in output growth rates instead of levels, the dynamics of the growth rates is better captured by the long order VAR. We provide this upon request. Within the same system, the hours worked response further illustrates our point that the longer order VAR is better able to mitigate trend misspecification.
hand, sometimes pick lag orders up to 12. In fact, while the AIC and BIC will almost exclusively choose 1 or 2 lags, the likelihood ratio tests picks a VAR lag order larger than 2 about 40% of the time. This result has implications for applied work. Much is known about how AIC and BIC selects between models if the correct lag order is in the model set. However, given we know the true lag order is theoretically a VAR(∞), and the true lag order is not in the model set, little is known how the AIC and BIC operates in such settings. As the AIC and BIC penalise additional parameters, it is not surprising they pick a low lag order. While a likelihood ratio test is more likely to pick a higher lag order, given that most of the lag parameters beyond a second lag are usually close to zero, rejecting the null and concluding the high lag order is statistically significant can still be challenging, with about 60% of our Monte Carlo trials picking a lag order of 1 or 2.

Empirical work in more recent years have adopted rule of thumb for lag orders included in a VAR model. For example, researchers working in quarterly data often choose 4 lags and monthly data choose 12 lags to adequately attempt to model one year’s worth of model dynamics. Within the realms of possible trend misspecification, such an approach appears to be a sensible compromise. Figure 6 shows the relative bias of an SVAR(4) to an SVAR(1) and SVAR(2) in finite samples when the trend is misspecified. While some isolated horizons may have a smaller bias with the SVAR(1) or SVAR(2), the SVAR(4) does in generally appear less biased when the trend is misspecified, suggesting there is some merit in researchers adopting rule of thumb lag orders rather than relying on lag order tests.

6 Conclusion

In this paper, we highlight that the choice of detrending is linked to SVAR identification. While identification in SVARs receive much attention given its role for sensible empirical analysis, trend misspecification remains a possible blindspot for researchers using SVARs.

In an illustrative example using a Monte Carlo study, we demonstrate that the biases directly attributable to trend misspecifications can be non-trivial relative to many commonly studied SVAR biases. We are deliberately minimalist with the SVAR structure. Our approach is largely designed for the purpose of isolating the different sources of biases by keeping any potential known biases to a minimum. This serves in keeping the analysis tractable while drawing attention to the role of trend misspecification. While our exercise can be construed as being model, or even parameter, specific, one cannot rule out the empirical possibility of significant biases emanating from trend misspecification as the key source of biases in the SVAR.

A practical solution is increasing the lag order can somewhat mitigate trend misspecification. This appears to be a more general, rather than model specific, point. A natural question is how important are then these sources of biases within richer and larger model environments. To this end, the decomposition of the biases which we put forward in this paper and also some of the suggested practical implications like lag order selection are useful tools for addressing these questions in future research.
References


Table 1: List of Parameters Specified in the Model

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>True Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ (Discount factor)</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha$ (Capital share)</td>
<td>0.33</td>
</tr>
<tr>
<td>$\delta$ (Depreciation rate of capital)</td>
<td>0.025</td>
</tr>
<tr>
<td>$\eta$ (Inverse short-run labor elasticity)</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma$ (Average growth rate of technology shock)</td>
<td>1.0074</td>
</tr>
<tr>
<td>$\rho_z$ (Persistence in technology shock)</td>
<td>0.25 (RBC-rw)</td>
</tr>
<tr>
<td>$\rho_B$ (Persistence in non-technology shock)</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_z$ (Standard deviation of technology shock)</td>
<td>0.0148</td>
</tr>
<tr>
<td>$\sigma_B$ (Standard deviation of non-technology shock)</td>
<td>0.009</td>
</tr>
</tbody>
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Table 2: Monte Carlo Simulation

<table>
<thead>
<tr>
<th>SVAR</th>
<th>DGP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RBC-rw: $Y_t \sim I(1)$</td>
</tr>
<tr>
<td>Long-Run Restrictions</td>
<td>Correct Trend Specification 1</td>
</tr>
<tr>
<td>Short-Run Restrictions</td>
<td>Trend Misspecification 2</td>
</tr>
</tbody>
</table>

Table 3: Coverage Rates of Impulse Response Functions to a Technology Shock

<table>
<thead>
<tr>
<th>DGP</th>
<th>RBC-rw: $Y_t \sim I(1)$</th>
<th>RBC-dt: $Y_t \sim I(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVAR</td>
<td>Long-Run Restrictions (C1)</td>
<td>Short-Run Restrictions (M2)</td>
</tr>
<tr>
<td>Variable</td>
<td>Period</td>
<td>Coverage Rates</td>
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<tr>
<td>Output ($Y_t$)</td>
<td>0</td>
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<tr>
<td></td>
<td>1</td>
<td>0.6843</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.7725</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.8553</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.9045</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.9863</td>
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<tr>
<td>Hours Worked ($H_t$)</td>
<td>0</td>
<td>0.5470</td>
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<tr>
<td></td>
<td>1</td>
<td>0.5613</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.5880</td>
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<td></td>
<td>3</td>
<td>0.6018</td>
</tr>
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<td></td>
<td>4</td>
<td>0.6375</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.3768</td>
</tr>
</tbody>
</table>

Note: C1 and C2 are Correct Trend Specification 1 and 2 whereas M1 and M2 are Trend Misspecification 1 and 2 in the Monte Carlo exercises.
Figure 1: Monte Carlo Simulations Implementing Long-Run Restrictions Given RBC-dt as the DGP (Trend Misspecification 1)

(a) Impulse Response Functions to a Positive Technology Shock

(b) Total Bias Decomposition
Figure 2: Monte Carlo Simulations Implementing Short-Run Restrictions Given RBC-rw as the DGP (Trend Misspecification 2)

(a) Impulse Response Functions to a Positive Technology Shock

(b) Total Bias Decomposition
Figure 3: Bias Decomposition Given a Smaller Variance in Non-Technology Shock in the DGP

(a) Given RBC-dt as the DGP, Impose Long-Run Restrictions

(b) Given RBC-rw as the DGP, Impose Short-Run Restrictions
Figure 4: Impulse Response Functions to a Positive Technology Shock Imposing Theoretical Identification Given Trend Misspecification in SVAR

(a) Given RBC-dt as the DGP, Impose Long-Run Restrictions

(b) Given RBC-rw as the DGP, Impose Short-Run Restrictions
Figure 5: Lag Length Test

Note: Trend Misspecification 2 refers to having RBC-rw as the DGP and imposing short-run restrictions in an estimated SVAR. Trend Misspecification 1 refers to having RBC-dt as the DGP and imposing long-run restrictions in an estimated SVAR.
Figure 6: Relative Bias in Impulse Response Estimates With Respect to SVAR(4) when the Trend is Misspecified

(a) Given RBC-dt as the DGP, Impose Long-Run Restrictions

(b) Given RBC-rw as the DGP, Impose Short-Run Restrictions
Supplementary Material for "Structural VARs, Deterministic and Stochastic Trends: Does Detrending Matter?"

Varang Wiriyawit*        Benjamin Wong†

The supplementary appendix provides detailed explanation on the theoretical framework’s First Order Conditions. Within this appendix, we also discuss alternative experiments for robustness and completeness of the paper. The main findings of the paper are shown to be robust to changes in degree of persistence in the permanent technology shock and even if we use an SVAR as a Data Generating Process (DGP) instead of a Real Business Cycle (RBC) model.

A1 First Order Conditions of RBC Model

Given an RBC framework presented in the paper, this section goes through the first order conditions for households’ and firms’ maximisation problems. Competitive equilibrium under this framework is also defined.

In this theoretical model, households maximise their utility by choosing each period consumption ($C_t$), hours worked ($H_t$) and next-period capital holdings ($K_{t+1}$) subject to their budget constraint, a capital accumulation equation and a stationary AR(1) exogenous process ($B_t$). Let $I_t$ be investment. The First Order Conditions for households’ utility maximisation are then given by

\[ \frac{1}{C_t} = \beta \mathbb{E}_t \left\{ \frac{1}{C_{t+1}} \left( R_{t+1} + 1 - \delta \right) \right\} \]
\[ \frac{1}{C_t} = \frac{H_t^\eta}{B_t^{1+\eta} W_t} \]

where $\beta \in (0,1)$ is a discount factor, $\eta > 0$ is an inverse Frisch elasticity of labour supply, $\delta \in (0,1)$ is a depreciation rate of capital, $\rho_B \in (0,1)$ is a measure of persistence and $\epsilon_{B,t+1} \sim \mathcal{N}(0,\sigma_B^2)$ is an exogenous Gaussian shock process.

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These necessary conditions characterise optimal decision rules for households. Equation (1) is an Euler equation for consumption stating that the marginal rate of substitution between consumption at period \(t\) and consumption at period \(t+1\) equals the return of capital. Equation (2) is a labour supply equation stating that the marginal rate of substitution between consumption and leisure must equal the wage rate.

Firms maximise their profit subject to the labor-augmenting Cobb-Douglas production function and a technology innovation to productivity. The First Order Conditions for firms’ profit maximisation are then given by

\[
R_t = \alpha K_t^{\alpha-1} (Z_t H_t)^{1-\alpha} \\
W_t = (1-\alpha) Z_t^{1-\alpha} K_t^{\alpha} H_{t-\alpha}.
\]

where \(\alpha \in (0,1)\) is a capital share and \(Z_t\) is a technology innovation to productivity.

Equations (3) and (4) imply that the rental rate of capital \((R_t)\) and the wage rate \((W_t)\) are set equal to the marginal productivity of an additional unit of capital and labour respectively.

**Competitive Equilibrium Definition**

The competitive equilibrium is defined as follows. In a competitive equilibrium, households choose allocations of \(\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}\) and firms will choose allocations of \(\{K_t, H_t\}_{t=0}^{\infty}\) such that, given a sequence of prices \(\{W_t, R_t\}_{t=0}^{\infty}\) and exogenous shocks to \(\{Z_t, B_t\}_{t=0}^{\infty}\), households and firms optimise their utility and profit respectively with the market clearing such that \(Y_t = C_t + I_t\).

**A2 Alternative Experimental Set-up**

We examine alternative experimental set-ups to explore scenarios that an econometrician may face. These settings range from setting different parameter values used in the DGPs to alternative detrending strategies the econometrician might employ.

**Degree of Persistence in the Permanent Technology Shock**

In contrast to the theoretical responses we have in the paper, some empirical studies suggest that a positive permanent technology shock should lead to a fall in hours worked instead of an increase (see e.g. Galí 2004; Francis and Ramey 2005; Kimball et al. 2006). As shown by Lindé (2009), a reparameterisation of the model by increasing the persistence of the permanent technology shock can produce a negative response of hours worked to a technology shock. We therefore repeat the simulation exercises with RBC-rw as the DGP and increase the persistence in the permanent technology shock from 0.25 to 0.5. Other parameter values remain the same as used in the paper. In this subsection, the SVAR with long-run restrictions is the correct trend specification.

Figure A1 presents the result if we misspecify the trend as a deterministic trend and use the Cholesky decomposition to identify the technology shock. The output response to a technology shock is badly biased in Figure A1a. The results also reveal that none of the impulse response functions are able to capture the fall in hours worked as per the reparameterised DGP. The
decomposition reveals that the predominant sources of biases is the indirect detrending bias for hours worked and a mix of both types of detrending bias for output. However, the absolute bias is much larger than the specification with less persistence in the growth rate of the technology shock. Note that there is a large degree of non-detrending bias. This suggests that even if the trend was properly specified, the total bias would still be large. This is not entirely surprising. Given a more persistent growth rate of the shock in the DGP, it is going to be the case that a higher lag order is needed to fully model the dynamics. The truncation of the lag length is thus going to be significant, and so unsurprisingly feeds into the non-detrending bias component. Even so, the source of this bias is still not as dominant as the ones induced from mishandling the trend.

**Alternative Detrending Strategies**

The Hodrick-Prescott (1997) (HP) filter is a widely-used tool in empirical studies to extract the cyclical component of the series. It is therefore worth investigating the performance of the filter in estimating the true impulse response functions when the econometrician does not have adequate information regarding the underlying trend process in output. We consider a case where an econometrician employs the HP filter on the output series as a detrending methodology, and then implements short-run restrictions to identify structural shocks in the system. Figure A2 plots the total bias given the DGPs of interest with different degrees of persistence in the technology process. We find that the performance of the HP filter depends on the degree of persistence in the technology shock. As discussed by Canova and Ferroni (2011), the relatively persistent shock process in an RBC model would produce the variability of the series at longer horizons. The HP filter however attributes the low frequency cycle as the non-cyclical component, and thus measures the true cyclical component with error. As a consequence, the higher the persistence in the technology shock, the larger the detrending bias induced by the HP filter. Another way of thinking about this is if there is a persistent component in the cycle, the HP filter will mistake part of this persistent cycle as a trend and systematically filters it. If one expects the persistence of technology shocks to be large empirically, we should not expect the performance of the filter to be satisfactory.

At this stage, one might suspect that estimating all series in differences may be flexible enough to consider both transitory and permanent shocks without the econometrician taking a stand. While differencing does conceptually produce permanent shocks, it is possible that empirical exercises may produce impulse response functions which are transitory. We therefore consider a common empirical strategy of just first differencing and imposing a Cholesky decomposition to identify the technology shock. Note that such an approach has no theoretical support as both shocks are permanent. We therefore generate the RBC model with two transitory shocks, RBC-dt, and then first difference output and take a Cholesky decomposition to explore whether the original transitory technology shock can be recovered. Figure A3

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1 As we simulate quarterly data, the smoothness parameter for the HP filter is set to 1600 as Hodrick and Prescott (1997) recommend.

2 In this exercise, the degrees of persistence used under RBC-dt are 0.25 and 0.9 for low and high persistence respectively. Under RBC-rw, 0.25 and 0.5 are set respectively for low and high persistence in the growth rate of a technology shock.
suggests that confidence in recovering underlying transitory shocks despite first differencing may be misplaced. In particular, none of the impulse response functions are able to recover the underlying transitory response of output. Moreover, the impulse response functions are badly biased, missing both the short and long-run properties of the underlying DGP. The intuition is similar to that of Fisher et al. (2014). By not distinguishing between permanent and transitory shocks, the identification mixes up both the transitory and permanent components. This reveals an uncomfortable reality of SVAR practitioners. Taking a stand on the underlying shock properties and modelling them as such is mandatory. As this is an identification exercise, the data cannot speak without imposing any structure. First differencing as an empirical strategy, with the intention of being agnostic about the transitory or permanent nature of shocks, is unfortunately misguided.

A3 Using an SVAR DGP

We also consider experiments in which our DGP is an SVAR instead of an RBC. The true parameter values are from an estimation of U.S. quarterly data from 1948 - 2014. We repeat the same simulation setup switching the RBC model with the SVAR DGP. More specifically, we generate quarterly data from the true SVAR model assuming either a stochastic trend or a deterministic trend. We then estimate an SVAR on the simulated data and misspecify the trend. Note that, in the SVAR DGP setup, we assume that econometricians are able to identify the number of lags they should include in the estimated model correctly, which we have set to $p = 4$. Therefore, the non-detrending bias in this experiment is due to other factors such small sample bias.

The findings presented in Figures A4 and A5 arrive at the same conclusion as having an RBC as the DGP. Incorrectly detrending output once again causes the shock properties to deviate from the true DGP. For example, first differencing output once again results in permanent effect on estimated impulse response of output from a technology shock even though, in DGP with a deterministic trend, the true impulse response would have a transitory response. Once again, the bias decomposition also reveals that the detrending bias is non-trivial and remains a potential source of biases arising from trend misspecification in an estimated SVAR model.

This alternative experiment set-up thus confirms the main results we present in the paper.

Moreover, we also investigate the impact of trend misspecification on the estimation of impulse response functions given an SVAR framework which has both deterministic and stochastic trends as the DGP. The Monte Carlo simulations and total bias decomposition assuming either a stochastic trend or a deterministic trend in an estimated SVAR model are presented in Figures A6 and A7 respectively. In the case of implementing long-run restrictions, we do not have indirect detrending bias arising from imposing incorrect identification strategy following an incorrect trend assumption. This is because we also assume long-run restrictions in the DGP. According to Figure A7b, the size of the non-detrending bias is larger than the direct detrending bias. Assuming a stochastic trend thus seems to be a flexible trend specification to detrend series which contains both trends. On the other hand, assuming a deterministic trend in an estimated SVAR model induces a relatively large detrending bias and consequently large downward total bias in the estimated impulse response functions.
References


Figure A1: Monte Carlo Simulations Implementing Short-Run Restrictions Given RBC-rw as the DGP (Trend Misspecification 2) where Hours Worked Responses Negatively to a Positive Technology Shock

(a) Impulse Response Functions to a Positive Technology Shock

(b) Total Bias Decomposition
Figure A2: Total Bias Using the HP Filter and Implementing Short-Run Restrictions

Figure A3: Monte Carlo Simulations Using First Difference and Implementing Short-Run Restrictions
Figure A4: Monte Carlo Simulations Implementing Long-Run Restrictions Given SVAR with a Deterministic Trend as the DGP

(a) Impulse Response Functions to a Positive Technology Shock

(b) Total Bias Decomposition
Figure A5: Monte Carlo Simulations Implementing Short-Run Restrictions Given SVAR with a Stochastic Trend as the DGP

(a) Impulse Response Functions to a Positive Technology Shock

(b) Total Bias Decomposition
Figure A6: Monte Carlo Simulations Implementing Long-Run Restrictions Given SVAR with both Stochastic and Deterministic Trends as the DGP

(a) Impulse Response Functions to a Positive Technology Shock

(b) Total Bias Decomposition
Figure A7: Monte Carlo Simulations Implementing Short-Run Restrictions Given SVAR with both Stochastic and Deterministic Trends as the DGP

(a) Impulse Response Functions to a Positive Technology Shock

(b) Total Bias Decomposition