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Using estimated models to assess nominal and real rigidities in the United Kingdom*

Güneş Kamber and Stephen Millard†

Abstract

This paper aims to contribute to our understanding of inflation dynamics in the United Kingdom by estimating two dynamic stochastic general equilibrium models and assessing the role of nominal and real rigidities within them. We first obtain an empirical representation of the monetary transmission mechanism in the United Kingdom and then estimate the models by minimising the difference between this representation and its model equivalents. We find that both models can explain the data reasonably well without relying on undue amounts of price and wage stickiness.

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1 Introduction

Most monetary policy makers focus on achieving price stability: typically defined as low and stable inflation. But in order to achieve price stability, it is important to understand what the dynamics of prices (and inflation) are, what drives them and, perhaps most importantly, how monetary policy fits into this, ie, how the monetary transmission mechanism works. For many years, the traditional Phillips curve provided the standard framework for understanding inflation dynamics. But this is a reduced-form approach. This led to the development of models with explicit microfoundations of optimising behaviour, imperfect competition, and 'sticky' prices at the microeconomic level. The most popular of these models has been the Calvo pricing model (based on Calvo (1983)), where individual companies have an exogenous probability of being able to change prices in any given period. Because of this fixed probability, companies who are changing their prices have to consider what future prices are (and will be) optimal in case they do not get the chance to change prices again for some time. This intuition results in a derived 'New Keynesian' Phillips Curve (NKPC) that relates inflation this period to expected inflation in the next period, and to the deviation of real marginal cost from trend.

But, although the NKPC is a useful framework for thinking about the monetary transmission mechanism and how various shocks might affect inflation, it cannot be used to provide quantitative predictions unless it forms part of a general equilibrium model. Estimating the key parameters within the general equilibrium model allows us to assess the uncertainty around the parameters themselves and, hence, predictions generated by the model. This has led to a number of recent papers in which authors have used different techniques to estimate dynamic stochastic general equilibrium (DSGE) models based around the NKPC. For example, Smets and Wouters (2003), Smets and Wouters (2007) and Harrison and Oomen (2010) use Bayesian techniques to estimate a New Keynesian model on data from the euro area, the United States and the United Kingdom, respectively. Gertler, Sala, and Trigari (2008) also used Bayesian techniques and US data to estimate a New Keynesian model with search and matching in the labour market. The key advantage of Bayesian techniques are that, in theory, they can provide a complete description of the data generating process and, so, allow you to test hypotheses within the DSGE models, evaluate their relative performance against each other, and use them to run forecasts. Against this, the parameter estimates seem to be driven by the priors and the choice of priors will also affect model comparisons. (See del Negro and Schorfheide (2008).)

These problems motivate an alternative approach. Initiated by Rotemberg and Woodford (1997), the 'minimum distance' approach has been widely used to assess the empirical performance of DSGE models. For example, Amato and Laubach (2003) analyse the welfare implications of various interest rate rules using an estimated model with sticky wages and prices. Boivin and Giannoni (2006) examine the change in the effectiveness of the monetary policy in the United States for the pre and post-Volcker periods. Meier and Muller (2006) quantify the role of financial frictions in the transmission of monetary policy shocks. The idea of this approach is to obtain values of the parameters so that the model matches as closely as possible those features of the data in which you are particularly interested.

In this paper, we use the minimum distance approach to estimate two DSGE models using UK data: the models of Smets and Wouters (2003) and Gertler et al. (2008). In both cases, we are interested in estimating the parameters of our models so as to match as closely as possible the responses of variables to a movement in interest rates. This is motivated by our particular interest in understanding inflation dynamics within the United Kingdom and the UK monetary transmission mechanism in particular. The Smets and Wouters (2003) model has become a 'workhorse' DSGE model and has been estimated using Bayesian methods on both US and euro-area data. But in the Smets and Wouters (2003) model, as is the case for most models based on the NKPC framework, the labour market is modelled as a spot market with no realistic distinction being made between heads and hours.[1] A long tradition in monetary economics, starting with Phillips (1958), has assigned labour market frictions and, in particular wage-setting frictions, a central role in inflation dynamics. This motivates consideration of the Gertler et al. (2008) model in which the labour market is modelled more explicitly within the New Keynesian framework. More specifically, the model appends a variant of the Mortensen and Pissarides (1994) model of search and matching frictions to the New Keynesian framework.

Our paper makes three contributions to the literature. First, estimating the Gertler et al. (2008) model on UK data and comparing our estimates with those obtained in the original paper using US data enables us to assess how similar the United Kingdom is to the United States and where differences may lie, eg, in the degree of nominal price and wage rigidity, or in the bargaining power of workers. Second, by comparing our results across the two models,

[1] In Smets and Wouters (2003), workers are assumed to have market power with the result that there is a difference between the amount of labour supplied in equilibrium and the amount that would be supplied if this distortion were not there.

we can assess the importance of explicitly modelling unemployment for understanding inflation dynamics; in particular, once you have controlled for total hours worked/employment, does unemployment/labour market tightness give you any additional information about the effects of movements in interest rates on inflation? The results of Gertler et al. (2008) suggest, that in the United States, the Smets and Wouters (2003) model explains inflation and output well enough and that the gain to introducing unemployment explicitly is solely that you can tell coherent stories about unemployment itself; a key point of our paper is to see whether or not this result holds for the United Kingdom. Third, we are able to assess the importance of nominal and real rigidities within the United Kingdom by comparing the fit of the two models with and without such rigidities.

The paper is structured as follows. We first use a structural vector autoregression (SVAR) approach to obtain an empirical representation of the monetary transmission mechanism, ie, how a movement in interest rates affects some important macroeconomic variables in the United Kingdom. We then discuss the two models we are going to estimate before moving on to discuss the estimation strategy. In brief, our aim is to obtain values for the parameters of the two models that enable them to replicate the empirical representation of the monetary transmission mechanism we found in Section 2. After discussing our estimation strategy, we present our results before concluding.

2 Monetary transmission in the United Kingdom

We estimate a nine-variable SVAR in order to identify the effects of a monetary policy shock on macroeconomic variables in the United Kingdom. DiCecio and Nelson (2007) find that the break date on a VAR similar to ours is located between 1977-81 and they argue that 1979 Q2 constitutes an important monetary and government policy regime change. Given that, our estimation period starts in the second quarter of 1979. Of course, there have been subsequent changes in the UK monetary policy regime; indeed, Nelson (2003) identifies regimes lasting from 1979-87, 1987-90 and 1992-97. But, given the problems with estimating a VAR on a short sample, we chose to follow DiCecio and Nelson (2007) and assume that these different monetary policy regimes were all compatible with the same implied policy reaction function.

In their work, DiCecio and Nelson (2007) estimated a six-variable VAR including real GDP, real consumption, real investment, labour productivity, the Treasury bill rate and retail price inflation. To these variables, we added capacity utilisation, the relative price of investment goods and the real wage.[2] This left us with a similar list to Altig, Christiano, Eichenbaum, and Linde (2010), although they also included the money supply.[3] We also took care to use variables in the VAR that were consistent with their model counterparts. So, we used consumption spending on non-durables per head of population for consumption, c. Investment, I, was defined as business investment plus consumption spending on durables per head of population. Output was defined as the sum of these two series ($y = c + I$). Our series for inflation used the implied output deflator, given our definition of output. We calculated our real wage series by dividing the nominal private sector wage per worker by this deflator. We calculated the relative price of investment goods implied by our investment and output series. To these series we added private sector employment per head of population – dividing our output measure by this variable to create a measure of productivity – capacity utilisation, and the Bank of England's official interest rate.[4]

In order to obtain sensible results from the VAR – and to make the variables used comparable to the model – it was necessary to detrend the variables in the VAR. We chose to assume a log-linear trend when detrending each of our variables as this seemed to fit the data reasonably well and ensured that the resulting data were stationary. We examined the robustness of our results to alternative assumptions about the trends; in particular, we found the results to be robust to using a quadratic trend, a cubic trend or an HP-filter to detrend the data. We also considered using the approach of Altig *et al* (2010). However, in our data set the consumption to output and investment to output ratios are trending, probably a result of the short sample period. So, using the Altig *et al* approach does not result in stationary variables in

[2] Given that the Gertler et al. (2008) model we consider is designed specifically to model frictions in the labour market, it would seem particularly important to include real wages and employment within our set of variables. Indeed, it could be argued that we should also include the unemployment rate, but this variable has no analogue within the Smets and Wouters (2003) model, so we leave it out.

[3] In addition, Altig et al. (2010) transform their variables so as to make them stationary; their final list is: Change in the relative price of investment goods, productivity growth, inflation, capacity utilisation, total hours worked per head of population, the labour share, the shares of conusmption and investment in output, the nominal interest rate and the change in the velocity of money.

[4] We used 'employment' rather than 'total hours worked' in order to keep our results comparable to those in DiCecio and Nelson (2007) and Altig et al. (2010).

our VAR.

In summary, we estimate a VAR with the following nine variables :

$$\mathbf{Y}_t = \begin{pmatrix} \ln\text{ (output)} \\ \Delta\ln\text{ (output deflator)} \\ \ln\text{ (consumption)} \\ \ln\text{ (investment)} \\ \ln\text{ (real wage)} \\ \ln\text{ (productivity)} \\ \text{capacity utilisation} \\ \ln\text{(relative investment price)} \\ \text{nominal interest rate} \end{pmatrix}$$

In order to identify monetary policy shocks, we follow the identification strategy used in Altig et al. (2010). The monetary authority is assumed to operate according to a rule which takes the following form:

$$1 + rg_t = f\{\Omega_t\} + \varepsilon_{r,t} \tag{1}$$

where rg_t is the nominal interest rate and Ω_t is the information set of the monetary authority as of time t.

The SVAR representation is given by:

$$\mathbf{A}_0\mathbf{Y}_t = \mathbf{A}(\mathbf{L})\mathbf{Y}_{t-1} + \varepsilon_t \tag{2}$$

We estimate the reduced-form VAR with the variables in $\mathbf{Y}_t$. That is:

$$\mathbf{Y}_t = \mathbf{B}(\mathbf{L})\mathbf{Y}_{t-1} + \mathbf{u}_t \tag{3}$$

where $\mathbf{u}_t$ are the reduced-form residuals. In order to recover the structural shocks, ε_t, we assume that the relationship between the reduced form and structural errors are given by:

$$\mathbf{u}_t = \mathbf{C}\varepsilon_t \tag{4}$$

where $\mathbf{C}$ is a lower triangular matrix. Since rg is ordered last in the vector of variables $\mathbf{Y}$, this identification strategy implies that none of the variables in our VAR respond contemporaneously to the monetary policy shock. With this assumption, the relationship between the parameters of the reduced-form and SVAR representations is given by:

$$\mathbf{C} = \mathbf{A}_0^{-1} \quad \mathbf{B}(\mathbf{L}) = \mathbf{A}_0^{-1}\mathbf{A}(\mathbf{L}) \tag{5}$$

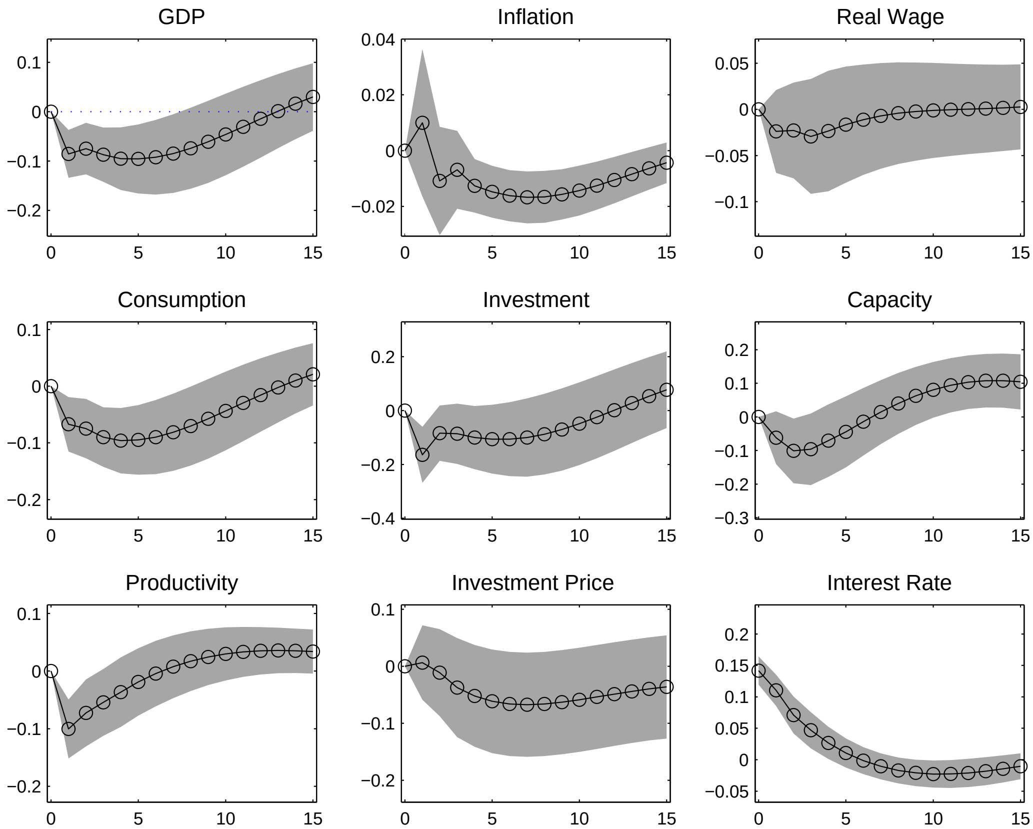

Figure 1
Impulse responses to monetary policy shock

Chart 1 displays the impulse response functions (IRFs) to a one standard deviation increase in the interest rate.[5] The solid line is the estimated response and the shaded areas correspond to 90% confidence intervals. We summarise our results by comparing them with the effect of monetary policy shocks in the United States.[6] The following results are similar:

- The responses of output, consumption, investment and capacity utilisation are hump shaped. The peak response of output occurs five quarters after the shock.
- The inflation response is hump shaped with a peak after two years and

[5] Reponses are measured as the percentage deviations from trend except for inflation and interest rates, which are measured as the percentage point deviation from base.

[6] Our comparison takes the results in Altig et al. (2010) as the 'benchmark' response to a monetary policy shock in the United States.

the effect on inflation of a monetary policy shock dies out after three years. There is also a price puzzle lasting one period, but this is not statistically significant.

- The responses of the relative price of investment and real wages are effectively zero.[7] For the relative price of investment goods we might expect this; for real wages, the result suggests that there are significant real wage rigidities in the United Kingdom.
- The peak response of productivity is one period after the shock. Given the response of GDP, this path suggests that the adjustment in labour input occurs with a lag relative to the response of output.
- Following a monetary policy shock, the investment response is only slightly higher than the response of output. Cyclical investment is however 2.2 times more volatile than cyclical output.[8]

In Chart 2, we present the IRFs of output, inflation, productivity and real wage from rolling sample estimates of the VAR. The responses of output, and productivity are broadly stable over time. The inflation response seems to display a larger 'price puzzle' towards the end of the sample. The real wage also increases after a positive interest rate shock at the end of the sample, but this effect is not statistically significant.[9]

3 Theoretical models

In this section, we discuss the two small-scale DSGE models that we will be estimating on UK data. The models are almost identical except for the functioning of the labour market. The first model, developed in Smets and Wouters (2003), assumes the household is a monopoly supplier of a differentiated labour service, while the second, Gertler et al. (2008), represents the labour market with search and matching frictions. This difference is key since it introduces 'unemployment' into the model in such a way as to match how unemployment is measured in the data. Shimer (2005) showed that the search model was unable to match the volatility of unemployment in the data

[7] We do not use the response of relative price of investment in the estimation as the theoretical models assume that this variable is not affected by the monetary policy shocks.
[8] We define cyclical investment and output as the logarithm of the quarterly investment and output series that are HP filtered with a smoothing parameter of 1600.
[9] The standard errors of individual IRFs are available from authors upon request.

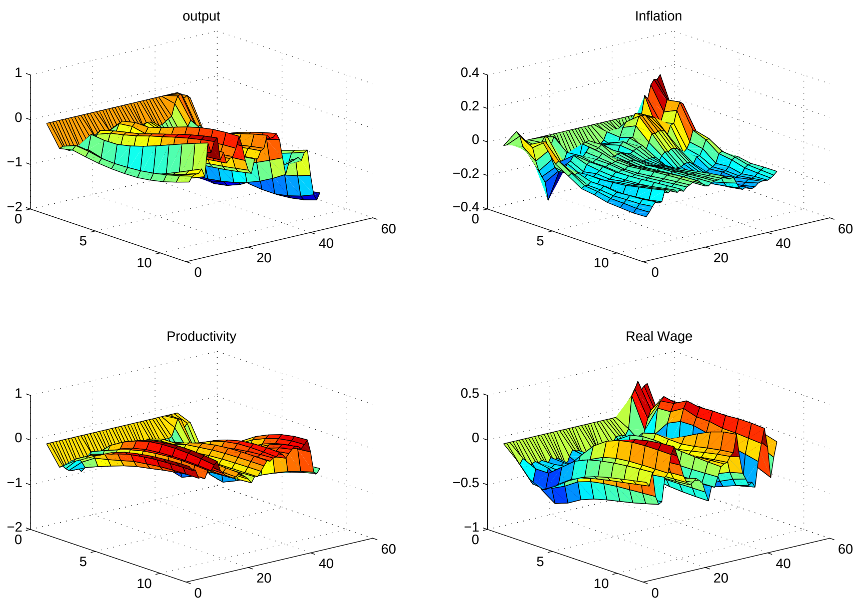

Figure 2
Recursive VAR estimates on rolling samples

and other papers, largely sparked by this critique, have sought to improve the modelling or calibration of the labour market in order to match better the unemployment data, eg, Gertler and Trigari (2009), Fujita and Ramey (2005) and Yashiv (2006). But we are more interested in whether including unemployment enables us better to match the empirical facts we established in the previous section. The rationale for thinking it might comes from the belief that sluggish responses in labour market variables to shocks are a natural place to look for the origins of the sluggish response of inflation to shocks. In terms of the New Keynesian framework, which nests both of these models, labour market frictions will alter aggregate marginal cost.[10]

3.1 The Smets and Wouters (2003) model

The model consists of three sectors: households, firms and a central bank. There are nominal rigidities in the goods and labour markets and real rigidi-

[10] Thus, the labour market was seen as a source of 'real rigidities'. For an overview of the extensive literature on real rigidities more generally, see Woodford (2003).

ties such as habit formation in consumption, investment adjustment costs and variable capital utilisation.

Households

Households consume the final good and supply differentiated labour to the firms. They are also assumed to own the capital stock and make decisions about capital accumulation and utilisation. This assumption, now standard in the business cycle literature, is necessary in order to simplify the firms' decision problem.

Each household maximises the discounted future flows of utility :

$$\max \sum_{s=0}^{\infty} \beta^s \frac{1}{1-\sigma}(C_{j,t} - \psi C_{t-1})^{1-\sigma} - \phi_h \frac{h_{j,t}^{1+\phi}}{1+\phi} \tag{6}$$

where C is consumption, $h_{j,t}$ denotes hours per worker, $\beta \in [0,1]$ is the discount factor, $\sigma \geq 0$ is the inverse of the intertemporal elasticity of substitution and $\psi \in [0,1]$ determines the degree of habit persistence in consumption. Steady-state hours worked are determined by the scaling parameter $\phi_h > 0$ and our utility function implies that the Frisch elasticity of labour supply is given by the inverse of $\phi \geq 0$.

The representative household maximise the objective function subject to an intertemporal budget constraint:

$$C_{j,t} + I_{j,t} + R_t \frac{B_{j,t}}{P_t} = \frac{B_{j,t-1}}{P_t} + D_{j,t} \tag{7}$$

where the household's total income ($D_{j,t}$) is composed of its wage earnings ($w_{j,t}$), rents on capital net of utilisation costs ($r_t^k z_t k_{t-1} - a(z_t)k_{t-1}$) and profits ($\Pi_{j,t}$):

$$D_{j,t} = w_{j,t}h_{j,t} + r_t^k z_{j,t}k_{j,t-1} - a(z_{j,t})k_{j,t-1} + \Pi_{j,t} \tag{8}$$

Households can vary their intensity of capital utilisation, (z_t) at a cost determined by the function $a(z_t)$. Each period the capital stock depreciates at rate $\delta \in [0,1]$ and the household undergoes investment adjustment cost ($S(I_t, I_{t-1})$):

$$k_{j,t} = (1-\delta)k_{j,t-1} + (1 - S(I_{j,t}, I_{j,t-1}))I_{j,t} \tag{9}$$

The investment adjustment cost is increasing with changes in investment. The assumption of investment adjustment costs, rather than capital adjustment costs, enables the model to capture the hump-shaped dynamics of investment.

The functional forms for adjustment costs are given by:

$$a(z_t) = \frac{a_o}{1+\sigma_z}(z_t^{1+\sigma_z} - 1)$$

for capital utilisation, where $a_0, \sigma_z \geq 0$ and

$$S(I_t, I_{t-1}) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

for investment adjustment. Investment adjustment costs satisfy the standard restrictions ($S(1) = S'(1) = 0$) as in Smets and Wouters (2003) and, as the log-linear form of this equation will make clear, $\kappa = S''(1) \geq 0$ captures the effects of investment adjustment costs on the model dynamics.

The modelling of the labour market implies that there will be a distribution of wages across households, since not all workers are able to optimally set their wage in every period. Since the objective of the paper is not the distributional issues which can emerge from heterogeneity within the labour market, we make the simplifying assumption that there is a perfect insurance market which enables agents to ensure themselves against idiosyncratic risks. Combined with our separable utility assumption, this will result in the equalisation of the marginal value of wealth across agents, and each household will be identical with respect to their consumption and asset holdings. We can therefore write the households' decision problems by solving the program of a representative household.

The household's optimal choices on bonds, consumption, capital, investment and capital utilisation can be summarised by the following five equations:

$$\lambda_t = (C_t - \psi C_{t-1})^{-\sigma} \tag{10}$$

$$\lambda_t = \beta E_t\left(\lambda_{t+1}\frac{R_t}{E_t\pi_{t+1}}\right) \tag{11}$$

$$p_t^k = \beta\frac{\lambda_{t+1}}{\lambda_t}\left[z_{t+1}r_{t+1}^k - a(z_{t+1}) + p_{t+1}^k(1-\delta)\right] \tag{12}$$

$$p_t^k\left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) = 1 + p_t^k\kappa\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right) - \beta\frac{\lambda_{t+1}}{\lambda_t}p_t^k\kappa\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right) \tag{13}$$

$$r_t^k = a'(z_t) = a_o z_t^{\sigma_z} \tag{14}$$

Equations 10 and 11 are the first-order conditions for bond holdings and consumption. Equation 10 relates the marginal utility of income to both current consumption and past consumption due to the presence of habit formation in consumption in the preferences. The marginal utility of income evolves according to standard intertemporal condition in 11.

Equation 12 gives the evolution of the value of installed capital stock. The price evolves according to the standard arbitrage rule. The cost of buying one unit of capital today is equalised to the discounted return on this unit of capital, net of utilisation costs and depreciation. Holding utilisation constant and assuming no adjustment cost ($p_t^k = 1$), this condition collapses to:

$$\frac{R_t}{E_t \pi_{t+1}} = r_{t+1}^k + 1 - \delta$$

which states that the return on capital net of depreciation should be equal to the prevailing real interest rate.

Equation 13 is the first-order condition for investment. It takes into account that one unit of investment produces an amount of capital net of investment adjustment costs and investment today will also affect investment next period. This intertemporal effect manifests itself by possible savings on future investment adjustment costs. Equation 14 determines how capacity utilisation varies in response to changes in rental rate of capital and $\frac{1}{\sigma_z}$ gives the elasticity of utilisation to the rental rate of capital.

Labour supply

Households supply differentiated labour services to the firms. Each household is a monopoly and has price-setting power. They are also subject to Calvo (1983) style nominal wage rigidities. Each period only a fraction, $(1 - \alpha_w) \in [0, 1]$, of households can adjust their wages. Wages that cannot be adjusted are indexed to past inflation.

Household specific labour services are aggregated to final labour input by the following technology:

$$L_t = \left(\int_0^1 (h_{j,t})^{1/(1+\lambda_w)} di \right)^{1+\lambda_w}$$

where $\lambda_w \geq 0$ is the steady-state mark-up of wages over the marginal disutility of work, and the demand for jth labour services is:

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-(1+\lambda_w)/\lambda_w} Lt$$

where W_t is the aggregate wage index. The relationship between individual and aggregate wage is:

$$W_t = \left(\int_0^1 (W_{j,t})^{-1/\lambda_w} di\right)^{-\lambda_w}$$

Wages that are not adjusted optimally are partially indexed to past level of inflation. We denote the degree of indexation by $\gamma_w \in [0,1]$. The optimal decision of a household that adjusts its wage then implies that the optimal wage, W_t^*, is given by:

$$\frac{W_t^*}{P_t} E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left(\frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+s}/P_{t+s-1}}\right) \frac{h_{j,t+s}\lambda_{t+s}}{1+\lambda_w} = E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s h_{j,t+s} \phi_h h_{j,t+s}^{\phi}$$

In order to interpret this equation, it is useful to define marginal rate of substitution between consumption and labour:

$$mrs_t = \frac{U_t^l}{\lambda_t} = \frac{\phi_h h_{j,t}^{\phi}}{(C_t - \psi C_{t-1})^{-\sigma}}$$

In the absence of nominal rigidities, the optimal wage equation collapses to:

$$\frac{W_t^*}{P_t} = (1+\lambda_w) mrs_{j,t}$$

where the wage is given as a mark-up over the marginal rate of substitution. With nominal rigidities households take into account the possibility that the mrs can change and sets its wage as a mark-up over the weighted sum of future marginal rates of substitution.

Given the definition of the wage index, the aggregate wage evolves according to:

$$W_t^{-1/\lambda_w} = \alpha_w (W_{t-1}\pi_{t-1}^{\gamma_w})^{-1/\lambda_w} + (1-\alpha_w)(W_t^*)^{-1/\lambda_w}$$

Production sector

Retailers combine the differentiated goods to produce the final good and sell it to the household. They operate in a perfectly competitive market. Wholesale firms operate in a monopolistically competitive market and produce using capital and labour. They are subject to nominal rigidities.

Retailers Retailers combine differentiated intermediate goods (y_j, t) according to a constant return to scale technology:

$$y_t = \left(\int (y_{j,t})^{1/(1+\lambda_p)} dj \right)^{1+\lambda_p} \tag{15}$$

where y_t is the final consumption good and $(1+\lambda_p)/\lambda_p$ is the elasticity of substitution between intermediate goods. ($\lambda_p \geq 0$ then denotes the steady-state mark-up of price over marginal cost.) Cost minimisation yields the following demand for each differentiated good where the demand for each intermediate good depends negatively on its relative price:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-(1+\lambda_p)/\lambda_p} y_t$$

where $P_t = \left(\sum_0^1 (P_{j,t})^{-1/\lambda_p} di \right)^{-\lambda_p}$ is the price of the final good and $P_{j,t}$ is the price of intermediate good j.

Wholesale firms Wholesale firms produce differentiated goods in a monopolistically competitive market. They produce according to the following technology:

$$y_{i,t} = (z_{i,t} K_{i,t-1})^{\alpha} (h_{i,t})^{1-\alpha} \tag{16}$$

where $\alpha \in [0, 1]$. Wholesale firms rent capital and labour in competitive markets. Cost minimisation implies the following relationship between marginal cost and the real wage and rental rate of capital.

$$mc_t = \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t^k}{\alpha} \right)^{\alpha}$$

and in a symmetric equilibrium all the firms have the same capital-labour ratio, which yields:

$$\frac{z_t K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}$$

The wholesale firms are subject to nominal rigidities *à la* Calvo (1983). Each period only a fraction $(1-\alpha_p) \in [0,1]$ of them are allowed to change their prices. The probability of being allowed to change price is independent of the pricing history of firms. When given the chance to adjust its price, a firm reoptimises it in order to maximise its discounted future flow of profits, while non-optimising firms index their prices to past inflation.

The problem facing a price-setting firm is:

$$\max_{p^*} E_t \sum_{s=0}^{\infty} \beta^s \alpha_p^s \lambda_{t+s} \left[\left(\frac{p_t^*}{P_t} \frac{(P_{t-1+s}/P_{t-1})^{\gamma_p}}{P_{t+s}/P_t} - mc_{t+s} \right) y_{j,t+s} \right] \tag{17}$$

where $\gamma_p \in [0,1]$ determines the degree of indexation. The term in brackets gives the period by period profit of the firm, by taking into account that the firm will be able to update its price with inflation indexation. The first term takes into account that firms discount future profits by β but also by α_p, as this gives the horizon during which its price will not be reoptimised. Finally, since the firms are owned by households, the profits are multiplied by the marginal value of income to express this value in utility terms.

As all the firms face the same problem, in the symmetric equilibrium they all choose the same price. The solution to this maximisation programme is given by:

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_p^s \lambda_{t+s} y_{t+s}^i \left(\frac{p_t^*}{P_t} \frac{(P_{t-1+s}/P_{t-1})^{\gamma_p}}{P_{t+s}/P_t} - (1+\lambda_p) mc_{t+s} \right) = 0 \tag{18}$$

Given the definition of the price index, the overall price level in each period can be expressed as a weighted sum of newly optimised prices and old prices updated by the past inflation:

$$P_t^{-1/\lambda_p} = \alpha_p (P_{t-1} \pi_{t-1}^{\gamma_p})^{-1/\lambda_p} + (1-\alpha_p)(P_t^*)^{-1/\lambda_p} \tag{19}$$

Resource constraint and monetary policy

Monetary policy is assumed to follow a variant of the Taylor (1993) rule with interest rate smoothing:

$$R_t = R_{t-1}^{\rho_r} \left(\frac{1}{\beta} \left(\frac{\pi_{t+1}}{\pi} \right)^{\rho_\pi} \left(\frac{y_t}{y} \right)^{\rho_y} \right)^{1-\rho_r} \exp(\varepsilon_t)$$

where ε_t is a monetary policy shock, assumed to be normally distributed with mean zero and variance σ_m^2, $\rho_r \in [0,1]$, $\rho_\pi > 1$, and $\rho_y \geq 0$.

Finally, the market for final goods clears in every period:

$$y_t = c_t + i_t + a(z_t)K_{t-1}$$

3.2 The Gertler, Sala and Trigari (2008) model

The Gertler et al. (2008) model is similar to the Smets and Wouters (2003) model except that labour market is characterised by search and matching frictions. As we said earlier, this allows us to assess whether or not the impact of unemployment on the monetary transmission mechanism is important in terms of enabling the model to fit better the data.

Households

Our modelling of the labour market implies that some members of the household will be unemployed. Using the same arguments as for the Smets and Wouters (2003) model, we assume that household members pool their income and there is complete consumption insurance.

The representative household maximises the discounted future flows of utility:

$$\max \sum_{s=0}^{\infty} \beta^s \frac{1}{1-\sigma} (C_t - \psi C_{t-1})^{1-\sigma} \tag{20}$$

subject to an intertemporal budget constraint:

$$C_t + I_t + R_t \frac{B_t}{P_t} = \frac{B_{t-1}}{P_t} + D_t \tag{21}$$

where the household's total income (D_t) is composed of its wage earnings of working members (w_t), unemployment benefits of its unemployed members (b_t), rents on capital net of utilisation costs ($r_t^k z_t k_{t-1} - a(z_t)k_{t-1}$) and profits ($\Pi_t$):

$$D_t = w_t n_t + (1 - n_t)b_t + r_t^k z_t k_{t-1} - a(z_t)k_{t-1} + \Pi_t \tag{22}$$

As a result, equations 10-14 fully describe the household's optimal decisions in this model.

Labour market

The formation of a job is a costly and time-consuming process. In order to create a productive job, firms must post vacancies, v_t, and workers must look for jobs. The number of new matches each period is determined by a matching function which relates new matches to existing vacancies and unemployed workers:

$$m_t = a_m u_t^{\sigma_u} v_t^{1-\sigma_u} \tag{23}$$

where $a_m > 0$ and $\sigma_u \in (0, 1)$.

As Hall (2005) points out, fluctuations in labour market flows are mainly driven by job creation. So we abstract out of job destruction decisions by assuming that, in each period, a fixed part of existing jobs are exogenously destroyed at rate $1 - \rho_n \in (0, 1)$. Employment evolves according to:

$$n_t = \rho_n n_{t-1} + m_t \tag{24}$$

The evolution of employment makes clear that new matches become productive within the same period. Accordingly, each period unemployment is given by:

$$u_t = 1 - \rho_n n_{t-1}$$

It is also useful to define transition probabilities. The probability for a firm to find a worker, q_t, and the probability for a worker to find a job, s_t are given by:

$$q_t = \frac{m_t}{v_t} \tag{25}$$

$$s_t = \frac{m_t}{u_t} \tag{26}$$

Our timing assumption implies successful matches become productive immediately. Since Gertler et al. (2008) assume that firms cannot alter their labour input via changes in hours, they need to allow firms to adjust at the extensive margin to shocks. Furthermore the more pronounced reaction of unemployment to shocks will mean that labour market tightness responds more to shocks and this, in turn, will lead to greater pressure on wages in response to shocks.

Wholesale firms

The production function of wholesale firms is given by:

$$y_{i,t} = (z_t K_{i,t-1})^{\alpha} n_{i,t}^{1-\alpha} \tag{27}$$

It is useful to define the hiring rate, $x_{i,t}$:

$$x_{i,t} = \frac{q_t v_{i,t}}{n_{i,t-1}} \tag{28}$$

and the firm pays a quadratic adjustment cost of hiring given by:

$$LAC = \frac{\kappa_v}{2} x_{i,t}^2 n_{i,t-1} \tag{29}$$

where $\kappa_v > 0$.

The firm maximises its value defined by:

$$F_{i,t} = p_t^w y_{i,t} - w_{i,t} n_{i,t} - \frac{\kappa_v}{2} x_{i,t}^2 n_{i,t-1} - r_t^k k_{t-1} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} F_{i,t+1} \tag{30}$$

where p^w is the price charged by wholesale firms, which will equal real marginal cost for the retailers.

The first-order condition for capital:

$$r_t^k = p_t^w \alpha \frac{y_t}{z_t k_{t-1}} \tag{31}$$

Because of the assumption that workers start working within the period, the expected value of opening up a vacancy will depend on current productivity and the current wage. Our assumption of labour adjustment costs implies that it is equivalent for the firm to set the number of vacancies or the hiring rate. Putting this together, the vacancy posting condition will be given by:

$$\kappa_v x_{i,t} = p_t^w(1-\alpha)\frac{y_t}{n_t} - w_{i,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa_v}{2}x_{i,t+1}^2 + \rho\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\kappa_v x_{i,t+1} \quad (32)$$

The value of an additional worker will determine the surplus of the firm when entering into the bargaining process. $J_{i,t}$ is defined as the value of a new worker at time t:

$$J_{i,t} = p_t^w(1-\alpha)\frac{y_t}{n_t} - w_{i,t} - \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa_v}{2}x_{i,t+1}^2 + \beta E_t \frac{n_{t+1}}{n_t}\frac{\lambda_{t+1}}{\lambda_t}J_{i,t+1} \quad (33)$$

Workers

$V_{i,t}$ and U_t are defined to be the value of being employed at firm i and the value of being unemployed, respectively. $V_{i,t}$ is given by:

$$V_{i,t} = w_{i,t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}[\rho V_{i,t+1} + (1-\rho)U_{t+1}] \quad (34)$$

A worker value depends on the current wage plus the discounted future value of being employed and unemployed, weighted by their respective probabilities.

As there is a wage dispersion across firms, define $V_{x,t}$ as the average value of employment conditional on being a new worker at time t:

$$V_{x,t} = \int_0^1 V_{i,t}\frac{x_{i,t}n_{i,t-1}}{x_t n_{t-1}}di \quad (35)$$

The idea is that the workers do not know the exact level of wages in each firm. Since there is no directed search in the sense that workers cannot choose to look for high wages firms, Gertler et al. (2008) argue that, as the contract differentials, due to nominal rigidities, are transitory, the gain from directed search may not be large. Then U_t is given by:

$$U_t = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}[s_{t+1}V_{x,t+1} + (1-s_{t+1})U_{t+1}] \quad (36)$$

with b representing unemployment benefits. In our estimation, we estimate, $\overline{b}$, the flow value of unemployment relative to the flow value of a worker to the firm at the steady state, defined as:

$$\bar{b} = \frac{b}{p^w mpl + \beta \frac{\kappa_v}{2} x^2} \in (0,1)$$

The value for the worker of finding a job relative to his value when unemployed is given by:

$$H_{i,t} = V_{i,t} - U_t \tag{37}$$

$$H_{x,t} = V_{x,t} - U_t \tag{38}$$

Nash bargaining and wage dynamics

The model departs from the standard Nash bargaining framework by assuming that each period a firm has a fixed probability $(1 - \alpha_w)$ that it may renegotiate the wage. Otherwise, the firms index their wages to past inflation. As we do not have trend growth or inflation, the indexation rule is given by:

$$W_t^n = W_{t-1}^n \pi_{t-1}^{\gamma_w} \tag{39}$$

The nominal contract wage, W_t^{n*}, is chosen to solve:

$$\max H_{i,t}^{\eta} J_{i,t}^{1-\eta} \tag{40}$$

where $\eta \in (0,1)$ denotes the bargaining power of the worker, subject to

$$W_{i,t+j}^n = \begin{array}{cc} W_{i,t+j-1}^n \pi_{t+j-1}^{\gamma_w} & \text{with probability } \lambda_w \\ W_{i,t+j}^{n*} & \text{with probability 1-}\lambda_w \end{array} \tag{41}$$

The first-order condition is given by:

$$\eta \frac{\partial H_{i,t}}{\partial W_{i,t}^n} J_{i,t} + (1-\eta) \frac{\partial J_{i,t}}{\partial W_{i,t}^n} H_{i,t} = 0 \tag{42}$$

The marginal values of the worker's and firm's surplus with respect to the real wage, $\Delta_t = \frac{\partial H_{i,t}}{\partial W_{i,t}^n / p_t}$ and $\Sigma_t = -\frac{\partial J_{i,t}}{\partial W_{i,t}^n / p_t}$, are given by:

$$\Delta_t = 1 + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (\rho \alpha_w) \frac{p_t}{p_{t+1}} \pi_t^{\gamma_w} \Delta_{t+1} \tag{43}$$

and

$$\Sigma_t = 1 + E_t \frac{\lambda_{t+1}}{\lambda_t} (\rho + x_{t+1} \pi_t^{\gamma_w} W_{i,t}^n)(\beta \lambda_w) \frac{p_t}{p_{t+1}} \pi_t^{\gamma_w} \Sigma \pi_t^{\gamma_w} W_{i,t}^n \tag{44}$$

Then the first-order condition for wages can be rewritten as:

$$\chi_{i,t} J_{i,t} = (1 - \chi_{i,t}) H_{i,t} \tag{45}$$

with

$$\chi_{i,t} = \frac{\eta}{\eta + (1-\eta)\Sigma_{i,t}/\Delta_t} \tag{46}$$

Gertler et al. (2008) show that this all implies the following log-linearised equation for the aggregate real wage:

$$\widehat{w}_t = \gamma_b \left(\widehat{w}_{t-1} - \pi_t + \gamma_w \pi_{t-1}\right) + \gamma_o w_{i,t}^o + \gamma_f E_t \left(\widehat{w}_{t+1} + \pi_{t+1} - \gamma_w \pi_t\right) \tag{47}$$

where the target real wage $w_{i,t}^o$ will be given by:

$$\begin{aligned} w_{i,t}^o \;\; = \;\; & \varphi_a(\widehat{p}_t^w + \widehat{mpl}_t) + (\varphi_s + \varphi_x) E_t \widehat{x}_{t+1} + \varphi_s E_t \widehat{s}_{t+1} + \left(\varphi_s + \frac{\varphi_x}{2}\right) E_t \left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\right) \\ & + \varphi_x \left(\widehat{\chi}_t - (\rho - s)\, \beta E_t \widehat{\chi}_{t+1}\right) \end{aligned} \tag{48}$$

4 Estimation

We evaluate the models following the minimum distance estimation strategy developed in Rotemberg and Woodford (1997), Altig et al. (2010), Boivin and Giannoni (2006) and Meier and Muller (2006). As Smets and Wouters (2003) stress, this strategy helps to focus on empirical properties that the model has been developed to explain. The objective is to minimise the difference between empirical and model-based impulse responses.

Formally, define $\mathbf{J}$, the vector containing the empirical impulse responses resulting from our VAR estimation and $\mathbf{J}(\theta)$ the vector of theoretical impulse responses of the DSGE model where the vector θ contains the parameters we are looking to estimate.

$$L = \min_{\theta} [(\mathbf{J} - \mathbf{J}(\theta))' \mathbf{W}^{-1} (\mathbf{J} - \mathbf{J}(\theta))'] \tag{49}$$

where $\mathbf{W}$ is a diagonal weighting matrix which contains the variance of estimated impulse responses. This weighting matrix gives more weight to more precisely estimated impulse responses and ensures that the resulting model-based impulse responses lies within the estimated confidence intervals.

4.1 The Smets and Wouters (2003) model

Following DiCecio and Nelson (2007), we use the minimum distance approach to estimate the following vector of parameters:

$$\theta = \{\psi, \phi, \gamma_p, \gamma_w, \alpha_p, \alpha_w, \kappa, \sigma_z, \rho_R, \rho_\pi, \rho_y, \sigma_m\} \tag{50}$$

The remaining parameters are important for determining steady-state relationships rather than for the dynamics of the model and so we use values that can be inferred either from the steady-state relationships or from the microeconomic studies. In particular, we fix the discount rate, β to 0.99 implying a steady-state annual nominal interest rate of about 4%. We fix $\alpha = 0.36$ and $\delta = 0.025$, values commonly used in the literature, including by DiCecio and Nelson (2007). We adopt a log utility function ($\sigma = 1$). Finally, the wage mark-up, λ_w, is set to 0.5 following Smets and Wouters (2003) and the price mark-up , λ_p, is set to 0.2 following the results reported in Macallan, Millard, and Parker (2008).[11] For all parameters that were restricted by theory to lie between zero and one, we allowed our estimates to take values between 0.01 and 0.99. For the other parameters we considered values within large ranges: for the investment adjustment cost parameter, κ, we considered values between 1 and 40, for the response of interest rates to inflation, ρ_π, we considered values between 1 and 5, for the response of interest rates to output, ρ_y, we considered values between 0 and 0.5, for the inverse of the Frisch elasticity of labour supply, ϕ, we considered values between 0.5 and 5, for the capacity utilisation parameter, σ_z, we considered values between 0.001 and 10 and for the standard deviation of the monetary policy shock, we considered values between 10^{-6} and 0.1.

Table A presents the estimated values for the parameters in our benchmark model using information contained in all the empirical IRFs. Our estimate for the habit formation parameter is somewhat higher than others (eg, Altig et al. (2010), DiCecio and Nelson (2007), Fuhrer (2000) and Harrison and Oomen (2010)) and it implies a substantial role for backward-looking behaviour in consumption. The lower bound fixed for the parameter σ_z is binding, which means that the elasticity of capital utilisation with respect to the rental rate of capital tends toward infinity. This finding is in line with the previous estimation results for the United States, though completely

[11] The price mark-up does not feature in any of the dynamic equations of the model. The wage mark-up affects the slope coefficient in the wage Phillips curve, but cannot be identified separately to the degree of wage rigidity. Given that, we fix both of these parameters in the estimation.

out of line with that of DiCecio and Nelson (2007). The reason for this difference is that, in addition to the empirical IRFs used by DiCecio and Nelson (2007) in their estimation, we have also sought to match the empirical IRF of capacity utilisation. Our estimates indicate that we match this response quite well. But, to do this we need the elasticity of utilisation to the rental rate to be infinite, compared with DiCecio and Nelson (2007)'s estimate of zero. As Altig et al. (2010) points out, variable capital utilisation helps the model to match observed inflation persistence by lowering the elasticity of rental rate to monetary policy shocks. Our estimate for the investment adjustment cost is higher than in the United States and the euro area but lower than the previous UK estimate. The reason for this result is that, although investment is more volatile than output at business cycle frequencies, we find, in line with DiCecio and Nelson (2007), that the investment response after a monetary policy shock is not large. Our estimate for the parameter ϕ is 0.78. It implies that the Frisch elasticity of labour supply is slightly over one. This value is close to the values generally used in the DSGE literature and it is well known that it is not in accordance with the micro estimates that suggest very low labour supply elasticities. However, we find that this parameter is not estimated very precisely as it has a large standard deviation.

Our results for the parameters governing the nominal side of the economy suggest that the average duration of wages is ten months whereas the average duration of prices is almost one year and a half. The estimated price rigidity is slightly higher than the recent survey evidence on price durations reported in Greenslade and Parker (2008). This result is in line with Smets and Wouters (2003) and DiCecio and Nelson (2007) who estimate the price rigidities to be higher than wage rigidities for the euro area and the United Kingdom, respectively. However, DiCecio and Nelson (2007) find that, for the 1979 Q2-2005 Q4 period, there is no nominal wage rigidity while they estimate nominal rigidities in the goods market to be very high, with an average price duration of three and a half years. Again, we are trying to match the IRF for the real wage, in addition to those IRFs matched by DiCecio and Nelson (2007); it is likely that this explains the different result we obtain for the extent of nominal wage rigidity.

Finally, γ_p and γ_w are estimated to be equal to the upper bound of one, implying full indexation both in the goods and the labour market. This finding contrasts with the estimates of Smets and Wouters (2003), Smets and Wouters (2007) and Groth, Jaaskela, and Surico (2006) who find much lower values for the euro area, United States and United Kingdom, respectively.

Our results confirm earlier findings in the United States, euro area and the

Table 1
Estimated parameter values for the Smets and Wouters (2003) model

Parameter		Value
ψ	Habit formation in consumption	0.85
		(0.05)
γ_p	Degree of price indexation	1.00
		(-)
γ_w	Degree of wage indexation	1.00
		(-)
α_p	Probability of not being able to reset prices	0.83
		(0.14)
α_w	Probability of not being able to reset wages	0.70
		(0.15)
ϕ	Inverse of labour supply elasticity	0.78
		(1.71)
κ	Elasticity of investment adjustment costs	6.17
		(4.11)
σ_z	Elasticity of capacity utilisation costs	0.00
		(-)
ρ_R	Persistence parameter in Taylor rule	0.78
		(0.10)
ρ_π	Coefficient on inflation in Taylor rule	1.25
		(0.76)
ρ_y	Coefficient on output in Taylor rule	0.21
		(0.34)
σ_m	Standard deviation of monetary policy shock	0.0014
		(0.0002)

United Kingdom that monetary policy exhibits high interest rate smoothing.[12] The parameters governing the response of the central bank to inflation and output are not very precisely estimated. This is likely a result of the changes in monetary policy regime over our sample period which we discussed earlier.

Chart 3 displays the empirical IRFs and the IRFs from the model obtained using the estimated parameter values. Our model does well in explaining the dynamic responses of macroeconomic variables in the United Kingdom to a monetary policy shock. The model predicts negative hump-shaped

[12] See, eg, Clarida, Galí, and Gertler (1998) and Nelson (2003).

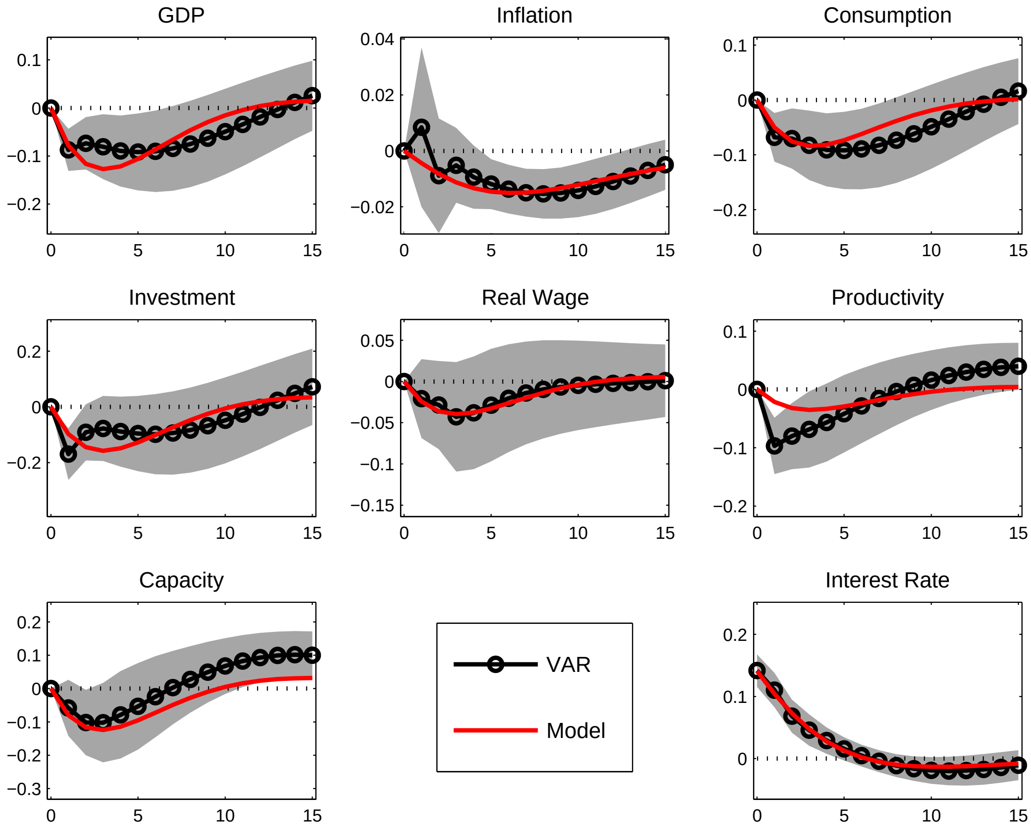

Figure 3
Impulse response functions for the Smets and Wouters (2003) model

responses for output, inflation, consumption, investment, productivity and capacity utilisation; it also predicts a near zero response of the real wage. All the model IRFs lie within the 90% confidence intervals apart from the response of productivity, which is too persistent. This result suggests the need for a better specification of the labour market in the model and helps motivate our estimation of the Gertler et al. (2008) model in Section 4.2.

4.2 The Gertler, Sala and Trigari (2008) model

The Gertler et al. (2008) model differs from the Smets and Wouters (2003) model only in its specification of the labour market. In this case the vector of parameters we wish to estimate is given by:

$$\theta = \{\psi, \gamma_p, \gamma_w, \alpha_p, \alpha_w, \kappa, \sigma_z, \rho_R, \rho_\pi, \rho_y, \overline{b}, \eta, \sigma_m\} \tag{51}$$

We again allowed our estimates of those parameters restricted by theory to lie between zero and one to take values between 0.01 and 0.99 and used the same ranges for the remaining parameters. Again, we were not able to estimate all the parameters of the model so, following Gertler et al. (2008), we used other evidence to set these parameters. Given the lack of direct evidence on the parameters governing labour market flows, we had to calculate these parameters using UK labour market data.

Specifically, we estimated a matching function for the 2001-08 period in order to infer about the elasticities of the matching function with respect to unemployment and vacancies. Our estimation takes a standard approach, described in Petrongolo and Pissarides (2001). We estimate a log-linear matching function where the dependant variable is outflows from unemployment. In theory, the matching function gives the number of new hires in terms of workers looking for jobs and vacancies. However, the data on unemployment may not reflect the real number of job searchers, as some workers may go from inactivity to activity without declaring themselves as unemployed. But as in Blanchard and Diamond (1990), we assume that, for the United Kingdom, the unemployment rate measured by those claiming unemployment benefit may be a good proxy for all job seekers. We also report our estimates using unemployment measured by the Labour Force Survey (LFS).

As in Blanchard and Diamond (1990), we estimate the following equation using OLS :

$$ln(M_t) = \alpha_1 + \alpha_2 ln(U_t) + \alpha_3 ln(V_t) + \alpha_4 Trend + \varepsilon_t \tag{52}$$

We use monthly data and our estimation period covers 2001:6-2008:6.

Table B presents our estimation results. The estimated elasticities of matches with respect to unemployment and vacancies are significant and positive. We also find a small but negative coefficient for the time trend, which implies a decrease over time in the efficiency of the matching technology. As Petrongolo and Pissarides (2001) point out, the estimated weight on unemployment is higher in the United Kingdom than in the United States. This finding is in line with previous matching function estimates of Hellwig et al. (1986) and Burda and Wyplosz (1994) for the United Kingdom. One noticeable point is that the estimated values are sensitive to the measure of unemployment we use in the estimation. The LFS unemployment measure is always higher than claimant count unemployment and it also yields higher

Table 2
Estimated parameter values: matching function

Parameter	Claimant Count		LFS	
	Unrestricted	Restricted	Unrestricted	Restricted
α_1	-0.46 (0.74)	-0.47 (0.02)	-0.31 (0.38)	-0.52 (0.02)
α_2	0.55 (0.13)	0.55 (0.05)	0.68 (0.07)	0.72 (0.05)
α_3	0.44 (0.15)		0.23 (0.09)	
α_4	-0.0004 (0.0001)	-0.0004 (0.0001)	-0.00011 (0.0001)	-0.00011 (0.0001)
CRS test's P-value	0.99		0.57	
R^2	0.41	0.42	0.63	0.64

estimates for the elasticity of matches to unemployment. The estimates remain, however, close to the 0.5-0.7 range considered in the literature.

Most of the empirical studies conclude that a constant return to scale matching function describes the data well. We also tested this assumption. The restriction that the elasticities sum up to 1 is not rejected only when we use claimant count unemployment. When we re-estimate the model imposing constant return to scale, the estimated values remain the same. We can therefore confidently set the parameter σ_u in our estimation to some value between 0.5 and 0.7.

The other two parameters that we can get from data are s, the probability of finding a job for an unemployed worker and ρ, the ratio of surviving jobs at each period (or one minus the separation rate).

We calculate the probability of an unemployed worker finding a job by dividing unemployment outflows by unemployment. This calculation yields a value of 0.55 for s, implying an average duration of unemployment of approximately five months. The unemployment series are not, however, consistent with our model as we are not modelling the labour market participation decision explicitly. In reality, some of the unemployment outflows will be into non-participation and so the implied time taken by unemployed workers to find a job may be a bit higher. Given this, we adopt the slightly lower value of 0.5 for s.

To calibrate the job-separation rate, we use data on unemployment inflows. This calculation indicates that each quarter, the inflow to unemployment is just 1% of total employment. This would imply a value of 99% for $\rho = 0.99$. Since, we do not have data on workers leaving a job and going to inactivity, we revise our calculation upward. In our simulation, we set ρ to 0.95. Finally, our steady state, when $s = 0.5$ and $\rho = 0.95$ imply that the steady-state unemployment rate is 9.1%. This value is higher than what we observe in the data. Our model, however, does not explicitly model the participation decision, ie, the possible transitions from inactivity to employment or unemployment. Our higher unemployment rate can be seen as a result of this difference between the model and the data.

Chart 4 displays the empirical IRFs and the IRFs from the model obtained using the estimated parameter values. The model again does well in explaining the dynamic responses of macroeconomic variables in the United Kingdom to a monetary policy shock with negative hump-shaped responses for output, inflation, consumption, investment and capacity utilisation and a near zero response in real wages. But, it seems to do less well at explaining the response of productivity to the shock than the Smets and Wouters (2003) model: essentially, the search frictions result in the productivity response to the shock being dampened, though it is more persistent. Given that the difference between the two models relates to how the labour market is modelled, this result is a little disappointing.

Table C presents the estimated values for the parameters in our model. In terms of the parameter estimates for the Gertler et al. (2008) model, the key difference to those for the Smets and Wouters (2003) model is the degree of wage stickiness we estimate. In this case, the average duration of wages is estimated to be approximately seven and a half months. In order to match the empirical IRFs, the Gertler et al. (2008) model needs less nominal wage rigidity due to the presence of search and matching frictions in the labour market. We again estimate wages and prices to be fully indexed and the

Table 3
Estimated parameter values (Gertler *et al* (2008))

Parameter		Value
ψ	Habit formation in consumption	0.90
		(0.04)
γ_p	Degree of price indexation	1.00
		(-)
γ_w	Degree of wage indexation	1.00
		(-)
α_p	Probability of not being able to reset prices	0.82
		(0.16)
α_w	Probability of not being able to reset wages	0.60
		(0.19)
κ	Elasticity of investment adjustment costs	7.33
		(4.77)
σ_z	Elasticity of capacity utilisation costs	0.00
		(-)
ρ_R	Persistence parameter in Taylor rule	0.77
		(0.09)
ρ_π	Coefficient on inflation in Taylor rule	1.45
		(0.93)
ρ_y	Coefficient on output in Taylor rule	0.13
		(0.22)
η	Workers' bargaining power	0.75
		(0.48)
b	Flow value of being unemployed	0.93
		(0.12)
σ_m	Standard deviation of monetary policy shock	0.0014
		(0.0002)

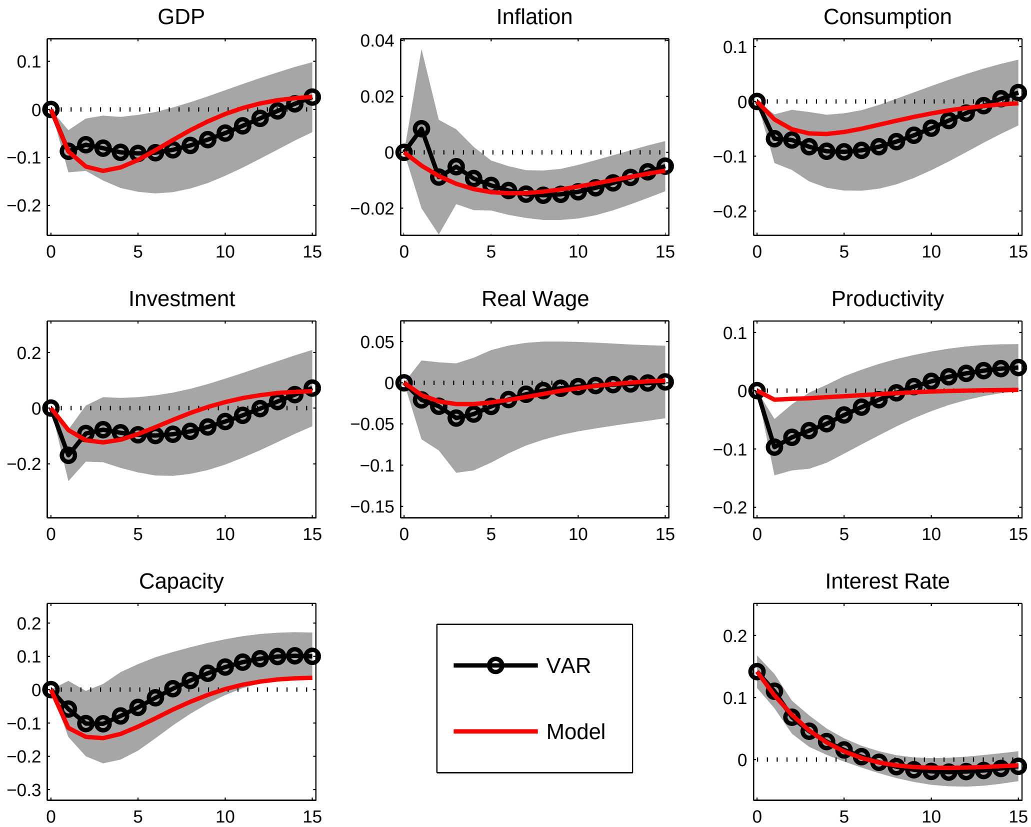

Figure 4
Impulse response functions for the Gertler *et al* (2008) model

lower bound for the elasticity of capital utilisation costs parameter is binding. Within this model, we estimate two additional parameters relative to the Smets and Wouters (2003) model: the bargaining power of workers and the flow value of being unemployed. We estimate the workers' bargaining power to be equal to 0.75 and the flow value of unemployment relative to the flow value of a worker to the firm at the steady state to be 0.93. This is a high value but, as argued by Hagedorn and Manovskii (2008) such a high value is necessary for the model to match the relative responses of employment and real wages to shocks.

Table 4
Loss values

	SW	GST
Benchmark	53.1	68.02
No price or wage indexation	109.3	140.5
No price indexation	105.6	122.7
No wage indexation	68.4	111.4
No nominal rigidities	687.5	425.7
No price rigidities	129.3	162.0
No wage rigidities	208.4	191.4

4.3 Assessing the role of frictions

We can use the values of our estimated loss function – equation 49 – to compare the fit of the two models and assess the role of nominal rigidities. A lower loss value implies that the theoretical monetary transmission mechanism in the estimated variant is closer to the empirical one.[13]

The first line of Table 4 shows the loss values for the benchmark models estimation. As we argued earlier, the Gertler et al. (2008) model does slightly less well than the Smets and Wouters (2003) model in minimising the distance between the empirical and theoretical IRFs. The inclusion of labour market rigidities via search and matching frictions does not improve the fit of the model. The model has, however, the advantage of allowing us to quantify the effects of shocks on unemployment and job and worker flows.

Next, we turn to the analysis of the role of nominal frictions in the models' ability to match the data. First, we calculate the loss value of the models in which neither wages nor prices are indexed to lagged inflation. For both models, the loss is almost two times the benchmark case, suggesting that indexation is important in explaining the response of variables to a monetary policy shock. In order to assess whether it is price or wage indexation that drives this result, we also re-calculate the loss values for versions of the models in which wage (price) indexation was assumed to be zero and the degree of price (wage) indexation was set to its estimated value. The absence of indexation in price-setting deteriorates the models' fit more than the absence

[13] In future work, we plan to estimate the distribution of this statistic using a bootstrap approach; given that, we would be able to assess the statistical significance of differences between the loss values across models. This approach is similar to that suggested by Fève, Matheron, and Sahuc (2009). For now, we simply report the statistics without being able to assess statistical significance.

of indexation in wage-setting, suggesting that indexation of prices to lagged inflation is the more important component in explaining the sluggish response of nominal variables to a monetary policy shock.

Finally, the last three lines of Table 4 aim to quantify the role of wage and price rigidities. When we restrict the Calvo parameter to be 0.1 in both the goods and labour markets, the loss value for each model increases dramatically. We then re-calculate the loss values when the Calvo parameter is restricted to be 0.1 in only one market at a time. Both price and wage rigidities improve substantially the models' performance compared to their low rigidity versions. For both models, the variant with only wage rigidities does a better job than the same model with only price rigidities. According to our analysis of the sensitivity of the loss function, nominal wage rigidities and indexation to past inflation in the price-setting play an important role in both models.

4.4 A closer look at inflation dynamics

In this subsection, we evaluate the contribution of the estimated parameters to inflation dynamics. We do this by using the Gertler et al. (2008) model with all parameters – other than that whose contribution we wish to understand – set at their estimated values. In Chart 5, the red lines represent the implied IRFs from our benchmark estimation. The blue lines correspond to the IRFs when we set some parameters to extreme values while keeping all the other parameters at their estimated values.

We first consider the impact of nominal price rigidities. The first graph on the first row displays the response of inflation when we set the Calvo parameter for price rigidities to 0.1. In this case, the implied average duration of prices is roughly one quarter. Since a higher proportion of firms can adjust their prices in each period, the response of inflation after the monetary policy shock is higher on impact and inflation comes back to its steady state more quickly. The second graph shows the response of inflation when there is no indexation to past inflation for firms that cannot adjust their prices optimally. In the absence of indexation, the Phillips curve is completely forward looking. In this case, inflation falls on impact to its lowest level before increasing back to steady state. In other words, the response of inflation is larger on impact but much less persistent. Taken together, these results imply that the combination of sticky prices and indexation helps to generate the small and delayed effect of monetary policy shocks on inflation.

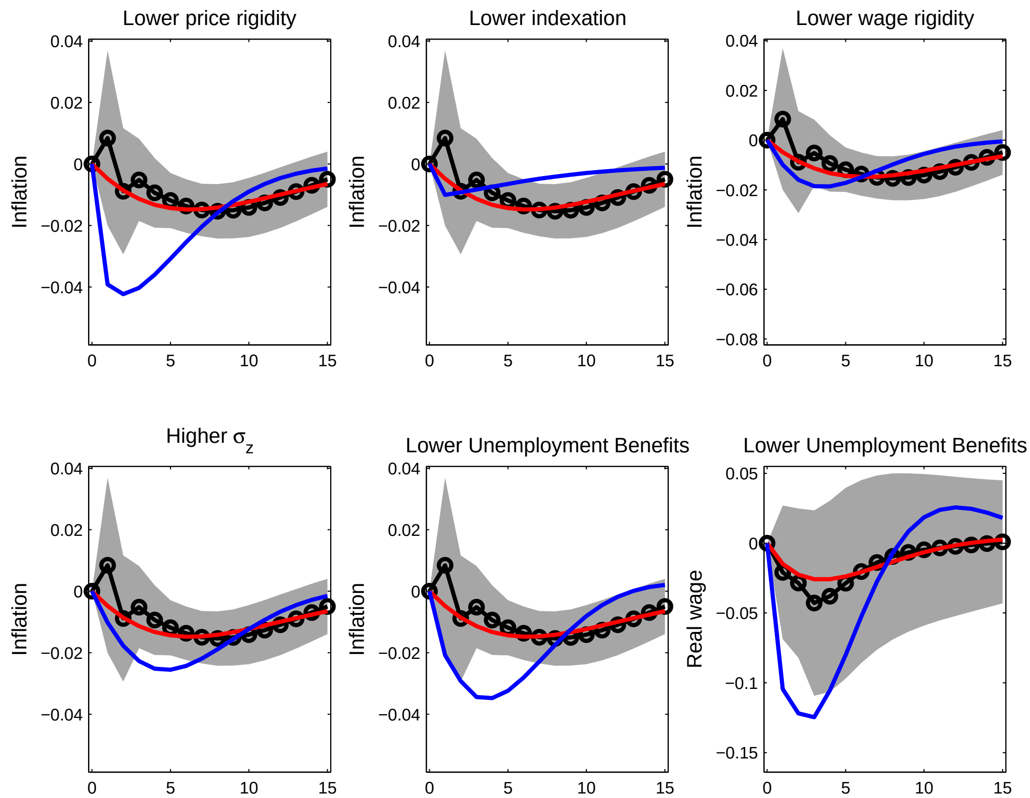

Figure 5
Impulse responses under different parameter settings

The last graph in the first row and the first one in the second row show how the persistence of marginal cost affects inflation dynamics. When there are low nominal wage rigidities ($\alpha_w = 0.1$), wages become more volatile. This makes marginal cost and, hence, via the Phillips curve, inflation more volatile than in the data. The peak response of inflation is almost four times larger than in the data. The same logic applies to an increase in the elasticity of utilisation to the rental rate of capital. Marginal cost becomes more sensitive to economic conditions, making inflation more responsive to monetary policy shocks.

Finally, the last two graphs show that if the flow value of being unemployed is lower, the response of wages to a monetary policy shock increases. This is the well-known result of Hagedorn and Manovskii (2008), who show that this particular calibration enables the model to generate increased volatility in unemployment. As Gertler et al. (2008) points out, higher unemployment benefits make labour supply more elastic. With wages not responding as much to a monetary policy shock, real marginal cost does not respond as much. This, in turn, lowers the response of inflation to the shock.

5 Conclusion

In this paper, we used the minimum distance approach to estimate the DSGE models of Smets and Wouters (2003) and Gertler et al. (2008) using UK data. This was motivated by our interest in understanding inflation dynamics and the monetary transmission mechanism. In particular, we were motivated by a belief that labour market frictions and, in particular wage-setting frictions, play a central role in the monetary transmission mechanism.

We first used a SVAR approach to obtain an empirical representation of the monetary transmission mechanism, ie, how a change in interest rates affects some important macroeconomic variables in the United Kingdom. We found that output, consumption, investment and capacity utilisation all fell in response to the shock and that the responses of all these variables were hump shaped. The peak response of output occurs five quarters after the shock. Inflation rose on impact (though this rise was not statistically significant) before falling to a trough two years after the shock. The effect on inflation of the shock dies out after three years. The relative price of capital and real wages fell in response to the shock, but these effects were not statistically significant. The peak response of productivity was one period after the shock. Given the response of output, this result suggested that the adjustment in labour input occurs with a lag relative to the response of output.

In terms of the models, we found that both were able to explain reasonably well the dynamic responses of the macroeconomic variables we considered in the United Kingdom to a shock to interest rates. In order to achieve this, our estimates implied that wages are reset about once every three quarters, and prices every one year and a half. The estimated price rigidity is slightly higher than the recent survey evidence on price durations reported in Greenslade and Parker (2008) but is in line with DiCecio and Nelson (2007) which estimates price rigidities to be higher than wage rigidities in the United Kingdom. But, in order to match the impulse responses, we also needed a large degree of indexation in price and wage-setting. This finding contrasts with the estimates of Groth et al. (2006) which finds much less evidence of wage and price indexation in their estimated UK Phillips curves.

Unfortunately, neither model was able satisfactorily to explain the response of productivity. An implication of this is that they were unable to explain the response of employment, given that they could explain the response of output. This suggests that it may be worth thinking more about the costs of adjusting labour input if we are to explain movements in employment as well as we can explain movements in output. We leave this for future research. More generally, our results leave us with a big question: given that the Smets and Wouters (2003) model was able to match the implied impulse response functions fairly well, what is the role, if any, of search and matching frictions and unemployment in the monetary transmission mechanism? We leave finding an answer to that question to future research.

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Appendices

A Appendix:Log-linear models

A.1 Smets and Wouters model

- Marginal utility of consumption

$$\widehat{\lambda}_t = -\sigma\frac{1}{1-\psi}(\widehat{c}_t - \psi\widehat{c}_{t-1}) \tag{53}$$

- Euler equation

$$\widehat{\lambda}_t = E_t\widehat{\lambda}_{t+1} + r_t - E_t\widehat{\pi}_{t+1} \tag{54}$$

- IS consumption

$$\widehat{c}_t = \frac{\psi}{1+\psi}\widehat{c}_{t-1} + \frac{1}{1+\psi}E_t\widehat{c}_{t+1} - \frac{1-\psi}{(1+\psi)\sigma}(r_t - E_t\widehat{\pi}_{t+1}) \tag{55}$$

- Capital

$$\widehat{p}^k_t = E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \frac{r^k}{r^k+1-\delta}\widehat{r}^k_{t+1} + \frac{1-\delta}{r^k+1-\delta}\widehat{p}^k_{t+1} \tag{56}$$

- Investment

$$\widehat{i}_t = \frac{1}{1+\beta}\widehat{i}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{i}_{t+1} + \frac{1}{\kappa}\frac{1}{1+\beta}\widehat{p}^k_t \tag{57}$$

- Capital accumulation

$$\widehat{k}_t = (1-\delta)\widehat{k}_{t-1} + \delta\widehat{i}_t \tag{58}$$

- Capital utilisation

$$\widehat{z}_t = \frac{1}{\sigma_z}\widehat{r}^k_t \tag{59}$$

- Wage Phillips curve (indexation to price inflation)

$$\widehat{\pi}^w_t - \gamma_w\widehat{\pi}_{t-1} = \beta E_t(\widehat{\pi}^w_{t+1} - \gamma_w\widehat{\pi}_t) + \frac{(1-\beta\alpha_w)(1-\alpha_w)}{\alpha_w(1+\frac{1+\lambda_w}{\lambda_w}\phi)}(\widehat{mrs}_t - \widehat{w}_t) \tag{60}$$

- Definition of real wage

$$\widehat{w}_t = \widehat{w}_{t-1} + \widehat{\pi}_t^w - \widehat{\pi}_t \tag{61}$$

- Definition of MRS

$$\widehat{mrs}_t = \phi \widehat{l}_t - \widehat{\lambda}_t \tag{62}$$

- Production function

$$\widehat{y}_t = (1+\lambda_p)(\alpha \widehat{k}_{t-1} + \alpha \widehat{z}_t + (1-\alpha)\widehat{L}_t) \tag{63}$$

- NKPC

$$\widehat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \widehat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \widehat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p} \frac{(1-\beta\alpha_p)(1-\alpha_p)}{\alpha_p} \widehat{mc}_t \tag{64}$$

- Definition of marginal cost

$$\widehat{mc}_t = \alpha \widehat{r}_t^k + (1-\alpha)(\widehat{w}_t + r_t) \tag{65}$$

- Capital labour ratio or labour demand

$$\widehat{L}_t + \widehat{w}_t = \widehat{z}_t + \widehat{k}_{t-1} + \widehat{r}_t^k$$

- Monetary policy

$$i_t = \rho_r i_{t-1} + (1-\rho_r)(\rho_\pi \pi_{t+1} + \rho_y y_t) + \varepsilon_t \tag{66}$$

- Real interest rate

$$f_t = r_t - E_t \widehat{\pi}_{t+1} \tag{67}$$

- Resource constraint

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + r^k \frac{k}{y}\widehat{z}_t$$

A.2 Gertler, Sala and Trigari model

- Household's FOC

$$\widehat{c}_t = \frac{\psi}{1+\psi}\widehat{c}_{t-1} + \frac{1}{1+\psi}E_t\widehat{c}_{t+1} - \frac{1-\psi}{(1+\psi)\sigma}(r_t - E_t\widehat{\pi}_{t+1}) \tag{68}$$

$$\widehat{p}_t^k = E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \frac{r^k}{r^k+1-\delta}\widehat{r}_{t+1}^k + \frac{1-\delta}{r^k+1-\delta}\widehat{p}_{t+1}^k \tag{69}$$

$$\widehat{i}_t = \frac{1}{1+\beta}\widehat{i}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{i}_{t+1} + \frac{1}{\kappa}\frac{1}{1+\beta}\widehat{p}_t^k \tag{70}$$

$$\widehat{z}_t = \frac{1}{\sigma_z}\widehat{r}_t^k \tag{71}$$

$$\widehat{k}_t = (1-\delta)\widehat{k}_{t-1} + \delta\widehat{i}_t \tag{72}$$

- Unemployment

$$u_t = -\frac{n}{u}\hat{n}_{t-1} \tag{73}$$

- Matching

$$\hat{m}_t = \sigma_m\hat{u}_t + (1-\sigma_m)\hat{v}_t \tag{74}$$

- Employment

$$\hat{n}_t = \rho\hat{n}_{t-1} + (1-\rho)\hat{m}_t \tag{75}$$

- Vacancies

$$\hat{x}_t = \hat{q}_t + \hat{v}_t - \hat{n}_{t-1} \tag{76}$$

- Transition probabilities

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t \tag{77}$$

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t \tag{78}$$

- Market tightness

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t \tag{79}$$

- Production function

$$\widehat{y}_t = \alpha\widehat{z}_t + \alpha\widehat{k}_{t-1} + (1-\alpha)\widehat{n}_t \tag{80}$$

- Capital demand

$$\widehat{r}_t^k = \widehat{p}_t^w + \widehat{y}_t - \widehat{z}_t - \widehat{k}_{t-1} \tag{81}$$

- Vacancy posting condition (also gives marginal cost)

$$(\kappa_v x)\widehat{x}_t = p^w mpl(\widehat{p}_t^w + \widehat{mpl}_t) - w\widehat{w}_t + (\kappa_v x)\beta E_t x_{t+1} + (\kappa_v x)(1+\rho)\beta/2(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) \tag{82}$$

- Marginal product of labour

$$\widehat{mpl}_t = \widehat{y}_t - \widehat{n}_t \tag{83}$$

- Phillips curve

$$\widehat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \widehat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\widehat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\alpha_p)(1-\alpha_p)}{\alpha_p}\widehat{p}_t^w \tag{84}$$

- Bargaining weights

$$\widehat{\chi}_t = -(1-\chi)(\widehat{\Sigma}_t - \widehat{\Delta}_t) \tag{85}$$

$$\widehat{\Delta}_t = \rho\lambda_w \beta E_t(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Delta}_{t+1}) \tag{86}$$

and

$$\begin{aligned}\widehat{\Sigma}_t = (x\lambda_w\beta)E_t\widehat{x}_{t+1} - (x\lambda_w\beta)(\frac{w}{\kappa_v x}\Sigma)\Sigma E_t(\widehat{w}_t - \widehat{w}_{t+1} - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t) \\ + \lambda_w\beta E_t(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Sigma}_{t+1})\end{aligned} \tag{87}$$

- Target wage

$$\begin{aligned}\widehat{w}_t^o = \varphi_{mpl}(\widehat{p}_t^w + \widehat{mpl}_t) + (\varphi_x + \varphi_s)E_t\widehat{x}_{t+1} + \varphi_s E_t\widehat{s}_{t+1} + (\varphi_x/2 + \varphi_s)E_t(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t) \\ + \varphi_\chi(\widehat{\chi}_t - \beta(\rho - s)E_t\widehat{\chi}_{t+1})\end{aligned} \tag{88}$$

$$\varphi_{mpl} = \tfrac{\chi p^w mpl}{w} \qquad \varphi_x = \tfrac{\chi\beta\kappa x^2}{w}$$

$$\varphi_s = \tfrac{\chi\beta s\kappa_v x}{w} \text{ as } H = \tfrac{\chi}{1-\chi}\kappa_v x \qquad \varphi_\chi = \tfrac{\chi\kappa_v x}{(1-\chi)w}$$

- Real wage

$$\widehat{w}_t = \gamma_b(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1}) + \gamma_o\widehat{w}_t^o + \gamma_f(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t) \tag{89}$$

$$\gamma_b = (1+\tau_2)\phi^{-1}$$
$$\gamma_f = (\tau\lambda_w^{-1} - \tau_1)\phi^{-1}$$

$$\phi = (1+\tau_2 + \varsigma + \tau\lambda_w^{-1} - \tau_1)$$

$$\tau_1 = (\tfrac{w}{\kappa_v x}\Sigma\varphi_x + \varphi_\varkappa(1-\varkappa)x\beta\lambda^w\tfrac{w}{\kappa_v x}\Sigma^2\rho\beta + \varphi_s\Gamma)(1-\tau)$$

$$\tau_2 = -\tfrac{w}{\kappa_v x}\Sigma\varphi_\chi(1-\chi)(x\beta\lambda^w)\Sigma(1-\tau)$$

$$\Gamma = (1-\eta x\beta\lambda^w\Sigma)\eta^{-1}\Sigma\tfrac{w}{\kappa_v x}$$

$$\gamma_o = \varsigma\phi^{-1}$$

$$\varsigma = (1-\lambda_w)(1-\tau)\lambda_w^{-1}$$
$$\tau = \tfrac{\chi\beta\lambda^w\Sigma + (1-\chi)\rho\beta\lambda^w\Delta}{1+\chi\beta\lambda^w\Sigma + (1-\chi)\rho\beta\lambda^w\Delta}$$

- Monetary policy

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\rho_\pi\pi_{t+1} + \rho_y\widehat{y}_t) + \varepsilon_t \tag{90}$$

- Real interest rate

$$\widehat{f}_t = r_t - E_t\pi_{t+1} \tag{91}$$

- Resource constraint

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + r^k\frac{k}{y}\widehat{z}_t + \frac{\kappa}{2}\frac{x^2n}{y}(2\widehat{x}_t + \widehat{n}_{t-1}) \tag{92}$$