Multi-period fixed-rate loans, housing and monetary policy in small open economies

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Multi-period fixed-rate loans, housing and monetary policy in small open economies*

Jaromír Beneš and Kirdan Lees†

Abstract

We investigate the implications of the existence of multi-period fixed-rate loans for the behaviour of a small open economy exposed to finance shocks and housing boom-and-bust cycles. To this end, we propose a simple and analytically tractable method of incorporating multi-period debt into an otherwise standard consumer problem. Our simulations show that multi-period fixed-rate contracts can help insulate the economy from the adverse effects of particular shocks. This insulating mechanism is particularly effective for countries with high debt positions exposed to foreign exchange fluctuations, or countries operating a fixed exchange rate regime.

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1 Introduction

The financial crisis has driven home to both policymakers (see Bernanke, 2009) and researchers (see Iacoviello, 2005; Iacoviello and Neri, 2010; Kannan et al, 2009, for example) the impact of housing booms and subsequent busts on the economy. While the financial crisis has its genesis in the bust of the housing market in the US and other developed countries, policymakers in both developed and developing countries have also been subject to shocks to country specific risk premia that have widened spreads between the rate of interest on debt and the risk-free rate.

Within this context, policymakers have also been confronted with a variety of environments in housing finance across countries (see Ahearne et al, 2005; Calza et al, 2009). One key feature of several mortgage markets is the existence of multi-period loans where households may take several years to repay debt associated with the purchase of a house. Mortgage contracts of 2-, 5- and 10-year duration are not uncommon.

In this paper, we examine the implications of multi-period loans under both inflation targeting and fixed exchange rate regimes. First, we examine the consumer’s problem and show that \textit{ex ante}, up to first order, multi-period loans do not affect the consumer’s intertemporal consumption decision. The consumers ex-ante decisions are, up to first order, the same as those that would prevail in an economy where only one-period debt exists.

However, \textit{ex post}, shocks to the economy generate valuation effects with real impact on consumer welfare. We shows that the denomination of locally held debt, in either foreign or domestic currency, plays a critical role in determining the impact on consumer welfare. Further, we show how outcomes under multi-period and single-period loans depend on the monetary policy regime and in particular, whether the monetary authority targets inflation or operates a fixed exchange rate regime.

To do this we develop an open economy model that allows households to undertake multi-period loans. We do not motivate the household decision to undertake long-term loans (Campbell and Cocco, 2003, model motivating factors including risk-aversion, risky income and default costs). The open economy model contains three goods and a housing sector. Erceg \textit{et al} (2006) and Barsky \textit{et al} (2007) show that the inclusion of a durable good sector can change the properties of monetary transmission extensively, and is generally more sensitive to interest rates than other sectors. Furthermore Monacelli (2009) introduces a collateral constraint to in part model the extra sensitivity.
of the durable sector to interest rate shocks.

However, mortgage interest rates often respond sluggishly to the official cash rate making monetary stabilisation less efficient (see Kobayashi, 2008, for example). Here we assume full pass-through to the marginal mortgage interest rate but focus on the existence of average mortgage contracts.\(^1\) While our specific application focuses on the consumer and household mortgage rates, our framework is a general one and could be used to address a variety of issues with maturity mismatch features.

Section 2 introduces multi-period fixed rate loans into the consumer’s problem while section 3 develops a complete small open economy model. Section 4 calibrates the model while the response of the economy to a housing boom and bust shock and a country-specific premium shocks are explored in section 5. Section 6 concludes the paper.

## 2 Multi-period fixed-rate loans

In this section, we modify an otherwise standard consumer problem to incorporate the effects of the existence of multi-period debt with fixed repayments. Our aim is not to address the consumer’s portfolio choice between various options (such as multi-period, fixed-rate loans versus one-period or variable-rate loans), but rather to draw out the implications of the existence of long-term debt, while keeping the framework as analytically tractable as possible.

Within the model, the consumer is a net debtor at all times and can only take a loan that is repaid in an infinite number of geometrically decaying payments, starting from next period. The repayments are determined at the time the loan is granted, and cannot be re-negotiated. Declining repayments help us mimic the observed range of maturities that typically exist at the aggregate level. As each cohort of loans (i.e. the loans with different maturities taken at a particular time) decays over time, an increasing number of loans mature and the amount being repaid by that cohort decreases. We can easily characterise the “average” time until maturity of such a geometric loan by Macaulay’s duration, that is, the weighted term to maturity of the cash flows from a loan, and match Macaulay’s duration to the aggregate duration

\(^1\) An early conference version of this paper explored the implications of sticky marginal mortgage rates.
of loans existing in the real world (or other forms of debt, for that matter). Moreover, the declining repayments do not contradict our effort to model fixed-rate loans: It is the fact that the repayment scheme is set and fixed at time $t$ and cannot be changed at any later time that matters.

From a modelling perspective, introducing an infinitely long loan with geometric repayments has two major practical advantages over a more common loan with a fixed, finite maturity and flat repayments throughout. First, the geometric distribution allows us to write all the summations involved in describing the problem in recursive form. In other words, it does not entail a (possibly very large) number of new state variables in models with very long maturities. Second, the average maturity can be calibrated using just one parameter, namely the rate at which the repayments decay, without changing the model’s structure in any other way.

We now describe the problem formally. A loan $L_t$ taken at time $t$ is paid back in repayments proportional to the amount borrowed and decays at a fixed rate $\phi \in (0, 1)$. This implies the following repayment schedule:

- Repayment due at $t + 1$: $Q_t L_t$,
- Repayment due at $t + 2$: $\phi Q_t L_t$,
- ...,
- Repayment due at $t + k$: $\phi^{k-1} Q_t L_t$, etc.

The consumer’s otherwise standard budget constraint can be written as

$$L_t = J_{t-1} - \Delta_t,$$

where $L_t$ is the amount currently borrowed, $J_{t-1}$ is the sum of all repayments due at $t$ associated with all past loans, that is:

$$J_{t-1} = \sum_{k=1}^{\infty} \phi^{k-1} Q_{t-k} L_{t-k},$$

and $\Delta_t$ is the consumer’s current income less current expenditures (not including debt service). Note first that $J_t$ can be written recursively as

$$J_t = \phi J_{t-1} + Q_t L_t,$$  \hspace{1cm} (1)

---

2 See Bierwag and Fooladi (2006) for an overview of duration analysis. We discuss the duration in more detail in section 3.7

3 For example, 80 new state variables would be needed in a quarterly model with 20-year debt contracts.
and that setting $\phi = 0$ exactly reproduces the standard problem with one-period debt.

Then, assigning the $t+k$ budget constraint a Lagrange multiplier $\beta^k \Lambda_{t+k}$, and the law of motion for $J_{t+k}$ another Lagrange multiplier $\beta^k \Lambda_{t+k} \Psi_{t+k}$, we can work out the first-order conditions with respect to $L_t$ and $J_t$, respectively:

$$L_t : 1 = \Psi_t Q_t,$$

$$J_t : \Psi_t = E_t \left[ \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1 + \phi \Psi_{t+1}) \right].$$

To understand these two conditions, we will now turn to how the repayments are determined. Suppose there is a risk-neutral agent who supplies the multi-period loans while refinancing through short-term (one-period) debt, and bearing all the maturity mismatch risk. In a competitive market, the present value of the repayments discounted by the short-term rate $R_t$ will equal the amount borrowed,

$$1 = Q_t \sum_{k=1}^{\infty} \phi^{k-1} E_t \left[ \frac{1}{R_t \cdots R_{t+k-1}} \right].$$

Expressing this condition recursively,

$$1 = \Omega_t Q_t,$$

$$\Omega_t = \frac{1}{R_t} E_t [1 + \phi \Omega_{t+1}],$$

and comparing Eqs. (4)–(5) with Eqs. (2)–(3), we can observe that $\Phi_t = \Omega_t$ at all times, and up to first order,

$$E_t \left[ \frac{\beta \Lambda_{t+1}}{\Lambda_t} \right] \approx \frac{1}{R_t}.$$

In other words, the usual one-period Euler consumption equation approximately holds also with multi-period loans, and is based on the one-period rate, i.e. the rate used to construct the price of the multi-period loan.

We can now summarise the two most important implications:

1. The consumer’s ex-ante decisions are, up to first order, the same as those that would prevail in an economy where only one-period debt exists. The existence of multi-period loans does not alter the way the consumer wishes to substitute intertemporally; the intertemporal substitution between $t$ and $t+1$ is still determined by the underlying one-period rate. Nor does it change the ex-ante path for the consumer’s shadow value of wealth.
2. The existence of multi-period loans matters for the consumers’ outcomes \textit{ex post}, through valuation effects, that occur when the economy is hit by unforeseen shocks. This can be seen from the budget constraint (shown in equation (1)), where only a small proportion of future repayments is affected by today’s conditions, whereas for a one-period loan, the entire $t + 1$ repayment is determined by today’s short-term rate. We will return to this finding when designing the complete model in Section 3.

Note that implication 1 is not specific to the design of our geometric loan; it is more general. Imagine a more realistic example where the consumer can choose any combination of all possible maturities between one and infinity. Although we would not be able to determine the individual amounts borrowed (without adding some more assumptions), the first-order conditions, which would effectively give rise to an expectations-based term structure of interest rates, would still include the intertemporal condition relating consumption at $t$ and $t + 1$ to a one-period rate.

3 The complete model

In order to explore the macroeconomic implications of multi-period loans, we build a simple model of a small open economy that includes a housing sector and three types of goods: (i) imported consumption goods; (ii) domestically produced non-tradables demanded as consumption and residential investment; and (iii) exogenous export goods. To give housing wealth a non-trivial role in our simulations of house price bubbles and busts, we introduce a premium over the world rate that is not modelled from first principles here. The premium is an increasing function of the aggregate loan-to-value ratio. In a small open economy model, this set-up has less direct implications for current account dynamics than the more common collateral constraint used by Iacoviello (2005) which has similar effects to the Bernanke \textit{et al} (1999) financial accelerator.\footnote{We do not, though, derive the premium from an explicit debt contract.}

To keep the model mechanisms as simple as possible and the number of parameters low, we do not explicitly model the local input factor markets. Instead, we use a roundabout production function, proposed by Basu (1995), in our non-tradables market. This is a convenient short-cut, which gives rise to a pro-cyclical real marginal cost.

For ease of notation, we assume that the loans are denominated in local
currency throughout this section; it is straightforward to modify the model’s equations to allow for foreign-currency denomination, or a combination of both. Also, we do not explicitly introduce the usual Dixit and Stiglitz (1977) monopolistically competitive markets when they are necessary (i.e. for costly price adjustments), but instead directly impose downward-sloping demand curves on representative agents that are consistent with the implications of such monopolistic market structures. To this end, quantities and prices taken as given, are denoted by a bar.

3.1 Consumers

The representative consumer purchases non-tradable consumption, \( C_{N,t} \), imported consumption, \( C_{M,t} \), and residential investment, \( I_t \), and takes multi-period, fixed-rate loans, \( L_t \) (whose structure is described in section 2), to maximise lifetime utility,

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \omega_N \log(C_{N,t} - \theta \bar{C}_{N,t-1}) + (1 - \omega_N) \log(C_{M,t} - \theta \bar{C}_{M,t-1}) + \log H_t \right] ,
\]

subject to a budget constraint,

\[
L_t = J_{t-1} - [\Pi_t + P_{X,t}X_t - P_{N,t}(C_{N,t} + I_{N,t}) - P_{M,t}C_{M,t}] ,
\]

\[
J_t = \phi J_{t-1} + Q_t L_t ,
\]

and a law of motion for the housing stock, \( H_t \),

\[
H_t = (1 - \delta)H_{t-1} + I_t(1 - h_{I,t}) ,
\]

where \( \Pi_t = \Pi_{N,t} + \Pi_{M,t} \) are the net profits received from the non-tradables producer and the importer, \( P_{X,t}X_t \) are exogenous export revenues, \( J_{t-1} \) is the sum of repayments associated with all existing loans not matured yet, \( Q_t \) determines the repayment scheme of the new loans, and \( h_{I,t} \) is an investment adjustment cost given by

\[
h_{I,t} = \psi_I \left( \Delta \log I_t - \Delta \log \bar{I}_{t-1} \right)^2 .
\]

We define the consumer’s habit not in aggregate consumption, but separately for non-tradables and imports, similarly to Ravn et al (2006). We use this feature to keep both the intertemporal elasticity of substitution and the intratemporal elasticity of substitution realistically low in the short run, but gradually increasing to one in the medium and long run.
For future reference, we denote the current-value Lagrange multiplier on the 
$t + k$ budget constraint by $\beta^k \Lambda_{t+k}$, and that on the $t + k$ housing stock 
equation by $\beta^k \Lambda_{t+k} \Phi_{t+k}$.

### 3.2 Non-tradables producer

The representative non-tradables firm combines a fixed amount of business 
capital (normalised to one) and non-tradable intermediate inputs, $N_t$, to 
produce its output 

$$Y_t = 1^{1-\gamma} N_t \gamma,$$

The firm possesses monopoly power $\mu$ when selling its goods to the consumers 
and other firms in the sector. It chooses $Y_t$, $N_t$, and the final price, $P_t$, to 
maximise its present value 

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[ P_{N,t} Y_t (1 - h_{N,t}) - \bar{P}_{N,t} Z_t \right] ,$$

subject to a downward-sloping demand curve 

$$Y_t = \left( \frac{P_{N,t}}{\bar{P}_{N,t}} \right)^{\frac{\mu}{\mu-1}} Y_t ,$$

and where the firm’s net revenues are diminished by price adjustment costs, $h_{N,t}$, given by 

$$h_{N,t} := \frac{\psi_N}{2} (\Delta \log P_{N,t} - \Delta \log \bar{P}_{N,t})^2 .$$

The adjustment costs are private, not social, costs, and hence do not appear 
in the market clearing condition (10).

### 3.3 Importers

The representative importer purchases foreign-produced goods at a world 
price $P_{W,t}^*$, and re-sells them locally with monopoly power $\mu$. The importer 
chooses the volume of imports, $C_{M,t}$, and the final price, $P_{M,t}$, to maximise 
its present value, 

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[ P_{M,t} C_{M,t} (1 - h_{M,t}) - S_t P_{W,t}^* C_{M,t} \right]$$

subject to a downward-sloping demand curve, 

$$C_{M,t} = \left( \frac{P_{M,t}}{\bar{P}_{M,t}} \right)^{\frac{\mu}{\mu-1}} \bar{C}_{M,t} .$$
The importer’s net revenues are diminished by price adjustment costs, $h_{N,t}$, given by

$$h_{M,t} = \frac{\psi_M}{2} (\Delta \log P_{M,t} - \Delta \log \bar{P}_{M,t})^2.$$

### 3.4 Supply of multi-period loans

The multi-period loans are supplied by a foreign-owned, risk-neutral intermediary, which refinances itself through short-term (one-period) foreign-currency debt at a risk-free world rate, $R^*_W,t$. The assumption that the intermediary is not owned locally is a critical one. As it is clear from section 2, the existence of multi-period, fixed-rate loans affects the outcomes mainly through valuation effects. If the maturity mismatches were local, then the consumer’s unexpected losses from holding multi-period debt would net out with the transfers of the intermediary’s reciprocal gains to the consumer’s budget, and vice versa.5

We now describe the behaviour of the intermediary when the loans supplied to the consumer are denominated in local currency; a modification with foreign-currency denomination, or allowing for a combination of both, is then rather straightforward. When setting the geometric repayments, $Q_t, \phi Q_t, \ldots$, associated with the loans granted at time $t$, the intermediary equalises their present value with the amount borrowed, using the world rate plus an ad-hoc premium, $g_t$, as the discount factor. The repayments are thus given implicitly by the following equation:

$$1/S_t = Q_t \sum_{k=1}^{\infty} \phi^{k-1} E_t \left[ \frac{1}{S_{t+k}} \sum_{j=1}^{\infty} \frac{1}{S_{t+j}} \right]$$

which can be thought of as a modified, multi-period uncovered interest parity.

The premium, $g_t$, is increasing in the country’s aggregate loan-to-value ratio, measured by the total value of outstanding debt, $S_t V^*_t$, relative to the total value of houses, $P_{H,t} H_t$ (both expressed in local currency),

$$g_t = \zeta \left( \frac{S_t V^*_t}{P_{H,t} H_t} - \lambda \right) + u_t,$$

where $u_t$ is an autonomous component of the premium, which is subject to shocks in our simulations. We set the house price, $P_{H,t}$, equal to the

5 Another option would be a locally owned intermediary with its own net worth.
consumer's shadow value of the housing stock, $\Phi_t$, in normal times, but introduce an exogenous, persistent wedge between the two (a bubble) in our housing boom and bust simulations; we explain the bubble in more detail in section 4.

The value of debt is, in turn, calculated as the present value of the repayments due at $t + 1$, $t + 2$, and so forth, associated with the current and past loans not matured yet.

$$V_t^* = J_t \sum_{k=1}^{\infty} \phi^{k-1} E_t \left[ \frac{1}{S_{t+k}} \left( \frac{1}{S_{t+k}} \left( R_{W,t}^* + g_t \right) \cdots \left( R_{W,t+k-1}^* + g_{t+k-1} \right) \right) \right] = \frac{J_t}{S_t Q_t},$$

where the last equality follows from Eq. (6).

Finally, notice that for $\phi = 0$, the pricing of the loans reduces to a rather standard, premium-augmented uncovered interest parity,

$$R_t = \frac{R_{W,t}^* + g_t}{E_t \left[ \frac{S_t}{S_{t+1}} \right]},$$

where we denote the one-period loan repayment, i.e. the short-term gross interest rate, by $R_t$ for future reference.

### 3.5 Monetary policy

We explore both inflation targeting and fixed exchange rate regimes in our simulations. To capture inflation targeting, we use a simple policy rule that responds to deviations of expected annual inflation from an inflation target and allows for some smoothing of interest rates by responding to the previous value of the interest rate:

$$\log R_t = \rho \log R_{t-1} + (1 - \rho) \left( \log \bar{R} + \kappa E_t [\Delta_4 \log P_{t+4} - \pi] \right), \quad (8)$$

where $\Delta_4$ is four-quarter difference divided by four, $P_t$ is a consumer price index defined by

$$\Delta \log P_t = \omega_N \Delta \log P_{N,t} + (1 - \omega_N) \Delta \log P_{M,t}, \quad (9)$$

and $\pi$ is the inflation target.

Although the rule does not contain an output gap or output growth term, the forward-looking nature of the inflation component encapsulates contemporaneous demand pressure. The rule is simple and parsimonious but has also
been used in medium scale DSGE models used for policymaking at central banks (see Beneš et al., 2009).

To capture countries that pursue fixed exchange rates, we examine an exchange rate peg, with the central bank setting \( S_t = \bar{S} \) at all times.

### 3.6 Aggregation and market clearing

The non-tradables market clears,

\[
Y_t = C_{N,t} + I_t + N_t,
\]

and the following conditions hold in symmetric equilibrium:

\[
\begin{align*}
\bar{Y}_t &= Y_t, \\
\bar{P}_{N,t} &= P_{N,t}, \\
\bar{C}_{M,t} &= P_{M,t}, \\
\bar{P}_{M,t} &= P_{M,t}, \\
\bar{C}_{N,t} &= C_{N,t}.
\end{align*}
\]

### 3.7 Model calibration

Since we consider the model to be representative of a generic small open economy, we do not calibrate or estimate the model to a particular dataset. Instead, we adopt the set of parameters shown in table 1 that implies relatively standard properties and behaviour of the model.\(^6\) First we choose plausible steady-state output growth, inflation and real interest rates before choosing parameters that govern the dynamic properties of the model. We check that these parameters generate plausible dynamics, for example, that a hump-shape in consumption is observed in response to many shocks. For example, we choose a value 0.75 for the habit parameter - a value slightly higher than that reported in the literature, for example the value of 0.7 reported in Bodrin et al (2001) for example. The habit parameter determines both the short-run trade-off between tradables and non-tradable goods and also the sensitivity of consumption to the real interest rate. We chose a quarterly discount factor of 0.97\(^{1/4}\), similar to the value of the patient households in Iacoviello and Neri (2010) and the standard value of 0.99 in the literature.

\(^6\) Simulation properties that document the behaviour of the model in more detail are available upon request from the authors.
Table 1
Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Steady-state parameter</th>
<th>Transitory parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>$0.97^{1/4}$</td>
<td></td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Share of non-tradables in the CPI</td>
<td>0.70</td>
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<tr>
<td>$\omega_h$</td>
<td>Housing preference parameter</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation of the housing stock</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Market power of producers and importers</td>
<td>1.20</td>
<td></td>
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</table>

Steady-state parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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Transitory parameters

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<td>$\chi$</td>
<td>Habit parameter</td>
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<td>$\xi_n$</td>
<td>Non-tradable adjustment costs</td>
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<tr>
<td>$\zeta$</td>
<td>Elasticity of interest rates to the debt-to-value ratio</td>
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<td>$\rho_r$</td>
<td>Policy rate smoothing</td>
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<td>$\kappa$</td>
<td>Policy reaction to 4-quarter ahead expected annual inflation</td>
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<td>$\phi$</td>
<td>Loan repayment parameter</td>
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<td>$\rho_t$</td>
<td>Autocorrelation in terms of trade</td>
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<tr>
<td>$\rho_{rw}$</td>
<td>Autocorrelation in world rate</td>
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<tr>
<td>$\sigma$</td>
<td>House price bubble growth rate while bubble persists</td>
<td>1.05</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of a house price bubble burst</td>
<td>0.10</td>
</tr>
</tbody>
</table>

and an inflation target of 2% p.a., the mid-point of the band for several inflation targeters. Both these numbers generate plausible real and nominal interest rates in the long run.

Although the policy reaction function does not respond to an output gap or growth rate, the policy function responds strongly ($\kappa = 5$, see Beneš et al (2009) for a similarly high value for $\kappa$) to a four-quarter ahead expectation of annual inflation, determined at least in part by the pressure on firms to meet excess demand. There is some policy smoothing ($\rho = 0.85$) that prevents excess volatility in the nominal interest rate. In terms of depreciation of the housing stock, we set $\delta = 0.02$, implying somewhat more rapid depreciation than Iacoviello and Neri (2010). Finally, we introduce persistence in the housing shock ($\rho_b$), terms of trade shock ($\rho_t$) and autocorrelation in the world interest rate ($\rho_{rw}$).

When calibrating the parameter $\phi$, which controls the distribution of the loan repayments over time, we can relate its value to the implied average maturity of the loan in the model’s non-stochastic steady state. We measure
the average maturity by Macaulay’s duration. Macaulay’s duration of an asset, or a stream of payments in general, is defined as the present-value-weighted time to receive each cash flow. Formally, let $PV_t$ be the present value of a payment receivable at time $t$; the duration is given by

$$d = \frac{\sum_{k=1}^{\infty} k PV_{t+k}}{\sum_{k=1}^{\infty} PV_{t+k}}.$$

The general definition of duration does not prescribe how to calculate the present value of the cash flows. In the context of our model, we use the short-term rates to discount future payments. The non-stochastic steady-state duration of the geometric loan is then a function of $\phi$ and the steady-state short-term interest rate, $R$:

$$d = \frac{\sum_{k=1}^{\infty} k \frac{\phi^{k-1}}{R^k}}{\sum_{k=1}^{\infty} \frac{\phi^{k-1}}{R^k}} = \frac{R}{\frac{1}{R-\phi}} = \frac{R}{R - \phi}.$$

Given our other parameters that determine $R$ (namely $\beta$, $\pi$, and $\lambda$) we can choose the desired duration, and set $\phi$ accordingly. In Table 2, we show the values for a range of durations given our calibration of the steady-state short-term rate (approximately 5 % p.a.).

Note also that out of steady state, with the short-term interest rate fluctuating around their long-run levels, the duration of the loans varies over time. We simulated the order of magnitude of these variations, and found them to be negligible for a realistic range of shocks. In other words, the duration stays very close to the steady-state calibrated duration, and thus the character of the fixed-rate loans does not change in our dynamic simulations.

<table>
<thead>
<tr>
<th>Duration $d$</th>
<th>1 qtr</th>
<th>1 year</th>
<th>2 yrs</th>
<th>5 yrs</th>
<th>10 yrs</th>
<th>15 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repayment parameter $\phi$</td>
<td>0</td>
<td>0.7595</td>
<td>0.8861</td>
<td>0.9620</td>
<td>0.9873</td>
<td>0.9958</td>
</tr>
</tbody>
</table>

### 4 Simulation experiments

Our simulations explore several two specific shocks, designed to capture the experiences of several countries in both the run up to financial crisis and

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7 The concept of duration and the ways it has been used in macroeconomics and finance are briefly described in Weil (1973).
its subsequent aftermath. We further focus on the role of foreign currency denominated debt relative to local currency denomination, and whether the monetary authority operates an inflation targeting regime, or targets a fixed exchange rate. These factors have critical implications on the extent to which multi-period loans can insulate the economy from these shocks.

Our first shock details a housing boom, where house prices increase by 20 percent over four quarters, before ending abruptly when house prices return to zero in the fifth quarter. While the timing of the boom phase is clearly truncated, many developed and developing countries faced large increases in house prices and a subsequent fall in the run up to the financial crisis.

We also examine an exogenous risk premium shock, where international investors demand a country-specific premium — over and above the premium suggested by the country aggregate loan-to-value rate — to risk lending to domestic borrowers. The effects of this shock are detailed in our second set of simulations.

4.1 Simulating house price bubbles

Before turning to the simulation results, we describe what we mean by a “house price bubble” and more technically, how we simulate such a bubble. We closely follow Bernanke and Gertler (1999) such that the observed (or market) price of the houses, $P_{H,t}$, which enters the endogenous part of the finance premium, (7), can deviate from its fundamental level, i.e. the shadow value of the housing stock, $\Phi_t$. We call the logarithmic discrepancy between the two a bubble, $b_t$,

$$b_t = \log P_{H,t} - \log \Phi_t.$$ 

Whenever a bubble, $b_t \neq 0$, occurs at time $t$, it is modelled to grow at a rate $\sigma > 1$ with probability $(1 - \theta) \in [0, 1)$, where $(1 - \theta)\sigma < 1$, or burst ($b = 0$) with probability $\theta$ at $t + 1$. In other words, the conditional one-period-ahead expectations are

$$E_t[b_{t+1}] = (1 - \theta)\sigma b_t,$$

while the actual path for $b_t$ is $b_{t+1} = \sigma b_t$ so long as the bubble persists. Technically, we can simulate this kind bubble as follows. We introduce an autoregressive process for $b_t$,

$$b_t = \rho_b b_{t-1} + \epsilon_{b,t},$$

Note that in Bernanke and Gertler (1999), the bubble is the difference between $P_{H,t}$ and $\Phi_t$, not their logarithms.
with \( \rho_b := (1 - \theta)\sigma \). This autoregression guarantees that the conditional expectations will evolve according to the assumptions we made above. To make the bubble temporarily follow an explosive path, we include a series of \textit{unexpected} shocks \( \epsilon_{b,t} \) computed so that \( b_t = \sigma b_{t-1} \) during the house price bubble episode.

### 4.2 Housing boom and bust

Figure 1 shows impulse responses for selected macroeconomic variables from the house boom and bust. Note that the impulse responses are for a specific sequence of house price shocks, designed to return the house price track depicted in the bottom left panel of the figure. Through the boom phase (shaded grey in the figure), as the house price increases, the loan-to-value ratio falls, reducing the real interest rate the consumer faces, thus stimulating consumption. Since we have shown that existence of multi-period loans does not matter for the consumer’s \textit{ex ante} decision, only valuation affects will alter the paths under one-period loans and multi-period loans. Given that the loans are denominated in domestic currency and are thus not subject to valuations effects stemming from changes in the nominal exchange rate, the paths of consumption and indeed all the key macroeconomic variables in the figure are very close under both single-period and multi-period loans. In contrast figure 2 shows that when the loans are denominated in foreign currency, the valuation effects are large, and marked differences occur between one-period loans and the multi-period case. Over the house price boom, the nominal exchange rate appreciates, generating a decrease in the total repayments required on the debt incurred by the household. The presence of multi-period loans implies some future repayments are less affected by the exchange rate fluctuations and thus the economy is relatively insulated from the house price boom.

Figure 3 depicts the house boom under the fixed exchange rate regime with loans denominated in the local currency. Since monetary policy cannot adjust optimally to the accommodate the shock, movements in consumption and output are more pronounced than the case for the inflation targeting regime. Relative to the inflation targeting case, the real interest rate moves through a relatively large range and this generates sufficient valuation effects to generate differences in the impulses under the single- and multi-period loan scenarios. The total repayments are larger under the single-period case. The case where loans are denominated in foreign-currency under a fixed exchange rate is, of course, identical to the case of domestic currency loans and is depicted in
4.3 Country premium shock

The country premium shock increases the repayments required by foreign investors to compensate them for country specific risk. The shock is temporary and assumed to affect the premium at time $t$. The shock effectively increases the interest rate and households reduce consumption. Residential investment falls since there is short run reduction in demand for housing services. While multi-period contracts smooth through some of the effects of the shock, reducing total repayments under multi-period contracts that fall after the initial impact of the shock (see the bottom right panel of figure 5), the paths of key macroeconomic variables of output, inflation and interest rate are in fact very similar under single- and multi-period loans.

However, when the loans are denominated in foreign currency, figure 6 shows dramatic differences that occur between single-period and multi period loans. The large devaluation of the domestic currency under the shock implies that total repayments under the single period case are much larger than under multi-period loans. This drives a large fall in consumption, residential investment and output under the single-period case. Clearly, the presence of multi-period loans can help minimise large fluctuations in the economy when loans are denominated in foreign currency.

There are also large differences between single-period and multi-period loans under an exchange rate peg. Under this monetary policy regime, figure 7 shows that the total repayments increase substantially relatively to the multi-period counterpart, generating concomitant decreases in consumption and output. Finally, figure 8 shows that the paths of the impulse responses under the foreign currency denominated loans are identical since the exchange rate is fixed.

5 Conclusion

Mortgage debt contracts in many countries are characterised by multi-period fixed-rate loans rather than single period loans that are simply rolled over each period. To mimic this behaviour, that is present in several countries, we develop a model that allows households to be net debtors at all times and hold multi-period loans.
We show that we can match the long run implications of debt and also use the model to produce both the observed range of maturities that typically exist at the aggregate level, and the average time to maturity of debt, while keeping the framework as analytically tractable as possible. While our specific application is to mortgage markets, the multi-period loan framework is flexible and in future work could be applied to maturity mismatch problems more generally.\footnote{For example, currently we are studying the effects of maturity mismatches on the balance sheets of financial institutions to examine the rationale for liquidity regulations.}

Fixed multi-period loans are shown to not influence the consumer’s intertemporal decision problem \textit{ex ante}. But \textit{ex post}, we show that the existence of multi-period loans can prove a useful buffer in terms of insulating the economy from the effects of selected shocks. In particular, we show that multi-period loans can help stabilisation of an economy in response to a housing boom-bust cycle and a country risk premia shock — shocks that characterise the global financial crisis.

What proves critical for the insulating effects of multi-period loans is the monetary policy regime. If an exchange rate peg is in operation, the economy moves through more pronounced consumption and output cycles than under the standard case of single period loans. Further, if the debt held by households is denominated in foreign rather than domestic currency, multi-period loans assist in helping policy stabilise the macroeconomy. Thus the model has very broad implications for the implementation of monetary policy in an environment characterised by shocks that describe the global financial crisis.


Appendices
Figure 1
Housing boom and bust: Inflation targeting, local-currency loans.
Figure 2
Housing boom and bust: Inflation targeting, foreign-currency loans.
Figure 3
Housing boom and bust: Exchange rate peg, local-currency loans.
Figure 4
Housing boom and bust: Exchange rate peg, foreign-currency loans.
Figure 5
Country premium shock: Inflation targeting, local-currency loans.
Figure 6
Country premium shock: Inflation targeting, foreign-currency loans.
Figure 7
Country premium shock: Exchange rate peg, local-currency loans.
Figure 8
Country premium shock: Exchange rate peg, foreign-currency loans.