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# Evaluating a monetary business cycle model with unemployment for the euro area*

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## Abstract

This paper estimates a medium-scale DSGE model with search unemployment by matching model and data spectra. Price markup shocks emerge as the main source of business-cycle fluctuations in the euro area. Key for the propagation of these disturbances are a high degree of inflation indexation and a persistent response of monetary policy to deviations of inflation from the target.

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# 1 Introduction

What are the salient features of aggregate fluctuations in the euro area? And what are the main shocks and frictions behind these salient features?

To uncover some stylized facts of euro area business cycles, this paper examines the spectral densities of seven key macro variables for the period 1984 - 2005. The main finding emerging from this exercise is that the spectral densities of output growth, consumption growth, investment growth and wage growth all exhibit a single peak at period 26 quarters.

To understand which shocks and frictions are key in shaping the spectral densities of the euro area macro variables, the paper estimates a dynamic stochastic general equilibrium model (DSGE) that features equilibrium unemployment as well as several shocks and frictions. The estimation technique, called spectra matching (henceforth $SM$), is designed to maximize the model's ability to fit the spectral densities of the data. In particular, $SM$ has two main characteristics: (1) $SM$ weights each frequency proportionally to its contribution to the variance of the data; (2) $SM$ focuses on auto-covariances and ignores cross-covariances. This approach is suited to estimate a stylized model of the business cycles.

Shocks to the firms desired markup emerge as the main source of business-cycle fluctuations in the euro area. In particular, price-markup shocks are key to generating the 26-quarter cycles in output growth, consumption growth and wage growth. The features of the model which are crucial for the propagation of these cost-push disturbances are a high degree of backward-lookingness in price-setting behavior and a persistent and aggressive response of monetary policy to deviations of inflation from the target.

Another interesting finding is the remarkable ability of the model to fit the spectral density of the vacancy/unemployment ratio by involving three types of disturbances: price-markup shocks account for the bulk of business-cycle volatility while wage-markup shocks and preference shocks inject variance at low frequencies. Importantly, this good performance of the model is consistent with a good fit of the spectrum of real wages and it does not rely on incredible values for the unemployment benefit and the worker bargaining power. Unfortunately however, the model tends to underestimate the variance of investment growth at business-cycle frequencies.

Section 2 presents some stylized facts of the euro area business cycles. Section 3 discusses the spectra matching estimation strategy. Section 4 sets out the DSGE model. Section 5 gives the main results on the role of various shocks and frictions in shaping fluctuations in the euro area. Section 6 checks for robustness to an alternative estimation technique. Section 7 concludes.

# 2 Some stylized facts of euro area business cycles

To extract salient features of aggregate fluctuations in the euro area, I estimate the spectral density of seven key macro variables: the yearly growth rate of real output, the yearly growth rate of real consumption, the yearly growth rate of real investment, the yearly growth rate of real wages, the yearly inflation rate, the short-term nominal interest rate and the vacancy/unemployment ratio which summarizes the tightness of the labor market and plays an important role in the Mortensen-Pissarides model. I use quarterly observations from 1984:QI to 2005:QIV on these seven variables. The appendix describes the database in details.

The data are expressed in log deviations from sample mean. I estimate the spectrum of the transformed data using a VAR with four lags. The short length of the sample prevents a non-parametric estimation of the data spectrum. Confidence bands are computed using the parametric bootstrapping methods described by Ohanian (2004). Figure 1 plots the data together with their estimated spectral densities.[1] We see that investment growth has a much larger variance than output growth while the variance of consumption growth is smaller than the one of output growth. The variance of wage growth is roughly equal to the variance of output growth. The variance of the vacancy/unemployment ratio is by far the largest and stems from both business-cycle and low frequencies. Both inflation and the interest rate are trending downward over the sample period. Hence, not surprisingly, these two variables have monotonously decreasing spectra. Interestingly, the spectral densities of the yearly growth rates of output, consumption, investment and wages are all characterized by a single peak at period 26 quarters. Figure 2 compares the spectral densities of euro area and U.S. data for the period 1984 - 2005. In the U.S., the spectral density of output growth exhibits two peaks which corresponds to a 38-quarter cycle and an 18-quarter cycle. The 38-quarter cycle stems from investment growth while the 18-quarter cycle is inherited from consumption growth. The presence of a dominant cycle of period 6 1/2 years, common to output, consumption, investment and wage growth stands as a distinguishing feature of macroeconomic fluctuations in the euro area.

[1] The spectral density matrix $F(\omega)$, $\omega \in (0, \pi)$ decomposes the covariance matrix frequency by frequency. Let $f_{ii}(\omega)$ denote a generic main-diagonal element of $F(\omega)$. $f_{ii}(\omega)$ corresponds to the variance of variable $X_i$ at frequency $\omega$.

## 3 The spectra matching estimation technique

The spectrum and the covariance generating function are two alternative ways to summarize the complete set of second moments. The spectrum is convenient to analyse cyclical fluctuations. This paper applies an estimation technique which minimizes a distance between the spectrum of the model and the spectrum of the data. This *spectra-matching* approach makes it straightforward to weight frequencies differently in estimation. As stressed by Diebold, Ohanian and Berkowitz (1999), this feature is appealing to estimate a stylized model which is designed to explain primarily some frequencies of interest. The implementation of spectra matching in this paper follows Wen (1998).

Let $\theta$ denotes a column vector stacking the DSGE model's parameters to be estimated. The spectra-matching estimator is defined by

$$\widehat{\theta}_W = \arg\min_{\theta} \left[G_W(\theta)\right] \tag{1}$$

where the distance $G_W(\theta)$ is given by,

$$G_W(\theta) = \mathrm{tr}\left[D(\theta)\right], \tag{2}$$

with

$$D(\theta) = \sum_{j=1}^{T} W(\omega_j) \odot \left| F_m(\omega_j;\theta) - \widehat{F}_d(\omega_j) \right|, \ \omega_j \in (0.01, \pi). \tag{3}$$

Here $\odot$ denotes the element-by-element multiplication, $T$ denotes the sample size and $\left\{ \omega_1, \ \omega_2, \ \ldots \ \omega_T \right\}$ is a grid over $(0.01, \pi)$. The function $W(\omega_j)$ weights the absolute value of the difference between the model spectrum, $F_m(\omega_j;\theta)$, and a consistent estimate of the data spectrum, $\widehat{F}_d(\omega_j)$, across frequencies $\omega_j \in (0.01, \pi)$. I compute the theoretical spectrum $F_m(\omega, \theta)$ directly from the state-space representation of the solution to the log-linearized DSGE model. As already explained in the previous section, the spectrum of the data is estimated using a VAR with four lags.

Importantly, by taking the trace of matrix $D(\theta)$, I completely disregard the off-diagonal elements which contain information about cross-covariances. Hence, the distance $G_W(\theta)$ focuses precisely on the information summarized in the left column of Figure 1.

The weighting function $W(\omega_j)$ takes the following expression

$$W(\omega_j) = \widehat{F}_d(\omega_j) \oslash \left[ \sum_{j=1}^{T} \widehat{F}_d(\omega_j) \right], \tag{4}$$

where $\oslash$ denotes the element-by-element division. The first term $\widehat{F}_d(\omega_j)$ weights frequency $\omega_j$ according to its contribution to the total variance of the data. The term in squared brackets approximates the covariance matrix of the data. It acts as a scaling factor that prevents the minimization routine concentrating on the variables with the largest variances. Looking at figure 1, we see that this weighting scheme implies that the estimation strategy aims primarily to fit the variances of the four yearly growth rates and the vacancy/unemployment ratio at business-cycle frequencies and the variances of inflation and the interest rate at low frequencies. Figure 3 illustrates the fact that the variance of yearly growth rates is mainly concentrated at medium frequencies whereas quarterly growth rates exhibit considerable power at high frequencies. Hence, considering yearly growth rates instead of quarterly growth rates affects the shape of the weighting scheme.[2]

# 4 Model

Following Krause and Lubik (2007*b*), Trigari (2009), and Walsh (2005), I introduce the search and matching model of equilibrium unemployment due to Diamond, Mortensen and Pissarides into a sticky-price model of the business cycle. Apart from the labor market block, the model is similar to the one estimated by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003,2007) (henceforth CEE and SW respectively). In particular, the model features capital accumulation with investment adjustment costs and variable capital utilization. Capital accumulation introduces an additional state variable (the stock of physical capital) and therefore affects the dynamics of the endogenous variables as well as the propagation of the various disturbances. Moreover, the presence of variable capital utilization matters for the determination of marginal costs and thereby inflation dynamics. Finally, as in SW, the law of motion of capital is buffeted by an investment-specific technology shock that affects the rate at which the consumption good is transformed into capital. Christoffel, Kuester and Linzert (2009) (henceforth CKL) estimate a sticky-price model with unemployment on euro area data. However, their model does not feature capital accumulation. Similarly to Krause and Lubik (2007*b*), I opt for "Rotemberg pricing" instead of "Calvo pricing" in order to avoid the technical issues that arise in the presence of firm-specific inputs and staggered

[2] Instead of using the weighting function $W(\omega_j)$, I could weight each frequency in inverse proportion to the uncertainty surrounding the spectrum of the data. However, I don't take this route as it would assign most weight to high frequencies. An interesting alternative would be to use a band-pass weighting scheme assigning zero weight to low and high frequencies and unit weight to business-cycle frequencies.

price setting.[3] Hence, the same firms face both sticky-price rigidities in their output markets and search frictions in the market for labor input.

Turning to the labor market block, hours per worker are constant, job destruction is exogenous and constant, labor force participation is constant and there is no on-the-job search.[4] Hence, firms adjust labor exclusively through job creation.and only along the extensive margin. Firms face convex hiring costs as in Yashiv (2006). This feature helps the model to capture the high persistence in vacancies and unemployment. Finally, as in Ravenna and Walsh (2008), new hires become productive immediately. Therefore the representative firm has three margins to adjust its output level instantaneously in response to shocks: (1) by changing its price, (2) by varying the intensity of capital utilization and (3) by adjusting its work force. This last *instantaneous margin* is reasonable when taking the model to quarterly data. Moreover, its absence would generate an excess volatility of capital utilization and / or prices.

Stochastic singularity is strongly at odds with the data. Hence the model incorporates as many shocks as observable variables. Fluctuations are driven by seven disturbances: a neutral technology shock, an investment-specific technology shock, a preference shock, a price-markup shock, a wage-markup shock, an exogenous spending shock and a monetary policy shock. The neutral technology shock follows a random walk with drift and the DSGE model is consistent with balanced growth. This feature allows me to detrend the actual data in a model-consistent way.

The economy consists of a representative family, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by $i \in [0, 1]$, a central bank and a government that sets monetary and fiscal policy respectively. I now describe the behavior of these agents.

## 4.1 The representative household

There is a continuum of identical households of mass one. Each household is a large family, made of a continuum of individuals of measure one. Each period, an individual is either working or searching for a job. Following Merz (1995), family members perfectly self-insure themselves against the unemployment risk by pooling their income (wage or unemployment benefits) each period. The head of the family chooses per capita consumption to maximize the expected utility of the representative family member. The representative household enters each period $t = 0, 1, 2, ...$, with $B_{t-1}$ bonds and $\overline{K}_{t-1}$ units of physical capital. At

[3]Sveen and Weinke (2008) explore the interactions of Calvo pricing and forward-looking hiring decisions with firm-specific labor.

[4]As shown by Shimer (2005), endogenous job destruction tends to deliver a counterfactual positive correlation between vacancies and unemployment.

the beginning of each period, bonds mature, providing $B_{t-1}$ units of money. The family uses some of this money to purchase $B_t$ new bonds at nominal cost $B_t/r_t^B$, where $r_t^B$ denotes the gross nominal interest rate between period $t$ and $t+1$. Each period, $N_t(i)$ family members are employed at intermediate-goods-producing firm $i \in [0,1]$. Each worker employed at firm $i$ works a fixed amount of hours and earns the nominal wage $W_t(i)$. $N_t$ denotes aggregate employment in period $t$ and is given by

$$N_t = \int_0^1 N_t(i)\, di. \tag{5}$$

The remaining $(1 - N_t)$ family members are unemployed and each receives nominal unemployment benefit $b_t$, financed through lump-sum taxes. Unemployment benefit $b_t$ is proportional to the nominal wage along the steady-state balanced growth path $W_t^{BGP}$,

$$b_t = \tau W_t^{BGP}, \tag{6}$$

where $\tau$ is the replacement ratio.

The household owns capital and chooses the capital utilization rate $u_t$, which transforms physical capital into effective capital according to

$$K_t = u_t \overline{K}_{t-1}. \tag{7}$$

The cost of capital utilization is $\kappa(u_t)$ per unit of physical capital. I assume the following functional form for the function $\kappa$,

$$\kappa(u_t) = \phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t - 1)^2, \tag{8}$$

and that $u_t = 1$ in steady state. The law of motion for physical capital is

$$\overline{K}_t \leq (1-\delta)\overline{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{9}$$

where $\delta$ denotes the depreciation rate. The function $S$ captures the presence of adjustment costs in investment, as in CEE. I assume the following functional form for the function $S$,

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2, \tag{10}$$

where $g_I$ is the steady-state growth rate of investment. Hence, along the balanced growth path, $S(g_I) = S'(g_I) = 0$ and $S''(g_I) = \phi_I > 0$. The investment-specific technology shock $\mu_t$ follows the stationary autoregressive process

$$\ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \varepsilon_{\mu t}, \tag{11}$$

where $\varepsilon_{\mu t}$ is $i.i.d.N\left(0, \sigma_\mu^2\right)$. The household rents $K_t(i)$ units of effective capital to intermediate-goods-producing firm $i \in [0,1]$ at the nominal rate $r_t^K$. The household's choice of $K_t(i)$ must satisfy

$$K_t = \int_0^1 K_t(i)\, di. \tag{12}$$

During period $t$, the representative household receives total nominal factor payments $r_t^K K_t + W_t N_t + (1 - N_t) b_t$. In addition, the household also receives nominal profits $D_t(i)$ from each firm $i \in [0,1]$, for a total of

$$D_t = \int_0^1 D_t(i)\, di. \tag{13}$$

In each period $t = 0, 1, 2, ...$,the family uses these resources to purchase finished goods, for both consumption and investment purposes, from the representative finished goods-producing firm at the nominal price $P_t$. For all $t = 0, 1, 2, ...$, the family's budget constraint is given by

$$\begin{aligned} P_t C_t + P_t I_t + B_t / r_t^B \leq\ & B_{t-1} + W_t N_t + (1 - N_t) b_t + r_t^K u_t \overline{K}_{t-1} \\ & - P_t \kappa(u_t) \overline{K}_{t-1} - T_t + D_t. \end{aligned} \tag{14}$$

The family's lifetime utility is described by

$$E_t \sum_{s=0}^{\infty} \beta^s a_{t+s} \ln(C_{t+s} - hC_{t+s-1}), \tag{15}$$

where $0 < \beta < 1$. The preference shock $a_t$ follows the autoregressive process

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \tag{16}$$

where $0 < \rho_a < 1$, and $\varepsilon_{at}$ is $i.i.d.N(0, \sigma_a^2)$. When $h > 0$, the model allows for internal habit formation in consumption and consumption responds gradually to shocks. Labor supply is fully inelastic as hours per worker are constant and there is no labor force participation decison. The head of the family chooses $C_t$, $B_t$, $u_t$, $I_t$, and $\overline{K}_t$ for each $t = 0, 1, 2, ...$ to maximize the expected lifetime utility (15) subject to the constraints (9) and (14). The first order conditions for this problem are

$$C_t : \Lambda_t = \frac{a_t}{C_t - hC_{t-1}} - \beta h E_t \left( \frac{a_{t+1}}{C_{t+1} - hC_t} \right), \tag{17}$$

where $\Lambda_t$ denotes the multiplier on (14) and measures the household's marginal utility of consumption in period $t$,

$$B_t : \Lambda_t = \beta r_t^B E_t \left( \Lambda_{t+1} \frac{P_t}{P_{t+1}} \right), \tag{18}$$

$$u_t : (\phi_{u1} - \phi_{u2}) + \phi_{u2} u_t = \widetilde{r}_t^K, \tag{19}$$

where $\widetilde{r}_t^K = \frac{r_t^K}{P_t}$ denotes the real rental rate for capital,

$$\begin{aligned} I_t \quad : \quad 1 = {} & \upsilon_t \mu_t \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - g_I \right)^2 - \phi_I \left( \frac{I_t}{I_{t-1}} - g_I \right) \left( \frac{I_t}{I_{t-1}} \right) \right] \\ & + \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \phi_I \left( \frac{I_{t+1}}{I_t} - g_I \right) \left( \frac{I_{t+1}}{I_t} \right)^2, \end{aligned} \tag{20}$$

$$\overline{K}_t : \upsilon_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[ (1 - \delta) \upsilon_{t+1} + \widetilde{r}_{t+1}^K u_{t+1} - a(u_{t+1}) \right] \right\}. \tag{21}$$

where $\upsilon_t$ is the marginal Tobin's Q in period $t$.

## 4.2 The representative finished goods-producing firm

During each period $t = 0, 1, 2, ...,$ the representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$, purchased at the nominal price $P_t(i)$, to produce $Y_t$ units of the finished good using the constant-returns-to-scale technology described by

$$Y_t \leq \left[ \int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t} di \right]^{\theta_t/(\theta_t - 1)}, \tag{22}$$

where $\theta_t$ governs the elasticity of substitution across differentiated finished goods. This elasticity evolves exogenously over time according to the autoregressive process

$$\ln(\theta_t) = (1 - \rho_\theta) \ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta t}, \tag{23}$$

where $0 < \rho_\theta < 1$, $\theta > 1$, and $\varepsilon_{\theta t}$ is $i.i.d.N(0, \sigma_\theta^2)$. As in Smets and Wouters (2003) and Ireland (2004), this exogenous stochastic process translates into a shock to the markup of price over nominal marginal cost. This shock shows up in the New Keynesian Phillips curve in a similar way as the cost-push shock introduced by Clarida, Gali and Gertler (1999).

Intermediate good $i$ sells at the nominal price $P_t(i)$, while the finished good sells at the nominal price $P_t$. Given these prices, the finished goods-producing firm chooses $Y_t$ and $Y_t(i)$ for all $i \in [0,1]$ to maximize its profits

$$P_t Y_t - \int_0^1 P_t(i)\, Y_t(i)\, di, \tag{24}$$

subject to the constraint (22) for each $t = 0, 1, 2, \ldots$. The first-order conditions for this problem are (22) with equality and

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t, \tag{25}$$

for all $i \in [0,1]$ and $t = 0, 1, 2, \ldots$. Competition in the market for the finished good drives the finished goods-producing firm's profits to zero in equilibrium. This zero-profit condition determines $P_t$ as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t}\, di\right]^{1/(1-\theta_t)}. \tag{26}$$

## 4.3 The representative intermediate goods-producing firm

Each intermediate goods-producing firm $i \in [0,1]$ enters period $t$ with $N_{t-1}(i)$ employees. At the beginning of period $t$, $\rho N_{t-1}(i)$ jobs are destroyed, where $\rho$ is the job destruction rate. The $\rho N_{t-1}$ workers who have lost their jobs start searching immediately. The stock of employees at firm $i$ evolves according to

$$N_t(i) = (1-\rho)\, N_{t-1}(i) + m_t(i). \tag{27}$$

$m_t(i)$ denotes the flow of new employees hired by firm $i$ in period $t$ and is equal to

$$m_t(i) = q_t V_t(i), \tag{28}$$

where $V_t(i)$ denotes vacancies posted by firm $i$ and $q_t$ is the aggregate probability of filling a vacancy. Workers hired in period $t$ take part to period $t$ production. As I have stressed above, this timing assumption prevents excess volatility in capital utilization and inflation. Aggregate employment $N_t$ evolves according to

$$N_t = (1-\rho)\, N_{t-1} + m_t, \tag{29}$$

where $m_t = \int_0^1 m_t(i)\,di$ denotes aggregate matches in period $t$. Similarly, the aggregate number of vacancies is equal to $V_t = \int_0^1 V_t(i)\,di$. The pool of job seekers in period $t$, denoted by $S_t$, is given by

$$S_t = 1 - (1-\rho) N_{t-1}. \tag{30}$$

Each period, some vacancies and job seekers remain unmatched due to search frictions. As a consequence, a firm-worker pair enjoys a joint surplus from the on-going relationship between the two parties. The matching process is described by the following constant-returns-to-scale function

$$m_t = \zeta S_t^{\sigma} V_t^{1-\sigma}, \tag{31}$$

where $\zeta$ is a scale parameter that captures the efficiency of the matching technology. The aggregate probability to fill a vacancy in period $t$ is given by

$$q_t = \frac{m_t}{V_t}. \tag{32}$$

The aggregate probability $s_t$ for a job seeker to find a job is

$$s_t = \frac{m_t}{S_t}. \tag{33}$$

Finally aggregate unemployment in period $t$ is defined as

$$U_t \equiv 1 - N_t. \tag{34}$$

During each period $t = 0, 1, 2, ...$, firm $i$ combines $N_t(i)$ employees with $K_t(i)$ units of capital to produce $Y_t(i)$ units of intermediate good $i$ according to the technology described by

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha}. \tag{35}$$

$A_t$ is a neutral technology shock whose growth rate, $z_t \equiv A_t/A_{t-1}$, follows the stationary process described by

$$\ln(z_t) = \ln(z) + \varepsilon_{zt}, \tag{36}$$

where $\varepsilon_{zt}$ is $i.i.d.N(0, \sigma_z^2)$ and $z > 1$ denotes the steady-state growth rate of the economy. Following Yashiv (2006), firms face convex hiring costs measured in terms of the finished good and given by

$$\frac{\phi_N}{2} [x_t(i)]^2 Y_t, \tag{37}$$

where $x_t(i)$ denotes the hiring rate at firm $i$,

$$x_t(i) \equiv \frac{q_t V_t(i)}{N_t(i)}.$$

Intermediate goods substitute imperfectly for one another in the production function of the representative finished goods-producing firm. Hence, each intermediate goods-producing firm $i \in [0,1]$ sells its output $Y_t(i)$ in a monopolistically competitive market, setting $P_t(i)$, the price of its own product, with the commitment of satisfying the demand for good $i$ at that price. To avoid the difficulties arising from the combination of firm-specific labor and staggered price setting, I introduce nominal rigidities through quadratic price-adjustment cost. Following Rotemberg (1982), each intermediate goods-producing firm faces costs of adjusting its nominal price between periods, measured in terms of the finished good and given by

$$\frac{\phi_P}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 Y_t, \tag{38}$$

where $\pi > 1$ denotes the steady-state gross rate of inflation. The labor and price adjustment costs make the firm's problem dynamic. The firm chooses $K_t(i)$, $N_t(i)$, $V_t(i)$, $Y_t(i)$ and $P_t(i)$ for all $t = 0, 1, 2, \ldots$to maximize its total market value,

$$E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s}\left[\frac{D_{t+s}(i)}{P_{t+s}}\right],$$

where $\beta^t \Lambda_t / P_t$ measures the household's utility of an additional euro and $D_t(i)$ denotes nominal profits,

$$\begin{aligned} D_t(i) &= P_t(i) Y_t(i) - W_t(i) N_t(i) - r_t^K K_t(i) \\ &\quad - \frac{\phi_N}{2}\left(\frac{q_t V_t(i)}{N_t(i)}\right)^2 P_t Y_t - \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)^2 P_t Y_t, \end{aligned} \tag{39}$$

subject to the constraints

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t, \tag{40}$$

$$Y_t(i) \leq K_t(i)^{\alpha}\left[A_t N_t(i)\right]^{1-\alpha}, \tag{41}$$

$$N_t(i) = \chi N_{t-1}(i) + q_t V_t(i), \tag{42}$$

where $\chi \equiv 1 - \rho$ is the job survival rate. The firm takes the nominal wage as given when maximizing discounted expected future profits.[5] The first order

[5] Krause and Lubik (2007*a*) show that intrafirm bargaining has negligible quantitative

conditions for the problem of the representative intermediate goods-producing firm are then given by

$$K_t(i) : \widetilde{r}_t^K = \xi_t(i)\left(\alpha A_t^{1-\alpha} K_t(i)^{\alpha-1} N_t(i)^{1-\alpha}\right), \tag{43}$$

$$\begin{aligned} P_t(i) \quad : \quad & 0 = (1-\theta_t)\left(\frac{P_t(i)}{P_t}\right)^{-\theta_t}\left(\frac{Y_t}{P_t}\right) + \theta_t \xi_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta_t - 1}\left(\frac{Y_t}{P_t}\right) \\ & -\phi_P\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)\left(\frac{Y_t}{\pi P_{t-1}(i)}\right) \\ & +\beta\phi_P E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1\right)\left(\frac{P_{t+1}(i)}{\pi P_t(i)}\right)\left(\frac{Y_{t+1}}{P_t(i)}\right)\right], \end{aligned} \tag{44}$$

$$\begin{aligned} N_t(i) \quad : \quad & \frac{\Psi_t(i)}{\Lambda_t} = \xi_t(i)\left[(1-\alpha) K_t(i)^{\alpha} A_t^{1-\alpha} N_t(i)^{-\alpha}\right] - w_t(i) \\ & +\phi_N\left(\frac{Y_t}{N_t(i)}\right)[x_t(i)]^2 + \beta\chi E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{\Psi_{t+1}(i)}{\Lambda_{t+1}}\right), \end{aligned} \tag{45}$$

$$V_t(i) : \left(\frac{\Psi_t(i)}{\Lambda_t}\right) = \phi_N\left(\frac{Y_t}{N_t(i)}\right) x_t(i), \tag{46}$$

$$\Psi_t(i) : N_t(i) = \chi N_{t-1}(i) + q_t V_t(i), \tag{47}$$

$$\Xi_t(i) : A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} Y_t, \tag{48}$$

where $w_t(i) = W_t(i)/P_t$ denotes the real wage, $\xi_t(i) = \Xi_t(i)/\Lambda_t$ measures firm $i$ 's real marginal cost in period $t$. $\Psi_t(i)$ measures the value to firm $i$, expressed in utils, of an additional job. Combining (46) and (47), we obtain the vacancy posting condition

$$\begin{aligned} \phi_N\left(\frac{Y_t}{N_t(i)}\right) x_t(i)\left[1 - x_t(i)\right] \quad = \quad & \left(\frac{\Xi_t(i)}{\Lambda_t}\right)\left[(1-\alpha) K_t(i)^{\alpha} A_t^{1-\alpha} N_t(i)^{-\alpha}\right] - \frac{W_t(i)}{P_t} \\ & +\beta\chi E_t\left[\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\phi_N\left(\frac{Y_{t+1}}{N_{t+1}(i)}\right) x_{t+1}(i)\right]. \end{aligned} \tag{49}$$

This condition tells us that the costs and benefits of hiring an additional worker must be equal.

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effects on labor market dynamics.

### 4.4 Wage setting

Jobs and workers at a given intermediate goods-producing firm are homogeneous. Let $W_t^{NB}(i)$ denotes the nominal wage paid for any job at firm $i$ in period $t$. Each period, $W_t^{NB}(i)$ is set through bilateral Nash bargaining between a worker and its employer:

$$W_t^{NB}(i) = \arg\max_{W_t(i)} \left[ S_t^W(i)^{\eta_t} J_t(i)^{1-\eta_t} \right], \tag{50}$$

where $S_t^W(i)$ and $J_t(i)$ denote respectively the surplus of the representative worker at firm $i$ and the surplus of firm $i$. Following Trigari (2009) $S_t^W(i)$ is defined as the change in the family's value function from having one additional member employed. Both $S_t^W(i)$ and $J_t(i)$ are expressed in nominal terms. $\eta_t$ denotes the worker's bargaining power which evolves exogenously over time according to

$$\ln \eta_t = \left(1-\rho_\eta\right) \ln \eta + \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta t}, \tag{51}$$

where $0 < \eta < 1$ and $\varepsilon_{\eta t}$ is $i.i.d.N\left(0, \sigma_\eta^2\right)$. The worker's surplus is given by

$$S_t^W(i) = W_t(i) - b_t + \beta E_t\left[\chi\left(1 - s_{t+1}\right)\right] \left[ \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \left(\frac{P_t}{P_{t+1}}\right) S_{t+1}^W(i) \right]. \tag{52}$$

Firm $i$ 's surplus is equal to $P_t \frac{\Psi_t(i)}{\Lambda_t}$ where $\Psi_t(i)$ denotes the marginal utility value of a job,

$$\begin{aligned} J_t(i) &= \left(\frac{\Xi_t(i)}{\Lambda_t}\right) P_t \left[ (1-\alpha) \frac{Y_t(i)}{N_t(i)} \right] - W_t(i) \\ &\quad + \left(\frac{\phi_N P_t Y_t}{N_t(i)}\right) \left[x_t(i)\right]^2 + \beta\chi E_t \left[ \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} J_{t+1}(i) \right]. \end{aligned} \tag{53}$$

The first order condition for (50) is given by

$$\left(1-\eta_t\right) S_t^W(i) = \eta_t J_t(i). \tag{54}$$

Substituting the expressions of the worker's and firm's surpluses into the first order condition and using $S_{t+1}^W(i) = \frac{\eta}{(1-\eta)} J_{t+1}(i)$, we obtain the following expression for the nominal Nash wage

$$\begin{aligned} W_t^{NB}(i) &= \eta_t P_t \left(\frac{\Xi_t(i)}{\Lambda_t}\right) \left[ (1-\alpha) \frac{Y_t(i)}{N_t(i)} \right] + \eta_t \left(\frac{\phi_N P_t Y_t}{N_t(i)}\right) \left[x_t(i)\right]^2 \\ &\quad + \eta_t \beta \chi P_t E_t \left[ \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \left(\frac{\phi_N Y_{t+1}}{N_{t+1}(i)}\right) s_{t+1} x_{t+1}(i) \right] + \left(1-\eta_t\right) b_t. \end{aligned} \tag{55}$$

The Nash wage is a convex combination of foregone unemployment benefits $b_t$ and the sum of the marginal value product of labor and current and future savings on labor adjustment costs. The stationarity of the unemployment rate hinges on the real wage growing along the balanced growth path. The real wage inherits the common stochastic trend from both real hiring costs and real unemployments benefits.

### 4.5 The empirical wage and inflation equations

Before taking the model to the data in the following sections, I follow Ireland (2004) by adding lagged inflation $\widehat{\pi}_{t-1}$ into the log-linearized New Keynesian Phillips curve. Hence, the *empirical* inflation equation is given by

$$\widehat{\pi}_t = \beta \left[ \gamma_\pi \widehat{\pi}_{t-1} + (1 - \gamma_\pi) E_t \widehat{\pi}_{t+1} \right] + \left( \frac{\theta - 1}{\phi_P} \right) \widehat{\xi}_t - \frac{1}{\phi_P} \widehat{\theta}_t. \quad (56)$$

The parameter $\gamma_\pi \in (0, 1)$ reflects the importance of backward-looking price setting behavior. Setting $\gamma_\pi = 0$, the Phillips curve collapses to its microfounded purely forward looking expression. By estimating $\gamma_\pi$, I will let the data choose the degree of inflation inertia. Similarly, following Hall (2005), I assume that the real wage $\widehat{w}_t$ is a convex combination of the previous period real wage $\widehat{w}_{t-1}$ and the current period real Nash wage $\widehat{w}_t^{NB}$:

$$\widehat{w}_t = \gamma_w \widehat{w}_{t-1} + (1 - \gamma_w) \widehat{w}_t^{NB}.$$

The parameter $\gamma_w \in (0, 1)$ governs the magnitude of real wage inertia. By estimating $\gamma_w$, I will let the data select the degree of real wage inertia. Shimer (2005) and Hall (2005) have shown that real wage stickiness helps the search and matching model to replicate the volatility of the vacancy/unemployment ratio.

### 4.6 Government

The central bank adjusts the nominal interest rate $r_t^B$ following a Taylor rule

$$\ln \left( \frac{r_t^B}{r^B} \right) = \rho_r \ln \left( \frac{r_{t-1}^B}{r^B} \right) + \rho_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \rho_y \ln \left( \frac{Y_t}{Y_t^N} \right) + \varepsilon_{rt}, \quad (57)$$

where $Y_t^N$ denotes the target level of output, $\pi_t = P_t / P_{t-1}$ is the quarterly gross inflation rate and $\varepsilon_{rt}$ is an $i.i.d.N\left(0, \sigma_r^2\right)$ monetary policy shock. The target level of output $Y_t^N$ is defined as the level of output in the flexible-price economy ($\phi_P = 0$) with no markup shocks $\left( \widehat{\theta}_t = 0 \text{ and } \widehat{\eta}_t = 0 \text{ for all } t \right)$. The government budget constraint is given by

$$P_t G_t + (1 - N_t)\, b_t = \left( \frac{B_t}{r_t^B} - B_{t-1} \right) + T_t, \tag{58}$$

where $T_t$ denotes nominal lump-sum transfers. Public spending is an exogenous time-varying fraction of GDP given by

$$G_t = \left( 1 - \frac{1}{g_t} \right) Y_t, \tag{59}$$

where $g_t$ evolves according to

$$\ln g_t = \left(1 - \rho_g\right) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{gt}. \tag{60}$$

The exogenous spending shock $\varepsilon_{gt}$ is $i.i.d.N\left(0, \sigma_g^2\right)$.

## 4.7 Symmetric equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $Y_t(i) = Y_t$, $P_t(i) = P_t$, $N_t(i) = N_t$, $V_t(i) = V_t$, $K_t(i) = K_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \ldots.$. Moreover, workers are homogeneous and all workers at a given firm $i$ receive the same nominal wage $W_t(i)$, so that $W_t(i) = W_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \ldots.$ The aggregate resource constraint is obtained by aggregating the household budget constraint over all intermediate sectors $i \in [0, 1]$,

$$\left[ \frac{1}{g_t} - \frac{\phi_N}{2} x_t^2 - \frac{\phi_P}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 \right] Y_t = C_t + I_t + a(u_t)\, \overline{K}_{t-1}. \tag{61}$$

## 4.8 Model solution

Output, consumption, investment, capital and the real wage share the stochastic trend induced by the unit root process for neutral technological progress. In the absence of shocks, the economy converges to a steady-state growth path in which all stationary variables are constant. I first rewrite the model in terms of stationary variables, and then loglinearize the transformed model economy around its deterministic steady-state. This approximate model can then be solved using standard methods.

# 5 Econometric strategy and results

The macroeconometric model contains 34 free parameters. I calibrate seven parameters prior to estimation (see Table 1). The quarterly depreciation rate $\delta$ is set equal to 0.025, a value commonly used in the literature. Following CKL, I set the vacancy filling rate $q$ equal to 0.7. I set the government spending/output ratio $G/Y = 0.20$. Finally, the steady-state values of the unemployment rate $U$, the rate of inflation $\pi$, the nominal interest rate $r^B$, and the growth rate of output $z$, are set equal to their respective sample averages. Using the seven time series plotted in Figure 1, I estimate the remaining 27 parameters by spectra matching. Within the estimation algorithm, I restrict the parameter space to the determinacy region.

## 5.1 Sources of business-cycle fluctuations in the euro area

What are the sources of business cycles in the euro area. Figure 4 plots the spectral densities of the seven observables in the estimated model and shows the contribution of each shock. Price-markup shocks emerge as the main source of business-cycle fluctuations in the euro area. In particular, price-markup shocks are key to allowing the model to replicate the characteristic spectral shape of euro area macro variables: Price-markup shocks generate the dominant cycle of 26 quarters that we observe in the growth rates of output, consumption and wages and also account for the main cycle of 32 quarters in tightness. These disturbances explain about one half of the variance of these four variables. The model attributes the remaining half mainly to wage-bargaining shocks, and to a lesser extent, monetary policy shocks. Hence, the two main sources of business-cycle fluctuations in the euro area are supply shocks that move output and inflation in opposite directions. This result is in line with both Smets and Wouters (2007) and Den Haan (2000) who report similar evidence for the U.S. economy. Figures 5 and 6 plot impulse responses to a price-markup shock and a wage-bargaining shock respectively. Interestingly, both disturbances generate a positive comovement between output, consumption, investment and tightness. These two shocks mainly differ in their effects on wages: price markup shocks trigger procyclical variations in wages while wage bargaining shocks cause countercyclical wages movements. Both disturbances however fail to generate large fluctuations in investment.

How to interpret the finding that price-markup shocks emerge as the main source of business-cycle fluctuations in the euro area? Taking the DSGE model seriously, a strict interpretation is that these shocks correspond to exogenous variations in the degree of firms market power. For example, cost-push shocks will account for changes in the degree of competition caused, for example, by new legislations related to entry. For future research, it would be interesting to look for some narrative evidence regarding episodes of product markets deregulation in the euro area and to compare this narrative evidence with the shock estimates and historical decompositions obtained from the model.[6] In addition, in such a stylised DSGE model we can also allow for a broader interpretation of these disturbances. For example, the model may use cost-push shocks to account for variations in oil and commodity prices (see Blanchard and Gali (2008)).

## 5.2 Key frictions for the propagation of shocks in the euro area

Table 3 reports estimates of the structural parameters obtained by spectra matching ($SM$) as well as 90% bootstrapped confidence intervals. Two estimates stand out. First, the investment adjustment cost is one order of magnitude greater than the value obtained by SW (2003) for the euro area. This large value explains why the model underpredicts the variance of investment. Second, the coefficient in front of lagged inflation in the hybrid new Keynesian Phillips curve is close to 1, thereby strongly rejecting a purely forward-looking specification. According to the model, inflation nearly behaves as a unit root process driven by price-markup disturbances. The estimated high degree of backward-lookingness in price setting generates very persistent dynamics in inflation However, the actual behavior of inflation and the interest rate creates an obstacle for estimation: In Figure 1 we see that the inflation rate and the interest rate share a common downward-sloping trend over the sample period. Instead, the model assumes that both inflation and the interest rate are stationary around a constant mean. Introducing a permanent shock to the central bank's inflation target should therefore improve the congruency of the model with the data. This modification would probably affect the estimate of the coefficient in front of lagged inflation.[7] However, because it disregards information related to frequencies smaller than 0.01, the spectra matching estimation technique is likely to be more robust to this kind of misspecification than a full-information likelihood-based method.

The following estimates are broadly in line with the existing evidence. The price adjustment cost $\phi_P = 285$ implies that prices in the euro area are reop-

[6] Ebell and Haefke (2009) use a New Keynesian model with labor search to study the relationship between product market entry regulation and equilibrium unemployment in the US.

[7] See Smets and Wouters (2003), Ireland (2007) and Cogley and Sbordone (2008).

timized on average once every eight quarters.[8] This figure is in line with SW (2003). The data point towards a very high degree of interest-rate smoothing. The reaction to the output gap is basically zero. The low degree of habit persistence is associated to a highly persistent preference shock. The elasticity of the matching function to unemployment is in line with the existing literature (See CKL and Petrongolo and Pissarides (2001)). The replacement rate, $\tau = 0.6$, is consistent with the evidence provided by CKL on unemployment insurance in the euro area. The output share of hiring costs is one order of magnitude smaller than the calibrated value used by Walsh (2005). The quarterly job destruction rate $\rho = 10\%$ is close to the value of 8% computed by Davis, Haltiwanger and Schuh (1998) for the US. The implied steady-state markup is roughly equal to 20%. The model needs a high degree of real wage inertia to fit the persistence in real wages.

To shed light on the mechanisms at work in the propagation of price-markup shocks, Figure 7 shows the shock-decomposition of the model spectrum under four calibrations. Each calibration modifies the spectra-matching estimates in one particular dimension. The first calibration sets the price-stickiness parameter equal to a value commonly found in the literature for the US, $\phi_P = 50$ (See Ireland (2004, 2007) and Krause and Lubik (2007*b*)). The second calibration reduces the value of the coefficient in front of lagged inflation in the hybrid NKPC, $\gamma_\pi = 0.40$, so that the forward-looking component becomes prevailing. The third calibration modifies the degree of interest rate smoothing, using $\rho_r = 0.70$ instead of $\rho_r = 0.98$. Finally, the fourth calibration sets the investment-adjustment cost and the elasticity of the rental rate close to the estimates obtained by SW(2003) for the euro area, $\phi_I = 8$ and $\frac{\kappa''}{\kappa'} = 0.1$. In the first column of Figure 7, we see that decreasing the degree of price stickiness reduces considerably the period of the cycle in consumption growth, tightness and inflation. Figure 8, which compares the impulse responses to price-markup shocks across the alternative calibrations clearly shows that the cycle generated by price-markup shocks becomes shorter as prices become more flexible. The second column of figure 7 establishes that the backward-looking component in the NKPC plays a key role in the propagation of price-markup shocks: By reducing $\gamma_\pi$ from 0.95 to 0.40, the contribution of price-markup disturbances virtually disappear while the contribution of the other shocks is not much affected. Figure 8 shows that the responses of all variables to markup shocks become much less persistent when $\gamma_\pi = 0.40$. The third column of Figure 7 shows that the contribution of markup shocks to aggregate fluctuations diminishes substan-

[8] Setting $\beta = 1$, the slope of the NKPC $\varrho$ in a model with Calvo pricing and without indexation is given by $\varrho = (1-\varphi)^2/\varphi$, where $1/(1-\varphi)$ denotes the average duration of the price contracts. Using Rotemberg adjustment costs, the slope of the NKPC is equal to $\varrho = (\theta - 1)/\phi_P$.

tially when monetary policy becomes less persistent. Hence, monetary policy is key in transmitting the effects of a price-markup shock from inflation to real variables. Price-markup shocks emerge as the main source of business cycles because: (1) they have long-lasting effects on inflation and, (2) the response of monetary policy to the inflation gap is highly persistent. Finally, the fourth column of Figure 7 shows that reducing investment- and utilization-adjustment costs mainly enhances the contribution of preference shocks to investment fluctuations.

## 5.3 Explaining the variance and the persistence of tightness

Shimer (2005), and Hagedorn and Manovskii (2008) examine the so-called "volatility puzzle", i.e. the failure of the search and matching model to replicate the large variance of the vacancy/unemployment ratio. Hagedorn and Manovskii (2008) solve the volatility puzzle by using an extremely high value of the unemployment benefit and a low value of the worker bargaining power. Instead, Shimer (2005) suggests that the high volatility of the Nash wage annihilates most of the firms's incentives to hire new employees. Yashiv (2006) and Fujita (2004) stress the inability of the search and matching model to generate persistent fluctuations in tightness and propose using convex hiring costs. Following this set of advice, the model features both real-wage inertia and convex hiring costs. Indeed, in Figure 4, we see that the model fits the *spectrum* of tightness remarkably well, meaning that the model is able to explain both the variance and the persistence of this variable. The model achieves this good performance by *combining three shocks*: (1) price-markup disturbances generate the bulk of business-cycle-frequency variance while wage-bargaining and preference shocks inject low-frequency variance. Hence, as suggested by Rotemberg (2008), price-markup shocks help to solve the volatility puzzle. In line with Shimer (2005) and Hall (2005), the model relies on a substantial degree of real wage inertia. Importantly, the model fits the spectrum of real wages relatively well. Finally, the unemployment benefit is estimated to equal 60% of the steady-state wage, while the worker bargaining power is 0.75. These estimates seem reasonable for the euro area (see CKL). Moreover, they do not support the values proposed by Hagedorn and Manovskii (2008).

## 5.4 Evaluating the fit of the model

Watson (1993) and Söderlind (1994) apply frequency-domain tools to assess the fit of a calibrated version of the basic RBC model. Wen (1998) estimates an RBC model using spectra-matching techniques and then compares the model's and data 's spectral densities and coherences. Following this literature, I first evaluate the ability of the model to match the main diagonal elements of the data spectrum reported in Figure 1. I then check the ability of the model to account for some comovement patterns by comparing the model's and data's coherences.[9]

#### 5.4.1 Spectral decomposition of variances

How good is the model at replicating the stylized facts summarized in Figure 1, i.e. the variance and the persistence of each observable? Figure 9 plots the spectrum of the data against the model's 90% confidence bands. The model fits the spectral densities of output, consumption, wage growth and tightness quite well. It slightly underpredicts the variance of investment growth at business-cycle frequencies and fails to replicate the high persistence of inflation.

#### 5.4.2 Spectral decomposition of squared correlation

To what extent is the model able to replicate comovement patterns observed in the data? This question is interesting as the spectra-matching technique disregard information on comovements. Figure 10 confronts six data coherences with the model's 90% confidence bands. The two coherences between output growth and inflation and between tightness and inflation carry information about the Phillips curve in the euro area. Both coherences share a similar shape with a marked peak around the 26-quarter period. The model replicates this stylized fact fairly well .

In the data, the coherence between output and tightness is strong at business-cycle frequencies and weak at higher frequencies, meaning that in the short run, firms adjust their production by varying *instantaneous* margins other than employment.[10] The model captures this coherence at low-to-medium frequencies quite well, but overpredicts its strengh at medium-to-high frequencies. The assumption that hours per worker are constant may be responsible for this failure of the model. Alternatively, this failure may be an artefact of the limited-information estimation approach.

The model substantially underpredicts the coherence between inflation and the interest rate at frequency zero. Finally, the high correlation at business-cycle frequencies between consumption growth and investment growth suggests that a single shock drives the bulk of fluctuations in investment and consumption. However, the model significantly underestimates the coherence between

[9] A coherence is an $R^2$ decomposed frequency by frequency.

[10] It would be interesting to investigate the pattern of the coherence between output growth and hours per worker. Unfortunately, there are currently no series available for hours worked per employee in the euro area.

consumption growth and investment growth at business-cycle frequencies. The preference shock and the investment shock both tend to move consumption and investment in opposite directions. Therefore the model cannot easily generate a strong positive comovement at business-cycle frequencies.[11]

# 6 Robustness analysis: Comparing spectra matching and maximum likelihood

To understand whether the failure of the model to match some comovement patterns is due to a specification problem, or instead is an artefact of the limited-information estimation strategy, I reestimate the model by maximum likelihood. Spectra matching ($SM$) and maximum likelihood ($ML$) differ in the quantity of information they consider and in the way they use this information. First, $ML$ exploits the information embedded in cross-spectra, whereas $SM$ does not. Second, $ML$ minimizes the one-period-ahead prediction errors whereas $SM$ mainly cares about the disagreements at frequencies receiving high weights. For example, the balanced growth hypothesis and the stationarity of inflation in the model are poor approximations of the euro area data's low-frequency behavior. $SM$ partially ignores these tensions as its weighting scheme assigns zero weight to frequencies smaller than 0.01. Hence, $ML$ is likely to be more sensitive to misspecification at low frequency than $SM$. Finally $ML$ minimizes the sampling uncertainty by giving more weight to the frequencies which are more precisely estimated, whereas $SM$ gives more weight to the frequencies where the variance comes from.

Tables 3 and 4 compare the estimates obtained using $ML$ and $SM$. The two estimation techniques most often yield significantly different estimates. $ML$ finds 4 near-unit root shocks (investment, price markup, bargaining power and government spending) compared to a single one (preference) for $SM$. The discrepancy between $SM$ and $ML$ is most strikingly visible for the following five deep parameters: (1) the degree of internal habit formation in consumption $\left(h^{sm} = 0.14, h^{ml} = 0.93\right)$, (2) the investment adjustment cost ($\phi_I^{sm} = 57$, $\phi_I^{ml} = 0.001$), (3) the price adjustment cost $\left(\phi_P^{sm} = 285,\ \phi_P^{ml} = 0.8\right)$, (4) the degree of real wage inertia $\left(\gamma_w^{sm} = 0.77,\ \gamma_w^{ml} = 0.01\right)$, and (5) the degree of interest rate smoothing $\left(\rho_r^{sm} = 0.98,\ \rho_r^{ml} = 0.01\right)$. The high value of the $ML$ estimate for the habit persistence parameter $\left(h^{ml} = 0.93\right)$ is associated with a white noise preference shock $\left(\rho_a^{ml} = 0.02\right)$. On the contrary, $SM$ picks a very persistent preference shock and a low value for habit $(h^{sm} = 0.14,\ \rho_a^{sm} = 0.98)$.

[11] SW (2007) use a risk-premium shock instead of the discount factor shock. They stress that this latter disturbance helps reproduce the positive comovement between consumption and investment.

$SM$ accounts for the persistence in investment through a large investment adjustment cost whereas $ML$ uses a persistent investment disturbance. $SM$ fits the persistence in real wages through a high degree of real wage inertia whereas $ML$ uses a very persistent bargaining power shock.

Figure 11 compares the ability of the two techniques to fit spectral densities. $ML$ fits investment growth, inflation and the interest rate well at all frequencies but fails to generate a hump-shaped spectra for output growth, consumption growth, real-wage growth and tightness. $ML$ significantly overpredicts some low-frequency variances and mainly generates low-frequency fluctuations instead of business-cycle fluctuations. None of the two approaches allow the model to match simultaneously the low volatility of consumption and the high volatility of investment at business-cycle frequencies. In Figure 13, we see that neutral technology shocks are mainly responsible for the excessive low-frequency volatility in the growth rates of output, consumption and wages. The high degree of habit persistence prevents the model from generating hump-shaped spectra conditional on technology shocks. The excessive persistence in tightness and inflation is inherited from wage bargaining shocks.

Figure 12 compares the ability of the two techniques to fit coherences. Confidence bands of coherences are larger with $SM$ than with $ML$ since $SM$ ignores information about comovement. $ML$ tends to generate monotonously decreasing coherences. The large number of near-unit root shocks and the small values of the various adjustment costs found by $ML$ explain this tendency. $ML$ fails to match the hump-shaped coherence between tightness and output growth. Hence, relaxing the assumption that hours per worker are constant seems a priority for future research on euro area data. $ML$ replicates the high correlation between inflation and the interest rate at frequency zero. This good performance is due to the fact that $ML$ uses the same shock to accounts for the persistence in both inflation and the interest rate (see Figure 13). Finally, similarly to $SM$, $ML$ misses the shape of the coherence between consumption growth and investment growth. Hence, the effects of replacing the discount factor shock by a risk-premium shock as in SW (2007) should be explored in future research.

# 7 Concluding Remarks

In this paper, I have showed that macroeconomic fluctuations in the euro area are mainly generated by a cycle of 6 1/2 years. Through the lens of a medium-scale DSGE model estimated by spectra matching, price-markup shocks emerge as the main source of the 6 1/2 year cycle. Moreover, as suggested by Rotemberg (2008), these disturbances greatly help the model to match the variance of the vacancy/unemployment ratio at business-cycle frequencies. A high degree of inflation indexation as well as a very persistent and aggressive response of monetary policy to the inflation gap are the most crucial ingredients for the propagation of cost-push shocks. Future research should assess the robustness of these findings to the introduction of a permanent shock to the central bank's inflation target. Two other extensions that are likely to improve the congruency of the DSGE model to euro area data are: (1) to allow for variations in hours per worker, (2) to replace the discount factor shock with a risk-premium shock.

Finally, this paper has illustrated the pervasiveness of misspecifiation and the advantages of using an estimation technique that focuses on the dimensions of the data that the model is primarily designed to explain. Following Watson (1993) this paper has also demonstrated the usefulness of comparing the model's and data's spectra to assess the fit of a DSGE model.

# 8 Appendix: Description of the database

All data, except for vacancies, are taken from the AWM database downloadable from the EABCN website. Vacancies are measured by the number of vacancies in Germany, multiplied by the average coefficient of proportionality between the German and euro area labor force. The data on vacancies in Germany are downloaded from the OECD website. There is no series available for vacancies in Italy. Data on French vacancies are not available before 1989. Per capita real GDP, consumption and investment are obtained by dividing the real series by the labor force. Real wages correspond to nominal hourly compensation per employee, divided by the GDP deflator. Yearly inflation is the yearly growth rate of the GDP deflator.

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Table 1: Calibrated parameters

| | | |
|---|---|---|
| Capital depreciation rate | $\delta$ | 0.025 |
| Probability to fill a vacancy within a quarter | $q$ | 0.70 |
| Exogenous spending/output ratio | $G/Y$ | 0.20 |
| Unemployment rate | $U$ | 0.0926 |
| Quarterly growth rate of output | $z$ | 1.0036 |
| Quarterly inflation rate | $\pi$ | 1.0078 |
| Quarterly nominal interest rate | $r^B$ | 1.0156 |

Table 2: Parameters implied by steady state

| | | $SM$ | $ML$ |
|---|---|---|---|
| Discount factor | $\beta$ | 0.996 | |
| Quarterly net real rental rate of capital | $\widetilde{r}^K$ | 0.0327 | |
| Capital utilization cost first parameter | $\phi_{u1}$ | 0.0327 | |
| Mean of exogenous spending shock | $g$ | 1.250 | |
| Employment rate | $N$ | 0.907 | |
| Job survival rate | $\chi$ | 0.895 | 0.850 |
| Hiring rate | $x$ | 0.105 | 0.150 |
| Job finding rate | $s$ | 0.508 | 0.595 |
| Bargaining power parameter | $\eta$ | 0.747 | 0.827 |
| Employment adjustment costs parameter | $\phi_N$ | 1.502 | 0.889 |
| Real marginal cost | $\xi$ | 0.830 | 0.933 |
| Capital/output ratio | $K/Y$ | 5.484 | 11.41 |
| Investment/output ratio | $I/Y$ | 0.157 | 0.326 |
| Consumption/output ratio | $C/Y$ | 0.635 | 0.464 |
| Employees' share of output | $\widetilde{W}N/Y$ | 0.650 | 0.560 |

Table 3: Deep parameter estimates

| | | $SM$ | $ML$ |
|---|---|---|---|
| Capital share | $\alpha$ | 0.21 [0.18,0.22] | 0.40 [0.35,0.40] |
| Elasticity of substitution btw goods | $\theta$ | 5.87 [5.10,9.76] | 14.90 [9.5,15.0] |
| Job destruction rate | $\rho$ | 0.10 [0.08,0.14] | 0.15 [0.05,0.15] |
| Replacement rate | $\tau$ | 0.59 [0.46,0.59] | 0.25 [0.25,0.44] |
| Hiring cost/output ratio | $\frac{\phi_N}{2}x^2$ | 0.008 [0.005,0.009] | 0.01 [0.009,0.01] |
| Habit persistence in comsump. | $h$ | 0.14 [0.12,0.63] | 0.93 [0.51,0.94] |
| Elasticity of matches to unemp. | $\sigma$ | 0.76 [0.49,0.79] | 0.73 [0.42,0.74] |
| Investment adjustment costs | $\phi_I$ | 56.91 [10.2,93.4] | 0.001 [0.001,0.01] |
| Capital utilization cost | $\phi_{u2}$ | 2.14 [1.56,2.20] | 4.99 [4.6,5.0] |
| Price adjustment costs | $\phi_P$ | 284.67 [260,293] | 0.80 [0.2,10.3] |
| Wage inertia parameter | $\gamma_w$ | 0.77 [0.36,0.87] | 0.01 [0.01,0.05] |
| Inflation inertia parameter | $\gamma_\pi$ | 0.95 [0.93,0.98] | 0.75 [0.01,0.98] |
| Monetary policy inertia | $\rho_r$ | 0.98 [0.91,0.99] | 0.01 [0.01,0.01] |
| Response of policy to inflation | $\rho_\pi$ | 0.51 [0.50,0.52] | 2.70 [1.6,3.9] |
| Response of policy to output gap | $\rho_y$ | 0.004 [0.001,0.007] | 0.10 [0.005,0.22] |

90% bootstrapped confidence intervals in brackets.

Table 4: Shocks' parameter estimates

| | | $SM$ | $ML$ |
|---|---|---|---|
| Std dev of technology shock | $\sigma_z$ | 0.0011 [0.0008,0.0012] | 0.008 [0.008,0.017] |
| Std dev of monetary policy shock | $\sigma_r$ | 0.0006 [0.0005,0.0007] | 0.007 [0.006,0.014] |
| Persistence of investment shock | $\rho_\mu$ | 0.18 [0.04,0.22] | 0.96 [0.86,0.98] |
| Std dev of investment shock | $\sigma_\mu$ | 0.087 [0.06,0.09] | 0.011 [0.003,0.014] |
| Persistence of preference shock | $\rho_a$ | 0.98 [0.48,0.98] | 0.02 [0.01,0.02] |
| Standard deviation of pref. shock | $\sigma_a$ | 0.013 [0.001,0.03] | 0.074 [0.015,0.098] |
| Persistence of markup shock | $\rho_\theta$ | 0.11 [0.03,0.26] | 0.99 [0.86,0.99] |
| Std dev of markup shock | $\sigma_\theta$ | 0.09 [0.06,0.10] | 0.073 [0.066,0.098] |
| Persistence of bargaining shock | $\rho_\eta$ | 0.25 [0.11,0.32] | 0.99 [0.93,0.99] |
| Std dev of bargaining power shock | $\sigma_\eta$ | 0.02 [0.001,0.03] | 0.004 [0.004,0.013] |
| Persistence of exog. spend. shock | $\rho_g$ | 0.29 [0.26,0.33] | 0.99 [0.91,0.99] |
| Std dev of exog. spending shock | $\sigma_g$ | 0.0018 [0.0006,0.0026] | 0.004 [0.004,0.010] |

90% bootstrapped confidence intervals in brackets.

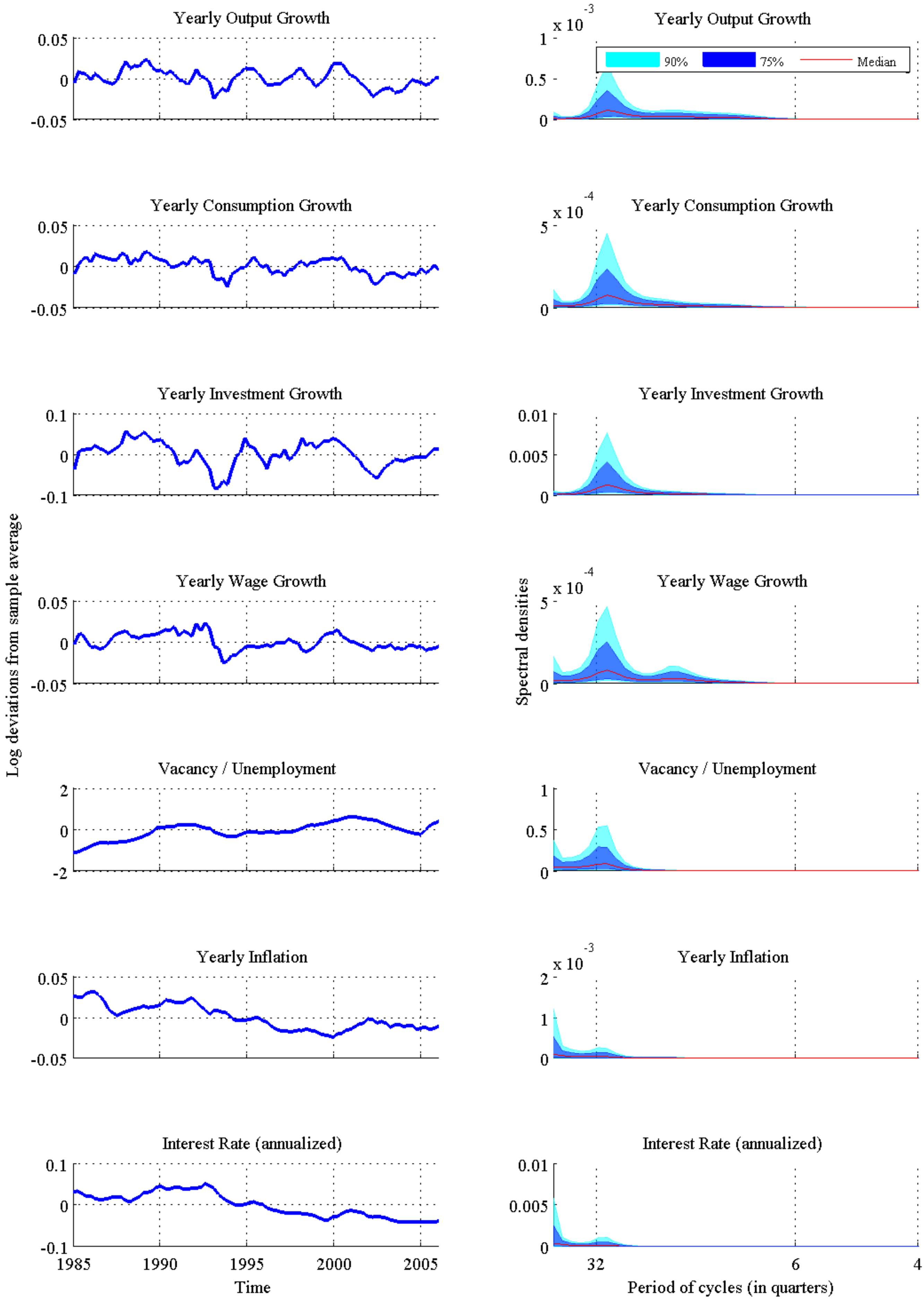


Figure 1. Euro Area data (expressed in log-deviation from sample mean) with their spectral densities. Spectral densities are estimated using a VAR with 4 lags. Confidence bands are computed by parametric bootstrap.

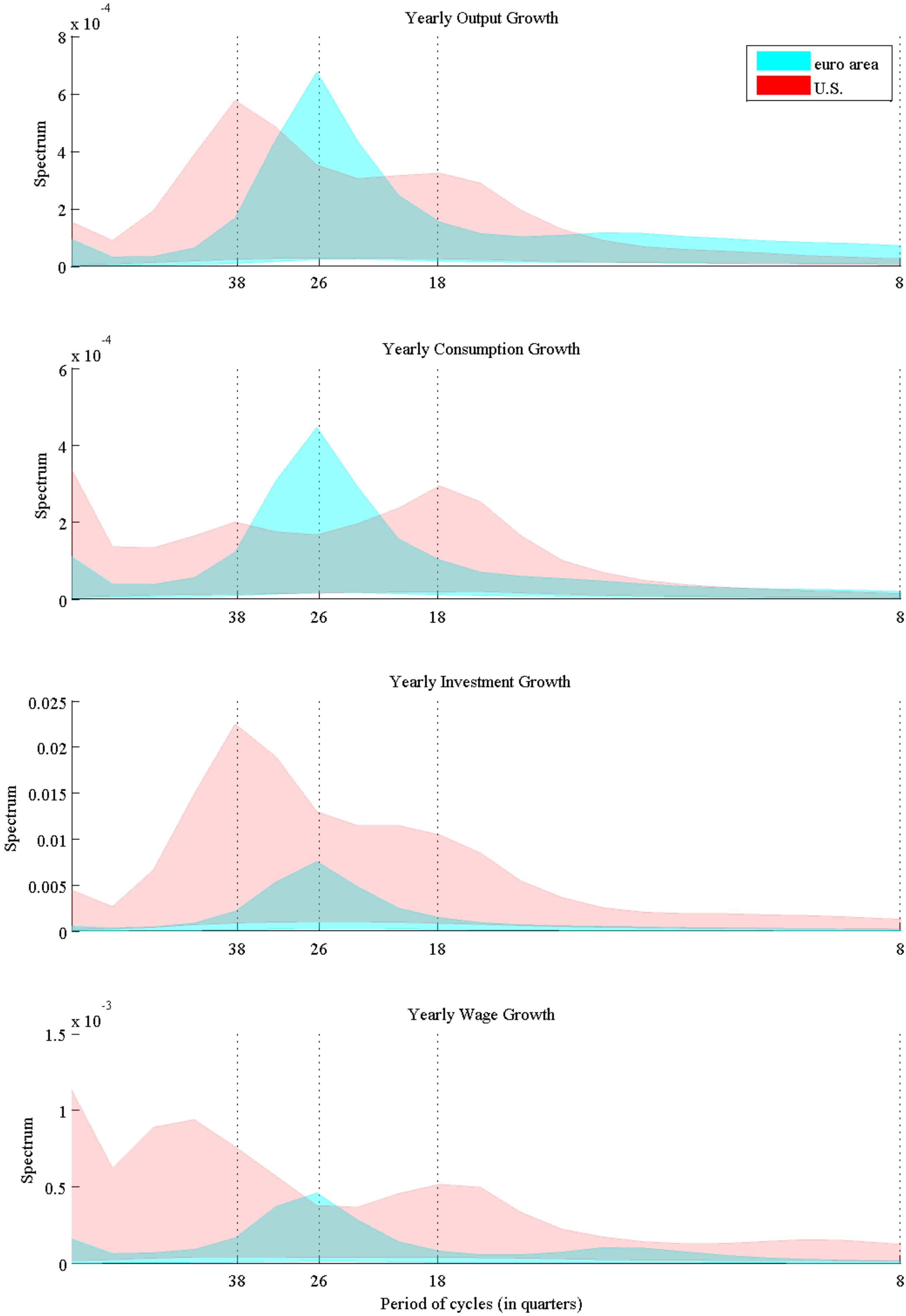


Figure 2. 90% confidence bands of euro area and U.S. data spectral densities.
Confidence bands are computed by parametric bootstrap.

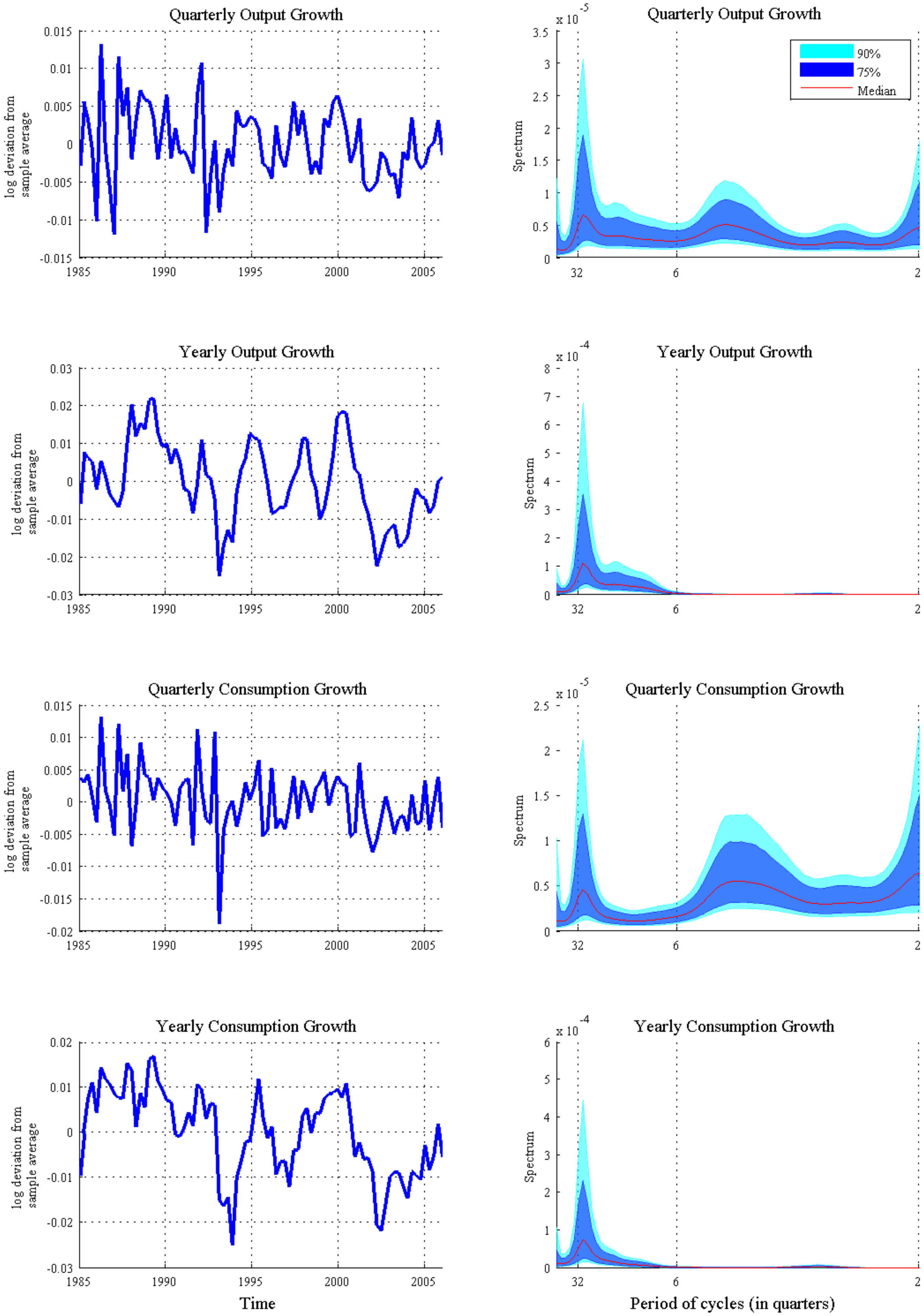


Figure 3. Time series plots and spectral densities of quarterly and yearly growth rates of output and consumption. Spectral densities are estimated using a VAR with 4 lags. Confidence bands are computed by parametric bootstrap.

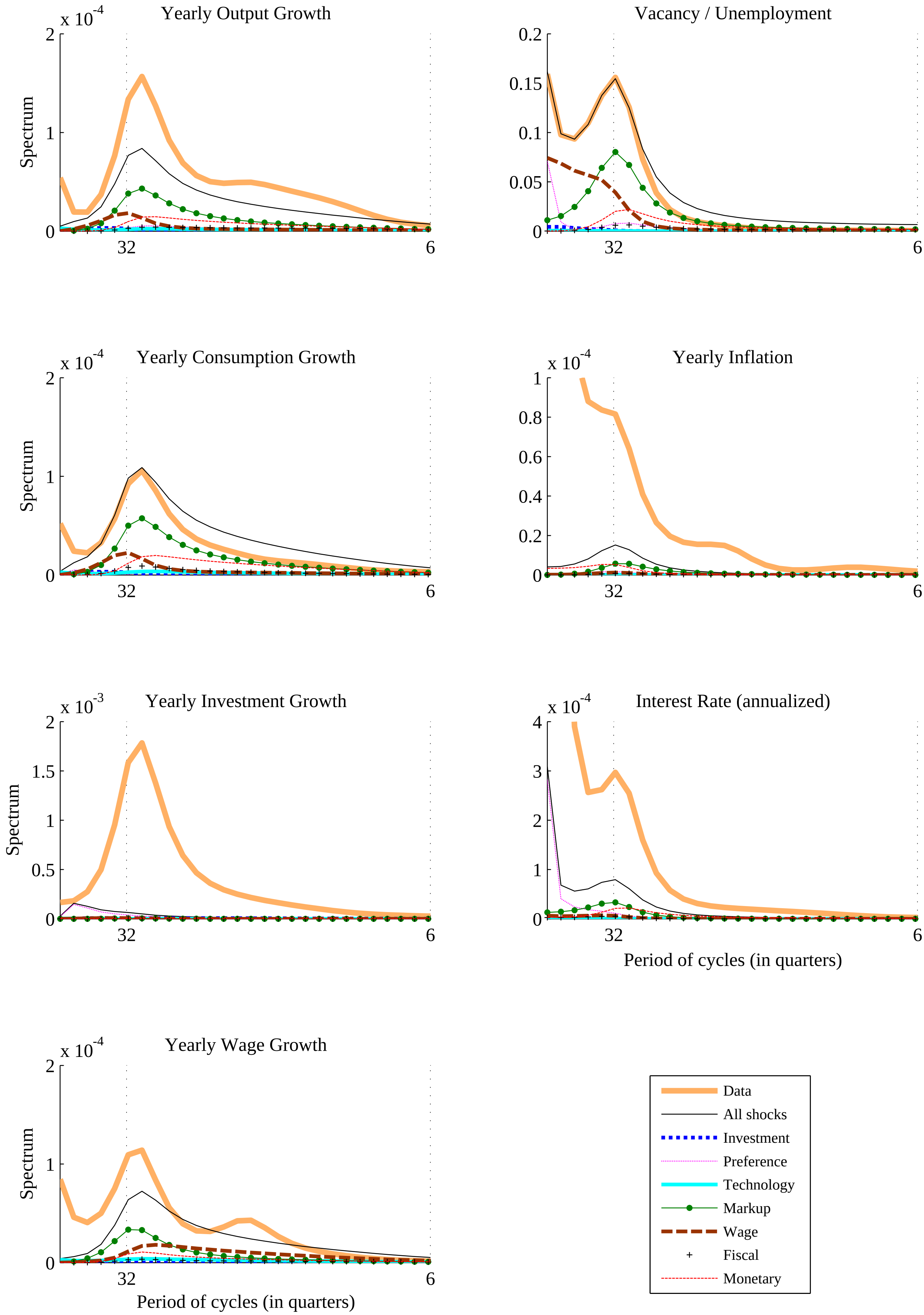


Figure 4. Model spectrum conditional on one shock at a time, computed at mode by successively setting equal to zero the standard deviations of all shocks but one.

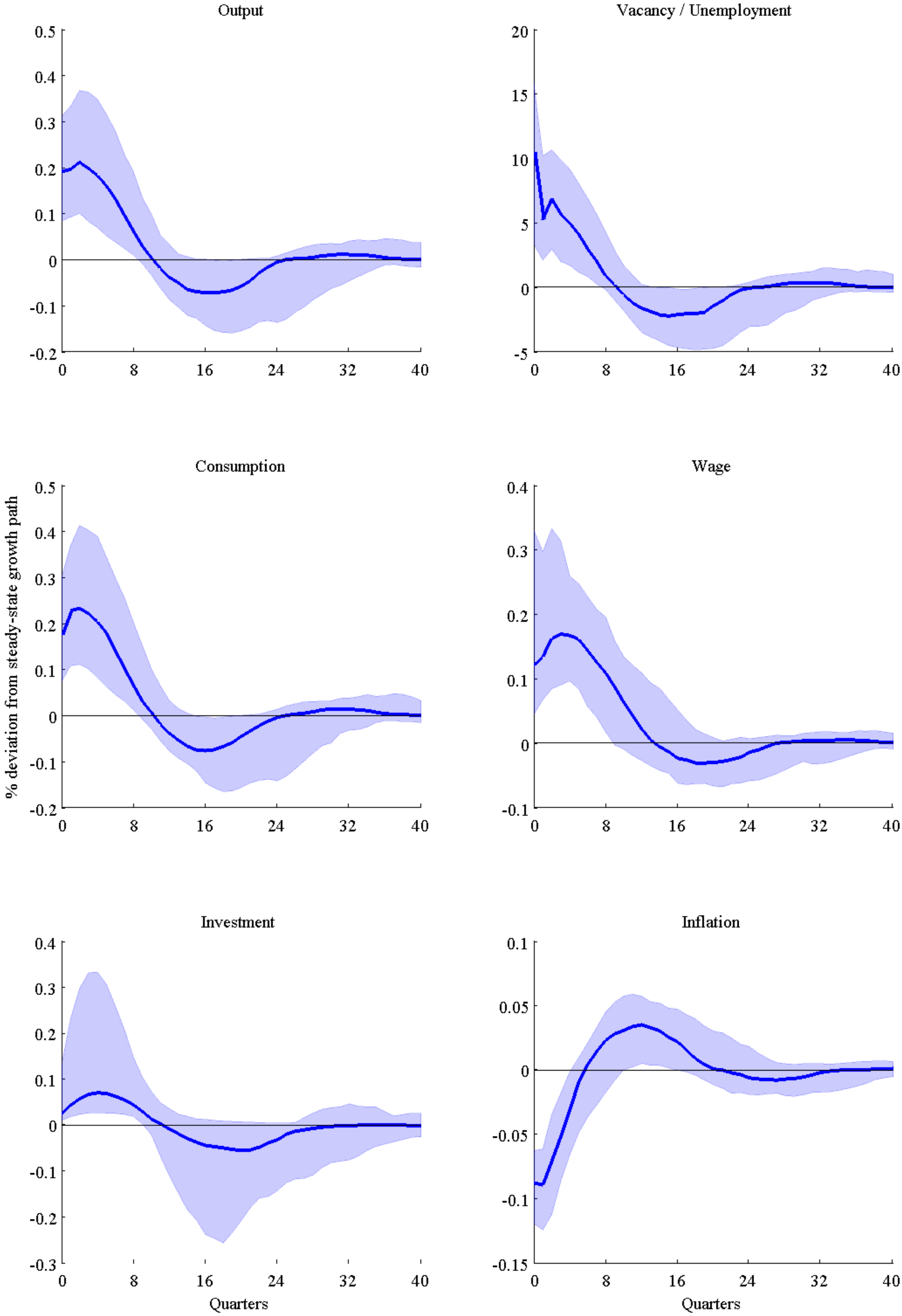


Figure 5. Median impulse responses to a one-standard-deviation price markup shock and 90% bootstrapped confidence bands.
All variables are expressed in percent deviations from the steady-state growth path. Inflation is annualized quarter-to-quarter inflation.

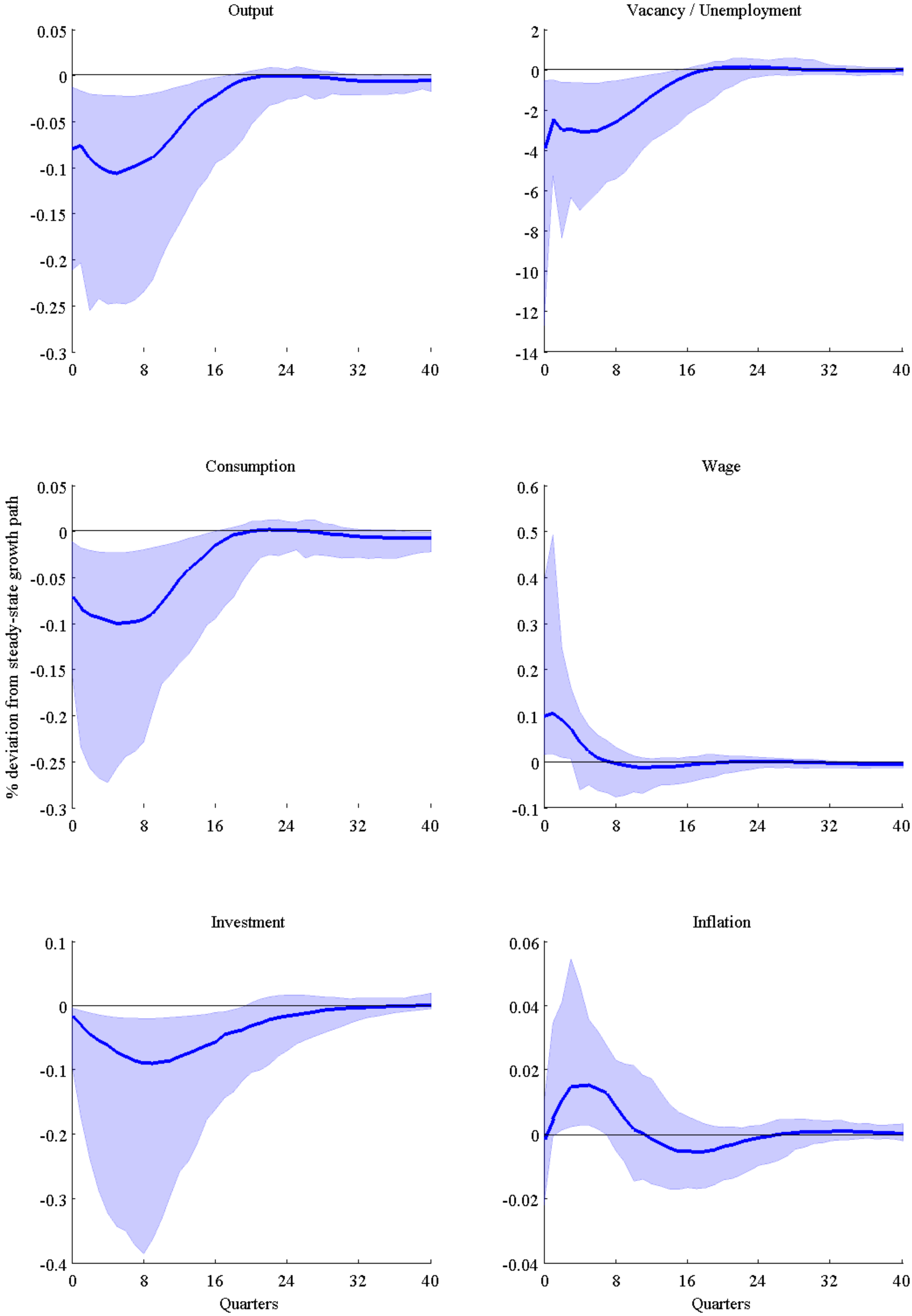


Figure 6. Median impulse responses to a one-standard-deviation wage bargaining power shock and 90% bootstrapped confidence bands. All variables are expressed in percent deviations from the steady-state growth path. Inflation is annualized quarter-to-quarter inflation.

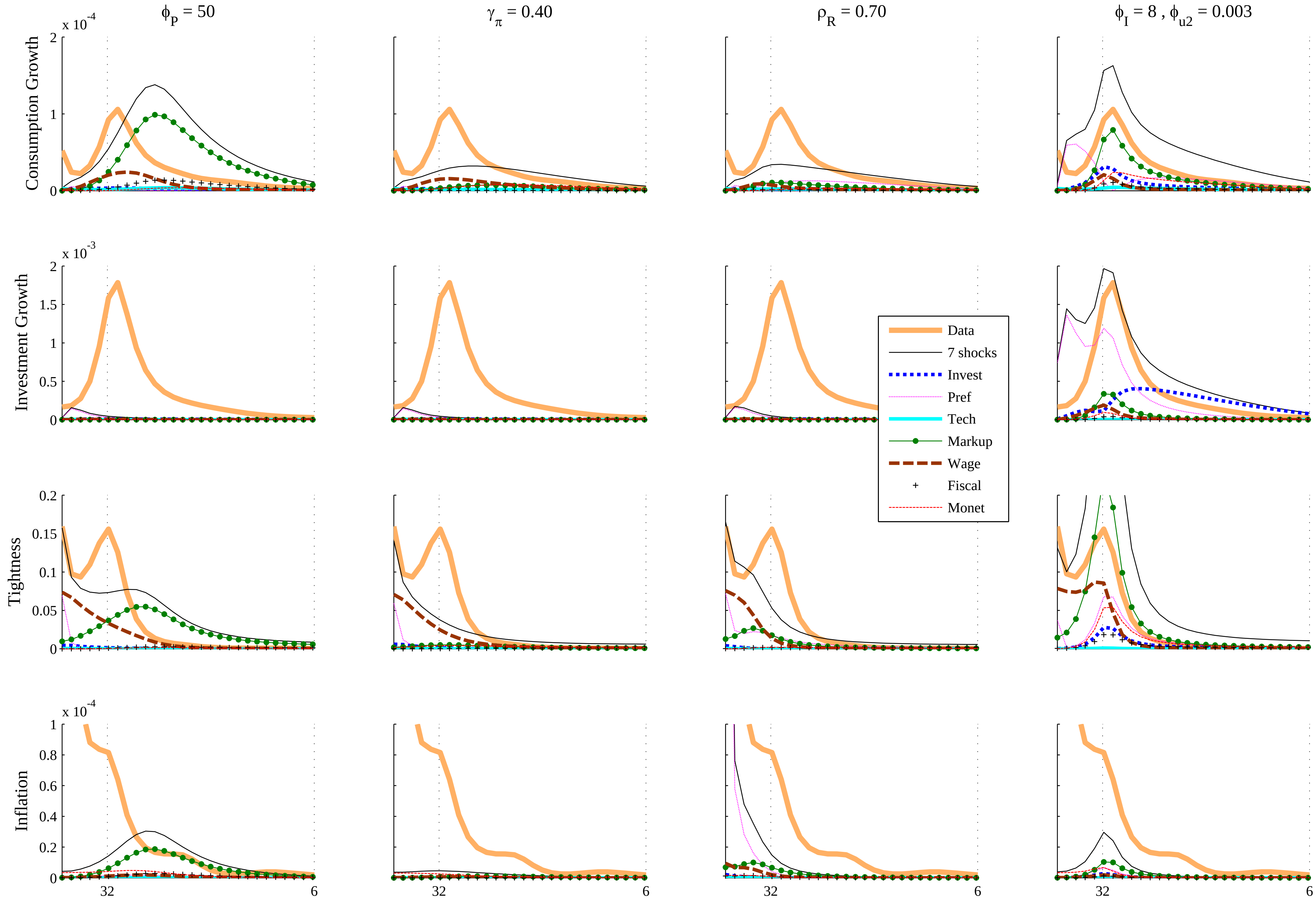


Figure 7. Model spectra conditional on one shock at a time for various calibrations.
Each column corresponds to a calibration which departs from the spectra matching estimates in one particular dimension.

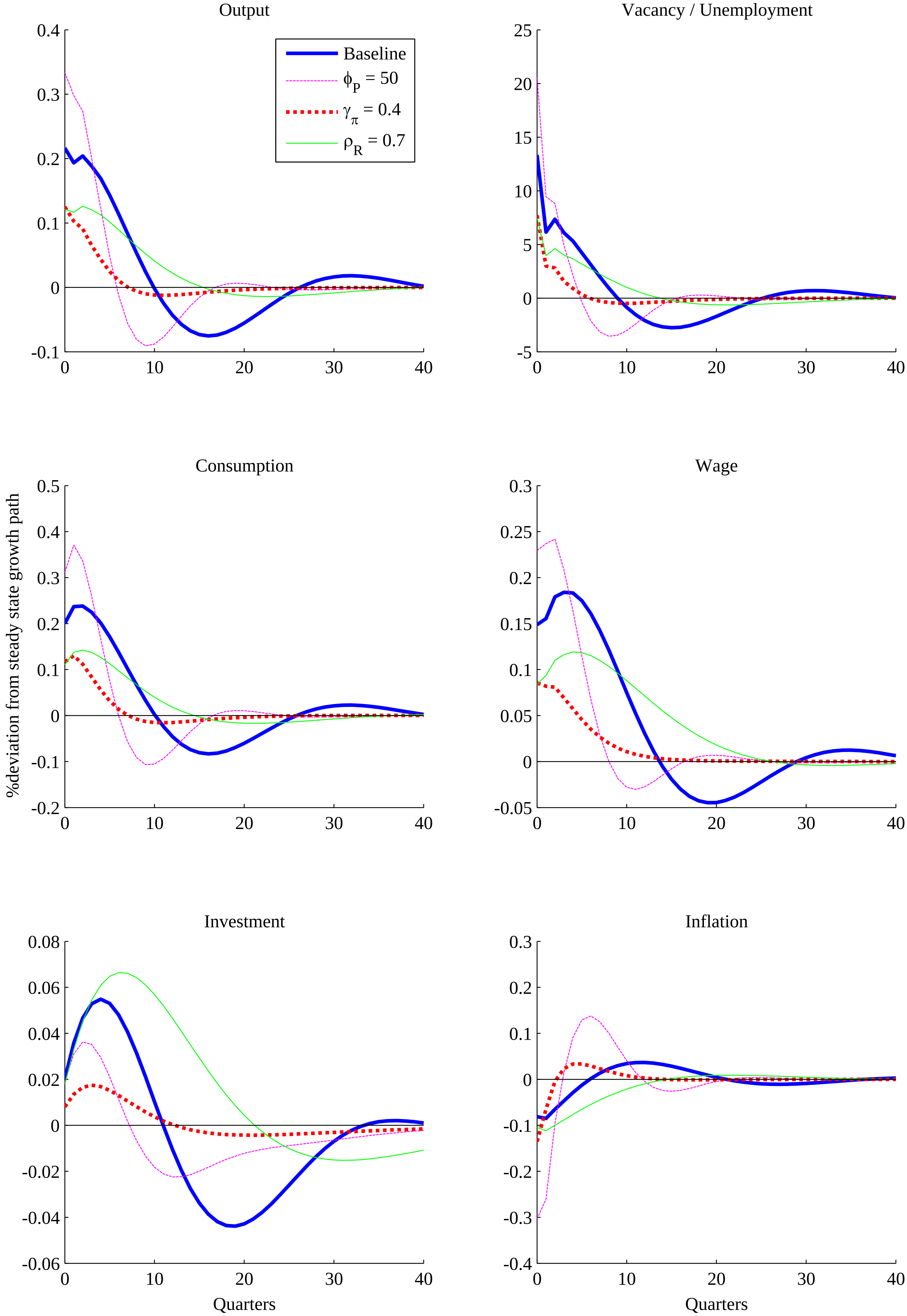


Figure 8. Impulse responses to a one-standard-deviation price markup shock under alternative calibrations.
All variables are expressed in percent deviations from the steady-state growth path.
Inflation is annualized quarter-to-quarter inflation.

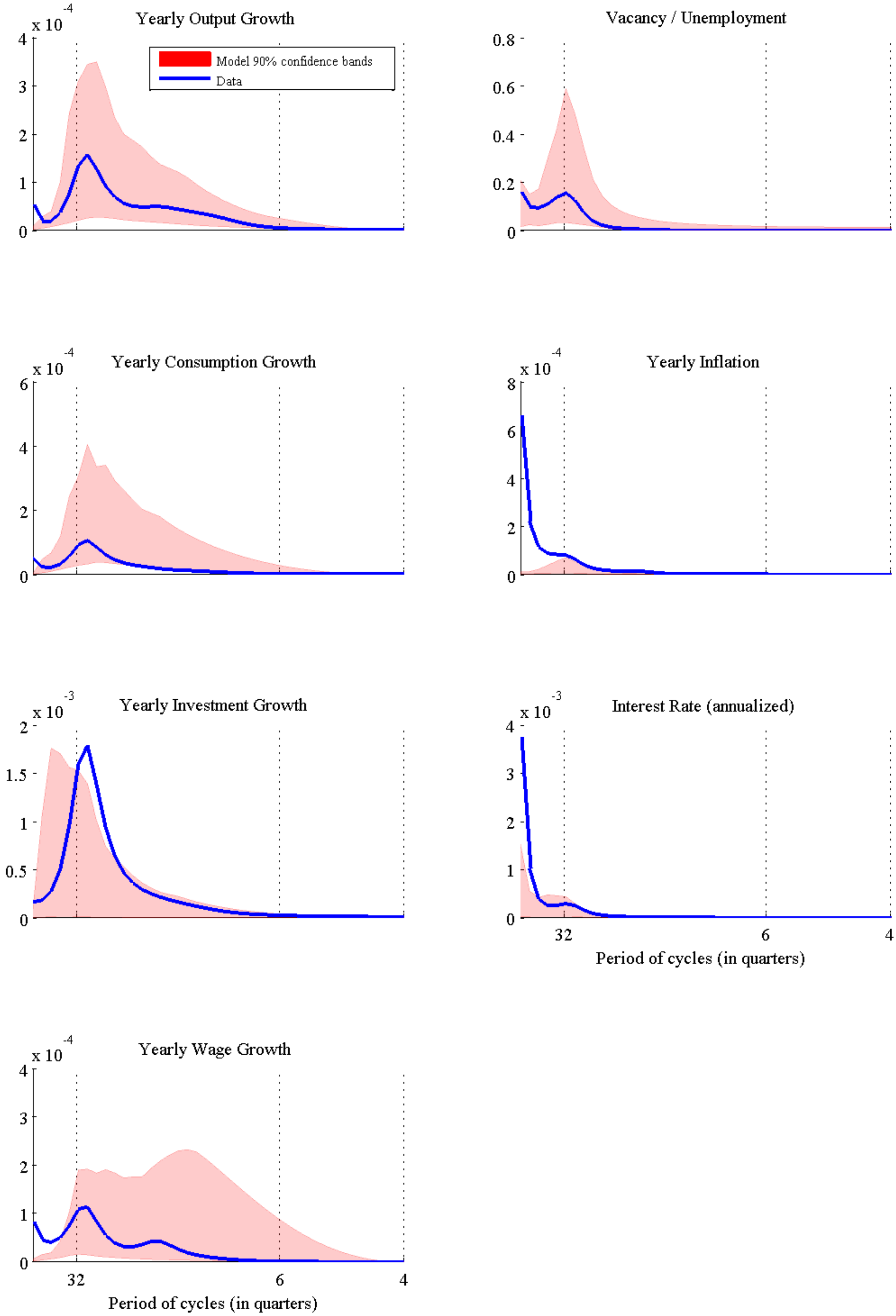


Figure 9. Data spectrum and model's 90% bootstrapped confidence bands. All variables are expressed in log-deviations from sample mean. Data spectrum is estimated using a VAR with 4 lags. Model's theoretical spectrum is computed directly from the state-space representation of the solved log-linearized DSGE model.

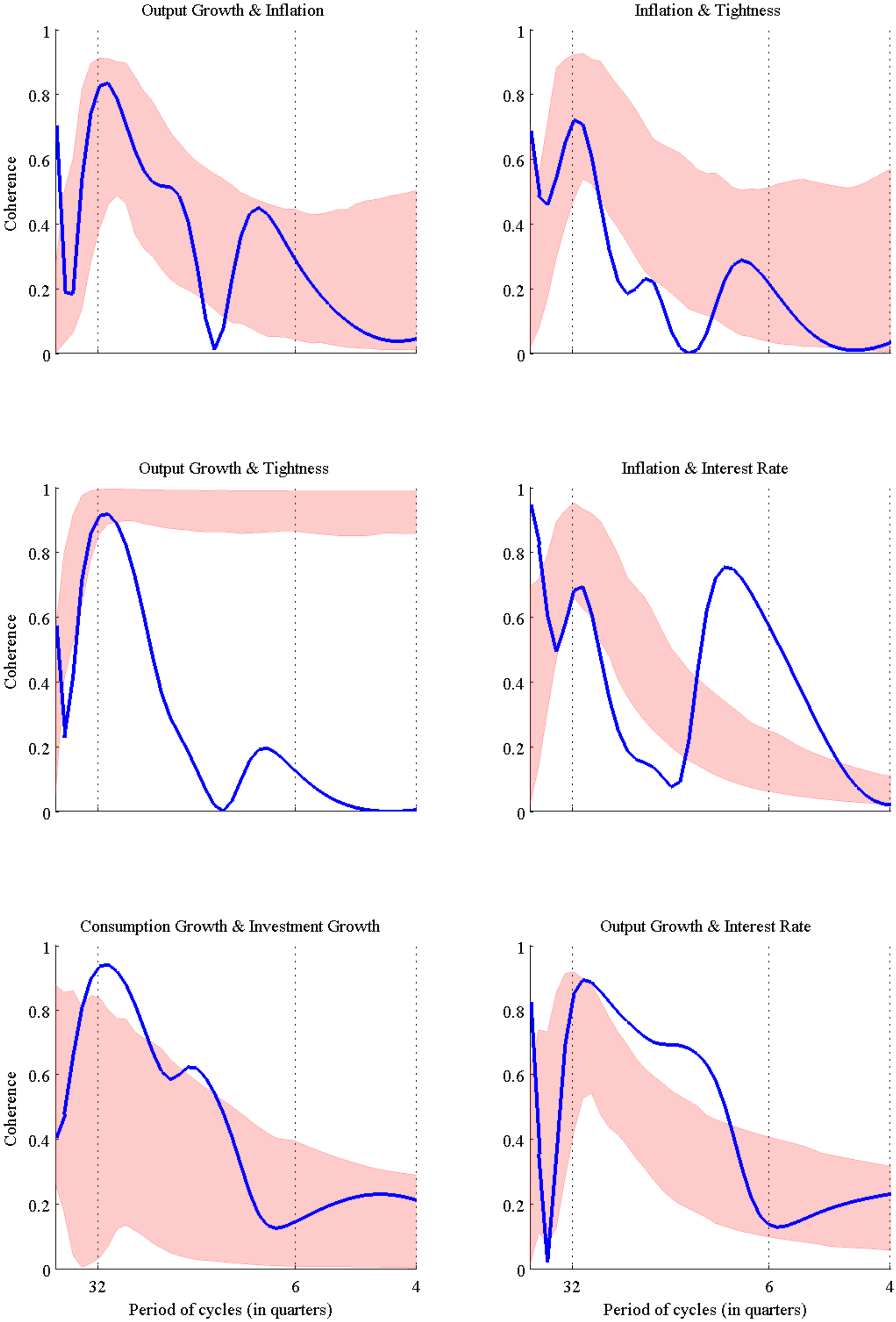


Figure 10. Data coherences (blue line) and model's 90% confidence bands. All variables are expressed in log-deviations from sample mean. Data coherences are estimated using the seven-dimensional VAR with 4 lags. Model's confidence bands are computed by parametric bootstrap using spectra matching estimation. Theoretical coherences are computed directly from the state-space representation of the DSGE model.

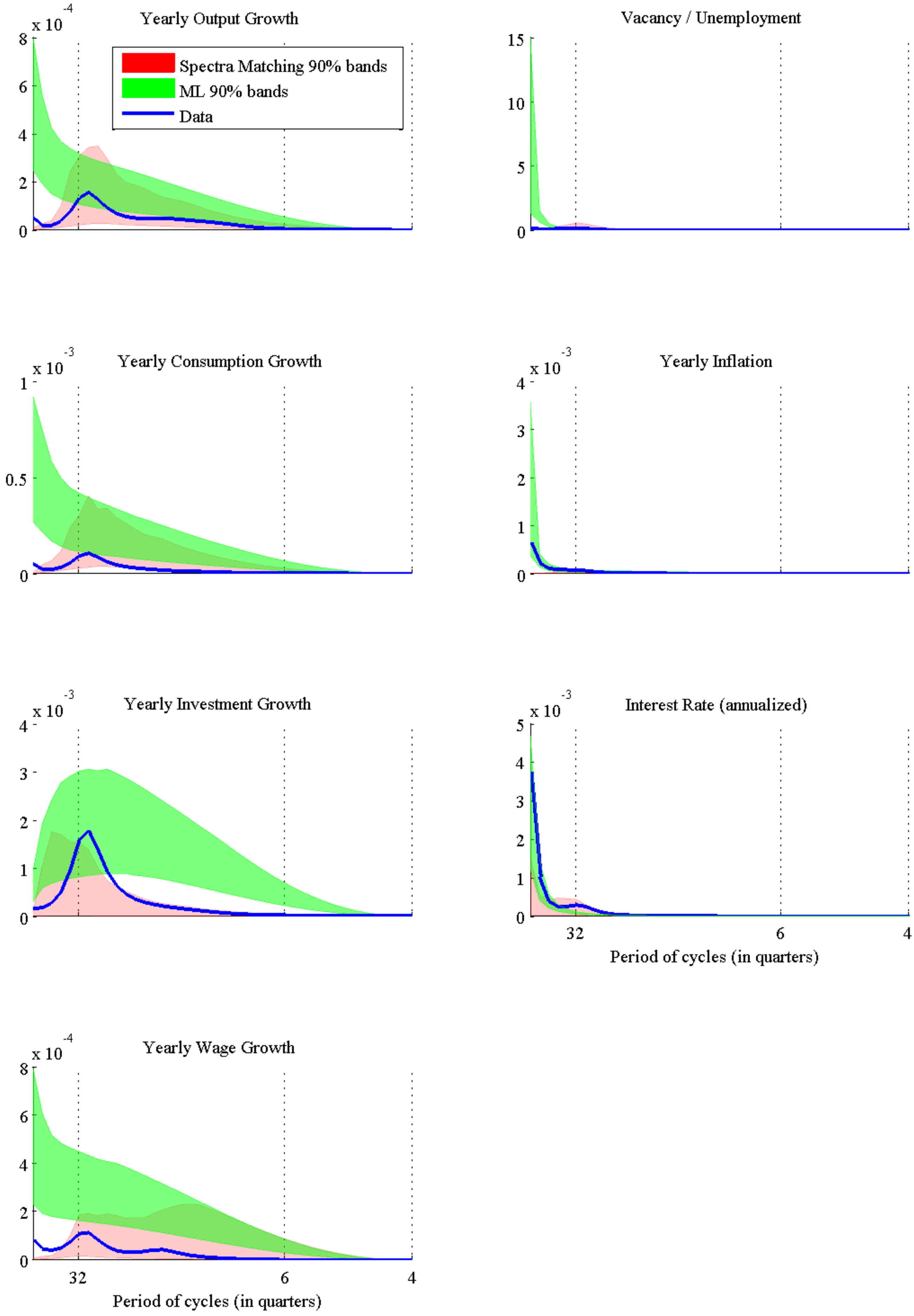


Figure 11. Data spectrum (thick blue line) and 90% confidence bands of model spectrum estimated by (1) spectra matching (shaded red), (2) maximum likelihood (shaded green). Data spectrum is estimated using a VAR with four lags. Confidence bands are computed by parametric bootstrap and using the theoretical model spectrum.

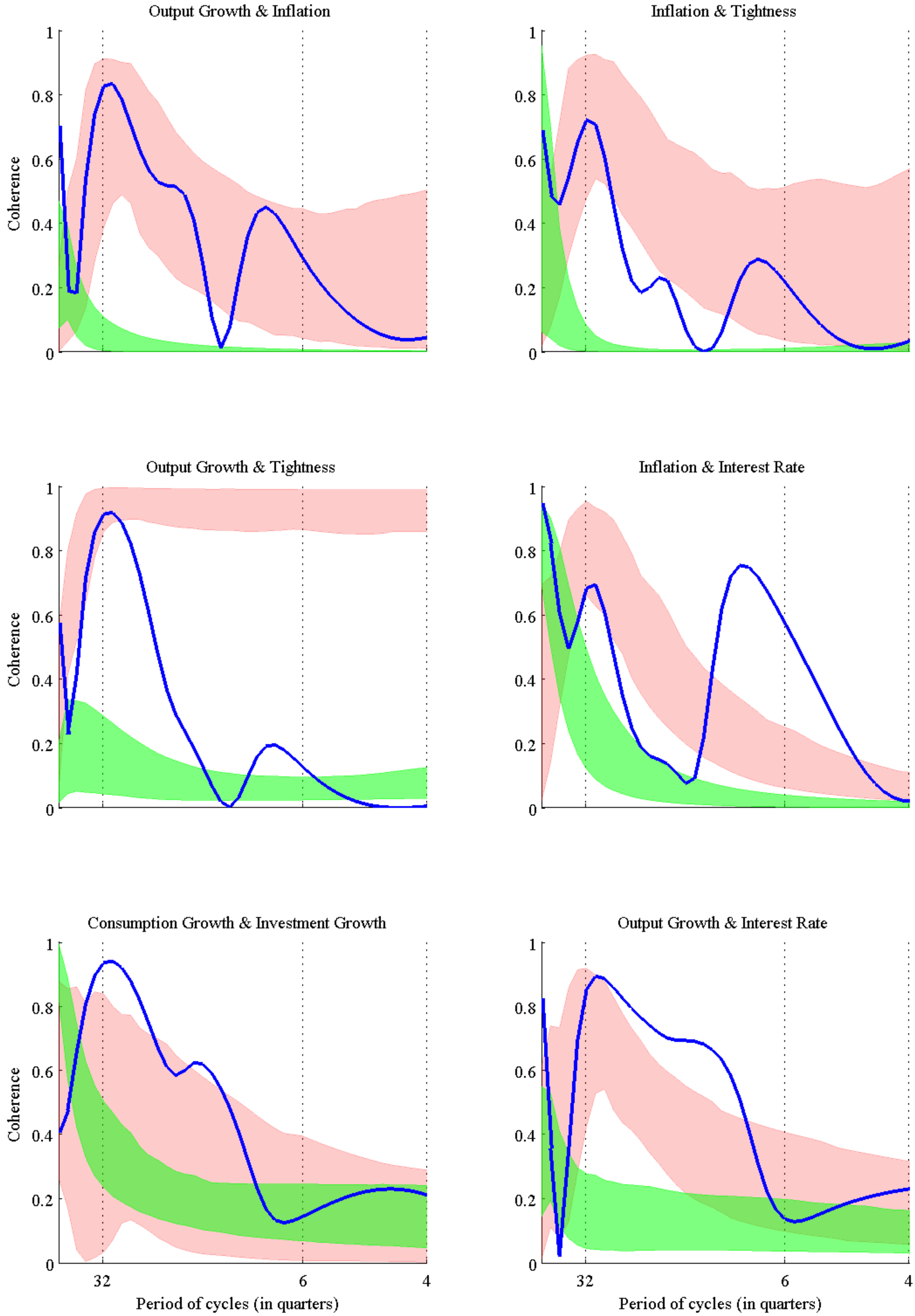


Figure 12. Data coherences and model's 90% bootstrapped confidence bands estimated by (1) spectra matching (red shaded) (2) maximum likelihood (green shaded). Variables are expressed in log-deviations from sample mean. Data coherences are estimated using a seven dimensional VAR with 4 lags. Theoretical coherences are directly computed from the state-space representation of the DSGE model.

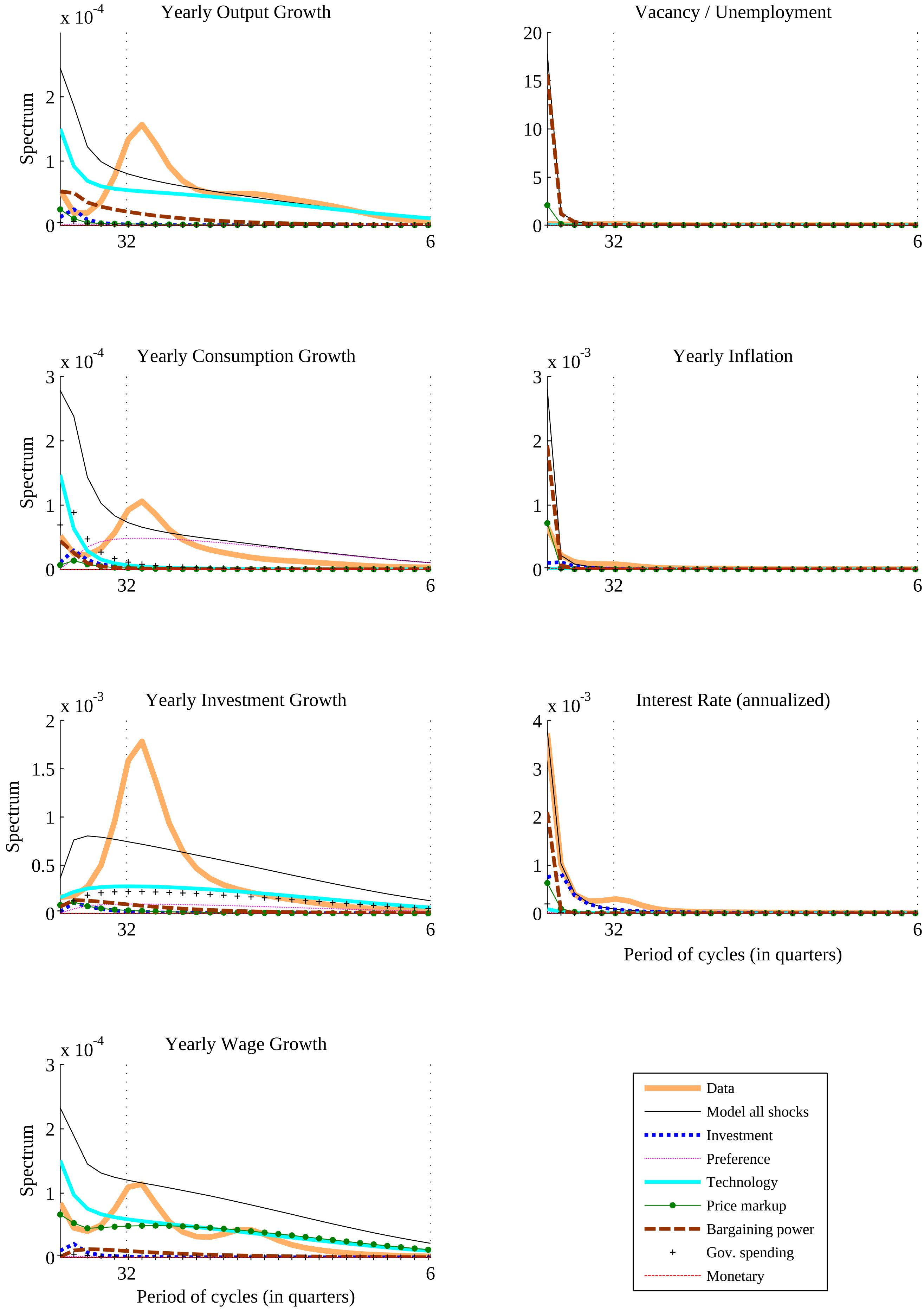


Figure 13. Spectral density decomposition of the DSGE model estimated by maximum likelihood. Spectra conditional on one shock at a time are computed at the ML estimate by successively setting equal to zero the standard deviations of all shocks but one.