Does natural rate variation matter? Evidence from New Zealand

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Abstract

Natural rates are an important concept within the new Keynesian models often used for monetary policy advice. However, many of these models rely on demeaned interest rate and inflation data. Thus, they implicitly impose the strict assumption that the natural rates of these series are constant. Using New Zealand data and a small open-economy new Keynesian model with time-varying parameters, we estimate the natural real rate of interest, inflation target, potential output, and neutral real exchange rate. We find that the model estimates of the natural real rate of interest and neutral exchange rate display noticeable time variation and considerable uncertainty, while the inflation target has been relatively stable over the sample period. We also compare the results of this model to a model with time-invariant natural rates. The comparison reveals the data prefers the fit of the time-varying model. It also shows that allowing the natural rates to vary over time has implications for the persistence parameters and impulse responses of the model.
1 Introduction

New Keynesian models are a popular tool for monetary policy advice. Typically, these models are estimated on demeaned interest rate and inflation data, implicitly assuming a constant natural real rate of interest and inflation target. However, if these concepts in fact vary over time, working with demeaned data implies the dynamics parameters that are encapsulated within the new Keynesian model will be biased and — importantly for central banks — this may have strong implications for the setting of monetary policy.

Most central banks use the short-term interest rate as their monetary policy instrument. By controlling the short-term interest rate they are able to influence (at least in the short run) the real interest rate that, economic theory suggests, drives our economic decisions. However, it is often difficult to know whether a particular real interest rate level is either contractionary or expansionary, let alone what degree of contractionary or expansionary pressure it is putting on the economy.

In this context, the natural real rate of interest is a useful concept. The natural real rate of interest was originally defined by Wicksell as “a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them” (Williams 2003). In other words, the natural real rate of interest provides a benchmark level for the real interest rate where monetary policy is neither contractionary nor expansionary. Therefore, understanding where the real interest rate is relative to the natural rate is of great interest to policymakers when setting monetary policy. However, like the concept of potential output, the natural real rate of interest is unobservable and must be estimated.

Within the open-economy new Keynesian literature, there has been little (explicit) focus given to the estimation of a time-varying natural real rate of interest (or inflation target). However, microfounded open-economy models, such as those in the seminal papers of Gali and Monacelli (2005) and Monacelli (2005), often allow for a time-varying natural real rate of interest that can be expressed in structural terms. According to both the specifications in Gali and Monacelli (2005) and Monacelli (2005) the natural real rate of interest depends upon both domestic technology and expected world output growth (with the degree of openness influencing the sensitivity to world output growth).

Estimating the natural rates using a microfounded new Keynesian model
has the advantage of providing economic intuition for changes in the natural rates. However, these estimates of the time-varying natural rates are often highly dependent upon the microfoundations and assumptions made in the model (Giammariola and Valla 2004). Therefore, an alternative approach used in some of the literature is to combine a small macroeconomic model with statistical filtering techniques. Though lacking the economic intuition for changes in the natural real rate of interest, this ‘semi-structural’ approach “seems to be more tractable in practice and hence more widely accepted” (Mésonnier and Renne 2007). However, previous papers that use this semi-structural approach focus on estimating the natural rates using closed-economy models (see Laubach and Williams 2003, and Benati and Vitale 2007 as examples).

New Zealand’s long history of inflation targeting provides a useful test case for examining the impact of working with demeaned inflation and interest rate data (and hence, assuming the natural real rate and inflation target are constant) in an open economy context. Since the adoption of inflation targeting in February 1990, the midpoint of the inflation target has shifted from 1 percent, up to 2 percent (its current level since September 2002). The shifts in the midpoint of the inflation target have been in accordance to changes made to the Policy Targets Agreement that encapsulates the agreed objectives for monetary policy. Thus, a priori, we expect some variation in the inflation target and the trend nominal interest rate (the natural rate of nominal interest).

Furthermore, earlier research using simple filters and Taylor rules points to evidence of some variation in New Zealand’s natural real rate of interest (see Plantier and Scrimgeour 2002, and Basdevant, Björksten, and Karagedikli 2004). More recently, Schmidt-Hebbel and Walsh (2007) included New Zealand in their estimates of the natural real rate of interest (and other variables) for various countries. Using a backwards-looking closed-economy model, they found that the natural real rate of interest in New Zealand has varied since the 1980’s, showing small but persistent deviations from a stable level around 5 percent. Note that generally speaking, natural rate estimates do not account for the differentials or spreads between the OCR, 90-day rate or the effective mortgage rate, and our new Keynesian model continues in this tradition.

Time-varying natural rates can have implications for the estimation of new Keynesian models. Both Sbordone (2007) and Benati (2008) find that allowing for persistence in the inflation target (sometimes referred to as trend inflation) affects the degree of intrinsic persistence within a hybrid new Key-
nesian Phillips curve. This can have serious implications for the dynamics and impulse responses of the model. In particular, if we use demeaned inflation data, variation in the inflation target can be misconstrued as intrinsic persistence in the inflation dynamics, suggesting monetary policy must work harder to control inflation. If this situation holds for inflation, it could also hold for other natural rates.

This issue is particularly relevant for the Reserve Bank of New Zealand which, like many other small open-economy inflation targeters, uses new Keynesian DSGE models estimated on demeaned or detrended data to inform policy (see Liu 2006, Stephens 2006, and Matheson 2006a as examples). Thus, there is the potential that the policy advice from these models may be biased due to the models overstating the structural persistence in the data.

In this paper, we follow the semi-structural approach by using a small open-economy new Keynesian model with time-varying parameters to endogenously estimate the natural real rate of interest, inflation target, potential output, and neutral real exchange rate for New Zealand. We refer to these collectively as ‘natural rates’. Furthermore, we develop a version of the model in which these natural rates are time-invariant (constant), and compare this to the time-varying model. This not only allows us to test whether the time-varying model is a better fit to the data, but we are also able to examine how allowing for time variation in the natural rates affects the model’s dynamic parameters and impulse responses.

We find that the time-varying model fits the data significantly better than the time-invariant model. The time-varying model estimates that the natural real rate of interest has been increasing over the last few years following a noticeable decline over the period from 1998 to 2004. The endogenous inflation target has also increased from slightly above 2 percent at the start of our sample period, to around 2.5 percent by the end of our sample period. The output gap estimated by our model is similar to the output gap used in FPS, the Reserve Bank’s core forecasting model over most of the sample period. Finally, the estimate of the neutral real exchange rate shows that the real exchange rate was approximately 20 percent above its neutral rate at the start of 2008.

The results relating to the dynamics of the model suggest that allowing for time variation in the natural rates reduces the persistence parameters of the

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1 Sbordone (2007) concludes that inflation deviations from trend show no intrinsic persistence once persistence in the trend is allowed for. The perceived persistence in inflation is caused by persistent deviations in trend inflation.
nominal interest rate and inflation rate, but it does not have a significant impact on the persistence parameter of the output gap. The impulse responses of the two models show that allowing for time-variation has some impact on both the magnitude and persistence of the shocks within the model. For most of the shocks in the model, allowing for time-varying natural rates reduces the persistence of the impulse responses.

The remainder of this paper is organised as follows. Section 2 outlines the model used in this analysis. Section 3 gives details on the data and estimation method. Section 4 discusses the results from the estimation of the model and the robustness tests. Section 5 provides a brief conclusion.

2 The model

The model used in our analysis is adapted from the small open-economy new Keynesian model developed by Berg et al (2006). We follow the structure of their model closely, with a few notable exceptions. We make adjustments to the Phillips curve so that today’s annualised inflation rate is driven by the annualised rate of future and lagged periods inflation, rather than future and lagged annual inflation rates. The Phillips curve specification based on annualised inflation is standard in the literature and avoids the possibility of introducing an MA term in the error of the Phillips curve equation. We also redefine the real exchange rate (and associated parameters) such that an appreciation increases the real exchange rate. Finally, we specify the natural rate processes to be random walks.

By employing a simple model that has features commonly found in the literature, we aim to ensure that our conclusions are applicable to a wide range of open-economy new Keynesian models. Our model follows the standard two country framework, with the domestic economy assumed to be a small open economy who is a price taker on the world market. The foreign economy (representing the rest of the world) is a large economy whose choices and decisions influence the smaller, domestic economy. Finally, to close off the two country model structure, an exchange rate relationship between the two countries is specified in real terms. The exchange rate we specify implicitly assumes complete pass-through (similar to other models such as Gali and Monacelli 2005). However, this assumption may be too simplistic for more sophisticated models, in which case it may be more appropriate to assume incomplete pass-through such as in Monacelli (2005).
The complete log-linearised model is presented below. Unlike other small open-economy New Keynesian models, we use time-varying parameters to explicitly model time variation in the natural real rate of interest, inflation target, growth rate of potential output, and neutral real exchange rate. We refer to this model as the \textit{time-varying model}. Using the Kalman smoother, we are able to back-out estimates of these unobservable natural rate series once the model is estimated.

In addition, we also develop a restricted \textit{time-invariant model}, in which the natural rates mentioned above are assumed to be constant over time. Using these two models, we are able to isolate the effects of allowing for time variation within the model.

\subsection{Domestic economy}

We specify an IS relationship for the output gap in equation 1. Similar specifications of the IS relationship can be found in Svensson (2000), Leitemo and Söderström (2005), and Buncic and Melecky (2008). The IS relationship states that today’s output gap \((x_t)\) is dependent upon its expected value next period \((E_t x_{t+1})\) and its lagged value \((x_{t-1})\).

\[x_t = (1 - \beta_x) E_t x_{t+1} + \beta_x x_{t-1} - \beta_r \tilde{r}_{t-1} - \beta_z \tilde{z}_{t-1} + \beta_f x_f^t + \varepsilon^x_t \quad (1)\]

The real interest rate gap \((\tilde{r}_t)\), and the exchange rate gap \((\tilde{z}_t)\) are defined as the difference between the observed levels \((r_t\) and \(z_t\)) and their natural rates \((r^*_t\) and \(z^*_t\)), and the neutral level of the real exchange

\footnote{Although, our model may not be considered a true DSGE model, it does share some similarities. In a true DSGE model, such a forward-looking term might be derived from a consumption Euler equation where agents are forward looking. And the lagged term could be derived from habit formation.}
rate $z_t^*$.\(^3\)

\[ \tilde{r}_t = r_t - r_t^* \tag{2} \]
\[ \tilde{z}_t = z_t - z_t^* \tag{3} \]

For the time-varying model, we assume that the natural real rate of interest ($r_t^*$) and the neutral level of the real exchange rate ($z_t^*$), both follow a random walk process.

\[ r_t^* = r_{t-1}^* + \varepsilon_t^r \tag{4} \]
\[ z_t^* = z_{t-1}^* + \varepsilon_t^z \tag{5} \]

For the time-invariant model, we assume that both the natural rates are constant over time, and equations 4 and 5 are replaced with equations 4’ and 5’ for the time-invariant model.

\[ r_t^* = \bar{r} \tag{4’} \]
\[ z_t^* = \bar{z} \tag{5’} \]

The output gap ($x_t$) is defined as the difference between actual output ($y_t$) and its potential level ($y_t^*$).

\[ x_t = y_t - y_t^* \tag{6} \]

We assume that the level of potential output grows at an annualised rate of $g_t^*$.\(^4\)

\[ 400(\Delta y_t^*) = g_t^* \tag{7} \]

The growth rate ($g_t^*$) above, is assumed to follow a random walk process in the time-varying model.

\[ g_t^* = g_{t-1}^* + \varepsilon_t^{g*} \tag{8} \]

In the time-invariant model, the growth rate is assumed to be constant at the rate $\bar{g}$.

\[ g_t^* = \bar{g} \tag{8’} \]

\(^3\) The real interest rate is calculated using the Fisher equation: $r_t = i_t - E_t\pi_{t+1}$. Where $i_t$ is the nominal interest rate, and $E_t\pi_{t+1}$ is the expected, annualised inflation rate next period.

Inflation within the domestic economy is modelled using a hybrid new Keynesian Phillips curve (9) in a similar fashion to Svensson (2000) and Giordani (2004). The current level of annualised inflation \((\pi_t)\) depends not only upon expected future inflation \((\mathbb{E}_t\pi_{t+1})\), which in a micro-founded model is introduced through staggered price setting behaviour (e.g. Calvo pricing), but also the previous period’s inflation rate \((\pi_{t-1})\). The introduction of the lagged inflation term into the Phillips curve of micro-founded models, comes from partial indexation to last periods inflation by those firms who do not adjust their prices to the optimal level. We assume that the output gap with a lag of one period \((x_{t-1})\), and the change in the real exchange rate \((\Delta z_t)\), which captures the direct impact from changes in the price of imported goods and services.

\[
\pi_t = (1 - \alpha)\pi \mathbb{E}_t \pi_{t+1} + \alpha_x \pi_{t-1} + \alpha_x x_{t-1} - \alpha_x \Delta z_t + \epsilon^\pi_t \tag{9}
\]

To complete the core structure of the domestic economy, and anchor inflation to a stable level, a monetary policy reaction function is defined using the following forward-looking Taylor-type rule (10):

\[
\begin{align*}
    i_t &= \gamma_i i_{t-1} + (1 - \gamma_i) \left[ r^*_t + E_t \pi^T_{t+1} + \gamma_x E_t \left( \pi^A_{t+4} - \pi^{A,T}_{t+4} \right) + \gamma_x x_t \right] + \epsilon^i_t \tag{10}
\end{align*}
\]

The Taylor-type rule includes interest rate smoothing (controlled by the parameter \(\gamma_i\)) in the level of the nominal interest rate \((i_t)\). The monetary authority moves the nominal interest rate \((i_t)\) away from its natural rate — the natural rate of nominal interest \((r^*_t + E_t \pi^T_{t+1})\) — in response to deviations in expected, annual inflation from its annual target \((E_t(\pi^A_{t+4} - \pi^{A,T}_{t+4}))\), and the contemporaneous output gap. FPS, the Reserve Bank’s current macroeconomic model, is also relatively forward looking with a focus on annual inflation six to eight quarters ahead (see Black et al 1997).

The (annualised) inflation target \((\pi^T_t)\) represents the implicit inflation target of the monetary authority, implied by its behaviour and actions. In the time-varying model we assume the implicit inflation target follows a random walk process.

\[
\pi^T_t = \pi^T_{t-1} + \epsilon^\pi^T_t \tag{11}
\]

The inflation target is assumed to be constant in the time-invariant model (equation 11 is replaced by equation 11')

\[
\pi^T_t = \bar{\pi} \tag{11'}
\]
The annual inflation rate ($\pi_A^t$) can be found as:

$$\pi_A^t = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \quad (12)$$

Likewise, the annual inflation target ($\pi_A^{A,T}$) is given by the following identity:

$$\pi_A^{A,T} = (\pi_t^T + \pi_{t-1}^T + \pi_{t-2}^T + \pi_{t-3}^T)/4 \quad (13)$$

### 2.2 Exchange rate relationship

Typically in the literature, models based on the small open-economy framework rely on an uncovered interest rate parity (UIP) condition to model the exchange rate. However, most empirical studies reject the UIP condition as a poor fit to actual data (see Froot and Thaler 1990). Therefore we follow the approach of Berg et al (2006) and use the modified UIP condition given in equation 14.

$$z_t = z_{t+1}^e + (r_t - r_f^* + \rho^*)/4 + \varepsilon_t^z \quad (14)$$

Where $z_t$ is the real exchange rate, $z_{t+1}^e$ is the expected real exchange rate next period, $\rho^*$ is the equilibrium risk premium, and $\varepsilon_t^z$ is a shock to the risk premium.$^5$

Expectations in the exchange rate market ($z_{t+1}^e$) are formed by a weighted average between forward-looking, rational expectations ($E_t z_{t+1}$) and adaptive (backwards-looking) expectations ($z_{t-1}$), as defined in equation 15. When $\delta_z = 1$, expectations are fully rational and we obtain the standard UIP condition.

$$z_{t+1}^e = \delta_z E_t z_{t+1} + (1 - \delta_z) z_{t-1} \quad (15)$$

The equilibrium risk premium ($\rho^*$) is defined as:$^6$

$$\rho_t^* = 4[z_t^* - \delta_z E_t z_{t+1}^* - (1 - \delta_z) z_{t-1}^* - r_t^* + r_f^*] \quad (16)$$

---

$^5$ In the UIP condition (14), the real interest rate terms in the UIP condition, are divided by four because they are expressed in annual terms, while the UIP condition is for quarterly data.

$^6$ The equilibrium risk premium equation given in Berg et al (2006) does not contain a lagged neutral real exchange rate term (or the $\delta_z$ weighting between the forward and lagged terms). We have included the lagged term so that if the equilibrium risk premium equation (16) is substituted into the UIP condition (14), we obtain a UIP condition in ‘gap’ form.
Where $z_t^*$ is the neutral level of the real exchange rate (defined in equation 4 for the time-varying model, and equation 4' for the time-invariant model), $r_t^*$ is the neutral real interest rate of the domestic economy (defined in equation 5, and equation 5'), and $r_t^{f*}$ is the neutral real interest rate of the foreign economy.\(^7\)

### 2.3 Foreign economy

For simplicity, the foreign economy is modelled as a detrended closed-economy version of the domestic economy. Therefore, all of the natural rates in the foreign economy equations are assumed to be equal to zero.

The core equations of the foreign economy in the model are given by an IS relationship (17), hybrid new Keynesian Phillips curve (18), and monetary policy rule (19).

\[
x_t^f = (1 - \beta_x^f)E_t x_{t+1}^f + \beta_x^f x_{t-1}^f - \beta_r^f r_{t-1}^f + \varepsilon_t^x
\]

\[
\pi_t^f = (1 - \alpha_\pi^f)E_t \pi_{t+1}^f + \alpha_\pi^f \pi_{t-1}^f + \alpha_x^f x_{t-1}^f + \varepsilon_t^\pi
\]

\[
i_t^f = \gamma_i^f i_{t-1}^f + (1 - \gamma_i^f) \left( \gamma_x^f E_t \pi_{t+4}^A + \gamma_x^f x_t^f \right) + \varepsilon_t^i
\]

Where the foreign real interest rate ($r_t^f$) is given by the Fisher equation:

\[
r_t^f = i_t^f - E_t \pi_{t+1}^f
\]

And the annual inflation rate ($\pi_t^A$) is given by the identity:

\[
\pi_t^A = (\pi_t^f + \pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f)/4
\]

### 3 Estimation

For each model, the parameters are estimated using Bayesian estimation. Bayesian estimation has become a popular approach amongst central banks to take new Keynesian DSGE models to the data. It has a number of advantages including allowing us to compare the fit of models using the posterior odds ratios, and allowing us to use prior information we may have to help ‘pin

\(^7\) As discussed below, the foreign economy data has been detrended. Therefore, $r_t^{f*} = 0$. 

9
down’ weakly identified parameters. We use the IRIS toolbox for MATLAB to carry out the Bayesian estimation of the two models.\(^8\)

### 3.1 Data

We estimate the models using quarterly data for New Zealand (the domestic economy) and the United States (a proxy for the foreign economy) from 1992Q1 to 2008Q1.

Although New Zealand began targeting inflation at the end of 1989, we ignore the disinflation period between 1989 and 1991 (characterised by a large recession) and focus our attention on the period from 1992Q1, where inflation was at a relatively low and stable level.\(^9\)

For the domestic economy (New Zealand), output \((y_t)\) is measured as the log of seasonally-adjusted real GDP. The nominal interest rate \((i_t)\) is defined as the 90-day bank bill rate. Inflation \((\pi_t)\) is an annualised measure derived from the consumer price index (CPI).

In 1999Q3, the official (headline) CPI measure was adjusted to exclude components that relate to interest charges. The CPI series we use adjusts the headline CPI series prior to 1999Q3 to exclude these same components. This ensures the CPI series is comparable over time. In addition, we adjust the inflation rate in 2001Q1 to match the inflation rate found for the same period using the a measure of ‘CPI excluding central and local government charges.’ This adjustment was made to remove an outlier, because in 2001Q1, the government moved from charging market-rate to income-based rents on state housing. This resulted in a sharp one off, fall in the rent component of CPI for that quarter.\(^10\)

We use the United States to proxy the foreign economy, and detrend all the observable series using an HP filter. The output gap \((x^f_t)\) is calculated on the log of seasonally-adjusted real GDP. Foreign interest rates \((i^f_t)\) are calculated using the 90-day bank bill rate. Foreign annualised inflation \((\pi^f_t)\) is calculated using core CPI (excluding food and energy).\(^11\)

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\(^8\) The IRIS toolbox was created by Jaromír Beneš, and is available from: [http://www.iris-toolbox.com/](http://www.iris-toolbox.com/)

\(^9\) See Matheson (2006b) for the use of a similar sample period.

\(^10\) The model was also estimated using the the unadjusted CPI series. However, this had very little impact on the results.

\(^11\) This core CPI measure is the inflation measure that the Federal Reserve focuses on.
The real exchange rate series \((z_t)\) is derived as
\[
z_t = 100 \times log \left( e_t \times \frac{CPI_t^{NZ}}{CPI_t^U} \right)
\]
where \(e_t\) is the nominal exchange rate (US$/NZ$), and the CPI measures for New Zealand \((CPI_t^{NZ})\) and the United States \((CPI_t^U)\) are the measures used in the inflation rate calculations above.

### 3.2 Identification

Because we have extended the small open-economy model to explicitly model the natural rates, the model may not identify, or only weakly identify, the value of some parameters (we have seven observable series and 11 shock terms). In these cases, our priors become important in anchoring the parameter values. To test the identification of the parameters in our model, we use the Fisher information matrix.

The Fisher information matrix tests the full information likelihood function, to identify if it is relatively flat in any dimension (and thus weakly or not identified in that dimension). If the likelihood does have identification problems, it is a result of the structure of the model. Focusing on the dimensions where there is weak or no identification, we are able to look at the weights of each parameter that contribute to the identification problem in each dimension. We use these weightings to identify those parameters for which the model’s structure limits the data’s ability to provide information on.

The Fisher information matrix finds that the time-varying model has 11 dimensions in which the full information likelihood is weakly identified, and none that are unidentified.\(^{12}\) To identify those parameters the model will struggle to identify, we look at the parameters that have a particularly large weighting in one dimension, and those that have relatively large weights in multiple dimensions. From this criteria, the following parameters can be considered to be weakly identified:

\(^{12}\) The likelihood of the model has 29 dimensions in total as we have 29 different parameters in the model.
The sensitivity of the output gap to the real exchange rate gap ($\beta_z$), the sensitivity of the output gap to the foreign demand conditions ($\beta_f$), monetary policy’s responsiveness to the deviation in annual inflation from its target ($\gamma_\pi$), the standard deviation of shocks to the growth rate of potential output ($\sigma_g^*$), the standard deviation of shocks to the inflation target ($\sigma_\pi^T$), and the standard deviation of shocks to the neutral real exchange rate ($\sigma_z^*$).

Therefore, it is particularly important that we understand the impact our choices of priors for these parameters have on the parameter estimates.

3.3 Priors

To estimate the parameters using Bayesian estimation we must specify prior distributions for each parameter in the model. Table 1 provides a summary of the prior used in the estimation of the time-varying model. The choice of priors was influenced by a range of previous models of the New Zealand economy and models of other small open economies. Of particular importance is our choice of priors that the Fisher information matrix noted has weak identification.

The prior on the sensitivity of the domestic economy to the real exchange rate gap ($\beta_z$) is distributed around a mean of 0.01. Relative to the parametrisation of similar models, this value is low. Our motivation for this comes from analysing the TWI (exchange rate) and output gap series used in the Reserve Bank’s FPS model. Over our sample period, the TWI measure used in FPS shows large volatility, moving as much as 20 percent above and below the average TWI. Meanwhile, the deviations in the output gap used in FPS is never larger than a few percentage points. Therefore, we expect $\beta_z$ to be fairly insensitive (small). The variance of the prior is set to provide a rather diffuse prior to reflect that uncertainty we have over this parameter.

Although the United States is a large export market for New Zealand, it is not dominant enough that minor changes in the demand pressures would significantly impact on the demand pressures in New Zealand. Therefore, we choose the prior for $\beta_f$, the sensitivity of the domestic economy to foreign demand conditions, to also be relatively low (0.05).

In the domestic Phillips curve, we set the mean of the prior on $\alpha_x$, the effect of the output gap on inflation, equal to 0.1. In other small open-economy literature with similar Phillips curve specifications, this parameter value ranges in size from 0.0011 (Buncic and Melecky 2008) to 0.22 (Harjes
and Ricci 2008). It is therefore difficult to form a tight prior on what an appropriate value should be.

We choose the prior $\alpha_z = 0.075$ (the sensitivity of domestic inflation to an appreciation in the real exchange rate) based on two observations. First, the United States is not a relatively large source of imports for New Zealand. And second, given the large movements in New Zealand’s real exchange rate, the Reserve Bank has been reasonably successful at maintaining a low and stable inflation rate over the years. Despite believing $\alpha_z$ is low, we are still uncertain exactly how low it is. Therefore, we choose a relatively diverse prior.

We set the prior on $\gamma_\pi = 2$ (monetary policy’s responsiveness to expected inflation deviations from target). This value matches that used by Berg et al (2006) and Harjes and Ricci (2008), and suggests that the monetary authority responds rather aggressively towards deviations in inflation from its target. The distribution of our prior on $\gamma_\pi$ is also more diffuse than in Harjes and Ricci (2008). Also in the monetary policy rule, we set the sensitivity to the output gap ($\gamma_x$) equal to 1, noting that the Reserve Bank is required to give consideration to the output gap under clause 4b of its Policy Targets agreement.

The mean of our prior on the standard deviation of shocks to the annualised growth rate of potential output ($\sigma_g$) is set equal to $0.1$. This value is close to that obtained if we fitted equation 7 (the potential output growth equation) to the potential output series from FPS and an HP filtered series, using maximum likelihood.

The mean of our prior for the standard deviation of shocks to the inflation target ($\sigma_{\pi T}$) is set to $0.15$. This value is very close to the standard deviation found if we fit a random walk equation to the midpoint of the inflation target series. This gives us a ratio between standard deviation of shocks to the inflation target and the inflation level ($\sigma_{\pi T}/\sigma_{\pi}$) of $0.3$ which seems reasonable.

Finally, we set the mean of the prior on $\sigma_{zt}$, the standard deviation of shocks to the neutral real exchange rate, equal to one. It is difficult to find other estimates to inform our prior, but we expect that the shocks to the real exchange rate would be significantly larger than the the shocks to to neutral real exchange rate. Choosing $\sigma_{zt} = 1$ sets the ratio of standard deviations between neutral real exchange rate shocks and real exchange rate shocks ($\sigma_{zt}/\sigma_z$) to $0.5$.

The priors for the time-invariant model are presented in appendix A. Where
the two models share the same parameters, the same priors were used.

4 Results

4.1 Posteriors

The means and 90 percent confidence intervals from the posterior distributions of the time-varying model parameters are also presented in table 1. Likewise, the posterior means and confidence intervals for the time-invariant model are presented in appendix A. Plots of the posterior distributions in each model are presented in appendices B and C. Our main focus for this paper is on the posterior distributions of the domestic parameters in the time-varying models. Therefore, we only discuss these in this section.

The results reveal there is a relatively high degree of persistence in the IS relationship ($\beta_x = 0.726$). The estimation also shows that the domestic output gap is relatively insensitive to the real exchange rate gap ($\beta_z = 0.006$) and foreign demand conditions ($\beta_f = 0.042$), similar to our prior beliefs. Although this insensitivity should be interpreted with caution given that the real exchange rate has undergone large changes in valuation over the sample period (see figure 5).

The domestic Phillips curve is predominantly forward looking ($\alpha_{\pi} = 0.194$). Like the IS relationship, the Phillips curve also shows low sensitivity to the real exchange rate ($\alpha_z = 0.031$). Also of interest is the fact that the posterior mean of the inflation rate shows less sensitivity to the domestic output gap ($\alpha_x = 0.056$) than our prior suggested.

The Taylor-type rule for monetary policy demonstrates a high degree of persistence ($\gamma_i = 0.778$). Therefore, the model suggests the Reserve Bank seeks to smooth changes to the interest rate over time. This behaviour could be the result of the Reserve Bank facing uncertainty over optimal policy and the current economic situation, or their wish to reduce interest rate volatility (one of the requirement outlined in the Policy Targets Agreement). The estimation results also show that monetary policy is slightly more aggressive towards deviations in inflation from its target ($\gamma_{\pi} = 2.148$) and slight less aggressive towards output deviations ($\gamma_x = 0.808$) than prior belief. Although, the plot of the posterior in appendix B shows $\gamma_{\pi}$ is not well identified, as suggested by the Fisher information matrix.

The standard deviation of the shocks to the growth rate of potential output
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Mean</th>
<th>S. D.</th>
<th>Dist.</th>
<th>Range</th>
<th>Mean</th>
<th>90% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic economy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_x) IS: weight on lag</td>
<td>0.4</td>
<td>0.15</td>
<td></td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.726</td>
<td>[0.618, 0.834]</td>
</tr>
<tr>
<td>(\beta_r) IS: effect of real int rate gap</td>
<td>0.1</td>
<td>0.05</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.065</td>
<td>[0.018, 0.108]</td>
</tr>
<tr>
<td>(\beta_z) IS: effect of real exch rate gap</td>
<td>0.01</td>
<td>0.005</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.006</td>
<td>[0.002, 0.011]</td>
</tr>
<tr>
<td>(\beta_f) IS: effect of foreign output gap</td>
<td>0.05</td>
<td>0.015</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.042</td>
<td>[0.022, 0.061]</td>
</tr>
<tr>
<td>(\alpha_{pi}) PC: weight on lag</td>
<td>0.5</td>
<td>0.15</td>
<td></td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.194</td>
<td>[0.099, 0.284]</td>
</tr>
<tr>
<td>(\alpha_{ix}) PC: effect of the output gap</td>
<td>0.1</td>
<td>0.035</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.056</td>
<td>[0.028, 0.083]</td>
</tr>
<tr>
<td>(\alpha_{iz}) PC: effect of change in real exch rate</td>
<td>0.075</td>
<td>0.05</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.031</td>
<td>[0.003, 0.061]</td>
</tr>
<tr>
<td>(\gamma_{i}) MP: smoothing parameter</td>
<td>0.7</td>
<td>0.2</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.778</td>
<td>[0.688, 0.874]</td>
</tr>
<tr>
<td>(\gamma_{pi}) MP: responsiveness to inflation</td>
<td>2</td>
<td>0.5</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>2.148</td>
<td>[1.263, 2.983]</td>
</tr>
<tr>
<td>(\gamma_{iz}) MP: responsiveness to output gap</td>
<td>1</td>
<td>0.3</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.808</td>
<td>[0.439, 1.154]</td>
</tr>
<tr>
<td>(\delta_{x}) UIP: forward looking weight</td>
<td>0.75</td>
<td>0.15</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.475</td>
<td>[0.403, 0.542]</td>
</tr>
<tr>
<td><strong>Exchange rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{f}x) IS: weighting on lag</td>
<td>0.4</td>
<td>0.15</td>
<td></td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.54</td>
<td>[0.436, 0.655]</td>
</tr>
<tr>
<td>(\beta_{f}r) IS: effect of real int. rate gap</td>
<td>0.1</td>
<td>0.05</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.033</td>
<td>[0.006, 0.060]</td>
</tr>
<tr>
<td>(\alpha_{fj}) PC: weight on lag</td>
<td>0.5</td>
<td>0.15</td>
<td></td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.172</td>
<td>[0.086, 0.252]</td>
</tr>
<tr>
<td>(\alpha_{fi}) PC: effect of the output gap</td>
<td>0.1</td>
<td>0.035</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>0.047</td>
<td>[0.024, 0.073]</td>
</tr>
<tr>
<td>(\gamma_{fj}) MP: smoothing parameter</td>
<td>0.7</td>
<td>0.2</td>
<td></td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.775</td>
<td>[0.704, 0.850]</td>
</tr>
<tr>
<td>(\gamma_{pi}) MP: responsiveness to inflation</td>
<td>1.75</td>
<td>0.5</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>1.844</td>
<td>[1.004, 2.517]</td>
</tr>
<tr>
<td>(\gamma_{iz}) MP: responsiveness to output gap</td>
<td>1</td>
<td>0.3</td>
<td></td>
<td>Gamma</td>
<td>[0,∞)</td>
<td>1.295</td>
<td>[0.853, 1.738]</td>
</tr>
<tr>
<td><strong>Standard deviations of shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_x) Std dev: output shock</td>
<td>0.5</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.363</td>
<td>[0.235, 0.487]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_{pi}) Std dev: inflation shock</td>
<td>0.5</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.616</td>
<td>[0.393, 0.826]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_i) Std dev: interest shock</td>
<td>0.5</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.415</td>
<td>[0.260, 0.577]</td>
<td></td>
</tr>
<tr>
<td>(\sigma^g) Std dev: growth rate shock</td>
<td>0.1</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.097</td>
<td>[0.028, 0.172]</td>
<td></td>
</tr>
<tr>
<td>(\sigma^g_{T}) Std dev: inflation target shock</td>
<td>0.15</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.098</td>
<td>[0.035, 0.164]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_f) Std dev: neutral real rate shock</td>
<td>0.2</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.241</td>
<td>[0.048, 0.525]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_{iz}) Std dev: real exch. rate shock</td>
<td>2</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>1.066</td>
<td>[0.535, 1.551]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_{iz}) Std dev: neutral real exch. Rate shock</td>
<td>1</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>1.319</td>
<td>[0.266, 2.426]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_f) Std dev: foreign output shock</td>
<td>0.5</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.18</td>
<td>[0.124, 0.238]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_{pi}) Std dev: foreign inflation shock</td>
<td>0.5</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.379</td>
<td>[0.248, 0.508]</td>
<td></td>
</tr>
<tr>
<td>(\sigma_f) Std dev: foreign interest shock</td>
<td>0.5</td>
<td>Inf</td>
<td>Inv. G.</td>
<td>[0,∞)</td>
<td>0.274</td>
<td>[0.185, 0.362]</td>
<td></td>
</tr>
</tbody>
</table>
\( \sigma_g^* = 0.097 \) is very close to our initial prior (0.1). This is a result of the weak identification of the parameter as highlighted by the Fisher information matrix.

The standard deviation of shocks to the inflation target \( \sigma_{\pi_T} = 0.098 \) is lower than our prior of 0.15. This suggests the inflation target has been relatively more stable than the midpoint of the inflation target band.

Unexpectedly, the posteriors mean of the standard deviation of shocks to the neutral real exchange rate \( \sigma_{z^*} = 1.319 \) is larger than the standard deviation of shocks to the UIP condition \( \sigma_z = 1.066 \).

The estimated time-invariant model shows a relatively high constant inflation target of \( \bar{\pi} = 2.2 \) percent (see appendix A). While this is close to the current midpoint of the target band (2 percent), it is noticeably higher than the mean of the inflation target’s midpoint over the sample period (1.51 percent). Likewise, the average (annualised) growth rate of the model \( \bar{g} = 3.26 \) is higher than our prior of 2.5. This result however, is likely to be sensitive to our choice of sample period. If our sample period includes more business cycle upswings than downswings, the estimate of the average growth rate will likely be biased upward.

\section*{4.2 Fit}

Bayesian estimation lends itself naturally to comparing the fit of models. By taking the marginal data densities from the models, we are able to compute the posterior odds ratio. According to Bayesian estimation, a posterior odds ratio \( P_{O_{ij}} \) greater than one favours model \( i \) over model \( j \). That is to say, model \( i \) is a better fit to the data than model \( j \). While a posterior odds ratio less than one favours model \( j \).

The posterior odds ratio is computed as:

\[
P_{O_{ij}} = \frac{p(M_i|y)}{p(M_j|y)}
\]

Where \( p(M_i|y) \) is the marginal data density of model \( i \).

From our Bayesian estimation we obtain the log marginal data densities of the time-varying and time-invariant models. These are presented in table 2.

Using the above formula, the posterior odds ratio between the time-varying and time-invariant models is calculated as 107098\( (= exp(11.58)) \). Therefore,
Table 2
Model fit

<table>
<thead>
<tr>
<th>Model</th>
<th>Log marginal data density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time varying</td>
<td>-303.94</td>
</tr>
<tr>
<td>Time invariant</td>
<td>-315.52</td>
</tr>
</tbody>
</table>

according to the posterior odds ratio, the time-varying model is a significantly better fit to the data than the time-invariant model.

Furthermore, from the posterior odds ratio we are also able to gauge the strength of the preference for the time-varying model. The posterior odds ratio of 107098 states that in order for us to choose the time-invariant model over the time-varying model (i.e. go against what the Bayesian estimation suggests is the better model), our prior belief that the time-invariant model is the correct model, would need to be 107098 times greater than our prior belief that the time-varying model is the correct model. Therefore, the posterior odds ratio is overwhelmingly in favour of the time-varying model.

Such a large posterior odds ratio seems almost implausible given the similarities between our two models. However, according to Sims (2003), when the set of models being compared is too sparse, the results from the Bayesian model comparison will tend to be implausibly sharp. This ‘misbehaviour’ occurs as a result of the discrete collection of models serving as a proxy for a more realistic continuous parameter space. Therefore, if we were to introduce more models that spanned the region between our time-varying and time-invariant models, we would have a better proxy for the continuous space, and it is unlikely the posterior odds ratio would favour the time-varying model so strongly above all the rest.

4.3 Model estimated natural rates

Using the Kalman smoother we are able to extract the paths of the unobservable variables within a model. We use this approach to find the time-varying estimates of the natural real rate of interest, inflation target, natural rate of nominal interest, output gap (driven by the models estimate of potential output), and neutral real exchange rate. The results are plotted in figures 1 to 5.

The estimated natural real rate of interest in figure 1 shows that between 1992 and 1998 the natural real rate of interest was relatively stable around 5.25
percent. However, after 1998 the natural real rate began trending downwards, reaching almost 3 percent in 2004 before rising to its current level of around 4.3 percent.

Basdevant et al (2004) estimated the natural real rate of interest for New Zealand over the period 1992 to 2004 using several statistical and semi-structural approaches. Their results are much smoother than our estimate, but the trends and magnitudes are broadly for the range where sample periods overlap. However, our results contrast with Schmidt-Hebbel and Walsh (2007) who using a semi-structural approach with a backwards-looking close-economy new Keynesian model, estimate that the natural real rate of interest has been relatively stable around 5 percent (with only small, but persistent deviations) between 1986 and 2006.

The rise in the natural real rate since 2004 may be one of the contributing factors as to why the Reserve Bank has found it more difficult to control inflationary pressures in the latest cycle. If the policymaker’s estimate of the natural rate remained relatively constant since 2004, the time-varying model suggests that they would have over predicted the contractionary strength of monetary policy. In other words, the monetary policy the Reserve Bank was running was not as tight as policymakers would have believed.
According to the time-varying model, the (annualised) inflation target has been relatively stable over the whole sample period (see figure 2). Prior to 2000, the Reserve Bank was targeting inflation in the medium term at a rate slightly above 2 percent. Around 2000, the inflation target increased to a new rate of 2.5 percent, where it has stayed close to for the remainder of the sample period. This suggests that prior to 1997 when the target band was 0-2 percent, the Reserve Bank was targeting annualised inflation just above the top of the band. Given that the average annualised inflation rate in figure 2 is close to 2 percent over this period, we should not be surprised by this discrepancy between the model’s estimated inflation target and the mid-point of the target band.

The natural rate of nominal interest (shown in figure 3) is found using the natural real rate of interest and the expected inflation target. The model attributes some of the change in nominal interest rates, not explained by the Reserve Bank responding to the output gap or deviation in inflation from its target, as changes in the natural rate of nominal interest. Our estimate suggests that the Reserve Bank has historically been very persistent in its movements away from the natural rate of nominal interest since 1992. However, as the 90 percent confidence intervals show, there is a high degree of uncertainty surrounding the estimate of the natural rate of nominal interest.
Figure 4 shows the output gap estimate from the time-varying model alongside the output gap used in FPS and the output gap estimated by an HP filter. The model estimates an output gap very similar to the FPS output gap between 1993 to 1998 and 2003 to 2007. However, in between these two periods (1998 to 2003), the output gap is estimated to be lower than the output gap used in FPS and estimated by the HP filter. The model estimates an output gap close to zero percent for 1992Q1. This is similar to the estimate from the HP filter, but noticeably different from the output gap used in the FPS model (-2 percent). This difference is driven by the fact that the output gap used in FPS takes into account observations of output prior to 1992Q1, while the Model and HP filter do not have any information prior to 1992Q1.

Figure 5 shows the model estimate of the neutral real exchange rate. That is, the exchange rate at which no pressure is put on the domestic output gap or inflation rate. The neutral rate is estimated to have been steadily increasing since 2002. At the beginning of 2008, the real exchange rate between New Zealand and the United States was slightly over 20 percent above its neutral level.

13 The HP filter is estimated on our sample period from 1992Q1 to 2008Q1. No adjustment is made for any end point issues.
Figure 4
Model estimate of the output gap ($x_t$)

Figure 5
Model estimate of the neutral real exchange rate ($z^*_t$)
4.4 Dynamics

We examine the implications time-varying natural rates have on a model’s dynamics using two main measures. First, we examine how the structural persistence parameters differ between the time-invariant and time-varying models. This follows similar analysis from previous literature which investigates inflation persistence and its implication for monetary policy. And second, we compare the impulse responses of the time-invariant and time-varying models to a variety of shocks.

Persistence Parameters

Recent international literature (such as Sbordone 2007 and Benati 2008) has suggested that the inflation persistence we usually observe in a hybrid, new Keynesian Phillips curve may overstate the true level of structural inflation persistence. Sbordone (2007) defines structural inflation persistence as the persistence that is a structural feature of the economy, and not a consequence of the way monetary policy has been conducted. This distinction is important for policymakers as trend inflation (or the inflation target) is ultimately determined by policymakers’s actions and therefore, is not taken as given when setting monetary policy. For our model, we extend the analysis to compare the posterior distribution of the three parameters that control persistence in the domestic economy’s inflation rate ($\alpha_\pi$), output gap ($\beta_x$), and interest rate ($\gamma_i$), to assess if allowing for time-variation in the natural rates has any significant impact on the persistence within the model.

The first panel in figure 6 shows the posterior distributions of the parameter $\alpha_\pi$, the weighting on the lagged inflation term in the Phillips curve, which measures the persistence in inflation. We can see that the posterior distribution for the time-varying model is slightly lower (to the left) than the time-invariant model. This means that allowing for time-variation in the natural rates decreases the persistence of inflation, but only slightly. While this result is similar to Sbordone (2007), unlike Sbordone (2007) we still see some persistence in the Phillips curve.

The difference between our results and those in Sbordone (2007) comes from the fact that trend inflation (the inflation target) in Sbordone (2007) tracks actual inflation quite closely, much more so than the case for New Zealand (see figure 2). Therefore, the inflation target is our model has significantly less persistence that that in Sbordone (2007). By not having as much persistence in the inflation target, deviations in inflation from its target will by
Figure 6
Posterior distributions of persistence parameters

Over time, construction be more persistent.

From the plots in the second panel of figure 6 we can see that the posterior distribution of $\beta_x$ (the persistence parameter in the IS relationship) is virtually identical under the time-varying and time-invariant models. This means the persistence parameters in the two models are not significantly different.

Finally, in the third panel of figure 6, we examine the posterior distribution of the interest rate smoothing parameter ($\gamma_i$). Our results show that the posterior distribution of $\gamma_i$ under the time-varying model is centered slightly to the left of the distribution under the time-invariant model. Therefore, allowing for time-variation in the natural rates, reduces the persistence of interest rates. In terms of the Taylor-type rule, the monetary authority giving less emphasis to interest rate smoothing and behaves more aggressive to deviations in inflation from its target and the output gap.

Impulse Responses

For the impulse responses, we focus on the major domestic and foreign shocks. Figures 7 to 9 show the impulse responses of the time-varying and time-invariant models to a one unit shock to domestic output, domestic inflation, and domestic interest rates. Figures 10 to 12 show the impulse responses of a one unit shock to foreign output, foreign inflation, and foreign interest rates. Finally, figure 13 shows the impulse responses to a one unit real exchange rate shock.

In response to a domestic output shock (see figure 7), both models show
Figure 7
Impulse responses to a domestic output shock

an increase in the domestic interest rate. The time-invariant model however, shows a more muted response, driven by the fact that under the time-invariant model inflation is much smaller. In fact, inflation is initially driven lower in the time-invariant model. This is the result of a number of factors including inflation’s sensitivity to changes in the real exchange rate rate ($\alpha_z$) being lower in the time-invariant model. The output gap itself, shows noticeably more persistence in the time-invariant model. Likewise, the time-invariant model has more persistence in the real exchange rate (compared to the time-varying model), although both models still show very large changes in the real exchange rate in response to a domestic output shock.

For both domestic inflation and interest rate shocks (see figures 8 and 9), the time-invariant model shows more persistence in the domestic output gap and real exchange rate. The time-varying model is back close to the natural rate after 40 quarters, while the time-invariant model takes longer to converge. The magnitudes of the domestic output gap and real exchange rate responses to these shocks are also noticeably larger under the time-invariant model.

On the foreign shock side, the domestic output gap, interest rate, and real exchange rate of the time-varying model, all display a more cyclical response
Figure 8
Impulse responses to a domestic inflation shock

to a foreign output shock (figure 10). However, the time-varying model’s domestic inflation impulse is less cyclical to this shock. The persistence between the two models is broadly similar, with both models being back close to their natural rates after 40 periods.

The domestic impulses of the time-varying and time-invariant models show very different paths in response to a foreign inflation shock (figure 11). While the persistence of the two models is similar, the time-invariant model shows much shorter cycles in the domestic variables. However, the magnitudes of the impulses for all the domestic variables (and the real exchange rate) suggest that in this model, foreign inflation does not have a large impact on the domestic economy.

The domestic output gap of the time-invariant models show a much larger (and more persistent) response to foreign interest rates than the time-varying model (figure 12). However, the domestic interest and inflation rates are still broadly similar between the two models. For all of the foreign shocks, the impulses of the foreign economy are virtually identical under both models.

In response to a real exchange rate shock (figure 13), the two modes show a
noticeable difference in the persistence and magnitudes of all four variables. The domestic inflation rate under the time-invariant model initially responds more strongly to the real exchange rate appreciation. It also shows more sensitivity when the real exchange rate starts to fall back down to its neutral rate. The difference in inflation responses between the time-varying and time-invariant models produces very different responses from monetary policy (the domestic interest rate). Which, in turn, leads to very different domestic output gap profiles. For all four variables, the time-invariant model shows more persistence than the time-invariant model and is not always close to converging after 40 periods.

Overall, we can see that allowing for time-varying natural rates can have large impacts on the impulse responses of the model to various domestic and foreign shocks. For most shocks the time-varying model displays less persistence than the time-invariant model. However, the results are highly dependent upon which individual variables and shocks are examined.
Figure 10
Impulse responses to a foreign output shock

Domestic output gap ($x_t$)

Domestic interest ($i_t$)

Domestic inflation ($\pi_t$)

Real exchange rate ($z_t$)

Foreign output gap ($x^f_t$)

Foreign interest rate ($i^f_t$)

Foreign inflation ($\pi^f_t$)
Figure 11
Impulse responses to a foreign inflation shock
Figure 12
Impulse responses to a foreign interest rate shock
Figure 13
Impulse responses to a real exchange rate shock

4.5 Robustness

Whenever estimating unobservable natural rates, there is a large amount of uncertainty related to model and data specification. Other model frameworks such as the Reserve Bank’s FPS model, or simple time series approaches, will generate different estimates and in practice, policymakers apply judgment and draw on many sources of information when using natural rate concepts in decision making. To test model robustness within our specific new Keynesian framework, we perform a number of robustness checks to the model. In particular, we test how robust the model is to: (i) changes in priors; (ii) changes in annual inflation expectations; and (iii) an alternative output gap measure.

Priors

As a test of how robustness the natural rates are to the priors we specify, we re-estimate the model after doubling our initial priors on the standard deviation of shocks to the natural real rate (σ_τ*) and the standard deviation
of shocks to the inflation target ($\sigma_{\pi T}$). This gives us priors of $\sigma_{\pi T} = 0.3$, and $\sigma_{\pi r} = 0.4$.

We choose to test the robustness of the results to larger priors for two main reasons. First, the natural real rate of interest, inflation target, and natural rate of nominal interest all depend directly upon our choice of these priors. And thus, the priors are relatively important for the results we obtain. And second, There is a large amount of uncertainty around the ‘true paths’ of these natural rates. Using larger priors is less restrictions on the natural rate paths, allowing them to vary more.

**Expectation data**

As part of a regular survey of expectations, the Reserve Bank asks respondents what they expect annual inflation to be in one years time. We take the mean response from this survey ($\pi^S_t$) and use it to inform the rational expectations in the model. More precisely, we introduce the following identity:

$$\pi^S_t = E_t\pi_{t+1}^{A} = E_t(\pi_{t+4} + \pi_{t+3} + \pi_{t+2} + \pi_{t+1})/4$$

Therefore, the model’s rational expectations for inflation over the next four quarters, must be consistent with survey measure over the same horizon. Including this extra observable series should assist in the identification of the parameters within the model. It is important to note here that we do not consider any issues relating to the representational quality of the survey. If the survey is of a small sample or biased in some way, the responses collected may not reflect the true inflation expectations that people use in their day to day decision making process.

**Alternative output gap**

To test how sensitive the other natural rates are to the estimate of potential output, we replace the model’s endogenously estimated output gap with the output gap used in the Reserve Bank’s FPS model. Therefore, the equations that determine potential output within the model (equations 6 and 7) and the output series ($y_t$) are redundant. We remove these equations before estimating the model with FPS’s output gap.

By removing these equations, and assuming the output gap is observable, the model no longer has to try and estimate the weakly identified parameter $\sigma_{g^*}$.
(the standard deviation of shocks to the growth rate of potential output). This may also have the benefit of improving the model’s identification of other weakly identified parameters.

Results of the robustness tests

The natural rate estimates resulting from all three robustness tests can be seen in figures 14 to 18, alongside the original estimates and 90 percent confidence intervals from the time-varying model.

The plots of the natural real rate of interest in figure 14 show that the model’s estimate is relatively robust to the three tests. Between 1992 and 1998 the tests suggest some upside risk to the model’s estimate. And between 1998 and 2006 the tests suggest some downside risk to the estimate of the natural real rate of interest. As we would expect, doubling the prior on the standard deviation of shocks to the natural real rate of interest ($\sigma_r^*$) produces a more volatile series. In fact, our model shows the highest sensitivity to this particular test. However, the natural real rate of interest under all three robustness tests fall within the 90 percent confidence intervals, and are generally very close to the original estimate.

The robustness test results for the inflation target (figure 15) suggest the model’s inflation target track ($\pi_T^T$) is fairly robust to the use of larger priors and the alternative output gap, with the inflation target estimates being fairly close to our original estimate. On the other hand, the results from the robustness tests using expectations survey data show the inflation target is rather sensitive to the expectations of inflation within the model. The inflation target found under this robustness tests shows significantly more volatility than our original model estimate. It does not fit in with out prior view that while the inflation target may have varied over time, the change is likely to be slow and gradual.

Figure 16 shows the natural rate of nominal interest estimated under our original model and the robustness tests. Using larger priors, or the survey of inflation expectation data produces the largest deviations from our original estimate. This is not surprising as these two robustness tests showing the largest deviations when we examined the natural real rate of interest and the inflation target estimate. Overall, the natural rate of nominal interest under all three robustness tests are within the 90 percent confidence interval (apart from one minor breach in 1994).

The model’s estimate of the output gap also appears relatively robust to our
Figure 14
Robustness of the natural real rate estimate ($r^*_t$)

tests (see figure 17). Using larger priors on the standard deviations of the inflation target and natural real rate of interest have an insignificant impact on the model’s estimate of the output gap. Using the survey data of inflation expectations produces an output gap that between 1994 and 2007 is slightly higher than our original estimate. The estimate is very close to the upper bound of the 90 percent confidence interval, but is still within it. However, it is still lower over this period than the output gap estimate used in FPS.

Figure 18 shows the estimate of the neutral real exchange rate ($z^*_t$) under our various robustness tests. From the graph we can see that using larger priors for the standard deviation of shock to the natural real rate and inflation target has a negligible effect on the neutral real exchange rate estimate. However, using the survey of inflation expectations data produces a neutral real exchange rate that is the higher than our original estimate between 1992 and 2007 (most of our sample period). The FPS output gap robustness test suggests the neutral real exchange rate has actually declined over our sample range. During the whole sample period, all three robustness tests fall within the 90 percent confidence intervals of our original estimation.

Overall, our qualitative results are fairly robust to the three tests we perform.
Figure 15
Robustness of the inflation target estimate ($\pi_T^t$)

Figure 16
Robustness of the natural rate of nominal interest estimate
The use of larger priors on the standard deviation of shocks to the natural real rate ($\sigma_{r^*}$) and inflation target target ($\sigma_{\pi T}$), and to the use the FPS output gap do not have significant impacts on our model estimate. The model is more sensitivity to the use of the inflation expectation data, especially in the estimates of the inflation target and neutral real exchange rate. However, it is not enough to change the overall picture.

5 Conclusion

Small new Keynesian models have become a popular tool to assist and inform monetary policy decisions. However, these models are often estimated on demeaned interest rate and inflation data. If the natural real rate of interest and inflation target are not constant, the dynamics of the model will be biased.

We estimate (using Bayesian techniques) a small open-economy new Keynesian model to endogenously model the natural real rate of interest, inflation target, potential output, and neutral real exchange rate as time-varying pa-
Figure 18
Robustness of the neutral real exchange rate estimate ($z_t^*$)

Parameters. This time-varying model was compared to a time-invariant model in which these natural rates were assumed to be constant.

Using the posterior odds ratio, we are able to test the models based on their fit to the data. The posterior odds ratio shows that the time-varying model is a significantly better fit to the data. We are also able to back out via the Kalman smoother estimates of the time-varying natural rates. The time-varying model suggests there has been noticeable variation in the natural real rate, potential output, and neutral real exchange rate over the sample period. The model also suggests that the inflation target has been increasing from around 2 percent at the start of our sample to around 2.5 percent by the end of our sample.

We found that allowing for time variation in the natural rates only slightly decreases the persistence parameter for the nominal interest rate and inflation processes (but not the output gap). We also found the difference between the impulse responses of the time-varying and time-invariant model can be quite large, with the time-varying model generally displaying less persistence to shocks. However, the individual results vary greatly for the different shocks we consider.
The robustness tests we perform on the model show that the estimates are fairly robust to the use of larger priors on the standard deviations of shocks to the natural real rate of interest and the inflation target, the use of inflation expectations data, and using FPS’s output gap.

Overall, our analysis suggests that working with demeaned data, and hence implicitly assuming the natural rates are constant, does matter in the contexts of a small open-economy new Keynesian models. When we apply our model to New Zealand data, allowing for time variation in the natural rates has a noticeable impact on the model and its dynamics. Therefore, policymakers should consider what implicit assumptions they are making about the natural rates when using or analysing new Keynesian models based on demeaned or detrended data.
References


## Appendices

### A  Time-invariant priors and posteriors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>S. D.</th>
<th>Dist.</th>
<th>Range</th>
<th>Mean</th>
<th>90% CI</th>
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<tbody>
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<td><strong>Domestic economy</strong></td>
<td></td>
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<tr>
<td>$\beta_x$ IS: weight on lag</td>
<td>0.4</td>
<td>0.15</td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.735</td>
<td>[0.630, 0.844]</td>
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<tr>
<td>$\beta_r$ IS: effect of real int rate gap</td>
<td>0.1</td>
<td>0.05</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.073</td>
<td>[0.030, 0.113]</td>
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<tr>
<td>$\beta_z$ IS: effect of real exch rate gap</td>
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<td>0.005</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.004</td>
<td>[0.001, 0.007]</td>
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<tr>
<td>$\beta_f$ IS: effect of foreign output gap</td>
<td>0.05</td>
<td>0.015</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.042</td>
<td>[0.022, 0.062]</td>
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<tr>
<td>$\alpha_x$ PC: weight on lag</td>
<td>0.5</td>
<td>0.15</td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.232</td>
<td>[0.140, 0.324]</td>
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<td>$\alpha_x$ PC: effect of the output gap</td>
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<td>0.035</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.048</td>
<td>[0.024, 0.071]</td>
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<tr>
<td>$\alpha_x$ PC: effect of change in real exch rate</td>
<td>0.075</td>
<td>0.05</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.053</td>
<td>[0.026, 0.081]</td>
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<td>$\gamma_i$ MP: smoothing parameter</td>
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<td>Gamma</td>
<td>(0,¥)</td>
<td>0.806</td>
<td>[0.738, 0.873]</td>
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<tr>
<td>$\gamma_x$ MP: responsiveness to inflation</td>
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<td>0.5</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>2.325</td>
<td>[1.497, 3.158]</td>
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<tr>
<td>$\gamma_x$ MP: responsiveness to output gap</td>
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<td>0.3</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.899</td>
<td>[0.515, 1.263]</td>
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<tr>
<td>$\bar{g}$ Time-invariant growth rate</td>
<td>2.5</td>
<td>0.625</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>3.262</td>
<td>[3.101, 3.419]</td>
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<td>$\bar{\pi}$ Time-invariant inflation target</td>
<td>1.75</td>
<td>0.4</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>2.204</td>
<td>[1.836, 2.569]</td>
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<td>$\bar{r}$ Time-invariant real interest rate</td>
<td>5</td>
<td>1.25</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>4.600</td>
<td>[4.022, 5.174]</td>
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<td>$\delta_z$ UIP: forward looking weight</td>
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<td>0.15</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.437</td>
<td>[0.387, 0.488]</td>
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<tr>
<td>$\bar{\varepsilon}$ Time-invariant neutral real exch. rate</td>
<td>-55</td>
<td>5</td>
<td>Normal</td>
<td>(-∞,∞)</td>
<td>-56.344</td>
<td>[-63.479, -49.192]</td>
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<td><strong>Foreign economy</strong></td>
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<tr>
<td>$\beta_f^l$ IS: weighting on lag</td>
<td>0.4</td>
<td>0.15</td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.522</td>
<td>[0.427, 0.624]</td>
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<td>$\beta_f^r$ IS: effect of real int. rate gap</td>
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<td>0.05</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>0.030</td>
<td>[0.006, 0.056]</td>
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<tr>
<td>$\alpha_f^l$ PC: weight on lag</td>
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<td>0.15</td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.175</td>
<td>[0.094, 0.260]</td>
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<td>$\alpha_f^r$ PC: effect of the output gap</td>
<td>0.1</td>
<td>0.035</td>
<td>Gamma</td>
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<td>0.051</td>
<td>[0.024, 0.077]</td>
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<td>$\gamma_f^l$ MP: smoothing parameter</td>
<td>0.7</td>
<td>0.2</td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.781</td>
<td>[0.710, 0.853]</td>
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<tr>
<td>$\gamma_f^r$ MP: responsiveness to inflation</td>
<td>1.75</td>
<td>0.5</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>1.821</td>
<td>[1.019, 2.494]</td>
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<tr>
<td>$\gamma_f^x$ MP: responsiveness to output gap</td>
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<td>0.3</td>
<td>Gamma</td>
<td>(0,¥)</td>
<td>1.285</td>
<td>[0.848, 1.739]</td>
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<td><strong>Standard deviations of shocks</strong></td>
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<tr>
<td>$\sigma_x$ Std dev: output shock</td>
<td>0.5</td>
<td>Inf.</td>
<td>Inv. G.</td>
<td>(0,¥)</td>
<td>0.364</td>
<td>[0.235, 0.497]</td>
</tr>
<tr>
<td>$\sigma_x$ Std dev: inflation shock</td>
<td>0.5</td>
<td>Inf.</td>
<td>Inv. G.</td>
<td>(0,¥)</td>
<td>0.615</td>
<td>[0.390, 0.833]</td>
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<td>Inf.</td>
<td>Inv. G.</td>
<td>(0,¥)</td>
<td>0.438</td>
<td>[0.281, 0.587]</td>
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<td>$\sigma_z$ Std dev: real exch. rate shock</td>
<td>2</td>
<td>Inf.</td>
<td>Inv. G.</td>
<td>(0,¥)</td>
<td>1.508</td>
<td>[0.972, 2.034]</td>
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<td>$\sigma_f^l$ Std dev: foreign output shock</td>
<td>0.5</td>
<td>Inf.</td>
<td>Inv. G.</td>
<td>(0,¥)</td>
<td>0.175</td>
<td>[0.117, 0.232]</td>
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<td>$\sigma_f^r$ Std dev: foreign inflation shock</td>
<td>0.5</td>
<td>Inf.</td>
<td>Inv. G.</td>
<td>(0,¥)</td>
<td>0.366</td>
<td>[0.235, 0.493]</td>
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<tr>
<td>$\sigma_f^x$ Std dev: foreign interest shock</td>
<td>0.5</td>
<td>Inf.</td>
<td>Inv. G.</td>
<td>(0,¥)</td>
<td>0.265</td>
<td>[0.173, 0.353]</td>
</tr>
</tbody>
</table>

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B Posterior of time-varying model

Presented below are the posterior distributions plotted in black. As a comparison, the priors from table 1 are plotted and grey, and the green dashed line represents the numerical optimisation of the posterior kernel.
C Posterioris of time-invariant model

Presented below are the posterior distributions plotted in black. As a comparison, the priors from appendix A are plotted and grey, and the green dashed line represents the numerical optimisation of the posterior kernel.