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Heterogeneous Expectations, Adaptive Learning, and Forward-Looking Monetary Policy*

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Abstract

In this paper, I examine the role of monetary policy in a heterogeneous expectations environment. I use a New Keynesian business cycle model as the experiment laboratory. I assume that the central bank and private economic agents (households and producing firms) have imperfect and heterogeneous information about the economy, and as a consequence, they disagree in their views on its future development. I facilitate the heterogeneous environment by assuming that all agents learn adaptively. Measured by the central bank's expected loss, the two major findings are: (i) policy that is efficient under homogeneous expectations is not efficient under heterogeneous expectations; (ii) in the short and medium run, policy that is excessively responsive to inflation increases inflation and output volatility, but in the long run such policy lowers economic volatility.

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1 Introduction

Mishkin and Schmidt-Hebbel (2006) provide evidence that long-term inflation expectations are anchored to the target in inflation targeting regimes. On the other hand, short-term inflation expectations over one to two years, which is considered to be the time horizon at which monetary policy is most effective, are typically more volatile. There is a high degree of expectations heterogeneity.[1] Central bankers face the problem of trying to anchor these short-run expectations to announced inflation targets. This raises the question of what effects arise when private agents have different expectations to the central bank.

The contribution of this paper is in its focus on short-run transitional dynamics, from imperfect and heterogeneous expectations to a perfectly homogeneous (rational) expectations environment. In this paper there are two groups of agents resulting in a simple heterogeneous expectations environment. The private agents (comprising households and firms) and the central bank are assumed to have imperfect information, and they form different expectations. I am interested in how forward-looking monetary policy can influence the speed and volatility of the convergence process in a one to ten year time horizon in such an environment. Orphanides and Williams (2003), and Ferrero (2007) show that a central bank, operating in an environment featuring imperfect knowledge but homogenous expectations, may potentially improve the process. I extend this problem to the heterogeneous expectations case. I build on Honkapohja and Mitra (2005) who show the conditions under which a heterogeneous expectations economy can converge to a stationary, rational expectations equilibrium (REE). I use numerical analysis to study the convergence process under these conditions.

I find that if private agents and the monetary authority disagree about the expected inflation rate then, in an inflation targeting regime, a central bank should not respond aggressively to deviations from an inflation target that it itself anticipates. Weaker responses improve economic stability in the short run. Less responsive policy leads to falls in inflation and output volatility, and in the central bank's expected loss. This is in contrast to the findings for the imperfect knowledge, homogeneous expectations environment (e.g., Orphanides and Williams 2003, and Ferrero 2007).

Heterogeneous expectations cause a mismatch in subjective real interest rates. The mismatch leads to higher volatility in both inflation and output than would occur

[1] E.g. see Mankiw and Wolfers (2003).

when expectations are homogeneous across the economy. A scenario in which private agents predict less inflation than the central bank, yields exceptionally high losses, prompting the central bank to raise the policy interest rate. For private agents, who expect lower inflation, the ex ante real interest rate is higher. Higher real rates cause private agents to substitute away from current consumption so that aggregate demand drops. But consumption drops more than it would have if the private agents expected the same inflation rate as the central bank, since it is the subjective real rate that is relevant to the consumption decision. So the effect of monetary policy is stronger than the central bank itself intends. A similar situation, but with opposite implications, arises when the central bank expects low future inflation, and private agents expect high inflation: implied, subjective real interest rates are low for private agents, which results in the economy growing at the cost of unnecessarily high inflation.

The role of monetary policy is complex in a heterogeneous expectations environment. Central banks' aversion to price inflation implies strong policy responses to deviations of expected inflation from the desired target. But if the central bank is too responsive, it magnifies the effect of the mismatch in the real rates even more. In the short run, the mismatch matters most for monetary policy. In the medium to long run this phenomenon naturally disappears due to adaptive learning, and optimal monetary policy is standard as in a homogeneous expectations environment.

The following text describes a simple numerical analysis of the dynamics of a New Keynesian model under imperfect and heterogeneous knowledge on the part of economic agents. A particular focus is on the implications that heterogeneous expectations have for the optimal behavior of the central bank. The next section sets up the experiment laboratory: a workhorse model, the adaptive learning mechanism, and the source of expectations heterogeneity. In the third section, the dynamics of the model environment are studied, and basic observations are summarized. Section four provides economic intuition for the results. The last part concludes with a general discussion of the results and the lesson for monetary policy.

2 The model

The New Keynesian business cycle model is used as an approximation of the economy. As an extension to the standard model, the assumption that monetary policy is perfectly credible is relaxed. As a result, private firms and households – as one economic group – form different expectations relative to the central bank. All agents use an adaptive (econometric) learning mechanism to learn about the actual structure of the economy, and they may disagree in their views. The only source of expectations heterogeneity in my set-up is that the private agents and the central bank give different weights to past forecasting errors. They hold different opinions about how much of the innovation is due to the fundamental error in their forecasting model – in other words, about how much they should update the model structure – and how much of it is due to an unanticipated shock that hits the economy.

The basic model is standard (eg, see Walsh 2003 (ch. 5.4) or Honkapohja and Mitra 2005). The aggregate dynamics are given by the IS curve (1), which is the representative household's Euler equation, linearised around a flexible price equilibrium; and the Phillips curve (2), which is derived from firms' pricing rules. In a perfect-knowledge environment, the model is

$$x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + g_t, \quad (1)$$
$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t, \quad (2)$$

where x_t is the output gap, defined as the deviation of actual output from the output arising in a flexible price environment; π_t is the inflation rate; and i_t is the interest rate set by the central bank. g_t and u_t are demand and cost-push shocks, respectively, assumed to follow AR(1) processes. β, σ, and λ, are the parameters for household's rate of time preference, risk aversion, and for the elasticity of inflation to the output gap, respectively.

The nominal side of the economy is anchored at a zero inflation target rate by a discretionary, expectations-based policy rule:

$$i_t = \theta_0 + \theta_\pi E_t \pi_{t+1} + \theta_x E_t x_{t+1}. \quad (3)$$

i_t is the nominal interest rate set by the central bank. θ_0 collects constant terms like the equilibrium real interest rate, and inflation target; and θ_π and θ_x are the policy weights on inflation and the output gap, respectively. Their optimal values are derived in Appendix A. Here, the central bank *cannot* observe the shocks $\{g_t, u_t\}$ when making policy decisions.[2] The central bank minimises the following

[2] Evans and Honkapohja (2003b) derive an expectations-based rule where a central bank observes shocks in the current period.

quadratic loss function

$$\min_{\{x_t,\pi_t\}} \qquad V = \frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\alpha x_{t+i}^2 + (1-\alpha)(\pi_{t+i}-\pi^*)^2\right]\right\}, \qquad (4)$$

where $\alpha \in (0,1)$ is the CB's preference parameter. If $\alpha < 0.5$, the central bank prefers inflation stabilisation. If $\alpha > 0.5$, the central bank prefers output stabilisation.

All expectations operators $E_t(.) = E_t(.|\Omega_t)$ are applied in an environment of perfect knowledge and with rational expectations. Ω_t is the perfect-knowledge information set:

$$\Omega_t = \{\beta, \lambda, \sigma, \theta_0, \theta_\pi, \theta_x, g_t, u_t, g_{t-1}, u_{t-1}, ...\}.$$

Definition 1 *Economic agents have perfect knowledge if an information set Ω_t is available at time t, where*

$$\Omega_t = \{\beta, \lambda, \sigma, \theta_0, \theta_\pi, \theta_x, u_t, g_t, u_{t-1}, g_{t-1}, ...\}.$$

The information set contains the true values of the structural parameters, and the current and past exogenous shocks u and g.

Definition 2 *Economic agents have imperfect, homogeneous knowledge if all agents share the same, imperfect information set $\hat{\Omega}_t$ at time t, where*

$$\hat{\Omega}_t = \{\hat{\Theta}_t, \kappa_t, u_t, g_t, u_{t-1}, g_{t-1}, ...\}.$$

$\hat{\Theta}_t$ is the imperfect, time varying belief about the true structural parameters $\{\beta, \lambda, \sigma, \theta_0, \theta_\pi, \theta_x\}$, and κ_t represents the information gain.

Definition 3 *There are two groups of agents: (P) private agents, and (CB) the central bank. The private agents and central bank have imperfect, heterogeneous knowledge if the information available to each group differs, and if it is not perfect, $\hat{\Omega}_t^P \neq \hat{\Omega}_t^{CB}$.*

The workhorse model The key assumption of this paper is that the perfect information set Ω_t is not available to agents. Agents have imperfect and heterogeneous knowledge, which leads to a heterogeneous expectations formation. The workhorse model takes the form

$$x_t = \hat{E}_t^P x_{t+1} - \sigma \left(i_t - \hat{E}_t^P \pi_{t+1} \right) + g_t, \quad (5)$$
$$\pi_t = \beta \hat{E}_t^P \pi_{t+1} + \lambda x_t + u_t, \quad (6)$$
$$i_t = \theta_0 + \theta_\pi \hat{E}_t^{CB} \pi_{t+1} + \theta_x \hat{E}_t^{CB} x_{t+1}, \quad (7)$$

where $\hat{E}_t^P(.) = E_t(.|\hat{\Omega}_t^P)$ are the subjective, imperfect-knowledge expectations of private agents, and $\hat{E}_t^{CB}(.) = E_t(.|\hat{\Omega}_t^{CB})$ are the subjective, imperfect knowledge expectations of the central bank. The individual imperfect information sets $\hat{\Omega}_t^P$ and $\hat{\Omega}_t^{CB}$ are subsets of the perfect knowledge set, $\{\hat{\Omega}_t^P, \hat{\Omega}_t^{CB}\} \subset \Omega_t$.

Agents in this set-up are naive. To deviate from the homogeneous expectations case (the pooling-information assumption), and at the same time to avoid the problems of infinite-order expectations due to “forecasting others’ forecasts", as raised by Towsend (1983), I suppose that agents believe that everyone shares their own expectations, and that they do not learn from experiences other than their own. Though this may not seem to be a very realistic assumption, it is useful. It sets bounds for the results that one might expect for the convex combinations of two extreme assumptions – this one, and the imperfect homogeneous knowledge assumption.

Adaptive learning mechanism Expectations heterogeneity is driven by an adaptive learning technology. The learning mechanism described below reflects the assumption about the agents’ knowledge. Honkapohja and Mitra (2005) show that the move from the perfect knowledge model to the imperfect and heterogeneous knowledge model is possible under Euler-equation learning. If all agents are learning (using recursive least squares, and the E-stability conditions hold), the originally heterogeneous knowledge information sets $\hat{\Omega}_t^P$ and $\hat{\Omega}_t^{CB}$ are enriched over time so that they converge to the perfect knowledge set Ω_t. This convergence happens despite the very restrictive assumptions that agents believe that everyone shares their expectations and that they only trust their own experiences.

The adaptive learning methodology relies on agents learning about a reduced form model. Substituting (7) into (5) and solving for rational expectations, the minimum-state representation of the structural model (5)-(7) is

$$Y_t = a + b s_t.$$

The minimum-state variable representation is also sometimes called as the actual law of motion. Y_t is the vector of endogenous variables, $(x_t, \pi_t)'$, s_t is the vector of exogenous shocks $(g_t, u_t)'$, and a and b are the matrices collecting structural parameters. Their derivation is in Appendix B.

The (P) private agents' and (CB) central bank's *perceived law of motion* (PLM) for the economy (5)-(7) is assumed to be

$$\hat{Y}_t = \hat{a}_t^i + \hat{b}_t^i s_t,$$

where $\{\hat{a}_t^n, \hat{b}_t^n\} \in \Omega_t^n$, and $n = \{P, CB\}$ are the time-varying matrices of the model primitives, representing beliefs about the true structure $\{a, b\}$. Implicitly, in this framework, agents have perfect knowledge about the structure of the economy, but they have imperfect knowledge about the true values of some of the structural parameters. Consequently, private agents and the central bank both learn about the structural matrices $\{a, b\}$ over time. The learning behaviour takes the form of econometric learning (recursive least squares). In the adaptive learning literature, it is believed that such a mechanism resembles the actual behaviour of agents very closely (see Evans and Honkapohja (2001)). The recursive algorithm is

$$\xi_t^i = \xi_{t-1}^i + \kappa_t^i (R_t^i)^{-1} X_t (Y_t - X_t' \xi_{t-1}^i), \quad (8)$$

$$R_t^i = R_{t-1}^i + \kappa_t^i (X_t X_t' - R_{t-1}^i). \quad (9)$$

where $n = \{P, CB\}$, $\xi_t^n = [\hat{a}_{11}^n, \hat{a}_{21}^n, \hat{b}_{11}^n, \hat{b}_{12}^n, \hat{b}_{21}^n, \hat{b}_{22}^n]'$ is the vector of individual PLM parameters. X_t is the matrix of appropriately stacked exogenous shocks s_t, and κ_t^n is the information gain. I also call this gain the willingness to learn, or the sensitivity to new information, this is the only source of heterogeneity as defined in definitions 2 and 3. R_t^n is the information matrix available at time t to a group n.

In Appendix C, I state the conditions that have to be met in order the REE to be determined and learnable (E-stabile) under the learning algorithm (8) and (9).

3 Model dynamics

This section analyses the dynamics of the model. The goal is to assess the implications that expectations heterogeneity has in a forward-looking monetary policy regime for short-run economic fluctuations. I focus on two questions. The first question is: what is the contribution of expectations heterogeneity to inflation and output volatility? The benchmark is the standard, rational expectations model with optimised monetary policy. The second question is: how can a central bank minimise the fluctuations in heterogeneous expectations environments?

To address both questions, I perform an intervention analysis. I expose the model economy to a one-period unitary cost-push shock, a one-period unitary demand shock, and to a combination of the two. The REE serves as a benchmark for the model's dynamics. There are no monetary policy shocks. I use the central bank's expected loss (4) to summarize the results. I report (i) the half life of the shock to the central bank's expected loss (the half-life is the time it takes for the amplitude of the shock to decay to less than half the size of the initial shock), and (ii) the amplitude of the deviation of the response to that under rational expectations dynamics (that is, the maximum deviation of imperfect knowledge dynamics from REE dynamics). If the amplitude is positive, the adaptive learning (AL) economy is more responsive to the shock than the economy under rational expectations (RE); if the amplitude is negative, the AL economy is less responsive to the shock than the RE economy.

Model calibration I adopt the Clarida, Gali, and Gertler (2000) calibration of the model. The calibrated values are: $\sigma = 1$, $\beta = 0.99$, and $\lambda = 0.3$. Optimal weights are derived (for the policy rule equation (3) in Appendix A. I assume that a central bank puts $1/3$ weight on output stabilization, and $2/3$ weight on inflation stabilisation yielding $(\theta_\pi^*, \theta_x^*) = (1.5, 1)$. For comparison purposes, I also use two sets of non-optimal policy weights: $(\theta_\pi, \theta_x) = (1.3, 1), (2.5, 1)$. In all the simulations, I assume an econometric learning algorithm, which means that whenever a new piece of information (an observation) arrives, the agents re-estimate their forecasting models. The recursive econometric learning is represented by (8) and (9) with $\kappa_t^n = c_n(t-15)^{-1}$, where t denotes time, and $n = \{CB, P\}$; c_n is a positive constant and represents a bias in the information gain. If $c_n = 1$, κ_t^n is the recursive least squares technique. If $c_n > 1$, there is a greater willingness to update than under standard econometric learning. However, $\kappa_t^n \to 0$ as $t \to \infty$, thus the effect of $c_n \neq 1$ matters only initially. Next, I calibrate the autocorrelation in demand and cost-push shocks to be 0.2. The reason for such a small number is that high persistence in the output gap and inflation is delivered by adaptive learning (see for instance Milani (2007)). The value is set to replicate the empirical volatility of inflation and output. All the simulations are initialized from steady state values: $\xi_0^n = [a_{11}^n, a_{21}^n, b_{11}^n, b_{12}^n, b_{21}^n, b_{22}^n]'$, for $n = \{P, CB\}$. R_0^n is an identity matrix.[3]

[3] The Matlab code to replicate the simulation results can be obtained from the author upon request.

Experiment description First, the reference results start with the case where knowledge is imperfect but homogeneous. Both the private agents and the central bank have the same sensitivity to new information/innovations, $\kappa_t^P = \kappa_t^{CB} = \kappa_t$. Then, the same technology is used to study the heterogenous expectations case. To determine what contribution expectations heterogeneity makes to inflation and output volatility, the two groups of agents are assumed to have different sensitivity to new information, $\kappa_t^P \neq \kappa_t^{CB}$. The focus is on the instances in which (i) private agents are more sensitive than the central bank, and (ii) private agents are less sensitive then the central bank. To explore how a central bank can minimise the fluctuations under setups (i) and (ii), I compare the effects of monetary policies which are (a) optimal in the rational expectations (RE) environment, (b) more, and (c) less responsive to inflation than under the optimal (RE) setting.

3.1 The central bank's expected loss in the homogenous case

Table 1 and 2 give a representative set of results.[4] The elements in bold show the results for the homogeneous knowledge case. The remaining results illustrate the outcomes for heterogenous knowledge, and are discussed in the next subsection. Table 3 summarizes the impulse responses for the perfect knowledge case (i.e., with RE-consistent dynamics).

The contribution of adaptive learning to volatility Looking at the results for the case of homogeneous knowledge, we can clearly see that adaptive learning increases overall economic volatility. In all cases considered, $c_P = c_{CB} \in \{0.8, 1, 1.2\}$, the amplitude (table 1), and the half life (table 2) is a positive number, which means that the impulse response of the CB's expected loss is bigger than under the RE dynamics for all t. For example, if a demand shock hits the economy, $u_0 = 1$, and $c_P = c_{CB} = 0.8$ and $\theta_\pi = 1.3$, the central bank's expected loss is higher by 0.192 basis points. The total loss amplitude is then 1.172, the sum of the RE response, 0.98, and the contribution of adaptive learning, 0.192. The half-life of the shock exceeds the RE case by more than 1000 periods (1000+). If $\theta_\pi = 2.5$, then the contribution to the response amplitude is 0.136, and the total amplitude is 1.116. The half-life is only 488 periods longer than in the RE case (table 3), where it takes the economy only 7 periods to converge to its steady state.

[4] A full grid search was performed for all possible combinations of policy parameters ($\theta_\pi \in (1,5)$ and $\theta_x = 1$, and the information gain parameters $\{c_p, c_{CB}\} \in (0.8, 1.2)$. Due to their complexity, I only present a representative set. The full set of results can be obtained upon request.

Table 1
The amplitude of the central bank's expected loss to demand and cost-push shocks relative to the REE

			θ_π								
			1.3			1.5			2.5		
c_P	Shock		c_{CB}								
	g_0	u_0	0.8	1	1.2	0.8	1	1.2	0.8	1	1.2
0.8	1	0	**0.055**	0.050	0.047	**0.043**	0.039	0.037	**0.031**	0.028	0.018
	0	1	0.192	0.188	0.184	0.162	0.159	0.157	0.136	0.133	0.130
	1	1	0.232	0.221	0.214	0.194	0.188	0.183	0.187	0.182	0.179
1	1	0	0.077	0.066	0.062	0.056	0.050	0.046	0.040	0.035	0.032
	0	1	0.242	**0.235**	0.230	0.200	**0.196**	0.193	0.166	**0.162**	0.159
	1	1	0.307	0.281	0.270	0.244	0.235	0.229	0.230	0.224	0.219
1.2	1	0	0.109	0.084	0.077	0.073	0.062	0.057	0.049	0.042	0.037
	0	1	0.295	0.284	0.277	0.239	0.234	0.229	0.197	0.192	0.187
	1	1	0.407	0.344	**0.328**	0.339	0.311	**0.297**	0.272	0.265	**0.258**

Note: For shocks “0" means no shock, “1" is a unitary shock.

Table 2
The half life of the central bank's loss function response to demand and cost-push shocks, relative to the REE

			θ_π								
			1.3			1.5			2.5		
c_P	Shock		c_{CB}								
	g_0	u_0	0.8	1	1.2	0.8	1	1.2	0.8	1	1.2
0.8	1	0	1000+	990	981	993	971	970	488	742	720
	0	1	1000+	994	993	1000+	1000+	1000+	222	225	233
	1	1	997	996	995	1000+	1000+	1000+	153	149	151
1	1	0	1000+	995	993	996	993	977	227	285	389
	0	1	1000+	995	994	859	816	812	140	137	139
	1	1	1000+	997	993	1000+	821	867	102	96	95
1.2	1	0	607	995	1000+	430	644	925	153	151	192
	0	1	1000+	1000+	1000+	504	462	454	102	97	97
	1	1	1000+	1000+	1000+	993	996	1000+	78	71	69

Note: For shocks “0" means no shock, “1" is a unitary shock.

We can also see that with higher sensitivity to new information, $c_i > 0.8$, the half-

Table 3
A summary of impulse responses for the rational expectations

Shock		Periods to	Response amplitude		
g_0	u_0	convergence	θ_π =1.3	θ_π =1.5	θ_π =2.5
1	0	7	0.98	1.02	1.29
0	1	7	0.72	0.52	0.43
1	1	7	1.49	1.52	1.58

life shortens. Examining the last three columns of table 2 for the demand shock again, we see that the half life drops from 222 periods, through 137, to only 97 periods, as $c_P = c_{CB}$ increases from 0.8 to 1.2. The same results hold for a cost-push shock, and for cost-push and demand shocks occurring jointly, $g_0 = u_0 = 1$. (Note, the decline in half-life is not monotonic when $\theta_\pi = 1.3$.)

Remarkably, efficient policy under rational expectations does not perform very well in an imperfect knowledge case. It is a finding similar to that of Orphanides and Williams (2003). From Tables 1 and 2 it follows that optimal (RE) policy is outperformed by the more inflation-responsive policy rule.[5]

The effects of monetary policy The key result for the case of homogeneous knowledge is that monetary policy can effectively influence both economic variability and the speed of learning (the convergence to the REE). The numbers in tables 1 and 2 demonstrate that increasing the inflation responsiveness from $\theta_\pi = 1.3$ to 2.5 lowers the deviation from RE dynamics. The shock response amplitude decreases in all three cases. Also the speed of learning improves significantly. Its relation to the policy reactiveness is highly non-linear. All these results confirm the findings made by Orphanides and Williams (2002), and Ferrero (2004). Orphanides and Williams (2003, p.26) write, “Policy should respond more aggressively to inflation under imperfect knowledge than under perfect knowledge ... in order to anchor inflation expectations and foster macroeconomic stability".

The results for the imperfect, homogeneous knowledge case can be summarised in two points:

[5] "...policies that would be efficient under rational expectations can perform poorly when knowledge is imperfect", *from* Orphanides and Williams (2003,p.26).

- overall volatility increases with higher sensitivity to new information, but the increase in volatility is offset by faster learning;
- if monetary policy reacts aggressively to inflation, the central bank's expected loss decreases and the speed of learning increases.

3.2 The central banks's expected loss in the heterogeneous case

The results under heterogenous expectations differ dramatically from the benchmark case. The summary of the central bank's expected loss characteristics is again in tables 1 and 2. The heterogeneous information cases are not in bold. There are two dimensions to the results: the effect of different sensitivities to new information ($\{c_P, c_{CB}\} \in \{0.8, 1, 1.2\} \times \{0.8, 1, 1.2\} : c_P \neq c_{CB}$), and the policy inflation reactiveness ($\theta_\pi = \{1.3, 2.5\}$). We can read the following story from tables 1 and 2:

The effect of expectations heterogeneity

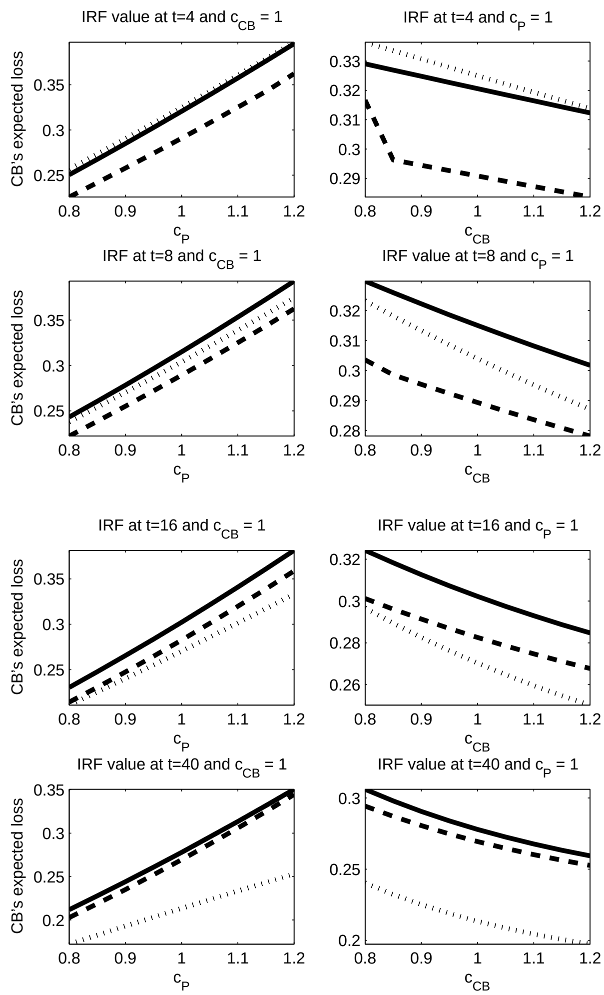
- As the private sector becomes more sensitive to new information, economic variability increases. Fixing c_{CB}, the impulse amplitude of CB's expected loss increases with c_P.
- As the central bank becomes more sensitive to new information, economic variability decreases. Fixing c_p and varying c_{CB}, the amplitude falls.
- As the central bank and the private sector both become more sensitive to new information (c_i increases), and as monetary policy is less responsive to inflation ($\theta_\pi = 1.3$), volatility falls.
- As the private sector becomes more sensitive to new information, the speed of convergence increases.
- As the central bank becomes more sensitive to new information, and at the same time the policy is excessively responsive, the speed of convergence decreases. If policy is less responsive than the optimal RE policy, the results are inconclusive.

The effect of monetary policy

- As policy becomes more responsive to inflation, economic volatility decreases.

- The relative reduction in volatility depends on the degree of expectations heterogeneity (the ratio of c_P to c_{CB}).
- As policy becomes more responsive to inflation, the speed of convergence significantly increases.

Figure 1
Impulse response of the central banks' expected loss



Note: The *dashed line* is the response when $\theta_\pi = 1.3$, the *continuous line* is for $\theta_\pi^* = 1.5$, and the *dotted line* is for $\theta_\pi = 2.5$. The economy is subject to a unitary cost-push and demand shock ($u_0 = g_0 = 1$), and the impulse response function is evaluated in 3 dimensions: time, and information gain bias c_p and c_{CB}. The graphs above show the impulse response value at times $t = \{4, 8, 16, 40\}$.

The aggregated results indicate that, in the short run, monetary policy has inverse implications than in the long run. In figure 1, I plot the impulse response function of the central bank's expected loss for policies that differ in their reactiveness to inflation. The impulse responses are presented in a static form, for the horizons $t = \{4, 8, 16, 40\}$, and are plotted for different information gain biases, $\{c_P, c_{CB}\}$. Periods 4 to 8 represent the short run. The medium run and long run are represented by periods 16 and 40, respectively.

Figure 1 reveals that, in the short run, monetary policy that is less responsive to inflation ($\theta_\pi < 1.5$), delivers the lowest expected loss for almost all combinations of $\{c_P, c_{CB}\}$ – the dashed line lies at the bottom at $t = 8$. However, this is not a long-lasting phenomenon. In the medium run, policy that is more responsive to inflation ($\theta_\pi = 2.5$) results in lower expected loss. At $t = 16$, it depends on the degree of bias in the information gain, but the dotted line shows that the policy that is more responsive to inflation performs the best in most cases. In the long run, for $t \geq 40$, the policy that is responsive to inflation shocks is dominant, and is always associated with the lowest expected losses.

Similarly, as noted for the case of homogeneous expectations case, optimal (RE) monetary policy does not perform very well under heterogeneous expectations. In Figure 1, the continuous line, where $\theta_\pi^* = 1.5$, is not connected with the lowest expected loss. Even though it can be the second best option in some cases, it is always dominated by policy that responds more or less than the optimal RE policy.

Robustness The robustness of results is checked by changing the relative sizes of the shocks. The basic results remain mostly unchanged. As the variance of the demand shock g_t gets bigger in relative terms, there is a polarization of the policy effect at the short and long horizon. At $t = 20$, policy that is relatively more responsive to inflation clearly dominates. On the other hand, as the variance of cost-push shock u_t gets bigger, in relative terms – approximately twice bigger – the picture slightly changes. At the short horizon, there is already a region of expectations heterogeneity in which inflation-responsive policy is preferred, and over time it dominates.

4 Some intuition behind the results

One of the characteristics of adaptive learning is that, eventually, the boundedly rational equilibrium path converges to the REE (Evans and Honkapohja 2003a). Even though the private agents and central bank are assumed not to communicate with one another, they can each individually attain homogeneous and perfect knowledge. Over time, both groups, end up with the same forecasting model and the same expectations as a result of their own forecasting errors. This is why we observe from figure 1 that policy that is relatively more responsive to inflation starts to dominate after 16 quarters. This is because expectations become homogeneous, and the economic environment evolves towards the RE equilibrium. An important difference is that this does not hold early on, and excessively responsive policy can actually considerably destabilise the economy. This observation leads to the conclusion that when expectations are heterogeneous, monetary policy should not be "too active" in order to improve stability in the short-term.

To understand the results it is helpful if the model (5)-(7) is rewritten in a more suitable form so that we can see the effects of expectations heterogeneity more easily. For simplicity, I also assume that $\hat{E}_t^P x_{t+1} - \hat{E}_t^{CB} x_{t+1} = 0$. Then the workhorse model can be written as

$$
\begin{aligned}
x_t &= -\sigma\theta_0 + \sigma\theta_\pi(\theta_\pi^{-1}\hat{E}_t^P\pi_{t+1} - \hat{E}_t^{CB}\pi_{t+1}) + g_t, \\
\pi_t &= \lambda\sigma\theta_0 + \lambda\sigma\theta_\pi\left[\frac{\lambda\sigma+\beta}{\lambda\beta\theta_\pi}\hat{E}_t^P\pi_{t+1} - \hat{E}_t^{CB}\pi_{t+1}\right] + u_t + \lambda g_t.
\end{aligned}
$$

Demand shock A demand shock first hits the output gap, and then affects inflation. Beginning in the REE, agents expect equilibrium values of inflation and the output gap. In this RE and persistence-free environment, the shock has just a one period impact. Under adaptive learning it influences expectations in subsequent periods. Being surprised, agents update their forecasting models. The policy rate is set so that it neutralises the shock. A positive demand shock will cause an upward correction in the parameters for the perceived law of motion, which will yield higher predictions of inflation and the output gap for future periods. The policy rate reacts to those values. Because increasing c_{CB} causes higher expected values for inflation and output gap, monetary policy is suddenly more restrictive – the policy rate increases. This is why the CB's expected loss declines as c_{CB} increases.

Using the same logic, we can interpret the effect of increasing private sector sensitivity to new information. A demand shock's future effects are transmitted via expectations. Private agents update their model in a similar way to the central bank. Their expectations, however, influence the economic dynamics directly. A positive shock motivates model updates, yielding higher inflation and output gap forecasts in the future. Higher output gap expectations imply a higher current output gap, and consequently higher inflation. Higher inflation expectations have a direct effect on inflation, which increases, and an indirect effect on the output gap via a decrease in the real interest rate, which influences the output gap positively.

Cost-push shock Assuming no persistence, a cost-push shock has an immediate impact on contemporaneous inflation and expectations, via which it is transmitted further. In the next period, since no other shock occurs, inflation should return to the REE. But because private agents and the central bank update their model by biasing their expectations upward, the inflation rate and the output gap increase above the RE values. The mechanism of monetary policy is the same as in the case of previous shock. Policy that responds aggressively to inflation, pushes inflation down to the REE, the output gap decreases further, and becomes more responsive.

The central bank's sensitivity to new information decreases the inflation response to a cost-push shock, but increases the responsiveness of the output gap. Again, monetary policy becomes more restrictive than under the RE, since the central bank predicts higher inflation due to the model updates, increases interest rates, which closes the output gap, and the inflation rate returns to the RE dynamics. Thus by changing c_{CB}, we can explain the decrease in the responsiveness of inflation, accompanied by the increase in the responsiveness of the output gap.

The private sector's sensitivity to new information helps the cost shock to propagate to inflation. As private agents become more sensitive to innovations, they anticipate higher inflation than under full knowledge, and thus increase the actual inflation rate. With higher values of c_P, agents update their models more, and produce higher forecasts of inflation. This immediately increases inflation due to higher expected inflation in the future. Agents also update their forecasts of the output gap. They anticipate the reaction of the central bank, which they assume has similar expectations, leading to a policy rate adjustment. Since c_P will bias a policy reaction upwards, private agents will assume a lower output gap than under RE. This explains why the output gap becomes more reactive if the private sector is more sensitive to new information. This phenomenon is particularly observable when the central bank is excessively responsive to inflation.

5 Concluding discussion

The world is simpler if knowledge and beliefs are homogeneous. If knowledge is homogeneous, a central bank's aversion to price inflation helps to decrease inflation variability and speeds up learning. The speed of learning affects the persistence of inflation and its variability. If a central bank wishes to minimize its expected loss, it is desirable that agents learn about the economy's actual law of motion as quickly as possible. If knowledge and beliefs are heterogeneous, the central bank should not be excessively anti-inflationary, because if the bank is less responsive to inflation, short-run economic stability improves. Thus, in a heterogeneous expectations world, the first goal of the central bank should be to make expectations homogeneous across the economy, in order to minimise (inflation) target and output volatility. Once expectations have become homogeneous, the standard policy recommendations apply.

How can a central bank make expectations homogeneous in the short run? The expectations homogeneity is closely related to enhancing policy effectiveness. In this simple model, there are two ways this may work. The central bank either learns and adopts private agents' expectations, or alternatively, private agents get to know and acquire the central bank's expectations. (And of course the two processes could be combined, with both sets of expectations converging on each other). In practice, neither is simple. The first will require reliable measures of private sector expectations. Central banks usually have surveys of private sector expectations on future economic developments. But the information that such surveys yield might be unreliable. The data collected may not truly represent market expectations, which drive agents' market behaviour – they could be subject to systematic measurement errors (perhaps due to inaccuracies or collusive, game-playing responses). These considerations suggest that it might be better for private agents to adopt central bank expectations, than the reverse. But how can this be done? And can it be relied upon? Central bank communications, through publications, speeches, and press conferences, clearly provide a crucial educational function. But when credibility is absent, the logic of this paper is that the central bank really must furnish evidence of its commitment and capability to turn its expectations into reality.

A major challenge for future research is obtaining an analytical solution to the problem addressed in this paper. Even with a simple model, heterogeneous expectations and adaptive learning create a modeling environment that is not readily analytically tractable. Model transition functions are highly non-linear, which complicates and limits a comparative statics analysis. Analytical evaluation of the speed of learning, as in Ferrero (2007), also seems complex and so, at present, numerical analysis similar to that presented in this paper seems to be the most viable approach.

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Appendix A

Optimal Expectations-Based Policy Rule

The central bank minimises a quadratic loss function

$$\min_{\{x_t,\pi_t\}} \qquad V = \frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\alpha x_{t+i}^2 + (1-\alpha)(\pi_{t+i}-\pi^*)^2\right]\right\}$$

subject to

$$\begin{aligned} x_t &= \hat{E}_t^{CB}x_{t+1} - \sigma\left(i_t - \hat{E}_t^{CB}\pi_{t+1}\right) \\ \pi_t &= \lambda x_t + \beta\hat{E}_t^{CB}\pi_{t+1}. \end{aligned}$$

Note that the central bank assumes that private sector agents trust the bank's expectations and adopts them for their own decisions. The central bank *a priori* assumes that monetary policy is credible. Further, we assume the bank does not observe current period exogenous shocks u_t and v_t.

The first order condition to the problem is

$$\alpha x_t + \alpha(1-\alpha)(\pi_t - \pi^*) = 0.$$

Using the FOC, the Phillips curve and IS curve to solve for i_t, we obtain the optimal policy rule under discretion. When we assume that the inflation target π^* is zero, then the expectations-based policy rule takes the form

$$i_t = \theta_0 + \theta_\pi\hat{E}_t^{CB}\pi_{t+1} + \theta_x\hat{E}_t^{CB}x_{t+1},$$

where $\theta_\pi = 1 + \frac{(1-\alpha)\lambda\beta}{\lambda^2(1-\alpha)+\alpha}$, and $\theta_x = \frac{1}{\sigma}$, $\theta_0 = 0$.

Appendix B

MSV representation

Using the method of undetermined coefficients, we derive the exact form of the minimum state variable (MSV) representation for the model considered in the text. Starting with the reduced form and assuming rational expectations, i.e., $\hat{E}_t^P(.) = \hat{E}_t^{CB}(.) = E_t(.)$, we get

$$Y_t = M_0 + (M_1 + M_2)E_t Y_{t+1} + P\varepsilon_t, \tag{10}$$

where

$$\varepsilon_t = F\varepsilon_{t-1} + \eta_t,$$

and η_t is N(0,σ_η^2). Now assume the MSV form takes the form

$$Y_t = a + b\varepsilon_t. \tag{11}$$

Taking the appropriate expectations needed in (10) one obtains

$$E_t Y_{t+1} = a + bF\varepsilon_t, \tag{12}$$

Plugging these expectations back into (10) yields

$$Y_t = M_0 + (M_1 + M_2)a + [(M_1 + M_2)bF + P]\varepsilon_t. \tag{13}$$

Using the method of undetermined coefficients, it follows that the MSV solution must satisfy

$$\begin{aligned} M_0 + (M_1 + M_2)a &= a, \\ (M_1 + M_2)bF + P &= b. \end{aligned}$$

Solving for the matrices a, and b we get

$$\begin{aligned} a &= (I - M_1 - M_2)^{-1} M_0, \\ vec(b) &= [\mathbf{I} - F' \otimes (M_1 + M_2)]^{-1} vec(P). \end{aligned} \tag{14}$$

Appendix C

Determinacy and E-stability

To analyse the conditions under which the incomplete knowledge model (5)-(9) converges to the true model REE form, the methodology developed by Evans and Honkapohja (2001) is employed. In principle, the methodology consists of two parts. First, the rational expectation equilibrium of the model is examined. I look for conditions under which the REE is *determined*. In the adaptive-learning terminology, the REE is said to be determined if it is found to be unique. Second, I check for the learnability of the REE. The question is, if economic agents have incomplete knowledge, can they learn the REE? The conditions that guarantee the REE is attainable under the adaptive learning mechanism are called the *E-stability conditions*.[6]

REE Determinacy

To examine the rational expectation equilibrium of the model (5)-(7), we begin by rewriting the model in a matrix *reduced form*

$$\hat{Y}_t = M_0 + M_1 \hat{E}_t^P \hat{Y}_{t+1} + M_2 \hat{E}_t^{CB} \hat{Y}_{t+1} + Ps_t, \tag{15}$$

where $\hat{Y}_t = [x_t, \pi_t]$, $s_t = [g_t, u_t]$, M_0 is an intercept vector. Because of a zero inflation target, all intercepts are zero. For that reason I omit M_0 in further derivations.

$$M_1 = \begin{bmatrix} 1 & \sigma \\ \lambda & \beta + \lambda\sigma \end{bmatrix}, M_2 = \begin{bmatrix} -\sigma\theta_x & -\phi\theta_\pi \\ -\lambda\sigma\theta_x & -\lambda\sigma\theta_\pi \end{bmatrix}, P = \begin{bmatrix} 1 & 0 \\ 1 & \lambda \end{bmatrix}.$$

To analyse the REE determinacy, we will assume for now a complete knowledge environment, $\hat{E}_t^P(.) = \hat{E}_t^{CB}(.) = E_t(.)$. Then rearranging the reduced form one obtains

$$\tilde{Y}_t = ME_t\tilde{Y}_{t+1} + Ps_t, \tag{16}$$

where $M = M_1 + M_2$.

[6] For details on the methodology, I refer to Evans and Honkapohja (2001) and Evans and Honkapohja (2003a), where adaptive learning in a homogeneous environment is explained, and to Honkapohja and Mitra (2005) for an extension to heterogeneous learning.

Proposition 1 *The model (5)-(7) has a unique and stable rational expectations equilibrium if the eigenvalues of matrix M in (16) have real parts lest than one.*

Proof Standard outcome of the difference equation theory.

E-Stability

The second issue is to analyse the conditions under which the REE is learnable. We already know when the REE exists and is unique. We are now interested in whether, having incomplete knowledge, we can learn such a REE eventually. If the REE is determined, the model has the *minimum state variable* (MSV) representation

$$Y_t = a + bs_t. \tag{17}$$

a, and b are the (3x1) and (3x3) matrices of the model primitives. Their exact form is derived in Appendix B.

We recall that the *perceived law of motion* (PLM) is

$$\hat{Y}_t = \hat{a}_t^i + \hat{b}_t^i s_t. \tag{18}$$

$i = \{P, CB\}$. The subscript t on the matrices indicates the time dependence of the matrices as the agents learn using (8) and (9). s_t follows an AR(1) process, $s_t = F s_{t-1} + e_t$, where e_t is white noise. The private agents and central bank use their PLMs to form expectations

$$\hat{E}_t^i \hat{Y}_{t+1} = \hat{a}_t^i + \hat{b}_t^i F s_t. \tag{19}$$

Substituting (19) back into the reduced form (17), one obtains the economy's *actual law of motion* (ALM)

$$Y_t = \left(M_1 \hat{a}_t^P + M_2 \hat{a}_t^{CB}\right) + \left(P + M_1 \hat{b}_t^P F + M_2 \hat{b}_t^{CB} F\right) s_t. \tag{20}$$

The mapping from PLM to ALM is formalized to

$$T[a,b] = [M_1 \hat{a}_t^P + M_2 \hat{a}_t^{CB}, P + M_1 \hat{b}_t^P F + M_2 \hat{b}_t^{CB} F] \tag{21}$$

where $T : R^2 \to R$ is a map between perceived parameters and their true (equilibrium) values.

We are interested in its fixed point. Honkapohja and Evans (2002) show that E-stability is achieved if the steady state in the following differential equation is locally stable

$$\frac{d}{d\tau}(a,b) = T[a,b] - (a,b). \tag{22}$$

Furthermore, Honkapohja and Mitra (2005) and Evans and Honkapohja (2003a) show that the map under heterogeneous and homogeneous expectations is equivalent. Using their result I rewrite (21) by equating $\hat{j}_t^P = \hat{j}_t^{CB} = \hat{j}_t$ for $j = \{a,b\}$. Hence, (21) becomes

$$T[a,b] = \left[(M_1+M_2)\hat{a}_t, P + (M_1+M_2)\hat{b}_t F\right], \tag{23}$$

and can be easily assessed.

Proposition 2 *The REE of the model (5)-(9) is E-stable under heterogeneous expectations if and only if the corresponding model with homogeneous expectations is E-stable. Hence the real parts of the eigenvalues of*

$$\begin{aligned} DT_a(a) &= I \otimes (M_1+M_2) \\ DT_b(b) &= F' \otimes (M_1+M_2) \end{aligned}$$

must be less than one, and $\otimes$ is the Kronecker product.[7]

Proof See Evans and Honkapohja (2003a) for the proof.

[7] Having the map from the PLMs to ALM

$$T[a,b] = \left[M_0 + (M_1+M_2)\hat{a}_t, P + (M_1+M_2)\hat{b}_t F)\right].$$

we take derivatives with respect to $\hat{a}_t$ and $\hat{b}_t$. Using the rules for the derivatives of matrices we get

$$\begin{aligned} DT_a(a) &= \frac{d}{d\hat{a}_t}\left[M_0 + (M_1+M_2)\hat{a}_t\right] = I \otimes (M_1+M_2), \\ DT_b(b) &= \frac{d}{d\hat{b}_t}\left[P + (M_1+M_2)\hat{b}_t\right] = F' \otimes (M_1+M_2). \end{aligned}$$