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Abstract

Real Business Cycle (RBC) and Dynamic Stochastic General Equilibrium (DSGE) methods have become essential components of the macroeconomist's toolkit. This literature review stresses recently developed techniques for computation and inference, providing a supplement to the Romer (2006) textbook, which stresses theoretical issues. Many computational aspects are illustrated with reference to the simple divisible labour RBC model. Code and US data to replicate the computations are provided on the Internet, together with a number of appendices providing background details.

1 Introduction

"Real business analysis now occupies a major position in the core curriculum of nearly every graduate program. At a recent National Bureau of Economic Research conference, a prominent Cambridge economist of the New Keynesian school described the RBC approach as the new orthodoxy of macroeconomics, without raising a challenge from the audience."
King and Rebelo (1999, p 930)

This review examines the Real Business Cycle (RBC) and Dynamic Stochastic General Equilibrium (DSGE) methods for analysing business cycle behaviour that have become necessary tools for graduate-trained macroeconomists. The default Master's level text, Romer (2006), provides an insightful characterisation of the RBC literature, based heavily on Campbell (1994). However, the Romer text says little about the computational side of the RBC literature, to the frustration of researchers interested in using the models for business cycle analysis.

This paper describes the computational steps commonly used by RBC practitioners. In particular, it stresses recently developed (often Bayesian) techniques for computation and inference. A number of computational issues are discussed in the context of the simple divisible labour RBC model, which is often used for pedagogical purposes. Code and US data to replicate the computations for this paper are provided on the Reserve Bank of New Zealand's website.[2] Our aim is to describe the empirical tools required by the reader of Romer (2006) to build DSGE models. The paper also provides numerous Internet links to helpful code for business cycle computation (in Table 1).

Section 2 provides a brief overview of RBC origins, including the contribution of Kydland and Prescott (1982). Section 3 presents a simple RBC model and considers solution methods. Section 4 examines how the model can be used to analyze the data by a variety of methods, including Bayesian estimation. Section 5 outlines several recent research agendas. Appendices to the discussion paper version of this paper provide details of background steps for our RBC example (see Table 2).

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[2] http://www.rbnz.govt.nz/research-and-publications/discussion-papers/supporting-files-for-dp2007-15

2 An overview of RBCs

The RBC approach is a flexible framework for quantitative business cycle analysis which owes much to the pioneering work by Kydland and Prescott (1982). Their research agenda proposed novel techniques to examine both the theory and empirics of fluctuations. Prescott (1986) provides a summary. Rebelo (2005) surveys the literature from a modern perspective, but does not focus on the computational issues that are the subject of this review.

Kydland and Prescott's theoretical framework was based on the idea that the Neoclassical growth model could be used to study business cycles, following Brock (1974). Kydland and Prescott's use of stochastic technology and rational expectations produced a model that adhered to the Lucas micro-foundations research agenda. Like Lucas (1972, 1973), agent behaviour in the RBC model was governed by the optimisation under uncertainty framework straight from the micro-economist's toolbox.[3] Elements of this approach were foreshadowed by Robertson (1915), who noted that technology disturbances "inventions" contributed to business cycles, and Frisch (1933), who studied business cycles within an optimising framework. Unfortunately, optimisation in the Neoclassical growth model yields non-linear behaviour, ruling out analytical solutions in general cases. The common approach is to linearise the model about the steady state of the system and consider an approximate solution. Researchers use computer programs such as GAUSS or MATLAB to solve and analyse these linearised systems.

The Kydland and Prescott (1982) approach to business cycle empirics became known as calibration. This involves choosing parameters on the basis of long-run data properties and judgment (sometimes guided by microeconomic evidence). Judicious parameter selection is computationally convenient given numerical solution methods.

More recently, many RBC practitioners have turned to Bayesian methods to allow for a range of parameter values and to check the empirical validity of

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[3] The highly influential Lucas (1976) critique suggested that the empirical failures of modified IS-LM models, such as Mundell (1968) and Fleming (1962), stemmed from the absence of micro-foundations. Sims (1980) showed that the ad hoc nature of identification in their empirical counterparts made the parameter estimates unreliable.

models. The Bayesian distributions are often estimated by Monte Carlo Markov chain (MCMC) simulation techniques: by choosing suitable transition functions for Markov chains it can be shown that the posterior distribution of a model's parameters coincides with the stationary distribution of the chain. Consequently, the posterior can be approximated by sampling from a suitably long realisation of the chain. (See Gelman, Carlin, Stern and Rubin (2003) and Koop (2003) for a discussion of MCMC methods for Bayesian analysis.) However, these techniques are very computationally intensive: the chains may need to be iterated for a very long time to approximate the stationary distribution. Other methods for RBC inference, such as limited information methods and classical maximum likelihood estimation with full information, are also often burdensome because they require the researcher to solve the forward-looking systems and may require numerical optimisation techniques. In contrast to the impression left by Romer's textbook, analysis with modern macroeconomic models requires computational techniques.

The Kydland-Prescott model has just one source of uncertainty (a technology disturbance). Many practitioners inferred that the early vintage RBC models were inconsistent with the sample data, and subsequent models increased considerably in complexity to cope with the criticisms that arose. By the late 1990s, the RBC literature produced models with, for example, multiple shocks, price rigidities, and monetary and fiscal policies. Since the term RBC was associated with real disturbances, the DSGE label became popular for these larger, computationally-demanding models with multiple disturbances.

It is worth mentioning that some models drawing on RBC techniques abstract from the micro-foundations initially at the heart of the RBC research agenda. RBC may indeed be "the new orthodoxy of macroeconomics" as King and Rebelo (1999) claim, but some DSGE models are more RBC than others. In particular, many of the models now incorporate the types of rigidities emphasised by the New Keynesian literature. With this point made, hereafter we simply use the term "DSGE" to refer to an RBC model with multiple shocks.

3   A simple RBC model

The computational steps for theory and inference with RBC models are discussed below in relation to an explicit simple divisible labour example,

encountered by many graduate students in studying (among others) King, Plosser and Rebelo (1988), Christiano and Eichenbaum (1992), Campbell (1994) and Romer (2006).[4] Given the functional forms that are employed in our simple model, analytical solutions are achievable for the log-linear approximation. However, with less restrictive assumptions on functional forms, computational techniques will typically be required. Since the theoretical specification −covered in detail by Campbell (1994) and Romer (2006) − will be familiar to most readers, a concise treatment will suffice.

We model a simple Walrasian economy subject to technological disturbances in discrete time. Following the literature, we make specific assumptions about the functional forms followed by preferences and constraints of the (here, but not necessarily) representative agent. These assumptions sometimes trouble students new to the RBC approach; the functional forms are unlikely to be exact. The keep-it-simple approach owes something to the need for tractability. It also reflects the desire by RBC researchers to analyse the sample data by using incremental deviations from the well-understood Neoclassical growth model.

The perfectly competitive economy contains a representative household that maximizes utility given an initial stock of capital. The household simultaneously participates in the goods, capital and labour markets. The economy also contains a representative firm, which sells output produced from capital, labour and technology.

The representative firm has a constant returns-to-scale Cobb Douglas production function:

$$Y_t = (A_t N_t)^{\alpha} K_t^{1-\alpha} \tag{1}$$

where Y_t is output in time t, A_t is technology, N_t is the number of labour hours worked, K_t is the capital stock, and $0 < \alpha < 1$. Capital accumulates according to:

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t \tag{2}$$

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[4] The Christiano and Eichenbaum (1992) model has technology and government expenditure shocks; we abstract from the latter.

where δ is the depreciation rate of the stock of capital, and C_t is consumption.

The household has log utility in consumption and power utility in leisure:

$$\sum_{i=0}^{\infty} \beta^i [\log(C_{t+i}) + \theta \frac{(1-N_{t+i})^{1-\gamma}}{1-\gamma}] \tag{3}$$

where the elasticity of intertemporal substitution of leisure is $\sigma = 1/\gamma$ and β is the discount factor. The mechanism of intertemporal labour supply is similar to that of Lucas and Rapping (1969) and labour is divisible. (Romer, 2006, p 210, discusses the indivisible labour case analysed by Hansen, 1985, and Rogerson, 1988.)

We define the gross rate of return on investment capital, R_{t+1}, to be the marginal product of capital plus undepreciated capital:

$$R_{t+1} = (1-\alpha)\left(\frac{A_{t+1}N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1-\delta). \tag{4}$$

We consider a social planner maximizing the expected utility of the representative individual by picking a path for consumption and leisure subject to the two constraints (1) and (2). Stokey and Lucas (1989) and Ljungqvist and Sargent (2004) discuss a dynamic programming approach and the Bellman's equation. A simple introduction to intertemporal optimisation is provided in supplement A of Obstfeld and Rogoff (1997). Note that the unique solution to the social planner's problem is the competitive equilibrium from the welfare theorems. Given the functional forms that have been assumed for preferences, the necessary first order conditions for the problem can be written:[5]

$$C_t^{-1} - \beta E_t[C_{t+1}^{-1}R_{t+1}] \tag{5}$$

and

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[5] We are presuming that the shock process for technology takes the form given in equation (17).

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$$\theta(1-N_t)^{-\gamma} = \frac{W_t}{C_t} - \alpha \frac{A_t^{\alpha}}{C_t}\left(\frac{K_t}{N_t}\right)^{1-\alpha} \tag{6}$$

where the marginal utility of leisure is set equal to the real wage W_t times the marginal utility of consumption. Given a competitive labour market, the real wage also equals the marginal product of labour. Note that equation (5) reflects the between-periods aspect of the problem, so that labour supply is dictated by intertemporal substitution.

Following Campbell (1994) and Romer (2006), we assume that in the (unique) steady state, technology, capital, output, and consumption all grow at the common constant rate, $G = A_{t+1} A_t^{-1}$. The gross rate of return on capital is also constant in the steady state, denoted R. (We will consider the importance of this definition of the steady state below.)

The first order condition (5) becomes:

$$G = \beta R. \tag{7}$$

In logs (denoted by lower case letters):

$$g = \log(\beta) + r. \tag{8}$$

The definition of the return to capital (4) and the first-order condition (7) imply that the steady-state technology-capital ratio is constant:

$$\frac{A}{K} = \left(\frac{G/\beta - (1-\delta)}{1-\alpha}\right)^{1/\alpha} \frac{1}{N} \approx \left(\frac{r+\delta}{1-\alpha}\right)^{1/\alpha} \frac{1}{N} \tag{9}$$

with $R \approx 1 + r$. The production function and the technology-capital ratio also imply a constant steady-state output-capital ratio:

$$\frac{Y}{K} \left(\frac{AN}{K}\right)^{\alpha} \approx \left(\frac{r+\delta}{1-\alpha}\right). \tag{10}$$

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The steady-state consumption output ratio is:

$$\frac{C}{Y} \quad \frac{C/K}{Y/K} \approx 1 - \frac{(1-\alpha)(g+\delta)}{r+\delta}. \tag{11}$$

Departures from the steady state can be modelled as a system of non-linear equations in the logs of technology, capital, output, labour and consumption. In some special cases, the model becomes linear. See, for example, McCallum (1989). Non-linear variants can be log-linearised using first-order Taylor series expansions around the steady state. The non-steady state behaviour of the resulting linear system approximates the original non-linear specification.

The first-order log-linearisation approximation can be inaccurate, especially when the economy is some distance from the steady state; see, for example, Den Haan and Marcet (1994). A number of researchers have considered higher order Taylor series approximations, including Collard and Juillard (2001) and Schmitt-Grohé and Uribe (2004a,b). Aruoba *et al* (2006) describe less restrictive but typically more computationally burdensome approaches and horse-race a variety of methods. Fernández-Villaverde and Rubio-Ramírez (2006) provide a nice outline of their research agenda on estimating and analyzing non-linear and non-normal DSGE models. (Table 1 provides a link to Rubio-Ramírez's webpage.) A particularly appealing feature of their particle filter approach is that general equilibrium models can be estimated allowing for parameter change (see the discussion in Section 5). Nevertheless, for consistency with the Romer (2006) representation of RBC modelling, in this paper we use the standard linear approximation procedure.

Appendix A (available on the Internet; see Table 2) provides details of the somewhat time-consuming log-linearisation of equations (1), (2), (4), (5), and (6) using first-order Taylor expansions around the steady state. The handy rules in Uhlig (1999) make the process less burdensome than working through Campbell (1994) suggests. This approach yields the following linear (in logs) system:

$$y_t = \alpha(a_t + n_t) + (1-\alpha)k_t \tag{12}$$

$$k_{t+1} = \lambda_1 k_t + \lambda_2(a_t + n_t) + (1 - \lambda_1 - \lambda_2)c_t \tag{13}$$

$$r_{t+1} - \lambda_3 \left(a_{t+1} + n_{t+1} \quad k_{t+1} \right) \tag{14}$$

$$E_t \Delta c_{t+1} = \sigma \lambda_3 E_t [a_{t+1} + n_{t+1} - k_{t+1}] \tag{15}$$

$$n_t = v[(1 - \alpha)(k_t - n_t) + \alpha a_t - c_t] \tag{16}$$

where:

$$\lambda_1 = \frac{1+r}{1+g}, \quad \lambda_2 = \frac{\alpha(r+\delta)}{(1-\alpha)(1+g)}, \quad \lambda_3 = \frac{\alpha(r+\delta)}{r+1}, \text{ and } v - \frac{(1-N)}{N}\sigma.$$

For simplicity, we omit all constants; the variables can be thought of as zero-mean deviations from the steady state growth path.

The log technology process follows:

$$a_t = \phi a_{t-1} + \varepsilon_t \tag{17}$$

where ϕ measures the persistence of technology shocks, $-1 < \phi < 1$, and ε_t is an idiosyncratic disturbance. For simplicity, the technology process specified in equation (17) rules out unit root behaviour that would cause the model to be non-stationary.

The 'technology shocks' label is controversial. In this highly abstract model, any disturbances that affect the supply side–other than changes in capital and labour will be lumped together under the label 'technology'. See Summers (1986) for a critique of the contribution of technology in early RBC models. King and Rebelo (1999) argue that the RBC research agenda requires resuscitation because one-shock models require implausibly large and frequent technology shocks.[6]

For consistency with the Romer (2006) treatment of RBCs, we de-trend the by defining a deterministic steady state. Many RBC researchers prefer not to

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[6] King and Rebelo (1999) point to procyclical capacity utilisation as a way of reconciling the one-shock model with the data. Long and Plosser (1983) offer an alternative way to build multi-shock models, exploiting disaggregate shocks. For simplicity we do not follow this approach in our example. As noted earlier, most DSGE researchers prefer a multi-shock specification.

linearise about a deterministic steady state; see, for example, Cogley and Nason (1995a,b). Fukač and Pagan (2008) review methods for de-trending models using a recent DSGE example.

Our system of linear difference equations (12)-(17) can be solved using the method of undetermined coefficients as described by Campbell (1994) and others. The researcher guesses the functional form of the solution and then verifies. The approach was developed in the rational expectations literature; see Blanchard and Fisher (1989, p 261-266) for a textbook treatment. Binder and Pesaran (1995), Uhlig (1999) and Sims (2002) discuss more general vector-based approaches for solving systems of linear difference equations; Blanchard and Kahn (1980) and Anderson and Moore (1985) provide the building blocks.[7] Stokey and Lucas (1989), Cooley (1995), Ljungqvist and Sargent (2004) and Canova (2007) contain useful discussions. Anderson (2006) compares the Anderson-Moore approach with a number of common alternatives. He finds the Anderson-Moore approach to be superior in numerical accuracy and computational efficiency.

Sims (2002) discusses conditions required to solve expectational difference equations, relating to the eigenvalues of the model. Note that there may be many solutions in complex DSGE models. In our simple example, we discard the explosive eigenvalue case since the competitive economy should have a unique solution. More complex models could give either only explosive solutions (usually interpreted as a misspecified model), or many stable solutions (which might make sense with multiple equilibria, or sunspot equilibria).

For our simple model, the researcher can, like Campbell (1994), derive expressions for the dynamics of consumption, output, capital and labour conditional on the 'deep' parameters. In a multi-shock model, these derivations by hand would be extremely time consuming, and unnecessary given that the model properties could simply be simulated.

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[7] Juillard (1996) provides the MATLAB- and GAUSS-based DYNARE package, which automates first and second order log-linearisation if required; see Table 1 for the DYNARE webpage address.

4   Data analysis

To solve the model by the methods described above, one requires values for the deep parameters (and hence the reduced-form parameters, the a's) which specify the log-linear system. The early RBC literature focused on 'calibration', rather than estimation, of these key parameters.

This controversial step is driven by three main concerns. First, identification of deep parameters in estimated large dynamic systems can be troublesome. Second, there is often a conceptual mismatch between the theoretical variables and their sample counterparts. (For example, what is the sample equivalent of the 'rate of return on one period real government debt' if bonds are denominated nominally in the sample data?) And third, in practice the classical estimation of macro models can be plagued by badly-behaved likelihoods – the researcher typically has very few business cycle fluctuations with which to estimate the highly abstract model.

Recent literature has focused more on estimation of the key parameters, often by Bayesian methods. The use of off-model or 'prior' information allows the researcher to mitigate, but not eliminate, the first and third of these issues (we return to these matters in section 5). Not much can be done about conceptual mismatch in highly abstract aggregate models; the researcher simply hopes that the selected values of the parameters move the model closer to the data.

We start our review of empirical methods by describing a Bayesian full information approach, keeping our review closer to more recent literature, and move on to discuss other estimation-based procedures. Then we describe and contrast the calibration approach and what we term 'Bayesian calibration'. The last of these deals with model uncertainty in a more formal way than calibration, but the full model is not estimated.

Bayesian estimation

Returning to our simple RBC model, there are a number of unknown parameters in equations (12)–(16), namely (α, δ, σ, r, N, g), and the parameters of the technology process, ϕ and the variance of ε, denoted $\text{var}(\varepsilon) = \sigma_{\varepsilon}^2$. Since nearly all capital stock data are constructed by making assumptions about the rate of depreciation, most researchers fix δ (at a

quarterly rate of around two percent). The remaining parameters can be estimated.

Koop (2003) provides an excellent general introduction to Bayesian estimation, highlighting the differences from the classical approach; see also Gelman *et al* (2004). Since neither of these Bayesian textbooks consider explicitly DSGE estimation, the descriptions given by Lubik and Schorfheide (2005), An and Schorfheide (2007) and Canova (2007) are particularly welcome. Key influential papers include DeJong *et al* (2000), Schorfheide (2000), Otrok (2001), and Rabanal and Rubio-Ramírez (2005).

To fix ideas without being too specific, consider a researcher wishing to estimate deep parameters of interest using additional 'off-model' information. The researcher's priors encompass all the information that is not from the sample data. To implement the Bayesian approach, the researcher re-weights the likelihood using the 'off-model' priors and maximizes the re-weighted likelihood.

Since $P(Data)$ is simply a constant, Bayes' rule implies that the posterior distribution over the parameters of model i, θ_i, given the *Data* is proportional to the marginal likelihood, $p(Data | \theta_i)$, multiplied by the prior, $p(\theta_i)$,

$$ p(\theta_i | Data) \quad \propto \quad p(Data | \theta_i)\, p(\theta_i). \tag{18} $$

Maximizing this re-weighted likelihood thus delivers the most likely values of the parameters given the data and the prior. In practice, the posterior mode for the parameters is often computed using the methods described by Schorfheide (2000):

1.   Solve the linear rational expectations system
2.   Use the Kalman filter to find the numerical values which maximise the sum of the likelihood and the prior.

Given that the shape of the posterior density is determined by the prior and the likelihood, the posterior density for each parameter can be constructed by using a Monte Carlo Markov chain, typically the Metropolis-Hastings (MH) algorithm.

Juillard's (1996) DYNARE package allows Bayesian estimation along these lines using MATLAB. The researcher can input the model in linearised form or in levels, with some higher-order approximation methods as options. Appendix B (see Table 2) describes the results from the estimation of our simple RBC model using US data from 1959Q1 to 2006Q1. The DYNARE code and US data to replicate our results are provided on the website that accompanies this paper.

The main advantage of the Bayesian estimation approach is that the researcher can make (post-data) probabilistic statements about model parameters and related events of interest. To illustrate this, Figure 1 below displays the prior (lighter line) and posterior densities for the parameters ϕ and σ analysed in our simple RBC example. In each case, the posterior mode value is shown by the green dashed line. In the Romer (2006) textbook, the student is invited to explore the implications of different parameterizations of the model. But the student has no formal means to assess what values are reasonable, given the data. However, armed with the predictive densities for the parameters, the researcher can simulate the model at the parameter values which are supported by the data.

It is important to note that the posterior densities shown in Figure 1 are conditional on the priors for all the parameters, not just those for ϕ and σ. (Appendix B to this paper contains a complete list of the priors used for our analysis.) Some users find it difficult to specify priors since their specification require a great deal of information for each parameter: the type of distribution, the support for the distribution, the mean of the distribution and perhaps higher moments too. In practice, prior elicitation can be troublesome. In response, DSGE practitioners sometimes resort to justifying their priors for a particular parameter with reference to other studies, even though the earlier research may have utilized a different model. Parameter priors are also typically assumed to be independent, even though the parameters may not necessarily be so.

In practice, the researcher also has to grapple with two other contentious issues: uninformativeness and Markov chain convergence problems. The former often causes the results to exhibit prior sensitivity. Consider the right hand panel of Figure 1. The posterior density is pretty close to the prior density, often taken as a 'warning indicator' that the data are relatively uninformative about this parameter. In principle, prior sensitivity can be assessed by re-estimating the model using 'reasonable' (but not first choice)

priors. Of course, there are many feasible prior densities. And the prior sensitivity for a specific parameter depends on the other priors in the system. So a thorough assessment carries a considerable computational burden.

Figure 1
Example RBC prior and posterior densities
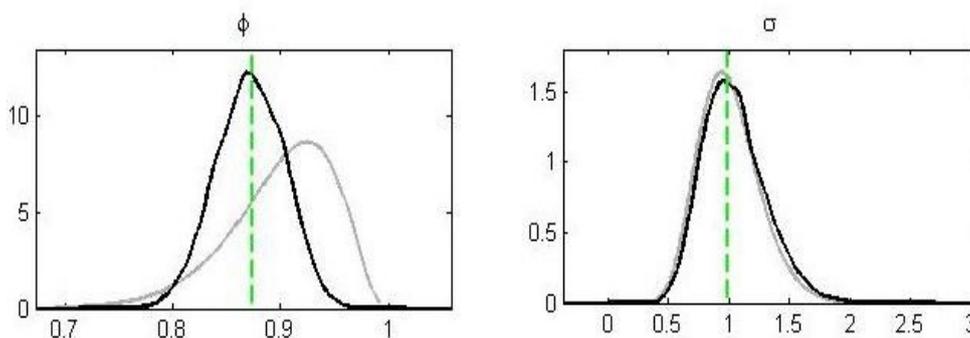
 A further worry is that a close match between the prior and the posterior of a particular parameter is neither necessary nor sufficient for prior sensitivity to be a problem. Canova and Sala (2006) discuss these and identification-related issues for DSGE modelling; see also, An and Schorfheide (2007), with discussion, rejoinder, plus GAUSS and MATLAB replication programs available from Schorfheide's webpage (see Table 1 for a link). These uninformativeness problems occur with other types of modelling of course. Poirier (1998) provides a Bayesian perspective with examples based on different models. No doubt the DSGE literature will investigate more formal approaches to dealing with these issues in the future.

The second contentious issue is that chain convergence problems sometimes pollute reported posterior densities. Can the researcher be sure that Figure 1 is based on appropriate draws from the Markov chain? Ideally, the researcher should be convinced that the Markov chain has iterated long enough for the sample of parameters at the end of the simulated chain to approximate the stationary (posterior) distribution with sufficient accuracy. There are, however, two problems. First, has the chain run for long enough so that the effects of initial conditions have dissipated (an issue of bias)? And second, is the sample of simulated parameters from the Markov chain large enough to ensure that the properties of the stationary distribution are adequately captured (an issue of variance)?

Cowles and Carlin (1996), Guihenneuc-Jouyaux *et al* (1998), and Brooks and Roberts (1999) survey convergence diagnostics in general. A common approach is to employ a variety diagnostics, including graphical representations of the Markov chains. Appendix C (see the associated website) shows how to implement a selection of diagnostics for our simple RBC model. (The DYNARE programs from the website accompanying this paper also supply some diagnostics.)

To summarise, although Bayesian estimation provides the researcher with formal tools to make probabilistic statements about parameter values (and other features of interest), there are drawbacks. Would noninformative priors classical estimation – be a better option?

Arguably, noninformative priors misrepresent the researcher's knowledge. For example, most researchers have some view of what constitutes a likely number for the elasticity of intertemporal labour substitution. Nevertheless, there is a substantial literature using classical maximum likelihood methods which pre-dates the Bayesian approach. Kim and Pagan (1995) review the early literature. Hansen and Sargent (1980), Sargent (1989), Altug (1989), McGrattan (1994) and Ireland (2001) are key papers in the development of the approach. McGrattan's webpage contains many useful programs (see Table 1 for a link). Convergence problems are also sometimes a practical difficulty with classical estimation, even for relatively simple DSGE models.

An alternative strand of the DSGE estimation literature has focused on limited information methods. These include minimum distance estimation based on the discrepancy between VAR and DSGE model impulse response functions, used for example by Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005). Christiano's webpage provides the code necessary for replications (see Table 1 for an Internet link). Hall *et al* (2007) propose an information criterion approach to facilitate matching. Ruge-Murcia (2007) argues that limited information methods are more reliable than either Bayesian or classical approaches to estimation of DSGE models in the presence of stochastic singularity. Measurement errors are typically added to the models to facilitate the implementation of Bayesian and classical full information methods; see also, Bierens (2007) on singularity.

Calibration

Until the late 1990s, calibration was the most popular method for empirical analysis with DSGE models. Romer (2006, p 208-209) provides an introduction to calibration. In a nutshell, the researcher picks plausible values for the parameters by looking at the data or referring to other empirical studies. Ideally this process should use features of the data other than those to be subsequently studied. The researcher simulates the model and examines additional properties of the model to assess its merits. For example, Kydland and Prescott (1982) employed the Neoclassical growth model calibrated on 'great ratios', etc. and used the model to explore business cycle phenomena. Typically, the researcher compares the variance (or covariance) of the simulated data with equivalent sample statistics.

In the early literature, calibration was sometimes argued to be concerned with 'measurement' rather than 'inference'. The exchange between Kydland and Prescott (1995), Hansen and Heckman (1996) and Sims (1996) captures nicely the 1990s debate about empirical methods. King and Rebelo (1999) also note the differences between calibration and traditional econometric methods. Cooley (1997) argues that estimation is actually complementary, since the former can help guide calibration choices. Cooley also emphasizes the bidirectional interplay between measurement and theory.

From a Bayesian perspective, picking plausible model parameters raises many of the issues encountered in prior mean elicitation. But where a Bayesian would pick the combination of parameter values that reflect off-model beliefs, a calibrator would select a set of parameters that matches the sample data (albeit, not the same features intended to be studied with the calibrated model). In this sense, the Bayesian counterpart to calibrated values is a set of parameter values comprising (some measure of) the centre of the posterior density for each parameter. Papers in the calibration tradition typically justify the selected parameters with reference to the model and the sample data.

Notice that the pedagogical paper by Campbell (1994) understates considerably the attention given to parameter selection in the literature. Cooley and Prescott (1995) provide a more typical treatment. Many parameters are selected by matching the balanced growth path of the model to the long-run sample features (e.g. sample averages). Some calibrators refer to earlier papers; others use microeconometric evidence in calibration. In contrast, technology parameters are matched to the Solow residual (which

captures total factor productivity). This step requires the researcher to carry out a growth accounting exercise; see Romer (2006, p 29-31 and 298).

Returning to our simple RBC example, the posterior means shown in Figure 1 (others are reported in appendix B) are very close to the calibrated values selected by Campbell (1994), which we used to determine the prior means. (Campbell calibrates a subset of the parameters considered in our Bayesian estimation example; we used off-model judgment to complete the prior specification.) There is little evidence to suggest that Campbell's preferred parameters are implausible, given the data and our prior specification.

Once the parameters have been chosen, the model can be solved and simulated. For researchers interested in experimenting with various calibrations in well-known models, Uhlig's DYNAMO package, see Table 1 for the link, provides menu-driven MS-WINDOWS compatible software.

A controversial issue in the calibration literature is how to evaluate the resulting model. The early RBC literature judged model performance from the ability to match particular 'stylised facts' of the sample data. In particular, Prescott (1986) stressed that the one-shock RBC model produced a variance for real output not much smaller than in the detrended sample data. Early practitioners used the Hodrick and Prescott (HP) (1997) filter to extract a smooth non-linear trend from the sample data. Prescott (1986) argued that the HP filter resembles an approximate high-pass filter designed to eliminate stochastic components with periodicities greater than thirty-two quarters. Harvey and Jaeger (1993) and Cogley and Nason (1995a) note the sensitivity of the sample data characteristics to the filtering technique. Canova (1998a) and Burnside (1998) provide differing perspectives on the controversy surrounding HP filtering in the 1990s. The academic tension regarding filtering was sufficient for Canova to be labelled "a skunk in a rose garden"; see Canova (1998b, p 534). Fukač and Pagan (2008) discuss the implications of filtering with reference to more recent DSGE models.

The covariance statistics discussed in early vintage RBC models are just one measure of model fit. An influential paper by Cogley and Nason (1995b) shows that several well-known models from the early 1990s fail to reproduce the persistence and depth of business cycle fluctuations. A common interpretation is that the early models required more disturbances and frictions to match the data. (We return to this issue in the subsequent section.)

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Another criticism of calibration is that although the parameters are selected with model uncertainty in mind, very little sensitivity analysis appeared in the published papers. Kim and Pagan (1995) discuss this in detail. A number of researchers responded by formalising the contribution of model uncertainty by using Bayesian Monte Carlo techniques for calibration: a Bayesian calibration. Notice that this is distinct from the Bayesian estimation approach described above there is no posterior simulation. So issues about the informativeness of the data, which often blight Bayesian estimation of DSGE models, do not arise. Instead, the researcher generates (a large number of) draws from the priors– prior simulation – and solves the model for each draw. A good early reference is DeJong et al (1996); a more recent example is Nason and Rogers (2006).

Bayesian calibration practitioners often assess model performance by estimating less restrictive models, such as Bayesian vector autoregressions (BVARs), on the data simulated from the model and the sample. Geweke (2007) discusses the relationship with posterior odds comparisons for DSGE models relative to the reference model. The BACC package developed by Geweke and co-authors can be used with MATLAB and GAUSS to conduct prior simulation (see Table 1) and to carry out Bayesian estimation in a wide range of reduced-form specifications.

5 Some DSGE research agendas

In the final section of this paper, we give a sense of the direction of DSGE modelling by describing some (very broadly defined) research agendas, stressing computational issues.

More detailed models

The early RBC models, with just one source of disturbance, were very parsimonious. A common perception was that the early models were too abstract for analysis of many economic issues. Subsequent models have been used to conduct experiments with multiple shocks, heterogeneous agents, wage and price rigidities, and monetary and fiscal policies. The literature review in Cooley (1995) describes in detail many of the different aspects of this research initiative during the 1990s.

Rather than attempt a comprehensive review of the diverse strands of this research agenda, we limit our discussion to DSGE models for monetary

policy analysis. Chari and Kehoe (1999) provide an overview of the late 1990s' optimal monetary (and fiscal) policy literature. Smets and Wouters (2007) summarise the more recent monetary policy related literature which, following Taylor (1993), has emphasized monetary policy rules that often' perform 'well', rather than optimally. Kremer *et al* (2006) provide a more detailed review of DSGE models as policy tools.

 Smets and Wouters' own research design is summarised on the webpage listed in Table 1; see, also Sims (2007b) for a critique. Smets and Wouters' ambitious agenda involves the construction of considerably larger models than those in Schorfheide (2000), but they utilize a similar Bayesian estimation strategy. There are similar initiatives to build 'medium-sized' models for monetary policy analysis at the International Monetary Fund (see Bayoumi *et al*, 2004), the Bank of Canada (Murchison and Rennison, 2006), Sveriges Riksbank (Adolfson *et al*, 2007), the Norges Bank (Brubakk *et al*, 2006), the Reserve Bank of New Zealand and at other central banks. The rubric NOEM, New Open Economy Models, is sometimes used to describe open economy DSGE models, typically with New Keynesian elements. Examples include Lubik and Schorfheide (2005, 2007), and Justiniano and Preston (2008). Other interesting avenues include the introduction of fiscal policy, eg Schmitt-Grohé and Uribe (2004a,b), and the consideration of profound parameter uncertainty within the DSGE framework by Levin *et al* (2005) and Edge *et al* (2008).

A brief description of the Smets-Wouters (2003) model gives a feel for the scale, and illustrates some of the common features, of Bayesian DSGE models used for policy purposes. Their model has three agents: households, firms and the central bank. The policymaker sets the short term interest rate in response to deviations of output from its flexible price level (potential output) and inflation from its target rate. There are both nominal and real frictions: monopolistic competition, sticky wages and prices, partial indexation of prices and wages, cost adjustment in capital accumulation, habit formation and variable capital utilisation. The eight disturbances are to general technology, labour supply, preferences, investment-specific technology, government consumption, price mark-ups, wage mark-ups, the equity premium, interest rates, and the inflation target.

The search for richer dynamics has also focused on the labour market (for example, Lubik and Krause, 2006, 2007), and the microfoundations of money (for example Lagos and Wright, 2005). In both cases, the new

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generation aims to recast some of the ad hoc elements in current DSGE models within an optimising framework. These features seem likely to be incorporated in large-scale macro models in the future.

Fit assessment with DSGE models

The fit of DSGE models has played an important part in the development of the literature. The early papers by Kydland and Prescott focus on matching the variances and covariances of key macroeconomic aggregates. Cogley and Nason (1995b) drew attention to the limited internal propagation mechanisms within 1990s' vintage models by comparing the impulse response functions from a structural VAR and autocorrelation functions estimated on model-generated and US sample data. (Appendix D to this paper on the Internet discusses the fit of our simple example RBC model.) Watson (1993) provides a tool for assessing the fit of DSGE models related to the familiar R^2 statistic. More recently, Fukač and Pagan (2008) propose limited information techniques as supplements to measures of system fit.

A popular new tool for DSGE fit assessment comes from Del Negro and Schorfheide (2004a,b). They estimate a structural vector autoregression using priors based on a simple three equation New Keynesian DSGE model. (GAUSS code for replication can be obtained from Schorfheide's webpage; see Table 1.) The intuition behind their approach is that the theoretical model can generate synthetic data. And then a VAR can be estimated on sample and synthetic data, controlling the ratio of simulated and sample data with a 'tightness' hyper-parameter; see also Ingram and Whiteman (1994).

In practice, the hybrid DGSE-VAR is estimated by Bayesian methods. Del Negro and Schorfheide note that for their US data the resulting hybrid model betters the out-of-sample forecasting performance of their simple VAR. Lees, Matheson and Smith (2007) consider a NOEM variant on New Zealand data. Subsequent work by Del Negro *et al* (2007) reinterprets the tightness hyper-parameter as a measure of in-sample fit of the DSGE model, which could be used to guide DSGE model builders. Alternatively, Bayesian posterior odds rank candidate models; see Rabanal and Rubio-Ramírez (2005) and the small open economy application by Matheson (2006). Both Sims (2007a) and Del Negro and Schorfheide (2006) argue that posterior odds can overstate the difference in fit between candidate models within a sparse model space.

A common feature of many assessments of fit is that a constant parameter VAR is used as a benchmark. This choice can create difficulties however. For example, under certain conditions a DSGE model does not have a reduced form VAR representation; see Fernández-Villaverde *et al* (2007), Chari *et al* (2007b), and Christiano and Davis (2006). Another problem is that constant parameters may not be a very good restriction (a point considered further below).

Forecasting performance is often used to assess model fit, but by this metric it is not obvious that an unrestricted VAR should be used as a benchmark against which to assess DSGE models. Kilian (2007) notes the superior predictive properties of Bayesian VARs and factor models (among other techniques); see also the reply by Del Negro *et al* (2007). Since factor models have also been shown to have relative good forecasting performance, Giannone *et al* (2006) argue that they provide a natural benchmark for DSGE validation. Boivin and Giannoni (2006) suggest that DSGE models should use factor model representations to aid measurement of the imperfectly observed theoretical variables.

Consideration of structural breaks

Since a number of papers in the Bayesian VAR literature argue that the Great Inflation and Great Moderation are distinct US regimes (see, for example, Sims and Zha, 2006, and Cogley and Sargent, 2005), it is troubling that most DSGE models maintain the assumption of no structural change. Most existing DSGE models cannot handle expectations of parameter change. Reflecting the desire of researchers to replicate the business cycle characteristics of the recent US data, this area is already receiving a great deal of attention.

One strand of the research examines the impact of switches in policymaking regimes. For example, Lubik and Schorfheide (2004) and Lubik and Surico (2008) split the sample under consideration into sub-samples and consider DSGE parameters for each sub-sample. The forward-looking agents form expectations as if they did not expect the parameters to change. An alternative approach, influenced by the work of Svensson and Williams (2007), uses minimal state variable solutions. Farmer, Waggoner and Zha (2006) use this method to represent structural models with regime-switching VARs. Fernández-Villaverde and Rubio-Ramírez (2008) apply a particle filter to estimate a growth model with stochastic volatility; see also

Justiniano and Primiceri (2008). A handy technical appendix provided by Fernández-Villaverde and Rubio-Ramírez (2008) reviews methods for estimating non-linear and non-normal general equilibrium models.

Analysis of historical episodes

Prescott (2002) makes the case for using the Neoclassical growth model as a lens to analyse historical episodes. Although many recent papers examining modern business cycle behaviour have utilized Bayesian methods, most of the existing papers in this field do not. Given that analyses of historical episodes are often based on a very small number of observations, it would seem that Bayesian methods are particularly well suited to the task.

Some notable analyses of historical episodes utilise a deterministic growth model. Prominent examples include Cooley and Ohanian (1997), who analyse post World War II economic growth in the UK and the legacy of Keynes, the analysis of the Korean war by Ohanian (1997), and the accounts of the Great Depression from Cole and Ohanian (2002, 2004). Pensieroso (2007) surveys the causes of the Great Depression in the RBC literature. The survey encompasses deterministic models and discusses the macroeconomic implications in detail. Here we focus more on distinguishing between the empirical methodologies.

Adopting a stochastic framework, Christiano *et al* (2005) use the limited information estimation approach to match the impulse responses of a structural VAR to their growth model of the US Great Depression. Harrison and Weder (2006) and Weder (2006) provide further analysis of the same episode using calibrated DSGE models with sunspot equilibria.

Chari *et al* (2002, 2007a) take a classical maximum likelihood approach to estimating a DSGE model of the US Depression. Chari *et al* (2007a) argue that their prototype DSGE model, with time varying efficiency, labour and investment wedges should be used as a complement to growth accounting to provide a starting point for analysis of historical episodes. That is, as a precursor to a more complete DSGE analysis. Chari *et al* (2007b) argue that structural VARs are ill-suited to this task; Christiano and Davis (2006) take the opposite view, drawing attention to model uncertainty issues in the classical estimation approach adopted by Chari *et al* (2007a). Subsequent work in this area will doubtless extend to more formal treatments of model uncertainty in the analysis of historical episodes.

6   Conclusions

This review has examined the tools required by the reader of Romer (2006) to build modern DSGE models. Whereas the textbook treatment in Romer (2006) characterises the RBC literature, this supplementary review has examined the computational issues commonly encountered by model builders, as well as providing insight into the future of DSGE research. The simple divisible labour RBC model has been used to illustrate a number of computational issues; code and US data have been provided to aid replication and further study. A number of popular research agendas have been described which, in addition to illustrating the computational subtleties of modern DSGE research, highlight the diverse methods for inference used by DSGE practitioners.

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Tables

Table 1
Sources of Useful Code

BACC	http://www2.cirano.qc.ca/~bacc/
DYNARE	http://www.cepremap.cnrs.fr/dynare/
IRIS	http://www.iris-toolbox.com/
QM RBC	http://dge.repec.org/
R software	http://www.r-project.org/
Anderson and Moore	http://www.federalreserve.gov/pubs/oss/oss4/about.html
Christiano	http://faculty.wcas.northwestern.edu/~lchrist/
Fernández-Villaverde	http://www.econ.upenn.edu/~jesusfv/companion.htm
LeSage	http://www.spatial-econometrics.com/
McGrattan	http://ideas.repec.org/e/pmc46.html
Rubio-Ramírez	http://www.econ.duke.edu/~jfr23/
Schorfheide	http://www.econ.upenn.edu/~schorf/research.htm
Sims	http://www.princeton.edu/~sims/
Smets	http://www.ecb.int/home/html/researcher_swm.en.html
Söderlind	http://home.tiscalinet.ch/paulsoderlind
Uhlig	http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm

Table 2
Files to replicate results and appendices
http://www.rbnz.govt.nz/research-and-publications/discussion-papers/supporting-files-for-dp2007-15

Solving for dynamics	(No applicable software files)
Bayesian estimation of model	Replication.zip
Markov chain diagnostics	McmcConvergence.zip
Assessing model fit	Replication.zip (simulate.m)

Appendices

A   Solving for dynamics

When the model is outside its steady state, it is a system of non-linear difference equations in g, k, y, n and c. These non-linearities are caused by time variation in the consumption-output ratio and by incomplete capital depreciation. While an exact analytical solution to the model is available only under particular circumstances (where capital depreciates fully in one period and where agents have log utility), an approximate solution to the system can be obtained by using log-linear difference equations. For simplicity, Campbell suppresses constants, and the linear difference equations can be thought of as deviations from the steady state balanced growth path.

Log-linearising

Log-linearising the production function is easy:

$$y_t = \alpha\left(a_t + n_t\right) + (1-\alpha)k_t \tag{A1}$$

Log-linearising the other non-linear equations in the system is, however, more difficult.

The cornerstone of the log-linearisation technique employed in Campbell (1994) is the Taylor series. Briefly, if $f\left(x\right)$ has derivatives of all orders at some point s, then the Taylor series expansion for $f\left(x\right)$ about s is:

$$f(x) = \sum_{t=0}^{\infty} \frac{f^k(s)}{k!}(x - s)^k \tag{A2}$$

where $f^k(s) - \left.\dfrac{\partial f^k(x)}{\partial x^k}\right|_{x=s}$. The first order Taylor approximation of the function f(x) at some point s is then:

$$f(x) \approx f(s) + f^1(s)(x - s) \tag{A3}$$

The capital accumulation equation (2) is not log-linear. Campbell thus uses a first order Taylor series expansion to approximate the function. To begin, divide the capital accumulation equation by K_t, and take $(1-\delta)$ to the left-hand side of the equality. Then take logs of both sides, ie,

$$
\begin{aligned}
\frac{K_{t+1}}{K_t} - (1-\delta) &= \frac{(1-\delta)K_t + [Y_t - C_t]}{K_t} \\
&= \left[\frac{Y_t}{K_t} - \frac{C_t}{K_t}\right] \\
\log\left[\exp\left[\log\left[\frac{K_{t+1}}{K_t}\right]\right]1(-\delta)\right] &= \log\left[\frac{Y_t}{K_t} - \frac{C_t}{K_t}\right] \\
\log[\exp(\Delta k_{t+1}) - (1-\delta)] &= \log\left[\frac{Y_t}{K_t}\left[1 - \frac{C_t}{K_t}\right]\right] \\
&= y_t - k_t + \log[1 - \exp(c_t - y_t)]
\end{aligned}
\tag{A4}
$$

Both sides of this equation contain non-linear terms. Campbell linearises the model by taking first-order Taylor approximations of the left- and right-hand sides of this equation around the steady state.

First define the two functions:

$$
f_1(\Delta k_{t+1}) \equiv \log[\exp(\Delta k_{t+1}) - (1-\delta)]
\tag{A5}
$$

and

$$
f_2(c_{t+1} - y_{t+1}) \equiv \log[1 - \exp(c_t - y_t)]
\tag{A6}
$$

Denote the steady-state values of c_t, k_t, y_t, and Δk_t as c, k, y, and g, respectively. The first-order Taylor approximations of the two functions above around the steady state are:

$$
f_1(\Delta k_{t+1}) \approx f_1(g) + f_1'(g)(\Delta k_{t+1} - g)
\tag{A7}
$$

and

$$
f_2(c_{t+1} - y_{t+1}) \approx f_2(c - y) + f_2'(c - y)(c_t - y_t - (c - y))
\tag{A8}
$$

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To make these approximations, take the derivatives of (A5) and (A6) evaluated at the steady states, which yields (since from a first order Taylor series expansion $\exp(\varepsilon) \approx 1+\varepsilon$):

$$f_1'(g) \approx \frac{\exp(g)}{\exp(g)-(1-\delta)} \approx \frac{1+g}{\delta+g}$$

(A9)

and:

$$f_2'(c-y) = \frac{-\exp(c-y)}{1-\exp(c-y)} \approx \left(\frac{\frac{(g+\delta)(1-\alpha)-1}{r+\delta}}{1-\left(1-\frac{(g+\delta)(1-\alpha)}{r+\delta}\right)} \right)$$

$$\approx 1-\frac{r+\delta}{(1-\alpha)(g+\delta)}$$

(A10)

Substituting the approximations into the log-capital accumulation equation yields

$$\log\left[\exp(g)-(1-\delta)\right]+\frac{1+g}{\delta+g}\left(\Delta k_{t+1}-g\right) \approx y_t-k_t+\log[1-\exp(c-y)]$$

$$+\left[1-\frac{r+\delta}{(1-\alpha)(g+\delta)}\right]\left(c_t-y_t-(c-y)\right)$$

(A11)

Dropping the constants yields:

$$k_{t+1} \approx k_t+\frac{\delta+g}{1+g}\left(y_t-k_t+\left(1-\frac{r+\delta}{(1-\alpha)(g+\delta)}\right)(c_t-y_t)\right)$$

(A12)

Using the log-production function to substitute out y_t yields:

$$k_{t+1} \approx$$

$$k_t + \frac{\delta + g}{1+g}\left[\alpha(a_t + n_t) + (1-\alpha)k_t - k_t + \left[1 - \frac{r+\delta}{(1-\alpha)(g+\delta)}\right](c_t - (\alpha(a_t + n_t) + (1-\alpha)k_t))\right]$$

$$(A13)$$

Collecting terms and simplifying:

$$k_{t+1} \approx \frac{1-r}{1+g}k_t$$
$$+ \frac{\delta + g}{1+g}\left[\alpha - \alpha\left[1 - \frac{r+\delta}{(1-\alpha)(g+\delta)}\right]\right](a_t + n_t) \qquad (A14)$$
$$+ \frac{\delta + g}{1+g}\left[1 - \frac{r+\delta}{(1-\alpha)(g+\delta)}\right]c_t$$

Simplifying yields the log-linearised capital accumulation equation:

$$k_{t+1} \approx \lambda_1 k_t + \lambda_2(a_t + n_t) + (1-\lambda_1-\lambda_2)c_t \qquad (A15)$$

where:

$$\lambda_1 = \frac{1+r}{1+g} \quad \text{and} \quad \lambda_2 = \frac{\frac{r}{\alpha}+\delta}{(1-\alpha)(1+g)} \qquad (A16)$$

If the variables on the right-hand side of the Euler equation (5) are jointly log-normal and homoscedastic then the log-linearised first order condition is:

$$E_t \Delta c_{t+1} = \sigma E_t[r_{t+1}] \qquad (A17)$$

where $r_{t+1} \equiv \log(R_{t+1})$.

From equation (4), the log of the returns equation is a non-linear function, ie

$$r_{t+1} = \log[1 - \delta + (1-\alpha)\exp(\alpha(a_{t+1} + n_t) - \alpha k_{t+1})] \qquad (A18)$$

Define:

$$f_3(a_{t+1}+n_{t+1}-k_{t+1})=\log\left[1-\delta+(1-\alpha)\exp(\alpha(a_{t+1}+n_{t+1})-\alpha k_{t+1})\right] \quad \text{(A19)}$$

The first-order Taylor approximation of this function around the steady state (ie when $r_t=r$, $a_t=a$, $n_t=n$ and $k_t=k$) is:

$$f_3(a_{t+1}+n_{t+1}-k_{t+1})\approx f_3(a+n-k)+f_3'(a+n-k)((a_t+n_t-k_t)-(a+n-k)) \quad \text{(A20)}$$

Taking the derivative of (A19) at the steady state yields:

$$f_3'(a+n-k)\frac{\alpha(1-\alpha)\exp(\alpha(a+n-k))}{(1-\delta)+(1-\alpha)\exp(\alpha(a+n-k))}$$

$$=\frac{\alpha(1-\alpha)\left[\dfrac{AN}{K}\right]^{\alpha}}{(1-\delta)+(1-\alpha)\left[\dfrac{AN}{K}\right]^{\alpha}} \quad \text{(A21)}$$

Substituting in the equilibrium productivity-capital ratio (equation 10) yields:

$$f_3'(a+n-k)\approx\frac{\alpha(1-\alpha)\left[\dfrac{r+\delta}{1-\alpha}\right]}{(1-\delta)+(1-\alpha)\left[\dfrac{r+\delta}{1-\alpha}\right]}$$

$$\approx\frac{\alpha(r+\delta)}{r+1} \quad \text{(A22)}$$

Now using the first-order Taylor series approximation to (A19), the equation for log-returns becomes:

$$r_{t+1}=\log\left[1-\delta+(1-\alpha)\exp(\alpha(a+n)-\alpha k)\right]$$

$$+\frac{\alpha(r+\delta)}{r+1}\left(a_{t+1}+n_{t+1}-k_{t+1}-(a+n-k)\right) \quad \text{(A23)}$$

Dropping the constants:

$$r_{t+1} = \frac{\alpha(r+\delta)}{r+1}(a_{t+1} + n_{t+1} - k_{t+1})$$
$$= \lambda_3(a_{t+1} + n_{t+1} - k_{t+1})$$

(A24)

where $\lambda_3 = \frac{\alpha(r+\delta)}{r+1}$.

Substituting this into the log first-order condition (A17) gives:

$$E_t \Delta c_{t+1} = \sigma \lambda_3 E_t \left[(a_{t+1} + n_{t+1} - k_{t+1})\right]$$

(A25)

To close the model, Campbell assumes that log technology follows a first-order autoregressive process:

$$a_t = \phi a_{t-1} + \varepsilon_t$$

(A26)

Equation (6) specifies

$$\theta(1-N_t)^{-\gamma} = \alpha \frac{A_t^{\alpha}}{C_t}\left(\frac{K_t}{N_t}\right)^{1-\alpha}$$

(A27)

Take logs of both sides:

$$\log(\theta) - \gamma \log(1 - \exp(n_t)) = \log(\alpha) + \alpha \log(A_t) - \log(C_t) + (1-\alpha)\log\left(\frac{K_t}{N_t}\right)$$

(A28)

Let

$$f_4(n_t) = \log(\theta) - \gamma \log(1 - \exp(n_t))$$

(A29)

The right hand side is linear in logs and does not need a first order approximation. As before, lowercase letters denote the logs of

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corresponding uppercase letters. The derivative of $f_4(n)$ evaluated at the steady state is:

$$f_4'(n) = \frac{\gamma \exp(n)}{(1-\exp(n))} \tag{A30}$$

The first order Taylor approximation of (A28) evaluated at the steady state is thus:

$$\log(\theta) - \gamma \log(1 - \exp(n)) + \frac{\gamma \exp(n)}{(1-\exp(n))}(n_t - n) \approx \tag{A31}$$
$$\log(\alpha) + \alpha a_t - c_t + (1-\alpha)(k_t - n_t)$$

Dropping constants, this approximation implies that:

$$\frac{\gamma \exp(n)}{(1-\exp(n))} n_t \approx \alpha a_t - c_t + (1-\alpha)(k_t - n_t) \tag{A32}$$

Since

$$\frac{\exp(n)}{(1-\exp(n))} \approx \frac{N}{1-N} \tag{A33}$$

equation (A32) can be rewritten as:

$$n_t = \nu[(1-\alpha)(k_t - n_t) + \alpha a_t - c_t] \tag{A34}$$

where $\nu \equiv \dfrac{(1-N)}{N}\sigma$, since $\sigma = 1/\gamma$.

Equations (A15), (A24), (A25), and (A34) thus replicate equations (13)-(16). These, together with the log production function and the equation (A26) for log technology, yield a system of log-linear (expectational) difference equations that can be solved using the method of undetermined coefficients.

Undetermined coefficients

Campbell adopts the notation η_{yx} for the partial elasticity of y with respect to x. For simplicity, as in Campbell (1994), we describe the method of undetermined coefficients when labour supply is fixed, and normalised to equal 1 (so $\log(n_t) = 0$). To solve the model, first pose a putative solution for log consumption which takes the form:

$$c_t = \eta_{ck}k_t + \eta_{ca}a_t \tag{A35}$$

where η_{ck} and η_{ca} are unknown but assumed to be constant. This 'guess' is then verified by finding values for η_{ck} and η_{ka} that satisfy the restrictions of the approximate log-linear model.

Using a simplified version of equation (A15) the conjectured solution can be written in terms of the capital stock:

$$k_{t+1} = \eta_{kk}k_t + \eta_{ka}a_t \tag{A36}$$

where:

$$\eta_{kk} \equiv \lambda_1 + (1 - \lambda_1 - \lambda_2)\eta_{ck}, \quad \eta_{ka} \equiv \lambda_2 + (1 - \lambda_1 - \lambda_2)\eta_{ca} \tag{A37}$$

Substituting the conjectured solution into (A25) yields:

$$\eta_{ck}\Delta k_{t+1} + \eta_{ca}E_t[\Delta a_{t+1}] = \sigma\lambda_3 E_t[a_{t+1}] - \sigma\lambda_3 k_{t+1} \tag{A38}$$

Now, use the fact that $E_t[a_{t+1}] = \phi a_t$ and substitute (A36) into (A38) to get an equation in only two state variables:

$$\eta_{ck}(\eta_{kk} - 1)k_t + \eta_{ck}\eta_{ka}a_t + \eta_{ca}(\phi - 1)a_t = -\sigma\lambda_3\eta_{kk}k_t + \sigma\lambda_3\phi a_t - \sigma\lambda_3\eta_{ka}a_t \tag{A39}$$

Substituting the coefficients (A37) into (A39) yields:

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$$\eta_{ck}[\lambda_1+(1-\lambda_1-\lambda_2)\eta_{ck}-1]k_t +$$
$$\eta_{ck}[\lambda_2+(1-\lambda_1-\lambda_2)\eta_{ca}]a_t+\eta_{ca}(\phi-1)a_t =$$
$$-\sigma\lambda_3[\lambda_1+(1-\lambda_1-\lambda_2)\eta_{ck}]k_t +$$
$$\sigma\lambda_3\phi a_t-\sigma\lambda_3[\lambda_2+(1-\lambda_1-\lambda_2)\eta_{ca}]a_t$$

$$(A40)$$

Now equate the coefficients on the left- and right-hand sides of (A40). Begin by equating the coefficients on k_t to find η_{ck} and then equate the coefficients on a_t to find η_{ca}. Equating coefficients on k_t yields a quadratic equation:

$$\eta_{ck}[\lambda_1+(1-\lambda_1-\lambda_2)\eta_{ck}-1]=-\sigma\lambda_3[\lambda_1+(1-\lambda_1-\lambda_2)\eta_{ck}] \qquad (A41)$$

Gathering terms, this can be expressed as:

$$Q_2\eta_{ck}^2+Q_1\eta_{ck}+Q_0\lambda_1=0 \qquad (A42)$$

where

$$Q_2 \equiv 1-\lambda_1-\lambda_2$$
$$Q_1 \equiv \lambda_1-1+\sigma\lambda_3(1-\lambda_1-\lambda_2) \qquad (A43)$$
$$Q_0 \equiv \sigma\lambda_3\lambda_1$$

The quadratic equation has two solutions for η_{ck}. But Campbell shows that for the steady state to be locally stable η_{ck} must be:

$$\eta_{ck}-\frac{1}{2Q_2}\left(-Q_1-\sqrt{Q_1^2\quad 4Q_0Q_2}\right) \qquad (A44)$$

Equation (A44) has η_{ck} solely as functions of the model parameters. The solution of the model is completed using η_{ck} from (A41) to equate coefficients on a_t in (A40), and thus find a solution for η_{ca}. Equating coefficients on a_t in (A40) gives:

$$\eta_{ck}\left[\lambda_2 + (1 - \lambda_1 - \lambda_2)\eta_{ca}\right] + \eta_{ca}(\phi - 1) =$$
$$\sigma\lambda_3\phi - \sigma\lambda_3\left[\lambda_2 + (1 - \lambda_1 - \lambda_2)\eta_{ca}\right] \tag{A45}$$

which implies:

$$\eta_{ca} - \frac{-\eta^{ck}\lambda 2 + \sigma\lambda 3(\phi - \lambda 2)}{\phi - 1 + (1 - \lambda_1 - \lambda_2)(\eta_{ck} + \sigma\lambda_3)} \tag{A46}$$

Thus, we have a solution for the unknown parameters in (A35) as a function of the deep model parameters. The parameters in equation (A36) can be obtained by substituting (A35) back into (A15).

Similar substitutions can be employed to show that the dynamic behaviour of the economy can be characterised by the following equations:

$$y_t = \eta_{yk}k_t + \eta_{ya}a_t \tag{A47}$$

$$c_t = \eta_{ck}k_t + \eta_{ca}a_t \tag{A48}$$

$$k_{t+1} = \eta_{kk}k_t + \eta_{ka}a_t \tag{A49}$$

$$a_t = \phi a_{t-1} + \varepsilon_t \tag{A50}$$

When labour supply is allowed to vary, then additionally

$$n_t = \eta_{nk}k_t + \eta_{na}a_t \tag{A51}$$

Appendix B of Campbell (1994) provides a derivation of the dynamics for the variable-labour model.

B   Bayesian estimation

To ensure that the model does not imply a stochastic singularity for the observable variables, we set the number of observable variables to be equal to the number of shocks in a model.[8] Thus, because the Campbell model only has one shock (technology), we require only one observable variable to estimate the model. We choose to estimate the model using output y, which we proxy using quarterly United States data for real Gross Domestic Product (GDP) over a sample of 189 observations ranging from 1959Q1 to 2006Q1. The remaining variables c, k, n, a are treated as being unobserved.

This simple model is expressed in terms of deviations from steady states. The data must be detrended to be consistent with the model. The quantitative properties of the data, however, may depend on the de-trending method that is employed. Canova (2007) suggests that, at a minimum, the data should be passed through a variety of filters to check the robustness of empirical results.

There are a variety of de-trending methods that are used in the DSGE literature including: removing linear trends; HP filtering; Band Pass filtering; stochastic de-trending (removing a common technology trend from all variables), and (log) first differencing allowing steady state rates of growth to be estimated along with the parameters of the model (see Nason and Rogers, 2006, and An and Schorfheide, 2007). Following the treatment in Romer (2006) and the early RBC literature, we simply de-trend (log) GDP using the HP filter.

Our priors and the results of the Bayesian estimation are displayed in table B1. Where possible, the prior means are taken from Campbell (1994). We simulate three chains of 50,000 from the posterior distribution using the Metropolis Hastings posterior simulator. The first 25,000 draws from each chain are discarded to mitigate bias arising from the starting point of the chain (see appendix C). The statistics displayed in the table are based on the last 25,000 draws from each chain, and the hope is that this sample is sufficiently large to capture the properties of the underlying distribution (the issue of variance). The prior and posterior distributions for the parameters are displayed in figure B1.

---

[8] A stochastic singularity occurs when there exists a linear combination of observables that can be predicted without error.

Table B1
Bayesian estimation

	Prior Distribution	Support	Prior Std Dev	Prior Mean	Posterior Mean	90% Posterior Probability Interval
α	Beta	[0,1]	0.050	0.666	0.685	[0.607, 0.767]
σ	Gamma	[0, ∞)	0.250	1.000	1.042	[0.610, 1.435]
ϕ	Beta	[0,1]	0.050	0.900	0.872	[0.819, 0.922]
r	Gamma	[0, ∞)	0.005	0.015	0.015	[0.007, 0.023]
g	Gamma	[0, ∞)	0.001	0.005	0.005	[0.003, 0.007]
δ	Beta	[0,1]	0.010	0.025	0.025	[0.008, 0.041]
N	Beta	[0,1]	0.050	0.333	0.326	[0.244, 0.406]
σ_ε	Inv. Gamma	[0, ∞)	4.000	0.010	0.007	[0.006, 0.009]

Figure B1
Priors (grey), posteriors (black), and posterior means (green)

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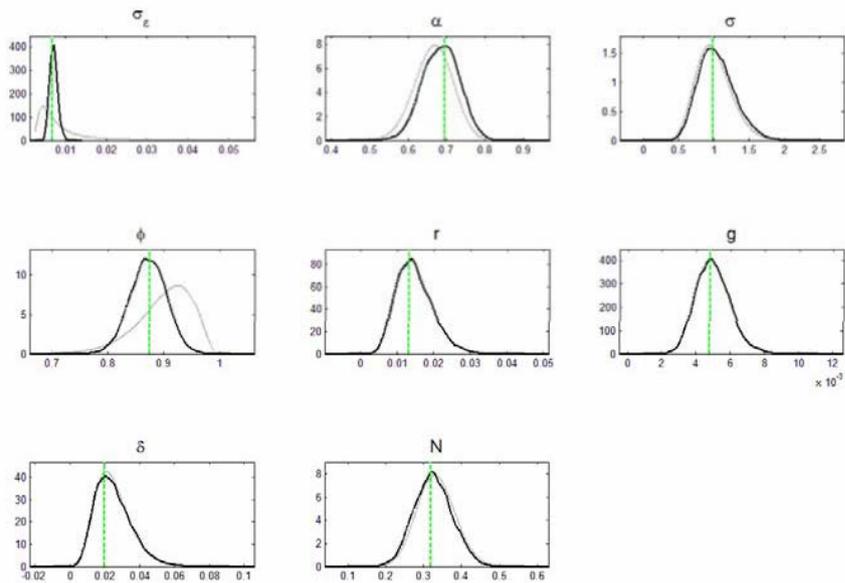
Most of the distributions of our estimated parameters are very similar to our prior distributions, suggesting that the data are uninformative with respect to these particular parameters. However, one parameter for which the data do seem informative is the autoregressive coefficient attached to the technology shock, ϕ. The data suggest that this coefficient is slightly lower than our prior mean of 0.9.

Prior sensitivity

How sensitive are the posterior estimates to our choice of priors? For our example, it seems likely that our posterior estimates are simply a reflection of our priors, and that the data provide very little 'new' information that can aid us in estimating many of the parameters of this model.

Suppose, for illustrative purposes, that we are comfortable with our prior distributions for all of the parameters with the exception of the elasticity of substitution, σ, and the autoregressive coefficient attached to the technology shock, ϕ. We can then evaluate how sensitive our estimates of these parameters are to the priors by re-estimating the model with different priors.

For this illustration, we change the means of our alternate prior distributions for σ and ϕ, and make the distributions more 'diffuse'. In particular, we double the mean and standard deviation of our prior for σ, and we impose a uniform distribution on the interval between 0 and 1 for ϕ. The results from re-estimating the model with these priors are displayed in table B2 and figure B2.

Table B2
Bayesian estimation (alternative priors)

	Prior Distribution	Support	Prior Std Dev	Prior Mean	Posterior Mean	90% Posterior Probability Interval
α	Beta	[0,1]	0.050	0.666	0.684	[0.604, 0.762]
σ	Gamma	$[0, \infty)$	0.500	2.000	2.052	[1.249, 2.870]
ϕ	Uniform	[0,1]	0.289	0.500	0.855	[0.795, 0.913]
r	Gamma	$[0, \infty)$	0.005	0.015	0.015	[0.007, 0.023]
g	Gamma	$[0, \infty)$	0.001	0.005	0.005	[0.003, 0.007]
δ	Beta	[0,1]	0.010	0.025	0.025	[0.009, 0.041]
N	Beta	[0,1]	0.050	0.333	0.327	[0.245, 0.409]
σ_ε	Inv. Gamma	$[0, \infty)$	4.000	0.010	0.006	[0.005, 0.008]

Figure B2
Priors (grey), posteriors (black), and posterior means (green): Alternative priors
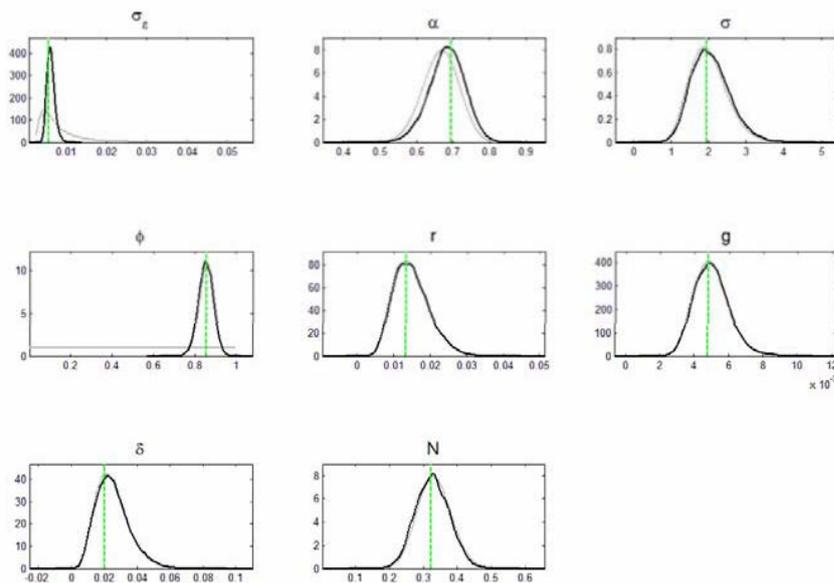
The posterior distributions for most of our parameters are very similar to those estimated above. However, the posterior distribution for the elasticity of substitution b has changed to closely match our updated prior. This suggests that the data contain very little information that can aid us in estimating the elasticity of substitution b and, as a consequence, that Bayesian estimates of this parameter are very sensitive to the prior beliefs of the researcher. In contrast, the autoregressive coefficient attached to the technology shock ϕ appears to be robustly estimated, with the posterior distribution of this parameter being relatively insensitive to our choice priors; the 90 per cent posterior probability range for a Beta prior with a mean of 0.9 and a standard deviation of 0.05 is [0.819, 0.922], very similar to the range for the Uniform prior over the interval between 0 and 1, [0.795, 0.913].

C   Markov chain diagnostics

We do not provide an exhaustive description of diagnostics, but instead apply a number of diagnostic tests to the model outlined in equations (12)-(16). Readers looking for a more in-depth treatment should consult the references, particularly the three surveys by Cowles and Carlin (1996), Guihenneuc-Jouyaux *et al* (1998), and Brooks and Roberts (1999).

The CODA and BOA packages available in the shareware statistical software R provide an easy way to implement a number of popular diagnostics.[9] These packages have comprehensive help files that detail the individual procedures.[10] CODA and BOA provide graphical output that can be used to summarise the chains, as well as graphical and numeric diagnostic statistics. There are also functions to manipulate the chains, eg to thin the chains, to subset them (concentrating on specific columns/parameters), or to create functions from the underlying parameters.[11]

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[9] See www.r-project.org. The *Bugs* program, which can be used for Gibbs sampling, can also be linked to *R*. http://www.mrc-bsu.cam.ac.uk/bugs/.

[10] Rather unhelpfully, the BOA *manual* in the BOA zip file has the same file-name as the *R* help file. The latter solely details the functions without providing any example code.

[11] In a time series context, for example, one might consider convergence of an impulse response function, being some nonlinear function of the model parameters. Thus, one could explicitly focus on convergence of the quantity of interest, rather than simply the parameters in the underlying Markov chain.

The simplest option in R is to use codamenu() or boa.menu() to apply the diagnostic functions to the data loaded into R. The analyst can then work through a pre-programmed variety of actions that include direct (including graphical) analysis of the chains, and diagnostics. An alternative to using the pre-programmed menus is to write a script file that achieves the same outcomes. Although more time consuming, scripts provide greater control over the diagnostic output. An example of such a script file is provided on the Internet with the other programs used in this paper.

The statistics implemented in CODA and BOA can be used with any samplers, eg the Gibbs sampler or other MCMC methods, and are applicable to models that have both discrete and continuous supports. Unlike a number of other tests, they do not have hard-to-verify conditions. The diagnostics in CODA and BOA include those from Geweke (1992), Gelman and Rubin (1992), Raftery and Lewis (1992), and Heidelberg and Welch (1983).

Application 1: An initial Markov chain with 10,000 observations

A typical starting point is to plot the Markov time-series, depicting the realisation for each parameter in the vector being simulated. Such plots can determine whether there are any unwanted trends in the chains; such trends clearly indicate a lack of convergence. So-called trace plots can be used for the same purpose, and may be useful if the simulated chains are very long. Cowles and Carlin (1996) recommend plotting multiple chains from dispersed chains together, to visually record whether the chains are traversing the same support. (To conserve space we do not depict these graphs.)

One can also look at the autocorrelation functions, to get an idea of just how problematic the persistence in the chain is, and obtain basic data summaries. Chains with a high degree of persistence will converge more slowly to the stationary distribution of interest. CODA's cumuplot command can be used to graph the evolution of quantiles. It is also usual to graph histograms of the parameters' unconditional distributions to gain understanding of the support of the distributions and to see where probability mass predominates. Cross-correlation plots from the chains may, in some circumstances, indicate whether the model suffers from identification problems. One can also establish whether the acceptance rate of the chains, the conditional probability of moving away from a current state, is around 0.25 (a recommended level when the chain has three or more dimensions).

Once the chains' basic properties have been established, one can calculate the diagnostic statistics. Raftery and Lewis' diagnostic is intended to be run on a short pre-sample to ascertain how long the chain needs to be run to achieve a desired level of accuracy. The diagnostic determines the number of iterations required to estimate a quantile q to q±r with a desired degree of probability. (Quantiles near the median are harder to estimate than those near the tails of the distribution.) Positive autocorrelation increases the required number of iterations; a dependence factor I indicates strong autocorrelation, and values of I greater than 5 suggest the chain has not been run long enough. The diagnostic output also provides an estimate of the required burn-in – the number of observations that need to be discarded.

For the first of our small chains with 10,000 iterations the output for the Raftery-Lewis diagnostic is:

Quantile (q) = 0.5
Accuracy (r) = +/- 0.005
Probability (s) = 0.95,

You need at least 38415 observations in the Markov chain with these values of q, r and s.

The minimum required sample size is a best-case scenario, when there is no autocorrelation in the chain. (More output is provided when this minimum size is exceeded.)

Many statistics are now graphically depicted for subsets of the chain, to assess whether their properties are sample-specific. The Gelman and Rubin statistic based on multiple chains examines whether the individual chains have mixed enough to look like the mixture of all chains. The 'potential scale reduction factor' indicates how much the estimated scale of the distribution could change. For our three chains with 10,000 rows the resulting 'potential scale reduction factors' are:

If the chains have converged then the point estimates should be around 1. One might be prepared to accept convergence if the statistics were around 1.05. Certainly, the statistics reported for σ_ε and δ would not generally be considered acceptable. The plot corresponding to the Gelman and Rubin statistic is depicted in figure C1.

Table C1
Potential scale reduction factors

Parameter	Point Estimate	97.5% Quantile
σ_ε	1.88	4.23
α	1.90	3.15
σ	1.02	1.06
ϕ	1.01	1.04
N	1.04	1.13
r	1.02	1.04
g	1.19	1.55
δ	1.39	2.16

Figure C1
Evolution of Gelman and Rubin's potential scale reduction factors
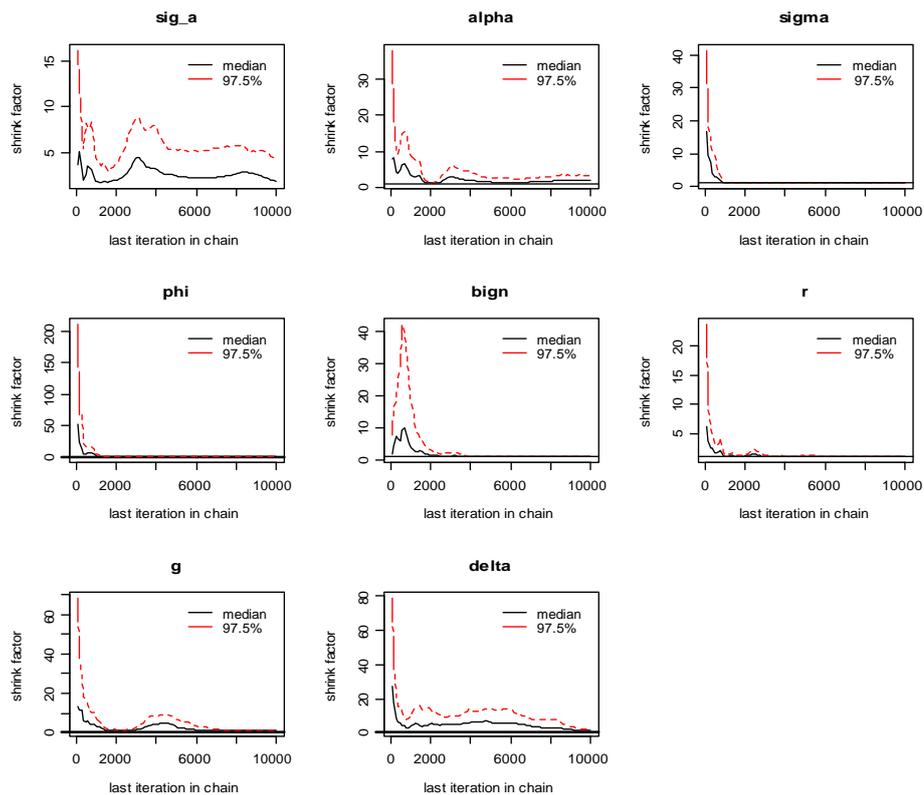
NB In the figure sig_a refers to σ_ε

Geweke's diagnostic looks at the means of two (separated) sub-samples taken from the chain. The default setting is to compare the means from the first 10 percent of the sample with the means from the latter 50 percent of the sample (ie means are calculated for each column). Using our example chain with 10,000 iterations, Geweke's diagnostic cannot even be calculated. This is another indication of convergence failure.

Heidelberg and Welch's diagnostic is also based on sample means and like Geweke makes use of spectral estimates of the variances. Confidence intervals are calculated for each parameter's mean. If the confidence interval is a sufficiently small (pre-specified) proportion of the mean then the diagnostic test is passed. The diagnostics are calculated after burning 10 percent, then 20 percent, ... up to 50 percent of the sample chain. If 50 percent of the chain is discarded then Heidelberg and Welch recommend running the chain for at least twice as long and repeating the test all over again. Like Geweke's diagnostic, the Heidelberg and Welch diagnostic cannot be calculated for our small sample.

Application 2: Extended chains with 50,000 iterations

The diagnostic statistics from our initial run prompt us to extend the length of the Markov chain to 50,000 observations. The Raftery and Lewis diagnostics on our first chain are not that favourable regarding convergence. The dependence factors are smaller than previously, but still well above 5.

Table C2

Raftery and Lewis Statistic

Quantile (q)=0.05 Accuracy (r) = +/- 0.005 Probability (s) = 0.95	Burn-in	Total	Lower bound	Dependence factor
	(M)	(N)	(Nmin)	(I)
σ_ε	30	62350	7299	11.7
α	40	84410	7299	14.9
σ	30	63596	7299	16.9
ϕ	63	128394	7299	24.8
r	31	65969	7299	21.7
g	40	87690	7299	18.8
δ	29	61619	7299	21.2
N	40	84852	7299	18.8

The picture obtained from the Potential scale reduction factors (PSRFs) is more sanguine. All of the PSRFs indicate that the chains have converged.

Table C3
Potential scale reduction factors

Parameter	Point Estimate	97.5% Quantile
σ_ε	1.00	1.00
α	1.00	1.00
σ	1.00	1.00
ϕ	1.00	1.00
r	1.00	1.00
g	1.00	1.00
δ	1.00	1.00
N	1.00	1.00
Multivariate PRSF	1+0i	

The Geweke statistic presents a more mixed perspective. Most of the dimensions of the chain appear to converge, though delta's t-statistic is still well under -2.

Table C4
Geweke Statistics

Parameters	σ_ε	α	σ	ϕ	r	g	δ	N
T-statistic	-0.592	0.793	1.030	0.434	-0.192	1.2312	2.313	1.056

Fraction in 1st window = 0.1; Fraction in 2nd window = 0.5

The output from the Heidelberg-Welch diagnostics are also supportive of convergence for this longer sample.

Table C5
Heidelberg-Welch Statistics

	Stationarity test	Start Iteration	p-value
σ_ε	Passed	1	0.518
α	Passed	1	0.869
σ	Passed	1	0.614
ϕ	Passed	1	0.923
R	Passed	1	0.526
G	Passed	1	0.585
δ	Passed	1	0.1
N	Passed	1	0.691
	Halfwidth test	Mean	Halfwidth
σ_ε	Passed	0.00727	3.56E-05
α	Passed	0.68595	1.81E-03
σ	Passed	1.04035	1.41E-02
ϕ	Passed	0.87337	1.86E-03
r	Passed	0.01494	2.56E-04
g	Passed	0.00499	4.61E-05
δ	Passed	0.02469	4.72E-04
N	Passed	0.32763	2.25E-03

The statistics indicate that the ' half-width' (half the confidence interval) is indeed less than ten percent of the mean value. For example, the mean of σ_ε is 0.00727 and the half-width is .0000356, which is less than ½ a percent of the mean value, much lower than the 10 percent (eps=0.1) that was initially specified for the diagnostic.

In summary, the diagnostics above provide a somewhat mixed perspective, and a degree of judgment will need to be employed by the researcher. Clearly the initial chains with only 10,000 observations were too short for convergence to be assured. The chains with 50,000 observations could be interpreted as having converged, though this conclusion depends on the weight that one attaches to the different diagnostics. Thus, the researcher needs to weigh computational costs against the benefits of extending the chains further.

D   Assessing model fit

Campbell (1994) solves the model analytically using the method of undetermined coefficients, and shows the dynamic behaviour of the economy can be characterised by the following equations:

$$y_t = \eta_{yk}k_t + \eta_{ya}a_t \tag{D1}$$

$$c_t = \eta_{ck}k_t + \eta_{ca}a_t \tag{D2}$$

$$k_{t+1} = \eta_{kk}k_t + \eta_{ka}a_t \tag{D3}$$

$$n_t = \eta_{nk}k_t + \eta_{na}a_t \tag{D4}$$

$$a_t = \phi a_{t-1} + \varepsilon_t \tag{D5}$$

where the reduced form elasticities η are combinations of the deep parameters of the model. These equations can be written more conveniently in VAR form:

$$A_0X_t = A_1X_{t-1} + A_2e_t \tag{D6}$$

where:

$$X_t = \begin{pmatrix} y_t \\ c_t \\ k_t \\ n_t \\ a_t \end{pmatrix},\ A_0 = \begin{pmatrix} 1 & 0 & -\eta_{yk} & 0 & -\eta_{ya} \\ 0 & 1 & -\eta_{ck} & 0 & -\eta_{ca} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\eta_{nk} & 1 & -\eta_{na} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},\ A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\eta_{kk} & 0 & -\eta_{ka} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \text{ and } e_t = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_t \end{pmatrix}.$$

The reduced-form VAR is then:

$$X_t = MX_{t-1} + We_t \qquad\qquad (D7)$$

where $M - A_0^{-1}A_1$ and $W = A_0^{-1}A_2$.

DSGE models typically imply parameter restrictions on more generously parameterised statistical models. From a classical perspective one can test whether such restrictions are supported by the data, for example by using likelihood ratio tests. Are, for example, the zero restrictions in equation (D6) consistent with the data? See Lippi and Neri (2007) for an application of such techniques.

Another way of assessing whether a model is broadly consistent with the data is through simulation techniques. One can, for example, simulate a reduced form VAR and then ascertain whether the highest probability reduced form properties of a DSGE model─ such as means, standard deviations, autocorrelations, and cross-correlations─fall within the VAR distribution. Precisely how one simulates the VAR depends on the methodological preferences of the researcher. For example, one might perform a conventional bootstrap, re-sampling from the residuals and estimating (pseudo) reduced form parameters and properties (moments and the like). Alternatively, if the VAR was estimated using Bayesian techniques one might simulate from the posterior distribution of VAR parameters (possibly also simulating from the distribution of shocks). DeJong, Ingram and Whiteman (1996) simulate both a DSGE model and a statistical model and then derive a statistic that reflects the degree of overlap from the distributions obtained from each model.

If Bayesian techniques have been used to estimate the DSGE model, one can simulate parameters from the posterior distribution of the DSGE model parameters, and then ascertain whether the sample moments lie near the

centre of the probability mass obtained from the simulation of the DSGE model (see for example figure 2 of Lees, Matheson and Smith, 2007). One can extend the simulation by also sampling from the estimated shock distributions. Although prior and posterior distributions are commonly reported for parameters in DSGE models, it is rather less common to report the extent to which the models capture the moments of the data.

**A simple example: Assessing fit based on simulation uncertainty**

This example performs an assessment of the empirical fit of the Campbell model. As described above, posterior distributions of the autocorrelation function (ACF) and the standard deviation of output can be simulated from the posterior distribution of parameters (allowing for uncertainty in the parameters and in the stochastic process). These distributions can be evaluated against autocorrelations and the standard deviation estimated from actual data.

The implementation proceeds as follows:

1.   Draw a set of reduced form parameters i from the posterior distribution and compute M_i and W_i .

2.   Draw a set of 189 (the size of the sample) technology shocks e_t from $N(0, \sigma_{ei}^2)$.

3.   Use M_i , W_i , and the shocks to simulate the reduced-form VAR representation of the model, equation (D7), for 189 periods.

4.   Estimate the autocorrelation function of the simulated output data and its standard deviation. Save the results.

5.   Repeat steps 1-4 for N parameter draws from the posterior density.

The simulated distributions of the first three ACFs and the standard deviation of output are displayed in figure D1, along with the same statistics estimated from our sample of data. The model matches the data in terms of the ACFs and the standard deviation.

Figure D1
Simulated distributions and empirical moments of output
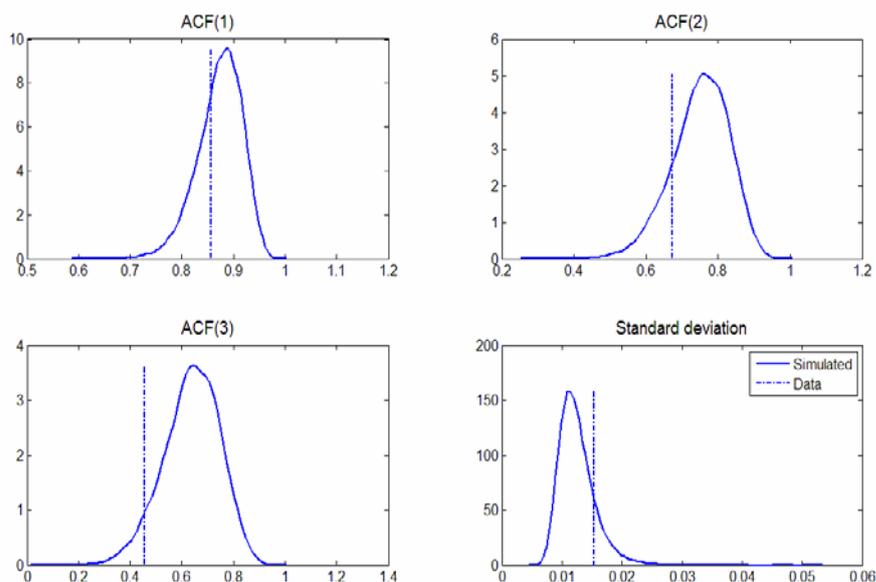
 Fukač and Pagan (2006) examine the ability of Lubik and Schorfheide's (2005) model to reproduce the cross- and auto-correlations inherent in the observable data. Fukač and Pagan also have a nice general discussion of efforts to assess model fit. These include minimum distance methods to estimate DSGE model parameters with respect to VAR impulse response functions (such as Rotemberg and Woodford 1997 and Christiano, Eichenbaum and Evans 2005) and efforts to quantify the discrepancy between the model and the data, such as Watson (1993).

If multiple models are under consideration (such as in Matheson 2007 for example), then posterior odds or Bayes factors are often used to assess the relative merits of the models. These statistics can be considered a measure of fit relative to the data (and priors).

Finally, it is also worth noting that DSGE models, particularly in policy-making institutions, are often evaluated with reference to their forecasting performance (as in Smets and Wouters, 2003 or DeJong, Ingram Whiteman, 2000). Such evaluations can be thought of as a measure of fit or out-of-sample cross-validation. As Geweke (2005, p 67) notes, the log-likelihood

can be broken down into the sum of one-step ahead predictions, and thus there is a close correspondence between prediction and fit.

Assessments of fit in the context of multivariate models can be difficult in that researchers may have a preference to focus on particular dimensions. Standard statistical measures of fit may not coincide with such preferences. Schorfheide (2000) provides a formal loss function based mechanism for assessing fit, though Murchison and Rennison (2006) suggest it may be difficult to specify an objective function that fully captures the researchers, or policymakers ' preferences.

In summary, this appendix has provided a flavour of the many different methods that are employed in the DSGE literature to evaluate fit, including classical and Bayesian methods, full and limited information approaches, and analytical and simulation techniques.