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Satisficing Solutions for New Zealand Monetary Policy*

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Abstract

Computing the optimal trajectory over time of key variables is a standard exercise in decision-making and the analysis of many dynamic systems. In practice however, it is often enough to ensure that these variables evolve within certain bounds. In this paper we study the problem of setting monetary policy in a 'good enough' sense, rather than in the optimising sense more common in the literature. Important advantages of our *satisficing* approach over policy optimisation include greater robustness to model, parameter, and shock uncertainty, and a better characterisation of imprecisely defined monetary policy goals. Also, optimisation may be unsuitable for determining prescriptive policy in that it suggests a unique 'best' solution while many solutions may be satisficing. Our analysis frames the monetary policy problem in the context of viability theory which rigorously captures the notion of satisficing. We estimate a simple closed economy model on New Zealand data and use viability theory to discuss how inflation, output, and interest rate may be maintained within some acceptable bounds. We derive monetary policy rules that achieve such an outcome endogenously.

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1 Introduction

Viability analysis is is a relatively young area of continuous mathematics that rigorously captures the essence of *satisficing* – a theory of choice behaviour first articulated by Herbert A. Simon, the 1978 Economics Nobel Prize Laureate. The theory contends that humans are only sufficiently rational, and consequently seek to attain some minimum level of a goal variable, rather than the constrained optimum.[1] In the literature, such behaviour is often paralleled with 'bounded rationality', according to which agents process complex problems only partially given computation costs and limitations on the collection of information.

In this paper we aim to use viability theory[2] for the analysis of an economic state-constrained control problem.[3] Specifically, we derive satisficing solutions for monetary control by considering a standard interest rate adjustment problem as faced by the Reserve Bank of New Zealand. We believe that the Simon prescription brings economic modeling closer to how agents actually behave and offers an alternative framework for macroeconomic study to the more standard analysis of model equilibria and system dynamics.

By applying the principles of viability theory to a central bank's macroeconomic stabilisation problem we are able to evaluate the economic system's evolution within certain normative constraints (such as a desired inflation band).[4] On the basis of the system's evolution we propose some robust, satisficing, monetary policies.[5] Policies obtained through viability analysis are 'robust' (or precautionary or preventative) in that they are based on the economic system's inertia making them naturally forward looking. This is so because knowledge of the system's inertia

[1] See Simon (1955). It has also been contended that satisficing is a better alternative to the theory of the firm than profit maximisation. Further, it can be shown that optimisation exercises that account for *all* costs, including those associated with the collection of information necessary for optimisation and with the optimisation calculation itself, yield satisficing solutions.

[2] See Aubin (1997) and Aubin, Da Prato, and Frankowska (2000) for an introduction to viability analysis and its apparatus.

[3] The paper draws from Krawczyk and Kim (2006) and extends Krawczyk and Kim (2004). An earlier version of this paper was presented as Krawczyk and Sethi (2006).

[4] In contrast, optimising approaches focus evaluation of a model's properties on an assessment of its return to a steady state in the face of economic shock.

[5] Actually, we propose various *alternative* robust strategies. For the minimax approach applied to the design of *robust* monetary policies, see Hansen, Sargent, and Turmuhambetova (2006), and Žaković, Rustem, and Wieland (2002). Strategies delivered by viability theory are alternative as they are based not on minimax optimisation but on an evolutionary analysis of admissible system trajectories, and so yield equivalently satisfactory outcomes.

enables detection and avoidance of regions where prevailing economic conditions (such as a large output gap or accelerating inflation) make system control difficult or impossible.

The paper proceeds as follows. Given the relative newness of viability analysis to macroeconomics, we begin with a brief, self-contained summary of the basics of the theory. In sections 4 and 5, we motivate and estimate a simple New Keynesian monetary model on New Zealand data.[6] The model is completed with a description of the constraints on the evolution of the goal variables and the policy instrument in section 6. In sections 7 and 8 we apply tools from viability theory to the model, determine satisficing outcomes for monetary policy, and construct interest rate rules that achieve such outcomes. We end with concluding remarks and directions for future research.

2 What is viability theory?

Viability theory is a new area of mathematics concerned with describing the evolution of controlled dynamic systems. It may be helpful to informally consider the important elements of theory before we offer precise definitions. In broad terms, for any given system of variables it is often desirable (or necessary) to prescribe bounds within which the system must evolve for a certain period of time. If there exists a system trajectory such that the constraints are obeyed then the system evolution can be termed 'viable'. The chief aim of viability theory is to establish whether a control strategy exists that retains the system within the constraint set. The initial conditions that the system must satisfy if it is to evolve viably is called the viability kernel.

[6] Notwithstanding this macroeconomic application, viability theory can be readily applied to other problems where the uniqueness of the optimal strategy is not of foremost concern. See Saint-Pierre (2001) and Clément-Pitiot and Saint-Pierre (2006) for studies on endogenous business cycles and Clément-Pitiot and Doyen (1999) for an analysis of viable exchange rate dynamics. Applications of the theory to environmental economics may also be found in Bene, Doyen, and Gabay (2001) and Martinet and Doyen (2007); and to financial analysis in Pujal and Saint-Pierre (2006) and the references therein.

2.1 Definitions

Consider a dynamic economic system with several state variables. At time $t \in \Theta \equiv [0, T] \subset \mathbb{R}^+$, where T can be finite or infinite, the state variables are

$$x(t) \equiv [x_1(t), x_2(t), \ldots, x_N(t)]' \in \mathbb{R}^N, \qquad \forall t \in \Theta,$$

and the controls (or actions) are

$$u(t) \equiv [u_1(t), u_2(t), \ldots, u_M(t)]' \in \mathbb{R}^M, \qquad \forall t \in \Theta.$$

Note that the state vector may be generalised to a *meta*-state that includes flow variables along with the usual stock variables. If so, the system's controls will be the velocities (or growth rates) of the instruments.[7]

We impose normative constraints on the systems and controls such that

$$x(t) \in K \quad \text{and} \quad u(t) \in U, \quad \forall t \in \Theta,$$

where K and U are compact sets of constraints that the state and control variables are required to to satisfy.[8] The control set U may be split into an arbitrary number of subsets if there is more than one player to decide upon the actions on the system.[9]

The state vector evolves according to system dynamics $f(\cdot,\cdot)$ and controls $u(t)$ as follows:

$$\dot{x}(t) = f(x(t), u(t)) \qquad \forall t \in \Theta, \quad x(t) \in K \subset \mathbb{R}^N, \quad u(t) \in U \subset \mathbb{R}^M, \qquad (1)$$

and so the system's evolution – the growth rate or velocity given by $\dot{x}(t)$ – depends both on the state of the economy and on the action $u(t)$. We may say that the velocity of $x(t) \in K$, for any $t \in \Theta$, is governed by the correspondence:

$$F(x) \equiv \{f(x,u), u \in U\}. \qquad (2)$$

This is the set of feasible velocities for the system given controls that are restricted to some available domain. The system's dynamics can then be rewritten in the form of a differential inclusion:[10]

$$\dot{x}(t) \in F(x(t)), \quad \text{for almost all} \quad t \in \Theta, \qquad (3)$$

[7] See the system of equations (25)-(27) later in the paper where the instrument is a nominal interest rate and the control is the *change* in this interest rate.

[8] In general, the control constraints depend on x; however, for simplicity, we avoid the notation $u(x(t),t)$ in this paper.

[9] See Krawczyk and Kim (2004).

[10] A differential inclusion can be thought of as a generalized differential equation, wherein the solution to the problem is a set of reachable states rather than a single trajectory.

which determines the range of growth rates of the state variables at $x(t)$.

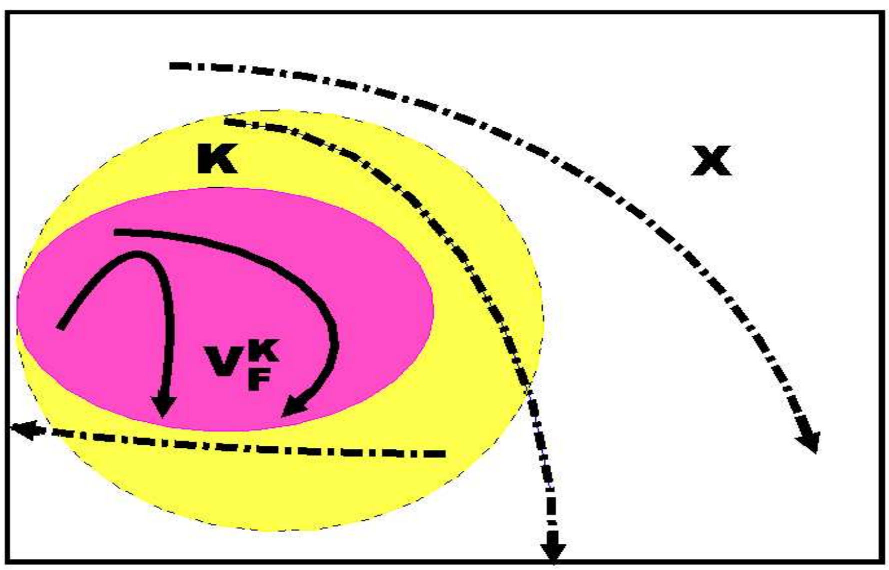
In economic terms, the last relationship states that at time t, for a given composition of x (capital, labour, technology, *etc.*), the extent of growth (or decline) in these variables, or in their steady state stability, is dependent on the map $F : \mathbb{R}^N \rightarrow \mathbb{R}^N$ the values of which are limited by the scope of the system's dynamics f and the set of available controls, U.

Viability analysis attempts to establish the non-emptiness of a *viability kernel*, a collection of loci of initial conditions for which at least one viable evolution, $x(t), t \in \Theta$, exists.

Definition 2.1 *The T-**viability kernel** of the constraint set K for the control set U is the set of initial conditions $x_0 \in K$, denoted $V_F^K(T)$, and defined as follows:*

$$V_F^K(T) \equiv \{x_0 \in K : \exists x(t) \text{ solution to (3) with} x(0) = x_0 \text{ s.t. } x(t) \in K, \forall t \in \Theta = [0,T]\}. \quad (4)$$

Figure 1
Viable and non-viable trajectories when $T = \infty$



In other words, we know that if a trajectory begins inside the viability kernel $V_F^K(T)$ then we have sufficient controls to keep this trajectory in the constraint set

K for $t \in \Theta$. See figure 1 for an illustration of the viability kernel concept when $T = \infty$.

The constraint set K for the state vector is represented by the yellow (or light shaded) round shape contained in the state space (here, $X \equiv \mathbb{R}^2$). The solid and dash-dotted lines are possible trajectories for the evolution of the system.

Given K, controls from the set U, and the economic model (or system dynamics) F, the viability kernel $V_F^K(T)$ is the pink (darker shaded) ellipse. Trajectories that start inside the kernel (dashed lines) are viable as they remain in K. This is not the property of the other trajectories (dash-dotted lines) that start outside the kernel and leave K in finite time.

Definition 2.2 *Given the system correspondence $F(\cdot)$ (i.e., given the system dynamics f and sets of constraints K and U), the associated viability problem consists of establishing existence of the viability kernel $V_F^K(T)$.*

Remark 2.1 *If the kernel is nonempty $V_F^K(T) \neq \emptyset$, we say that the viability problem has a solution; otherwise, the viability problem has no solution.*[11]

Returning to our economic example, suppose x = capital, labour, technology, etc. we might reasonably believe that if these variables are initially close to their long-term steady state values $\bar{x}$ – that is if, in some measure, $x_0 \simeq \bar{x}$ – then there exist policy actions from U which return $x(t)$ to $\bar{x}$ in finite t when the system is buffeted by economic shocks. In this case, the viability kernel, $V_F^K(T)$, is the region in the neighbourhood of $\bar{x}$. In general, the viability kernel might be much larger than this neighbourhood, depending on the instruments available at x_0.

If policy instruments are time-dependent – that is, if $u = u(x(t),t)$ rather than $u = u(x)$ – then the state vector may evolve such that it satisfies the imposed constraints for all $t \leq T$, but exceeds the kernel's bounds for some $t > T$. In general, dynamic economic problems can be non-stationary. Solutions to such problems depend on when control is first applied to the evolution of the system.

2.2 Geometric characterisation

For simple dynamic systems, viability kernels can be described graphically.[12]

[11] If $V_F^K(T) = \emptyset$ then K is termed a repeller.

[12] See Aubin (1997), Aubin, Da Prato, and Frankowska (2000) and Cardaliaguet, Quincampoix, and Saint-Pierre (1999).

Consider a system that features linear dynamics defined as:[13]

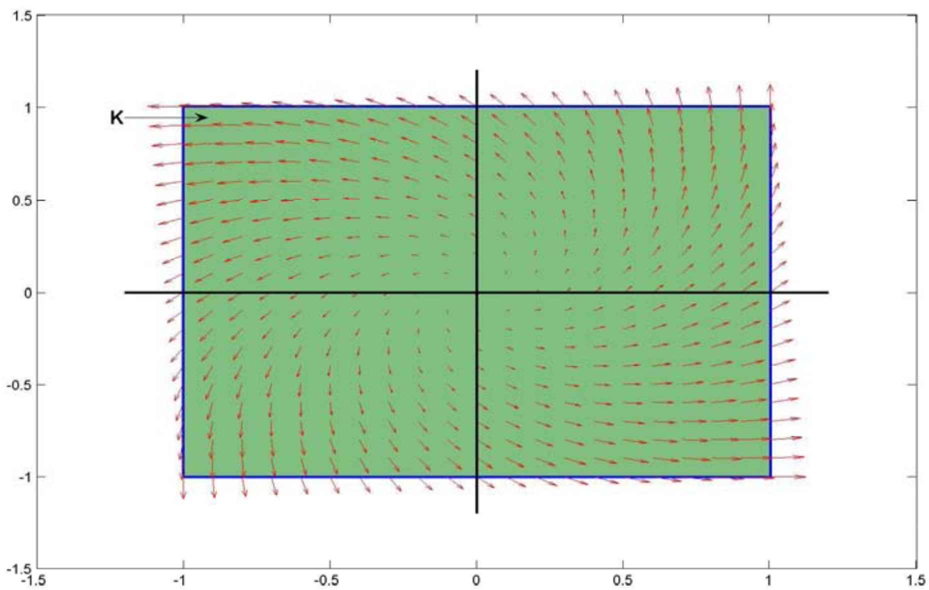
$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix}. \tag{5}$$

The state variables are (x,y) which are required to be constrained within the rectangle

$$K \equiv \left\{(x,y) \,:\, \max(|x|,|y|) \leq 1,\, (x,y) \in \mathbb{R}^2\right\}.$$

A vector field for the uncontrolled system (5) is shown in figure 2.

Figure 2
A vector field for the uncontrolled dynamic system (5)



From figure 2, we see that this system is inherently unstable – dynamic evolutions beginning at any point in K (excluding the origin) eventually lead to states outside K. In fact, the further (x,y) is from the origin the faster it leaves the constraint set.

Suppose (v_x, v_y) are controls available such that

$$U \equiv \left\{(v_x, v_y) : v_x^2 + v_y^2 \leq 1, \quad (v_x, v_y) \in \mathbb{R}^2\right\} \tag{6}$$

[13] Note that (5) evolves slower than the analogous examples in Cardaliaguet, Quincampoix, and Saint-Pierre (1999) and Krawczyk and Kim (2004). The change in the systems dynamics consists of dividing the equations' right hand sides by $\sqrt{2}$. We do it here for pedagogical reasons, to produce figures 3 and 4.

where v_x increases the speed of the state in direction x and v_y increases speed in direction y.

In an economic context if the state variables (x, y) denote inflation and the output gap respectively then, in the north-east corner of the state-space, the dynamics of the system above describe an 'overheating' economy that delivers output at levels well-above its potential.[14] A large output gap yields high inflation which results in a larger still output gap, all else equal. The standard monetary policy response in such a situation is to increase interest rates and thus provide restraint to the economy. However, such an action must be implemented in a timely fashion, before the output gap increases to such a large magnitude that it becomes an unstable process that is inadequately constrained by the set of available interest rate controls. The viability kernel for the system delineates those points in the (x, y)-space within which interest rate actions have a sufficiently meaningful influence on the output gap such that the economy does not 'boil over'.[15]

We now derive a kernel for the discrete version of the above system:

$$\left.\begin{array}{rcl} x(t+h) & = & \left(1+\frac{h}{\sqrt{2}}\right)x(t) - \frac{h}{\sqrt{2}}y(t) + hv_x(t) \\ y(t+h) & = & \frac{h}{\sqrt{2}}x(t) + \left(1+\frac{h}{\sqrt{2}}\right)y(t) + hv_y(t) \end{array}\right\} \tag{7}$$

where h is the time interval.

Figure 3, where B is a point on the frontier of the rectangle and figure 4 where A is an interior point of the rectangle, provide an example of how viability kernels can be determined. For this illustration we assume that $h = 1$ ie, the system moves discretely and the controls cannot be adjusted more often than at times $t = 1, 2, \ldots$.

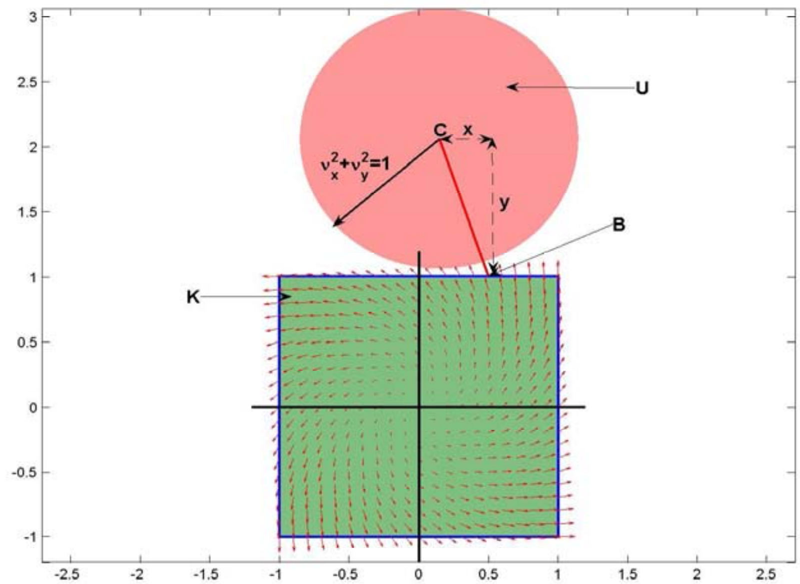
We first check if the rectangle K is itself a viability kernel.

In the absence of any control applied to the system, with $h = 1$, (x, y) evolve to the point at the end of the thick line starting at B, see figure 3. The set of points to which (x, y) can move, when driven by the controls available at B for $h = 1$ is denoted by the red-shaded circle. We see that there is no (ν_x, ν_y) control that returns (x, y) to K – there is no overlap between the control circle and the rectangular constraint. The same reasoning can be repeated at many points of rectangle K with the conclusion that the rectangle cannot be the viability kernel.

[14] We adopt the common interpretation of the output gap in this paper. It is the log deviation of actual output from a trend or potential level.

[15] In this example, the economy may be taken to 'boil over' whenever y exceeds 1.

Figure 3
A geometric characterisation of viability



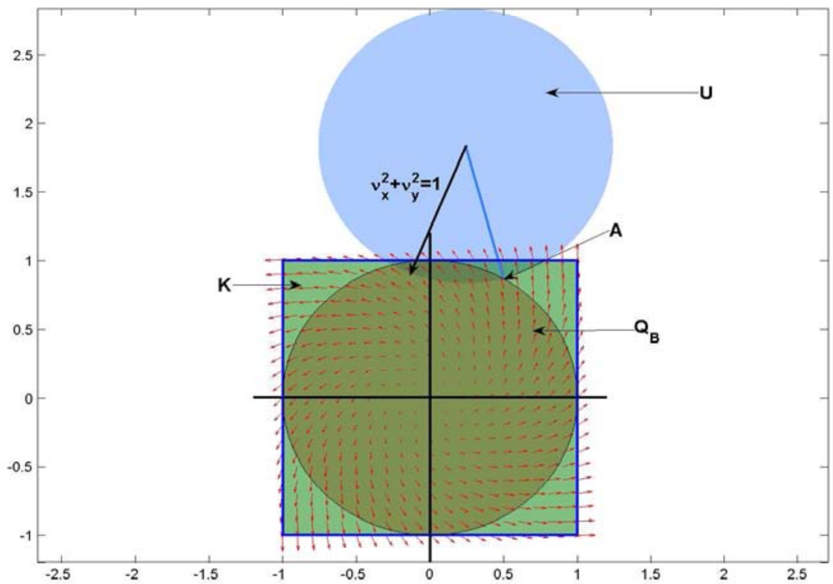
However, we can prove that the disc Q_B

$$Q_B \equiv \left\{(x,y) : x^2+y^2 \leq 1, (x,y) \in \mathbb{R}^2\right\}$$

(delimited by the circle of radius 1 centered at origin, see figure 4) is a viability kernel. From point A on the boundary of this circle the uncontrolled system evolves to the point at the end of the thick blue line. (See figure 4.) The set of points enabled by system controls available at A is denoted by the blue circle. There exist several $(v_x, v_y) \in U$ pairs that generate trajectories yielding evolutions to points inside K, and indeed some evolutions to points inside Q_B. This reasoning can be repeated at any point of disc Q_B. We can also prove that no viable trajectories begin at any point $(x,y) \in Q \backslash Q_B$ and so conclude that the disc Q_B is a viability kernel.

A comparison between sets K and Q_B (the latter is a viability kernel the former is not) reveals that at **B**, the system moves too 'fast' to be controlled through $(v_x, v_y) \in U$. However, the same control set contains elements that are sufficient to restrain the system, should we apply them 'early enough' *i.e.*, when the process is within Q_B.

Figure 4
A geometric characterisation of viability *continued*



3 Viable and optimising economic policy compared

In this section we illustrate the concepts discussed above in the context of a macroeconomic decision-making problem. Specifically, we use the analysis of monetary policy as a typical example from this class of problems to highlight both how viable policies may be established and how these differ from more-standard, optimisation-based analysis.

Consider economic situations in which state variables, control variables, and the planner who exercises the controls to manage the states can all be readily identified. As an example, standard monetary theory posits that societal preferences over intertemporal consumption – manifested implicitly as preferences over the state variables of inflation, output, employment and others – can be managed by a monetary authority exercising control over the money supply (either directly, or indirectly through reserve requirements or interest rates on short-term debt).

In this setup, the planner's problem is to manipulate the supply of money to maximise societal welfare or – as it is more commonly framed in the literature – to minimise a weighted loss function. In general equilibrium economic models, such optimisation yields prescriptive rules for interest rates that are ordinarily functions of the state variables. Several features of such optimising rules are worthy of note. First, it is obvious that these rules are optimal only in the context of a given economic model and are therefore unlikely to prove robust to uncertainty about the choice of model. Second, the estimates or the calibration of the parameters of such rules are sensitive to the parameterisation of the central bank's loss function, which is not known with any great precision.

Some of these concerns can be alleviated by considering a more restricted class of interest rate rules – the so-called simple rules (such as Taylor- and speed-limit-rules) where the policy interest rate is a function of one or two carefully chosen variables only. These variables are typically inflation and the output gap (for a Taylor rule) or the changes therein (for speed-limit or momentum rules). However, even these restricted rules suffer from the problem that they do not, by construction, allow for the incorporation of off-model information or policymaker judgment. Viable policies can help mitigate many of these problems.

How might we go about establishing and choosing between viable policies? Following the process introduced in section 2.2 we begin by establishing bounds on the evolution of the state and control variables. Next, a viability kernel is determined, after which they choice of a satisficing policy is itself simple – any one will do, so choose the one that best satisfies any off-model concerns.

More specifically, let us suppose that the desired constraints on the evolution of the economy can be summarised (at least in a two-dimensional sense) by the rectangle in figure 2. In the terminology of the geometric characterisation in the previous section, the interest rate controls are represented by the tuple (v_x, v_y). We claim that there exists a rule $(v_x, v_y)|v_x^2 + v_y^2 \leq 1$, which keeps the state (x, y) in $Q_B \subset K$. In particular, we know that even if $(x(t), y(t))$ is at the frontier of the viability kernel Q_B, the 'outermost' or *extreme* control is sufficient to prevent the system trajectory from leaving K.[16] It is obvious that for $(x(t), y(t))$ *inside* Q_B there are infinitely many (v_x, v_y) pairs that satisfy the criterion of being viable policies in that they deliver satisficing results.

If we denote $\mathsf{fr}\, D$ as the frontier of the viability kernel D and $x(t)$ as the entire state vector then, for deterministic models, the following satisficing policy prescription

[16] We use the term *extreme* for a control from set U that belongs to this set's boundary. The meaning of this definition is made clear every time a viability kernel will be determined. For example, in a continuous time version of figure 3, presented in Krawczyk and Kim (2006), the element of U that guarantees viability of Q_B is *extreme* because $v_x^2 + v_y^2 = 1$.

follows from the above:

$$\begin{cases} \texttt{if} \quad x(t) \in D \backslash \mathsf{fr} D, & \texttt{apply} \quad \underline{\texttt{any}} \ \texttt{control} \quad \in U; \\ \texttt{if} \quad x(t) \in \mathsf{fr} D, & \texttt{apply} \quad \underline{\texttt{extreme}} \ \texttt{control} \quad \in U \end{cases} \tag{8}$$

Optimisation by the central bank yields outcomes similar to that of the application of any of the satisficing policies: x is maintained in K. However, as we have seen, fewer (subjective) parameters are needed to establish D than to compute a minimising solution to the bank's loss function. Also, the 'relaxed' approach advocated by (8) (the first "`if`") offers the planner the possibility to strive to achieve other goals (eg, political), which are not used for the specification of K.[17] This is not the case for optimal solutions as they remain optimal for the original problem formulation only.[18]

4 The problem: Viable monetary control

4.1 The problem

It is not unreasonable to characterise a typical central bank as one that seeks to maintain a few selected macroeconomic variables within some bounds. This claim finds support in legislation, in government-delegated mandates, and in loss-functions for central banks commonly employed for monetary analysis. These target variables ordinarily include inflation and output and perhaps exchange and interest rates. Precise goals for these variables may often be specified in both levels and in growth rates.

As noted above, central banks usually realise their multiple targets using optimising solutions that result from minimisation of loss functions. The loss function typically includes penalties for violating an allowable inflation band, and may sometimes also include explicit penalties for excess volatilities in output and exchange rates, and for interest rate adjustment. The policy solution, which minimises the loss function, is unique for a given parametrisation of the bank's loss function. As such, alternative strategies are not allowed for.

[17] Perhaps such goals are difficult to specify mathematically or arise only after the viability kernel has been established.

[18] Also, note the precautionary character of the satisficing policies; see Krawczyk and Kim (2006) for further detail.

We now turn to the main goal of this paper: to establish a viability kernel for a stylised monetary policy problem as faced by the Reserve Bank of New Zealand. We establish the economic kernel, the subset of the constraint set K, inside which the evolution of the economy can be contained, given a set of reduced-form economic dynamics and instruments available to the central bank.

In the next section we describe a monetary rules model (based on that of Walsh 2003 and Svensson 2000) and estimate the parameters using historical New Zealand data on inflation, output gap and nominal interest rates. We then show how viable, satisficing, policies can be obtained in practise for such a model. Expanding on themes covered in the previous section, we also show in more detail how the solutions obtained through viability theory do not suffer from drawbacks typical of their optimising counterparts.

4.2 A model

The Policy Targets Agreement (PTA) negotiated between the government and the governor of the Reserve Bank of New Zealand currently requires the latter to maintain inflation between 1 and 3 per cent on average over the medium-term. Maintenance of this goal is subject to the further requirements that unnecessary volatility in output, interest and exchange rates be avoided. In the model below, we assume that the central bank uses a short-term nominal interest rate $i(t)$ as a policy instrument to control inflation $\pi(t)$ and, to a lesser extent, the output gap $y(t)$. Given our focus on the application of viability theory we use a popular, but simple, new Keynesian model. See Walsh (2003) for a derivation of this model from microeconomic constructs and an examination of its properties.

The model, a simplified version of Rudebush and Svensson (1999), is given by the equations

$$
\begin{aligned}
y(t) &= a_1 y(t-h) - a_2\Big(i(t-h) - E_{t-h}\pi(t)\Big) + \eta_y(t) && (9)\\
\pi(t) &= \pi(t-h) + \gamma y(t) + \eta_\pi(t) && (10)
\end{aligned}
$$

where $y(t)$ is the output gap, $\pi(t)$ is inflation, $i(t)$ is a nominal interest rate and $\eta_y(t), \eta_\pi(t)$ are serially uncorrelated zero-mean disturbances. The parameters a_1, a_2 and γ are estimated.

Equation (9) is an aggregate demand relationship. It corresponds to a traditional IS-curve where demand is inversely related to the real interest rate $r(t-h) =$

$i(t-h) - E_{t-h}\pi(t)$. Note that, in our aggregate spending specification, time-t spending depends on the lagged value of the real interest rate. Since monetary policy affects aggregate demand via the real interest rate, the assumption that time-t spending depends on the lagged real interest rate will imply a lagged response of output changes to monetary policy. This reflects a long-standing view that many macroeconomic variables do not respond instantaneously to monetary policy shocks. The interest rate relevant for aggregate spending decisions is the long-term rate, which is related to the short-term rate via the term structure relationship. To minimise the number of variables in our exposition, we do not distinguish between long-term and short-term interest rates.

Equation (10) captures an inflation adjustment process that is driven by the size of the output gap. In the canonical New Keynesian specification, current inflation $\pi(t)$ depends on expected future inflation $E_t\pi(t+h)$. Furthermore, in the Fuhrer and Moore (1995) model of multi-period, overlapping nominal contracts, current inflation depends on both past inflation and expected future inflation. However, several empirical works suggest that the expected inflation term is empirically unimportant once lagged inflation is included in the inflation adjustment equation.[19] In light of this equivocal result and to simplify our exposition, we ignore the expected inflation term.[20]

By applying the expectation operator E_{t-h} to both (9) and (10), we obtain

$$E_{t-h}y(t) = a_1 E_{t-h}y(t-h) - a_2\Big(E_{t-h}i(t-h) - E_{t-h}\pi(t)\Big) \tag{11}$$

$$E_{t-h}\pi(t) = E_{t-h}\pi(t-h) + \gamma E_{t-h}y(t) \tag{12}$$

At time $t-h$, expectations are identical with observations so,

$$E_{t-h}y(t) = a_1 y(t-h) - a_2\Big(i(t-h) - E_{t-h}\pi(t)\Big) \tag{13}$$

$$E_{t-h}\pi(t) = \pi(t-h) + \gamma E_{t-h}y(t). \tag{14}$$

Assume differentiability of the inflation and output gap processes. If so, for small h

$$E_{t-h}y(t) = y(t-h) + \dot{y}h \tag{15}$$

$$E_{t-h}\pi(t) = \pi(t-h) + \dot{\pi}h \tag{16}$$

[19] See, for example, Fuhrer (1997).

[20] Given that the output gap enters contemporaneously in the Phillips curve, it is necessary to ensure that our system is identified. We verify that both the order and rank conditions for identification is satisfied. Note that we have imposed the accelerationist property on the Phillips curve in that the lag of inflation is restricted to have a co-efficient of one. The implication, therefore, is of a Phillips curve that is vertical in the long run.

where the derivatives are computed at time $t-1$. From these it is apparent that agents form expectations by extrapolating from past values of the state variables. This corresponds to a basic learning process.[21]

Substituting in (13) and (14) (and omitting the time index $t-h$) yields:

$$y+\dot{y}h = a_1 y - a_2\Big(i-(\pi+\dot{\pi}h)\Big) \quad (17)$$

$$\pi+\dot{\pi}h = \pi+\gamma(y+\dot{y}h). \quad (18)$$

From (18), $\dot{\pi}h=\gamma y+\gamma\dot{y}h$. Defining $\alpha h = a_1-1, \quad \xi h = a_2, \quad \zeta h=\gamma$, dividing by h and taking the limit $h\to 0$ we arrive at the following differential equations which describe the dynamic evolution of inflation and the output gap:

$$\frac{dy}{dt} = \alpha y(t) - \xi\Big(i(t)-\pi(t)\Big) \quad (19)$$

$$\frac{d\pi}{dt} = \zeta y(t). \quad (20)$$

These equations are continuous time equivalents of the aggregate demand equation (9) and the Phillips curve (10). They state that the output gap evolves as a "sticky" process (19) driven by the real interest rate and that inflation (20) changes proportionally with the output gap.

The model embodied by (19)-(20) provides insight into the formation of expectations of inflation and the output gap by private sector agents. For example, positive and increasing output gaps imply that the excess of $i(t)$ above its neutral rate will cause the output gap to close (and eventually turn negative). Further, inflation will not begin to decrease as long as the output gap remains positive.[22] Note that it is possible for the central bank to exploit the inertia of controlled processes to constrain their evolution.

5 Estimating the model

There are severe limitations in taking the above model to the New Zealand data. The equations describe a closed economy, with no role for exchange rates or other external economy influences – omissions that are ordinarily untenable in models

[21] See Honkapohja and Mitra (2006).

[22] In this simple model, inflation grows for any positive output gap. However, adding an exchange rate to the model as in Krawczyk and Kim (2004) shows that this may not always be the case.

of the New Zealand economy. However, we persist with this model for two reasons. First, this note presents an illustrative application of viability theory and, as such, we are keen to reduce the dimensionality of the problem to its bare minimum to allow for clearer exposition. Second, we believe that even as a very basic representation of the New Zealand economy, viability analysis applied to the model above yields lessons that are informative for central bank decision-makers.

5.1 New Zealand historical data

There are several options available in moving from the theoretical model to its empirical counterpart. We initially considered defining π_t as non-traded inflation on the basis of the closed nature of the model. Traded and non-traded components of inflation also exhibit very different dynamics, and the use of non-traded inflation in estimation would likely yield better fit since it is more immune to exchange rate effects, which have been ignored in this model.[23] However, one important disadvantage of using non-traded inflation rather than the aggregate CPI is that the former does not map directly to the Bank's inflation target.

We choose to use the output gap resulting from a multivariate-filter (MV) rather than a more standard HP-filtered gap. The MV-gap incorporates a HP-type filter together with an unemployment gap and the level of capacity utilisation.[24]

For inflation expectations we have the choices of using either data from the Reserve Bank's Survey of Expectations, *assuming* some learning about the policy formation process, or alternatively the choice of assuming rational expectations. There also remain the possibilities of assuming perfect foresight or that the expectations formation process is entirely backward looking. It is important to note that surveyed expectations measure CPI inflation rather than non-traded inflation. Given the desire to keep the model dynamics simple, the assumption of rational expectations is unsuitable since it adds an extra state variable to the analysis.

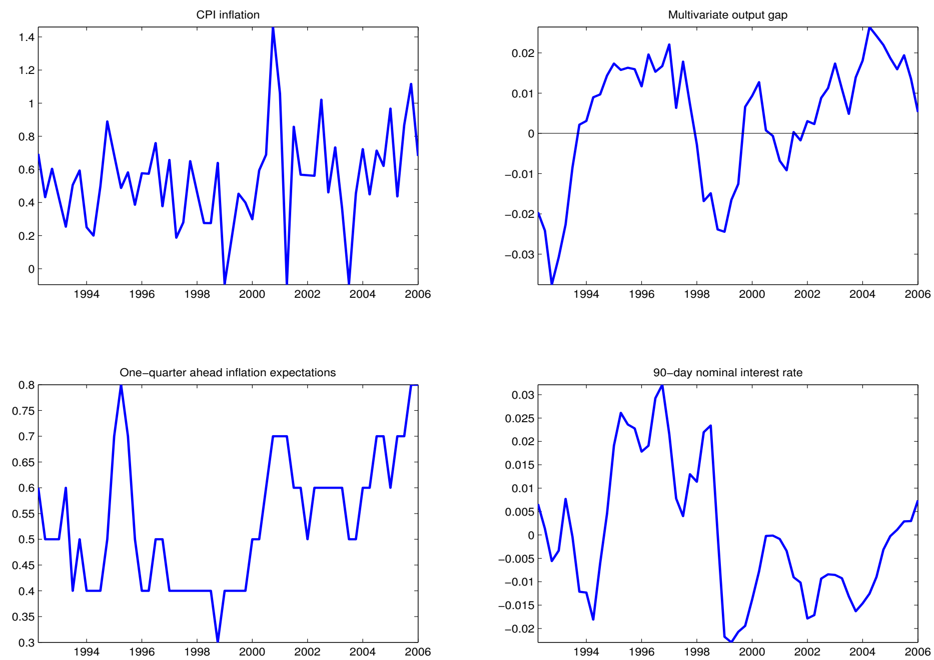
With these points in mind, the following data definitions are used: $\pi(t)$ is CPI inflation, $y(t)$ is a multivariate (MV) output gap, and $i(t)$ is the nominal 90-day interest rate which is a good proxy for the actual policy interest rate, the official

[23] There is also a well-documented link between non-traded inflation and the the Reserve Bank's multivariate output gap (see Hargreaves, Kite, and Hodgetts 2006.) This relationship is a cornerstone of the Bank's main macro-model for forecasting and policy analysis. However, the relationship aggregate CPI and the output gap is less obvious. In future research we intend to extend the model by including the exchange rate.

[24] See Conway and Hunt (1997).

cash rate. Finally, given the discussion above on the learning elements in the formation of expectations, we use one-quarter ahead inflation expectations from the Reserve Bank's survey.[25] Figure 5 charts the series over the sample period 1992Q1 to 2005Q4.

Figure 5
Estimation data



The model is framed in gap terms and a simple de-meaning of the data is used to render the model stationary. However, this mean is latter added back to the fitted variables in our analysis of the viability kernel since we prefer to use these variables in levels as they relate more closely to those that the policymaker actually has preferences over.

[25] We do not observe any significant difference in the estimation results from using backward looking expectations or perfect foresight. Additional estimation details are available on request.

Table 1
Estimated parameters

Parameter	Estimate	T-stat	PC summary statistics		IS summary statistics	
a_1	0.8897	15.04	R-squared	0.0343	R-squared	0.7961
a_2	0.1208	-1.93	SE of regression	0.1497	SE of regression	0.4849
γ	0.047	1.43				

5.2 Estimation results

We estimate the model using OLS. Results are presented in table 1 and the fit of the data to the two model equations (together with estimated residuals) is shown in figure 6.

The parameter estimates for the Phillips curve are lower than previously estimated on New Zealand data. For example, Lees (2003) finds that the coefficient on the output gap in the Phillips curve is 0.15 when estimating a Phillips curve with similar long run properties to ours.[26] Note the unsatisfactorily low explanatory power of the curve, which we believe is likely due to the closed economy nature of the model, and also perhaps to the absence of a forward-looking element. Future work that includes the exchange rate as a state variable may help improve the fit of the Phillips curve.

The estimated aggregate demand equation delivers much better fit and is broadly plausible. Other Reserve Bank studies report similar estimates.[27]

5.3 The residuals

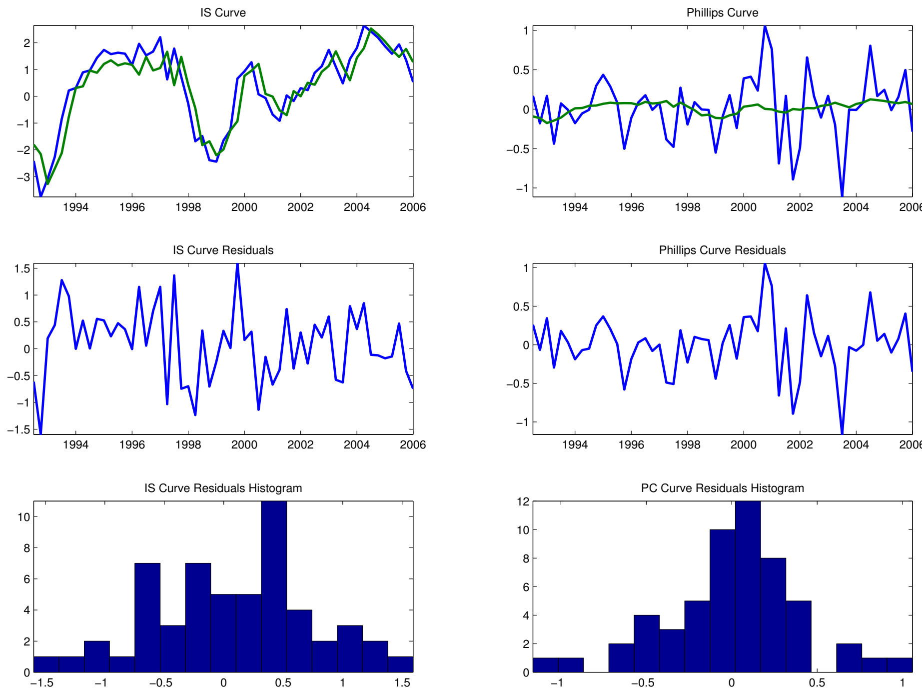
The estimated size of the residuals standard error – 0.5 percent for the IS curve and 0.15 percent for the Phillips curve – matches well with the empirical standard deviations of the output gap and inflation, and thus provides mild assurance that the model is a reasonable first approximation to the New Zealand economy.

Consistent with the fact that inflation has been more volatile in the second-half of our sample period, the Phillips curve residuals too exhibit greater variation over

[26] More specifically, Lees (2003) uses GMM and estimates a coefficient of 0.077 on both the contemporaneous and the first lag of output, yielding a total coefficient 0.152 on the output gap.

[27] See Hargreaves, Kite, and Hodgetts (2006) and Lees (2003).

Figure 6
Model fit and residuals



this period. Given the relative absence of clear outliers in the residuals, we do not believe that any one observation is overly distorting the fit of the model.[28] Tests of first-order autocorrelation in the residuals for both curves are rejected at the five per cent level.

[28] This claim is supported by calculating the leverage statistic – that is the contribution that each observation makes to the vector of parameter estimates – using the method of Davidson and McKinnon (1993). Eight percent of all observations have a leverage that is greater than the suggested threshold value of 0.1.

6 Completing the model: Constraints and controls

6.1 The constraints

As we have noted, the Reserve Bank is charged with maintaining inflation between 1 and 3 per cent on average and over the medium term. We impose the more convenient – and more stringent – requirement that this target must be met in every period rather than as a multi-period average.

Unlike inflation, the Bank has no precise politically mandated goals with respect to the output gap, other than a 'secondary' requirement that unnecessary instability in output be avoided as the Bank seeks to meet its primary inflation target goal. Consequently, we assume a rather wide interval for output gap $y(t) \in [-0.04, 0.04]$ to reflect this secondary concern.[29]

As with the desired size of the output gap, there is no formal goal for interest rates that is delegated to the Reserve Bank. We assume that $i(t) \in [0, .077]$ which reflects both the theoretical zero lower-bound on nominal interest rates and a 'reasonable' upper limit.[30]

The constraint set $K \subset \mathbb{R}^3$ defined as

$$K \equiv \{(y(t), \pi(t), i(t)) : \tag{21}$$

$$\{-0.04 \leq y(t) \leq 0.04, 0.01 \leq \pi(t) \leq 0.03, 0 \leq i(t) \leq 0.077\}, \tag{22}$$

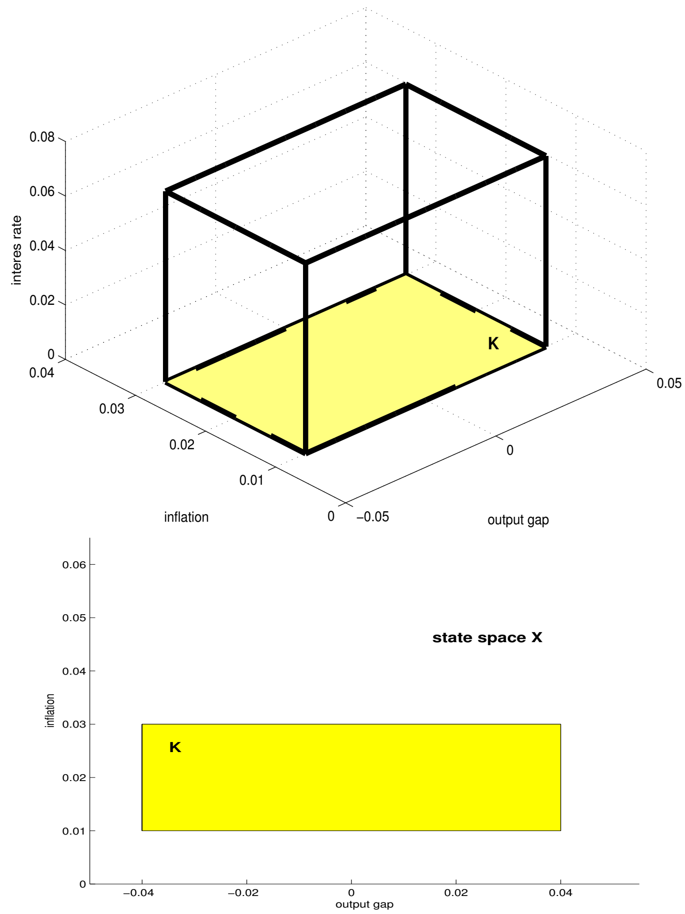
summarises the constraints within which the trajectory of the meta-system is required to be contained. This is shown in figure 7, with a two-dimensional projection in (π, y)- space in the bottom panel.

Together with preferences over inflation and the output gap, central banks also typically have concerns about the degree of interest rate smoothness (see e.g., Amato and Laubach 1999). That concern is usually modelled by adding the term

[29] To more accurately capture the spirit of the Policy Targets Agreement, this constraint might perhaps be better stated in terms of the first-difference of the output gap rather than its level.

[30] Nominal 90-day interest rates have touched 10 percent within the sample period, in June 1996; and the average of the series is 6.75 percent with a standard deviation of 1.5 percent. We use an 'intermediate' upper bound of 7.7 percent on interest rates. At first glance, this is an odd choice of an upper bound given that it is within one standard deviation of the sample mean. However, an upper bound of 7.7 percent on the interest rate means that the fastest drop policy leads to steady-state equilibrium even when the economy 'begins' evolution at a point where the output gap is at its allowable maximum. Consequently, choosing a higher value for the upper bound on interest rates is meaningless in the context of our parameter estimates.

Figure 7
Constraint set over inflation, output and interest rates and its 2-dimensional projection



$w\Big(i(t)-i(t-h)\Big)^2$, $w>0$ to the loss function. In continuous time, limiting the interest rate changes (or 'velocities')

$$u \equiv \frac{di}{dt} \tag{23}$$

to a "small" interval produces a smooth time-profile for $i(t)$. The Reserve Bank's policy announcements relate to changes in interest rates and we treat u as the bank's control variable. Changes in the policy instrument are usually made every six weeks and a typical change, if made, is 0.25 percent. We use quarterly time divisions in this analysis and the bank's control set U can thus be defined as

$$U \equiv \{u : u(t) \in [-.005, .005]\} \tag{24}$$

i.e., the interest rate can move by a maximum of 0.5 percent in every quarter.

Hence, to analyse the relationship between the interest rate, inflation and output gap, the dynamic system is augmented with the interest rate velocity constraint. Allowing for the estimation results of section 5.2 and the relations between the discrete-time and continuous-time model parameters (see section 4.2)

$$\alpha = -.1103, \quad \xi = 0.1208, \quad \zeta = 0.047$$

we get the following inflation and output gap dynamics:

$$\frac{dy}{dt} = -.1103\,y(t) - 0.1208\,(i(t) - \pi(t)) \tag{25}$$

$$\frac{d\pi}{dt} = 0.047\,y(t). \tag{26}$$

$$\frac{di}{dt} = u \in [-.005, .005]. \tag{27}$$

Having fully specified the setup we see that a viability model does indeed require fewer subjectively assessed parameters than the corresponding optimisation model. In particular, it does not require the explicit specification of the weights on the Reserve Bank's loss function. Further, a discount rate on future losses is not required and no parameters on a policy reaction need be estimated. The bounds of the constraint set are either legislated or identifiable in a reasonably non-controversial manner. If there is less concern for limits of a particular variable, say, for exchange rate volatilities, then they can be set to be arbitrarily 'large'.

Consequently, the boundaries of the constraint set convey information about the desired evolution of the economy in an objective and straightforward fashion.

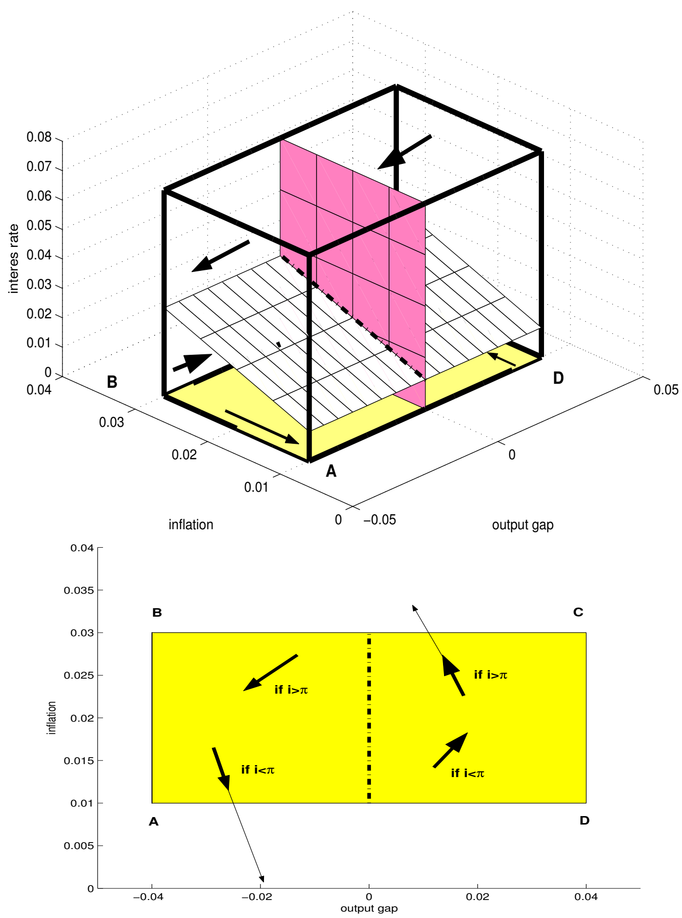
7 Viable solutions for a deterministic economy

In this section we assume that the economy described by the equations (25)-(27) is deterministic and show how viability theory can be usefully employed to analyse two common monetary control problems. Then, in section 8 we consider an economy with the same system dynamics, but this time subject it to shocks that are distributed as identified in section 5.

7.1 Steady state and transition analysis

Examining (25), (26), and (27) it is apparent that the output gap must close in steady state and that the nominal interest rate must equal the inflation rate. In the context of the desired bounds on $\{\pi, y, i\}$, represented by the cuboid K in Figure 7, we add the planes $y = 0$ and $i = \pi$ to graphically assess the steady state. The intersection of the planes (marked by the dash-dotted line) is the steady-state of the model economy and the phase diagram in the bottom panel summarises the system's evolution in the (π, y) plane.

Figure 8
Constraint set K and steady states.



At any point where the output gap is negative ($y < 0$) inflation decreases (arrows point south); conversely, inflation increases if the output gap is positive (arrows point north). When $i > \pi$ interest rates dominate inflation causing the output gap to decrease (arrows point left above $i = \pi$). In sum, for moderate (non-accelerationary) values of y, the output gap diminishes whenever the real interest rate is positive.

Two interesting monetary control problems can be analysed with the help of figure 8. First, we consider a stretched economy where the output gap is positive and inflation and interest rates are near their respective upper bounds.

Second, we consider the situation wherein the output gap is negative and inflation is close to its lower limit such that there is little prospect of the output gap closing of its own accord. If the bank lowers a high nominal interest rate 'too late', a positive real interest rate will cause a further decrease in the output gap and inflation may decrease to zero – a region of the state-space where no instrument exists to lift output. If so, the economy may experience a *liquidity trap*, remaining mired in an area where output gap is negative and inflation is close to zero.[31]

7.2 A viability kernel for the New Zealand economy

We now apply some principles from viability theory to analyse these two situations in greater detail and begin by determining the collection of points where the control policy, chosen from U, is sufficient to retain the economy inside K – the constraint set.

An overheating economy

We characterise an 'overheating' economy as one that unsustainably operates at levels above its potential; that is, one that features a large positive output gap together with inflation and interest rates significantly higher than their steady-state norms. Such a situation is depicted in the north-east corner of the projection of the Reserve Bank's constraint set K in (π, y)-space, near point (C). (See figures 8 and 9.)

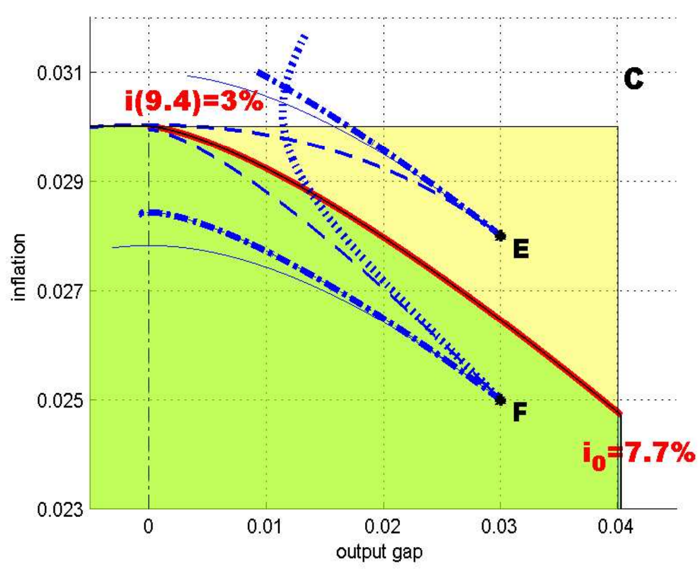
What should the Reserve Bank's time-T target be in such situations? We assume that the Bank is interested in engineering a 'soft landing' for the economy – an output gap that closes to zero without slipping into negative territory for an extended period (which may pre-date a possible recession) and an inflation rate that

[31] See McCallum (2004) for an analysis of a liquidity trap problem performed through an established method. Also see Nishiyama (2003) for a study where a liquidity trap problem is analysed in state space.

is less than its 3 percent upper bound. Since the output gap will continue to shrink for as long as the real interest rate is positive, forward-looking monetary control that seeks to achieve the above target requires gradual and continuing decreases in the interest rate to slow the *rate* at which the output gap closes as it approaches zero. In fact, when $i(T) = \pi(T) = 0.03$, the Bank arrives at a steady state at which pressures on output vanish. To summarise, whenever the economy is at $\pi(0) \simeq \bar{\pi}, y(0) >> 0, i(0) \simeq \bar{i} >> \bar{\pi}$, the Bank's 'soft-landing' target can plausibly be $\pi(T) \leq \bar{\pi}, y(0) \simeq 0, i(T) \simeq \pi(T)$ where $\bar{z}$ denotes the upper bound on variable z.

How can the policy instrument – the change in the interest rate, u – be used to obtain such an outcome? Recall that in the north-west corner of the constraint set under consideration, $i > \pi$ and that if this inequality is maintained, the output gap will eventually become negative, violating the Bank's time-T target. The Bank must therefore determine the set of strategies that decrease i down to π while ensuring that $y(T) \to 0$. The maximum allowable interest rate decrease in every period is $u = -.005$. To calculate the limiting bound of the viability kernel we apply this maximum-drop strategy and run (25)-(27) backwards from $(\pi(T) = .03, y(T) = 0, i(T) = .03)$ with $u(t) = 0.005$. This limiting bound is the solid red line in figures 9 and 10.

Figure 9
Evolution trajectories for an overheating economy.



That this line is the limiting bound is easy to see: system trajectories starting at all points in K above the red line, violate the upper bound on inflation in finite time. For example, if at point (E), where $(\pi(0) = 2.8\%, y(0) = 3\%, i(0) = 7.1\%)$, we apply the fastest-drop interest rate policy then the economy exceeds the inflation upper bound of 3 percent (in less than three quarters). See the dash-dotted trajectory starting at (E) in figures 9 and 10. There is no policy compatible with the Bank's interest rate smoothing preferences that can stabilise the economy if it has reached (E). If the interest rate remains unchanged, the economy also leaves K (in less than two quarters); see the thin solid line in figures 9 and 10. To keep inflation in check, the nominal interest rate will need to *rise* to 14 percent at the point when $y = 0$ (in about 2.5 quarters), in which case the economy follows the dashed trajectory originating at (E), see figures 9 and 10. However, this is not a viable trajectory as the increase of the interest rate is faster than 0.5 percent a quarter. Further, the positive real interest rate soon prompts a large negative output gap.

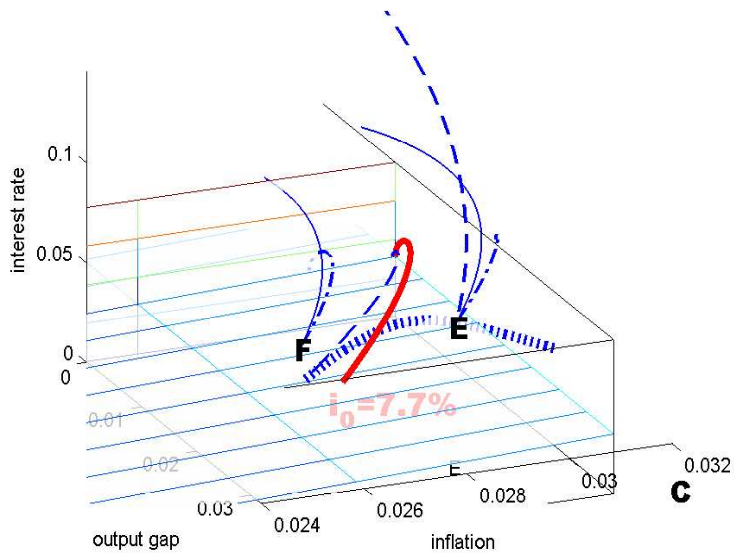
Alternatively, the desired stable target $y(T) = 0, \pi(T) = .03, i(T) = .03$ is achievable via a variety of trajectories that start from points below the red line. At point (F) where inflation is only 0.3 percent less than at (E) (i.e., i(0)=7.1%), the fastest-drop interest rate policy achieves $y = 0$ in less than 7 quarters with $\pi(6.75) = 2.84\%$. Then, the economy very slowly crosses the neutral output gap and loops back. This is because the real interest rate becomes marginally negative, which stimulates output gap to grow. For this evolution, see the thick dash-dotted line emanating from (F) in figures 9 and 10. If the interest rate remains unchanged, the economy reaches $y = 0$ in 4.5 quarters. This is faster than before because the real interest rate decreases more slowly, thus supressing the output gap faster. [32]

A smooth evolution of the economy from (F) to the steady state can be achieved if the interest rate is lower than 7.1 percent at $t = 0$. The dashed line shows a trajectory from F (where $i(0) = 5.23\%$) that leads to steady state equilibrium in about 12 quarters. The interest rate decreases by 18 basis points per quarter, slower than the maximal allowable decrease.

Interestingly, there also exist trajectories starting at (F) that violate the inflation upper bound. This can happen when monetary policy is eased excessively fast: the real interest rate in turn quickly becomes negative and the output gap grows. Inflation increases and pushes the economy away from the target and outside the constraint set. See the dashed trajectory beginning at (F).

[32] Once the output gap is closed, the bank may choose to start decreasing the interest rate of 7.1% and to achieve a sign reversal of real interest rate. From this point on, the output gap will grow.

Figure 10
3-dimensional evolution trajectories for an overheating economy.



To summarise, we see that the solving for the viability kernel explicitly accounts for all constraints that we wish to impose on the evolution of the state variables, and then demarcates the set of policies which achieve a desired outcome. The actual policy chosen from this set is not important in itself and, as mentioned earlier, can be determined on the basis of other goals.

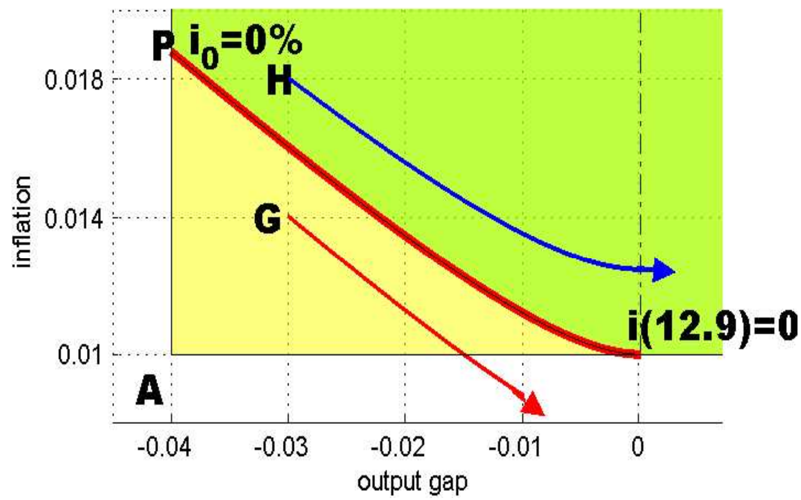
Liquidity trap

As another example, we now determine the viability kernel's boundaries in the south-west corner of K, in the region of point (A) in figure 8. As before, we begin with a discussion of the possible goals of the Reserve Bank when the economy is under threat of 'recession' ($y(t) << 0$), and when inflation and interest rates are low such that the realisation of a sizeable stimulatory negative real interest rate is difficult. Given the structure of the economy and the parameter estimates, $y(0) < 0$ corrects to $y(T) = 0$ only for large T when real interest rates are negative but close to zero.

However, even if the nominal interest rate is zero, small positive inflation creates a small negative real interest rate, which can stimulate growth. Of interest to the

Bank thus is the identification of economic states from which the limiting zero nominal interest rate policy guarantees a recovery without prompting a slide into deflation.

Figure 11
Evolution trajectories for an economy facing a liquidity trap.



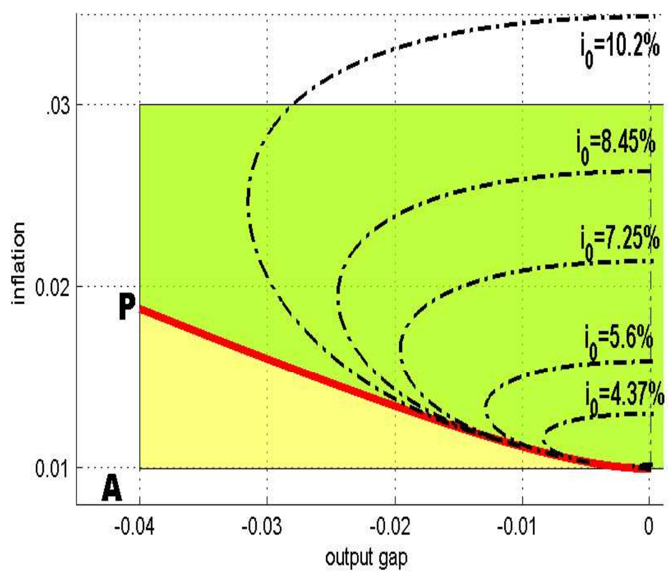
In figure 11 the thick red line denotes the limiting bound to the kernel computed by applying the zero interest rate policy i.e., $i_0 = i(T) = 0$. Along this bound, the economy evolves from $y(0) < 0$ to $y(T) \geq 0$ where $T = 12.9$ quarters and inflation decreases from about 1.9 percent to 1 percent, its lower limit. To calculate the bound, the model economy given by the equations (25)-(27) is 'run' backwards from $y(T) = 0, \pi(T) = .01, i(T) = .0$ with $u = 0$ until the lower bound on the outgap gap is exceeded. The bound delimits south-west portion of the viability kernel (in green). Inside this kernel, the application of the zero interest rate policy guarantees achievement of a positive output gap in finite time. Any state to the left of the solid line does not have this property.

At point (G), $(\pi = 1.4\%, y = -3\%)$ the largest (negative) real interest rate that can be achieved is -1.4 percent – too little to stimulate the economy given the inertia in the demand process. The thin red solid line beginning at (G) illustrates this evolution, and leaves the constraint set K with $y < 0$ in about 4 quarters.

Conversely, at point (H), $(\pi = 1.6\%, y = -3\%)$, we apply the same policy and the output gap is closed in 10 quarters. After that, inflation increases and output gap grows further as shown by the thin solid (blue) line in figure 11.

However, the ‘current’ interest rate (that is, the one from which the Bank begins its fight against recession) may be ‘high’ and dropping it to zero may create an undesirable shock if (27) is not satisfied. If such a shock is to be avoided by smoothing interest rate adjustment, then a negative real interest rate cannot be realised instantaneously and further shrinking in the output gap is likely to take place. Depending on the initial interest rate, the degree to which cuts in the interest rate are smoothed, and on the strength of the response of the output gap to the interest rate, this secondary shrinkage phase in the output gap may encompass an especially lengthy period. In fact, violation of the lower bound on the output gap is imminent if the initial interest rate is high relative to inflation measured at the same time. It is therefore important for the Bank to identify the states of the economy from which the fastest-drop interest rate policy moves the economy to point (P) (or to the separating trajectory (P)$\rightarrow$ 0.01 with a final interest rate equal to zero. From (P) or from the separating trajectory, the zero interest rate policy will lead the economy to recovery.

Figure 12
Evolutions from alternative starting points for maximum-speed recovery.



Consider the economy’s evolution from initial situations that feature low inflation, dominant interest rates, and small negative output gaps. Figure 12 shows several trajectories that bring the economy from various high interest rates to a zero interest rate with maximal speed. For all such trajectories, the output gap shrinks before it starts growing again. Finally, we note as an aside that keeping the economy within the viability kernel may help prevent a liquidity trap. This is perhaps made clearer by considering the situation in figures with an additional interest rate dimension.

In figure 13 the cuboid is the constraint set K for the desired evolution of the system (25)-(27) as before and the thick red lines belong to the viability kernel. These were obtained for the two situations – an overheating economy and a liquidity trap – that we have just considered.

Figure 13
Sample evolutions: a 3D perspective

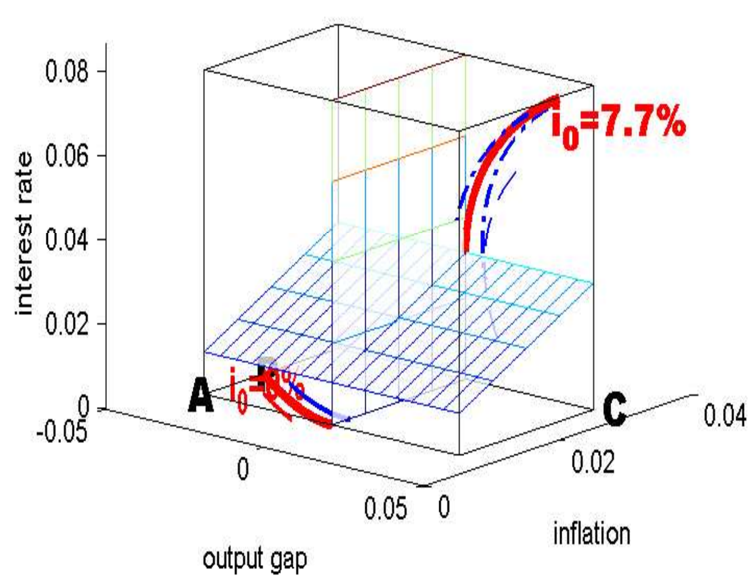
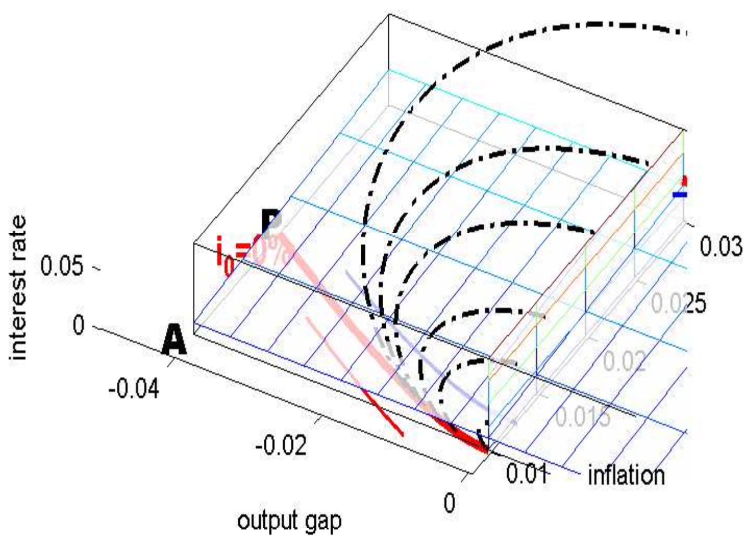


Figure 14 is the 3-dimensional analogue of figure 12 and provides an enlargement of the set K around corner (A). Various trajectories commence from different initial interest rates, showing the state of economy as it is controlled towards a zero interest rate, building up a 'cone of viability'. If the economy is to recover speedily from liquidity trap-type situations then the state must remain inside the intersection of the cone with K.

In other words, the cone's interior contains those points of K, from which viable evolutions are possible. Hence the interior of the cone is the (projection in $\mathbb{R}^2$) of the viability kernel near corner (A).

A caution for the conduct of monetary policy is apparent from the above analysis: in the context of this admittedly simple model, even mildly negative output gaps can lead to a liquidity trap if inflation is low relative to the interest rate. This is manly due to inertia of the economic processes considered here. Keeping the process evolution inside a viability kernel guarantees that the instrument (here: the nominal short term interest rate) will be applied sufficiently early so that uncontrollable fallout can be avoided.

Figure 14
A 'cone' of viability



8 Viable solutions for an economy subjected to shocks

8.1 Time-line for a stochastic economy evolution

Conditional on the estimated distribution of the shocks to the inflation and output gap processes the following plan for the management of a stochastic economy may be appropriate:

- at time t, data on output gap, inflation and nominal interest rates are known;[33]
- conditional on the data, the increment in interest rates for the next period is decided on;
- the economy evolves according to (25)-(27);
- at time $t+1$ the economy is subjected to a two-dimensional shock $\eta_\pi(t+1), \eta_y(t+1)$ from the distribution of the residuals (see section 5.2).

Let the deterministic values of the output gap and inflation at time $t+1$ be denoted $\underline{y}(t+1)$ and $\underline{\pi}(t+1)$, respectively. Then, the distributions of the states $y(t+1)$ and $\pi(t+1)$ at $t+1$ are simply those of the residuals centred at $\underline{y}(t+1), \underline{\pi}(t+1)$.

By restricting attention to the shock realisations from the intervals $\pm\sigma_y$ and $\pm\sigma_\pi$ where σ_y and σ_π denote estimates of the standard deviations of the shocks, it can be claimed that, at $t+1$, the economy will quite likely be at:

$$\tilde{y} \equiv \underline{y}(t+1) \pm \sigma_y, \qquad \tilde{\pi} \equiv \underline{\pi}(t+1) \pm \sigma_\pi. \tag{28}$$

If the stochastic economy is viable then we should be able to choose a path for the interest rate $i(\tau), \tau \in [t, t+1]$ such that $[\tilde{y}, \tilde{\pi}, \tilde{i}] \in K$ (where $\tilde{i} = i(t+1)$). In other words, a sufficient condition for the viability of the economy is that $\forall t \in \Theta$, there exists an interest rate path such that

$$[\tilde{y}, \tilde{\pi}, \tilde{i}] \in V_F^K \subset K\,. \tag{29}$$

The feasibility of this approach depends on how large σ_y, σ_π are relative to $V_F^K \subset K$.

In the next section we present an example of a viable policy analysis of the New Zealand economy represented by the simple model considered in the paper.

8.2 Monetary policy making in a stochastic environment

Table 2 presents the observed New Zealand data on inflation and interest rates between 2004Q2 and 2004Q4, together with *ex post* estimates of the prevailing multivariate output gap.

[33] Given lags in the publication of economic, for example, inflation data is known only with a one quarter lag, and output data with a two quarter lag. However, it is not unreasonable to assume that a policymaker will have ‘good’ estimates of these variables available in the near term.

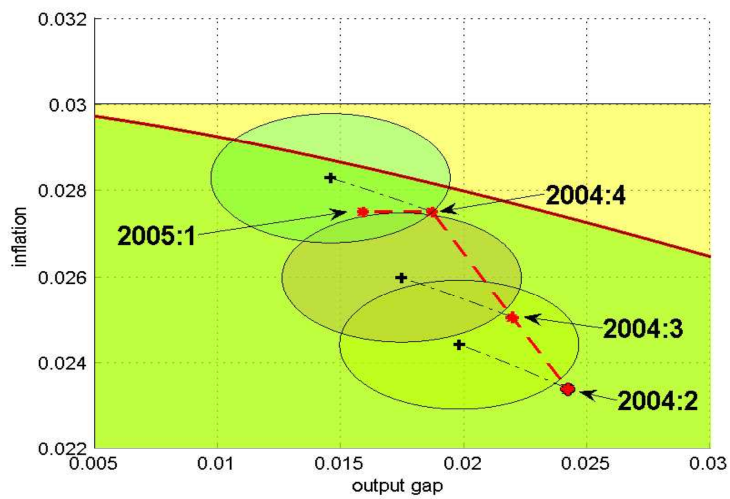
Table 2
Historical data 2004Q2 — 2004Q4.

quarter t	Output Gap $y(t)$	Inflation $\pi(t)$	Interest Rate $i(t)$
2004Q2	.024	0.023	.0585
2004Q3	.021	0.025	.0644
2004Q4	.018	0.027	.0672

The shock standard deviations σ_y and σ_π estimated in section 5.2 are 0.0048 and 0.0015, respectively.

The last point on the right in figure 15, marked '$*$' (red), denotes New Zealand inflation and output in the second quarter of 2004.[34] All subsequent '$*$' (red) indicate actual outcomes while all '$+$' (black) represent the model (deterministic) economy with no shocks.

Figure 15
Viable monetary policy for New Zealand in 2004



The nominal interest rate change between 2004Q2 and 2004Q3, at 0.58 percent,

[34] This image is a projection of the 3D representation pictured in Figure 16. Note that we have added back the mean inflation rate of 2.11 percent. Therefore the real interest rate which characterises the economy at this quarter, is $.0585 - .0211 = 0.0374$

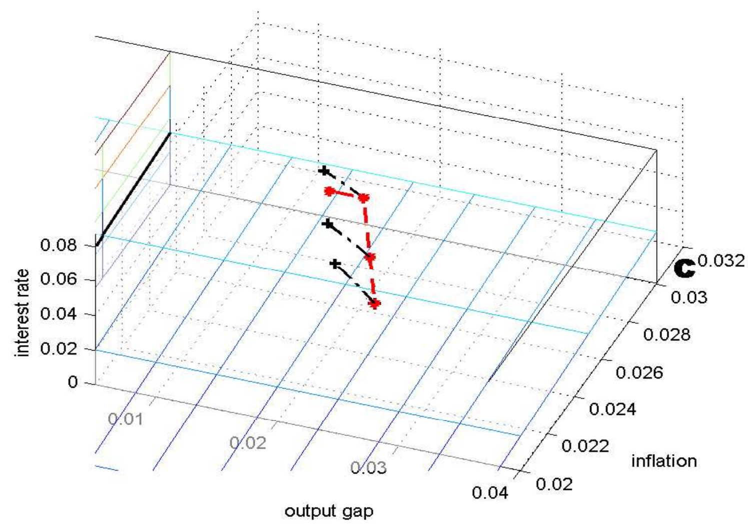
was only slightly above the constraint (24).[35] According to the model the deterministic economy moves to the point marked by the closest '+' in the next quarter. At this time, the economy is subjected to the shocks $(\eta_\pi(2004Q3), \eta_y(2004Q3))$. Assuming that the actual shock sizes are confined to within one estimated standard deviation, we expect the real economy to be in the yellow ellipse, whose major and minor axes equal σ_y, σ_π, respectively. The observed historical state of the economy in 2004Q3 is represented by the red '$*$'. We see that in this period the real (ie, stochastic) economy is within the one-standard deviation 'ball' of the historical shocks.

From this figure it appears that the interest rate decision in 2004Q2 was 'correct' insofar as a slower increase in the interest rate would have been likely to render the economy dangerously close to the viability kernel boundary. As noted in section 7.2, a deterministic economy which crosses the kernel boundary is destined to violate the constraint set K. In a stochastic sense, it is even more important for an economy to remain 'well-inside' the kernel boundary.

Now consider the evolution of the economy in the subsequent quarter i.e., from 2004Q3 to 2004Q4. The nominal interest rate change is 0.29 percent, well below the constraint (24). The model (25)-(27) dictates that the "deterministic" economy moves to the point marked by the next "+". At this time, the economy is subjected to the shock $(\eta_\pi(2004Q4), \eta_y(2004Q4))$. Again, assuming that the shock size is confined to within one standard deviation, we expect the real economy to be in the purple (darker) ellipse. The observed historical state of the economy in 2004Q4 is represented by the next red '$*$': the economy is within the expected range but very close to the boundary of the viability kernel. Over this period, the central bank signalled a pause in the cycle, believing that monetary settings were appropriate in the current environment in terms of ensuring that the medium term inflation goal of the PTA was met. As mentioned before, this is not captured in our analysis given that the horizon of our model is essentially just one quarter. As judged by the next '$*$', which is well under the separating trajectory, we can conclude that the Bank's decision was justified in hindsight.

[35] This is almost entirely due to the fact that we are using a 90-day rate as a proxy for the actual policy instrument, the Official Cash Rate. Over this period, even though the OCR itself moved by only 0.5 percent, the 90-day rate moved by more because several further increases in the OCR were signalled by the Bank.

Figure 16
Viable monetary policy for New Zealand in 2004



In Figure 16 we can follow the evolution of the NZ economy described above, in the 3D space with the interest rate added on the vertical axis.

9 Concluding remarks

This paper has considered a simple estimated macroeconomic model for viability analysis of New Zealand monetary policy conducted by the Reserve Bank of New Zealand. Policy advice may be established under reasonably general conditions:

(I) `if` $y(t), \pi(t)$ `are` *well inside* V_F^K `apply` $i(t) + \Delta i(t)$,
$\Delta i(t) \in [-.005, .005]$ `for every time interval` $t \in \Theta$

(II) *otherwise* `apply` $i(t) - .005$ `if` $y < 0$ `or` $i(t) + .005$
`if` $y > 0$.

These recommendations are in line with policy (8); in particular, the policy is *extreme* in that it calls for maximal interest rate changes when (II) occurs.

The proposed policy schedule distinguishes between two states: an economy can be (i) *well inside* the viability kernel or (ii) close to its boundaries. An assessment of this will obviously depend on the policymaker's judgment. We contend that viability analysis is helpful in this assessment since it allows the policymaker to determine where the economy is *expected* to move to, given current conditions and the applied instruments. If at time $t+h$, the economy is expected to remain in the constraint set then the economy state at time t is *well inside* the kernel and a natural set of control policies suggest themselves.

We believe that policy decisions made in this manner permit explicit incorporation of judgment and are less arbitrary than "optimal" ones that rely on subjective choices regarding loss function weights and discount rates.

We have shown how satisficing policy choices can be modified to allow for measurement errors and/or parameter uncertainty. A ball around each point of the trajectory on the (y,π) plane (see figure 9) was constructed where the size of the ball is 'proportional' to the magnitude of uncertainty. Rules (I) and (II) may be modified: apply (I) if the ball does not intersect with the viability kernel's boundary; else apply (II).[36]

In general, a policy based on a viability analysis is precautionary in that it directs the system away from regions of adverse economic conditions (like large negative output gaps or accelerating inflation) where control of the system is difficult or impossible. Hence a viable policy is 'naturally' forward looking and appealing under uncertainty. Policy advice stemming from viability analysis offers a satisfactory compromise between the timing of change in an instrument and its strength.

In future work we aim to focus on open economy models, which will take exchange rate uncertainty into account. Clément-Pitiot and Doyen (1999) provide a starting point for such analysis by computing viable policies that can keep an exchange rate in a target zone; also, see Krawczyk and Kim (2004) where the exchange rate enters the model as a control variable. We expect that extensions of the model will produce viable policies robust to private sector agents' reactions and hence be even more resistant to the Lucas critique than those computed in this paper.

[36] An alternative risk-minimising interpretation: choose an interest rate at t such that the economy (together with likely shocks) at $t+1$ is inside the viability kernel.

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