How costly is exchange rate stabilisation for an inflation targeter? The case of Australia

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How costly is exchange rate stabilisation for an inflation targeter? The case of Australia*

Mark Crosby, Tim Kam and Kirdan Lees†

Abstract

This paper quantifies the costs of mitigating exchange rate volatility within the context of a flexible inflation targeting central bank. Within a standard linear-quadratic formulation of inflation targeting, we append a term that penalises deviations in the exchange rate to the central bank’s loss function. For a simple forward-looking New Keynesian model, we show that the central bank can reduce volatility in the exchange rate relatively costlessly by aggressively responding to the real exchange rate. However, when we append correlated shocks – to better match summary statistics of the Australian data – we find that the costs associated with reducing exchange rate volatility are larger: output volatility increases substantially. Finally, we apply our method to a variant of a small backward-looking New Keynesian model of the Australian economy. Under this model, large increases in inflation and output volatility accrue if the central bank attempts to mitigate exchange rate volatility.

* The views expressed in this paper are those of the author(s) and do not necessarily reflect those of the Reserve Bank of New Zealand. We thank Jonathan Kearns and Christie Smith for useful comments.
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1 Introduction

Most central banks within the OECD currently have as their main target of policy a low inflation rate, with other possible policy objectives relegated to second place. In most of the theoretical literature regarding central bank behaviour the two variables that enter a central bank’s loss function are the rate of inflation and the level of output growth (or the output gap). This is despite the fact that a number of other candidate objectives might reasonably be expected to be of interest to central banks.

In this paper we explore the idea that central banks are often interested in limiting exchange rate variability. Central banks may desire exchange rate stability – above and beyond the role of exchange rate movements in determining inflation – to fulfil legislated objectives, for reasons of political economy, and to reduce uncertainty for the plans of firms and households within the economy. Obviously different central banks have different objectives in this regard, ranging from strong limits to exchange rate variability as in China, to heavy interventions but some exchange rate variation as in Singapore, to the Reserve Bank of Australia’s approach of leaning against large movements in either direction of the exchange rate but otherwise not intervening against exchange rate movements.

The issue of whether or not central banks should more actively manage the exchange rate is an open question. Researchers have suggested that In this paper we simply take it as given that the central bank may want to limit exchange rate volatility. There is now a vast literature that examines the empirical impact of exchange rate volatility on the real economy, with the findings being mixed over whether such volatility affects real variables at the macroeconomic level (see Arize, Osang and Slottje 2000 and Crosby 2004 for recent papers on this topic and for a summary of the evidence).

While the evidence at the macroeconomic level tends to give mixed results, models developed from the microfounded behaviour of optimising agents frequently suggest that there exist significant welfare gains to mitigating exchange rate volatility. Agents may be prepared to give up a certain fraction of consumption in each period in return for reduced exchange rate volatility that otherwise increases risk and uncertainty around the economic decisions (see Obstfeld and Rogoff 1998,
and de Paoli, 2004). Central banks may also desire exchange rate stability – above and beyond the role of exchange rate movements in determining inflation – to promote financial stability, fulfil legislated objectives and for reasons of political economy.

This paper seeks to quantify the costs of mitigating exchange rate volatility for a small open economy with an inflation targeting central bank. The central bank sets interest rates to minimise deviations of inflation from target in the context of a flexible, market determined, exchange rate regime. The central bank is flexible in their inflation targeting and seeks to reduce volatility in output, interest rates and the exchange rate.

We view the Reserve Bank of Australia (RBA) as a prime example of a small open economy inflation targeter operating within the framework specified. The specific policy experiment we explore is increasing the weight on exchange rate stabilisation relative to other macroeconomic objectives.

We present results for a simple forward-looking New Keynesian model. To presage the results, this model suggests mitigating exchange rate volatility can be achieved relatively costlessly by responding more aggressively to the real exchange rate only. However, this model fails to replicate the correlation in the data.

To address this failure, we consider two other models that better fit the Australian data. First, we append the simple New Keynesian model with a shock correlation matrix derived from Australian data in the same manner as West (2003). This model suggests some costs associated with reducing exchange rate volatility.

Second, the robustness of these results are checked for a small variant of the backward-looking model due to Beechey et al (2000), that has been used for policy simulations at the Reserve Bank of Australia. This model also suggests large costs to reducing exchange rate volatility.

The following section presents the forward-looking model, how the correlation matrix associated with the shocks is appended to the model and the small variant

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1 Bergin and Tchakarov (2003) present an alternative view that the costs from exchange rate volatility are small. The aim of this paper is not quantify the welfare costs of exchange rate volatility but rather to explore the trade-offs an open economy central bank faces in attempting to reduce exchange rate volatility.

2 We focus solely on monetary policy and abstract from intervening in the market for domestic currency. Intervention may be effective at mitigating exchange rate volatility at high (daily) frequencies (see Kearns and Rigobon 2005) but is unlikely to be effective, on its own, in standing against large swings in the exchange rate.

2 Three small open economy models

In the following section we consider the three model economies that will be used in our analysis. The first two economies can be represented in the canonical forward looking structure:

\[ A_0X_t + A_1E_tX_{t+1} = U_t \]

where \( X = (y, \pi, i, q, \pi^*, i^*) \) contains the small open economy’s output gap, inflation rate, nominal interest rate, real exchange rate, foreign output gap, foreign inflation and foreign nominal interest rate, respectively. The vector of exogenous random variables \( U = (u_y, u_\pi, u_m, u_r, u_{\pi^*}, u_{i^*}, u_{i^*}) \) collect the respective shocks to each equation in the system ordered according to the vector \( X \). The difference between the first and the second small open economies will be in the stochastic processes for \( U \). In the former, \( U \) will be an independently and identically distributed random vector, while in the latter, \( U \) will be a Markov process whose dynamics are determined jointly by the structural model’s calibration and the data.

We will compare our results for the new Keynesian models with a third model that has purely backward-looking dynamics that contrasts with the forward looking aspect of the first two economies. This third model takes a backward-looking structural VAR form:

\[ X_{t+1} = AX_t + U_t \]

and the elements of \( X \) in this case include a richer set of variables including domestic good inflation and the domestic terms of trade.

2.1 A forward-looking New Keynesian model

Our first model is a variant of the New Keynesian model used in West (2003). The model is extremely simple. This simplicity yields a lack of dynamics but offers

\footnote{Note that there is a structural shock attached to the interest rate equation that can be considered to represent imperfect control over the ninety day interest rate and is useful for ensuring exact structural identification of the shocks in the model.}
analytical solutions for optimal simple rules. These solutions clarify the mapping from the weights on stabilisation objectives within the central bank’s loss function to response coefficients within the central bank’s reaction function. The model is characterised by the following IS curve:

\[ y_t = \alpha_y y^*_t - \alpha_q q_t - \alpha_r (i_t - E_t \pi_{t+1}) + u_{y,t} \]  

(3)
a simple Phillips curve equation:

\[ \pi_t = \beta_\pi E_t \pi_{t+1} + \beta_y y_t + u_{\pi,t} \]  

(4)
and the real interest rate parity condition:

\[ q_t = E_t q_{t+1} + (i_t - E_t \pi_{t+1}) + (i^*_t - E_t \pi^*_{t+1}) + u_{q,t}. \]  

(5)

We follow West (2003) and close the model by assuming the interest rate is set according to the following monetary policy rule with no interest rate smoothing:

\[ i_t = \gamma_\pi \pi_t + \gamma_y y_t + \gamma_q q_t \]  

(6)

where \( \gamma_\pi, \gamma_y > 0 \) and \( \gamma_q < 0 \).

All variables are as noted in the previous section; the output gap is represented with \( y_t \), \( q_t \) is the real exchange rate, \( i_t \) is the monetary policy instrument (assumed to be the ninety day interest rate), \( \pi_t \) is domestic consumer price inflation, while \( y^*_t \), \( i^*_t \) and \( \pi^*_t \) represent the foreign output gap, the foreign nominal interest rate, and foreign inflation respectively. The foreign sector is assumed to be exogenous and is determined by the following shock processes:

\[ (y^*_t, \pi^*_t, i^*_t)' = (u^*_y, u^*_\pi, u^*_i)' \].

As West (2003) notes, the model has much in common with the New Keynesian models derived from explicit microfoundations (for example, Gali and Monacelli 2005, and Rotemberg and Woodford 1998). A clear departure from these models is the lack of forward-looking behaviour in the output gap equation. McCallum and Nelson (1999) show that optimising behaviour on the part of households implies that the consumption Euler equation contains the expectation of future consumption. However, based on US data, Fuhrer and Rudebusch (2004) find little to recommend using expectations of future output to determine current output and Estrella and Fuhrer (2002) note that a purely forward-looking New Keynesian model cannot replicate the US data.

The Phillips equation is closely related to the derivations in Clarida, Gali and Gertler (1999) and represents the summation of firms’ pricing decisions. There is disagreement in the literature regarding the degree of forward-looking behaviour.
Lindé (2002), Dennis (2004) and Söderlind, Söderström and Vestin (2005) find a fairly limited role for expectations. Initially we explore a purely forward-looking Phillips relationship and then consider a more persistent process for inflation in subsequent models.

The real exchange rate is modelled by assuming that uncovered real interest rate parity holds. Empirically, this equation is difficult to maintain – the real exchange rate typically moves through cycles largely unexplained by exchange rate arbitrage. To address this, we model the persistence in the exchange rate in subsequent representations of the economy.

The model contains no lagged processes, though lags are necessary for the model to replicate the autocorrelation typically observed in key macroeconomic series such as inflation, the output gap, and the real exchange rate. The following subsection extends the baseline model by specifying processes for the foreign variables and allowing the data to determine the degree of correlation in the residuals of equations (3)–(6).

### 2.2 A model with persistence

Our second model allows the structural shocks to take a VAR(1) specification:

\[
U_t = \Phi U_{t-1} + W_t
\]

where the matrix \( \Phi \) and the variance-covariance matrix \( \Omega_W \), associated with the shock process \( W_t \), are determined by the data. West (2003) details how iterating on discrete Lyapunov equations produces a numerical solution for \( \Gamma_X \), the variance-covariance matrix of the state variables, \( X_t \), and the method is presented in the technical appendix.

When the model allows the data to determine the process of the structural shocks (encapsulated in the matrix \( \Phi \)), and allows autocorrelation in the foreign sector, the optimal policy rule is no longer restricted to only the three state variables in equation (6). Instead, the rule will be a function of all the state variables, including the foreign sector and the structural shocks. Rather than pursue fully optimal rules we determine the dynamics and variances of key macroeconomic variables under

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4 Meese and Rogoff (1983) find that UIP cannot beat a random walk in forecasting the exchange rate and West (2003) appears correct in relabelling the UIP condition “uncertain interest rate parity”.

a simple policy rule that takes its arguments from equation (6). That is, the rule responds contemporaneously to inflation, the output gap and the real exchange rate. The response coefficients can no longer be solved analytically but a simple numerical algorithm searches for optimal coefficients until gains from search are negligible.

We use Australian data over the period 1990Q1 to 2003Q4; the data are detailed in the data appendix to the paper. The sample period represents a flexible exchange rate regime, where the interest rate was manipulated with the primary goal of achieving inflation objectives.\(^5\)

We HP filter ($\lambda = 1600$) Australian and US output and construct the output gap as output relative to the appropriate HP filtered trend series. Australian consumer price inflation is annual inflation, constructed from summing four quarters of quarterly headline CPI inflation.\(^6\)

We adjust the inflation series with an additive dummy variable to account for the introduction of the goods and services tax in September 2000. The real exchange rate is expressed as a percentage deviation from its mean over the entire data period. The quarterly ninety day interest rate is constructed by averaging monthly data. The US federal funds rate is also the quarterly average of monthly data. US consumer price inflation is constructed as the annualized quarterly increase in the consumer price index.

Obtaining the variance-covariance matrix of the structural shocks (and thus the variance-covariance of the state variables) is achieved by obtaining the variance-covariance of the reduced-form residuals (see the technical appendix). To this end, we estimate an unrestricted reduced-form VAR model and include a constant that effectively removes variable means that are unimportant for determining the dynamics and variances of state variables.\(^7\)

First-pass estimation of the VAR(1) model returns residuals for the interest rate equation that are severely non-normal – the Jarque-Bera test of normality is rejected at the 1 percent level. This is attributable to the increases in the Australia

\(^5\) The Reserve Bank of Australia suggests that inflation targeting was officially adopted in the first half of 1993 (Stevens 2003). However, Bernanke et al (1999) note that interest rates rose dramatically in the late 1980s with no apparent increase in inflation and conclude the RBA possessed inflation objectives earlier than the announced adoption of inflation targeting.

\(^6\) The Reserve Bank of Australia targeted underlying CPI inflation until 1998. We focus on headline inflation over the entire sample, abstracting from comparing the costs of exchange rate volatility relative to the two different measures.

\(^7\) Thus, over the period the mean of the exchange rate is treated as the equilibrium exchange rate.
ninety day interest rate in 1994Q4 and 1995Q1, which the model fails to replicate. Bernanke et al (1999) note that this period represents the first acid test of the RBA’s commitment to inflation targeting and note the RBA raised the cash rate 100 basis points on both October 24 and December 14.

Our model cannot replicate the rapid increased in interest rates over this period, when the RBA was surprised about both the strength of the economy and inflation expectations, at a time that called for a sign of commitment to the new inflation targeting regime. We use an additive dummy that takes the same value in both 1994Q4 and 1995Q1 to account for this non-normality. The VAR(1) representation of the model (excluding constant terms) is presented below.

\[
\begin{pmatrix}
  y_{t+1} \\
  \pi_{t+1} \\
  i_{t+1} \\
  q_{t+1} \\
  y^*_{t+1} \\
  \pi^*_{t+1} \\
  i^*_{t+1}
\end{pmatrix} =
\begin{pmatrix}
  0.746 & -0.010 & -0.002 & 0.005 & 0.080 & -0.143 & -0.016 \\
  (0.102) & (0.102) & (0.132) & (0.020) & (0.064) & (0.323) & (0.353) \\
  0.068 & 0.741 & 0.122 & 0.042 & 0.164 & 0.022 & -0.091 \\
  (0.110) & (0.110) & (0.142) & (0.021) & (0.069) & (0.347) & (0.380) \\
  0.116 & 0.176 & 0.684 & -0.020 & 0.044 & 0.064 & 0.051 \\
  (0.054) & (0.054) & (0.070) & (0.010) & (0.034) & (0.172) & (0.188) \\
  -1.198 & 0.142 & -1.426 & 0.763 & 1.247 & 2.250 & 0.825 \\
  (0.547) & (0.548) & (0.709) & (0.105) & (0.543) & (1.730) & (1.893) \\
  0.110 & -0.034 & -0.065 & -0.028 & 0.837 & -0.103 & -0.045 \\
  (0.086) & (0.087) & (0.112) & (0.016) & (0.054) & (0.273) & (0.299) \\
  -0.183 & -0.082 & 0.287 & 0.023 & 0.158 & 0.337 & 0.011 \\
  (0.091) & (0.091) & (0.118) & (0.017) & (0.057) & (0.287) & (0.314) \\
  0.179 & 0.046 & -0.212 & -0.045 & 0.139 & 0.149 & 0.973 \\
  (0.067) & (0.067) & (0.066) & (0.013) & (0.042) & (0.211) & (0.231)
\end{pmatrix}
\begin{pmatrix}
  y_t \\
  \pi_t \\
  i_t \\
  q_t \\
  y^*_{t} \\
  \pi^*_{t} \\
  i^*_{t}
\end{pmatrix} +
\begin{pmatrix}
  \epsilon_{yt} \\
  \epsilon_{\pi t} \\
  \epsilon_{it} \\
  \epsilon_{qt} \\
  \epsilon_{y^*t} \\
  \epsilon_{\pi^*t} \\
  \epsilon_{i^*t}
\end{pmatrix}
\]

The diagonal elements of the VAR(1) representation are relatively high and generally significant, suggesting most of the explanatory power of each variable is contained within lags of the left hand side variable in question. This is particularly true of annual inflation.

The diagnostics associated with the VAR(1) model are presented in table 1 on the following page. The VAR(1) returns high $R^2$ values across each equation. The standard error of the real exchange rate is 1.860 – much higher than the standard errors associated with the other variables.

According to the Jarque-Bera test, the errors are all normal at the 5 percent level of significance. However, the Ljung-Box statistics, which test for autocorrelation in the residuals with up to four lags, shows the model may be susceptible to correlation in the US inflation and interest rate series.

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8 De Brouwer and Gilbert (2005) express a similar concern.
As a final check on the policy experiment we use a simplified version of the Beechey et al (2000) model used in the past at the RBA. The model is close to the simplification of the Beechey et al model used in Dennis (2003). The aim here is to explore whether the costs of exchange rate stabilisation are similar within a reasonable backward-looking representation of the economy. Our simplified model takes the form:

\[
\begin{align*}
    y_t &= 0.75y_{t-1} - 0.1q_t + 0.05s_{t-1} - 0.22(i_{t-1} - \pi_{c t-1}^f) + \varepsilon_{yt} \\
    q_t &= 1.09\Delta s_t + 0.63q_{t-1} + 0.25s_{t-1} + 0.66(i_{t-1} - \pi_{c t-1}^f) + \varepsilon_{qt} \\
    s_t &= 1.68s_{t-1} - 0.81s_{t-2} + \varepsilon_{st} \\
    \pi_t^d &= \pi_{t-1}^d + 0.64y_{t-1} + \varepsilon_{\pi_t^d} \\
    \pi_t^f &= 0.5\pi_{t-1}^f + 0.5\Delta q_t + \varepsilon_{\pi_t^f} \\
    \pi_t &= 0.5\pi_t^d + 0.5\pi_t^f \\
    i_t &= 1.5\pi_t^d + 0.5y_t
\end{align*}
\]

Table 1

<table>
<thead>
<tr>
<th>Equation</th>
<th>( R^2 )</th>
<th>se</th>
<th>JB</th>
<th>LB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{t+1} )</td>
<td>0.720</td>
<td>0.481</td>
<td>5.143 (0.076)</td>
<td>1.766 (0.622)</td>
</tr>
<tr>
<td>( \pi_{t+1} )</td>
<td>0.892</td>
<td>0.517</td>
<td>2.105 (0.349)</td>
<td>4.453 (0.217)</td>
</tr>
<tr>
<td>( i_{t+1} )</td>
<td>0.989</td>
<td>0.256</td>
<td>0.049 (0.976)</td>
<td>0.754 (0.861)</td>
</tr>
<tr>
<td>( q_{t+1} )</td>
<td>0.892</td>
<td>2.576</td>
<td>0.693 (0.707)</td>
<td>1.014 (0.798)</td>
</tr>
<tr>
<td>( y_{t+1}^* )</td>
<td>0.797</td>
<td>0.407</td>
<td>2.835 (0.242)</td>
<td>1.194 (0.754)</td>
</tr>
<tr>
<td>( \pi_{t+1}^* )</td>
<td>0.813</td>
<td>0.428</td>
<td>3.725 (0.155)</td>
<td>17.160 (0.001)</td>
</tr>
<tr>
<td>( i_{t+1}^* )</td>
<td>0.969</td>
<td>0.314</td>
<td>3.398 (0.183)</td>
<td>11.350 (0.010)</td>
</tr>
</tbody>
</table>

NB. se = standard errors; JB = Jarque-Bera; LB = Ljung-Box statistic

2.3 A backward-looking model

As a final check on the policy experiment we use a simplified version of the Beechey et al (2000) model used in the past at the RBA. The model is close to the simplification of the Beechey et al model used in Dennis (2003). The aim here is to explore whether the costs of exchange rate stabilisation are similar within a reasonable backward-looking representation of the economy. Our simplified model takes the form:

\[
\begin{align*}
    y_t &= 0.75y_{t-1} - 0.1q_t + 0.05s_{t-1} - 0.22(i_{t-1} - \pi_{c t-1}^f) + \varepsilon_{yt} \\
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    s_t &= 1.68s_{t-1} - 0.81s_{t-2} + \varepsilon_{st} \\
    \pi_t^d &= \pi_{t-1}^d + 0.64y_{t-1} + \varepsilon_{\pi_t^d} \\
    \pi_t^f &= 0.5\pi_{t-1}^f + 0.5\Delta q_t + \varepsilon_{\pi_t^f} \\
    \pi_t &= 0.5\pi_t^d + 0.5\pi_t^f \\
    i_t &= 1.5\pi_t^d + 0.5y_t
\end{align*}
\]

The full model was used for actual policy simulations within the RBA.
Note that $\pi^d_t$ is domestic good inflation, $\pi^f_t$ is foreign good inflation, $\pi^c_t$ is consumer price inflation, $s_t$ is the terms of trade and $q_t$ is the real exchange rate. Equations (8) to (10) are identical to the representation of the small RBA model in Dennis (2003). We abstract from the change in labour costs and the error correction mechanism between the prices of consumption goods, import good and labour costs in the Dennis (2003) model such that equations (11) to (13) represent a simplified process for consumer price inflation. Consumer price inflation is the equally-weighted average of domestic good inflation (which is driven by the output gap) and foreign good inflation (which contains imperfect exchange rate pass-through). Finally, we close the model with the Taylor rule for the purpose of assessing model fit only.

### 2.4 Model fit

It is natural to evaluate alternative models based on fit. To this end, we match a number of summary statistics (standard deviations, autocorrelations and cross-correlations of key macroeconomic variables) implied by each model, under a specific loss function, to the summary statistics obtained from the data. Rather than construct summary statistics from a specific data sample we use a Bayesian approach to characterize parameter uncertainty. We estimate a VAR and draw from the distribution of VAR parameters to construct distributions for key summary statistics. The VAR(3) is estimated over the output gap, inflation, the interest rates and the exchange rate. The companion form of the VAR(3) can be represented:

$$\xi_t = A\xi_{t-1} + \epsilon_t$$ (15)

We define $g(A) = \prod_i g(A_i)$ to be an uninformative prior density for $A$ where $i = 1, 2, \ldots, \infty$. If $g(A)$ is uninformative and $X_t$ is normally distributed, $A$ will take a Normal-Wishart distribution (see Schorfheide 2000 and Canova 2005).

We draw 5,000 sets of stationary parameters estimates from the Normal-Wishart distribution for $A$, using an indicator function that removes parameter draws that imply nonstationarity in the VAR model. For each parameter draw, $A_i$, the variance-covariance matrix of the simulated data $y_i$ is:

$$\Sigma_\xi = A_i\Sigma_\xi A_i' + \Sigma_\epsilon$$ (16)
where $\Sigma_{\varepsilon}$ is the variance-covariance matrix of the errors of the companion form VAR. Rather than simulate data we solve for the variance-covariance matrix with the vec operator that stacks columns of $A$ in a single vector:

$$vec(\Sigma_{\varepsilon}) = [I - (A_i \otimes A_i)]^{-1}vec(\Sigma_{\varepsilon})$$

(17)

We construct distributions of summary statistics from the variance-covariance matrix. Comparison of the summary statistics implied by the models to the distributions of the data yields a sense of distance regarding which summary statistics “miss” the data and which merely reflect genuine uncertainty in the data sample itself. Figure 1 depicts the distributions of the data implied by the VAR(3) model against summary statistics for all three models under a specific loss function where $L_t = \pi_t^2 + y_t^2 + 0.5l_t^2 + 0.1q_t^2$.\(^{10}\)

Firstly turning to the standard deviation of the output gap in the first cell of the figure, the models match the narrow range of estimates implied by the data. The mode of the distribution in the data peaks at 0.745. In comparison, the standard deviations of the models are 0.666, 0.690 and 0.678 for the forward looking model, West’s empirical model and the small RBA model respectively. Although these estimates are all below the mode, they are comfortably within the range of distribution of estimates. Thus all three models match output volatility particularly well.

Both the forward-looking and simple RBA model match the volatility of quarterly inflation relatively well. However, West’s empirical model suggests inflation is much less volatile than the data would suggest. Furthermore, West’s empirical model substantially underestimates the volatility of the exchange rate and implies the standard deviation of the nominal interest rate is about the same in the models as in the data. Summing across these volatility measures, the model suggests the shocks to the model are too small relative to the VAR(3) specification of the data.

In contrast, the simple RBA model variant predicts about twice as much interest rate volatility as the data. Interest rates must be manipulated relatively vigorously to achieve stability of output, inflation and the exchange rate in the variant of the RBA model.

The second row of the figure depicts the autocorrelations of the four key time

\(^{10}\text{Of course, other loss function specifications will yield different dynamics. Table 2 shows the effect of the choice of loss function on one particular summary statistic – the standard deviation of key macroeconomic variables.}\)
Figure 1
Summary statistics for three small empirical models
series. Note that because the theoretical model contains no lagged dynamics, the variables are functions of the i.i.d. shocks each period and thus the implied autocorrelation for each series is zero.

Both the empirical model and the small RBA model match the autocorrelation in output – which the data identifies clearly as very persistent. However, both models also predict highly autocorrelated inflation processes which is inconsistent with the data: the VAR(3) model suggests inflation is not particularly persistent. This marks the largest deviation of these models from the data. The task of reducing persistent inflation deviations appears particularly difficult under both West’s empirical model and the variant of the small RBA model, relative to the persistence observed in the data.

Although, the distributions for the autocorrelations evident in both the interest rate and the exchange rate are relatively tight, both the empirical West specification and the small RBA model also specify relatively high interest and exchange rate persistence.

Finally, the last row of the table displays four cross-correlations for key variables. The data are more agnostic relative to the autocorrelations for each variable, reflecting a higher degree of uncertainty in off-diagonal elements in the underlying VAR(3) model. However, although both the theoretical and empirical model appear to match the low correlation between the output and inflation, the simple RBA model predicts too much correlation. The theoretical West model fails to match the correlation between the output gap and the interest rate and the simple RBA model overestimates the low correlation between inflation and the nominal interest rate.

The next section undertakes our experiment across the three open economy models. The goal is to explore the extent of the trade-off between minimising deviations of the exchange rate from equilibrium and other macroeconomic objectives, within an inflation targeting framework.

3 Results

Our key experiment departs from West (2003) in describing the motivation for the behaviour of the central bank. We explicitly model the central bank’s problem as one of selecting an interest rate rule that minimises an objective function whereas West (2003) explores the implications of policy rules that include explicit
responses to the exchange rate.

We assume that the central bank is an inflation targeter and is thus concerned with the volatility of inflation but is flexible in its approach and in addition, is also concerned with volatility in the output gap, the interest rate and the real exchange rate. The key innovation in this paper is to uncover the implied behaviour for the central bank and the economy when the central bank possesses a concern for exchange rate stabilisation, over and above the concerns for macroeconomy objectives encapsulated by flexible inflation targeting. The simple rule is restricted to output gap, inflation and exchange rate arguments and can be considered a Taylor-type rule appended with a response to the contemporaneous exchange rate.\(^{11}\)

Specifically the central bank in each of the three economies seeks to minimize the lifetime loss criterion:\(^{12}\)

\[
W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t (y_t, \pi_t, q_t, i_t); \quad \beta \in (0, 1), \lambda_y, \lambda_i, \lambda_q > 0, \quad (18)
\]

\[
L_t (y_t, \pi_t, q_t, i_t) = \frac{1}{2} [\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 + \lambda_q q_t^2] \quad (19)
\]

subject to the constraints imposed by the models’ dynamics in (1) for the first two New Keynesian economies, or in (2) for the backward-looking economy. We assume that there is no discounting such that the lifetime loss criterion in the limit is:

\[
\lim_{\beta \to 1} W = \frac{1}{2} [\sigma_\pi^2 + \lambda_y \sigma_y^2 + \lambda_i \sigma_i^2 + \lambda_q \sigma_q^2]. \quad (20)
\]

If the central bank cares about the current and expected future continuation value of its policy program, they must commit to a policy rule that minimizes the expected present value of lifetime losses. This is equivalent to having asymptotic variances as arguments in the loss function, in the case of quadratic period losses.

Rather than finding the fully optimal solution to the central bank’s dynamic problem in each model, we restrict the central bank in all three candidate economies to solving the problem of committing to a simple rule that minimizes (20) subject to the model constraints each period. We restrict the class of optimal simple rules to Taylor-type rules of the form:

\[
i_t = \gamma_\pi \pi_t + \gamma_y y_t + \gamma_q q_t \quad (21)
\]

\(^{11}\)Taylor (2001) also discusses appending exchange rate arguments to Taylor-type rules.

\(^{12}\)Note, the targets are assumed zero for convenience. These can be easily recovered from the constants in the VAR solution by allowing for a VAR with a constant vector.
as in West (2003), with $\gamma_r, \gamma_i > 0$ and $\gamma_q < 0$. Thus the set of variables the central box may respond to is the same across all three models.

First, we report the results for the theoretical model, with no autocorrelation in the shock processes for the model equations (3) - (5). Recall that we can derive the optimal discretionary rule given the model and parameterisation of the central bank loss function.

The key parameter for our policy experiment is the relative weight the central bank places on stabilising the level of the real exchange rate relative to its equilibrium. This is restricted to the preference set $\lambda_q = \{0.0, 0.1, 0.2\}$. A small weight on the variance of the real exchange rate is entirely appropriate because the observed movement in the exchange rate from equilibrium (expressed in percentage terms) is larger, by an order of magnitude, relative to the deviation of inflation from target, the size of the output gap and deviations of the interest rate from neutral.

For West’s theoretical model, we set the variances of inflation, the output gap and the interest rate shocks to one and set the variance of the exchange rate to 8 in order to approximately match the variance of the key macroeconomic variables in the data. The variances of the shocks for West’s empirical model are data determined. For the small backward-looking model, the variances of the shocks are those from Dennis (2003) for the output gap, exchange rate and the terms of trade equations, while the variance of the domestic and foreign good inflation equations are 0.1.

Table 2 on the following page displays the results of the policy experiment for all three models with columns 2-4 of the table displaying the range of weights within the central banks loss function.

The right-most column displays the response coefficients for the simple rule that optimises the associated loss function in the left of the table. The cells in the middle columns of the table display the standard deviations of key macroeconomic variables and the associated loss of the central bank. Note that under the theoretical model, inflation expectations are predetermined, implying that movements in the nominal interest rate translate directly to movements in the real interest rate.

The analytical results for West’s theoretical model are presented in the top section of the table. As the weight placed on exchange rate stabilisation increases, the response to the inflation and the output gap remain unchanged, but the central bank responds more aggressively to movements in the real exchange rate. The central bank decreases the nominal interest rate by 40 basis points in response to a positive 1 percent deviation of the exchange rate relative to trend. This reduces the standard deviation of the real exchange rate in return for only small increases.
Table 2  
Standard deviations, loss, optimal rules for alternative preferences

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in the volatility of inflation and larger increases in the volatility of the interest rate. The volatility of output actually decreases.

To see this, examine the information in the I(i) and I(iii) rows of the table. The weight on exchange rate stability increases from 0 to 0.2. This is associated with an increase in the strength of the response to the real exchange rate of 0 to -0.4, which generates a decrease in the standard deviation of the real exchange rate from 3.056 to 2.398 – just under 25 percent. This is associated with a trivial increase in inflation volatility (less than a percent), a decrease in output volatility (about 5 percent) but a large increase in the interest rate volatility – the standard deviation of the interest rate increases by about 20 percent.

Similar tradeoffs accrue throughout the table. For example the last three rows associated with the forward-looking New Keynesian model place relatively large weights on output gap and interest rate stabilisation. For this set of preferences the concomitant trade-off across macroeconomic stabilisation objectives is similar to the first three rows – reducing real exchange rate volatility by 13 percent requires an increase in the volatility of the nominal interest rate of about 7 percent. Output gap volatility decreases by about 6 percent and there is an infinitesimal decrease in inflation volatility (obscured in the table by rounding) Thus under the New Keynesian model, it appears that if the central bank is willing to response aggressively to the exchange rate, exchange rate volatility can be mitigated with little cost in terms of output gap and inflation volatility.

For the empirical model, the costs associated with exchange rate volatility are markedly different than for the purely forward-looking specification. With low weights on other stabilisation objectives, contained in rows II(i) to II(iii), moving from a zero weight on exchange rate volatility to a weight of 0.2 reduces exchange rate volatility by about 8 percent. However, unlike West’s (2003) forward-looking model, this is associated with an increase in output volatility of about 5 percent and a decrease in inflation volatility of about 6 percent. Under the empirical formulation, the exchange rate channel of monetary policy appears to be particularly dominant. Increasing interest rate movements reduces exchange rate volatility and inflation volatility over and above volatility induced in inflation via increased output volatility.

Similar trade-offs are apparent throughout the other specifications. For example, rows II(x) and II(xii) show that a 4 percent reduction in the standard deviation of the real exchange rate is associated with a 3 percent decrease in inflation volatility, a 2 percent increase in the nominal interest rate and an increase in output volatility of 4 percent.
How does the central bank achieve desired reductions in the real exchange rate under the empirical model? The right-most columns show that the central bank responds less aggressively to the exchange rate to decrease volatility in the exchange rate. The associated response to inflation and the output gap falls. Note that the response to inflation actually falls below one for the preferences in row II(iii) with comparatively low weights on interest rate and output stability. Evidently the matrix $\Phi$ that determines the cross-correlation of the structural shocks (see equation 16) specifies a structure that does not require significant movements in the real interest rate to return key macrovariables to target.

Finally, for the variant of the small RBA model that simplifies Beechey et al (2000), the inflation volatility costs associated with exchange rate stabilisation are relatively large. Note that the Taylor rule that was used to map out the summary statistics in figure 1 is replaced with the optimal monetary policy implied by the loss functions in the first columns of the table. Results for the backward-looking model are presented in the final third of the table. The first three rows show that an increase in preference for exchange rate stabilisation, from $\lambda_3 = 0$ to $\lambda_3 = 0.2$, results in a small (less than 2 percent) decrease in the standard deviation of the real exchange rate. This is associated with small increases in inflation volatility (5 percent) and output gap volatility (1 percent). Assuming that the changes in volatility are linear, these results imply a 24 percent decrease in exchange rate volatility would be associated with a 70 percent increase in inflation volatility and a 14 percent increase in output volatility.

Similar results hold across the table although the exchange rate-inflation volatility tradeoff is somewhat less pronounced when there is already a large weight on other macro-stabilisation objectives. For example, moving from row III(x) to III(xii) increases the weight on exchange rate stabilisation and shows that reducing exchange rate volatility by 1.2 percent is associated with a 2.5 percent increase in inflation volatility – a two to one tradeoff.

Under the backward-looking model, the central bank achieves lower exchange rate volatility by increasing the response to the real exchange rate and reducing the response to the output gap and inflation. This results in a more volatile interest rate – interest rate volatility increases by about 15 percent for every 25 percent decrease in exchange rate volatility.
4 Conclusion

This paper explores the effectiveness of stabilizing the real exchange rate under inflation targeting. We show that under a forward-looking New Keynesian model, exchange rate volatility can be reduced with little cost in terms of the volatility of inflation and output if the central bank responds to the real exchange rate relatively aggressively. However, this result is not robust to two models that match the Australian data better.

When the forward-looking model is appended with correlated errors determined by the data, the costs to mitigating exchange rate volatility increase dramatically – a 25 percent decrease in the standard deviation of the exchange rate is associated with an increase in the standard deviation of output of 15 percent, though the standard deviation of inflation decreases.

An alternative backward-looking representation of the Australian economy, based on Beechey et al (2000), suggests higher costs to reducing exchange rate volatility: a 24 percent decrease in exchange rate volatility is associated with a 70 percent increase in inflation volatility and a 14 percent increase in output volatility. To achieve lower exchange rate volatility the central bank needs to reduce interest rates relatively aggressively when the exchange rate is overvalued relative to trend, and respond less aggressively to output and inflation.

Thus, within plausible representations of open economy models there appear to be large concomitant increases in macroeconomic volatility from mitigating exchange rate volatility. Inflation targeting central banks should proceed with caution when attempting to mitigate movements in the exchange rate. At the very least, central banks should have a strong understanding of how volatility maps into social welfare if attempting to control exchange rate variability.
References


Appendix

A Technical appendix

The structural parameters are

\[
A_0 = \begin{bmatrix}
1 & 0 & \alpha_r & -\alpha_q & -\alpha_y & 0 & 0 \\
-\beta & 1 & 0 & 0 & 0 & 0 & 0 \\
-g & -g & 1 & -g & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
0 & -\alpha_r & 0 & 0 & 0 & 0 & 0 \\
0 & -\beta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

And the structural shocks follow a VAR(1) specification:

\[
U_t = \Phi U_{t-1} + W_t
\]

\[
E [W_t W_t'] = \Omega_W.
\]

To solve the rational expectations system, guess a linear solution \( X_t = DU_t \). Substituting this into the system (1) we get

\[
[A_0 D + A_1 D\Phi] U_t = U_t
\]

which implies

\[
A_0 D + A_1 D\Phi = I \quad (22)
\]

We estimate from data the reduced form process for the state variables, \( X_t \):

\[
X_t = \hat{F} X_{t-1} + V_t \quad (23)
\]

\[
E [V_t V_t'] = \Omega_V.
\]

Comparing the reduced form and the structural model we can rewrite the reduced form as

\[
DU_t = \hat{F}DU_{t-1} + V_t
\]

or

\[
U_t = D^{-1} \hat{F}DU_{t-1} + D^{-1}V_t
\]
which implies that $\Phi = D^{-1}\tilde{F}D$ from comparing the previous equation to the VAR(1) specification of the structural shocks. Replacing this in (22) we get

$$
\begin{bmatrix} A_0 + A_1\tilde{F} \end{bmatrix} D = I
$$

and thus we can recover the estimate of the structural solution as

$$
\tilde{D} = \left[ A_0 + A_1\tilde{F} \right]^{-1}.
$$

Then we can also back out the estimated serial correlation matrix for the structural shocks

$$
\hat{\Phi} = \tilde{D}^{-1}\tilde{F}\tilde{D},
$$

and their variance covariance

$$
\hat{\Omega}_W = \tilde{D}^{-1}E\left[\hat{V}_t\hat{V}_t'\right]\tilde{D}^{-1'} = \tilde{D}^{-1}\tilde{\Omega}_V\tilde{D}^{-1'}.
$$

To generate unconditional moments without Monte Carlo simulations, we can exploit the structure of the model in equilibrium using recursive methods. Recall our model in equilibrium is described by the state-space VAR:

$$
U_t = \Phi U_{t-1} + W_t
$$

$$
X_t = DU_t.
$$

We can solve for the asymptotic unconditional moments, $\Gamma_U$, for the exogenous states $U_t$, as the fixed point of the following discrete Lyapunov equations

$$
\Gamma_U = \Phi \Gamma_U \Phi' + \Omega_W
$$

The fixed point that solves these equations can be found by obtaining the limit of the following recursion

$$
\Gamma_{U,j+1} = \Phi \Gamma_{U,j} \Phi' + \hat{\Omega}_W
$$

starting with some positive definite matrix for $\Gamma_{U,0}$ so that $\lim_{j \to \infty} \Gamma_{U,j} = \Gamma_U$ is a positive semidefinite matrix. Once, $\hat{\Gamma}_U$ is found, we can use the observation equation above to calculate the unconditional moments of the endogenous state variables:

$$
\hat{\Gamma}_X \equiv E\left[ X_t X_t' \right] = D\hat{\Gamma}_U D'.
$$

The diagonal of the matrix $\hat{\Gamma}_X$ contains the asymptotic variances of the endogenous states.
### B Data appendix

<table>
<thead>
<tr>
<th>variable</th>
<th>series</th>
<th>source</th>
<th>identifier</th>
</tr>
</thead>
<tbody>
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<td>$y_t$</td>
<td>Gross Domestic Product</td>
<td>RBA website</td>
<td>GGDPCVGGDP</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>Consumer Price Inflation</td>
<td>RBA website</td>
<td>GCPIAGQP</td>
</tr>
<tr>
<td>$q_t$</td>
<td>Real Exchange Rate</td>
<td>IFS</td>
<td>Q193L00REC.Q</td>
</tr>
<tr>
<td>$i_t$</td>
<td>Ninety Day Nominal Interest Rate</td>
<td>RBA website</td>
<td>FIRMMBAB90</td>
</tr>
<tr>
<td>$y_t^*$</td>
<td>US Gross Domestic Product</td>
<td>FRED II website</td>
<td>GDPC1</td>
</tr>
<tr>
<td>$\pi_t^*$</td>
<td>US Consumer Price Index</td>
<td>FRED II website</td>
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<tr>
<td>$i_t^*$</td>
<td>Effective US Federal Funds Rate</td>
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<td>FEDFUNDS</td>
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FRED II website: [http://research.stlouisfed.org/fred2/](http://research.stlouisfed.org/fred2/)
IFS: International Financial Statistics Database