RESERVE BANK OF NEW ZEALAND

DP2005/07

Discretionary Policy, Potential Output Uncertainty, and Optimal Learning

James Yetman

December 2005

JEL classification: E52

www.rbnz.govt.nz/research/discusspapers

Discussion Paper Series

DP2005/07

Discretionary Policy, Potential Output Uncertainty, and Optimal Learning

Abstract[1]

We compare inflation targeting, price level targeting, and speed limit policies when a central bank sets monetary policy under discretion, and must learn about the level of potential output over time. We show that if the central bank learns optimally over time, a speed limit policy dominates [is dominated by] a price level target if society places a high [low] weight on inflation stability. Inefficient learning on the part of the central bank can radically change this conclusion. A speed limit policy is favoured if the central bank places too much weight on recent data when estimating potential output, while a price level target is favoured if the central bank places too much weight on historical data.

---

1 The views expressed in this paper are those of the author(s) and do not necessarily reflect the views of the Reserve Bank of New Zealand. This paper was written in part while the author was visiting the Reserve Bank of New Zealand. I thank, without implication, the RBNZ for their hospitality, Ozer Karagedikli for providing the parameter values used in the New Zealand calibration, and Alex Ho for comments. Any remaining errors are the author's sole responsibility. School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong; phone (852) 2857-8506; fax (852) 2548-1152; email jyetman@econ.hku.hk.

1 Introduction

Dealing with uncertainty is fundamental to monetary policy. One important source of uncertainty is uncertainty as to the level of potential output, and therefore the output gap. The maximum capacity of the economy to increase output without generating inflation is unknowable in real time, complicating the operation of monetary policy. Orphanides and van Norden (2002) show that revisions to the output gap are of the same order of magnitude as the output gap itself, and are highly persistent.[2,3]

This uncertainty has important implications for the conduct of monetary policy. For example, Orphanides (2003b) demonstrates that potential output uncertainty can explain the 'Great Inflation' in the United States (US), and further that the performance of the US economy would have been substantially improved if the Federal Reserve had followed either an inflation target or a speed limit policy.[4] Orphanides et al. (2000), using the Federal Reserve's model of the US economy, demonstrate that the poor performance implied by potential output uncertainty could be reduced if the coefficient on the output gap in the central bank's reaction function is reduced,[5] or if the central bank responded to output growth rather than the output gap when setting policy.[6,7] In contrast to the current paper, all these

---

[2] Orphanides and van Norden identify the largest source of revisions as due to the "end-of-sample" problem, where final estimates are very poorly estimated, as opposed to the revision of published data. Comparing real-time with final estimates of the output gap, they find that the root mean square difference between the two varies between 1.17 percent and 2.64 percent with a first order autocorrelation between 0.80 and 0.96, depending on the method used for measuring potential output.

[3] See also Cayen and van Norden (2004) who investigate potential output uncertainty in Canadian data.

[4] Similarly, Orphanides and Williams (2002) show that responding to the change in the unemployment rate, rather than its level, reduces the problems imposed by imperfect measurement of the natural unemployment rate.

[5] Gaspar and Smets (2003) and Orphanides (2003a) also argue that a cautious response to noisy information is appropriate. Rudebusch (2001) shows that the policy response to potential output estimates should be the certainty equivalent response if the estimate is efficient (in the sense that revisions are correlated with the underlying true value, and are 'news'), but cautious if the estimate is inefficient (implying that revisions are correlated with the initial estimate, and are 'noise.' He argues that the empirical evidence favours the second interpretation.

[6] Orphanides et al (2000) make the simplifying assumption that measurement errors are uncorrelated with disturbances that hit the economy, an assumption that we will relax

papers treat the learning process, and resulting expectational errors of the central bank, as exogenous to the model; here we explicitly incorporate the learning process of the central bank within the model.

Elsewhere, several papers have focused on the potential benefits of the central bank targeting the price level (for example, Svensson 1999, Dittmar and Gavin 2000, Vestin 2005 and Yetman 2005a) or the economy's 'speed limit' (defined as the rate of growth in the output gap; see Walsh 2003 and Yetman 2005b) when it is constrained to operate under discretion, even if society's loss function is specified in terms of inflation and output gap variability. These papers illustrate that an appropriately delegated loss function for the central bank can help to ameliorate the time-inconsistency problem imposed by discretionary policy.[8] However, the models in these papers are typically based on the assumption that the central bank has complete information about the economy, and faces no uncertainty when setting policy.

Walsh (2004) shows that in the presence of output gap uncertainty, speed limit policies may outperform policies based on an inflation target. This is because errors in measuring the level of trend output, and hence in the output gap, are typically highly persistent, so that these errors will largely disappear once first differences are taken in the data. Walsh demonstrates the intuition for this is as follows: suppose that the central bank's measure of potential output, $\hat{y}_t^*$, is given by

$$\hat{y}_t^* = y_t^* + \varepsilon_t, \tag{1}$$

in our analysis.

7 From a theoretical perspective, how a policy maker should respond to uncertainty was first addressed by Brainard (1967), who showed that if the uncertainty is additive, a policy maker with a quadratic objective function should display certainty equivalence, while if it is multiplicative a more cautious policy is appropriate. Wieland (2000) shows that the desire for caution is partially mitigated if a more aggressive policy increases the ability of the central bank to reduce their future uncertainty. In the current paper, we abstract from multiplicative uncertainty by incorporating only additive shocks, and assuming all parameter values are known by the central bank.

8 Yetman (2005a,b) investigates the robustness of this result. He shows that while price level targets dominate a speed limit policy when the central bank enjoys perfect credibility and agents are fully rational, even small deviations from either rational expectations or perfect credibility results in a speed limit policy dominating a price level target.

where y_t^* is the true value of potential output, and ε_t is the measurement error. Suppose further that forecast errors follow an AR(1):

$$\varepsilon_t = \rho\varepsilon_{t-1} + \nu_t. \qquad (2)$$

Then the variance of the measurement error of the output gap is given by $\sigma_\nu^2/(1-\rho^2)$, while the variance of the measurement error in the change in potential output is $2\sigma_\nu^2/(1+\rho)$. The latter is smaller if $\rho > 0.5$, as is typically found in measures of potential output.[9] Walsh further considers one particular point on the policy frontier, and shows that a speed limit policy under discretion does almost as well as a commitment policy when the central bank has a standard quadratic objective in terms of output and inflation.

Here, as in Walsh (2004), we study the optimal response of monetary policy to potential output uncertainty in terms of the policy objectives of the central bank. We assume that potential output follows a random walk, and the central bank has difficulty estimating the level of potential output. We calibrate the degree of uncertainty to historical central bank forecast errors, assuming that these forecast errors result from optimal learning on the part of the central bank. We focus in particular on the case where the central bank cannot commit to following a monetary policy rule, so that monetary policy is confined to being discretionary. We then compare different reaction functions for the central bank. These reaction functions correspond to inflation targeting, price level targeting, or following a speed limit policy.

The next section develops the model, while section 3 describes the choice of parameter values for the calibration. Section 4 presents our results. Conclusions then follow.

---

[9] Walsh finds that revisions from real time to final estimates of trend output over the 1959-2003 period have a mean absolute value of 1.38 percent and a standard deviation of 0.84 percent; in contrast revisions of the change in trend output have a means absolute value of 0.29 percent and a standard deviation of 0.23 percent.

2 A simple New-Keynesian model

Here I describe a simple linear-quadratic, forward-looking model, similar to that in Vestin (2000), Walsh (2003) and Yetman (2005a,b). In this model price level targeting and speed limit policies can play a role in improving the trade-off between output and inflation variability faced by the central bank when monetary policy is set under discretion. The economy is assumed to consist of a Phillips curve given by

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(x_t + e_t) + u_t, \tag{3}$$

where π_t is inflation, x_t is the output gap, u_t is an exogenous shock term that is known by the central bank, and e_t is an unanticipated shock term.[10,11] Expectations of future inflation are assumed rational, and all variables are expressed in logs. For simplicity, the policy instrument of the central bank is the central bank's expected value of the output gap, given by $\hat{x}_t$.

Each period, the central bank (and agents) observe inflation and from that try to infer potential output (y_t^*), where $x_t = y_t - y_t^*$ is the output gap. Suppose potential output follows a random walk, so that

$$y_t^* = y_{t-1}^* + w_t. \tag{4}$$

As we show in Appendix 1, efficient learning will result in the central bank's estimate of potential output following the process outlined in equations (1) and (2) above where

$$v_t = (1-\rho)e_{t-1} - w_t, \tag{5}$$

---

[10] This Phillips curve can be derived from optimizing sticky price models; for example, see Woodford (2003, Appendix B).

[11] The anticipated shock u_t ensures that there exists a role for stabilization on the part of the central bank, while the unanticipated shock e_t ensures that the central bank cannot completely identify x_t from π_t. Both u_t and e_t are assumed to be identically and independently distributed.

$$\rho = \frac{2\sigma_e^2 + \sigma_w^2 - \sqrt{(\sigma_w^2)^2 + 4\sigma_e^2\sigma_w^2}}{2\sigma_e^2}. \tag{6}$$

An inflation-targeting central bank seeks to minimize a standard, quadratic loss function given by

$$L = (1-\beta)E_t \sum_{i=0}^{\infty} \beta^i L_{t+i},$$

$$L_{t+i} = \pi_{t+i}^2 + \lambda x_{t+i}^2 \tag{7}$$

subject to (3), where λ captures the relative weight of output versus inflation in the central bank's loss function.[12] Under discretionary policy, the central bank is assumed to lack the means to commit to future policy actions. Optimal monetary policy will therefore minimize the period loss function taking the form

$$x_t = \frac{-\kappa}{\kappa^2 + \lambda} u_t - \varepsilon_t, \tag{8}$$

while inflation evolves according to

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda} u_t + \kappa(e_t - \varepsilon_t). \tag{9}$$

To examine monetary policy with a price level target, note that (3) may be rewritten as

$$E_t(p_{t+1}) = E_t(p_t) + \frac{1}{\beta} p_t - \frac{1}{\beta}(p_{t-1} + u_t) - \frac{\kappa}{\beta}(x_t + e_t). \tag{10}$$

The appropriate quadratic period loss function is then given by

---

[12] Without loss of generality, the inflation target (and later the price level target) is assumed to be zero.

$$L_{t+i} = p_{t+i}^2 + \lambda x_{t+i}^2 , \qquad (11)$$

where λ is appropriately scaled to reflect the differences between the magnitude of price level and inflation rate volatility and the optimal degree of conservatism with a price level target.[13] In contrast to inflation targeting, today's policy affects losses in future periods, implying the presence of state variables in the model. Following the methodology of Currie and Levine (1993), the paths of output and inflation under optimal discretionary monetary policy may be defined by

$$x_t = \theta_1(p_{t-1} + u_t) + \theta_2 \varepsilon_t + \theta_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1}) , \qquad (12)$$

$$p_t = \phi_1(p_{t-1} + u_t) + \phi_2 \varepsilon_t + \phi_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1}) + \phi_4 e_t , \qquad (13)$$

where $\hat{\varepsilon}_{t-1} = E_t(x_{t-1}) - x_{t-1}$ and the solution values of the coefficients are given in Appendix 2.

To examine monetary policy with a speed limit policy, the appropriate quadratic period loss function is then given by

$$L_{t+i} = \pi_{t+i}^2 + \lambda (x_{t+i} - x_{t+i-1})^2 , \qquad (14)$$

where λ should again be scaled appropriately. As shown in Appendix 3, the paths of output and inflation under optimal discretionary monetary policy may be defined by

$$x_t = \xi_1 x_{t-1} + \xi_2 u_t + \xi_3 \varepsilon_t + \xi_4 \hat{\varepsilon}_{t-1} , \qquad (15)$$

$$\pi_t = \varsigma_1 x_{t-1} + \varsigma_2 u_t + \varsigma_3 \varepsilon_t + \varsigma_4 \hat{\varepsilon}_{t-1} + \varsigma_5 e_t . \qquad (16)$$

Finally, under commitment, the economy follows

---

[13] In what follows, we will consider a range of values of λ, $0 \le \lambda \le \infty$, and look at the corresponding policy frontiers that result. By comparing frontiers for different monetary policy targets, we are implicitly adjusting λ optimally between different targets. See also the discussion in Yetman (2005a).

$$\pi_t = \alpha_1(x_{t-1} + \hat{\varepsilon}_{t-1}) + \alpha_2\varepsilon_t + \alpha_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1}) + \alpha_4 u_t + \kappa e_t, \qquad (17)$$

$$x_t = (1 - \alpha_1\frac{\kappa}{\lambda})(x_{t-1} + \hat{\varepsilon}_{t-1}) - \varepsilon_t - (1 + \alpha_3\frac{\kappa}{\lambda})(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1}) - \alpha_4\frac{\kappa}{\lambda}u_t \quad (18)$$

for $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ given in Appendix 4.

3 Calibration

In order to calibrate the model, we interpret final estimates of the output gap as the true values of the output gap.[14] Then the difference between real time estimates and final estimates is due to shocks to potential output (*w*) and unobservable inflation shocks (*e*), combined with the learning process of the central bank.

There have been a number of studies estimating central bank uncertainty about potential output. Orphanides and van Norden (2002) and Cayen and van Norden (2004) compare mechanical estimates of potential based on various assumptions based on how potential is measured. However, mechanical rules may provide poor measures of a central bank's underlying estimate of potential output, since they inevitably exclude information available to the central bank that is relevant for estimating potential output. Instead we focus here on internal central bank estimates of potential at a quarterly frequency, using the parameter values estimated by Orphanides et al (2000) and Rudebusch (2001) for the United States and new estimates derived from Reserve Bank of New Zealand data.[15] These parameters, along with their theoretical counterparts in our model, are given in Table 1.

Descriptions of the US parameters are given in their source publications. We consider three parameterizations of potential output uncertainty based on US data, one from Rudebusch (2001) and two based on different historical periods reported in Orphanides et al (2000). We combine these parameter

---

14 In reality, the volatility of central bank estimates of potential output implied by this assumption represent lower bounds on the true volatility, as Cayen and van Norden (2004) argue.

15 Orphanides (2003a) instead uses semi-annual data.

estimates with the variances from inflation forecast errors and Phillips curve residuals given in Rudebusch (2001), to uniquely identify all parameters in our model.

The Reserve Bank of New Zealand data is constructed based on real time and final measures of output and inflation by Reserve Bank staff. Differences between real time estimates of potential output and final estimates of potential using the Bank's official internal measure of potential output are highly persistent, with a first order autocorrelation coefficient of 0.95. The variance of the difference between the real time and final estimates of potential is 1.86, while the variance of the first-difference in final potential output is only 0.11. Errors in estimating inflation at a horizon of 1 quarter have a variance (annualized) of 0.39, while inflation residuals from an estimated Phillips curve have a variance (annualized) of 4.31.

For New Zealand data, the parameters are over-identified. The first calibration endogenizes ρ conditional on the other parameter values, assuming optimal learning on the part of the central bank, implying $\rho = 0.68$. The second calibration relaxes the assumption that ρ represents optimal learning on the part of the central bank, but instead imposes $\rho = 0.95$, as is found in the historical output gap estimates. This calibration implies that the central bank makes forecast errors (or equivalently places too much weight on historical data when estimating potential output) that are more persistent than would be optimal. The final calibration considers a low value of ρ, implying that the central bank's forecast errors are not persistent enough.

A full set of parameterizations considered are given in Table 2. Comparing the two sets of calibrations across countries, inflation is more responsive to output in New Zealand than in the United States (κ is larger), and for most parameterizations, New Zealand is subject to larger nominal shocks, as would be expected of a highly open economy. Final estimates of potential output are more stable, and errors in estimating potential output in real time are larger and more persistent in New Zealand than in the United States, although this is likely to be due to differences in the way that potential output is estimated in the two central banks.

4 Results

We can now consider the policy frontiers obtained under the different policy targets. We construct the full policy frontiers across all possible weights of in the central bank's loss function ($0 \le \lambda \le \infty$), with $\beta = 1$.

Figure 1 contains the policy frontiers for the first calibration, but with potential output uncertainty and unanticipated inflation removed. An inflation target under discretion is clearly dominated by a speed limit policy, which is itself dominated by a price level target.[16] Further, as Vestin (2005) shows, the policy frontier with a price level target coincides with the commitment solution.

Figure 2 adds potential output uncertainty and unanticipated inflation. The effect of uncertainty moves both output and inflation volatility away from the origin, and ensures that it is now impossible for a central bank to fully remove inflation or output volatility. Here a speed limit policy dominates the other discretionary policies if the central bank places a high weight on inflation stability, while a price level target dominates if the central bank places a high weight on output stability. An inflation target is never optimal.

The stark contrast between Figure 1 and Figure 2 is due to the increased policy errors that inevitably result in an uncertain world, combined with the manner in which price level targets differ from speed limit policies. A price level target requires the central bank to correct for past inflationary errors, implying that the central bank must induce increased inflation volatility. If society places a high weight on inflation stability, this is sub-optimal. In contrast, a speed limit policy requires that the central bank propagate past output gap errors into the future. This increases output gap volatility, which is more costly the higher the weight society places on output stability.

Figures 3 and 4 illustrate the results for the other calibrations with US data. In all cases, a similar story unfolds, even for Calibration 3 where the central bank's measurement errors are relatively non persistent. In fact, we obtain similar results for all levels of ρ. This is because, in our analysis, we have assumed that the central bank learns optimally about the level of potential

[16] This is consistent with Yetman (2005b). Note that qualitatively similar figures are generated by all of the calibrations considered.

output over time. As a result, the variance of the central bank's estimate of the output gap may be represented as a function of the underlying parameters of the model as

$$V[E(x_t)-x_t]=\frac{(1-\rho)^2\sigma_e^2+\sigma_w^2}{1-\rho^2}. \tag{19}$$

By comparison, the variance of the central bank's estimate of the change in the output gap may be written as

$$V[E_t(x_t-x_{t-1})-(x_t-x_{t-1})]=\sigma_w^2, \tag{20}$$

which is unambiguously lower. This strengthens the intuition of Walsh (2004): now, even in the case where $\rho < 0.5$, the volatility of the change in the output gap is less than the volatility in the level of the output gap. This is because we have explicitly modeled the learning process of the central bank, allowing the central bank to update their estimate of last period's output gap this period, and condition this period's policy on the revised estimate.[17]

Figure 5 displays the equivalent results for New Zealand data. Figures 6 and 7 illustrate the importance of our assumption of optimal learning. In Figure 6, the persistence of the measurement error was imposed as 0.95, while the optimal ρ would be lower at 0.68. Thus the central bank places a sub-optimally low weight on the most recent data when estimating potential output, and correspondingly too high a weight on historical data. Now a price level target dominates the other discretionary policies across the full range of λ. In contrast, Figure 7 presents the results imposing $\rho = 0.40$, implying that the central bank places too high a weight on the most recent data and too little weight on historical data and thus inefficiently estimates potential output. Now a speed limit policy dominates the other discretionary policies.

Table 3 presents the results another way. It illustrates the minimum loss for each discretionary policy as a fraction of the loss from the optimal

---

[17] The example given by Walsh (2004) corresponds to the case where the central bank fails to update their estimate of last period's output gap. That is, $V[(E_t(x_t)-x_t)-(E_{t-1}(x_{t-1})-x_{t-1})]<V[E_t(x_t)-x_t]$ only if ρ >0.5.

commitment policy for several values of λ. With no potential output uncertainty (right hand columns), a price level target coincides with the commitment policy, a speed limit policy raises losses by approximately 4 percent, while an inflation target increases losses by 15-30 percent. With potential output uncertainty (left hand columns), a speed limit policy performs best for $\lambda \in \{0.1, 0.25\}$, with losses 2-4 percent higher than under commitment, but a price level target performs best for $\lambda \in \{0.5, 1.0\}$ with losses 1- 4 percent higher than under commitment.

Table 4 outlines the optimal weight in the central bank's loss function (λ) that coincides with each of the data points in Table 3. Potential output uncertainty has little effect on the optimal degree of conservatism with an inflation target. This is not surprising, since with a quadratic objective function and additive uncertainty, certainty equivalence may be expected to approximately hold.

With either a price level target or a speed limit, the optimal level of λ reflects both the optimal degree of conservatism, as well as the difference in the magnitude between different variables. That is, the level of variability in the inflation rate or the output gap will not generally coincide with the level of variability in the price level or the change in the output gap. An adjustment in the level of λ assigned to the central bank may correct this difference.

With a price level target, the optimal weight on output is generally greater with potential output uncertainty than without it. And with a speed limit policy, the results are mixed. In general, as society cares more about output (that is, as λ increases), a smaller response to output is optimal. But at low levels of λ, a stronger response to output is generally optimal.

5 Conclusions

Both price level targeting and speed limit policies have been shown to improve the performance of the economy when a central bank is constrained to set policy under discretion. However, the manner in which they do so differs. A price level target requires the central bank to correct for past errors in achieving desired inflation, while a speed limit policy requires the central bank to propagate past errors in achieving their output target into the future.

We have compared the two policies with an inflation target when the central bank is faced with a noisy measure of the output gap, and must learn about the output gap over time. If the central bank learns optimally about potential output over time, the change in the output gap will be measured with greater precision than the level of the output gap, further favouring inertial policies. However, a central bank that follows a price level target must correct for past inflationary errors due to past mis-measurement of the output gap, increasing inflation volatility. In contrast, a central bank that follows a speed limit policy does not correct for past inflation errors, but rather propagates output gap errors into the future, increasing output volatility.

After calibrating potential output uncertainty to real time measures from two central banks and incorporating optimal learning on the part of the central bank, we find that a speed limit policy dominates both an inflation target and a price level target if society places a relatively high weight on inflation stability. Conversely, a price level target dominates if society places a relatively high weight on output stability. However, if the central bank fails to learn optimally over time, a price level target will become more favourable if the central bank places too much weight on historical data, implying forecast errors that are too persistent. A price level target becomes less favourable if the central bank places too much weight on recent data, implying forecast errors are not persistent enough.

Appendix 1

Suppose potential output follows a random walk

$$y_t^* = y_{t-1}^* + w_t. \tag{A1}$$

Each period, the central bank tries to learn about potential output by observing the inflation rate. For simplicity, the central bank will set policy in period t having only observed π_{t-1}.

From π_{t-1}, the central bank observes the value of $y_t^* + e_{t-1} - w_t$, which provides a point estimate of y_t^* with a variance of $\sigma_e^2 + \sigma_w^2$. More generally, given π_{t-i}, the central bank observes the value of $y_t^* + e_{t-i} - \sum_{j=0}^{i-1} w_{t-j}$, which provides a point estimate of y_t^* with a variance of $V_i = \sigma_e^2 + i\sigma_w^2$.

Suppose the central bank constructs an unbiased estimate of y_t^* efficiently from these point estimates, with a weight on the point estimate constructed from π_{t-i} of f_i. Then their error in estimating potential output is given by

$$\begin{aligned} \varepsilon_t &= \hat{y}_t^* - y_t^* \\ &= \sum_{i=1}^{\infty} f_i (e_{t-i} - \sum_{j=0}^{i-1} w_{t-j}), \end{aligned} \tag{A2}$$

where unbiasedness implies that $\sum_{i=1}^{\infty} f_i = 1$.

We can use the variance of each of the point estimates of y_t^* and the covariance between them (where the covariance between the point estimates obtained using π_{t-i} and π_{t-j} is given by $C_{ij} = \min(i, j)\sigma_w^2$) to construct a Best Linear Unbiased Estimate of y_t^*. This is given by

$$f_i = \frac{1 + \sum_{k=1}^{\infty} \frac{1}{V_k} \sum_{j \neq k} f_j C_{kj} - \sum_{k=1}^{\infty} \frac{1}{V_k} \sum_{j \neq i} f_j C_{ij}}{V_i \sum_{k=1}^{\infty} \frac{1}{V_k}}$$

$$= \frac{1}{V_i} \left[\frac{\sum_{k=1}^{\infty} \frac{1}{V_k} \sum_{j \neq k} f_j C_{kj}}{\sum_{k=1}^{\infty} \frac{1}{V_k}} - \sum_{j \neq i} f_j C_{ij} \right]. \tag{A3}$$

Now, combining (A3) with itself iterated forward (f_{i+1}),

$$V_i f_i = V_{i+1} f_{i+1} + \sum_{j \neq i+1} f_j C_{i+1\,j} - \sum_{j \neq i} f_j C_{ij} \,, \tag{A4}$$

implying

$$f_i = f_{i+1} + \frac{\sigma_w^2}{\sigma_e^2} \sum_{j=i+1}^{\infty} f_j \,. \tag{A5}$$

Combining (A5) with itself iterated forward, we have a second order linear difference equation

$$f_i = \left[2 + \frac{\sigma_w^2}{\sigma_e^2} \right] f_{i+1} - f_{i+2}. \tag{A6}$$

The solution to (A6) takes the form $f_i = A_1(\delta_1)^i + A_2(\delta_2)^i$ where δ_1, δ_2 are the positive and negative roots respectively in

$$\delta = \frac{2\sigma_e^2 + \sigma_w^2 \pm \sqrt{(\sigma_w^2)^2 + 4\sigma_e^2\sigma_w^2}}{2\sigma_e^2}. \tag{A7}$$

The restriction that $\sum_{i=1}^{\infty} f_i = 1$ precludes the positive root (since $A_1(\delta_1)^i$ is explosive), and further implies the weights follow an exponential decay curve,

$$f_i = (1-\delta)\delta^{i-1}. \tag{A8}$$

Then it is straight forward to show that

$$\varepsilon_t = [(1-\delta)e_{t-1} - w_t] + \delta\varepsilon_{t-1}, \tag{A9}$$

implying that $\rho = \delta$ and $v_t = (1-\delta)e_{t-1} - w_t$ in equation (2).

Next, to solve the model with a speed limit, price level target, or under commitment, we require $E_t(y_{t-1}^*)$. As above, we assume expectations at time t are formed before π_t is observed. Following the same arguments as above, the central bank will impose the same exponential weights, implying

$$E_t(x_{t-1}) = x_{t-1} + (1-\delta)e_{t-1} + \delta\varepsilon_{t-1}, \tag{A10}$$

$$\hat{\varepsilon}_{t-1} = (1-\delta)e_{t-1} + \delta\varepsilon_{t-1}. \tag{A11}$$

Appendix 2

To solve the price level targeting model under optimal discretionary policy, optimal discretionary monetary policy will satisfy

$$V_t = \min E_t[L_t + \beta V_{t+1}], \tag{A12}$$

where the relevant term of V_t is

$$V_t = \gamma p_{t-1}^2 + \dots \tag{A13}$$

Following Currie and Levine (1993), the solution price-path may be written as

$$p_t = \phi_1(p_{t-1} + u_t) + \phi_2\varepsilon_t + \phi_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1}) + \phi_4 e_t\,, \qquad \text{(A14)}$$

implying that

$$E_t(p_{t+1}) = \phi_1^2(p_{t-1} + u_t) + \phi_1\phi_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1})\,. \qquad \text{(A15)}$$

Equations (10) and (A15) imply that

$$p_t = [1 + \beta\phi_1(\phi_1 - 1)](p_{t-1} + u_t) + \beta(\phi_1 - 1)\phi_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1}) + \kappa(x_t + e_t)\,. \qquad \text{(A16)}$$

Substituting (A13) (iterated forward) and (A16) into (A12) and differentiating with respect to x_t yields the optimal discretionary policy rule

$$x_t = \theta_1(p_{t-1} + u_t) + \theta_2\varepsilon_t + \theta_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1})\,, \qquad \text{(A17)}$$

$$\theta_1 = \frac{-\kappa(1+\beta\gamma)[1+\beta\phi_1(\phi_1 - 1)]}{\kappa^2(1+\beta\gamma)+\lambda};\ \theta_2 = -1;\ \theta_3 = \frac{-\beta\kappa(\phi_1 - 1)\phi_3(1+\beta\gamma)}{\kappa^2(1+\beta\gamma)+\lambda}.$$

Substituting (A17) back into (A16) yields

$$\phi_1 = \frac{\lambda[1+\beta\phi_1(\phi_1 - 1)]}{\kappa^2(1+\beta\gamma)+\lambda};\ \phi_2 = -\kappa\,;\ \phi_3 = \frac{\beta\lambda(\phi_1 - 1)\phi_3}{\kappa^2(1+\beta\gamma)+\lambda};\ \phi_4 = \kappa\,.$$

Finally, substituting (A13) iterated forward, (A17), and (A14) back into (A12) and equating the coefficient with (A13) yields

$$\gamma = (1+\beta\gamma)\phi_1^2 + \lambda\theta_1^2\,. \qquad \text{(A18)}$$

The parameter values may then be solved numerically.

Appendix 3

To solve the speed limit model, optimal discretionary policy will satisfy (A12) where the relevant term of V_t is

$$V_t = \gamma x_{t-1}^2 + \ldots \tag{A19}$$

Equations (3) and (16) imply that

$$E_t(\pi_t) = (\beta\varsigma_1 + \kappa)\hat{x}_t + u_t\,, \tag{A20}$$

$$E_t(\pi_{t+1}) = \varsigma_1 \hat{x}_t\,. \tag{A21}$$

Substituting (A19) (iterated forward) and (A20) into (A12), differentiating with respect to $\hat{x}_t$, and combining with $\hat{x}_t = x_t + \varepsilon_t$ yields

$$\xi_1 = \xi_4 = \frac{\lambda}{(\beta\varsigma_1 + \kappa)^2 + \lambda + \beta\gamma}\,;$$

$$\xi_2 = \frac{-(\beta\varsigma_1 + \kappa)}{(\beta\varsigma_1 + \kappa)^2 + \lambda + \beta\gamma}\,; \xi_3 = -1. \tag{A22}$$

Substituting (A22), (A21) and (A20) into (3) and equating coefficients with (16) yields

$$\begin{aligned}
\zeta_1 &= (\beta\zeta_1 + \kappa)\xi_1 \\
\zeta_2 &= (\beta\zeta_1 + \kappa)\xi_2 + 1 \\
\zeta_3 &= -\kappa \\
\zeta_4 &= (\beta\zeta_1 + \kappa)\xi_4 \\
\zeta_5 &= \kappa.
\end{aligned} \tag{A23}$$

Finally, substituting (A19) iterated forward, (15) and (16) into (A12) and equating the coefficient with (A19) yields

$$\gamma = \frac{\varsigma_1^2 + \lambda(\xi_1 - 1)^2}{1 - \beta\xi_1^2}\,. \tag{A24}$$

Appendix 4

To solve the model under commitment, note that the first order conditions of the central bank's problem are

$$E_t(\pi_t + \delta_t - \delta_{t-1}) = 0, \tag{A25}$$

$$E_t(\lambda x_t - \kappa\delta_t) = 0, \tag{A26}$$

where δ_t is the Langrangean multiplier associated with the t^{th} period Phillip's curve.

Assuming inflation follows (17) implies

$$E_t(\pi_t) = \alpha_1(x_{t-1} + \hat{\varepsilon}_{t-1}) + \alpha_3(\hat{\varepsilon}_{t-1} - \varepsilon_{t-1}) + \alpha_5 u_t, \tag{A27}$$

$$E_t(\pi_{t+1}) = \alpha_1(x_t + \varepsilon_t). \tag{A28}$$

Combining (A25), (A26), and (A27), we obtain (18). Then, substituting (18) and (A28) into the Phillip's curve and solving with $\beta \to 1$, we obtain

$$\begin{aligned}
\alpha_1 &= \frac{-\kappa + \sqrt{\kappa^2 + 4\lambda}}{2}; \\
\alpha_2 &= -\kappa; \\
\alpha_3 &= \frac{-(\kappa + \alpha_1)}{1 + (\kappa + \alpha_1)\kappa / \lambda}; \\
\alpha_4 &= \frac{1}{1 + (\kappa + \alpha_1)\kappa / \lambda}.
\end{aligned} \tag{A29}$$

References

Brainard, William (1967), "Uncertainty and the effectiveness of policy," *American Economic Review*, 57(2), 411-425.

Cayen, Jean-Philippe and Simon van Norden (2004), "The reliability of output gap estimates," Manuscript.

Craine, Roger (1979), "Optimal monetary policy with uncertainty," *Journal of Economic Dynamics and Control*, 1(1), 59-83.

Currie, David A. and Paul Levine (1993), *Rules, Reputation and Macroeconomic Policy Coordination*, Cambridge University Press, Cambridge.

Dittmar, Robert D and William T Gavin (2000), "What do New Keynesian Phillips curves imply for price level targeting?" *Federal Reserve Bank of St. Louis Review*, 82(2), 21-30.

Gaspar, Vitor and Frank Smets (2003), "Monetary policy, price stability and output gap stabilization," *International Finance*, 5(2), 193-211.

Jensen, Henrik (2002), "Targeting nominal income growth or inflation?" *American Economic Review*, 92(4), 928-956.

Orphanides, Athanasios (2003a), "Monetary policy evaluation with noisy information," *Journal of Monetary Economics*, 50(3), 605-631.

Orphanides, Athanasios (2003b), "The quest for prosperity without inflation," *Journal of Monetary Economics*, 50(3), 633-663.

Orphanides, Athanasios and Simon van Norden (2002), "The unreliability of output-gap estimates in real time," *Review of Economics and Statistics*, 84(4), 569-583.

Orphanides, Athanasios, Richard D. Porter, David Reifschneider, Robert Tetlow, and Federico Finan (2000), "Errors in the measurement of the output gap and the design of monetary policy," *Journal of Economics and Business*, 52(1-2), 117-141.

Orphanides, Athanasios and John C. Williams (2002), "Robust monetary policy rules with unknown natural rates," *Brookings Papers on Economic Activity*, 2002(2), 63-145.

Poole, William (1998), "A policymaker confronts uncertainty," *Federal Reserve Bank of St. Louis Review*, 80(4), 3-8.

Rudebusch, Glenn D. (2001), "Is the Fed too timid? Monetary policy in an uncertain world," *Review of Economics and Statistics*, 83(2), 203-217.

Svensson, Lars E.O. (1999), "Price-level targeting versus inflation targeting: A free lunch?" *Journal of Money, Credit, and Banking*, 31(3), 277-295.

Vestin, David (2005), "Price level targeting versus inflation targeting in a forward looking model," *Journal of Monetary Economics*, forthcoming. See also *Sveriges Riksbank Working Paper*, 106.

Walsh, Carl E. (2003), "Speed limit policies: The output gap and optimal monetary policy," *American Economic Review*, 93(1), 265-278.

Walsh, Carl E (2004), "Implications of a changing economic structure for the strategy of monetary policy," *Center for International Economics Working Paper Series*, Santa Cruz Center for International Economics, UC Santa Cruz,1023.

Wieland, Volker (2000), "Monetary Policy, Parameter Uncertainty and Optimal Learning," *Journal of Monetary Economics*, 46(1), 199-228.

Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton.

Yetman, James (2005a), "The credibility of the monetary policy 'free lunch.'" *Journal of Macroeconomics*, forthcoming.

Yetman, James (2005b), "Are speed limit policies robust?" *Journal of Macroeconomics*, forthcoming.

Table 1
Estimated parameter values

Source	Data	ρ	σ_ε^2 $V(x_t - \hat{x}_t)$	σ_ν^2 $V(\varepsilon_t - \rho\varepsilon_{t-1})$	σ_w^2 $V(y_t^* - y_{t-1}^*)$	$\kappa^2(\sigma_e^2 + \sigma_\varepsilon^2)$ $V(\pi_t - E_t(\pi_t))$	$\kappa^2\sigma_e^2 + \sigma_u^2$ $V(\pi_t - E(\pi_{t+1}))$
Rudebusch (2001)	US 1987:4-1992:4 US 1979:1-1993:4	0.75		0.70		0.12	1.01
Orphanides et al (2000)	US 1966:2-1994:4	0.60	1.08	0.69			
	US 1980:1-1994:4	0.52	0.69	0.50			
Reserve Bank of New Zealand	NZ 1984:1-2004:4 NZ 1994:4-2004:4	0.95	1.86		0.11	0.39	4.31

Table 2
Calibration parameter values

Calibration	Source	ρ	σ_ε^2	σ_v^2	σ_w^2	σ_e^2	σ_u^2	κ
1	Rudebusch (2001)	0.75	1.60	0.70	0.12	1.34	0.96	0.20
2	Orphanides et al (2000)	0.60	1.08	0.69	0.20	0.76	0.96	0.26
3	Orphanides et al (2000)	0.52	0.69	0.50	0.19	0.43	0.97	0.33
4	RBNZ	0.68	1.86	0.18	0.11	0.73	4.18	0.42
5	RBNZ	0.95	1.86	0.18	0.11	0.73	4.18	0.42
6	RBNZ	0.40	1.86	0.18	0.11	0.73	4.18	0.42

Table 3
Simulation results: Loss relative to commitment policy

Calibration	Source	λ	Potential Output Uncertainty			No Potential Output Uncertainty		
			L^{in} / L^{com}	L^{pr} / L^{com}	L^{sp} / L^{com}	L^{in} / L^{com}	L^{pr} / L^{com}	L^{sp} / L^{com}
1	Rudebusch	0.10	1.218	1.071	1.025	1.295	1.000	1.042
		0.25	1.171	1.039	1.033	1.249	1.000	1.042
		0.50	1.121	1.022	1.050	1.200	1.000	1.041
		1.00	1.075	1.010	1.083	1.155	1.000	1.040
2	Orphanides et al	0.10	1.202	1.098	1.025	1.296	1.000	1.040
		0.25	1.182	1.054	1.040	1.280	1.000	1.043
		0.50	1.137	1.032	1.065	1.237	1.000	1.042
		1.00	1.088	1.017	1.109	1.188	1.000	1.041
3	Orphanides et al	0.10	1.181	1.131	1.022	1.275	1.000	1.037
		0.25	1.195	1.072	1.038	1.296	1.000	1.042
		0.50	1.163	1.044	1.060	1.268	1.000	1.042
		1.00	1.114	1.025	1.100	1.222	1.000	1.042

Table 4
Simulation results: Optimal conservatism

Calibration	Source	λ	Potential Output Uncertainty			No Potential Output Uncertainty		
			λ^{in}	λ^{pr}	λ^{sp}	λ^{in}	λ^{pr}	λ^{sp}
1	Rudebusch	0.10	0.105	0.203	0.104	0.101	0.164	0.098
		0.25	0.265	0.741	0.314	0.250	0.538	0.304
		0.50	0.549	2.117	0.698	0.507	1.426	0.745
		1.00	1.150	6.234	1.323	0.990	3.743	1.897
2	Orphanides et al	0.10	0.103	0.179	0.090	0.099	0.147	0.085
		0.25	0.257	0.574	0.261	0.247	0.463	0.267
		0.50	0.509	1.499	0.552	0.494	1.150	0.641
		1.00	1.067	4.437	1.005	0.966	3.034	1.522
3	Orphanides et al	0.10	0.102	0.165	0.081	0.101	0.133	0.078
		0.25	0.254	0.487	0.237	0.247	0.397	0.241
		0.50	0.507	1.215	0.497	0.502	0.961	0.557
		1.00	1.031	3.174	0.947	0.975	2.497	1.363

Figure 1
Calibration 1: Excluding potential output uncertainty

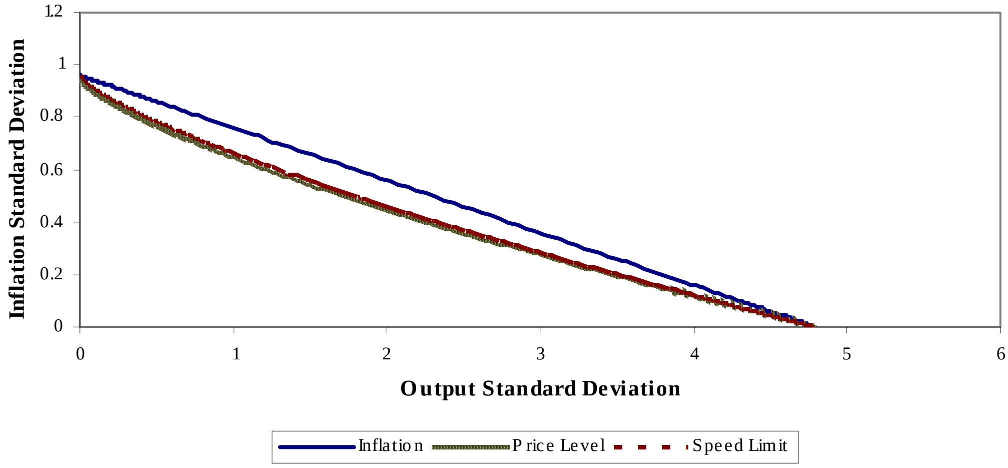

Figure 2
Calibration 1: Including potential output uncertainty

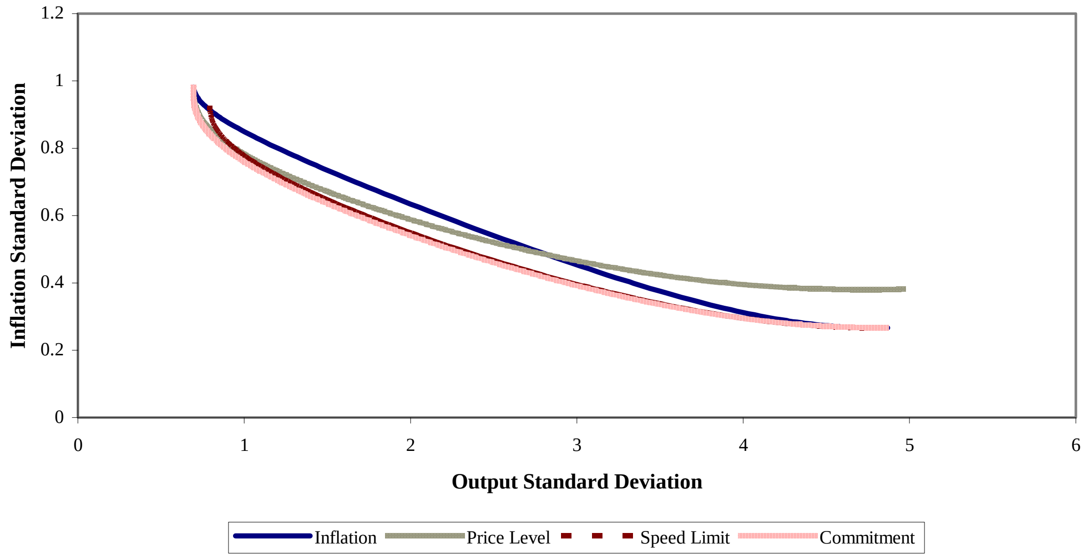

Figure 3
Calibration 2

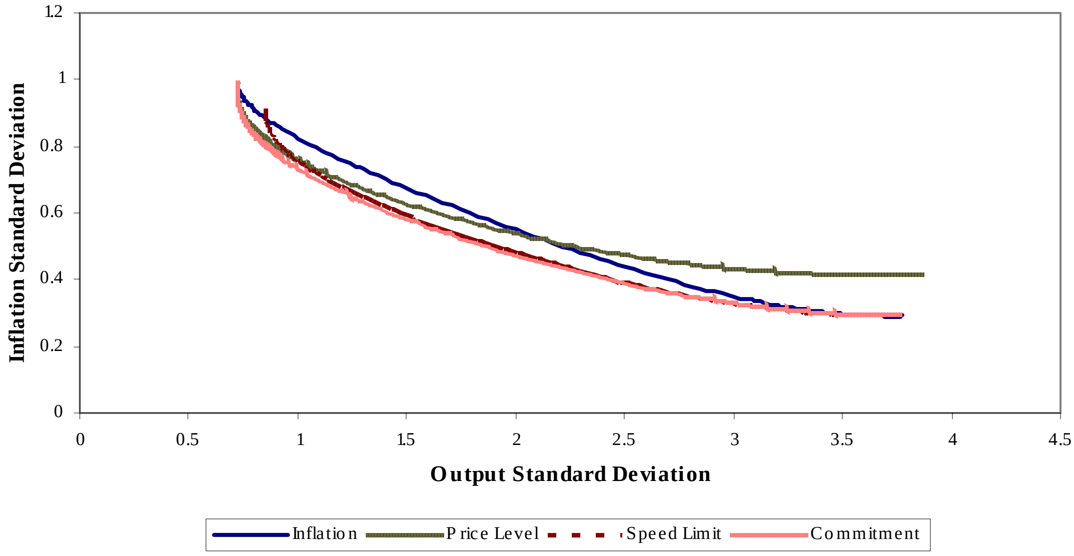

Figure 4
Calibration 3

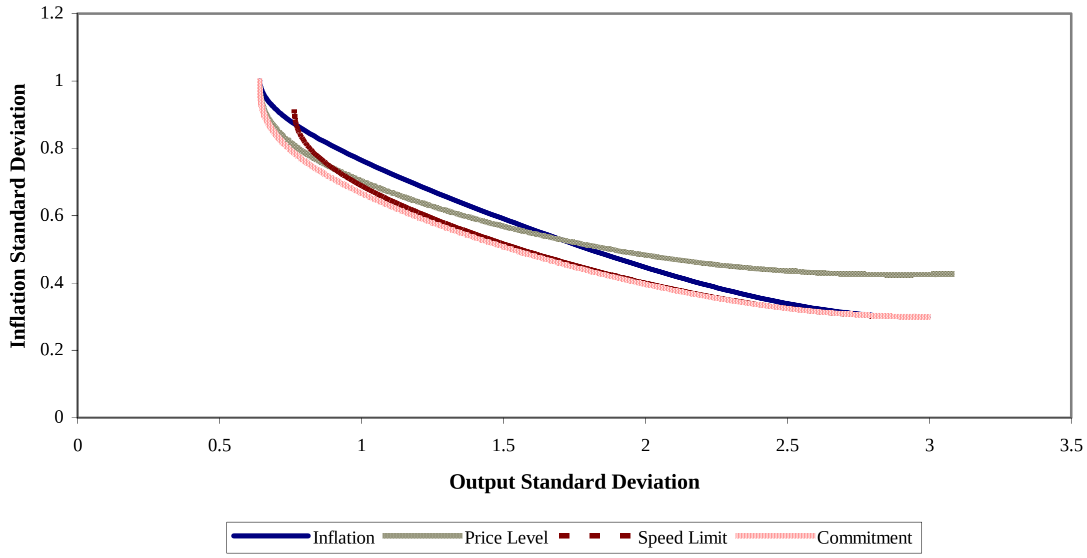

Figure 5
Calibration 4: New Zealand data, optimal ρ

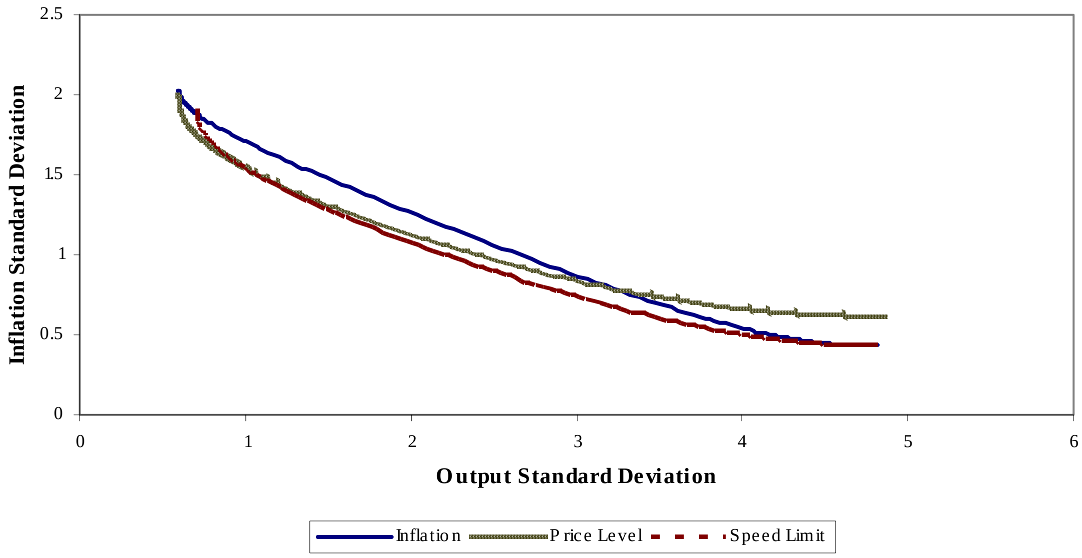

Figure 6
Calibration 5: New Zealand data, imposing $\rho = 0.95$

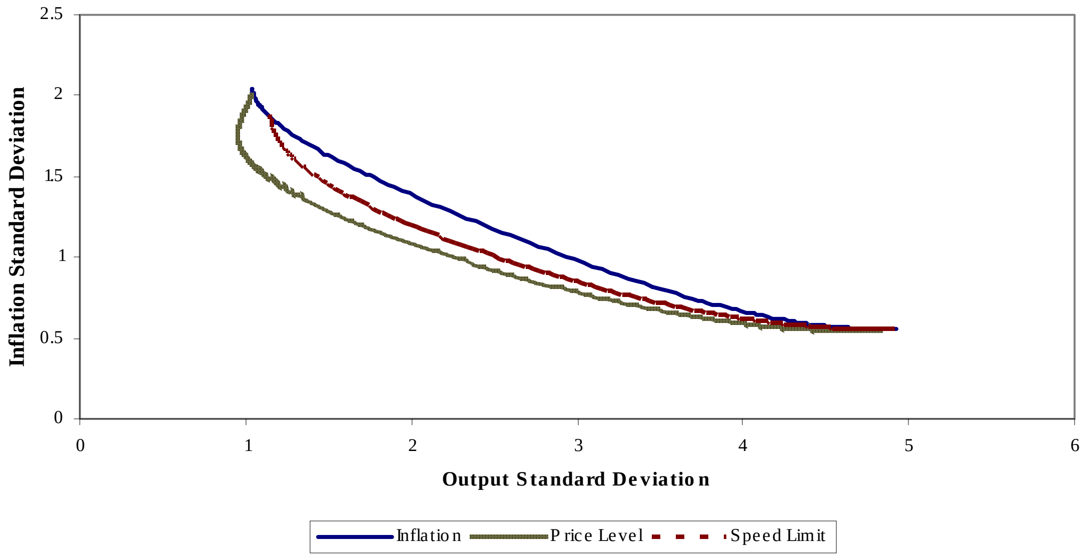

Figure 7
Calibration 6: New Zealand data, imposing $\rho = 0.40$