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Examining finite-sample problems in the application of cointegration tests for long-run bilateral exchange rates

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Abstract[1]

Numerous empirical studies investigate whether exchange rates are related to 'economic fundamentals' in the long-run and find a range of relationships through cointegration analysis. We report similar cointegrating relationships for the value of the New Zealand dollar relative to the US dollar (NZD/USD) and for the value of the New Zealand dollar relative to the Australian dollar (NZD/AUD). These include determinants such as commodity prices, 90-day interest rate differentials, and inflation and growth differentials. However, Godbout and van Norden (1997) demonstrate that finite-sample problems may have affected the conclusions of such cointegration studies. Through a simple Monte Carlo study, we consider whether the cointegration coefficients can reasonably be interpreted as 'long-run' elasticities of the exchange rate to changes in fundamental variables. The simulation results suggest that given a relatively short span of data it is possible for cointegration analysis to indicate that a long-run relationship has been found when in fact there is only a cyclical relationship. Therefore caution is advised when interpreting the empirical results and making policy assessments about the nature of exchange movements relative to its broad trend.

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1 Introduction

The behaviour of the exchange rate is an area that is heavily researched and discussed in the literature, but still not particularly well understood. For a small open economy like New Zealand, which experiences considerable exchange rate volatility and large cycles, having a reasonable understanding of the behaviour of the exchange rate is vital for policymakers in managing the economy.

As Black (1994) explains, the central idea underlying the equilibrium exchange rate framework and estimation methods is that the exchange rate moves around and at times drifts away from the level that is justified by economic or behavioural fundamentals (the 'equilibrium' exchange rate).[2]

A plethora of empirical evidence exists and many combinations of 'fundamentals' have been tested.[3] The main objective of this paper is not to present another model that has improved properties. Rather, we demonstrate that while different models can be estimated for the real NZD/AUD and NZD/USD exchange rates (using the single-equation regression method), such models are unfortunately susceptible to capturing short-run cyclical influences instead of long-run influences. Our work is in the spirit of Godbout and Van Norden (1997), who find that some important empirical results regarding exchange rate behaviour may simply reflect finite sample size distortions.[4] This risk implies that the model-based long-run or equilibrium exchange rate may be more cyclical or variable than the true equilibrium.

2 To be clear, there are other factors besides fundamental economic forces that are assumed to dominate exchange rate movements over short horizons, such as excess speculation and institutional customer or hedge fund manipulation (see Cheung and Chinn's (2001) survey of US foreign exchange traders). Throughout this paper, the equilibrium exchange rate is defined as the level consistent with fundamentals over a long horizon and therefore not considered a cyclical or short-term concept.

3 New Zealand examples include Conway and Franulovich (2002), MacDonald (2001), Stephens (2004), Wren-Lewis (2004).

4 See also Elliott (1998) and Haldrup and Jansson (1999) for complementary studies of the implications of near-integration in exactly cointegrated models and near-cointegration, respectively. Haldrup and Jansson (1999) point out that while the researcher may reject the null hypothesis of no cointegration when using cointegration methods, the results should be interpreted carefully as rejection of the null hypothesis may not by default indicate cointegration as cointegration tests cannot be expected to detect departures from exact cointegration.

The paper is organised as follows. In section 2 we outline some possible exchange rate drivers and examine the time-series properties of the data. The estimation method and regression results are presented in section 3. Then in section 4 we report the results of a Monte Carlo study that counsels caution in interpreting cointegration results, particularly given a relatively short sample period. Finally, we summarise and make some concluding comments in section 5.

2 The data and empirical model specification

2.1 Possible exchange rate drivers

Essentially, we posit that the long-run real exchange rate, z_t, relates to a set of fundamentals f_t:

$$z_t = \alpha + f_t + \varepsilon_t \tag{1}$$

where f_t includes variables such as commodity export price, short-term interest rate differential, output differential, inflation differential and output growth rate. Apart from the long-run or permanent influences of such fundamentals on the 'equilibrium' exchange rate, changes in these fundamentals may also reflect business cycles, pushing the actual exchange rate around its 'equilibrium' level. First, we outline the priors or basis for modelling the NZD/AUD and NZD/USD exchange rates with these variables.

Commodity export prices

Given that primary commodity products still account for over half of total NZ exports, movements in world prices for commodity export prices are likely to have a significant impact on the exchange rate. Chen (2002) and Chen and Rogoff (2003) find that country-specific commodity prices help to explain the long-run exchange rate of some commodity currencies (eg AUD and NZD) against the USD. In periods when we have experienced high commodity prices we have tended to also observe appreciation of the NZD.

Short-term interest rate differential

The uncovered interest parity (UIP) condition is a hypothesis that relates exchange rates to interest rate differentials. If the interest rate in the home country is lower than that in the foreign country and market participants believe that they will achieve a positive return even after the foreign exchange transactions, they will buy the foreign currency (increasing the demand for it and hence its price) to invest in the higher-yielding asset until the positive expected return is fully arbitraged away. This is fundamentally a cyclical rather than long-run relationship, since permanent real interest rate differentials are likely to represent compensation for risk rather than excess cyclical return.

Inflation differential

The concept of relative purchasing power parity (PPP) suggests that absent of price rigidities and transaction costs, the nominal exchange rate should adjust fully to the inflation differential between two countries to hold the real exchange rate constant. The inclusion of the differential in the real exchange rate models tests whether relative PPP holds.[5]

Output differential

When NZ's output increases relative to that of the foreign country, it may be a result of increased trade receipts that NZ firms have to repatriate, leading to an exchange rate appreciation. Put another way, as suggested in Conway and Franulovich (2002), the exchange rate acts as a re-adjustment tool to balance aggregate demand and supply or offset cyclical movements in net international trade. Alternatively, the output differential may proxy future monetary policy actions to manage each country's output relative to its potential output.

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[5] See Munro (2004) for a review of the Bank's recent exchange rate research and a discussion of concepts such as UIP and PPP.

Output growth rate
Historically, it appears that a higher output growth rate of foreign countries, especially our main trading partners, may have a positive influence on their demand for our exports and hence appreciate the NZD. However, this relationship may simply reflect the dependency of NZ's economy on the rest of the world's economic cycle and obviously would not hold for all countries.

2.2 The data

We model the real NZD/AUD and NZD/USD exchange rates, which are calculated using consumer price indices, and use the following notation:

- *lzau* is the real NZD/AUD exchange rate,
- *lzus* is the real NZD/USD exchange rate,
- *lcpusdnz* is the ANZ commodity price index,
- *ydifau* and *ydifus* are the real output differentials for Australia-NZ and US-NZ, respectively,
- *pidifau* and *pidifus* are the inflation differentials for Australia-NZ and US-NZ, respectively,
- *rdifau* and *rdifus* are the nominal short-term interest rate differentials for Australia-NZ and US-NZ, respectively,
- *apcyau* and *apcyus* are the annual GDP growth rates for Australia and NZ, respectively.

All variables are in logarithms except for the interest rates, output growth rates and inflation rates[6] and are quarterly data. The sample period is from 1986Q1 to 2003Q2 as it is the period where data is available for all the variables discussed.[7] Figures 1a and 1b plot the log of bilateral exchange rates with the variables discussed above.

2.3 Unit root and cointegration tests

We test to see if the variables contain unit roots using the augmented Dickey-Fuller (ADF) test and the results in levels are presented in table 1. Apart from the annual US GDP growth series (*apcyus*) and perhaps interest differential between Australia and NZ (*rdifau*), the results indicate that the variables appear to be unit root processes.

DeJong, Nankervis, Savin and Whiteman (1992) demonstrate that unit root tests have low power against plausible trend stationary alternatives and that the powers of both unit root and stationary tests may not be high enough to conclusively determine whether a series is integrated or not. In view of such uncertainties empirical cointegration studies tend to assume the presence of unit roots and then apply cointegration tests.

Consequently, for illustration purposes we also assume that the series are I(1) and apply Johansen's cointegration test procedure to test the null hypothesis that there is no cointegration between the variables. There are a few combinations of cointegrating relationships but for brevity table 2 shows the combination that we later report in the estimated model for each bilateral exchange rate equation. At the 1 per cent level we reject the null hypothesis of no cointegration between the variables tested. However, as shown by the estimated cointegration parameters (see the footnotes of table 2), Johansen's procedure gives widely dispersed coefficient estimates, which aren't generally consistent with our theoretical predictions.[8]

Gonzalo and Lee (1998) recommend using both Engle-Granger ADF *t*-test and Johansen's tests to test for cointegration. Thus we also apply Engle and Granger's residual-based test for the same combinations of variables and the results are displayed in table 3. Applying both tests shows that, as is typical in the practice of econometric analysis, we are given contrasting results.[9]

Nonetheless, Johansen's test offers some supporting evidence of cointegration among the variables and to develop the main argument of this paper we assume cointegration and continue estimation of the cointegrating vectors using the single-equation regression method in the next section. In section 4 we consider the validity of interpreting our regression results as cointegrating relationships.

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6 The definitions of the variables are described in more detail in the data appendix.

7 For some variables, eg interest rates and inflation rates, the sample period begins earlier (from 1983Q1) for unit root tests. For cointegration and single-equation regression methods discussed in the next section, the sample periods begin from 1986Q1 and are adjusted for leads and lags in the estimated equation.

8 See Maddala and Kim (1999, p 173) for a discussion of some drawbacks associated with the Johansen procedure including a tendency to produce outliers and to find spurious cointegration in small samples.

9 Facing this situation, apart from applying these tests the researcher can only hope, as Maddala and Kim (1998) suggest, that powerful cointegration tests are devised to overcome the deficiencies or problems of existing tests.

3 Estimation of the model, and the results

3.1 Estimation method

As discussed in Stock and Watson (2003), if z_t and f_t are cointegrated, then the OLS estimator of the cointegrating regression in equation (1) is consistent.[10] However, given the nonstationary nature of the time-series, the estimates will have inconsistent standard errors, which make standard hypothesis testing of the long-run elasticities of fundamental variables inaccurate.

An approach to address this problem is dynamic ordinary least squares (DOLS) (Stock and Watson, 1993). It involves adding leads and lags of first differenced explanatory variables in the regression, which in combination with the Newey-West heteroskedasticity and autocorrelation consistent (HAC) procedure mops up serial correlation, produces consistent standard errors and resolves the issue of endogeneity in single-equation methods.

We report here a set of econometric results for each bilateral exchange rate model.

The DOLS specification for NZD/AUD is:

$$lzau_t = \alpha + \beta f_t + \sum_{j=-p}^{p} \sigma_j \Delta f_{t-j} + \varepsilon_t \qquad (2)$$

where the fundamentals f_t are the ANZ commodity price index (*lcpusdnz*), output differential (*ydifau*), inflation differential (*pidifau*) and 90-day interest rate differential (*ridifau*).

The DOLS specification for NZD/USD is:

$$lzus_t = \alpha + \beta f_t + \sum_{j=-p}^{p} \sigma_j \Delta f_{t-j} + \varepsilon_t \qquad (3)$$

where the fundamentals f_t are the ANZ commodity price index (*lcpusdnz*), annual percentage change in real US output (*apcyus)*, inflation differential (*pidifus*) and 90-day interest rate differential (*rdifus*).

3.2 Point estimates of cointegrating variables

The hypothesis test results, coefficient estimates, and standard errors for equations (2) and (3) are displayed in tables 4 and 5, respectively.

The long-run coefficient estimates for the NZD/AUD are of the expected signs. The regression results suggest that a 1 per cent increase in the ANZ commodity price index would lead to approximately a 0.5 per cent appreciation of the NZD against the AUD. If Australia's output is 1 per cent higher than that of NZ, then in the long-run this differential would make our dollar depreciate around 0.4 per cent against the AUD.

The point estimate of 0.03 on the inflation differential suggests that the NZD would appreciate by 3 per cent if Australia's rate of inflation is persistently higher than NZ's by 1 per cent. And lastly, if NZ's short-term interest rate is 100 basis points higher than that of Australia the regression indicates that the NZD would appreciate by 6 per cent against the AUD in the long-run.

Like those in the NZD/AUD model, the long-run coefficients for NZD/USD are of the expected signs. The commodity price index is an influential fundamental variable on the long-run exchange rate, where a 1 per cent increase in the index leads to about a 1.6 per cent appreciation in the NZD/USD. As a comparison, Chen and Rogoff (2003) find the commodity price elasticity between 0.5 and 1 and relatively robust to different assumptions about the time-series properties of the data. Higher US growth also appears to have positive effects on the NZD but the impact on the long-run exchange rate is very small with a 1 per cent higher US output growth implying a 0.1 per cent higher real exchange rate.

The point estimate of 0.15 on the inflation differential suggests that the NZD would appreciate by 15 per cent if the US's rate of inflation is persistently higher by 1 per cent than that of NZ. As for the influence of interest rate differentials, the point estimate of 0.06 indicates that if NZ's short-term interest rate is 100 basis points higher than that of the US the NZD would appreciate by 6 per cent in the long-run.

[10] That is, the probability that the sample mean value of z_t is within a small interval of the true value approaches 1 as the sample size increases.

Although the NZD/AUD model appears well-specified, the model for the NZD/USD seems to suffer from a serial correlation problem based on the Breusch-Godfrey LM test. But serial correlation does not appear to be a problem when we estimate the same model using a different estimator next.

3.2.1 Sensitivity check of point estimates

As Stock and Watson's (1993) Monte Carlo results and empirical investigation show, it is useful to apply more than one asymptotically efficient estimator to estimate cointegrating vectors. Hence we check the sensitivity of the point estimates using another estimator of the cointegrating coefficient— the two-sided non-linear dynamic least squares estimator (P-L) described in Phillips and Loretan (1991). The estimator is given by:

$$z_t = \alpha + \beta f_t + \sum_{j=-p}^{p} \sigma_j \Delta f_{t-j} + \rho(z_{t-1} - \alpha - \beta f_{t-1}) + \varepsilon_t \qquad (5)$$

It includes lagged equilibrium relationships as regressors, consequently requiring the use of nonlinear least squares. Phillips and Loretan (1991) mention that this nonlinear estimator with its use of lagged equilibria (eg $(z_{t-1} - \alpha - \beta f_{t-1})$ shown in equation (5)) tends to proxy the past history of the stationary deviations about the long-run cointegrating relationship better than the use of lagged differences of the dependent variables. Similar to DOLS, this regression also eliminates the endogeneity problem of single-equation methods. Furthermore, the test statistics of P-L's regression are asymptotically normal, which means the t-statistics also do not need to be adjusted before we make inferences. Tables 6 and 7 show the estimation results.

For ease of comparison, table 8 below reproduces the point estimates from DOLS and P-L's nonlinear method shown in previous tables. Apart from the insignificance of the P-L estimates for *ydifau* and *pidifau* at the 5 per cent level, the P-L estimates are around the same order of magnitude as those derived from DOLS and statistically significant.

The long-run estimates show that the commodity price index explains more of the NZD/USD than the NZD/AUD. This difference may be due to the overlap of our exports with those of Australia — in times of higher commodity prices both the NZD and AUD tend to appreciate, reducing the explanatory power of the ANZ commodity price index.[11]

As for the extent short-term interest rate differentials influence the exchange rate in the long-run, the estimation results for bilateral exchange rates give estimates that are consistent with the capital flow interpretation discussed in McDonald (2002). That is, a higher domestic interest rate relative to that of the foreign country attracts more capital, increasing demand for the NZD and consequently appreciating the currency. Here the results suggest that if our nominal short-term interest rate is persistently higher than that of the US or Australia by 100 basis points, the NZD would appreciate by around 6 per cent (or as high as 10 per cent based on P-L estimates for NZD/USD).

Overall, this section demonstrates that long-run cointegration analysis can be conducted to give elasticities that appear to be statistically significant, as numerous empirical investigations in the literature have reported. However, the short sample span typical of NZ data with most only available from mid-1980s may cast doubt on the validity and size of the estimated coefficients. Therefore, in the next section we examine this issue via a Monte Carlo study.

4 Monte Carlo experiments

The empirical results above indicate that the explanatory variables influence the long-run level of the real exchange rate but in this section we posit that such an outcome may partly come from a small sample bias. Cointegration analysis as a long-run concept implies the need for a long sample period to uncover underlying trend(s) from cyclical movements. With a relatively short sample period of around 60 quarters in the cointegration test and coefficient estimation above, it is possible that the estimated coefficient on the interest rate differential is erroneously picking up the cyclical impact of the interest rate on the exchange rate rather than capturing the 'long-run' or structural effect. We test for this possibility by conducting a Monte Carlo study.

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11 The Reserve Bank of Australia's commodity price index for Australia was included in other specifications but the coefficient estimates were statistically insignificant.

Essentially, we set up this Monte Carlo study to examine the following hypotheses:

(1) In small sample investigations of the uncovered interest parity (UIP) relationship we may falsely conclude that interest rates and exchange rates are cointegrated,[12] and

(2) In this case, the long-run relationships being reported may simply represent the cyclical effect of monetary policy. Consequently the results are subject to the Lucas critique,[13] meaning that as the behaviour of policymakers shifts across time (altering the persistence of monetary policy) the structural parameters will change and the estimated long-run relationships may no longer hold.

4.1 The underlying model or 'true' data generating process

The model we use assumes the following exchange rate process

$$e_t = E[e_{t+1} \mid I_t] + r_t + u_t$$

where the current exchange rate e_t is dependent on the expected exchange rate next period with the information set I_t, the interest rate differential r and u a risk premium. Thus expected exchange rates are given by

$$E[e_{t+1} \mid I_t] = E[e_{t+2} \mid I_t] + E[r_{t+1} \mid I_t] +$$
$$E[u_{t+1} \mid I_t] = E[e_{t+2} \mid I_t] + E[r_{t+1} \mid I_t]$$

Repeating n-1 times implies

$$e_t = E[e_{t+n} \mid I_t] + r_t + E[r_{t+1} + r_{t+2} + ... + r_{t+n-1} \mid I_t]$$

[12] Elliott (1998) shows that in small samples if variables are near unit roots and not exact unit roots then size distortions can be very large. See also Haldrup and Jansson (1999) for a discussion of the possibility of near cointegration when the currently available cointegration tests reject the null hypothesis of no cointegration.

[13] For example, see Lucas (1976).

Agents know the true process for interest rates, ie

$$r_t = \rho_1 r_{t-1} + \varepsilon_t$$

where ρ_1 is the autoregressive parameter and ε the innovations that are normally distributed with mean zero and unit variance. Put another way, the interest rate differential at time *t* is defined as being partially dependent on its value at time *t-1* plus some random error term that on average will equal zero with its variance scaled at one.[14]

So

$$E[r_{t+1} \mid I_t] = \rho_1 r_t$$
$$E[r_{t+2} \mid I_t] = \rho_1^2 r_t$$

and so on, which implies that as n goes to infinity the expected exchange rate becomes closer and closer to zero ($e_{t+n}=0$ as $n \to \infty$). Thus the exchange rate path is approximated by summing all future expected interest rate differentials:

$$e_t = \frac{r_t}{1-\rho_1} + u_t \ ,$$

$$u_t = \rho_2 u_{t-1} + (1-\rho_2)\upsilon_t$$

where u_t models other components that affects the exchange rate. Specifically, it is assumed to be composed of two parts that are weighted by ρ_2 and (1- ρ_2)—one part is correlated with the previous shock ($\rho_2 u_{t-1}$) and the other part represents the agents' prediction errors ($(1-\rho_2)\upsilon_t$), where υ_t is generated around a mean of zero with a standard deviation of seven around the mean.[15]

[14] The correct interpretation of such an autoregressive process should be one of correlation rather than causation. That is, the interest rate differential is correlated with its past values (instead of being explained by its past values) because the underlying factors that caused last period's interest rate differential are still exerting a significant amount of influence on current period's interest differential.

[15] Empirically for the NZD/AUD, the ratio of the exchange rate equation's standard deviation of residuals over that of the interest rate differential AR(1) model is around 5. For the NZD/USD, the standard deviation ratio is around 13. For this experiment we adopt a ratio of 2 and 7 but the results and conclusions are not qualitatively different so

To emphasise, both the exchange rate and interest rate differential series are by construction stationary and hence an OLS regression between the two variables picks up the persistence of monetary policy. To confirm this characteristic, we perform experiments with a large sample size of 1000 observations (as opposed to 60) and the unit root and cointegration test results show that the series are stationary and therefore not cointegrated (ie exactly as one would expect given the 'true' data generating process described above).

4.2 Monte Carlo simulation methodology

We simulate artificial quarterly time series of interest differentials and exchange rates over a sample period of 60 quarters, which is fairly realistic in empirical studies with NZ data. The synthetic data series are generated with a *pseudo-random* number generator[16]. Every time we change a model parameter, eg ρ_1, we are performing a new experiment with each experiment replicating the following procedure 1000 times:

1. Set the model parameters.[17]
2. Draw pseudo-random residuals to form time series for the model.
3. Apply the ADF and Dickey-Fuller Generalized Least Squares (DF-GLS) unit roots and repeat them as cointegration tests and save the test statistics and critical values.
4. Do the regressions and save the β estimates, ie the estimated parameter on the interest rate differential. Apart from applying the OLS estimator we also estimate the interest differential coefficient on the exchange rate using the two single-equation regression methods discussed in section 3 – DOLS and Phillips and Loretan's two-sided nonlinear least squares.
5. Characterise the empirical distribution of the estimated results.

Compared to Godbout and Van Norden (1997), what we are doing here is postulating a specific (theory based) functional form for the data generating process, rather than simulating data based on the historical data set. Our approach allows us to assess how well the estimators would distinguish a short-run uncovered interest parity relationship from a long run relationship between interest rate differentials and the real exchange rate: specificially, whether there is a tendency to falsely conclude the variables are non-stationary and cointegrated.

4.3 Simulation results

Table 9 shows the acceptance rate of the ADF and the DF-GLS tests under the null hypothesis of a unit root at the 5 per cent statistical significance level. To be more specific, the p-value from each ADF test over a range of values for ρ_1 and ρ_2 is saved and we then calculate the percentage of test statistics that is greater than the critical value at the 5 per cent significance level (ie non-rejection of the null hypothesis of a unit root present). A high percentage implies that the researcher is likely to conclude that based on the ADF test a series appears to be nonstationary when it is in fact stationary (ie no unit root present). The acceptance rates shown suggest that in finite samples the more persistent the data the more likely one will falsely accept the null hypothesis of a unit root. The results confirm the low power of unit root tests in finite samples in general as it is difficult for the test to distinguish a highly persistent but stationary data series from a truly nonstationary series. The results show that the acceptance rates are lower using the DF-GLS test, which supports the case Maddala and Kim (1999) make in advocating the application of unit root tests such as DF-GLS (with better power properties) rather than the popular ADF test.

Next, we apply the E-G ADF and DF-GLS residual-based cointegration tests by regressing the interest rate differential on the exchange rate via the OLS estimator and then carry out the tests on the OLS residuals. Table 10 displays the rejection rates of the null hypothesis of no cointegration at the 5 per cent significance level, ie the likelihood that a researcher concludes that there is a long-run relationship between the artificial interest rate

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we only report the simulation results with a standard deviation ratio of 7. Indeed, Davidson and MacKinnon (1993, p 741) show that many test statistics for regression models are invariant when the variance of the error terms are altered. Consequently, the variance of the error terms can be set at an arbitrary level without affecting the finite-sample distribution.

[16] Once the model parameters are determined, changes in the data and the coefficient estimates only occur through changes in the normal pseudo-random residuals. Note that the residuals are pseudo-random, that is, not truly random because econometricians for practical purposes use computer-generated numbers (based on algorithms) that are indistinguishable from genuinely random numbers drawn from a normally distribution with zero mean and unit variance. For more details, see Davidson and MacKinnon (1993, p 734).

[17] We simulate the experiments with a range of autoregressive parameters, ρ_1 and ρ_2, from 0.5 to 0.9 in increments of 0.1. Estimating AR(1) models on the 90-day interest differentials shows the autoregressive parameters to be large – 0.94 between US and NZ and 0.86 between Australia and NZ. Given the assumed relationships specified in this experiment, estimate of ρ_2 is 0.74 and 0.87 for the NZD/AUD and NZD/USD models, respectively.

differential and the exchange rate series.[18] The results in table 10 suggests that when the data (particularly the exchange rate) is highly persistent, the residuals still contain a lot of the persistence not explained by the explanatory variable (in this case the interest rate differential). Consequently, the ADF and DF-GLS tests find the residual series nonstationary and hence leads the researcher to reach the verdict of no cointegration. The same logic explains why the residual-based test leads one to more readily reject the null hypothesis of no cointegration when the data series are less persistent. Again, the DF-GLS appears to perform better in terms of fewer cases where the researcher would falsely conclude that a cointegrating relationship exists.

We also compare the estimated coefficients on the interest rate differential from the DOLS and the Phillips and Loretan's two-sided nonlinear dynamic least squares estimator with those from the OLS method[19]. We only report the estimated coefficient for the experiment involving the most persistent series[20] because in other cases the econometrician would probably not continue with cointegration tests, given that the unit root tests suggest stationarity.

Figures 2 to 4 show histograms of the empirical distribution of simulation results as well as line graphs of the estimated parameters for OLS, DOLS and P-L estimates, in that order. These are all for the particular case where ρ1and ρ2 are equal to 0.9. Apart from the P-L estimates, the cointegration parameter estimates are close to the true parameter ($\frac{1}{(1-\rho_1)}$).[21] However, it is important to note that this parameter simply reflects the persistence or cyclical impact of monetary policy, so to interpret the estimates as long-run coefficients would be erroneous.

18 Note that the rejection rate shown in Table 10 calculates the number of replications out of 1000 where the residual-based tests first suggest the presence of a unit root and then cointegration.

19 If the interest rate differential and the exchange rate series are both I(1) and cointegrated, then OLS is a feasible estimation procedure just as it would be for stationary series. For the single-equation estimation of the generated series involving dynamic terms, the exchange rate series is regressed with one lead and lag of the first differenced explanatory variables and the P-L equations also include the first lag of the equilibrium relationship $e_{t-1} - \beta r_{t-1}$, *where* β stands for beta, the estimated coefficient).

20 The case where ρ_1 and ρ_2 are both 0.9.

21 The nonlinear estimation of P-L on a couple of occasions seems to have had difficulties reaching convergence, ending up with large negative parameter estimates. Thus figure 4 shows results where the starting value is set to the true parameter value for each replication.

Therefore, based on the results of the Monte Carlo study, it appears that it is plausible that a researcher could falsely conclude that interest rates and exchange rates are cointegrated when investigating the UIP relationship with a small sample. Moreover, the estimated long-run elasticities may simply represent the persistence of monetary policy in response to the state of the business cycle and not a permanent influence on the 'equilibrium' or long-run exchange rate.

5 Concluding comments

In this paper, as in numerous empirical studies, we model long-run relationships between bilateral exchange rates and economic fundamentals. We begin by examining the time-series properties of the data and using Johansen's cointegration method as well as Engle-Granger's ADF test to find evidence of cointegrating relationships. We continue with the assumption of cointegration and find that the analysis suggests the existence of a long-run relationship between the real exchange rate, NZ commodity prices, nominal interest rate differential, output differential and inflation differential between Australia and NZ. A long-run relationship also appears to exist between the exchange rate, NZ commodity prices, nominal interest differential, US output growth rate and inflation differential between the US and NZ.

Subsequently, we apply DOLS and Phillips and Loretan's nonlinear least squares estimators to obtain coefficient estimates of the cointegrating relationship for each real bilateral exchange rate series. The variables enter with *a priori* signs and are of plausible magnitudes. In particular, the coefficient estimates suggest that a 100 basis point interest rate differential would appreciate the NZD against the AUD or the USD by around 6 per cent.

After conducting a few of the popular estimation procedures for this type of empirical study, we perform a Monte Carlo study to test a couple of hypotheses. The simulation experiments illustrate two main issues with long-run cointegration analysis. Firstly, with a realistic sample size of 60 quarters it is possible for a researcher to incorrectly conclude that two stationary variables are cointegrated. Secondly, if a researcher subsequently carried on to estimate a cointegration equation with single-equation procedures like DOLS or Phillips and Loretan's nonlinear estimator, the results may not be indicative of the 'true' long-run relationship and may instead pick up cyclical correlations between variables.

Thus given the short supply of NZ data available for cointegration analysis, it is very difficult to make definitive statements about the trend or equilibrium exchange rate. We advocate taking care in interpreting results from similar cointegration-based exchange rate models. Furthermore, we recommend applying different methods to consider a range or band of estimates of where the equilibrium path might lie. For example, Wren-Lewis (2004) extends his 'Five Area Bilateral Equilibrium Exchange Rate' (FABEER) partial equilibrium model to include New Zealand and Australia and finds that both countries' equilibrium exchange rates are sensitive to persistent shifts in commodity prices, though less so than the estimate from the NZD/USD model in this paper.

To conclude, while the concept of long-run or equilibrium exchange rate is appealing, a research strategy that directs the emphasis away from the performance of one particular exchange rate model and applies a number of models and/or estimation methods (including those of a non-linear nature) may better inform policy assessments about the nature of exchange movements relative to its broad trend.

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Table 1:
Augmented Dickey-Fuller (ADF) unit root test results

		In levels	
Variable	**Lags†**	**ADF test statistic (p-value) Constant, no trend**	**ADF test statistic (p-value) Constant & trend**
lzau	0	-2.1610 (0.2221)	-2.1776 (0.4952)
lzus	1	-1.9957 (0.2882)	-2.0760 (0.5509)
lcpusdnz	1	-2.6327 (0.0915)	-2.6623 (0.2553)
ydifau	0	-2.1678 (0.2197)	-1.6537 (0.7609)
ydifus	0	-2.3738 (0.1528)	-1.5066 (0.8180)
pidifau	0	-2.1506 (0.2260)	-2.6322 (0.2677)
pidifus	0	-1.3305 (0.6116)	-1.8815 (0.6539)
rdifau	1	-3.1207* (0.0290)	-3.3200 (0.0704)
rdifus	2	-1.9046 (0.3287)	-3.1828 (0.0954)
apcyus	2	-4.1024** (0.0017)	-4.0712* (0.0104)

† Lags are selected automatically using EViews based on the Schwartz information criterion (SIC).

*(**) denotes rejection of the null hypothesis of a unit root at the 5 (1) per cent level.

Table 2:
Johansen cointegration test results‡

Variables	Lags	Max-eigenvalue test statistic	Trace test statistic	Number of cointegrating vectors
(1) *lzau, lcpusdnz, ydifau, rdifau, pidifau*	2	45.0714**	97.3000**	1
(2) *lzus, lcpusdnz, apcyus, rdifus, pidifus*	1	39.1278**	82.6678**	1

‡ The trend assumption is linear trend in the data, and an intercept but no trend in the cointegrating equation.

** denotes rejection of the hypothesis at the 1 per cent level

(1): The estimated cointegration parameters normalised on *lzau*:
Normalized cointegrating coefficients (standard error in parentheses)

LZAU	LCPUSDNZ	YDIFAU	RDIFAU	PIDIFAU
1.000000	-0.958571	-0.998778	0.037465	-0.015170
	(0.18013)	(0.24429)	(0.00668)	(0.01142)

(2): The estimated cointegration parameters normalised on *lzus*:
Normalized cointegrating coefficients (standard error in parentheses)

LZUS	APCYUS	LCPUSDNZ	PIDIFUS	RDIFUS
1.000000	-0.086913	-0.185701	-0.172232	0.125979
	(0.01768)	(0.34325)	(0.02209)	(0.01454)

Table 3:
Engle-Granger ADF cointegration test results[‡]

Variables	Lags[+]	ADF test statistic (P-value)
(1) *lzau, lcpusdnz, ydifau, rdifau, pidifau*	0	-3.3018 (0.2866)
(2) *lzus, lcpusdnz, apcyus, rdifus, pidifus*	0	-2.7119 (0.5703)

[+] Lag length automatically selected based on the Schwartz information criterion (SIC).

[‡] The critical values for multivariate ADF tests are different from those for bivariate cases. The finite-sample critical value at 5 per cent significance level is -4.2423, calculated using the computer program introduced in MacKinnon (1996) and is that of the no constant case and for 70 observations.

Table 4:
DOLS coefficient estimates and hypothesis test results against the AUD

$$lzau_t = \alpha + \beta f_t + \sum_{j=-4}^{4} \sigma_j \Delta f_{t-j} + \varepsilon_t$$

Adjusted sample: 1987Q2-2002Q4

	Coefficient estimates (standard errors)	t-statistic (p-value)
constant	3.0388** (1.0488)	2.8975 (0.0084)
lcpusdnz	0.4620** (0.1619)	2.8532 (0.0092)
ydifau	-0.3561* (0.1765)	-2.0175 (0.0560)
pidifau	0.0302** (0.0058)	4.9972 (0.0001)
rdifau	-0.0611** (0.0061)	-10.6198 (0.0000)
Std. error of regression	0.02	
Durbin-Watson statistic	1.69	
No serial correlation	3.92 (0.14)	

*(**) Indicates significance at the 5 (1) per cent level of significance. The coefficients on the first difference leads and lags are not displayed because the leads and lags serve to deal with the problem of potential endogeneity.

The standard errors reported are Newey-West heteroskedasticity and autocorrelation consistent (HAC) standard errors. Stock and Watson (2003) point out that the DOLS estimator is efficient and statistical inferences about the coefficients based on HAC standard errors are valid in large samples. Consequently, the t-statistics reported here do not undergo the transformation suggested by Hamilton (1994, pp 608-612).

The null hypothesis of no serial correlation was tested using the Breusch-Godfrey serial correlation LM test (the statistic is the number of observations multiplied by R^2). The lag length for this test was set to two.

Table 5:
DOLS coefficient estimates and hypothesis test results against the USD

$$lzus_t = \alpha + \beta f_t + \sum_{j=-5}^{5} \sigma_j \Delta f_{t-j} + \varepsilon_t$$ Adjusted sample: 1987Q3-2002Q3		
	Coefficient estimates (standard errors)	t-statistic (p-values)
constant	-3.4854 (1.7272)	-2.0180 (0.0665)
lcpusdnz	1.5700** (0.3863)	4.0639 (0.0016)
apcyus	0.0706* (0.0235)	3.0007 (0.0111)
pidifus	0.1484** (0.0258)	5.7462 (0.0001)
rdifus	-0.0575** (0.0176)	-3.2726 (0.0067)
Std. error of regression	0.04	
Durbin-Watson statistic	2.00	
No serial correlation	11.63 (0.00)**	

*(**) Indicates significance at the 5 (1) per cent level of significance. The coefficients on the first difference leads and lags are not displayed because the leads and lags serve to deal with the problem of potential endogeneity.

The standard errors reported are Newey-West heteroskedasticity and autocorrelation consistent (HAC) standard errors. Stock and Watson (2003) point out that the DOLS estimator is efficient and statistical inferences about the coefficients based on HAC standard errors are valid in large samples. Consequently, the t-statistics reported here do not undergo the transformation suggested by Hamilton (1994, pp 608-612).

The null hypothesis of no serial correlation was tested using the Breusch-Godfrey serial correlation LM test (the statistic is the number of observations multiplied by R^2). The lag length for this test was set to two.

Table 6:
P-L coefficient estimates and hypothesis test results against the AUD

$$lzau_t = \alpha + \beta f_t + \sum_{j=-3}^{2} \sigma_j \Delta f_{t-j} + \rho(lzau_{t-1} - \alpha - \beta f_{t-1}) + \varepsilon_t$$ Adjusted sample: 1987Q1-2003Q2		
	Coefficient estimates (standard errors)	t-statistic (p-value)
lcpusdnz	0.5329* (0.2237)	2.3821 (0.0226)
ydifau	-0.2783 (0.2971)	-0.9367 (0.3551)
pidifau	0.0098 (0.0158)	0.6203 (0.5390)
rdifau	-0.0478** (0.0080)	-6.0128 (0.0000)
ρ (lagged equilibrium)	0.6278** (0.1133)	5.5423 (0.0000)
Std. error of regression	0.02	
Durbin-Watson statistic	1.67	
No serial correlation	2.73 (0.26)	

*(**) Indicates significance at the 5 (1) per cent level of significance.

The coefficients on the first difference leads and lags are not displayed because the leads and lags serve to deal with the problem of potential endogeneity.

The null hypothesis of no serial correlation was tested using the Breusch-Godfrey serial correlation LM test (the statistic is the number of observations multiplied by R^2). The lag length for this test was set to two.

Table 7:
P-L coefficient estimates and hypothesis test results against the USD

$lzus_t = \alpha + \beta f_t + \sum_{j=-3}^{3} \sigma_j \Delta f_{t-j} + \rho(lzus_{t-1} - \alpha - \beta f_{t-1}) + \varepsilon_t$ Adjusted sample: 1987Q1-2003Q1		
	Coefficient estimates (standard errors)	t-statistic (p-value)
lcpusdnz	1.8741* (0.8266)	2.2672 (0.0305)
apcyus	0.0854* (0.0354)	2.4132 (0.0219)
pidifus	0.1369** (0.0482)	2.8404 (0.0079)
rdifus	-0.0986** (0.0315)	-3.1259 (0.0038)
ρ (lagged equilibrium)	0.8422** (0.0825)	10.2047 (0.0000)
Std. error of regression	0.03	
Durbin-Watson statistic	1.60	
No serial correlation	3.34 (0.18)	

*(**) Indicates significance at the 5 (1) per cent level of significance.

The coefficients on the first difference leads and lags are not displayed because the leads and lags serve to deal with the problem of potential endogeneity.

The null hypothesis of no serial correlation was tested using the Breusch-Godfrey serial correlation LM test (the statistic is the number of observations multiplied by R^2). The lag length for this test was set to two.

Table 8:
DOLS and P-L coefficient estimates

	Real NZD/AUD	
	DOLS estimates	**P-L estimates**
lcpusdnz	0.4620**	0.5329*
ydifau	-0.3561*	-0.2783
pidifau	0.0302**	0.0098
rdifau	-0.0611**	-0.0478**
	Real NZD/USD	
	DOLS estimates	**P-L estimates**
lcpusdnz	1.5700**	1.8741*
apcyus	0.0706*	0.0854*
pidifus	0.1484**	0.1369**
rdifus	-0.0575**	-0.0986**

Table 9:
Acceptance rates of two unit root tests

Interest rate differential (r)		
ρ_1	ADF unit root test % of t-stats ≥ c.v at 5%	DF-GLS unit root test % of t-stats ≥ c.v at 5%
0.5	1%	2%
0.6	4%	1%
0.7	18%	2%
0.8	51%	10%
0.9	84%	44%

Exchange rate (e)			
ρ_1	ρ_2	ADF unit root test % of t-stats ≥ c.v at 5%	DF-GLS root test % of t-stats ≥ c.v at 5%
0.5	0.5	1%	2%
0.5	0.6	3%	1%
0.5	0.7	7%	2%
0.5	0.8	11%	3%
0.5	0.9	7%	4%
0.6	0.5	2%	2%
0.6	0.6	5%	2%
0.6	0.7	11%	2%
0.6	0.8	14%	4%
0.6	0.9	11%	5%
0.7	0.5	6%	2%
0.7	0.6	12%	3%
0.7	0.7	18%	5%
0.7	0.8	22%	6%
0.7	0.9	22%	6%
0.8	0.5	30%	7%
0.8	0.6	39%	10%
0.8	0.7	47%	13%
0.8	0.8	51%	15%
0.8	0.9	52%	15%
0.9	0.5	79%	42%
0.9	0.6	81%	44%
0.9	0.7	83%	45%
0.9	0.8	84%	47%
0.9	0.9	84%	47%

Note:
This table shows the percentages of Augmented Dickey-Fuller and Dickey-Fuller Generalized Least Squares unit root test results (based on 1000 draws with a sample size of 60 observations) that accept the null hypothesis of a unit root process. The acceptance rates are displayed for every combination of data persistence (between 0.5 and 0.9) simulated. The ADF test includes no intercept and trend in the test equation. The finite-sample critical value at the 5 per cent level for each test is calculated with the program described in McKinnon (1996). The test lag length is automatically selected based on the SIC and not reported here.

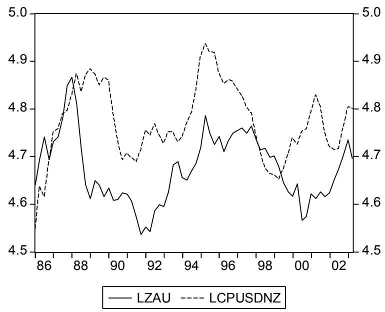
Table 10:
Rejection rates of the null hypothesis of no cointegration

ρ_1	ρ_2	ADF cointegration test	DF-GLS cointegration test
0.5	0.5	0%	0%
0.5	0.6	0%	0%
0.5	0.7	0%	0%
0.5	0.8	0%	0%
0.5	0.9	0%	0%
0.6	0.5	0%	0%
0.6	0.6	1%	0%
0.6	0.7	1%	0%
0.6	0.8	1%	0%
0.6	0.9	0%	0%
0.7	0.5	3%	0%
0.7	0.6	6%	0%
0.7	0.7	5%	1%
0.7	0.8	3%	1%
0.7	0.9	2%	0%
0.8	0.5	26%	3%
0.8	0.6	30%	4%
0.8	0.7	24%	4%
0.8	0.8	13%	3%
0.8	0.9	6%	1%
0.9	0.5	73%	33%
0.9	0.6	67%	33%
0.9	0.7	48%	30%
0.9	0.8	24%	19%
0.9	0.9	9%	7%

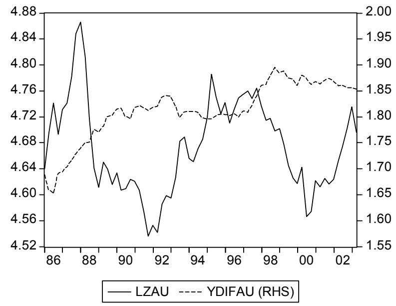
Note: This table shows the number of replications out of 1000 (with a sample size of 60 observations) where the residual-based tests (the Augmented Dickey-Fuller and Dickey-Fuller Generalized Least Squares test results first suggest the presence of a unit root and then cointegration at the 5 per cent significance level. The ADF test on the OLS residuals includes an intercept in the test equation and the test statistic is based on that in McKinnon (1996). The test lag length is automatically selected based on the Schwartz information criterion (SIC).

Figure 1a:
Real NZD/AUD exchange rate and other variables

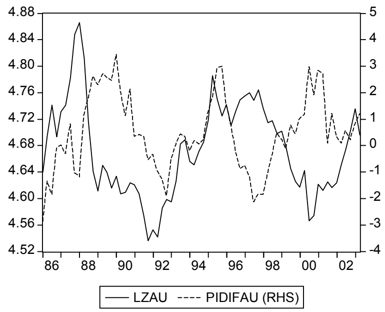
Real NZD/AUD exchange rate (lzau) and the ANZ commodity price index



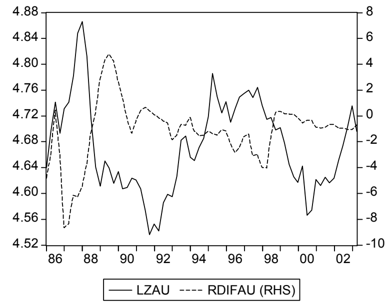
Real NZD/AUD exchange rate and output differential between AU and NZ



Real NZD/AUD exchange rate (lzau) and inflation differential between AU and NZ



Real NZD/AUD exchange rate (lzau) and interest rate differential between AU and NZ



Real NZD/AUD exchange rate (lzau) and Australian annual GDP growth

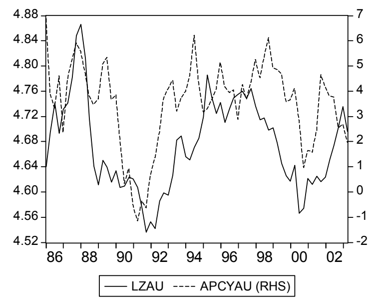
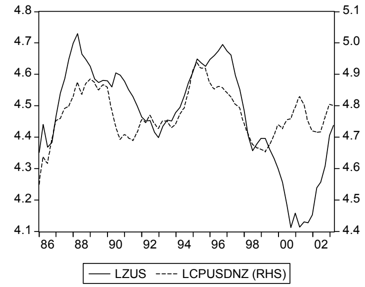
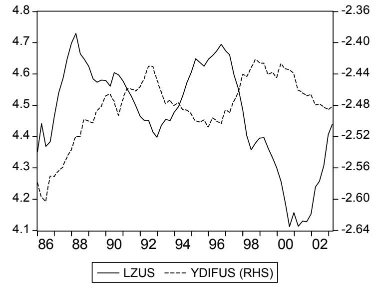


Figure 1b:
Real NZD/USD exchange rate and other variables

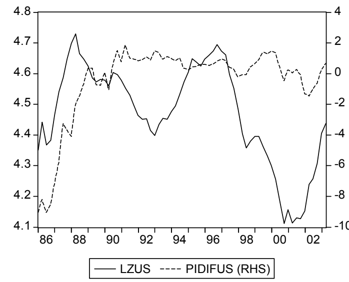
Real NZD/USD exchange rate (lzus) and the ANZ commodity price index



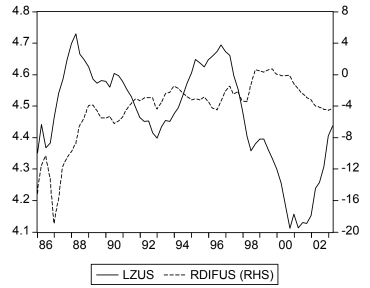
Real NZD/USD exchange rate (lzus) and output differential between US and NZ



Real NZD/USD exchange rate (lzus) and inflation differential between US and NZ



Real NZD/USD exchange rate (lzus) and interest rate differential between US and NZ



Real NZD/USD exchange rate (lzus) and US annual GDP growth

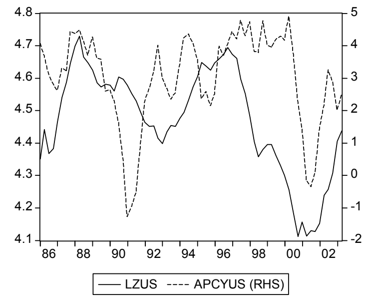
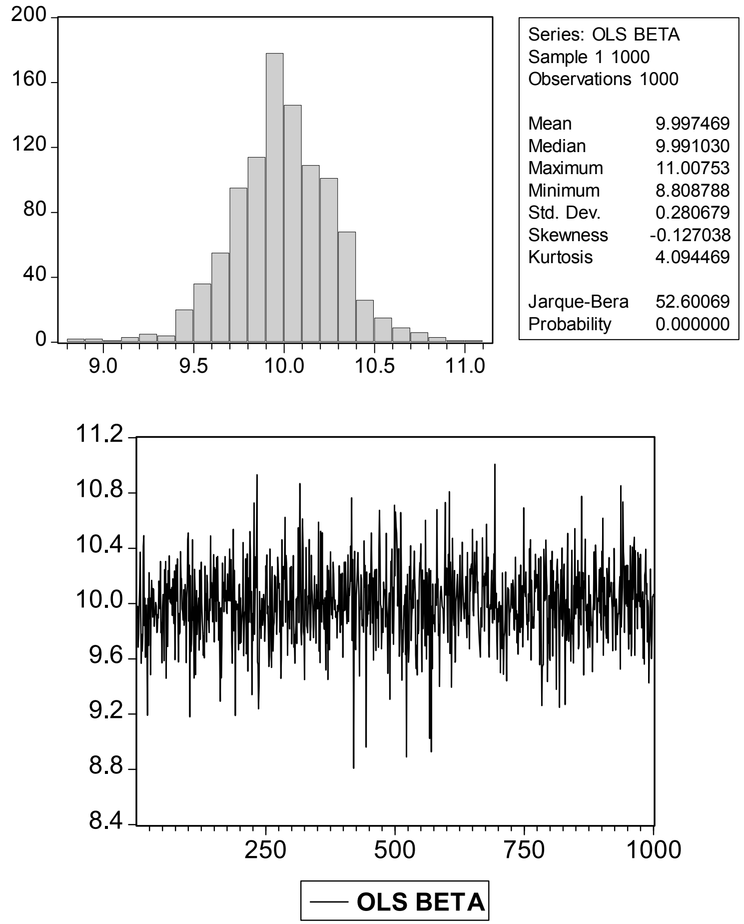
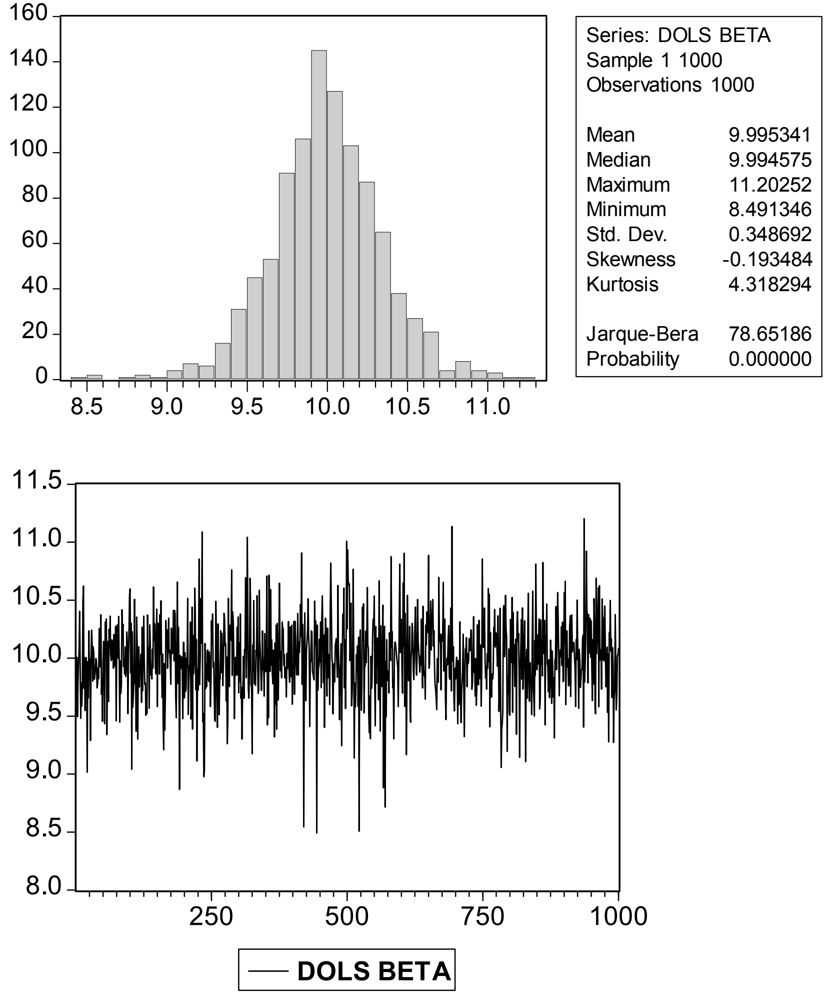


Figure 2:
Histogram and plot of Monte Carlo coefficient estimates via OLS (ρ1=0.9, ρ2= 0.9)



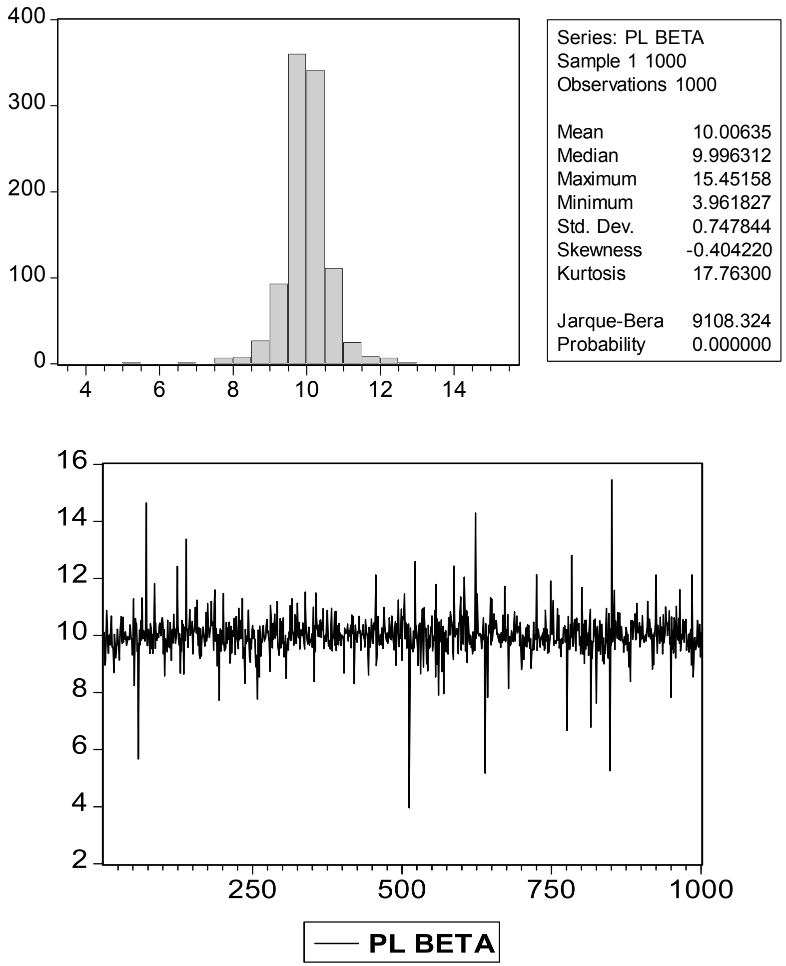
Note:
In the graph above, the X-axis identifies each of the 1000 draws or replications, while the Y-axis represents the coefficient value.

Figure 3:
Histogram and plot of Monte Carlo coefficient estimates via DOLS (ρ1=0.9, ρ2= 0.9)



Note:
In the graph above, the X-axis identifies each of the 1000 draws or replications, while the Y-axis represents the coefficient value.

Figure 4:
Histogram and plot of Monte Carlo coefficient estimates via P-L (ρ1=0.9, ρ2= 0.9)



Note:
In the graph above, the X-axis identifies each of the 1000 draws or replications, while the Y-axis represents the coefficient value.

Data Appendix

lzau: the log of the real NZD/AUD exchange rate.
lzus: the log of the real NZD/USD exchange rate.
lcpusdnz: the log of ANZ Commodity Price Index (world prices in USD).
rdifau: nominal short-term interest rate differential between Australia and NZ, each interest rate series is the 90-day interest rate.
rdifus: nominal short-term interest rate differential between the US and NZ, each interest rate series is the 90-day interest rate.
ydifau: the log of Australian real GDP minus the log of NZ GDP.
ydifus: the log of US GDP minus the log of NZ GDP.
pidifau: Australia's annual CPI inflation minus NZ's CPI inflation.
pidifus: The US's annual CPI inflation minus NZ's CPI inflation.
apcyus: The US's annual percentage change in real GDP.