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**Estimating a time varying neutral real
interest rate for New Zealand**

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Abstract^{1, 2}

The interest rate which corresponds to neutral monetary policy settings in New Zealand appears to have trended downwards since at least the stabilisation of inflation in 1992. We present several alternative estimates of a time varying neutral real interest rate (NRR) in state space models, which all show the same declining trend. We then test for a relationship between the Neo-Wicksellian real interest rate gap and future inflation. As in Neiss and Nelson (2003), these two are highly correlated.

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² The views expressed here are the views of the authors and do not necessarily reflect the views of the Reserve Bank of New Zealand or the World Bank. We have received many helpful comments from colleagues and friends. Specific thanks (without implication) go to David Law, Chris Plantier, Weshah Razzak and Matthew Shapiro. Remaining errors and omissions are our own. © Reserve Bank of New Zealand

1 Introduction

Over the past decade, New Zealand has experienced higher real interest rates than its main trading partners. At the same time, it seems that the interest rates required to maintain inflation within the target band are no longer as high as was the case a decade ago, and the interest rate cycles have diminished. These observations might be explained by a time varying and trend-declining neutral real interest rate (NRR), which nevertheless has been higher than in other countries. The main objective of this paper is to estimate the (unobservable) NRR that would be consistent with the observed data.³

The NRR concept itself dates back to Wicksell (1898),⁴ though a more modern version is expressed in Svensson (2001):

“Can the central bank indefinitely maintain a low instrument rate and a low exchange rate and in this way stimulate the economy indefinitely? The answer is definitely no. In the longer term, the central bank must set its instrument rate so that on average the short real rate is equal to the neutral real rate. The neutral real rate is the real rate that is consistent with output equal to potential output. It is largely determined by factors other than monetary policy. If the central bank tries to maintain a short real rate below the neutral real rate for too long, aggregate demand outstrips potential output, the economy becomes overheated, and inflation increases to high single digit, then double-digit inflation, and eventually hyperinflation.”

The neutral real rate has traditionally been thought of as the average real interest rate prevailing over the cycle, under conditions where both expected and actual inflation are stable, all else equal. During

³ Valentin (2002) argues that using a constant neutral real rate does almost the same job in the Taylor rule. However, his study is based on the US, where it does not move as much as other researchers found for New Zealand.

⁴ See Woodford (2002) for the so-called “neo-Wicksellian” framework

the inflation targeting period, however, such averaging leads to implausibly high estimates in New Zealand.

Various approaches exist to estimating a time-varying NRR. Laubach and Williams (2003) use the Kalman filter to jointly estimate the trend growth of the economy, the neutral real interest rate and potential output. They find a strong relationship between the trend growth rate and the neutral real interest rate in the United States. However, the approach taken by Laubach and Williams is better suited for a large closed economy than for a small open one. In addition, the combination of higher neutral real interest rates in New Zealand and flat productivity growth over the 1990s do not match the findings of Laubach and Williams.

Neiss and Nelson (2003) back out a neutral real interest rate series from a calibrated dynamic stochastic general equilibrium model for the UK economy. They then compare inflation outcomes to the gap between the actual real interest rate and their NRR estimate, and find some predictive power. The same methodology was recently used for the Euro area in Giammarioli and Valla (2003). Modelling the neutral real interest rates nevertheless relies on the correct specification of the model.⁵ In the New Zealand context the issue was conceptually presented in Archibald and Hunter (2001), who present several indicators yielding estimates of the equilibrium neutral real rate ranging from 2.8 per cent to 5.8 per cent. Plantier and Scrimgeour (2002) used the Taylor rule for New Zealand in state space to estimate a time varying neutral real interest rates. Their work suggested that the neutral real interest rate has been declining in New Zealand. Björkstén and Karagedikli (2003a,b) also found empirical support for a decline in New Zealand's neutral real interest rates. A corresponding result emerged in a cross-country study by Plantier⁶ (2003). In practice, the Reserve Bank of New Zealand continuously adjusts its view on the neutral real interest rate in a way

⁵ In particular how well it distinguishes persistent shocks from transitory blips.

⁶ Plantier (2003) looked at the neutral real interest rate from a cross-country perspective to explain the persistently high neutral real interest rate in New Zealand. However, this differential between New Zealand and the rest of the world is no the question we want to answer in this paper.

which is consistent with the findings of the studies mentioned above (Basdevant and Hargreaves 2003).

In this paper we derive estimates of a time-varying neutral real interest rate under several different specifications. We also derive the associated real interest rate gap (in the sense used by Neiss and Nelson 2003), which indicates in real time the amount of stimulus that monetary policy is imparting to the economy. We then use our estimates to compare the real interest rate gap to both inflation and inflation deviations from target, finding broadly that there is a correlation between the real interest rate gap and future inflationary pressure.

The remainder of the paper is structured as follows. Section 2 presents both a model-independent approach to estimating the neutral real interest rate in New Zealand, as well as multivariate approach with several alternative specifications. Section 3 looks at the relationship between the inflation deviations from target and the real interest rate gap (deviations of short term interest rates from the estimated neutral settings). Section 4 discusses policy implications and concludes.

2 Estimating a time-varying NRR with and without formal models

We start with a simple and intuitive non-model approach of deriving a time-varying NRR, by making use of market data that is contained in the yield curve. In order to directly estimate monetary policy deviations from neutral settings, we consider the extent to which the slope of the yield curve differs from “normal”, with a steeper-than-normal yield curve indicating expansionary monetary policy. The idea is that over the business cycle, monetary policy settings must on average be neutral if inflation is stable. Likewise, short and long interest rates move procyclically, but short rates move by more than long rates. Then, over the business cycle, the yield spread will revert to its mean. This analysis abstracts from level considerations of the interest rate, and instead gives an alternative measure of whether monetary policy is tight or loose.

Doing this for New Zealand, we derive a very simple indicator for when monetary policy has differed from neutral. The real interest rate gap series is computed as follows:

$$rirgap_t = r_t - R_t - (\bar{r} - \bar{R}) \quad (1)$$

where R is the 10-year nominal interest rate and r is the 90-day nominal rate. Taking the difference between the two nominal interest rates yields a term premium plus a term reflecting expected changes in inflation over the term. We make the assumption that inflation is stable, ie the inflation components of nominal short and long interest rates cancel each other out when one is subtracted from the other. Consequently, the real interest rate gap in equation 1 simply gives us the term premium at time t as compared to the average term premium over the period.

We selected for analysis the time period 1992-2002, for two important reasons. First, this corresponds to the time period during which inflation has been low and stable in New Zealand (at about two per cent). Second, we argue that the endpoints of this time period correspond to similar points in the business cycle, so that the average term premium over the time period corresponds to a cyclically neutral time premium. This assumption is tested, as described below.

Once a baseline yield curve slope has been established and a real interest rate gap computed from equation (1), a neutral real rate can be backed out of the analysis. This is done in equation (2), by subtracting the real interest rate gap from the 90-day rates to get the short term neutral nominal rate, then subtracting out expected inflation one year ahead.

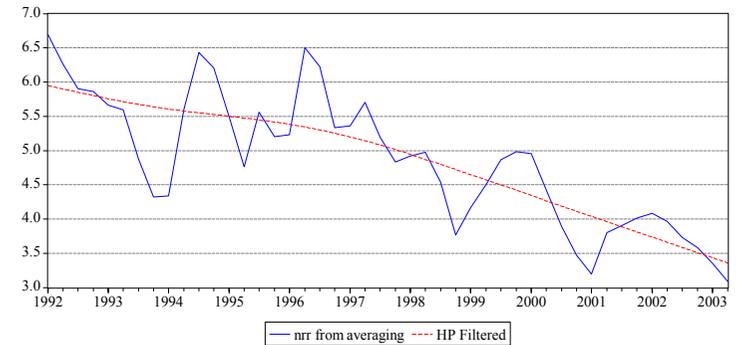
$$\begin{aligned} NRR_t &= r_t - rirgap_t - \pi_{t+1}^e \\ &= R_t - (\bar{R} - \bar{r}) - \pi_{t+1}^e \end{aligned} \quad (2)$$

Thus, the level of the short term neutral real interest rate is determined from the nominal 10-year interest rate, minus a constant

representing the sum of the average yield spread and expected inflation. This can be thought of as a forward-looking weighted average of expected future short real rates plus the deviation of the term premium from its mean (see Appendix 5.1).

Figure 1 below shows the calculated neutral real rates based on equation (2) and its HP filtered trend with a smoothing parameter of 1600. From this very simple and basic analysis, there appears to be a downward trend in New Zealand's neutral real interest rate.

Figure 1:
Implied NRR using average yield curve spread



Although the result of this exercise is suggestive of a systematic time-varying relationship, it makes several important assumptions, including mean-reversion of the term premium and stable inflation throughout the time period in question. In the following subsection we filter the NRR from short and long interest rates, without specifying a specific structure, using a state-space model. This approach has the advantage of being extendable to a more structural approach, as we do subsequently. We start with a simple specification that aims at understanding the NRR as a common trend between short and long term interest rates. We also discuss how this type of modelling makes it possible to address issues raised by the initial exercise.

2.1 Filtering the NRR with short and long term rates

The time-varying NRR can be estimated in at least two ways. Firstly, we can interpret an observed simultaneous level shift in both the long and the short interest rates (after cyclical fluctuations have been taken into account) as a shift in the neutral real rate. Alternatively, the trend decline in the neutral real rate can be modelled. Both approaches have strengths and weaknesses. In this subsection, we continue with the model-independent approach. In subsections 2.2 and 2.3 below we introduce more structural modelling frameworks as a robustness check.

We derive an initial Kalman filter based estimate by allowing both the neutral real rate and the equilibrium yield curve spread to fluctuate (see annex 5.2 for a discussion of the Kalman filter). Specifically, the signal equations are:

$$r_t = r_t^* + \pi_{t+1}^e + \varepsilon_{1,t} \quad (3)$$

$$R_t = r_t^* + \alpha_t + \pi_{t+1}^e + \varepsilon_{2,t} \quad (4)$$

and the state equations are given by:

$$r_t^* = r_{t-1}^* + \xi_{1,t} \quad (5)$$

$$\alpha_t = \delta_0 + \delta_1 \alpha_{t-1} + \xi_{2,t} \quad (6)$$

where r_t and R_t are 90 day and 10 year interest rates respectively, r_t^* is the NRR, π_{t+1}^e is the expectations for the period t+1 formed at period t and α is the term premium or yield curve spread. Once again, we base our analysis on the time period 1992-2002. We make the assumption that expected inflation over the time period has been

⁷ Although we allowed the term premium to be a time varying unobserved state variable, restricting it to be a constant did not change the results at all, suggesting the term premium has been rather flat.

stable⁸ and that long term interest rates can be derived from short term interest rates and thus from the NRR.

Our specification allows for cyclical fluctuations to the yield curve spread α , as well as possible productivity-related shifts associated with “new economy” effects. Allowing α to follow an AR(1) process has some nice features. For example, if δ_0 is zero, α follows a random walk and if δ_0 and δ_1 are jointly zero, the term premium is constant. As it happens, fixing the equilibrium yield curve spread did not change the profiles of the curves by much, since the yield curve spread appeared to be mean-reverting for this period. Table 1 shows the estimation results under the constant term premium assumption.

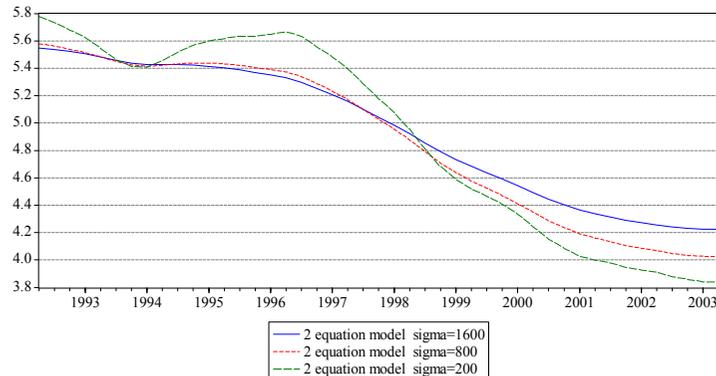
We used three different hyperparameters in this model (specifically 200, 800 and 1600), to see the difference these would make to the neutral real rate estimates.

⁸ Available survey data on inflation expectations support this assumption but this is not what has normally been experienced in other countries. We therefore test this assumption in subsequent robustness checks.

Table 1:
Estimation Results for 2-equation models

	Model 1		Model 2		Model 3	
	Coefficient	z-Stat	Coefficient	z-Stat	Coefficient	z-Stat
$\varepsilon_{1,t}$	1.012 (0.072)	13.922	1.231 (0.023)	52.274	0.539 (0.188)	2.863
α	0.110 (0.405)	0.272	0.175 (0.496)	0.352	0.131 (0.254)	0.517
$\varepsilon_{2,t}$	-1.609 (0.176)	-9.113	-1.196 (0.014)	-83.359	-1.661 (0.119)	-13.943
	Final State	Root MSE	Final State	Root MSE	Final State	Root MSE
r_t^*	4.22	0.143	4.02	0.202	3.83	0.226
	Log Likelihood	AIC	Log Likelihood	AIC	Log Likelihood	AIC
	-144.65	6.711	-133.47	6.203	-122.19	5.690

Figure 2:
Smooth estimates of neutral real rate



Estimating the model above in state space requires taking a position on the hyperparameters. Clearly the stiffness of the filter matters to

the end point result, and as figure 2 shows, a range of 3.8-4.3 is obtainable with this setup by just changing the filter's stiffness.

Freely estimating all of the hyperparameters may lead to implausible results, especially if the sample is relatively small, because final estimates of the parameters will depend quite heavily on the initial values set for them.

2.2 A semi-structural approach: the HPMV filter

An alternative is to recast the model as a Hodrick-Prescott Multivariate (HPMV) filter, which retains the flexibility of the state space form while also putting some constraints on the hyperparameters. These constraints are derived from the HP filter which, when put into state-space form (see Harvey 1985), calibrates the ratio of hyperparameters using the smoothing parameter proposed by Hodrick and Prescott (1997). One can also interpret this as the neutral real interest rate following an HP filter within the short rates. This is then augmented by information contained in the evolution of long rates.

To see this, recall that the HP filter gives an estimate of the unobserved variable as the solution to the following minimisation problem:

$$\underset{\{y_t^*\}}{\text{Min}} \sum_{t=1}^T \frac{1}{\sigma_0^2} (y_t - y_t^*)^2 + \frac{1}{\sigma_1^2} (\Delta^2 y_t^*)^2 \quad (7)$$

Where y is the observed variable, y^* is the unobserved variable being filtered, σ_0^2 is the variance of the cyclical component $y - y^*$ and σ_1^2 is the variance of the growth rate of the trend component. This problem is of course invariant to a homothetic transformation, therefore what matters is the ratio $\lambda_1 = \sigma_0^2 / \sigma_1^2$. Hodrick and Prescott suggest some parameterisation of λ_1 depending on the frequency of data.⁹ Following Harvey (1985), the HP filter can be

⁹ One hundred for annual data, 400 for semi-annual data and 1600 for quarterly data.

written in a state space form as follows. The measurement equation defines the observed variable as the sum of its trend and fluctuations around the trend:

$$y_t = y_t^* + e_t \quad (8)$$

with $e_t \sim N(0, \sigma_0^2)$. The state equations define the growth rate of the trend that is accumulated to compute the trend itself:

$$y_t^* = g_{t-1} + y_{t-1}^* + v_{1,t} \quad (9)$$

$$g_t = g_{t-1} + v_{2,t} \quad (10)$$

with $v_{1,t}$, $t=0$ and $v_{2,t} \sim N(0, \sigma_0^2/\lambda_1)$. $v_{2,t}$, t is the change in the growth rate of the filtered series or trend. In other words, the change in the trend follows a random walk. Then to get the HP filter estimate one has to use the whole set of information to derive y^* (as done in the minimisation problem in equation 7) to take the smoothed estimate provided by the Kalman filter.

The HPMV filter is an alternative way of estimating unobserved variables, developed at the Bank of Canada to estimate potential output (see Laxton and Tetlow (1992)). This method stems from use of the standard HP filter (see Hodrick and Prescott (1997)), augmented by relevant economic information. It has been used by the Central Banks of Canada and New Zealand to estimate potential output (see Butler (1996) and Conway and Hunt (1997)) and by OECD (1999) to estimate the NAIRU. Harvey (1985) explains how to reproduce the simple HP filter with the Kalman filter and Boone (2000) extends this to the HPMV filter. This is done in two steps. Firstly, the minimisation problem is written as a state space model. Secondly, restrictions are imposed on the variances of the equations of the state space model, to reproduce the balance between the elements of the minimisation programme.

The HPMV filter seeks to estimate the unobserved variable as the solution to the following minimisation problem:

$$\underset{\{y_t^*\}}{\text{Min}} \sum_{t=1}^T (y_t - y_t^*)^2 + \lambda_1 (\Delta^2 y_t^*)^2 + \lambda_2 \zeta_t^2 \quad (11)$$

With λ_1 and λ_2 given. This is a basic HP filter, augmented with the residuals ζ_t taken from an estimated economic relationship:

$$z_t = \beta y_t^* + dX_t + \zeta_t \quad (12)$$

where z is another explanatory variable can be explained by the unobserved variable y^* and X is a matrix of other exogenous variables. The residuals ζ are normal with a covariance matrix H .

As for the simple HP filter, the smoothing constants λ_1 and λ_2 reflect the weights attached to different elements of the minimisation problem. The estimated unobserved variable is not only a simple moving average going through the observed series, but is also modelled to give a better fit to the economic relationship. The HPMV filter can also be reproduced by a Kalman filter, following a similar methodology. In essence the problem is simply to add the additional structural equation in the set of measurement equations:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} 1 \\ \beta \end{pmatrix} y_t^* + \begin{pmatrix} 0 \\ d \end{pmatrix} X_t + \begin{pmatrix} e_t \\ \zeta_t \end{pmatrix} \quad (13)$$

with: $(e_t \quad \zeta_t)' \sim N(0, H)$. The covariance matrix H is defined as:

$$H = \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_0^2/\lambda_2 \end{bmatrix} \quad (14)$$

The two state equations are the same as for the standard HP filter (see equations (9) and (10)). The novelty is the term λ_2 , which gives a balance between the HP filter and the economic information embodied in the additional equation. A high value for λ_2 corresponds to a better fit of the economic relationship, and an unobserved variable that can depart significantly from the observed variable.

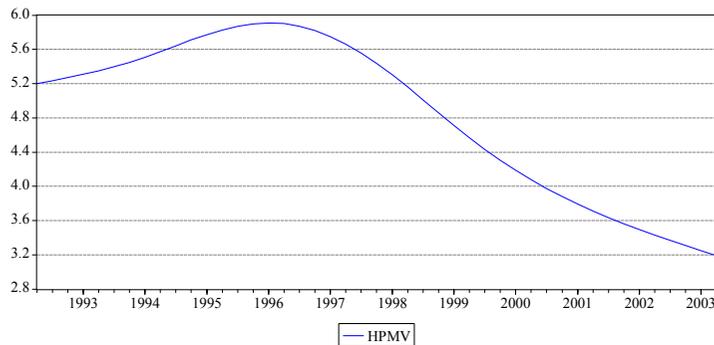
There are two advantages to be gained from reproducing an HPMV filter with the Kalman filter. Firstly, the method is done in one go (while the method proposed by Laxton and Tetlow (1992) is a multi-step procedure), and also allows estimation of the hyperparameters. Thus, although it is rarely done in practice, it is possible to freely estimate the parameters λ_1 and λ_2 instead of giving these essentially arbitrary values.

We did this exercise in order to check the robustness of the choice of hyperparameters. Because of our relatively small sample of quarterly data, we kept the smoothing parameter calibrated at 1600, but in a more general setting the smoothing parameter can be estimated. Our state equation r_t^* was determined in the following way:

$$r_t^* = r_{t-1}^* + g_{t-1} \quad (15)$$

$$g_t = g_{t-1} + \xi_{1,t} \quad (16)$$

Figure 3:
HPMV filtered neutral real interest rate



The results are plotted in figure 3, which illustrates well the difference between the HPMV and our original setup as plotted earlier in figure 2. This HPMV model assumes a signal to noise ratio of 1600 in short rates and freely estimates the variance of the long rates. The reason for doing this is that so far in this paper so far

we have assumed that long rates are a better indicator of the neutral real interest rates than are short rates, and hence should be given more weight. By freely estimating the variance of the long rates, we wanted to test the plausibility of this assumption. We also tried models with equal weights on long and short rates; they gave us an end point of around three per cent, which we consider to be unrealistically low. In particular, it gets too much “signal” from the short rates, which have been very low following the exceptional circumstances in the wake of 11 September 2001. This helps to confirm our priors that long rates contain more information when inferring neutral real interest rates, and hence should be given greater weight. Furthermore, these models generally failed to converge.

Table 2 below shows the estimation results with the HMPV filter. The final state is 3.12, which is well below the estimates we presented earlier. The negative g_t , which is the growth rate of the state variable, implies that the neutral real rate is declining. However, this coefficient is not significant. One can also see that the variance of the short rates is greater than that of the long rates, confirming our priors upon which we based our modelling in the earlier models.

Table 2:
Estimation Results of HPMV Model

	Coefficient	z-Stat		
$\varepsilon_{1,t}$	0.0007 (0.000)	167.438		
α	0.186 (0.271)	0.686		
$\varepsilon_{2,t}$	-0.001 (0.000)	-153.028		
	Final State	z-Stat	Log-likelihood	AIC
r_t^*	3.12	7.023	-140.604	6.527
g_t	-0.060	-0.781		

There are at least three caveats associated with using the yield curve approach to assess monetary policy settings. First, there is a circularity in the derivation of the benchmark, in that the Reserve Bank looks to financial markets for guidance into what constitutes neutral monetary policy settings, while financial market players act depending in part on their expectations of what the central bank is going to do with policy rates in the future.¹⁰ While the approach above interprets the term structure of interest rates as indicative of current policy tightness, the term structure can in principle also be interpreted as an indication of market expectations as to future monetary policy decisions.

A second shortcoming with the yield curve slope approach above is that the spread between long rates and policy rates is assumed to be mean-reverting. This appears to be the case for New Zealand and several other industrialised countries during the 1990s,¹¹ but if the sample period is changed by a few years, several of the series appear to be non-stationary. This could be an indication that structural changes have permanently affected the yield curve; on the other hand, it may also indicate that the sample does not start and end at the same point in a business cycle, and cyclical fluctuations are throwing off the stationarity tests. Careful thought has to be put into what reasons may exist for the spread between long and short rates to change, other than deviations of policy rates from neutral settings.

An obvious candidate is changing inflation expectations. Bomfim (2001) gets around this criticism for the US by looking at yields on

¹⁰ Normally long rates should reflect the unwinding of temporary imbalances in the economy over the time horizon, productivity developments, the evolution of yields on alternative investment opportunities, etc, though it is in theory possible that the effect on long rates of all of these factors can be swamped by second-guessing of what monetary policy authorities are likely to do.

¹¹ Siklos (2000) found a unit root in the series for the time period 1985-1997, but that time period included the high inflation years of the late 80s and the disinflationary period 89-91. ADF tests reject the null hypothesis of a unit root for the period 1992-2002, but the period is uncomfortably short for this test to be truly reliable. In addition, policy mistakes during the MCI period distort the series and thus reduce the reliability of unit root tests even further. The best that can really be said is that in a plot of the series, it looks like it might reasonably be mean-reverting.

indexed bonds, though data limitations of this approach are severe even in the case of the US and prohibitive for many countries, where markets for indexed bonds are illiquid or simply do not exist. In some cases, including in our analysis for New Zealand, the real interest rate gap measure also assumes a stable relationship between public and private sector risk. The 10-year rate is derived from government securities, whereas the 90-day rate reflects predominantly private sector borrowing.¹² To the extent that the creditworthiness of the public sector changes relative to the private sector, the indicator will be biased.

The third shortcoming concerns the extent to which the long rates in New Zealand are driven by factors outside of New Zealand, and associated noise. To the extent that long rates overseas exhibit cyclical fluctuations, and this moves around the slope of the New Zealand yield curve, the reliability of a yield curve based indicator of monetary policy stance will be diminished.¹³ Gürkaynak, Sack and Swanson (2003) have documented an excess sensitivity of US long term interest rates to surprises in macroeconomic data releases and monetary policy releases, from which the authors infer that expectations of the long run inflation rate are affected. Similar effects are not present in inflation-indexed debt or in the UK, where inflation targeting seems to have anchored expectations since 1997. If inflation expectations are equally well anchored in New Zealand, then local long debt should similarly not be overly sensitive to domestic surprises. Nevertheless, surprises reflected in noisy US long rates may carry over into New Zealand long rates.

¹² This is also a possible explanation for significant variation in average yield curve slopes across countries. In the case of New Zealand, for example, the average yield curve slope is close to zero, whereas in the US it is clearly positive.

¹³ For example, to the extent that yields on 10-year bonds move cyclically, a recession in the US and Europe coinciding with a boom in New Zealand (as was the case recently) may bias the NZ real interest rate gap in the direction of suggesting that monetary policy settings are tighter than they actually are, if long rates in New Zealand are driven by overseas rates to be artificially lower at the margin.

Because of these downsides and criticisms of the yield curve based approach above, we adapt our Kalman filter to add more economic structure. The approach we used in our two equation setup did not allow for any involvement of monetary policy. The short term interest rate equation in (3) simply stated that on average, short rates correspond to neutral monetary policy, rather than model the actual setting of policy rates in response to developments in the economy. In addition, it is plausible that in early stages of inflation targeting, the credibility of the Reserve Bank was less established and this would have been reflected in higher inflation expectations than what the survey data suggest. Because of the lack of consensus surrounding the way that inflation expectations are formed, we test the robustness of our results by allowing inflation expectations to be determined according to competing theories.

2.3 A slightly more structural approach to determining the NRR

We now improve the way we model the evolution of the neutral real interest rates and relax the constant inflation expectations assumption made above. We again estimated this model within an HPMV context. In addition to an HP filtered short term real interest rate, our analysis will rely on the following two equations to augment the HP filter:

$$r_t = r_t^* + \pi_{t+1}^e + \beta(\pi_t - \pi^*) + \tilde{\phi}_t + \varepsilon_{1,t} \quad (17)$$

$$R_t = r_t^* + \alpha + \pi_{t+1}^e + \varepsilon_{2,t} \quad (18)$$

Equation (17) is a Taylor rule without an interest rate smoothing term, where r_t^* is the time varying neutral real interest rate, π_{t+1}^e is the unobserved expected inflation, π_t is inflation, π^* is the inflation target, and $\tilde{\phi}_t$ is the output gap. Equation (18) is a non-arbitrage relation between short term financial assets and long term ones, which basically states that the nominal long term interest rate is equal to the short term nominal interest rate plus a premium α . In

this specification we directly replace the short term rate with the sum of the neutral real rate, r_t^* , and inflation expectations, π_{t+1}^e .¹⁴

The unobserved neutral real interest rate is modelled as a random walk:

$$r_t^* = r_{t-1}^* + g_{t-1} \quad (19)$$

where

$$g_t = g_{t-1} + \xi_{1,t} \quad (20)$$

The system formed by equations (17) through (20) is a state-space model, where the first two equations are the measurement ones, and the third equation defines the law of motion of the state variable. Thus, this model can be estimated with a Kalman filter algorithm.

Figure 4 shows our results for the neutral real interest rate for New Zealand.

The first variation of the model we estimate is a very simple one, where instead of allowing inflation expectations π_{t+1}^e to be an unobserved variable, we use the RBNZ one year ahead inflation expectations series. This reduces to one the number of unobserved variables we want to estimate, this being the neutral real interest rate. As a result, we would expect the standard errors to be tighter around the estimates. We then allowed inflation expectations to be an unobserved variable under different expectation formation assumptions, as described in table 3.

¹⁴ In models where we allow the inflation expectations to be an unobserved variable, we define the inflation expectations as in table 3.

Table 3:
Models analysed

Model 1	$\pi_{t+1}^e \approx \text{survey expectations}$
Model 2	$\pi_{t+1}^e = \pi_{t+1}$ (rational expectations)
Model 3	$\pi_{t+1}^e = \lambda \pi_{t+1} + (1 - \lambda) \pi_{t-1}$ (imperfectly rational)
Model 4	$\pi_{t+1}^e = \gamma \bar{\pi} + (1 - \gamma) \pi_{t-1}$ (perpetual learning)

Figure 4:
NRR estimates in HPMV filter with Taylor rule and long rate

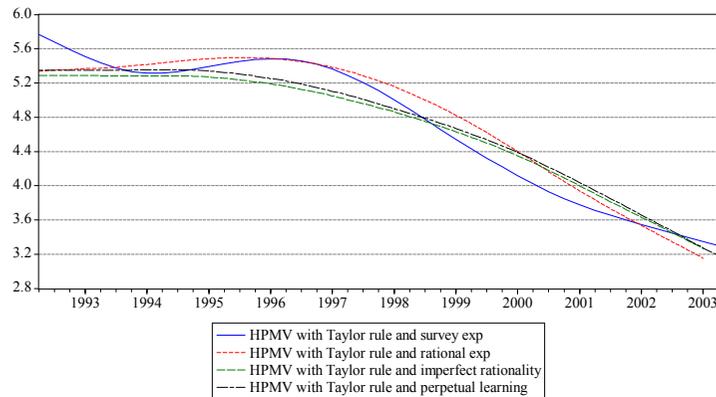


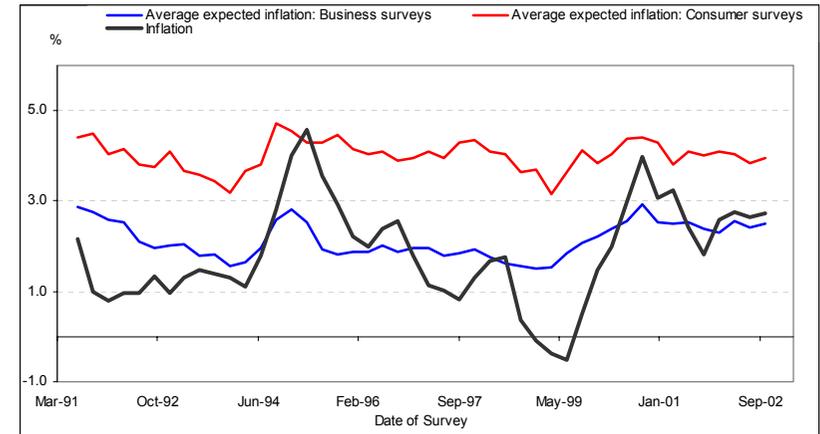
Figure 4 shows the plots of the smooth estimates of the neutral real rates and the estimation results obtained from the structural HPMV filters with different inflation expectations formations.

As can be seen from figure 4, the various different specifications for expectations do not make much difference to our estimates of the unobserved neutral real interest rate.¹⁵ It appears that inflation

¹⁵ However, we should note that adding more structure such as the Taylor rule will sometimes give implausible coefficient estimates. Nevertheless, our aim here is not forecasting or modelling. Our aim is to back out the historical time varying neutral real interest rate.

expectations have indeed been broadly stable throughout the time period analysed, which is supported by available survey data on inflation expectations (figure 5). Note, however, that modelling inflation expectations raises the end point of the HPMV-derived series from three per cent in figure 3 to somewhere closer to 3.5.

Figure 5:
Inflation expectations

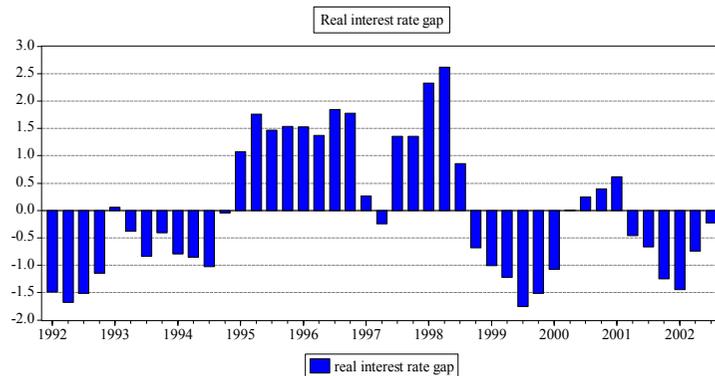


3 The real interest rate gap

One of the main reasons for deriving the NRR in the first place was to determine at what level of real interest rates monetary policy is neither adding nor subtracting stimulus to/from the economy. It is a simple step to compare this rate with actual interest rate settings to see exactly how much stimulus monetary policy was adding or subtracting at some given time. This difference between actual real interest rates and the NRR is what is called the real interest rate gap.

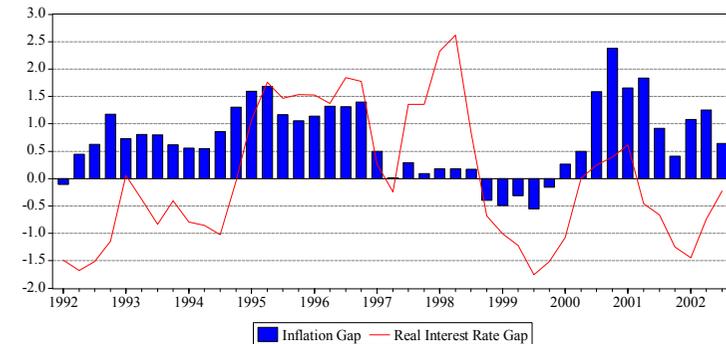
We compare the real interest rate deviations from the neutral levels to the output gap and to inflation, as one might normally expect a strong correlation between these variables.

Figure 6:
The real interest rate gap 1992-2002



The plausibility of the real interest rate gap as an indicator has to be assessed very critically. Figure 7 compares our estimate of the New Zealand real interest rate gap with inflation deviations from target. From figure 7, we observe that periods identified with expansionary monetary policy were followed by inflation above target, whereas the main period of contractionary monetary policy in the mid-1990s was followed by inflation close to or below target. Nevertheless, on balance over the period, monetary policy appears to have responded appropriately to inflation pressure as signalled by this indicator (Liu 2003).

Figure 7:
Real interest rate gap and inflation deviations from target



The series is given some additional credibility from this systematic relationship between the real interest rate gap and measures linked to the real economy. Moreover, since under stable inflation expectations the interest rate gap is the difference between two readily observable nominal numbers with no measurement error, it is a welcome reference indicator for use in real time policymaking.¹⁶ Neiss and Nelson (2003) found their real interest rate gap to be highly correlated with future inflation for the UK. They show cross correlations with lags up to 6 quarters.

¹⁶ See Christensen (2002) for a good discussion of this.

4 Conclusions

In real time, inflation targeting requires policy makers to take a position on the neutral real interest rate. Although there have recently been strong arguments made in favour of using monetary policy rules¹⁷ which do not rely heavily on unobserved variables, current practice still depends on the evolution of several such unobservables.

In this paper we tested a number of approaches in modelling the neutral real interest rate for New Zealand over the 1990s. While the different models all seem to suggest a similar downward trend in the neutral real interest rate, our different specifications still make some difference to point estimates of the NRR, especially at the end points. Nevertheless, the point estimates of different models are clustered very closely, lying within a 100 basis point band even at the endpoints. Current estimates of the neutral real interest rate lie between 3.25 and 4.25 per cent.

The assumptions on the signal to noise ratios matter to some extent. Another striking result from our models, although this is still early to conclude, is that the decline in the neutral real interest rate has levelled off in the past year or two at around the 3.25 – 4.25 per cent level. This is still well above what other developed countries seem to have. Explanatory factors for this difference have been examined in Plantier (2003), but there is still much work that can be done on this front.

Similarly to Neiss and Nelson (2003), we found a strong correlation between the real interest rate gap, defined as the difference between the current real rate and the NRR, with both the output gap and inflation deviations from target.

¹⁷ Orphanides and Williams (2003).

5 Annexes

5.1 The link between short term and long term interest rates

We can write out long interest rates generically as the weighted average of expected future short rates:

$$R_t = (1 - \rho) \sum \rho^i r_{t+i} + \alpha_t \quad (\text{A1})$$

where, as before, R_t is the long term nominal interest rate, r_t is the short term nominal interest rate, and α_t is the term premium. Nominal short rates in each future time period are conventionally understood to be real interest rates plus inflation:

$$r_{t+i} = \phi_{t+i} + \pi_{t+i}^e \quad (\text{A2})$$

where ϕ_{t+i} represents real interest rates. Substituting equation (A2) into (A1) gives the following:

$$R_t = (1 - \rho) \sum \rho^i \phi_{t+i} + (1 - \rho) \sum \rho^i (\pi_{t+i}^e) + \alpha_t \quad (\text{A3})$$

The long term nominal interest rate is now written as a combination of three terms: long term real interest rates as the weighted average of short real rates in the future, an inflation expectations component, and the term premium.

As mentioned, we assume that inflation is well-anchored. With stable inflation expectations, the long rate can be written as

$$R_t = (1 - \rho) \sum \rho^i \phi_{t+i} + \bar{\pi} + \alpha_t \quad (\text{A4})$$

Now we can rewrite equation (2) in the main text as

$$NRR_t = (1 - \rho) \sum \rho^i \phi_{t+i} + \alpha_t - \bar{\alpha} \quad (\text{A5})$$

Equation (A5) makes explicit that under stable inflation expectations, the neutral real rate is a forward-looking weighted average of expected future short real rates plus the deviation of the (cyclical) term premium from its mean. At any time t , the cyclical fluctuation in ϕ should be counterbalanced by the cyclical fluctuation in α .

5.2 The Kalman filter and the smoothed estimates¹⁸

Many dynamic models can be written and estimated in state-space form. The Kalman filter is the algorithm that generates the minimum mean square error forecasts for a given model in state space. If the errors are assumed to be Gaussian, the filter can then compute the log-likelihood function of the model. This enables the parameters to be estimated by using maximum likelihood methods.

For simplicity let us consider a measurement equation that has no fixed coefficients:

$$Y_t = \Gamma_t X_t + \varepsilon_t \quad (\text{A6})$$

where Y_t is a vector of measured variables, Γ_t is the state vector of unobserved variables, X_t is a matrix of parameters and $\varepsilon_t \sim N(0, H)$. The state equation is given as:

$$\Gamma_t = \Gamma_{t-1} + \eta_t \quad (\text{A7})$$

where $\eta_t \sim N(0, Q)$.¹⁹

Let γ_t be the optimal estimator of Γ_t based on the observations up to and including Y_t , $\gamma_{t|t-1}$ the estimator based on the information available in $t-1$, and $\gamma_{t|T}$ the estimator based on the whole sample.

¹⁸ This section draws heavily on Basdevant (2003).

¹⁹ Q and H are referred to as the hyperparameters of the model, to distinguish them from the other parameters.

We define the covariance matrix P of the state variable as follows:

$$P_{t-1} = E\left((\Gamma_{t-1} - \gamma_{t-1})(\Gamma_{t-1} - \gamma_{t-1})'\right) \quad (\text{A8})$$

The predicted estimate of the state variable in period t is defined as the optimal estimator based on information up to the period $t-1$, which is given by:

$$\gamma_{t|t-1} = \gamma_{t-1} \quad (\text{A8})$$

while the covariance matrix of the estimator is:

$$P_{t|t-1} = E\left((\Gamma_t - \gamma_{t|t-1})(\Gamma_t - \gamma_{t|t-1})'\right) = P_{t-1} + Q \quad (\text{A10})$$

The filtered estimate of the state variable in period t is defined as the optimal estimator based on information up to period t and is derived from the updating formulas of the Kalman filter:²⁰

$$\gamma_t = \gamma_{t|t-1} + P_{t|t-1} X_t' (X_t P_{t|t-1} X_t' + H)^{-1} (Y_t - X_t a_{t|t-1}) \quad (\text{A11})$$

and

$$P_t = P_{t|t-1} - P_{t|t-1} X_t' (X_t P_{t|t-1} X_t' + H)^{-1} X_t P_{t|t-1} \quad (\text{A12})$$

The smoothed estimate of the state variable in period t is defined as the optimal estimator based on the whole set of information, i.e. on information up to period T (the last point of the sample). It is

²⁰ This estimator minimises the mean square errors when the expectation is taken over all the variables in the information set rather than being conditional on a particular set of values (see Harvey 1989 for a detailed discussion). Thus the conditional mean estimator, γ_t , is the minimum mean square estimator of Γ_t . This estimator is unconditionally unbiased and the unconditional covariance matrix of the estimator is the P_t matrix given by the Kalman filter.

computed backwards from the last value of the earlier estimate $\gamma_{T|T}=\gamma_T$, $P_{T|T}=P_T$ with the following updating relations:

$$\gamma_{t|T} = \gamma_t + P_t^* (\gamma_{t+1|T} + \gamma_t) \quad (A13)$$

$$P_{t|T} = P_t + P_t^* (P_{t+1|T} + P_{t+1|t}) P_t^{*'} \quad (A14)$$

where

$$P_t^* = P_t P_{t+1|t}^{-1}.$$

Depending on the problem studied one can be interested in any one of those three estimates. In our particular case, looking at smoothed values is more appropriate, as the point is not to use the Kalman filter to produce forecasts but to give the most accurate information about the path followed by the time-varying coefficients. Therefore it is more informative to use the full dataset to derive each value of the state variables.

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