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Learning process and rational expectations: an analysis using a small macroeconomic model for New Zealand

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Abstract[1]

The nature of expectations matters when conducting monetary policy. Models with a learning process can exhibit very different properties from models with other types of expectations rules. This paper draws on the work of Orphanides and Williams (2002), extending it to allow for the possibility that the learning process may not be perpetual, but rather might be converging towards a rational expectations equilibrium. By modelling expectations using a learning process, we obtain evidence suggesting that inflation expectations in New Zealand are moving towards rational expectations. Theory suggests this will make it easier to control inflation after a temporary disturbance.

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The views expressed in the paper are those of the author and do not necessarily reflect those of the Reserve Bank of New Zealand.

1 Introduction

The nature of expectations matters when conducting economic policy, especially monetary policy. A particularly important aspect of forward looking behaviours is that they put at the centre of monetary policy issues such as credibility, commitment, reputation and time consistency. A crucial point in inflation targeting is not so much that the central bank has to directly target inflation instead of some intermediate objective, but more that it has to satisfy all of the requirements for being credible in achieving its goal, these being the adoption of an inflation target as the main objective, independence, the technical capability of forecasting inflation, and high levels of transparency. Therefore, inflation targeting can also be viewed as a monetary policy framework that explicitly integrates a forward-looking component in inflation.

If expectations were purely rational then inflation could arguably be reduced without any cost, provided the central bank is fully credible (see Ball 1994, 1995). As noted by Roberts (1997), reducing inflation is usually costly, mostly because there is some inertia in the inflation process. This inertia may arise either because of wage contracts that are set for several periods (see Taylor 1979, 1980 or Fuhrer and Moore 1992, 1995) or because expectations are not perfectly rational. What Roberts (1997) also emphasises is that in new Keynesian models inflation can be reduced at no cost, in spite of sticky prices, so long as inflation expectations are rational. It is important to understand where the stickiness in inflation comes from:

- If inflation is inherently sticky then reducing inflation would always imply some costs.

- If inflation is sticky because of expectations then reducing inflation could be costless in the medium term (or with substantially reduced costs) provided that agents change their expectation rule to learn a rational expectations equilibrium.

Thus, it matters how agents form their expectations, and also where inflation stickiness comes from. In this paper we propose to investigate what drives the inflation dynamic in New Zealand, first by analysing whether inflation is inherently sticky and then by examining whether or not expectations are rational. Although the results are to be treated with caution given the limited amount of data, the evidence so far suggests that the stickiness of inflation comes mostly from the dynamic of expectations.

The remainder of the paper is organised as follows: in section 2 we describe the general methodology used, as well as the economic and policy implications. Section 3 presents the results obtained for New Zealand, and section 4 concludes.

2 Modelling a learning processes: economic and practical issues

In this section we present how recent contributions analyse inflation using small macroeconomic models (section 2.1), and also how learning processes are modelled in practice (section 2.2). These discussions are of particular importance as the optimal policy may change when expectations are no longer rational.

2.1 Discussion of economic and policy implications

In recent years, many authors have investigated very compact macroeconomic models, and have discussed intensely the forward-looking nature of expectations.[2] A particular feature of those models (sometimes referred to as new Keynesian models) is that they have micro-foundations (see Roberts 1995, McCallum and Nelson 1999, Woodford 1996, Rotemberg and Woodford 1997) and therefore their parameters are structural and not subject to the Lucas critique (see Lucas 1976).

Basically those models can be represented as follows:

$$\begin{cases} y_t = \alpha y_{t+1}^e + \beta\left(i_t - \pi_{t+1}^e - \bar{r}\right) + \varepsilon_t^y \\ \pi_t = \pi_{t+1}^e + \gamma y_t + \varepsilon_t^\pi \\ i_t = (1-\mu)\left(\bar{r} + \pi_t + \lambda(\pi_t - \bar{\pi}) + \theta y_t\right) + \mu i_{t-1} + \varepsilon_t^i \end{cases} \quad (1)$$

[2] See Lindé (2001b) for a discussion of those different models.

where y_t is the output gap, π_t the inflation rate, i_t the short-term nominal interest rate, $\bar{\pi}$ the inflation target, $\bar{r}$ the neutral real interest rate, y_{t+1}^e and π_{t+1}^e are the expectations of the output gap and inflation rate, and α, β, γ, λ, θ and μ are structural parameters, all but β being positive. The first relation the aggregate demand (AD), the second relation is the aggregate supply (AS) and the third summarises the behaviour of the central bank using a Taylor rule with interest smoothing (μ being the smoother parameter).

In this set-up we do not assume anything regarding the rationality of expectations. They can be rational, adaptive, or derived from a learning process. As mentioned before, a first step is to investigate whether inflation inertia comes from the structure of the economy itself or from expectations. Roberts (1997) proposes a simple way to address where the inertia in inflation comes from, by using survey expectations in a model of the type of (1):

$$\begin{cases} y_t = \alpha y_{t+1} + \beta\left(i_t - \pi_{t+1}^e - \bar{r}\right) + \varepsilon_t^y \\ \pi_t = \delta\pi_{t-1} + (1-\delta)\pi_{t+1}^e + \gamma^1 y_t + \gamma^2 y_{t-1} + \varepsilon_t^\pi \\ i_t = (1-\mu)\left(\bar{r} + \pi_t + \lambda(\pi_t - \bar{\pi}) + \theta y_t\right) + \mu i_{t-1} + \varepsilon_t^i \end{cases} \quad (2)$$

The two main differences are that in this model π^e refers to survey expectations (therefore expectations are defined as opposed to the general formulation given in (1)) and the coefficient δ measures the inertia in the inflation dynamic. As δ is found not significantly different from 0 he concludes that in the US the inertia in inflation comes only from expectations that are not perfectly rational.

Several contributions propose hybrid models that nest backward and forward looking dynamics, and then try to evaluate the relevance of the forward looking dynamic (see Clarida et al 1999, Roberts 2001 or Rudebusch 2002). Empirical hybrid models give contrasting results. Among others, Fuhrer (1997) and Roberts (2001) find that forward looking behaviours are unimportant, while Gali and Gertler (1999) and Gali et al (2001) find the contrary, that forward looking behaviours are dominant. As pointed out by Gali and Gertler (1999), a possible explanation of those contrasting results is the choice of the tension variable entering in the Phillips curve. If one uses the output gap then the model tends to reject a forward looking nature of inflation, while models based on marginal cost exhibit the opposite result. Jondeau and Le Bihan (2001) have investigated those findings further by expanding the number of countries studied and by checking systematically the impact of different tension variables. They show that the observed differences are much more dependent on the structure of lags and leads than on the choice of the tension variable. They also accept a general hybrid model with three leads and lags, that places roughly equal weights on backward and forward dynamics. Another argument often put forward in favour of backward looking models is that empirically the Lucas critique does not seem to be relevant as parameters do not exhibit significant instability (see Ericsson and Irons 1995, Rudebusch and Svensson 1999). Nevertheless, those results have been criticised by Lindé (2001a), who points out that because of a low power on small samples the instability tests cannot correctly distinguish between changes in the policy from other shocks that affect the economy. Moreover, Lindé (2001b) suggests that both forward and backward looking models exhibit instability in parameters[3] which might be caused by a flawed measure of expectations.

What those findings reveal is that the nature of expectations formation is still under investigation, and the use of hybrid models is much more an ad hoc specification that acknowledges that expectations are neither totally rational nor totally adaptive. If we go back to the theoretical foundations the problem can be understood as follows:

- Adaptive expectations are an unsatisfactory concept mostly because they assume that agents do not react to systematic mistakes they make.

- Rational expectations have come under attack because they assume too much information on the part of agents.

As an alternative, a learning process in modelling expectations may be considered. It is then assumed that agents' expectations are on

[3] Thus even forward looking models should be tested for parameters instability (see also Estrella and Fuhrer 1999).

average correct, but that only a limited set of information is utilised. Hence a relatively simple representation of expectations is possible, avoiding systematic errors in a model similar to the one by Feige and Pearce (1976), in which agents are assumed to use a univariate model to form their expectations. While implementing a learning process it should be emphasised that agents adjust the expectation rule when they observe the errors they make, and the weights they assign to the different variables used are changing over time. An interesting feature of learning processes is that under some circumstances a rational expectations equilibrium (REE) may be learnt by agents. There is abundant literature relating rational expectations and the learning process (see Lucas 1986, Woodford 1990, Beeby et al 2001 or Orphanides and Williams 2002). Most of those contributions use least square estimations to simulate the learning process. Its convergence to REE will depend on the set of prior information that agents will consider when forming their expectations[4] (see Marcet and Sargent 1988, 1989a,b, Timmermann 1994 Sargent 1999 or Evans and Honkapohja 2001).

A major criticism addressed to learning is that the choice of the specification for the learning process is arbitrary.[5] Nevertheless, Garratt and Hall (1997) and Beeby at al (2001) investigate the impact of different learning processes, to find that at least for large macroeconomic models it makes little difference. Intuitively the idea is that learning processes extract information well enough so that the precise form of the learning is not crucial. Marcet and Sargent (1989a) also demonstrate that as long as the variables entering into the learning rule are correlated with the variables that explain the dynamic under REE, then the learning process will converge to REE. In a small macroeconomic model the problem is of a different nature: provided that the structure is simple enough, it is possible for agents to use the variables that are relevant in the REE

[4] Let A_t be the optimal estimate at date t of an unknown vector A. The learning process can be viewed as updating A_t using a simple rule of the type: $A_t = TA_{t-1}$. It will converge towards REE if the true value A is a solution of the updating equation, the initial value chosen for A is close to the true value and if the matrix T has its eigenvalues within the unit circle.

[5] As an example Woodford (1990) considers that agents could adopt a sunspot variable to form their expectations.

and then learn about the weights of each variable (see Orphanides and Williams 2002).

Beyond this criticism the most important issue is that a model with a learning process can exhibit very different properties from a model with a different type of expectations rule. Beeby et al (2001) show that if expectations are not fully rational then a model based on a learning process will provide simulation properties much closer to the true model than a model based on rational expectations. Following this perspective, Orphanides and Williams (2002) show in a small model of inflation that policies which are efficient under rational expectations are not when agents use a learning process. More precisely, the authors suggest that the optimal monetary policy under a learning process should be more aggressive and also narrowed to inflation stability as it is the main objective of the central bank under an inflation targeting system. These findings are linked to the nature of expectations formation: being aggressive towards inflation and focussing on one objective facilitates the learning process.

2.2 Discussion of implementation in practice

Having discussed the economic and policy implications of modelling expectations with a learning process in this section we turn to the description of how expectations are derived in the empirical literature. We also discuss under which conditions a learning process may eventually converge towards rational expectations.

Basically a learning process assumes that inflation expectations can be derived from the following rule:

$$\pi_{t+1} = a_{1,t} + a_{2,t}\pi_{t-1} + a_{3,t}z_{t-1} + \varepsilon_t \tag{3}$$

where π_t is the inflation rate and z_t is a vector of variables that agents use to form their expectations, ie the set of information they find relevant (besides the lagged value of inflation). Let X_t and A_t be the following vectors: $X_t=(1,\pi_{t-1},z_{t-1})$ and $A_t=(a_{1,t},\ a_{2,t},\ a_{3,t})'$.

$$A_t = A_{t-1} + \frac{\alpha_t}{t} R_t^{-1} X_t' \left(\pi_{t+1} - X_t A_{t-1}\right) \tag{4}$$

where $R_t = \frac{1}{t}\sum_{\tau=1}^{t} \alpha_\tau X_\tau' X_\tau$ and α_t a sequence of positive numbers.[6] This formula is actually a version of weighted least squares and if $\alpha_t=1$ the formula above corresponds to recursive least squares. An interesting feature is that this method of updating parameters can be cast into the Kalman filter formulae. Defining $P_t = \frac{1}{t} R_t^{-1}$ and $f_t = X_t P_{t-1} X_t' + \frac{1}{\alpha_t}$ it becomes (see Bullard 1992):

$$A_t = A_{t-1} + P_{t-1} X_t' f_t^{-1} \left(\pi_{t+1} - X_t A_{t-1}\right) \tag{5}$$

$$P_t = P_{t-1} - P_{t-1} X_t' X_t P_{t-1} f_t^{-1} \tag{6}$$

Which corresponds to the following state-space model:

$$\pi_{t+1} = a_{1,t} + a_{2,t}\pi_{t-1} + a_{3,t}z_{t-1} + \varepsilon_t \tag{7}$$

$$\forall i \quad a_{i,t} = a_{i,t-1} + \eta_{i,t} \tag{8}$$

with the hyper-parameters given by:

$$Var\left(\varepsilon_t\right) = \frac{1}{\alpha_t} \tag{9}$$

$$Var\left(\eta_t\right) = 0 \tag{10}$$

With least square estimation the learning process is not optimal in the sense that this method assumes that coefficients are stable while their estimates are time-varying. Put in other words, the results of Marcet and Sargent (1989a, b) on the convergence of learning process towards rational expectations hold only when the law of motions of parameters is viewed as invariant. Ljung and Söderstöm

6 R_t can also be derived according to the following formula: $R_t = R_{t-1} + \frac{\alpha_t}{t}\left(X_t' X_t - R_{t-1}\right)$, which is used by Orphanides and Williams (2002) and Honkapohja and Mitra (2002) for most recent contributions.

(1983) and Bullard (1992) show that when this assumption is relaxed the convergence property no longer holds. Intuitively the reason is rather straightforward: if $Var(\eta_t)\neq 0$ then P_t does not converge towards 0, and thus learning does not converge to rational expectations.

More precisely in a more general state-space form the coefficients would be derived as follows:

$$A_t = A_{t-1} + P_{t-1}X_t' f_t^{-1}\left(\pi_{t+1} - X_t A_{t-1}\right) \qquad (11)$$

$$P_t = P_{t-1} + Q_t - P_{t-1}X_t' X_t P_{t-1} f_t^{-1} \qquad (12)$$

with $Var(\varepsilon_t)=H_t$, $Var(\eta_t)=Q_t$ and $f_t = X_t P_{t-1} X_t' + H_t$.

The set of equations (11) and (12) has to be viewed as follows. First agents form their expectations for the value of inflation for the next period, before they observe its current value. Once it is known, they use this information to revise their belief in order to avoid systematic mistakes. Expectations are thus computed as the predicted estimate for π_{t+1}:

$$\hat{\pi}_{t+1} = a_{1,t|t-1} + a_{2,t|t-1}\pi_{t-1} + a_{3,t|t-1}z_{t-1} \qquad (13)$$

Sargent (1999), Evans and Honkapohja (1999, 2001) or Orphanides and Williams (2002) propose a simpler version of permanent learning, using the algorithm reproducing weighted least squares given in (4). Basically their methodology consists of setting a geometrical pattern for the weights: $\alpha_t=\kappa t$, where κ is set to an arbitrarily small value. The advantage of using the Kalman filter in empirical contributions is that it will give the optimal gain that agents apply when updating their parameters, and can also allow one to test whether or not the variance of the state variables is significantly different from zero, *ie* to test if the learning is perpetual or if it converges towards rational expectations.

2.3 The model

2.3.1 The general structure

The underlying model is composed by a set of three equations: an AD curve, an AS curve and a reaction function of monetary authorities. The starting point was a general model that includes both backward and forward looking behaviours, in order to have an encompassing approach. The structure tested was as follows:

$$\begin{cases} y_t = \alpha^1 y_{t+1} + \alpha^2 y_{t-1} + \beta\left(i_t - \pi^e_{t+1} - \bar{r}\right) + \varepsilon^y_t \\ \pi_t = \delta\pi_{t-1} + (1-\delta)\pi^e_{t+1} + \gamma^1 y_t + \gamma^2 y_{t-1} + \varepsilon^\pi_t \\ i_t = (1-\mu)\left(\bar{r} + \pi_t + \lambda(\pi_t - \bar{\pi}) + \theta y_t\right) + \mu i_{t-1} + \varepsilon^i_t \end{cases} \qquad (14)$$

Thus we basically consider a hybrid model, where the AD and AS curves have both forward looking dynamics (see McCallum and Nelson 1999 or Roberts 1995) and backward looking dynamics (see Svensson 1997, Rudebusch and Svensson 1999). This model was estimated with quarterly data, where (following the methodology adopted by Rudebusch and Svensson) π is the annualised inflation rate, i is the annual interest rate and y the quarterly output gap (see the appendix for a description of the data used). We could have considered a more general structure for lags and leads (see Jondeau and Le Bihan 2001), but because of the short sample available (approximately ten years of data) the specification was kept with just one lead and one lag in order to have enough degrees of freedom.

Table 1: Results of the general model

	Coefficient	Std. Error	t-Statistic	P-value
α^1	0.52	0.17	2.98	0.00
α^2	0.49	0.10	4.99	0.00
β	-0.02	0.19	-0.08	0.93
δ	-0.13	0.16	-0.80	0.42
γ^1	0.07	0.30	0.24	0.81
γ^2	0.02	0.33	0.06	0.95
μ	0.64	0.09	6.96	0.00
λ	1.19	0.51	2.33	0.02
δ	0.57	0.33	1.72	0.09
		$\bar{R}^2$		DW
AD		0.86		3.04
AS		0.31		2.08
Taylor		0.90		1.86

The main point in this exercise was to have an acceptable model econometrically and to test if δ is significantly different from zero. The results showed that the model behaves reasonably well in terms of residuals,[7] and coefficients are stable except for the parameter β, which is not significant.

The next step was to investigate how the model behaves when we add some constraints, eg when we impose $\delta=0$ or $\alpha^1=1-\alpha^2$ (as the sum of those two coefficients is not significantly different from one). What was found is that such a model passes most tests reasonably well, except that keeping a forward looking dynamic in the output gap tends to worsen results: either the interest rate in the AD curve is found insignificant with an extremely low value, or it has the wrong sign. Other studies of the same type of model in New Zealand also accept a pure backward looking dynamic in the output gap (see Razzak 2002 or NBNZ 2002), so we decided to exclude y_{t+1} from the model. Thus, the final accepted model was the following:

[7] The Durbin-Watson statistic for the AD curve is a bit high, but in the final specification the residuals are a lot more uncorrelated.

$$\begin{cases} y_t = y_{t-1} + \beta\left(i_t - \pi^e_{t+1} - \bar{r}\right) + \varepsilon^y_t \\ \pi_t = \pi^e_{t+1} + \gamma y_t + \varepsilon^\pi_t \\ i_t = (1-\mu)\left(\bar{r} + \pi_t + \lambda(\pi_t - \bar{\pi}) + \theta y_t\right) + \mu i_{t-1} + \varepsilon^i_t \end{cases} \quad (15)$$

Table 2: Results of the restricted model

	Coefficient	Std. Error	t-Statistic	P-value
β	-0.33	0.15	-2.23	0.03
γ	0.09	0.16	0.60	0.55
μ	0.64	0.08	7.74	0.00
λ	1.14	0.57	1.98	0.05
θ	0.61	0.28	2.14	0.03
		$\bar{R}^2$		DW
AD		0.73		1.99
AS		0.32		2.25
Taylor		0.90		1.83

This specification fits about as well as the unrestricted specification above. The fact that we can fit the data without a backward looking inflation term suggests that inflation may not be inherently sticky. Instead, inflation stickiness may come entirely from forward looking (but non-rational) inflation expectations. Another important point is that the dynamic of output exhibits strong persistence. Finally, the results obtained for the Taylor rule are consistent with those of Drew and Plantier (2000) and Plantier and Scrimgeour (2002). Regarding the policy implemented the results also suggest that there is a relatively high degree of interest smoothing, and the weight on inflation is relatively high, suggesting that the Reserve Bank of New Zealand reacts strongly to deviations from target inflation. Nevertheless the estimate of λ is not estimated with high precision (the standard error is 0.57), so it is difficult to infer strong conclusions from this result.

3 Investigating survey expectations

There are various ways of investigating whether surveyed expectations are rational. As an example, Roberts (1997) suggests estimating the following:

$$\pi_t^e = a + b\pi_{t+1} + \varepsilon_t \qquad (16)$$

If expectations are rational then one would expect to find *a=0* and *b=1*. Another possibility is to consider a hybrid model, which is often done in practice, and estimate the following:

$$\pi_t^e = \phi\pi_{t-1} + (1-\phi)\pi_{t+1} + \varepsilon_t \qquad (17)$$

Those two specifications were tried, and provided the following results: $a=0.01$, $b=0.64$, $\phi=0.64$. The assumption that $b=1$ or $\phi=0$ was strongly rejected, suggesting that expectations are not perfectly rational, which was not surprising. Nevertheless, this type of conclusion would be misleading if not integrating explicitly that the behaviour of agents is changing over time. In the case of New Zealand, changing expectations could be justified for two reasons: because of structural change that occurred, and because of a learning process, the latter being reinforced by the former.

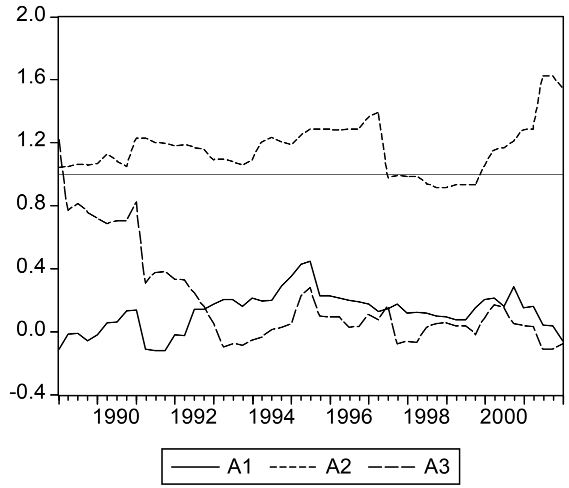
The next step consisted of estimating time-varying parameters using the model set up in equations (11) to (13). But we also tried to investigate whether the learning process is permanent or if it converges towards recursive OLS which would imply the convergence of the learning process towards rational expectations. As we mentioned before, the distinction between those two can easily be understood with the Kalman filter formulae: if the variance of hyper-parameters is significantly different from 0 then the learning process is perpetual, while if it is not then the learning process may converge towards REE. To model expectations, the following model was estimated:

$$\pi_t^e = a_{1,t}\pi_{t-1} + a_{2,t}\bar{\pi} + a_{3,t}y_{t-1} + \varepsilon_t \qquad (18)$$

where π_t^e is the survey expectation of inflation formed in period t for period $t+1$, and with: $\forall i \in \{1,3\} \quad a_{i,t} = a_{i-1,t} + \eta_t^i$. $\bar{\pi}$ denotes the inflation target, which we interpret as the midpoint of the target bands in the successive Policy Target Agreements signed since

1987. The remaining problem was to specify the functional form of hyper-parameters. As the dataset is relatively limited, we are not confident in correctly estimating a general structure for hyper-parameters. Instead we constrained the variance of each state variable to be identical.[8] Let Q_t be $Q_t = Var(\eta_t^i)$. In a standard state-space model Q_t would be estimated as a constant, which was done initially. More precisely we estimated μ as $Q = exp(\mu)$,[9] and it was found equal to *-4.74* with a standard error of *0.19*. This would suggest a permanent learning, as the coefficient is highly significant. The patterns followed by the three time-varying parameters were as follows:

Figure 1:



8 Although we tested an accepted the restriction that those variances were not significantly different.

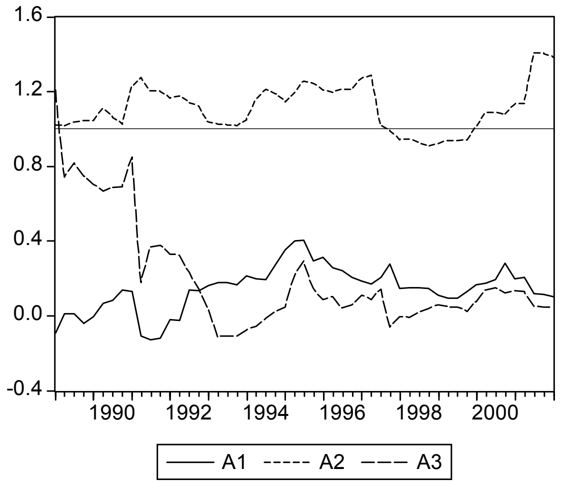
9 The estimations were performed with EViews 4.1.

and the final estimates were:

	Final State	Std. Error	t-stat	P-value
A1	0.00	0.14	0.03	0.98
A2	1.60	0.18	8.86	0.00
A3	-0.07	0.27	-0.26	0.79

Although the results seem to suggest perpetual learning it was also interesting to check if the hyper-parameter on the state decreases over time, which would imply that the learning process converges towards least squares estimates and thus moves towards a rational expectations equilibrium. Thus following the ideas discussed in Hall et al (1997) on convergence, we modelled Q_t as follows: $Q_t = Q_0\, exp(\mu t)$. In this case a negative and significant value for μ would imply that the process is converging towards recursive least squares. Estimating such a model gave a value of *-0.086* with a standard error of *0.0058*. Thus, although the coefficient found was relatively small it is still significantly different from *0* and backs the idea that expectations formation converges towards recursive least squares. The patterns followed by the three time-varying parameters were as follows:

Figure 2:



and the final estimates were:

	Final State	Std. Error	t-stat	P-value
A1	0.11	0.06	1.89	0.06
A2	1.39	0.08	16.75	0.00
A3	0.05	0.10	0.48	0.63

These results suggest several things about how expectations are formed. Among the common features we can see that basically the patterns followed by the state variables are similar in both of the exercises. In the first model (where the hyper-parameter is estimated as a constant) the respective weights on lagged inflation and the output gap converge towards zero, suggesting that expectations are becoming less responsive to those variables and more anchored to a constant. We have interpreted this constant as the inflation target with a time-varying coefficient. With this interpretation, the coefficient on the inflation target, *A2*, does not converge to one, which suggests that expectations are not fully rational. Another interesting point is to look at the pattern of *A2* as compared to the horizontal line corresponding to a constant weight of one. What we can see is that in both cases the weight on the target has been gradually increasing before a sharp fall in 1997, which corresponds to the shift from a 1 per cent mid-point to 1.5 per cent. Then the weight has continued to increase, until the PTA was again changed. Nevertheless all changes in the coefficient *A2* cannot be attributed solely to changes in the official target. The Kalman filter gives the optimal adjustment of coefficients given the shocks that are hitting the economy, so even if the weight on the target moves temporarily away from one it is not necessarily a sign that the target is not credible or achievable. Only a permanent deviation from one could be interpreted as belief that the target differs from the mid-point of the target band.

Another important point to notice is that in both cases the coefficients on lagged inflation and the output gap are not significantly different from zero, at least when testing at 5 per cent. Thus all these results suggest that the inflation target has a much greater role in explaining expectations formation than does the last lagged value of inflation. Put in other words, explicitly allowing for the possibility that agents may revise their expectation rule can explain that inflation expectations are closer to rational expectations than what fixed coefficients estimations suggest. Another interesting feature is that looking at the last value of *A2* in the second model we obtain a value of 1.39, which once multiplied by the mid-point during this period gives a value of approximately 2.1 per cent, which is very close to the current mid-point of 2 per cent. In the first model the coefficient is 1.60 but with a higher standard error, thus although it implies a perceived target of 2.4 per cent, it is not significantly different from the previous one. Thus in both cases expectations seem to be centring around a value near 2 per cent, which is the current mid-point.

Nevertheless, these findings have to be treated with caution. Although in the second model we can accept that the variance is decreasing over time and converges towards zero, the coefficient is small enough so that the convergence process occurs very slowly, and thus is not very different from the first specification where the learning process is perpetual. As a result, even though the coefficient *A2* was consistent with a target of 2 per cent, there is no reason why it should not change in the future, and likewise the other coefficients.

4 Conclusions

In this paper we have analysed the inflation expectations dynamic in the case of New Zealand. Using the methodology proposed by Roberts (1995), we find evidence for the possibility that New Zealand inflation is not inherently sticky, but instead its inertia derives from expectations. By modelling expectations using a learning process, we obtain that expectations seem to be becoming less responsive to lagged inflation and the output gap, which may mean they are moving towards rational expectations. These results have potentially strong implications for monetary policy. In particular, the more rational are expectations, the easier inflation can be reduced without incurring costs. If expectations were more adaptive, then monetary policy should be smoother in order to avoid excessively high costs in the short run together with undesirable economic variability. Moreover, to the extent that the learning process converges quite slowly towards rational expectations, then the policy implication, following the ideas discussed in Orphanides

and Williams (2002), is for policymakers to react more strongly to deviations from the target.

The specification we have utilised above is simple, and data are only available for a relatively short sample. The model also does not explicitly integrate the transmission of shocks from the rest of the world and the exchange rate. Nevertheless, our results suggest that it would be worthwhile to conduct further study into how expectations are formed, and also to investigate expectations formation within models that allow for their variation over time.

Appendix

The data used were defined as follows:

$$y = Ln(Y) - Ln(Y^*) \qquad (19)$$

where Y is the quarterly GDP seasonally adjusted, and Y^* is the potential GDP, estimated as the HP filtered Y.

$$\pi_t = Ln(P_t) - Ln(P_{t-1}) \qquad (20)$$

where P is the consumer price index, excluding interest payments and VAT.

The expectation π^e is computed using the CPI expectation one year ahead, taken from the Reserve Bank of New Zealand survey of expectations.

The interest rate used is the 90 days interest rate. The neutral interest rate is the HP filtered real interest rate over the sample. Thus it is assume to have changed over time. For more details about the neutral real interest rate the reader can refer to the work of Plantier and Scrimgeour (2002) who model a time-varying neutral interest rate and analyse in depth its policy implications.

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