Monetary policy and forecasting inflation with and without the output gap

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Abstract

Some observers have worried that under or over-estimating the output gap may unnecessarily induce tightening or loosening of monetary conditions, causing real fluctuations. To investigate the relationship between the output gap and inflation, we examine models of inflation that do and do not use the output gap. The Phillips curve, which relates inflation to real activity, is regarded as the maintained theory of inflation. Models of inflation without the output gap include the equation of exchange of the quantity theory of money, the real interest rate gap, and two versions of the P* model. Since none of these economic models are either totally wrong nor complete, it makes sense to diversify across models rather than relying on one model exclusively. The forecasts derived from different stable models can be combined through averaging, which offsets biases and reduces the forecast error variance. Such model diversification spreads the risks of errors (i.e., insurance about bad outcomes that arise from the reliance on a single model) and provides greater robustness for policy. This paper examines ten different models of inflation and estimates sixty-seven different specifications, some of which outperform others. Some explanatory variables like money and the real interest rate gap seem to provide more information about future inflation than does estimates of output gap.

1 Introduction

In general, the Phillips curve represents the relationship between the output gap (unemployment gap) and inflation (wage inflation). Most central banks rely in one way or another on forecasts of future inflation to set the interest rate – the policy instrument – using the Phillips curve as the maintained theory for inflation. When the output gap is positive, inflation accelerates and if central banks wish to reduce inflation they increase the interest rate. A higher interest rate reduces investment and consumption spending and in turn closes the output gap and hence reduces inflation.

The output gap cannot be observed directly, it has to be estimated. This is very hard to do accurately. Some observers have worried that under or over-estimating the output gap may result in unnecessarily tight or loose monetary conditions, causing harm to the real economy. The problem has recently been highlighted in research by Athanasios Orphanides. Orphanides (1998) demonstrates that “if policymakers mistakenly adopt policies that are optimal under the presumption that their understanding of the state of the economy is accurate when, in fact, such accuracy is lacking, they inadvertently induce instability in both inflation and economic activity”. In a recent paper, Orphanides (2001) shows that U.S. monetary policy during the pre-Volcker period was excessively “activist” in the sense that it responded to the output gap when the size of potential output was mismeasured.

This finding illustrates what the public is worried about in New Zealand. Brash (2001) responds to this question at length. This paper examines some non-trivial issues pertaining to the relationship between inflation and the output gap on one side, and forecasting and policy setting on the other.

This paper starts with the Phillips curve as the maintained theory of inflation. I describe various versions of the Phillips curve, then I
estimate different specifications of each of these models. When describing these models I discuss policy implications, and when estimating them I examine their stability and out-of-sample forecasting performances.

Then I discuss alternative models of inflation. These are the models that do not rely on the output gap as the main explanatory variable of inflation. For example, I discuss the equation of exchange of the quantity theory of money, various types of the $P^*$ model, and the real interest rate gap.

When describing these alternative models, I also discuss policy implications and contrast them with those derived from the different specifications of the Phillips curve. I then estimate many different specifications of these different alternative models of inflation, examine their stability and out-of-sample forecast abilities. Having done that, I contrast these models with the Phillips curve(s).

This paper argues that it is rather risky for policymakers to put all their eggs in one basket; ie to rely on only one version of the Phillips curve or one particular model to forecast inflation and to set monetary policy. The fact that both of the explanatory variables in the conventional Phillips curve, ie expected inflation and the output gap, are unobservable creates significant uncertainty about the forecast, especially around turning points. There are often large forecast errors that can translate into policy errors to the extent that the policymaker relies on these forecasts in setting policy. The stability of the model is also an important issue; policymakers ought to rely on stable relationships for forecasting and policy.

The paper proposes a strategy to deal with this uncertainty. Just as investment portfolios are diversified to reduce aggregate risk, policymakers can do the same with models. In the Phillips curve, uncertainty that stems from the size of the estimated output gap or expected inflation is irreducible beyond the natural variation of these time series. However, the risk that stems from this inherited uncertainty can be spread out.

We suggest that policymakers combine the outputs of as many stable models as they can, instead of searching for a single best model. Granger (2000) calls such model diversification ‘thick modelling’. In picking the best encompassing model some information in alternative models could be lost. Most economists agree that we don’t know the DGP of macroeconomic variables and we do not know when the shocks hit. Even after realising that a shock has hit, we spend some time trying to assess its permanency. We argue that more and different models, albeit incomplete descriptions of reality, contain more, different, and useful information about the economy than one single model can ever contain.

Moreover, it is probably beneficial to rely on more than one model for forecasting and policy advice. For example, if we use two models for forecasting, and the models’ outcomes are biased in two different directions, then the average is superior to either one. Averaging of forecasts and outcomes may reduce the error’s variance in general, which would probably reduce the policy errors too. Diversifying inflation models should thus add robustness to policy, and averaging the forecasts can reduce the error variance of the forecast. This paper will examine these two issues. Stock and Watson (1999) provide significant empirical evidence.

In this paper I estimate the Phillips curve(s) for the period 1992 onwards, which is the period of low and stable inflation in New Zealand. This is a period during which the RBNZ gained and maintained credibility as an inflation targeter. Razzak (2000).
annual change in the consumer price index (CPI). My hypothesis is that the correlation between inflation and the output gap deteriorates when inflation is successfully controlled. Further, I examine alternative specifications of the Phillips curve and alternative models (theories) of inflation that do not involve the output gap. I present ten models and estimate sixty-seven different specifications. These models are simple and commonly used in practice. My list of models is by no means exhaustive and only serves to illustrate my point. Policymakers can expand this list to include VARS or structural models, for example.

Because most central banks forecast the rate of change of the CPI every quarter and reset policy accordingly, I use all of these models to forecast inflation out-of-sample. It is generally difficult to forecast inflation out of sample. In the case of New Zealand it is especially difficult to fit the Phillips curve for the period of low and stable inflation. Past and expected inflation explain a significant portion of the variation in actual inflation. This paper provides evidence that the output gap does not have significant additional explanatory power beyond what expected and past inflation provide.

There are nine main findings. First, the output gap and many other explanatory variables like the price gaps, the real-money balances gaps, the marginal costs etc. do not provide information about current inflation defined as the annual rate of change of the CPI, beyond what is found in expected and past inflation. To be precise, they are statistically insignificant. Second, output, whether it is measured by the output gap or output growth, is not correlated with inflation conditional on expected and past inflation. Third, there is a large degree of forwardness in inflation. This means that expected inflation explains a larger portion of the variability of current inflation than past inflation does. Inflation’s inertia (past inflation) is nevertheless also significant. The restriction that inflation’s expectations are a linear combination of a forward-looking component measured by the RBNZ survey data and a backward-looking component measured by lagged inflation cannot be rejected by the data. Out of the sixty-seven different specifications estimated in this paper, the twelve specifications that outperform the naïve model out-of-sample are those that involve inflation’s inertia in addition to expected inflation. Fourth, past changes in oil prices have a small but nevertheless statistically significant impact on current inflation and account for supply shocks (cost-push). The coefficients of expected inflation, lagged inflation, oil price changes, money growth, and output growth are more precisely estimated than the coefficient of the real interest rate gap, which is less precise. Related to that, fifth, the equilibrium natural rate of interest is imprecisely estimated. Sixth, it is very difficult to explain and forecast inflation in New Zealand, especially during the period 1992 onwards, which is the period of maintained low and stable inflation. Seventh, all of the paper’s model forecasts miss the turning points in inflation that occurred in September 1999, March 2001 and September 2001. Eighth, a few models of inflation are identified that do not rely on the output gap and outperform many other models out-of-sample. In particular, the real interest rate gap and the quantity theory of money outperform the naïve model and even a forward-looking model. Nevertheless, these models miss all the turning points too. Ninth, some models seem to fit well in sample, but their out-of-sample forecasts are inefficient.

The remainder of the paper is structured as follows. In section 2, I outline the theories and discuss underlying assumptions of different models of the Phillips curve. Then I discuss alternative models of inflation, which do not include the output gap. In section 3 I estimate different specifications of these various models, forecast inflation, and discuss the results. Section 4 concludes.

2 The historical development of the Phillips curve

It is rather difficult to understand what people mean when they talk about the Phillips curve because there is more than one version of it. And each version is significantly different from the others. There are differences in the specifications and the policy implications.
Phillips’ (1958) article “The Relationship between Unemployment and the Rate of Change of Money Wages in the United Kingdom” dealt with the same phenomenon that Fisher’s (1926) article titled “The Statistical Relationship Between Unemployment and Price Changes” dealt with. Both articles show that, empirically, inflation (deflation) tended to be associated with low (high) levels of unemployment. However, the difference between Fisher’s 1926 article and Phillips’ 1958 is significant. Fisher took the rate of change of prices to be the independent variable, and he emphasised the difference between the rate of inflation and the changes in the rate of inflation and anticipated and unanticipated inflation. However, Phillips took the level of employment to be the independent variable – reversing the causality. Underlying Phillips’ system are Keynesian assumptions about price and wage stickiness, which is not the case in Fisher’s model. These differences have non-trivial policy implications for issues such as money neutrality and the long run versus the short run trade-offs between inflation and unemployment or output. Let us take one very simple example to motivate our interest in discussing various types of the Phillips curve.

Very briefly, suppose that the policymaker recognises that the economy is hit with an adverse supply shock. This shock shifts the aggregate supply curve to the left, and causes inflation to increase and output to fall below potential output. What should the policymaker do? A system like the one in Phillips assumes sticky prices and no role for expectations, so in order to bring output back to its potential, the central bank has to ease policy (lower the interest rate for example), which we call demand management. That increases inflation further. A system of flexible prices that allows for expectations to work on the other hand would imply a do-nothing policy because as output falls below potential inflation expectations will tend to change and supply curve will shift and market forces will restore equilibrium over time. In this case inflation does not increase, but output equilibrium might not be restored as quickly as we wish. These are some unresolved policy issues that motivate further discussion.

This paper will argue that the presence of differences in opinions about the underlying assumptions of the model of the economy (although much milder today than ever), the inability to timely determine the nature of shocks and risks arising from various uncertainties justifies thick modelling (model diversification) as a sensible strategy for policymaking.

2.1 Alternative Phillips curve models

2.1.1 The Phillips curve

The Phillips curve is due to Bill Phillips’ (1958) celebrated paper “The Relationship between Unemployment and the Rate of Change of Money Wages in the United Kingdom”. The curve represents an empirical relationship between wage inflation and unemployment, where there is trade-off between the unemployment deviations from its equilibrium value and wage inflation. The Phillips curve in this original paper implies that wages and prices adjust slowly to changes in aggregate demand. This trade-off has a policy implication of course. It says, to keep unemployment below its equilibrium inflation has to increase. It suggests that policymakers can choose the combinations of inflation-unemployment that they like. This kind of output/unemployment stabilisation policy was pursued by central banks in the past, and textbooks document its failure.

2.1.2 The Phelps-Friedman version of the Phillips curve

Edmund Phelps (1967) and Milton Friedman (1968) provided a critical addition to the theory of the original Phillips curve within a market-clearing framework. The idea behind the Phelps-Friedman thesis is that wages are flexible, but adjust slowly because expectations about the general price level (or inflation) are temporarily incorrect due to imperfect information about the nature of the shock. When a demand shock hits, the nominal wage goes up because the price level goes up, workers mistakenly believe that their real wage has increased so they supply more labour. In the short run, there will be lower unemployment and higher wage inflation. Once they realise their mistakes, workers reduce their supply of labour and unemployment returns to its “natural” level, the natural rate of unemployment. The longer it takes workers to realise their mistakes the longer unemployment will be away from its natural rate (ie the longer the economy will be in a recession or a
boom). The natural rate of unemployment is determined by real factors like capital accumulation, population growth and technological progress. So it is the real wage and the role of expectations that matter now and these were missing in Bill Phillips’ original curve. It is important to note that in the Phelps-Friedman work expectations were not necessarily rational in Muth’s (1961) sense.11

The policymaker can trade more inflation for lower unemployment in the short run, but unfortunately the policymaker can no longer choose the inflation rate that keeps unemployment below the natural rate in the long run (forever) as in the original formulation. Long run money neutrality holds in this new model. According to this model, if policymakers want to stabilise output or unemployment using the Phillips curve, expectations will shift it and in the long run, unemployment will be at its natural level, and the economy only ends up with a different level of inflation.

2.1.3 The Lucas version of the Phillips curve

Robert Lucas Jr (1973) extended the Phelps-Friedman hypothesis and used rational expectations (like in Muth, 1961) to build a model for the Phillips curve. In this Phillips curve it is assumed that workers do not know the price level at the time they have to decide how many hours of labour to supply given the market nominal wage rate. In order to decide how many hours to work, the worker has to compute the real wage and in the absence of information about the price level the worker has only one way to solve the problem: she must forecast it. When the nominal wage rate rises, workers guess that this rise can be due either to a higher general price level or to increasing demand for their type of labour that pushed their wage rate up; there is an inherent uncertainty. If the former is the cause for the increase in wages then their real wages have not changed at all and in this case they won’t sell more labour. If the latter is the reason for the increase in wages, they should supply more labour. Since workers cannot know at the time they have to decide whether to work more or not, they compromise by working longer hours. They would fully adjust labour supply if they were certain of the nature of the problem though. Thus, in the short run Lucas’ model predicts a lower unemployment rate associated with higher wage inflation. In the long run, however, workers understand what is going on, cut back their labour supply and unemployment returns to its natural rate.

To translate Lucas (1973) model to the Lucas supply function we write \( y_t = \phi P_t / E_t \), where \( y_t \) is real output, \( P_t \) is the price level, \( E \) is the expectations operator based on the information available at time \( t \), and \( \phi \) is a coefficient. The ratio \( P_t / E_t \) is a relative price. The Lucas supply function is then transformed into the so-called Expectations-Augmented Phillips curve, which is typically given by:

\[
y_t - y_t^* = \alpha (\pi_t - E_t \pi_0) + \phi \tag{1}
\]

Real GDP is denoted by \( y_t \) and \( y_t^* \) is the natural rate of output and the deviation \( y_t - y_t^* \) represents cyclical output. Inflation is denoted \( \pi_t \). The expectations are rational in Muth’s sense and made in time \( t \) (this is an important point). The date of the information set has serious implications for the dynamics of inflation as will be shown later. Given the complete model, we solve it and arrive at a system of equilibrium supply and demand relationships connected by testable cross-equation restrictions:12

\[
y_t - y_t^* = -\alpha_0 + \alpha_1 \Delta x_t + \alpha_2 (y_{t-1} - y_{t-1}^*) \tag{2}
\]

\[
\pi_t = -\alpha_0 + (1 - \alpha_1) \Delta x_t + \alpha_1 \Delta y_{t-1} - \alpha_2 \Delta (y_{t-1} - y_{t-1}^*) \tag{3}
\]

The equations above represent the interaction of aggregate demand and supply. The nominal demand shifter \( \Delta x_t \) shifts the demand curve, while aggregate supply shifts when expectations change. The system suggests that change in nominal demand such as changes in

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11 Under rational expectations, economic agents make forecasts based on all available information to them and, on average, they make no mistakes. The forecast errors ought to be serially uncorrelated, but serial correlation by itself does not necessarily imply irrationality because of information cost. Meltzer (1995) explains that due to uncertainty about the nature and permanency of the shock, producers take longer time to adjust prices.

12 Complete solution of the model is found in Razzak (2000).
money growth or export shocks have immediate impact on real cyclical output and lagged effects, which decay geometrically. The immediate effect on inflation is one minus the real output effect. So a demand shift increases output and inflation. The effect of the shock persists until the next period. This shift is mainly due to the misperception of demand shocks. Similarly, supply shocks result in a decline in output below its natural level and higher inflation.

The model predicts that when inflation is low and stable \( \alpha \), increases in magnitude. In equation (2), a rise in nominal demand would increase real output by \( \alpha \), but increases inflation in equation (3) by \( 1 - \alpha \), so when \( \alpha \) is large, the inflationary effect of nominal spending is small. The coefficient \( \alpha \) asserts the role of money neutrality. It is the mean of nominal GDP growth. To see that recall that in the steady state the output gap is equal to zero and \( \alpha \) is equal to its mean \( \alpha \). Similarly, \( \alpha \) has a negative sign. It represents trend output (secular or potential output defined as \( T_{\text{Trend}} \)), which is negatively related to current inflation (see Lucas, 1973).

**Discussion**

The natural rate of unemployment of the Phelps-Friedman-Lucas type is a fascinating concept, but this natural rate is an unobservable variable. One important question is how does this “unobservability” affect policy? Orphanides (1998 and 2001) demonstrated the problems that arise when using such a concept in policy. The natural rate of output is also confused (whether rightly or not) with many different concepts that are equally unobservable such as potential output and the NAIRU, the non-accelerating inflation rate of unemployment. Not all economists agree or realise that the Natural Rate of Unemployment is different from the NAIRU. To shed some light on these differences we need some simple algebra.

\[
\Delta \pi_t = -\alpha(u_t - u^*) + \varepsilon_t
\]  

Equation (4) is the Phillips curve. It says that the change in inflation \( \Delta \pi_t \) is negatively related to the deviation of the unemployment rate \( u_t \) from the NAIRU, \( u^* \). When inflation is neither accelerating nor decelerating, \( \Delta \pi_t = 0 \) and the NAIRU \( u^* \) is assumed to be a constant. The NAIRU may change over time though. Note that this formulation implies that for a given inflation rate, the unemployment rate can be kept below \( u^* \) forever. (Appendix 1 is a derivation of a simple Keynesian time-varying NAIRU-Phillips curve with expectations.)

The Phillips curve can also be written in terms of output using a price mark-up story and Okun’s law.

\[
\Delta \pi_t = \beta(y_t - y^*_t) + \eta_t
\]

Where \( y_t \) is the level of real GDP and \( y^*_t \) is the level of potential output. The deviation of real output from potential output is called the output gap. Potential output is a concept first introduced by Okun (1965). It measures the level of real GDP or GNP the economy can produce with full employment. So inflation is still determined by real variables here, ie the output gap. When the output gap is positive, the change in inflation is positive.

The difference between the original Phillips curve formulation and the Lucas formulation is clear. The former assumes sticky prices and wages while in the latter prices and wages are flexible. Although the rigidity in the economy is present in both models, they are present for different reasons: sticky prices in the Keynesian model and incomplete information about the nature of the shocks in the neoclassical model. Perhaps the truth is somewhere in between.

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13 Some argue that the NIRU (original name from Modigliani before Tobin came up with NAIRU) is the employment rate at which the Keynesian downward-sloping Phillips curve intersected a vertical line at Friedman’s natural rate of unemployment. Thus, the NIRU is equal to the natural rate. But while monetarists believed that there was no useful trade-off between inflation and unemployment, Modigliani and Papademos interpreted the NIRU as a constraint on the ability of policy-makers to exploit a trade-off that remained both available and helpful in the short run.

14 Unemployment is related to employment and full employment. Output is a function of labour or employment and the price level is related to unit labour cost via a simple mark-up mechanism.
The original Phillips curve did not include expectations, and certainly not rational expectations. And Okun’s potential output is replaced with the natural level of output in the Lucas model. The natural level of output in the Lucas model denotes the secular component of output that is determined by the accumulation of capital and population change. From the above discussion, it is not clear whether potential output and the natural rate of output are the same. This is a good example of model uncertainty.

The policy implications of these alternative models are different. In the Lucas model only unexpected monetary shocks (surprises) can affect real output. When workers expect inflation correctly on average, output is always at (or very close to) its natural rate. Or when the public expects inflation correctly on average, output is always at (or very close to) its natural level. Expectations are made at time $t$ for the period $t+1$, which is different from the Lucas model where expectations can either be dated at time $t$ for period $t$ or at $t-1$ for period $t$. The coefficient $\beta$ is less than one. It represents the discount factor usually found in the utility function. Unlike the original Phillips curve it has no inflation inertia so it is fully forward-looking. And when equation (6) is iterated forward, it yields a Phillips curve, where inflation depends on its future expected value while the current and the expected economic conditions are given by the output gap.

### 2.1.5 The new Phillips curve

In recent work by Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001), another new Phillips curve has emerged, which is actually a new New-Keynesian Phillips curve with a little difference. If equation (6) is iterated forward, the solution is interpreted as firms setting prices based on expected future marginal cost and the expected output gap is nothing but a proxy for marginal cost changes that are associated with excess demand. This is represented by equation (7).

$$\pi_t = E_t \sum_{\tau=0}^{\infty} \beta^\tau [a(y_{t+\tau}, y_{t+\tau}^e) + \eta_{\tau+1}]$$

The shock $\eta_{\tau}$ denotes cost-push and other factors that affect the expected marginal cost. Equation (7) says that inflation depends on current and expected future economic conditions (output gap). Firms set nominal prices based on the expectations of future marginal costs. Rotemberg and Woodford (1997) impose certain restrictions on technology and the labour market and show that the marginal cost is a function of the output gap.

Clarida et al (1999) argue that “the longer prices are fixed on average, the less sensitive is inflation to movements in the output gap”. I will show that inflation is insensitive to movements in the output gap during the period of low and stable inflation in New

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15 If the information set includes lagged inflation then some argue that the Phillips curve has inflation inertia.

16 See footnote number 14 page 1667.
Zealand. Whether this is because of price stickiness or successful inflation targeting is not clear, but it is highly probably related to the later. This paper will show that it is rather difficult to estimate the Phillips curve during the period 1992-2001 for New Zealand. The difference in the dating of the information set that is used to compute expected inflation plays a major role in determining inflation’s dynamic as emphasised by Clarida et al (2000) Lucas (1973) and Mankiw and Reis (2001).

In essence, equation (7) is similar to the Lucas model, where NRH implies that the unconditional expectations \( E(y_t - y^*_t) \) are unaffected by the choice of policy or the inflation rate in the long run. In other words, the central bank cannot choose a particular future path for inflation to keep output above its natural rate or unemployment below its natural rate in the long run or forever. The neutrality condition that holds in the Phelps-Friedman-Lucas Natural Rate paradigm does not hold in the NAIRU formulation and it may not hold in the Calvo model – with staggered contracts – either. However, the implications are rather different. The Taylor-Calvo-Rotemberg type Phillips curve with staggered contracts models imply that increasing inflation over time will tend to keep output permanently below its natural rate, McCallum (1999). This is not the case in the neo-classical type Phillips curve a la Friedman-Phelps-Lucas.

The interpretations of the Phillips curve – output gap framework has become even more complicated than central bankers ever wanted. This new Phillips curve formulation seems to suggest that inflation depends on the anticipated future movements of the output gap, whose current value is also unobservable. So which variable leads which variable?

Mankiw and Reis (2001) say that in the new Phillips curve of Gertler et al, inflation should respond quickly to monetary policy shocks. However, when the marginal cost replaces the output gap, Gali et al (2001) argue that Mankiw et al’s criticism does not follow because, unlike the output gap, the marginal cost responds slowly to the policy shock. Real marginal cost responds with a lag to the output gap.

2.1.5 The sticky information Phillips curve

Mankiw and Reis (2001) bring the New Keynesian Phillips curve much closer to the Phelps-Friedman-Lucas Phillips curve. They propose a replacement to the New Keynesian Phillips curve much in the spirit of Lucas (1973) combined with the Calvo’s price setting framework. They suggest that information about the state of the economy diffuses slowly through the population either because it is costly to acquire information or it is costly to re-optimize. They call the model the sticky-information model. Today’s price depends on expected prices that have been made in the past \( E(\pi^t, \pi_{t-1}^t) \), not so much because of labour contracts but because some firms are still using old information to set prices (Fischer, 1977). Quite similar to Lucas’ inflation presented earlier, inflation in Mankiw-Reis depends on output, expectations of inflation and expectations of output growth.

The story Mankiw and Reis tell is that firms set prices of their goods every period. The firms gather information and re-compute prices slowly over time. In each period, a fraction \( \lambda \) of firms obtain new information about the state of the economy and compute a new path of optimal prices. The other \((1-\lambda)\) firms continue to set prices based on the old plans. Following Calvo, each firm has the same probability of being one of the firms updating their pricing plans regardless of how long it has been since its last update.

The firm’s optimal price (profit-maximising price) is

\[
P_t^* = P_t + \alpha y_t.
\]

\(^{18}\) Mankiw and Reis (2001) do not cite Brunner, Cukierman and Meltzer (1983) paper, where the stickiness is due to information cost. Producers do not change prices immediately after the shock because it is costly to analyse the nature and permanency of the shock. But, it is really very close. Meltzer (1995) is a good discussion.
Where $P_t$ is the general price level in logs, and $y_t$ is output (potential output is normalised to zero). In other words, the firm’s desired relative price, $\hat{P}_t - P_t$, rises in booms and falls in recessions, $\alpha > 0$.\(^{19}\)

A firm that updated its pricing plan $j$ periods ago sets the price:

\[
x_t^j = E_{t-j} \hat{P}_t
\]

So $x_t$ is the adjustment price. The aggregate price is the average of the prices of all firms in the economy:

\[
P_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j x_t^j
\]

Equations (8), (9) and (10) solve for:

\[
P_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j}(P_t + \alpha y_t)
\]

Equation (11) says that output changes are associated with surprise movements in relative prices, a short-run Phillips curve of the sort described in Phelps-Friedman-Lucas. The inflation equation is given by:

\[
\pi_t = (\alpha \lambda / (1 - \lambda)) \dot{y}_t + \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} (\pi_t + \alpha \dot{y}_t)
\]

The sticky-information Phillips curve says that current inflation depends on output growth, anticipated (expected) inflation, and anticipated output growth.\(^{20}\) It is important to note that in this Phillips curve (like all other different Phillips curves introduced earlier) the date of the information set is important for the inflation’s dynamic. Here, past expectations of current economic conditions are important (like in the Lucas model). Recall that in the new Keynesian model the current expectations about future economic conditions are what mattered for current inflation. This difference, according to Mankiw-Reis and also McCallum, yields large differences in the dynamic pattern of prices and output in response to monetary policy.

Mankiw and Reis (2001) provide some simulation results such that their new Phillips curve matches the actual data, explains the observed persistence in inflation well and the positive correlation between contractionary monetary policy, output and inflation. In the sticky-price model, inflation responds quickly to monetary policy shocks – almost immediately. In their new model, inflation responds much slower. This is very consistent with the conventional wisdom, ie monetary policy lags are long and variable.

The model is useful because inflation is made a function of output growth (not the output gap), which is an observable variable, expected inflation and expected output growth, which although they are unobservable variables they can nevertheless be measured using observable survey data. In New Zealand both surveys are available. The survey data are actually released before the CPI is posted every quarter. Of course, there are some concerns related to the adequacy of the survey data. Alternatively, one can use all the survey data that the Bank typically uses in forecasting output as a proxy for expected real output growth.\(^{21}\)

There is also research on using capacity utilisation deviation from its equilibrium value or its mean as a regressor in the Phillips curve instead of the output gap. A very good recent reference about this issue is found in the *Journal of Economic Perspectives* (Winter 1997). The RBNZ uses capacity utilisation as one of the conditioning variables when computing potential output in its model (FPS) so it is implicit in the Phillips curve.

\[^{19}\] This a monopolistic competitive economy where in boom each firm faces high demand for its product, marginal cost rises with output, high demand implies each firm raises its relative price.

\[^{20}\] Take equation (11), re-write it as $P_t = \lambda \hat{P}_t + \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} (P_t + \alpha y_t)$, lag it once, and subtract current price from lagged price equation and with some manipulation get equation (12).

\[^{21}\] I do not use this data in this paper.
2.2 Alternative models

2.2.1 The P Star Model ($P^*$)

The poor empirical performance of various specifications of the Phillips curve lies in their inability to explain the persistence observed in inflation and the fact that the curve is unstable (the coefficients change over time). Combined with the ambiguity of the output gap as a concept and the fact that each Phillips curve can have different policy implications has led many researchers to look for different models.

The $P^*$ model is derived from the Quantity Theory of Money (QTM) under the assumption that the levels of velocity $V_t$ and real output $Y_t$, though they deviate from equilibrium for whatever reason, return reasonably quickly to their long-run equilibrium levels $V^*$ and $Y^*$. The Fed’s $P^*$ model assumes that velocity grows at a constant rate equal to its historical average and output grows at a smooth deterministic trend.

The price level $p_t$ can differ from its equilibrium value $p^*$ when $V_t$ and $Y_t$ diverge from their long-run equilibrium values $V^*$ and $Y^*$. If

\[ p_t = p^* \], inflation (rate of growth of $p_t$) settles at its long-run average. If $p$ is below $p^*$, inflation will rise over the next few years until $p_t = p^*$. If $p_t$ is above $p^*$, inflation will start to fall until $p_t = p^*$.

The model relates the inflation rate (growth rate of $p_t$) to the price gap or to the lagged value of price gap $(p^*_{t-1} - p_{t-1})$.

The inflation equation is given by the backward-looking model:

\[ \pi_t = A(L^*)\pi_{t-1} + B(L^*)(p^*_{t-1} - p_{t-1}) + \epsilon_t \]

Where $L$ is the lag operators and $\pi_t$ is the inflation rate or by the forward-looking equation:

\[ \pi_t = \pi^* + \beta(p^* - p_t) + \eta_t \]

I plot three different $p^*$ models based on three different measures of money, the monetary base (notes and coins in the hand of the public) $MB$, $M1$ and $M3$. In general, velocity is defined as $tV = p_t + y_t - m_t$ and $m_t$ is either $MB$ (adjusted for Y2K), $M1$ or $M3$. Testing for unit root using the ADF suggests that base velocity is probably not a unit root process, at least up to December 1997. When data up to 2001 are included the tests fail to reject the null hypothesis of unit root. Both $M1$ and $M3$ velocities probably have unit roots over the sample from 1992-2001. The estimated sample mean of velocity makes sense only when velocity is stationary. Thus, it is quite difficult to rationalise the value of $V^*$ (the mean of velocity) in the computation of $p_t$ when velocity is non-stationary or a unit root process.

To compute $y^*$, quarterly output is regressed on a linear trend from March 1975 to June 2001. The coefficient on the linear trend is estimated to be 0.004, which I assume to be 1.6 per cent per annum. However, GDP grew at a much faster rate (3.2 per cent) from March
1992 onwards, but these differences do not really affect the regression analyses presented in this paper.25

In figure 1, I plot log real GDP, y, and y∗. In figure 2 I plot the actual CPIX and p∗. The price level was above all three p∗ values from 1992 almost to the middle of the 1990s, while inflation fell substantially during this period. The price level was then below the p∗ (money base-dotted line) and p∗ (M1 -- dashed line) from late 1997-early 1998 to the end of the sample, which implies that inflation must have been increasing. A more dramatic picture emerges when we compare the price level to p∗ (M3 – thin line), where one can see that the price level has been below p∗ from mid 1990s, which implies that inflation has been increasing from 1995 to the end of the sample. This model is not criticism-free, but in our efforts to examine models that do not rely on the output gap this model will be examined rigorously.

2.2.2 The P Star model in terms of real-money balances gaps

This is essentially a p∗ model written in terms of the “real-money gap” instead of prices. Gerlach and Svensson (2000) argue that this model has a substantial predictive power for future inflation in Europe. The basic idea is to fit a demand for real-money balances function, use the estimated long-run elasticities (the income and the interest rate elasticities) to compute m∗t, then compute the money gap as the deviation of actual real-money from this long-run equilibrium. This is then used as an explanatory variable for inflation instead of p∗ or the output gap.26

\[ \pi_t = E, \pi_{t+1} + \beta (m_t - m^*_t) + \epsilon, \]  

Figure 3 plots real-money base and five different m∗. The latter are the long-run equilibrium values of real-money base balances estimated models. The model’s general form is given by equation (17).

\[ m^*_t / p_t = \alpha - \beta i_t + \gamma y_t + \epsilon, \]  

Money is the monetary base. The price level is the GDP deflator. Equation (17) is estimated five times, each time with the interest rate being measured differently. These different measures are the 90-day interest rate, the 10-year government bond rate, the yield gap, which is the difference between the 90-day rate and the 10-year government bond yield, the 90-day interest rate differential between New Zealand and the United States and the 10-year government bond yield differential between New Zealand and the United States. The interest rate differentials will account for the openness of the New Zealand economy. Output is production-based real GDP. The

25 Results of various different ways of defining p∗ are similar.

26 There are four main equations underlying Gerlach-Svensson’s model. First, the p∗ equation, which relates inflation to expected inflation and the real-money gap. Inflation expectations are modelled as deviations of inflation from the objective of the central bank. The demand for real-money balances is an error-correction model for money, where real money is a function of real GDP, the yield gap and inflation’s deviations from expected inflation. They make the following assumption. In the long run, inflation converges and equal to the inflation objective, the difference between the short and the long run interest rates is constant. And in the long run output is at potential.
equation is estimated using the Phillips-Loretan (1991) Two-Sided Dynamic Nonlinear Least Squares, which assumes that the variables have unit roots in the levels (stochastic trends) and are cointegrated (do not wander away from each other in the long run). The estimation covers the sample from March 1988 to December 1997 because the model becomes unstable in late 1998. The estimates of the long-run elasticities have the expected signs and magnitudes.

Figure 3

The real money base (the thickest black line) is below \( m' \) almost all the time when \( m' \) is based on the 90-day interest rate (thin dotted line), the 10-year government bond yield (dashed line), and the 90-day interest rate differential (solid thin line). This implies falling inflation. The real money base only slightly exceeds these \( m' \)'s in late 1999, which implies higher inflation. Real money base also lies below the \( m' \) that corresponds to the 10-year government bond yield differentials (thick dotted line) for most of the sample up until the December 1998, then exceeds it. This also implies higher inflation from 1998-2001. The money gap path based on the yield gap (thick dashed line) and real-money base suggests that New Zealand should have been facing rising inflation since mid 1997.

However, since we are trying to examine different models of inflation where the output gap is not the major explanatory variable this model is one of the alternative models. The model will be estimated and its out-of-sample performance will be examined.

2.2.3 Other variants of \( p' \)

Kool and Tatom (1994) suggested a \( p' \) model for open economies. As one might expect, the model includes the foreign \( p'^{\text{f}} \) and imposes strong assumptions. First, it assumes that the exchange rate is fixed. Second, \( p'=\frac{E_p}{E} \), where \( E \) is the fixed nominal exchange rate that is equal to the number of equilibrium domestic currency units per unit of foreign currency and \( E \) is the corresponding equilibrium real exchange rate. This model has two problems. The first problem is that we have to estimate another unobservable variable, ie the equilibrium exchange rate. The second problem is that the exchange rate is not fixed in New Zealand. Further, economists actually have made no progress on the question of exchange rate determination. All these challenges make this approach rather dubious. This model will not be estimated.

2.2.4 Interest rate as an explanatory variable of inflation

Christiano (1989) used past quarterly changes in the short-term nominal T-bill interest rate as an explanatory variable of the U.S. inflation. The nominal interest rate is the sum of the real interest rate, which is presumably determined by real factors (eg the output gap), and expected inflation. Thus, changes in the nominal interest rate are changes in real interest rate and the revision of expectations from one period to another (the change of the forecasts). When inflation is kept under control for a long period of time like in New Zealand, one does not expect major revisions in inflation expectations. Therefore most of the explanatory power of changes in short-term nominal interest rate is derived from the explanatory power of the changes in the real interest rate. Figures 4a and 4b plot...
the data for nominal interest rates with both quarterly and annual inflation rates. There seems to be some visual correlation between inflation and changes in the interest rate.

Christiano did not find significant differences between the forecasting performances of the $P^*$ model and the interest rate (T-bill) model in general. In this paper, I will also compare the forecasting ability of these two models for New Zealand. However, unlike the Christiano sample, which includes periods of variable inflation in the US, my sample will cover a period of low and stable inflation only (1992-2001).

2.2.5 Real interest rate gaps

Woodford (2000) uses Wicksellian ideas in a general equilibrium model and argues that deviation of the real interest rate from its natural level is a good predictor of inflation. Presumably the central bank is capable of controlling the real interest rate because prices are sticky. One could also argue that the central bank could control the real interest rate when inflation expectations are stationary or constant for a sufficiently long period of time. Empirically, there is huge uncertainty about how to measure (and define) the equilibrium real interest rate. Keep in mind that the ex ante real interest rate is an unobservable variable. Therefore, this variable will not provide a great help for forecasters and also it would not be more practical than the output gap. The natural real rate depends on trend or potential output, which is an unobservable variable itself.

Figure 5 plots the inflation rate and real 90-day interest rate’s deviation from its average over the period September 1987 to June 2001 lagged three-quarters. The average is 5.9 per cent. The real 90-day interest rate is defined as the nominal 90-day interest rate minus expected inflation for the next four quarters from the RBNZ Survey of inflation expectations. Then this real interest rate minus its average represents the deviation from equilibrium. This is the simplest measure of the real interest rate gap.29

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28 Later I will show that the estimation of this model is highly sensitive to the value of the mean of real interest rate. The average of real interest rate during the period from 1992 onwards drops to 5.2. The estimate will also depend on the sample span in New Zealand. The nominal interest rate dropped by half in 1998, which made the mean over the sample that followed very small compared to the mean over the sample from 1988 onwards.

29 Archibald and Hunter (2001) provide several different ways to compute the equilibrium real interest rates. Also see Neiss and Nelson (2000), Laubach and Williams (2001) and Plantier and Scrimgeour (2002).
2.2.6 The quantity theory of money (QTM)

The next model is the equation of exchange of the quantity theory of money where nominal income ($P_y$) is determined by the quantity of money ($m$). Solving for the price level, it says that the price level is proportional to the money stock in the long run. It is not quite clear what happens when we examine the inflation dynamics (the difference of the price level) and whether there is any significant relationship between inflation and money growth during the period of low and stable inflation from 1992 in New Zealand. Razzak (2001) shows that there was a strong correlation between the growth rate of the monetary base and CPIX inflation at high frequency. But there is no correlation between inflation and money growth at both business cycle and low frequencies during the period of low and stable inflation in New Zealand from 1992 onwards.

2.2.7 The New Keynesian system

The New Keynesian model is described in the system of stationary equations (18)-(20).

\[
\begin{align*}
\tilde{y}_t &= -a_1(r_{t-1} - \tilde{r}) + a_2\tilde{y}_{t-1} \\
i_{t} &= a_3\pi_{t-1} + a_4E_{t}\pi_{t-1} + a_5\tilde{y}_{t-1} \\
\Delta\pi &= \alpha(\bar{\pi} + E_{t-1}\pi_{t-1} + 0.5(\pi_{t-1} - 1.5) + 0.5\tilde{y}_{t-1} - i_{t-1}) + \phi\Delta\pi_{t-2}
\end{align*}
\]

Equation (18) is the IS curve. The output gap, denoted \(\tilde{y}_t\), depends negatively on the deviation of the real interest rate from its equilibrium value. The real interest rate \(r_t\) is the nominal 90-day interest rate minus expected inflation, where the latter is measured by the RBNZ survey data of one year ahead inflation expectations, and \(\tilde{r}\) is its equilibrium value (a constant to be estimated).

Equation (19) is the expectations-augmented Phillips curve described earlier. Equation (20) is the Taylor rule, which represents endogenous monetary policy and replaces the LM curve. This rule is expressed in terms of the first difference of the 90-day interest rate and incorporates interest rate smoothing and persistence. Note that the equilibrium real interest rate in the policy rule \(\tilde{r}\) is allowed to be different from equilibrium interest rate in the IS curve, \(\tilde{r}\). I am assuming that the central bank may have a different perception about the equilibrium real interest rate. Of course, this is a testable assumption, and it will be tested in this paper. It is quite possible that the equilibrium real interest rate is not time invariant and changes when the potential output growth or other factors change as in Laubach and Williams (2001) and Plantier and Scrimgeour (2002). However, because I am only interested in the period 1992-2001, where inflation has been stable and the effects of the reforms in New Zealand have settled already, a constant term is not a very bad proxy. The IS curve or aggregate demand where expected future output, which is the relevant argument, is suppressed and instead the lag is used for simplicity.

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30 High frequency fluctuations are those occurring within 2-5 quarters and shorter than the business cycle frequency.

31 The IS curve is actually fully forward-looking in theory. This is the empirical version.

32 There are many controversial issues regarding the parameters in the Taylor rule, such that the constancy of the equilibrium real interest rate, and whether the response coefficients are 0.5 or not, and whether they should be estimated from the data or not. These issues are beyond the scope of this paper and the coefficients are imposed as in Taylor. The main reason for imposing the response coefficients is because the sample is small and I would prefer to estimate fewer coefficients although the response parameters may well affect the size of the estimated \(\bar{\pi}\). There is also an argument about the inflation target. The inflation target used in this formulation is the mid-point of the target zone in New Zealand. However, the target-zone was changed from 0-2 to 0-3 per cent in 1996. Thus, the mid-point is 1 for the period up to 1996 and 1.5 thereafter. Whether this matters or not is also testable. In this paper I kept the midpoint of the target to be a constant equal 1.5.

33 A forward looking aggregate demand equation is consistent with utility maximisation. For proof of this claim see Woodford (1999a), Walsh (1998) and Clarida et al (2000) who show that this model seems to have foundations in dynamic general equilibrium models with price stickiness. Many economists in academia and central banks endorse it. Laurence Meyer (2001) calls it “the consensus macro model”. For more details about the model see also Svensson (1999b), McCallum and Nelson (1999a, b), Rotemberg and Woodford (1999b) and Taylor (1999). Fair (2001) argues that this model has no empirical validity, yet another example of uncertainty about the model of the economy.
3 Estimation

From the Phillips curves presented in section 2, it is clear that equations (1), (4) and (5) and models like the *P* model and interest rate gaps are also non-structural, reduced form equations. Single-equation estimation techniques pose the first econometric problem, yet almost everyone uses them. A system of equations like the Lucas model, the New Keynesian model and the Mankiw-Reis model (and appendix 1) are more appropriate provided that we do not face other problems such as the lack of degrees of freedom. I estimate single-equation specifications so the coefficients are not structural. The RHS variables are lags, which means they are predetermined, but this does not necessarily solve the single-equation bias problem. The system of equations that I estimate is perhaps less problematic in this regard.

I estimate ten models and sixty-seven different specifications. The dependent variable is the inflation rate, measured by annual CPI inflation, \((\ln P_t - \ln P_{-t}) * 100\). This measure is smoother than quarterly inflation and is less sensitive to one-off changes in the government’s fees and charges, taxes on tobacco etc. that strongly influence quarterly data.

Detailed results are reported in ten tables. Tables in this paper are designed similarly in the sense they contain the same numbers of rows and columns, and the same test and diagnostic procedures etc., except for the system of equations model, which is a little different. The independent variables (eg the output gap, *P* etc) are listed in the first column. There are four explanatory variables, three of them appear in all sixty-seven regressions. These are expected inflation, lagged inflation and the lagged rate of change of oil prices. The models differ only in the fourth explanatory variable(s), which are either the output gap, marginal cost, price gaps, money gaps etc. In the second column I report the estimated coefficients for the specification when expected inflation is forward-looking only. Expected inflation is measured using the RBNZ survey data of one-year ahead-expected inflation. In the third column I report the estimated coefficients when expected inflation is a linear combination of forward-looking and backward-looking components. The same two regressions are repeated in columns four and five, except that I include the rate of change of oil price in the regressions as an additional independent variable to account for supply shocks, which are otherwise left in the residuals. After experimenting with the lag structure, the oil price growth rate enters the regressions with a 6-quarter lag in each specification.

All regressions are bootstrapped and 95 per cent confidence intervals for all coefficients are reported. Hypotheses about the coefficients and restrictions are tested and the appropriate statistics are reported. Tables include diagnostic statistics. I compute a wide range of diagnostics, but only report the important ones and the statistics that do not require much space. Some tests, like the rolling Chow test, are not reported because their output is large. The estimation method for single-equation specifications is OLS, but I found that serial correlation is present in the residuals of most equations. It is hard to tell whether the errors are MA(3) or AR(1), because both models fit well. I use Pagan’s (1974) method to deal with this generalised serial correlation and use the ML estimation method to estimate the models. I modelled the error terms as AR(1) processes when necessary. Residuals are checked again to ensure whiteness.

In general, forecasting inflation is hard. It is even harder in credible inflation-targeting regimes (like the one in New Zealand) because the permanent component of inflation is actually kept relatively constant and the only variations we observe are frequent ups and downs representing other price changes. Although it is recognised that most forecasting models fail to forecast turning points, at turning points the forecast errors are large and they are likely to translate into policy errors in the process. Nevertheless, central banks carry out forecasts on a quarterly basis and use them to re-set

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34 Two hundred business leaders are asked questions about their expectations of future inflation (and GDP growth rate) among other questions. The response rate is 40 per cent. A private firm administers the survey on behalf of the Bank. Details are found on the Reserve Bank website: [http://www.rbnz.govt.nz](http://www.rbnz.govt.nz).

35 The oil price is the log of the US dollar Brent crude.

36 Zellner and Min (1999) use a Bayesian method and claim that they can forecast turning points of GDP in many countries.
policy instruments quarterly. Orphanides and van Norden (2001) also show that out-of-sample forecasts of inflation using real-time estimates of the output gap are often less accurate than forecasts that abstract from the output gap concept altogether.

That said, and although the purpose of this paper is to examine different ways to model inflation without the output gap, out-of-sample forecasts will also be computed. The main purpose of the forecasting exercise is to examine ways to reduce the forecast error variance, such as forecast pooling, which might aid policymakers to reducing policy errors. Diversifying the portfolio of forecasting models is also consistent with the fact that we do not know the true model, and this provides robustness to policy decisions. I examine the out-of-sample forecasting performances of the sixty-seven different specifications of the Phillips curve and the other alternative models, and show that they cannot forecast turning points. I forecast 10 quarters out-of-sample (2 and ½ years) from March 1999 to June 2001. The length of the out-of-sample horizon probably matters for the evaluations of models. However, 2½ years is a reasonable medium term policy horizon.

I choose all the specifications that outperform, what I call, the base model. In the base model, actual inflation depends only on two explanatory variables, expected inflation, and one lagged value of actual inflation. The coefficients on these two variables sum to one. The reason I choose this model as a base model and not the random walk is because I believe that during the period 1992-2001, which is the estimation sample, inflation in New Zealand is not a random walk. Inflation is most probably stationary.

A textbook prerequisite for forecasting is that the model’s coefficients are stable. In fact, Hendry and Clements (2001) label shifts in the coefficients of the deterministic terms as most pernicious and say that they induce systematic forecast failure. Thus, the main source of forecast error is the shift in the mean. Once estimation is complete, I test for stability using the rolling Chow test. I found significant instability in many models in late 1998. The instability might represent a structural break in the data. This is particularly true for models that include money or rely on money demand specifications and interest rate models. In New Zealand, the short-term interest rate dropped significantly (from almost 10 per cent to less than 5 per cent in one quarter) in December 1998 and stayed low, which I suspect is the main reason for this instability.

I re-estimated all models from the beginning of the sample to December 1998, saved the data from March 1999 to June 2001 then used them to compute dynamic inflation forecasts. The forecasts are not rolling forecasts – that is, the coefficients are not updated each quarter. I forecast the whole 2 ½ years in one go. For the models that outperform the base model, the forecast is computed for September 2001 (before the release of actual CPI) and in the Mankiw-Reis model, I was able to compute the forecasts for December 2001. Thus, these two forecasts are genuinely out-of-sample.

I report the ratio of the root mean squared errors (RMSE) of each model’s forecast to the root mean squared errors of forecasts obtained from the base model. This allows us to measure the contribution of the explanatory variables such as the output gap and, etc.

For example, suppose that the model under examination is the output gap model. In this model inflation depends on expected inflation, lagged inflation and the output gap. This model is estimated from March 1992 to December 1998. The second step includes using the model to forecast inflation, out-of-sample, from March 1999 to June 2001. The RMSE of this model is then computed. The third step is to compute the ratio of the RMSE of this model to that of the base model. For example, if the ratio is 1.8 it means that the forecast

37 I estimated the models with four lags of inflation. The first lag is always significant. On a few occasions, the fourth lag is found significant. The second and third lags are always insignificant.

38 The Dickey-Fuller test is not powerful against stationary alternatives, but there is a little doubt about its power when it rejects the null hypothesis of unit root. There is no consideration for power when the null is rejected.

39 It is possible that the specification of the dynamics change when I estimate the shorter sample and also the method of estimation that deal with the serial correlation.
error of this model is 80 per cent higher than that of the base model’s forecast error. A ratio of 0.9, for example, means that the output gap model outperforms the base model in the sense that the forecast errors are 10 per cent lower than those of the base model. Finally, the forecasts of all models whose RMSE ratios are less than or equal to 1 are then averaged. The average has a smaller forecast error variance than the individual models, which would perhaps help reduce policy error too.

The criteria I chose to evaluate the models based on out-of-sample forecasts may be questioned. Some argue that we can choose the models that do well in and out of sample. In this paper, all models performed equally well in-sample, having very similar $R^2$. So I had no choice but to use the out-of-sample forecasts to discriminate among them. Extreme models were thrown away because they influence the average RMSE.

All the models missed the decline in inflation in September 1999, in March 2001 and in September 2001 (see figure 6). The decline in March 2001 was related to change in government’s regulation of government-owned housing (1/5 of the housing market). The government dropped the rental value, which caused the CPI to fall. However, inflation was on a downward path regardless.

Next, and before I plunge into estimation, I repeat the summary of the results that I reported in the introduction so the readers who are not interested in technical details can read this part only.

**Summary of results**

First, the output gap and many other explanatory variables like the price gaps, the real-money balances gaps, the marginal costs etc do not provide information about current inflation, which is defined as the annual rate of change of the CPI, beyond what expected and past inflation provide. More precisely, they are statistically insignificant.

Second, output, whether it is measured by the output gap or output growth, is not significantly correlated with inflation during the period 1992-2001.

Third, there is a large degree of forwardness in inflation. But, inflation’s inertia is significant. The restriction that inflation expectations are a linear combination of a forward-looking component measured by the survey data and a backward-looking component measured by lagged inflation cannot be rejected by the data. Out of the sixty-seven different specifications estimated in this paper, the twelve specifications that outperform the naïve model out-of-sample are those that involve inflation’s inertia in addition to expected inflation.

Fourth, past changes in oil prices have a small, but statistically significant, impact on current inflation and account for supply shocks (cost-push). The coefficients of expected inflation, lagged inflation, oil price changes, money growth, and output growth are precisely estimated. But, the coefficient of the real interest rate gap is less precise.

Related to that, fifth, the equilibrium real rate of interest is imprecisely estimated.

Sixth, it is very difficult to explain and forecast inflation in New Zealand especially during the period 1992 onwards, which is the period of maintained low and stable inflation.

Seventh, all forecasts miss the turning points in September 1999, March 2001 and September 2001.

Eighth, a few models of inflation are identified that do not rely on the output gap and outperform many other models out-of-sample. In particular, the real interest rate gap and the quantity theory of money outperform the random walk (naïve model) and even a forward-looking model. However, these models miss all the turning points too.

Ninth, many models seem to fit well in sample, but their out-of-sample performances are inefficient.

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3.1 The Phillips curve

I estimate different specifications of the Phillips curve for the period March 1992 to June 2001. The period of low and stable inflation begins in 1992. In this paper expected inflation is always measured using the RBNZ Survey’s data on expectations for inflation one-year ahead (four quarters). Expected inflation is denoted $E_t \pi_{t+1}$. I always use two different specifications for expected inflation. The first is $E_t \pi_{t+1}$. The second specification includes past inflation in addition to $E_t \pi_{t+1}$. In other words, inflation expectation is a linear combination $\alpha \pi_{t-1} + \beta E_{t-1} \pi_{t+1}$ and the restriction that $\alpha + \beta = 1$ is tested every time. The idea of writing expectations in this form is consistent with bounded rationality (Bomfim and Diebold, 1997), though the RBNZ survey data pass rationality tests.41

The output gap $\bar{y}$, is always measured using two different filters. The Hodrick-Prescott filter (HP) with a smoothing parameter set equal to 1600 and the approximate Band-Pass filter (Baxter and King, 1995).42 The way the output gap is measured will not affect the significance of the estimated coefficients in the Phillips curve. Also, whether we measure the output gap with real-time data is not an issue over the sample from 1992 onwards because the sample is rather short and constitutes no significant changes in the monetary policy regime (ie inflation targeting). It would certainly matter if our sample were from 1970s, where the RBNZ had different policy objectives. The RBNZ started using the Phillips curve more formally in policy discussions in 1995/1996. Then, it was estimated as a single-equation model. Later, the Phillips curve played a major role in the RBNZ model (FPS). Before 1995, the inflation model was a single-equation mark up model (Mayes and Razzak, 1998). This information is important because successful use of the Phillips curve in policy may destroy the correlation between inflation and the output gap and renders it ineffective for policy in the sense that policymakers cannot continue to use it for forecasting and policy. Thus, policymakers pay the cost of their own success. However, Milton Friedman argues that when average inflation and expected inflation are stable the Phillips curve is stable.43 Nevertheless, the question is how would the effect of the interest rate on inflation be transmitted when the link between the output and inflation is broken?

The sample span is crucial for estimating the Phillips curve. The output gap in the Phillips curve may be significant in a sample that

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41 I test for the rationality of the survey data. I regress inflation on a constant term and the survey of inflation expectations using OLS. I found that the constant term is statistically insignificant. The hypothesis that the slope coefficient equals one cannot be rejected by the Wald statistic. The residuals of this regression are tested for serial correlation using many tests like LM(1), LM(4), and DW. I could not find any significant serial correlation. These two tests suggest the data are at least weakly rational. I then regressed the residuals on a constant, the contemporaneous and four lags of the output gap, changes in the 90-day interest rate, the rate of growth of the monetary base and the rate of growth of $M1$ and the rate of growth of $M3$ as proxy of the information set. For the survey to be strongly rational, the residuals must not be correlated with the information set. I found all coefficients to be insignificant except for the money base, its lags, and maybe one lag of $M1$. I then repeated the same regression and included lagged inflation as an additional regressor. Many people believe that the survey is highly correlated with lagged inflation. In this regression all the coefficients are statistically insignificant. I conclude that the survey data satisfy the criteria of rationality.

42 This does not mean that these methods of extracting the trend are the best. Trending time series is a very crucial issue in modelling and in forecasting, but it is beyond the scope of this paper. I acknowledge the problems of filters discussed in the literature, but I use these two filters for practical reasons. I find the correlation between these two filters and the MV filter used by the Bank very high and this has been reported repeatedly in all of the Banks’ research papers.
includes periods of high inflation. It has been argued that a shorter sample from 1994 onwards rather than from 1992 onward fits the Phillips curve best and produces a significant coefficient on the output gap in an expectations-augmented Phillips curve. The reason is that the RBNZ survey data (the first explanatory variable) varies closely with actual inflation in the longer sample and that this high correlation swamps the explanatory power of the output gap and renders it insignificant. Therefore, I estimated Phillips curves using both, short and long samples to test this proposition finding little support for it.

Generally speaking, single-equation estimation methods are subject to the endogeneity problem (single-equation bias). Although the output gap is always lagged in this paper, to indicate that it is a predetermined variable, this treatment does not solve the single-equation bias. Results are reported in table 1 (a to d).

Different specifications of the Phillips curve suggest that when the RBNZ survey data are used as a proxy for expected inflation the estimated coefficient is not significantly different from unity. And

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44 Claus, Conway and Scott (2000, p.39) is a major piece of RBNZ research on the output gap. They conclude that the output gap is a useful concept for policy. The authors use many different and interesting methods to estimate the output gap. They conclude that the output gap is a useful concept for policy. This conclusion is drawn from the evaluation of different policy rules under uncertainty using the Reserve Bank’s model (FPS) and the estimation of a single-equation Phillips curve. They fit a single equation Phillips curve of the form $\Delta \pi_t = \alpha + \beta (\pi_{t-1} - \pi_{t-2}) + \epsilon_t$ from 1971-1999 and show that the output gap is statistically significant. I speculate that the output gap is found to be significant because the sample ran through periods of high inflation. In their seminal research, Phelps and Friedman reported similar results. They found the fit of the Phillips curve improves when inflation is high. Note that the Phillips curve of Claus-Conway-Scott is not an expectations-augmented Phillips curve, but rather the accelerationist version. This Phillips curve is fully backward looking and consistent with the NAIRU not the Natural Rate Hypothesis. Policy implications are quite different as I explained earlier. Further, the sample includes episodes of policy changes not relevant to the setting of policy today and because of these structural changes the out-of-sample forecast can be ruined. This model cannot fit the New Zealand data during the period 1992 to date. I estimated this particular specification, which includes contemporaneous and four lags of the output gap by OLS from 1992-2001 and found all coefficients including the constant to be insignificant and adjusted $R^2$ is negative.

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45 Some argue that a sample from 1994 fits better than a sample from 1992 because the survey data of inflation expectations display variations similar to that of actual inflation in 1992-1993. This variation makes it difficult for the output gap to compete with the survey data as an additional explanatory variable.

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46 I also estimated the Phillips curve over the sample from March 1988 to December 1991 and over the sample from March 1988 to June 2001. The output gap is always insignificant.
3.2 The new Phillips curve

Table 2 reports the regression results of the new Phillips curve for New Zealand. I use different measures of marginal cost. I use Statistics New Zealand’s index measure of unit labour costs, taking its deviation from its own mean to proxy the average real marginal costs.\(^\text{47}\) Second, I compute unit labour cost as \(\left(\frac{w_t h_t}{x_t}\right)\), where \(w_t\) is hourly private sector wages, \(h_t\) is hours and \(x_t\) is nominal GDP.\(^\text{48}\) I then take the deviation from the mean. I also used real wage / (output per head) as a measure of the real marginal cost adjusted for productivity. I also tried its growth rate deviation from trend, where trend is estimated using the HP filter. I only report the results of the first measure. The rest of the results are not reported to save space. In general, I find no significant relationship between inflation and marginal cost under inflation targeting.

The coefficient on inflation expectations is not significantly different from unity. And when expectations are specified as a linear combination of forward and backward looking components, they coefficients of the two components sum to one. The marginal cost is an insignificant explanatory variable, both contemporaneously and at several lags. This is the case regardless of how expected inflation enters the regression. This model fails for New Zealand. The forecasting ability is just as bad as that of the original Phillips curve.

3.3 The \(p^*\) model(s)

I estimate the \(p^*\) model for the money base, which is basically notes and coins in circulation \(MB_t\), \(M1_t\) and \(M3_t\). The model for inflation is estimated using the original \(p^*\) formulation presented in section 2. I estimate three models. The models differ in the price gap, which is \((p_{t+1} - p_{t+1})^{100}\), where \(p_t\) depends on the measure of money, ie the money base \(MB_t\), \(M1_t\) and \(M3_t\). All models are estimated from March 1992 to June 2001 under the assumption that this period – when inflation is low and stable – is more relevant for policy and forecasting. Table 3(a-c) reports the results.\(^\text{49}\)

Again, expected inflation is the main significant explanatory variable. The estimated coefficient is not significantly different from one in all cases. The hypothesis that the sum of the coefficients on the forward-looking and backward-looking components is one could not be rejected in all 12 regressions reported in table 3 (a-c). The price gap is insignificant in all models. This is true whether the model is based on the money base \(MB_t\), \(M1_t\) or \(M3_t\). The out-of-sample forecasting performance is bad. The model cannot outperform the base model. The forecast errors are 8 to 54 per cent larger than those of the base model. In model 27 (table 3b) and 31 (table 3c), the errors were so enormous they were the worst among the sixty-seven specifications examined in this paper.

3.4 A real-money balances gap \(p^*\) model

I estimate a demand for real-money balances for the monetary base only. Money is deflated using the GDP deflator.\(^\text{50}\) I estimate a simple Keynesian demand for money function because it has only two coefficients to estimate and because we don’t have sufficient data to estimate a more elaborate function. Simply, real-money balances are functions of a constant term, real GDP and a nominal interest rate.

I used five measures for the interest rate. I used the 90-day interest rate and the 10-year government bond yield. I also used the yield gap – 90-day interest rate minus the 10-year government bond yield, the 90-day interest rate differential between New Zealand and the US, and the 10-year government bond yield differential between

\(^{47}\) The RBNZ code is llisai.

\(^{48}\) The RBNZ codes for wages are lqhoprz, hours are lhhwz, and nominal GDP is ngdppz.

\(^{49}\) For the US, Christiano (1989, p11) reports very similar magnitudes of coefficients.

\(^{50}\) Data for production GDP deflator are not readily available in New Zealand. Expenditure GDP data are annual. The quarterly data are interpolated by the Reserve Bank forecasting group. The deflator used in this paper is nominal expenditure GDP / real production GDP.
New Zealand and the US. Loosely speaking, these last two measures allow for openness.

The sample is March 1992 to December 1997, which is shorter than all other models because there seems to be some instability in the coefficients in 1998. The results are similar in all regressions and very consistent with theory. The long-run semi-interest rate elasticity of the demand for money is between –0.008 and –0.02, which is consistent with reported values in the literature (see section 2). The demand for real-money base is fairly interest inelastic. The income elasticity of the demand for money is 1.

I computed the money gaps \( \frac{(m/p) - (m/p)}{g43/g45} \) and used them as explanatory variables of inflation in the model. The variable \( (m/p) \) is \( \frac{tEi}{g112} \), where the coefficients are the estimated long-run elasticities mentioned above. Results are reported in tables 4a to 4e.

The hypothesis that the coefficient on expected inflation equals one cannot be rejected and the hypothesis that the sum of the coefficients on the forward-looking and backward-looking components sums to one cannot be rejected either. The money gaps however, turn out to be significant in many cases. The money gap is slightly significant (at the 10 per cent level) in models 33, 34 and 38 in tables 4a and 4b respectively, but much more significant (at the 5 per cent level) in models 41 and 43 in table 4c. In these models, the money gap is based on the demand for the money base when the interest rate is measured by the yield gap. Less significant money gaps are found in models 45 and 46 in table 4d when the interest rate is measured using the 90-day interest rate differential between New Zealand and the US. The out-of-sample forecasting performance is, however, worse than all previous models. The forecast errors are much larger than the forecast errors obtained from the base model.

### 3.5 Changes in nominal interest rates

Table 5 reports the regression results for the changes in the nominal 90-day interest rates model. The 90-day interest rate changes have a reasonably good explanatory power for inflation. In table 5, models 54, 55 and 56 show very significant coefficients. The out-of-sample forecasting performance is no worse than the base model. Just like the previous regressions, the hypotheses regarding expected inflation or lagged inflation held as well.

### 3.6 Real interest rate gap

One can measure the real interest rate gap in many different ways. Archibald and Hunter (2001) summarise various possible ways to compute the real interest rate gap for New Zealand. Nelson and Neiss (2001) also answer the question in the UK experience and Laubach and Williams (2001) do the same thing for the US. However, for all practical reasons I find evidence for using a real interest rate gap to predict future inflation in New Zealand.

I define the real interest rate gap as the deviation of \( tEi \) from its sample mean for the period 1987-2001, which is about 5.9. This is a constant, however, Laubach and Williams (2001) and also Plantier and Scrimgeour (2002) argue, convincingly, that it is time varying. It is quite plausible that the equilibrium real interest rate is not time invariant and changes when the structure changes. However, because I am only interested in the period 1992-2001 in New Zealand, where inflation has been stable and the effects of the structural reforms in New Zealand have settled already, a constant term will suffice. The stability of inflation and expected inflation implies that there have been no significant revisions in inflation expectations. Thus, deviations of the real interest rate from its own mean are highly correlated with the deviations of nominal interest rate from its own mean.

When I estimate the New-Keynesian

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51 The sample over which the mean is computed is longer than estimation period. The idea is that the average of a relatively longer sample is more precisely estimated than that of shorter sample. And, if there are some significant deviations from the average that occurred late in the sample (1992-2001), one would want to capture them.

52 I compare the real with nominal interest rate gaps during the period of low and stable inflation (1992-2001). I compute the nominal 90-day interest rate gap as the 90-day interest rate minus its own mean from March 1992 to June 2001, which is 7.11. The mean of this gap is zero. The standard deviation is 1.56. I compare this gap with the real interest rate gap defined as the real interest rate minus its own mean. The mean of the real interest rate gap is negative, but not
I will estimate the equilibrium real interest rate from the data.

Table 6 reports the regression results for the real interest rate gap model. The coefficients on the lags of the real interest rate gap are significant and negative as the model predicts and the forecasting performances are exceptionally good. In model 58 reported in column 3 in table 6, the ratio of the root mean squared error to the root mean squared error of the base model is 0.89, which implies that this model’s forecasts are superior to the base model’s over a two year horizon.

There is a paradox in this model. We know that policy responds to inflation’s deviation from the target by altering the short-term nominal interest rate, which supposedly alters the path of the real interest rate because prices are sticky in the short run. However, the model suggests that last period’s interest rate can predict inflation. It seems that there is identification or a causation problem – does inflation respond to interest rates or interest rates to inflation?

3.7 The equation of exchange of the quantity theory of money (QTM)

I estimated the equation of exchange of the quantity theory of money in first-differences. In this specification inflation depends on expected inflation, lags of the growth rate of the money base, lags of the growth rate of real GDP and the rate of growth of the Brent oil price lagged six periods. Just like in the real-money gap model, the sample size is shorter in this exercise because of the instability in the money-interest rate relationship in 1998 that I mentioned earlier. Results are reported in table 7. I find that money base growth has a significantly different from zero. The standard deviation is 1.47. The P value of the student t statistic to test for the equality of the means is 0.05, which is not very significant and perhaps cannot reject the equality. The P value of the F statistic to test for the equality of the standard deviation is 0.73, which cannot reject the hypothesis that the two standard deviations are equal. The real and nominal interest rate gaps are the same because the changes in expected inflation have zero a mean and a zero median and the standard deviation is 0.24.

3.8 The New Keynesian model

I estimated the New Keynesian model as a system of equations. The model in equations (18), (19) and (20) is estimated by the full information maximum likelihood method (FIML) for the period March 1992 to June 2001. Then it is re-estimated from 1992 to December 1998 and the observations from March 1999 are saved for forecasting. Then, the model is solved (simulated) for the period from March 1992 to June 2001. Results are reported in table 8.

I experimented with the lag of the real interest rate in the IS curve in equation (18). I found that all lags from $r_{t-1}$ to $r_{t-8}$ are significant. The magnitude of the coefficient and the significance level increase with time, peak at $r_{t-8}$ then start to drop and become insignificant when $r_{t-10}$ is used. For example, the coefficient of $r_{t-3}$ is 0.05, the coefficient of $r_{t-4}$ is 0.1, the coefficient of $r_{t-8}$ is 0.26 and the coefficient of $r_{t-10}$ is 0.05. So I picked $r_{t-8}$ and ran the system with it. Thus, current changes in real interest rate affect real output gap two years later. The magnitude of the coefficient is -0.26.

The equilibrium real interest rate in the IS curve is estimated to be 6 per cent. Recall that the equilibrium real interest rate also appears in the Taylor rule, but we assumed that the central bank is allowed to have a different view on its value. This was estimated to be 5 percent. The standard errors of these estimates are 0.5, so these two estimates are not significantly different from each other, and they are two-quarter lead on future inflation, and output growth has a one-quarter lead on inflation. The out-of-sample forecast is pretty good. It improves when expected inflation is measured by the sum of forward-looking and backward-looking components rather than a fully forward-looking specification, which again is an indication of the importance of inflation inertia. In model 62 in table 7 the forecast errors are 20 per cent smaller than those obtained from the base model. This model produces the smallest forecast error variance among all sixty-seven specifications.
not significantly different from the sample average for the period 1987-2001 or 1992-2001. The sum of the coefficients on expected inflation and lagged inflation is one. This is consistent with all single-equation models estimated earlier. The output gap (measured using the HP filter) in the Phillips curve is also insignificant, which is consistent with the regressions’ results reported earlier. I tried lags from 1 to 30 quarters, but none is found to be significant. The variance of the output gap is much larger than the variance of inflation and the correlation is pretty weak. Similar results are obtained when the output gap is measured using the BP filter.

Although the performance of the model out-of-sample is slightly better than the random walk model, and there is information beyond what expected and lagged inflation provide for future inflation, it is quite puzzling how this model can be used for policy analysis. The direction of causality in the model runs from the interest rate to the output gap then to inflation. But, the output gap is never significant in the Phillips curve. Changes in the nominal interest rate are divided into the real interest rate and anticipated inflation. Because the inflation-targeting regime in New Zealand succeeded in keeping inflation low and stable over the estimation sample (1992-2001), inflation expectations remained stable and changes in the nominal interest rate are mainly reflected in real interest rate changes as explained and showed evidence earlier. The problem is that the real interest rate affects the output gap, but the output gap is not correlated with inflation.

This question that remains to be answered: if there is no significant correlation between the output gap and inflation, how does the Taylor rule close the inflation gap in New Zealand?

3.9 The Lucas imperfect information Phillips curve

I estimate the system of equations in equation 2-3 from March 1992 to June 2001 using the full information maximum likelihood function estimation method. I impose the restrictions on the model. This model, however, required a little alteration. The inflation rate is measured as $\ln(\frac{P_t}{P_{t-1}})$ instead of $\ln(\frac{P_t}{P_{t-1}})$. The former is more volatile than the latter and this is required because the change in the output gap term that appears in the inflation equation (3) is quite volatile and does not fit the second measure of inflation, which is rather smooth. I report the results in table 9. The coefficients are significant and the signs are correct. I re-estimated the model up to December 1998 and used the data for solving forward up to June 2001. The data fit the output gap equation remarkably well, and is much better than fitting the inflation equation. The RMSE ratio is rather large for inflation. The main reason for this outcome is that the inflation equation does not depend, directly, on its own lagged value. Regardless of the restrictions imposed on the model, the dynamics of inflation and its persistence are not well captured by the model.

3.10 Mankiw-Reis sticky information Phillips curve

I estimate the sticky information Phillips curve in equation (12) for two periods, $j = 0, 1$ because we have a small sample size for estimation. Two parameters are estimated, $\alpha$ (the sensitivity of inflation to output and expected output growth) and $\lambda$ (the fraction of firms in the economy that adjust prices). I use output growth defined as $(\ln y_t - \ln y_{t-1})*100$ where $y_t$ is real GDP instead of the output gap (an observable variable). Expected inflation is measured using the RBNZ survey of expected inflation. In this model,
however, inflation also depends on expected output growth. I use two measures. One is the RBNZ survey data on expected output growth for one-year ahead. This data do not match well with very volatile GDP growth in New Zealand. The other measure is the lagged value of real GDP growth. Results of the two regressions are not significantly different.

The regression is estimated using a non-linear estimation method with the residuals modelled as an AR process from March 1992 to June 2001. Then it was estimated up to December 1998. Using the coefficient estimates for this sample, the model is solved (simulated) up to December 2001. The regression converged in 7 iterations and yielded estimates for $\alpha$ and $\lambda$ not significantly different from what Mankiw and Reis used in their calibration of the US data. The estimate of $\alpha$ is 0.1 and $\lambda$ is 0.33 while Mankiw-Reis used 0.1 and 0.25 for $\alpha$ and $\lambda$ respectively. Thus, the data suggest that about 33 per cent of firms in New Zealand change prices every quarter based on new information. The coefficients $\alpha$ and $\lambda$ are robust to whether we use the output gap or output growth and to the lag length. I experimented with the lag of output growth. I used the contemporaneous growth rate $y_t$, then used $y_{t-1}$, $y_{t-2}$ and $y_{t-3}$ instead. Each one of them is found significant, I chose the longer lag, $y_{t-3}$, so that I could simulate the model further into the future. The model can predict inflation in December 2001. Results are reported in table 10. The model’s RMSE ratio to the RMSE obtained from the base model is 1.0. This means that the model is as good as the simple expected inflation plus lagged inflation base model. Thus, conditional on expected and lagged inflation, output does not explain inflation in New Zealand whether it is the output gap or output growth.

Based on the RMSE ratios, about 12 models in total outperform the basic model (and the random walk or naïve model). These are the Phillips curves models 2, 4, 6, 8 and 10, the change in interest rate model 56, the real interest rate gap models 58 and 60, the equation of exchange of the quantity theory of money model 62, the New Keynesian system model 65 and the Mankiw-Reis model 67. These specifications resulted in RMSE ratios less than 1. In table 6, model 58 for example has a RMSE ratio of 0.89 and the quantity theory of money models (61 and 62) in table 7 have RMSE ratios of 0.93 and 0.82 respectively. The New Keynesian system of equations (model 65) in table 8 has a RMSE ratio of 0.98, and I included the Mankiw-Reis sticky information model (67) in table 10 although its RMSE ratio is 1. All of these models are used to forecast inflation from March 1999 to June 2001.

Further, all of those models are then used to forecast CPIX inflation September 2001 before the actual CPI was announced (it was 2.4). The RMSE ratios deteriorate. Model 61, the quantity theory of money, drops out because its RMSE ratio exceeded one. The RMSE ratios for the remaining models are 1.0, 1.05, 1.01 and 1.04 for the real interest rate gap, the equation of exchange of the quantity theory of money, the New Keynesian and the Mankiw-Reis models respectively. Their forecasts of CPIX inflation for September 2001 are 3.17, 3.41, 2.96 and 2.69 respectively. The average of all those forecasts is 3.05 and it has a smaller RMSE ratio, 0.97, than each individual model. I report the forecasts of September 2001 and December 2001, which could only be computed for the Mankiw-Reis model in the table below. The forecast of December 2001 is 1.9. Also, I plot the out-of-sample forecasts from March 1999 to September and December 2001 in figure 6. Superimposed on the graph is also the PCPITA, a measure of inflation used by the Bank recently.57

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56 When I use lagged real GDP growth as a proxy for expected real GDP growth the estimates are 0.05 and 0.33 respectively.

57 PCPITA is made up of different series spliced together. In the early to mid 1990s it was the Bank’s measure of underlying inflation. From December 1997 to June 1999 it is annual CPIX inflation (which excludes the effects of interest rate changes on the CPI). Since then it is annual CPI inflation.
4 Conclusions

In setting monetary policy, central banks put a lot of emphasis on a deliberate forecasting process. They forecast many macroeconomic variables, but the inflation rate and the output gap (or output growth) are perhaps the most important ones. To forecast inflation (the rate of change of the CPI) quarterly, most central banks use a version of the Phillips curve and it is considered the maintained theory of inflation. Generally, the Phillips curve relates current inflation to its own lags, anticipated or expected inflation, and a measure of cyclical real activity like the output gap. Thus, the Phillips curve describes the dynamic price adjustment in the economy, and except lagged inflation, the other two explanatory variables (expected inflation and the output gap) are themselves unobservable variables, and the central bank must come up with a view on their evolution.

Central banking involves much uncertainty. Central banks are uncertain about the true model of the economy and about the nature and permanency of shocks hitting the economy. Moreover, because potential output is unobservable, there is uncertainty about the level and growth rate of potential output. All this presents a major challenge to policymakers. Model uncertainty, shock uncertainty and measurement error, make forecasting inflation difficult. And because forecasts of inflation are a key input for policy decisions, forecast errors can become policy errors. Thus central banking involves risks.

To reduce the adverse effects of uncertainty it is recommended that central banks diversify their modelling efforts and not depend on one single model. This is like investors spreading the risks by diversifying their portfolios. However, for the models to be useful for policymakers they must have empirical support and must be stable. This paper has presented a range of alternative inflation forecasting models, with most of these avoiding use of the output gap. Averaging forecasts across satisfactory models results in a lower error variance than would otherwise result. Policy that relies on more than one model is a more robust policy. Thus, the diversification approach is likely to reduce the average size of policy errors.
References

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Woodford, M, A (2000), New-Wicksellian Framework for the Analysis of Monetary policy, in Interest and Prices (chapter 4), manuscript, Princeton University.


### Table 1a: The Phillips curve

<table>
<thead>
<tr>
<th>Sample (1992:1 – 2001:2)</th>
<th>Dependent Variable $\pi_t = (\Delta_t \ln p_t^e) * 100^A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1$^B$</td>
<td>Model 2</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$E_\tau \pi_{t,2}$</td>
<td>0.04 (0.27)$^B$</td>
</tr>
<tr>
<td>[0.09, 1.00]</td>
<td>[0.75, 2.62]</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>-</td>
</tr>
<tr>
<td>[0.76, 0.31]</td>
<td>[0.80, 0.05]</td>
</tr>
<tr>
<td>$\bar{y}_{t-1}$</td>
<td>0.04 (0.88)$^E$</td>
</tr>
<tr>
<td>[0.09, -0.007]</td>
<td>[0.09, -0.007]</td>
</tr>
<tr>
<td>$\Delta_oil_{t-6}$</td>
<td>-</td>
</tr>
<tr>
<td>[0.008, 0.002]</td>
<td>[0.007, 0.000]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.45 (3.15)$^F$</td>
</tr>
<tr>
<td>0.04 (3.15)$^F$</td>
<td>0.04 (3.31)$^I$</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.72</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>0.31</td>
</tr>
<tr>
<td>Dh / DW$^G$</td>
<td>0.59/1.83</td>
</tr>
<tr>
<td>RMSE$^L$</td>
<td>1.15</td>
</tr>
</tbody>
</table>

- **A** Inflation is tested for unit root using ADF from 1989-2001. The ADF $\phi(\sigma_0 = 0, \theta = 0, \delta = 4)$ is 7.5 and the 5 per cent critical value is 4.59. The ADF $\phi(\sigma_1 = 1)$ is -5.3. Thus, we reject the hypothesis that inflation has a unit root.
- **B** Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out. Model 2 is estimated by OLS.
- **C** This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.
- **D** The P value of the Wald statistic for testing the hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected.
- **E** The output gap is measured by the Band-Pass filter cycle of 6-32 quarters in length. The output gap is stationary by construction.
- **F** AR1 error term.
- **G** Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.
- **H** The hypothesis that $\Delta_{oil_{t-6}} = \pi_{t-1}$ cannot be rejected. The Wald statistic P value is 0.1356.
- **I** Log Brent crude oil in US dollars.
- **J** The P value of the Wald statistic for testing the hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected.
- **K** The hypothesis that $\pi_{t-1}$ $\pi_{t-1}$ cannot be rejected. The Wald statistic P value is 0.2494.
- **L** This is the ratio of the Root Mean Squared Errors of this model to Root Mean squaredErrors obtained from this model, $\sigma_t / (\sigma_t + \pi_{t-1} + \pi_{t-1})$. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.
- **()** t values are in parentheses.
- **[ ]** Squared brackets include 95 per cent interval of 1000 bootstrapped regressions.
- **{ }** P values of the Wald statistic.
- ***$** Significant at the 5 per cent level.
Table 1c: The Phillips curve
Sample (1994:1 – 2001:2) – Shorter Sample
Dependent Variable $\pi_t = (\Delta \ln p_t)^* 100$

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 9(^\text{B})</th>
<th>Model 10(^\text{B})</th>
<th>Model 11(^\text{B})</th>
<th>Model 12(^\text{B})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E, \pi_{t-5}$</td>
<td>1.04 (0.4541)</td>
<td>0.65 (3.37)</td>
<td>1.03 (0.4215)</td>
<td>0.89 (4.21)</td>
</tr>
<tr>
<td>$\sigma_{\pi_{t-5}}$</td>
<td>0.61 (2.75)</td>
<td>0.77</td>
<td>0.89</td>
<td>0.61</td>
</tr>
<tr>
<td>$Dh / DW$</td>
<td>1.16</td>
<td>0.97</td>
<td>1.00</td>
<td>1.06</td>
</tr>
</tbody>
</table>

A Inflation is tested for unit root using ADF from 1989-2001. The ADF $\Phi(0,0,0) = 0.61$ (log 4) is 7.5 and the 5% critical value is 4.59. The ADF $\rho(0,1)$ is -5.3. Thus, we reject the hypothesis that inflation has a unit root.

B Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.

C This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.

D The P value of the Wald statistic for testing the hypothesis that $K_{\pi_{t-5}} = 0$ is in parenthesis. It cannot be rejected.

E The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

F AR error terms. The roots are complex, and the AR process displays pseudo periodic behaviour with damped sine wave.

G Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey-Collier (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey-Collier (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

H The hypothesis that $K_{\pi_{t-5}} = 0$ cannot be rejected. The Wald statistic P value is 0.6188.

I Log Brent crude oil in US dollars.

J The P value of the Wald statistic for testing the hypothesis that $K_{\pi_{t-5}} = 0$ is in parenthesis. It cannot be rejected.

K The hypothesis that $E, \pi_{t-5}$ cannot be rejected. The Wald statistic P value is 0.3219.

L This is the ratio of the Root Mean Squared Errors of this model to Root Mean squared Errors obtained from this model, $n = (E, \pi_{t-5}, n_{-1})$. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

• t values are in parentheses.
• [] Significant at the 5% per cent level.
• * Significant at the 5% per cent level.
• # Significant at the 10% per cent level.

* Significant at the 5 per cent level.
{} The P value of the Wald statistic.
Table 1d: The Phillips curve
Sample (1994:1 – 2001:2) - Shorter Sample
Dependent variable $\pi_t = (\Delta \ln p_t) \times 100^A$

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 13$^B$</th>
<th>Model 14$^H$</th>
<th>Model 15$^B$</th>
<th>Model 16$^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\pi_t]$</td>
<td>1.04 (0.5118)$^B$</td>
<td>1.06 (3.44)$^H$</td>
<td>1.02 (0.5287)$^B$</td>
<td>0.89 (4.42)$^K$</td>
</tr>
<tr>
<td>$[0.90, 0.99]$</td>
<td>[0.90, 0.42]</td>
<td>[1.08, 0.99]</td>
<td>[1.25, 0.60]</td>
<td></td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>-</td>
<td>0.36 (1.96)$^H$</td>
<td>-</td>
<td>0.10 (0.53)$^K$</td>
</tr>
<tr>
<td>$[0.60, 0.13]$</td>
<td>[0.60, 0.13]</td>
<td>[0.60, 0.13]</td>
<td>[0.43, 0.19]</td>
<td></td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>0.07 (1.03)</td>
<td>0.06 (1.05)</td>
<td>$0.09 (1.67)^E$</td>
<td>$0.09 (1.60)^E$</td>
</tr>
<tr>
<td>$[0.03, 0.007]$</td>
<td>[0.15, 0.03]</td>
<td>[0.10, 0.05]</td>
<td>[0.10, 0.05]</td>
<td></td>
</tr>
<tr>
<td>$\Delta_oil_{t, t-1}$</td>
<td>0.52 (3.40)$^I$</td>
<td>-</td>
<td>0.007 (2.84)$^I$</td>
<td>0.007 (2.26)$^I$</td>
</tr>
<tr>
<td>$[0.01, 0.004]$</td>
<td>[0.01, 0.002]</td>
<td>[0.01, 0.002]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.76</td>
<td>0.77</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.31</td>
<td>0.30</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>$Dh / DW$</td>
<td>1.32/1.67</td>
<td>0.88/1.81</td>
<td>1.54/1.75</td>
<td>1.41/1.77</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.27</td>
<td>1.18</td>
<td>1.16</td>
<td>1.16</td>
</tr>
</tbody>
</table>

A: Inflation is tested for unit root using ADF from 1989-2001. The ADF $\phi(\alpha = 0, \rho = 0, \log = 4)$ is 7.5 and the 5 per cent critical value is 4.5. The ADF $\phi(\alpha = 1, \rho = -1)$ is -5.3. Thus, we reject the hypothesis that inflation has a unit root.

B: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.

C: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.

D: The $P$ value of the Wald statistic for testing the hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected.

E: The output gap is measured using the HP filter with $\lambda = 1600$.

F: The marginal cost is measured by real unit labour cost (index) deviations from its mean. The hypothesis that it has a unit root can be rejected by the ADF test. The ADF($\phi(\alpha = 0, \rho = 0, \log = 1)$ is 5.10. The 5 per cent critical value is 4.59.

G: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey-Collier (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.

H: The hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected. The Wald statistical $P$ value is

I: Log Brent crude oil in US dollars.

J: The $P$ value of the Wald statistic for testing the hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected.

K: The hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected. The Wald statistical $P$ value is 0.297.

L: This is the ratio of the Root Mean Squared Errors of this model to Root Mean square Errors obtained from this model, $\sqrt{\gamma_t}$ is in parenthesis. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

\(t\) values are in parentheses.

\(t\) The $P$ value of the Wald statistic.

\(t\) Squared brackets include 95 per cent interval of 1000 bootstrapped regressions.

* Significant at the 5 per cent level.

# Significant at the 10 per cent level.

Table 2: The New Phillips curve
Sample (1992:4 – 2001:2)
Dependent Variable $\pi_t = (\Delta \ln p_t) \times 100^A$

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 17$^B$</th>
<th>Model 18$^B$</th>
<th>Model 19$^B$</th>
<th>Model 20$^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\pi_t]$</td>
<td>1.03 (0.5187)$^B$</td>
<td>0.67 (3.77)$^H$</td>
<td>1.06 (0.1035)$^B$</td>
<td>0.90 (4.56)$^K$</td>
</tr>
<tr>
<td>$[0.88, 0.44]$</td>
<td>[1.10, 0.12]</td>
<td>[1.23, 0.57]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>-</td>
<td>0.37 (2.25)$^H$</td>
<td>-</td>
<td>0.15 (0.85)$^K$</td>
</tr>
<tr>
<td>$[0.60, 0.17]$</td>
<td>[0.47, 0.15]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$mc_{t-1}$</td>
<td>0.01 (0.67)</td>
<td>0.01 (0.75)</td>
<td>-0.005 (-0.27)</td>
<td>-0.001 (0.00)</td>
</tr>
<tr>
<td>$[0.04, -0.005]$</td>
<td>[0.03, -0.006]</td>
<td>[0.01, -0.02]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_oil_{t-1}$</td>
<td>0.51 (3.72)$^I$</td>
<td>0.32 (2.04)$^I$</td>
<td>0.42 (2.77)$^I$</td>
<td>0.38 (2.46)$^I$</td>
</tr>
<tr>
<td>$[0.009, 0.004]$</td>
<td>[0.009, 0.001]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.76</td>
<td>0.77</td>
<td>0.79</td>
<td>0.78</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.29</td>
<td>0.28</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>$Dh / DW$</td>
<td>0.27/1.87</td>
<td>0.68/1.87</td>
<td>1.34/1.79</td>
<td>1.21/1.83</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.16</td>
<td>1.08</td>
<td>1.44</td>
<td>1.35</td>
</tr>
</tbody>
</table>

A: Inflation is tested for unit root using ADF from 1989-2001. The ADF $\phi(\alpha = 0, \rho = 0, \log = 4)$ is 7.5 and the 5 per cent critical value is 4.59. The ADF $\phi(\alpha = 1, \rho = -1)$ is 5.3. Thus, we reject the hypothesis that inflation has a unit root.

B: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.

C: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.

D: The $P$ value of the Wald statistic for testing the hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected.

E: The marginal cost is measured by real unit labour cost (index) deviations from its mean. The hypothesis that it has a unit root can be rejected by the ADF test. The ADF($\phi(\alpha = 0, \rho = 0, \log = 1)$ is 5.10. The 5 per cent critical value is 4.59.

F: AR1 error term.

G: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey-Collier (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.

H: The hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected. The Wald statistical $P$ value is 0.1823.

I: Log Brent crude oil in US dollars.

J: The $P$ value of the Wald statistic for testing the hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected.

K: The hypothesis that $\pi_{t-1}$ is in parenthesis. It cannot be rejected. The Wald statistical $P$ value is 0.1195.

L: This is the ratio of the Root Mean Squared Errors of this model to Root Mean square Errors obtained from this model, $\sqrt{\gamma_t}$ is in parenthesis. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

\(t\) values are in parentheses.

\(t\) Squared brackets include 95 per cent interval of 1000 bootstrapped regressions.

* Significant at the 5 per cent level.

# Significant at the 10 per cent level.
### Table 3a: The \( p' \) Model
Sample (1992:1 – 2001:2)
Dependent Variable \( \pi_t = (\Delta_t \ln p_t) * 100\)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 21</th>
<th>Model 22</th>
<th>Model 23</th>
<th>Model 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{t-1} )</td>
<td>1.01 (0.7987)</td>
<td>0.54 (3.48)</td>
<td>1.00 (0.9140)</td>
<td>0.68 (4.02)</td>
</tr>
<tr>
<td>( \rho_{t-1} )</td>
<td>-</td>
<td>0.47 (3.18)</td>
<td>-</td>
<td>0.33 (2.03)</td>
</tr>
<tr>
<td>( \delta_{oil} )</td>
<td>0.01 (1.46)</td>
<td>0.01 (1.67)</td>
<td>0.01 (1.29)</td>
<td>0.01 (1.47)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.48 (3.79)</td>
<td>0.20 (1.20)</td>
<td>0.48 (3.46)</td>
<td>0.30 (1.95)</td>
</tr>
<tr>
<td>( \overline{R} )</td>
<td>0.74</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.30</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>( D_h / D_W )</td>
<td>0.30</td>
<td>1.90</td>
<td>0.51</td>
<td>1.95</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.33</td>
<td>1.09</td>
<td>1.54</td>
<td>1.13</td>
</tr>
</tbody>
</table>

A: Inflation is tested for unit root using ADF from 1989-2000. The ADF \( \Phi(\alpha = 0, \rho = 0, lag = 4) \) is 7.5 and the 5 per cent critical value is 4.59. The ADF \( \chi^2(p-1) = 5.3. \) Thus, we reject the hypothesis that inflation has a unit root.

B: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.

C: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.

D: The P value of the Wald statistic for testing the hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) is in parenthesis. The hypothesis is not rejected.

E: \( \rho_{t-1}^p \) is \( \bar{w}, \bar{v}^*, \bar{y}^* \), where \( \bar{v}^* \) is the mean of base velocity and \( \bar{y}^* \) is trend equal to 1.6% per annum over the period 1975-2001. The hypothesis that \( \rho_{t-1}^p = \rho \) has a unit root can be rejected by the ADF test. The ADF \( \Phi(\alpha = 0, \rho = 0, lag = 0) = 11.01 \).

F: AR1 error term.

G: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected.

H: The hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) cannot be rejected. The Wald statistic P value is 0.5688.

I: Log Brent crude oil in US dollars.

J: The P value of the Wald statistic for testing the hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) is in parenthesis. The hypothesis is not rejected.

K: The hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) cannot be rejected. The Wald statistic P value is 0.1745.

L: This is the ratio of the Root Mean Squared Errors of this model to Root Mean Squared Errors obtained from this model. \( \sigma = (N_t, \pi_{t-1} = 1) \). The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

\( * \) Significant at the 5 per cent level.

\( \# \) Significant at the 10 per cent level.

### Table 3b: The \( p' \) Model
Sample (1992:1 – 2001:2)
Dependent Variable \( \pi_t = (\Delta_t \ln p_t) * 100\)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 25</th>
<th>Model 26</th>
<th>Model 27</th>
<th>Model 28</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{t-1} )</td>
<td>1.02 (0.6986)</td>
<td>0.50 (3.10)</td>
<td>1.01 (0.8229)</td>
<td>0.65 (3.70)</td>
</tr>
<tr>
<td>( \rho_{t-1} )</td>
<td>-</td>
<td>0.51 (3.34)</td>
<td>-</td>
<td>0.36 (2.15)</td>
</tr>
<tr>
<td>( \delta_{oil} )</td>
<td>0.006 (0.78)</td>
<td>0.006 (1.36)</td>
<td>0.005 (0.71)</td>
<td>0.006 (1.16)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.50 (3.56)</td>
<td>0.20 (1.27)</td>
<td>0.50 (3.61)</td>
<td>0.31 (2.05)</td>
</tr>
<tr>
<td>( \overline{R} )</td>
<td>0.73</td>
<td>0.76</td>
<td>0.75</td>
<td>0.76</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.31</td>
<td>0.29</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>( D_h / D_W )</td>
<td>0.37</td>
<td>1.87</td>
<td>0.46</td>
<td>1.94</td>
</tr>
<tr>
<td>RMSE</td>
<td>1.39</td>
<td>1.08</td>
<td>3.76</td>
<td>1.20</td>
</tr>
</tbody>
</table>

A: Inflation is tested for unit root using ADF from 1989-2001. The ADF \( \Phi(\alpha = 0, \rho = 0, lag = 4) \) is 7.5 and the 5 per cent critical value is 4.59. The ADF \( \chi^2(p-1) = 5.3. \) Thus, we reject the hypothesis that inflation has a unit root.

B: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.

C: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.

D: The P value of the Wald statistic for testing the hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) is in parenthesis. The hypothesis is not rejected.

E: \( \rho_{t-1}^p \) is \( \bar{w}, \bar{v}^*, \bar{y}^* \), where \( \bar{v}^* \) is the mean of base velocity and \( \bar{y}^* \) is trend equal to 1.6% per annum over the period 1975-2001. The hypothesis that \( \rho_{t-1}^p = \rho \) has a unit root can be rejected by the ADF test. The ADF \( \Phi(\alpha = 0, \rho = 0, lag = 0) = 5.29 \).

F: AR1 error term.

G: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected.

H: The hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) cannot be rejected. The Wald statistic P value is 0.65784.

I: Log Brent crude oil in US dollars.

J: The P value of the Wald statistic for testing the hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) is in parenthesis. The hypothesis is not rejected.

K: The hypothesis that \( \varepsilon_t, \pi_{t-1} = 1 \) cannot be rejected. The Wald statistic P value is 0.6581.

L: This is the ratio of the Root Mean Squared Errors of this model to Root Mean Squared Errors obtained from this model. \( \sigma = (N_t, \pi_{t-1} = 1) \). The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

\( * \) Significant at the 5 per cent level.

\( \# \) Significant at the 10 per cent level.
Table 3c: The $p'$ Model

 Sample (1992-2001:2)

Dependent Variable $\pi_t = (\Delta \ln p_t) \ast 100$

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 29$^B$</th>
<th>Model 30$^H$</th>
<th>Model 31$^W$</th>
<th>Model 32$^K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{E}<em>{\pi</em>{t+1}}$</td>
<td>1.0 [0.9953]$^B$</td>
<td>0.58 [0.345]$^H$</td>
<td>0.98 [0.6519]$^W$</td>
<td>0.73 [4.15]$^K$</td>
</tr>
<tr>
<td>$[0.80, 0.95]$</td>
<td>[0.83, 0.32]</td>
<td>[1.04, 0.94]</td>
<td>[1.01, 0.46]</td>
<td></td>
</tr>
</tbody>
</table>

- $\pi_{t-1}$
- $0.44 (2.79)^B$
- $0.27 (1.63)^H$
- $0.01 (0.008)$

- $p_{t-1} - p_{t-1}$
- 0.007 [0.93] |
- 0.003 [0.54] |
- 0.007 [0.92] |
- 0.004 [0.69] |

- $\Delta_oil_{t-1}$
- -
- 0.005 [2.30]
- 0.004 [1.71]

- $\rho^F$
- 0.53 [3.87]
- 0.26 [1.73]
- 0.53 [3.94]
- 0.39 [2.65]

- $\tilde{\sigma}$
- 0.73
- 0.75
- 0.75
- 0.76

- $Dh / DW^G$
- 0.31 [1.88]
- 0.16 [1.92]
- 0.62 [1.83]
- 0.39 [1.89]

- RMSE$^L$
- 1.18
- 1.03
- 2.25
- 1.08

- A Inflation is tested for unit root using ADF from 1989-2000. The ADF $\Phi(\sigma = 0, \rho = 0, \gamma = 4) = 7.5$ and the 5 per cent critical value is 4.59. The ADF $\Phi(\sigma = 0.2, \rho = 0, \gamma = 4) = 5.3$. Thus, we reject the hypothesis that inflation has a unit root.
- B Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.
- C This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.
- D The P values of the Wald statistic for testing the hypothesis that $E_{\pi_{t+1}} = 1$ in parenthesis. The hypothesis cannot be rejected.
- E $p_{t-1} = \gamma(t_{t-1})$, where $\gamma$ is the mean of $ct$ velocity and $\gamma$ is trend equal to 1.6% per annum during the period 1975-2001. The hypothesis that $p_{t-1} - p_{t-1}$ has a unit root is tested for the period 1992-2000 using the ADF test. The ADF $\Phi(\sigma = 0.2, \rho = 0, \gamma = 0, \delta = 0)$ is 5.00. The 5 per cent critical value is 4.59. The hypothesis cannot be rejected. The hypothesis cannot be rejected statistically.
- F AR1 error term.
- G Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey-Colliver (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey) test and could not reject the null. A recursive Chow test shows parameter instability in late 1998.
- H The hypothesis that $E_{\pi_{t+1}} = 1$ cannot be rejected. The Wald statistic P value is 0.9089.
- I Log Brent crude oil in US dollars.
- J The P values of the Wald statistic for testing the hypothesis that $E_{\pi_{t+1}} = 1$ are in parenthesis. The unit root hypothesis is not rejected.
- K The hypothesis that $E_{\pi_{t+1}} = 1$ cannot be rejected. The Wald statistic P value is 0.8232.
- L This is the ratio of the Root Mean Squared Errors of this model to Root Mean squared Errors obtained from this model, $\pi_t = \{E_{\pi_{t+1}} \times \pi_{t+1}\}$. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

Table 4a: The $p'$ Model in terms of real money balances gap ($m^*_t, i^*$)

 Sample (1992-2001:2)

Dependent Variable $\pi_t = (\Delta \ln p_t) \ast 100$

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 33$^B$</th>
<th>Model 34$^H$</th>
<th>Model 35$^W$</th>
<th>Model 36$^K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{E}<em>{\pi</em>{t+1}}$</td>
<td>0.97 [0.713]$^B$</td>
<td>0.53 [0.349]$^H$</td>
<td>0.97 [0.705]$^W$</td>
<td>0.67 [3.94]$^K$</td>
</tr>
<tr>
<td>[1.04, 0.93]</td>
<td>[0.77, 0.30]</td>
<td>[1.04, 0.93]</td>
<td>[0.96, 0.41]</td>
<td></td>
</tr>
</tbody>
</table>

- $\pi_{t-1}$
- 0.46 [1.63]
- 0.31 [1.95]

- $m_{t-1} - m_{t-1}$
- 0.02 [1.68]
- 0.01 [1.65]
- 0.02 [1.36]
- 0.1 [1.38]

- $\Delta_oil_{t-1}$
- -
- 0.005 [2.07]
- 0.003 [1.36]

- $\rho^F$
- 0.51 [3.66]
- 0.20 [1.29]
- 0.51 [3.74]
- 0.32 [2.09]

- $\tilde{\sigma}$
- 0.30
- 0.28
- 0.28
- 0.28

- $Dh / DW^G$
- 0.11 [1.92]
- 0.40 [1.94]
- 0.38 [1.85]
- 0.40 [1.90]

- RMSE$^L$
- 1.55
- 1.47
- 1.42
- 1.65

- A Inflation is tested for unit root using ADF from 1990-2001. The ADF $\Phi(\sigma = 0, \rho = 0, \gamma = 4) = 7.5$ and the 5 per cent critical value is 4.59. The ADF $\Phi(\sigma = 0.2, \rho = 0, \gamma = 4) = 5.3$. Thus, we reject the hypothesis that inflation has a unit root.
- B Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.
- C This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.
- D The P values of the Wald statistic for testing the hypothesis that $E_{\pi_{t+1}} = 1$ in parenthesis. The hypothesis cannot be rejected.
- E $p_{t-1} = \gamma(t_{t-1})$, where $\gamma$ is the mean of $ct$ velocity and $\gamma$ is trend equal to 1.6% per annum during the period 1975-2001. The hypothesis that $p_{t-1} - p_{t-1}$ has a unit root is tested for the period 1992-2000 using the ADF test. The ADF $\Phi(\sigma = 0.2, \rho = 0, \gamma = 0, \delta = 0)$ is 5.00. The 5 per cent critical value is 4.59. The hypothesis cannot be rejected. The hypothesis cannot be rejected statistically.
- F AR1 error term.
- G Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey-Colliver (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey) test and could not reject the null. A recursive Chow test shows parameter instability in late 1998.
- H The hypothesis that $E_{\pi_{t+1}} = 1$ cannot be rejected. The Wald statistic P value is 0.9089.
- I Log Brent crude oil in US dollars.
- J The P values of the Wald statistic for testing the hypothesis that $E_{\pi_{t+1}} = 1$ are in parenthesis. The hypothesis cannot be rejected.
- K The hypothesis that $E_{\pi_{t+1}} = 1$ cannot be rejected. The Wald statistic P value is 0.8232.
Table 4b: The \( p^* \) Model in terms of real money balances gap (\( m_{t}^{\prime}(t^{(1)}) \))
Sample (1992:4 – 2001:2)
Dependent Variable \( \pi_{t} = (\Delta_{1} \ln p_{t})*100^{A} \)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 37B</th>
<th>Model 38B</th>
<th>Model 39B</th>
<th>Model 40B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E, p_{t} )</td>
<td>0.99 {0.9825} &amp; 0.52 (3.35) &amp; 0.99 {0.9258} &amp; 0.66 (3.88)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[1.06, 0.95]</td>
<td>[0.77, 0.27]</td>
<td>[1.05, 0.95]</td>
<td>[0.94, 0.41]</td>
<td></td>
</tr>
<tr>
<td>( \pi_{t-1} )</td>
<td>- &amp; 0.48 (3.27) &amp; - &amp; 0.34 (2.08)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.71, 0.26]</td>
<td>[0.59, 0.38]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m_{t-1} - m_{t-1} ) &amp; 0.02 (1.50) &amp; 0.01 (1.68) &amp; 0.01 (1.12) &amp; 0.01 (1.39)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.03, 0.00]</td>
<td>[0.02, 0.002]</td>
<td>[0.02, 0.002]</td>
<td>[0.02, 0.00]</td>
<td></td>
</tr>
<tr>
<td>( \Delta_{oil_{t-1}} ) &amp; - &amp; 0.005 (2.12) &amp; 0.003 (1.33) &amp; -</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.007, 0.002]</td>
<td>[0.006, -0.00]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho^F )</td>
<td>0.50 (3.60) &amp; 0.18 (1.18) &amp; 0.51 (3.66) &amp; 0.30 (1.96)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.43, -0.07]</td>
<td>[0.07, 0.007]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.74</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>( Dh/DW )</td>
<td>0.18/1.91</td>
<td>0.46/1.94</td>
<td>0.44/1.84</td>
<td>0.45/1.90</td>
</tr>
<tr>
<td>RMSE ( ^{L} )</td>
<td>1.70</td>
<td>1.45</td>
<td>1.66</td>
<td>1.55</td>
</tr>
</tbody>
</table>

- **A**: Inflation is tested for unit root using ADF from 1989-2001. The ADF (\( \phi = 0, \rho = 0, \lambda = 4 \)) is 7.5 and the 5 per cent critical value is 4.5. The ADF \( r(p) = 1 \) = -5.3. Thus, we reject the hypothesis that inflation has a unit root.
- **B**: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.
- **C**: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.
- **D**: The Wald statistic for testing the hypothesis that \( \pi_{t-1} = 0 \) cannot be rejected (borderline).
- **E**: \( p_{t} \) is written in terms of real money balances gap, \( m_{t-1} - m_{t-1} \). The variable \( m_{t} \) is real money balance defined as the money base / GDP deflator. The equilibrium \( \pi_{t} \) is the long-run linear combination of \( \Delta_{oil_{t-1}} \) and \( \Delta_{oil_{t-1}} \).
- **F**: AR1 error term.
- **G**: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey (1977). The hypothesis that they are homoscedastic could not be rejected using Harvey-Loretan and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.
- **H**: The hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) is rejected at the 5 percent level. The Wald statistic \( P \) value is 0.003.
- **I**: The P value of the Wald statistic for testing the hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) is in parenthesis. The hypothesis cannot be rejected.
- **J**: The hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) is rejected. The Wald statistic \( P \) value is 0.003.
- **K**: The hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) is rejected. The Wald statistic \( P \) value is 0.003.

---

Table 4c: The \( p^* \) Model in terms of Real Money Balances gap (\( m_{t}^{\prime}(t^{(1)}) \))
Sample (1992:4 – 2001:2)
Dependent Variable \( \pi_{t} = (\Delta_{1} \ln p_{t})*100^{A} \)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 41B</th>
<th>Model 42B</th>
<th>Model 43B</th>
<th>Model 44B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E, p_{t} )</td>
<td>0.75 {0.0575} &amp; 0.50 (2.84) &amp; 0.77 {0.0678} &amp; 0.66 (3.59)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.89, 0.62]</td>
<td>[0.78, 0.21]</td>
<td>[0.88, 0.66]</td>
<td>[0.95, 0.38]</td>
<td></td>
</tr>
<tr>
<td>( \pi_{t-1} )</td>
<td>- &amp; 0.39 (2.47) &amp; - &amp; 0.18 (1.09)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.63, 0.16]</td>
<td>[0.43, -0.07]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m_{t-1} - m_{t-1} ) &amp; 0.03 (2.31) &amp; 0.01 (1.46) &amp; 0.03 (2.10) &amp; 0.02 (1.63)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.05, 0.02]</td>
<td>[0.04, 0.01]</td>
<td>[0.04, 0.01]</td>
<td>[0.03, 0.006]</td>
<td></td>
</tr>
<tr>
<td>( \Delta_{oil_{t-1}} ) &amp; - &amp; 0.005 (2.12) &amp; 0.004 (1.78) &amp; -</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.007, 0.007]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho^F )</td>
<td>0.60 (4.53) &amp; 0.30 (1.95) &amp; 0.59 (4.58) &amp; 0.46 (3.25)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.43, -0.07]</td>
<td>[0.0007, 0.0037]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.75</td>
<td>0.76</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>( Dh/DW )</td>
<td>-0.10/1.98</td>
<td>0.05/1.95</td>
<td>0.18/1.89</td>
<td>0.18/1.91</td>
</tr>
<tr>
<td>RMSE ( ^{E} )</td>
<td>1.11</td>
<td>1.01</td>
<td>1.93</td>
<td>1.05</td>
</tr>
</tbody>
</table>

- **A**: Inflation is tested for unit root using ADF from 1989-2000. The ADF (\( \phi = 0, \rho = 0, \lambda = 4 \)) is 7.5 and the 5 per cent critical value is 4.5. The ADF \( r(p) = 1 \) = -5.3. Thus, we reject the hypothesis that inflation has a unit root.
- **B**: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.
- **C**: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.
- **D**: The Wald statistic for testing the hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) are in parenthesis. The hypothesis cannot be rejected (borderline).
- **E**: \( p_{t} \) is written in terms of real money balances gap, \( m_{t-1} - m_{t-1} \). The variable \( m_{t} \) is real money balance defined as the money base / GDP deflator. The equivalent \( \pi_{t} \) is the long-run linear combination of \( \Delta_{oil_{t-1}} \) and \( \Delta_{oil_{t-1}} \).
- **F**: AR1 error term.
- **G**: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected using Harvey (1977). Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.
- **H**: The hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) cannot be rejected at the 5 percent level. The Wald statistic \( P \) value is 0.3082.
- **I**: Log Brent crude oil in US dollars.
- **J**: The Wald statistic for testing the hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) is in parenthesis. The hypothesis cannot be rejected.
- **K**: The hypothesis that \( \pi_{t-1} = \pi_{t-1} = 1 \) cannot be rejected. The Wald statistic \( P \) value is 0.03.
- **L**: This is the ratio of the Root Mean Squared Errors of this model to Root Mean squared Errors obtained from this model, \( \pi_{t-1} = \pi_{t-1} = 1 \). The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.
- **M**: 0.1 values are in parenthesis.
- **N**: 0.1 values are in parenthesis.
Table 4d: The p’ Model in terms of Real Money Balances Gap

\[
(m_t^m, t^m_t - t^m_n)
\]
Sample (1992:4 – 2001:2)
Dependent Variable \( \pi_t = (\Delta \ln p_t) \times 100^A \)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 45^B</th>
<th>Model 46^B</th>
<th>Model 47^B</th>
<th>Model 48^B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_\pi )</td>
<td>1.00</td>
<td>0.89</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>( [0.88] )</td>
<td>0.80</td>
<td>0.78</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>( [0.97, 0.97] )</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>( \pi_{t-1} )</td>
<td>0.045</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>( [0.23] )</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>( t )</td>
<td>0.02 (1.80)</td>
<td>0.02 (1.40)</td>
<td>0.02 (1.40)</td>
<td>0.02 (1.40)</td>
</tr>
<tr>
<td>( [0.61, 0.41] )</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>( [0.03] )</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>( \Delta_oil_{t-0} )</td>
<td>0.005 (1.99)</td>
<td>0.003 (1.27)</td>
<td>0.003 (1.27)</td>
<td>0.003 (1.27)</td>
</tr>
<tr>
<td>( [0.007, 0.001] )</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>( \rho^F )</td>
<td>0.49 (3.52)</td>
<td>0.50 (3.63)</td>
<td>0.50 (3.63)</td>
<td>0.50 (3.63)</td>
</tr>
<tr>
<td>( [0.21] )</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.74</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>( Dfh/DWG^G )</td>
<td>0.17/1.92</td>
<td>0.42/1.94</td>
<td>0.42/1.94</td>
<td>0.42/1.94</td>
</tr>
<tr>
<td>RMSE^L</td>
<td>1.64</td>
<td>1.29</td>
<td>1.48</td>
<td>1.48</td>
</tr>
</tbody>
</table>

\* A Inflation is treated for unit root using ADF from 1989-2001. The ADF \( \Phi(\alpha = 0, \rho = 0, lag = 4) \) in 7.5 and the 5 per cent critical value is 4.59. The ADF \( t(\rho = 1) \) is -5.3. Thus, we reject the hypothesis that inflation has a unit root.

- B Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out.

- C This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.

- D The \( \rho \) values of the Wald statistic for testing the hypothesis that \( E_\sigma \pi_{t-1} \) are in parenthesis. The hypothesis cannot be rejected.

- E \( \rho^F \) is written in terms of real money balances gap, \( m_{t-1} - m_{t-1}^m \). The variable \( m_t \) is real money balance defined as the money base / GDP deflator. The equilibrium \( m_t^m \) is the long-run linear combination \( -0.77 - 0.016(\pi_{t-1} - \pi_{t-1}^m) + 0.93 \), where \( \pi_{t-1}^m \) is the US 90-day interest rate measured by the US bankers’ acceptance rate. The coefficients are estimated using the Phillips-Loretan Nonlinear Two-Sided Dynamic Least Squares (lags and leads regressions). The interest rate is the 10-year government bond rate and income is real production GDP.

- F AR1 error term.

- G Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homogeneous could not be rejected using Harvey-Collier (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.

- H The hypothesis that \( E_\sigma \pi_{t-1} \pi_{t-1}^m \) cannot be rejected at the 5 per cent level. The Wald statistic \( P \) value is 0.5100.

- I Log Brent crude oil in US dollars.

- J The \( \rho^F \) values of the Wald statistic for testing the hypothesis that \( E_\sigma \pi_{t-1}^m \) are in parenthesis. The hypothesis is not rejected.

- K The hypothesis that \( E_\sigma \pi_{t-1} \pi_{t-1}^m \) = 0 cannot be rejected. The Wald statistic \( P \) value is 0.5011.

- L This is the ratio of the Root Mean Squared Errors of this model to Root Mean Squared errors obtained from this model, \( \pi_t = (\Delta \ln p_t) \times 100^A \). The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.

- (i) Results are in parentheses.

- () \( \Delta \) Significant at the 5 per cent level.

* Significant at the 10 per cent level.
Table 5: The Change in Interest Rates Model  
Sample (1992:2 – 2001:2)  
Dependent Variable $r_t = (\Delta \ln p_t) \times 100$  

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 53</th>
<th>Model 54</th>
<th>Model 55</th>
<th>Model 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\pi_{it}}$</td>
<td>1.04 (0.3367)</td>
<td>1.03 (0.3707)</td>
<td>0.54 (0.52)</td>
<td></td>
</tr>
<tr>
<td>$\pi_{it}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\Delta i_{it}$</td>
<td>0.09 (1.56)</td>
<td>0.15 (2.93)</td>
<td>0.12 (2.17)</td>
<td>0.16 (3.18)</td>
</tr>
<tr>
<td>$\Delta oil_{it}$</td>
<td>0.45 (3.16)</td>
<td>NA</td>
<td>0.006 (2.73)</td>
<td>0.003 (1.43)</td>
</tr>
<tr>
<td>$\rho^F_t$</td>
<td>0.74</td>
<td>0.78</td>
<td>0.77</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.30</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>$Dh_t / DW_t$</td>
<td>0.08/1.89</td>
<td>0.26/1.91</td>
<td>0.39/1.86</td>
<td>0.36/1.80</td>
</tr>
<tr>
<td>RMSEFL</td>
<td>1.14</td>
<td>0.94</td>
<td>1.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>

A: Inflation is tested for unit root using ADF from 1989-2001. The ADF $\Phi(\alpha = 0, \rho = 0, lag = 4)$ is 7.5 and the 5 per cent critical value is 4.59. The ADF $\Phi(\alpha = 1, \rho = 0, lag = 2)$ is 6.72 and the 5 per cent critical value is 4.59.  
B: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out. Model 54 and model 56 were estimated by OLS.  
C: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.  
D: The $P$ values of the Wald statistic for testing the hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ are in parenthesis. The hypothesis is not rejected.  
E: The explanatory variable is $\Delta i_{it}$. The hypothesis that $\Delta i_{it}$ has a unit root can be rejected by the ADF test (1989-2001). The ADF $\Phi(0, \rho = 0, lag = 2)$ is 6.66 and the 5 per cent critical value is 6.25.  
F: AR1 error term.  
G: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected by using Harvey-Collier (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.  
H: The hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ cannot be rejected. The Wald statistic $P$ value is 0.1287.  
I: Log Brent crude oil in US dollars.  
J: The $P$ value of the Wald statistic for testing the hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ is in parenthesis. It cannot be rejected.  
K: The hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ cannot be rejected. The Wald statistic $P$ value is 0.0069.  
L: This is the ratio of the Root Mean Squared Errors of this model to Root Mean squared Errors obtained from this model, $\pi_{it} = (\Delta \ln p_{it} - \pi_{it} - 1)$. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample.  

Table 6: Real interest rate deviations from the natural rate  
Sample (1992:1-2001:2)  
Dependent Variable $\pi_t = (\Delta \ln p_t) \times 100$  

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 57</th>
<th>Model 58</th>
<th>Model 59</th>
<th>Model 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\pi_{it}}$</td>
<td>1.01 (0.7901)</td>
<td>0.34 (2.32)</td>
<td>0.99 (0.7795)</td>
<td>0.44 (2.85)</td>
</tr>
<tr>
<td>$\pi_{it}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(r - \overline{r})$</td>
<td>-0.08 (-1.62)</td>
<td>-0.09 (-2.84)</td>
<td>-0.13 (-2.58)</td>
<td>-0.11 (-3.26)</td>
</tr>
<tr>
<td>$\Delta oil_{it}$</td>
<td>0.02 (3.10)</td>
<td>0.007 (3.10)</td>
<td>0.003 (1.70)</td>
<td>NA</td>
</tr>
<tr>
<td>$\rho^F_t$</td>
<td>0.74</td>
<td>0.78</td>
<td>0.78</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.30</td>
<td>0.28</td>
<td>0.28</td>
<td>0.27</td>
</tr>
<tr>
<td>$Dh_t / DW_t$</td>
<td>0.06/1.91</td>
<td>0.45/1.88</td>
<td>0.53/1.81</td>
<td>0.52/1.72</td>
</tr>
<tr>
<td>RMSEFL</td>
<td>1.14</td>
<td>0.89</td>
<td>1.88</td>
<td>0.91</td>
</tr>
</tbody>
</table>

A: Inflation is tested for unit root using ADF from 1989-2001. The ADF $\Phi(\alpha = 0, \rho = 0, lag = 4)$ is 7.5 and the 5 per cent critical value is 4.59. The ADF $\Phi(\alpha = 1, \rho = 0, lag = 2)$ is 6.72 and the 5 per cent critical value is 4.59.  
B: Model estimated by ML and corrected for serial correlation using Pagan (1974). Six lags of each of the explanatory variables (other than expected inflation) and a constant are fit, but the insignificant lags and the constant are dropped out. Models 58 and 59 are estimated by OLS.  
C: This is measured by RBNZ survey data of one-year ahead (4 quarters) expected inflation.  
D: The $P$ values of the Wald statistic for testing the hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ is in parenthesis. It cannot be rejected.  
E: The real interest rate is the 90-day interest rate minus $E_{\pi_{it}}$, where the latter is RBNZ survey data of one-year ahead (4 quarters) expected inflation. The natural interest rate $\pi_t$ is the average from 1989-2001.  
F: AR1 error term.  
G: Also, the residuals are tested for whiteness using the Bartlett’s Kolmogorov-Smirnov test statistic and whiteness could not be rejected. The hypothesis that they are homoscedastic could not be rejected by using Harvey-Collier (1977), Harvey-Phillips (1974) and ARCH tests. I also tested for specification using a RESET (Ramsey test) and could not reject the null. A recursive Chow test shows parameter instability in late 1998.  
H: The hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ cannot be rejected. The Wald statistic $P$ value is 0.8577.  
I: Log Brent crude oil in US dollars.  
J: The $P$ value of the Wald statistic for testing the hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ is in parenthesis. It cannot be rejected.  
K: The hypothesis that $E_{\pi_{it}}(\pi_{it} - 1)$ cannot be rejected. The Wald statistic $P$ value is 0.9406.  
L: This is the ratio of the Root Mean Squared Errors of this model to Root Mean squared Errors obtained from this model, $\pi_{it} = (\Delta \ln p_{it} - \pi_{it} - 1)$. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved for out-of-sample forecasting. The forecast is from March 1999 to June 2001. For model 58, the forecast is extended to September 2001 (a genuine out-of-sample forecast).  

(i) $P$ values are in parentheses.  
[] Squared brackets include 95 per cent interval of 1000 bootstrapped regressions.  
[1] Significant at the 5 per cent level.  
[2] Significant at the 10 per cent level.
### Table 7: The Quantity Theory of Money and AR Models

**Dependent Variable**: \( \pi_t = (\Delta_t \ln(p_t))^*100 \)\(^A\)

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 61(^B)</th>
<th>Model 62(^B)</th>
<th>Model 63(^B)</th>
<th>Model 64(^B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{t-1} \pi_{t-1} )</td>
<td>0.80 ( {0.0019} )</td>
<td>0.44 (2.63) (^*)</td>
<td>-</td>
<td>0.47 (3.10)</td>
</tr>
<tr>
<td>( \pi_{t-1} )</td>
<td>-</td>
<td>0.40 (2.46) (^*)</td>
<td>1 (restricted)</td>
<td>0.55 (3.77)</td>
</tr>
<tr>
<td>( \Delta_t \pi_{t-1} )</td>
<td>0.05 (2.65) (^*)</td>
<td>0.04 (2.55) (^*)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Delta_t y_{t-1} )</td>
<td>0.05 (2.08) (^*)</td>
<td>0.04 (1.83) (^*)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Delta_t O_t )</td>
<td>0.005 (2.13) (^*)</td>
<td>0.005 (2.36) (^*)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.51 (3.17) (^*)</td>
<td>0.33 (1.89) (^*)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.63</td>
<td>0.67</td>
<td>0.68</td>
<td>-</td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>0.19</td>
<td>0.18</td>
<td>0.34</td>
<td>0.31</td>
</tr>
<tr>
<td>( Dh / DW )</td>
<td>1.33/1.63</td>
<td>0.78/1.73</td>
<td>/1.69</td>
<td>/1.55</td>
</tr>
<tr>
<td>RMSE (^L)</td>
<td>0.93</td>
<td>0.82</td>
<td>1.13</td>
<td>1.00</td>
</tr>
</tbody>
</table>

---

- **A**: Inflation is tested for unit root using ADF from 1989-2000. The ADF \( \Phi(\alpha = 0, \rho = 0, l_{AG} = 4) \) is 7.5 and the 5 per cent critical value is 4.95. Thus, we reject the hypothesis that inflation has a unit root.
- **B**: The model is estimated by ML with autocorrelation correction (Pagan, 1974). Six lags of money growth rates, and a constant are fitted and the insignificant lags and the constant are dropped out. Output growth is also found to be insignificant.
- **C**: The RBNZ Survey of one-year ahead inflation expectations.
- **D**: The hypothesis that \( E_{t-1} \pi_{t-2} = 1 \) is rejected. The Wald statistic \( 12^{*}x \) value is 0.0059.
- **E**: \( \pi_t \) is the money base.
- **F**: \( y \) is the log of real GDP.
- **G**: \( \rho \) is the log Brent crude oil in US dollars.
- **H**: AR1 error term.
- **I**: The hypothesis that \( E_{t-1} \pi_{t-3} = 1 \) is rejected. The Wald statistic \( 12^{*}x \) value is 0.0256.
- **J**: The Random Walk model estimated by OLS. Note that the sample is longer than the monetary models.
- **K**: Model estimated by OLS and the restriction \( E_{t-1} \pi_{t-3} = 1 \) is tested. The Wald statistic \( 12^{*}x \) value is 0.1061.
- **L**: This is the ratio of the Root Mean Squared Errors of this model to Root Mean squared Errors obtained from model 68 where the restriction \( E_{t-1} \pi_{t-3} = 1 \) is tested and imposed. The model is estimated to December 1998. Ten observations from March 1999 to June 2001 are saved then used for forecasting out-of-sample. The forecast is from March 1999 to June 2001 except for model 66, where the forecasts is extended to September 2001.
- **J**: \( r \) values are in parentheses.
- **P**: The \( P \) value of the Wald statistic.
- **[]**: Squared brackets include 95 per cent interval of 1000 bootstrapped regressions.
- *****: Significant at the 5 per cent level.
- **#**: Significant at the 10 per cent level.

\(^{*}\): Significant at the 5 per cent level.

### Table 8: The New Keynesian Model (model 65)

**Sample**: (1992:1 – 2001:2)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>( t )</th>
<th>( P ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{11} )</td>
<td>-0.26</td>
<td>-3.28</td>
<td>0.0010</td>
</tr>
<tr>
<td>( \hat{\rho} )</td>
<td>6.1 [0.54]</td>
<td>11.31</td>
<td>0.0000</td>
</tr>
<tr>
<td>( a_{21} )</td>
<td>0.73</td>
<td>7.78</td>
<td>0.0000</td>
</tr>
<tr>
<td>( a_{22} )</td>
<td>0.50</td>
<td>2.83</td>
<td>0.0047</td>
</tr>
<tr>
<td>( a_{23} )</td>
<td>0.52</td>
<td>2.75</td>
<td>0.0059</td>
</tr>
<tr>
<td>( a_{32} )</td>
<td>0.05</td>
<td>1.03</td>
<td>0.3000</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.27</td>
<td>3.15</td>
<td>0.0016</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>0.42</td>
<td>2.05</td>
<td>0.0400</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>5.04 [0.56]</td>
<td>8.88</td>
<td>0.0000</td>
</tr>
<tr>
<td>Log L</td>
<td>-89.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.98</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis that \( a_{11} + a_{22} = 1 \) cannot be rejected.

The Wald statistic’s \( P \) value is 0.2456.

The null hypothesis that \( \hat{\rho} = \hat{\rho} \) cannot be rejected.

The Wald statistic’s \( P \) value is 0.1689.

RMSE is the ratio of the root mean squared errors of the forecasts to RMSE obtained from this model \( \pi_t = f(E_t \pi_{t-1} + \pi_{t-1}) \).

The forecast is from March 1999 to September 2001.

The September’s forecast is a genuinely out-of-sample forecast.
Table 9: The Lucas Model (model 66)
Sample (1992:1 – 2001:2)

\[ \pi_t = (\ln P_t - \ln P_{t-1}) \times 400 \]
\[ y_t - y_t' = -\alpha_{10} \Delta x_t + \alpha_{11} \Delta x_{t-1} + \alpha_{12} (y_{t-1} - y_{t-1}') \]
\[ \pi_t = -\alpha_{20} + (1 - \alpha_{11}) \Delta x_t + \alpha_{11} \Delta x_{t-1} - \alpha_{12} \Delta (y_{t-1} - y_{t-1}') \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{10} )</td>
<td>-4.46</td>
<td>-6.95</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \alpha_{11} )</td>
<td>0.21</td>
<td>2.97</td>
<td>0.0029</td>
</tr>
<tr>
<td>( \alpha_{12} )</td>
<td>0.96</td>
<td>18.0</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \alpha_{20} )</td>
<td>-2.79</td>
<td>-3.02</td>
<td>0.0025</td>
</tr>
<tr>
<td>Log L</td>
<td>-102.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>1.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

RMSE is the ratio root mean squared errors of the forecasts of inflation to the RMSE of the base model.
The forecast is from March 1999 to June 2001.

Table 10: The Mankiw-Reis Model (model 67)

\[ \pi_t = \Delta \ln P_t \times 100 \]
\[ \pi_t = (\alpha \lambda)/(1 - \lambda) y_{t-1} + \lambda (E_{\pi_{t+1}} + \alpha \pi_{t+1} y_{t+1}) + \lambda (1 - \lambda) (E_{\pi_{t+2}} + \alpha \pi_{t+2} y_{t+2}) + \varepsilon_t \]
\[ \varepsilon_t = \rho_t \varepsilon_{t-1} + \rho_x \pi_{t-2} + \eta_t \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.12</td>
<td>3.2</td>
<td>0.0041</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.33</td>
<td>19.4</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>2.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

RMSE is the ratio of the root mean squared errors of the forecasts to RMSE obtained from this model \( \pi = f(E_{\pi_{t+1}} + \pi_{t+1}) \).
The model is estimated up to December 1998 and the observations from March 1999 to June 2001 are saved for the out-of-sample forecasts. The forecast is from March 1999 to December 2001.
Appendix 1: Deriving a New Keynesian Phillips Curve - NAIRU

Assume a wage equation like:

1. \[ W_t = \alpha_p E_t P_{t+1} + \alpha_r U_t + \alpha_z Z_t + \varepsilon_{w_t} \]
   Where \( W_t \) is the log of the wage rate, \( E_t P_{t+1} \) is the expected future price level, \( \varepsilon_{w_t} \) is the log of unemployment and \( Z_t \) is a vector of other explanatory variables. The shock term \( \varepsilon_{w_t} \) is \( N_i.d(0, \sigma^2) \).
   and a Price equation like:

2. \[ P_t = \beta_v W_t - \beta_p P_t + (1 - \beta_s)P_{m_t} + \beta_z X_t + \varepsilon_{p_t} \]
   where \( P_t \) is the log of the price level, \( \beta_v \) is log productivity, \( P_{m_t} \) is the log of import price, \( X_t \) is a vector of additional explanatory variables such as capacity utilisation, and \( \varepsilon_{p_t} \) is \( N_i.d(0, \tau^2) \). The usual adding up restriction is imposed on equation (2).

Substitute equation (1) in equation (2) and eliminate \( W_t \). We arrive at:

3. \[ P_t = \beta_w \alpha_p E_t P_{t+1} + \beta_w \alpha_r U_t + \beta_w \alpha_z Z_t - \beta_w P_t + (1 - \beta_s)P_{m_t} + \beta_w X_t + \beta_w \varepsilon_{w_t} + \varepsilon_{p_t} \]

Let \( X_t \), capacity utilisation, be a linear function of unemployment \( X_t = \gamma U_t \).

Add and subtract \((1 - \beta_s)E_t P_{t+1}\) from RHS of equation (3).

4. \[ P_t = \beta_w \alpha_p E_t P_{t+1} + (\beta_w \alpha_r + \beta_w \gamma) U_t + \beta_w \alpha_z Z_t - \beta_w P_t + (1 - \beta_s)P_{m_t} + \beta_w \varepsilon_{w_t} + \varepsilon_{p_t} \]

Subtract \( P_{t+1} \) from both sides of equation (4), call \( \Delta P_t \) inflation, \( \pi_t \)

5. \[ \pi_t = \beta_w \alpha_p E_t P_{t+1} + (\beta_w \alpha_r + \beta_w \gamma) U_t + \beta_w \alpha_z Z_t - \beta_w P_t + (1 - \beta_s)(P_{m_t} - E_t P_{t+1}) \]

and assume \( E_t P_{t+1} = \pi_t = 0 \) in equilibrium. The NAIRU is given by:

6. \[ U_t^* = -\beta_w \alpha_r - \beta_w \gamma \left[ \beta_w \alpha_z Z_t - \beta_w P_t + (1 - \beta_s)(P_{m_t} - E_t P_{t+1}) \right] \]

The short-run Phillips curve is given by:

\[ \pi_t = \lambda E_t \pi_t + \theta (U_t - U_t^*) + \xi_t \]

The coefficients \( \lambda \) and \( \theta \) are functions of \( \alpha_p, \alpha_r, \alpha_z, \beta_s, \beta_v \), and \( \gamma \). The error term \( \xi_t \) is a combination of the error terms \( \varepsilon_{w_t} \) and \( \varepsilon_{p_t} \).