Is the Taylor rule really different from the McCallum rule?

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Abstract

When base velocity is a stable function of the Federal funds rate (FFR), the money base-nominal GDP targeting rule (McCallum rule) can be re-parameterised and presented in terms of FFR as the policy instrument. Comparison of this McCallum modified policy rule with the popular Taylor rule suggests that these two rules and the FFR are actually cointegrated. Model-based evaluations of the two rules’ stabilisation properties indicate that the modified McCallum rule is similar to the Taylor rule. The key to this result is the degree of interest rate smoothing applied to the policy rules.

1 Introduction

The Taylor rule (Taylor, 1993) has become very popular in recent years. The rule represents an automatic response of the Federal funds rate (FFR) to any deviation of the inflation rate from a desired target value and to the output gap (deviation of real GDP from its potential). In essence, the rule involves changes of the ex-ante real interest rate, relative to its equilibrium value.

The money base-nominal GDP targeting rule (McCallum, 1988) is also an adaptive policy formula, but with a different policy instrument and a different underlying theory of transmission mechanism. The policy instrument is the money base instead of the FFR. With this rule, the money-base growth rate changes in response to deviation of the nominal GDP growth rate (or the level) from a desired target value that grows at a specified rate. The rule also allows for gradual changes in base velocity. Nominal GDP targeting and money base targeting are equivalent when changes in base velocity are not large.

Meltzer (1987), Gordon (1985), Hall and Mankiw (1994) and Feldstein and Stock (1994) recommended a nominal GDP targeting rule for monetary policy. However, many researchers suggest that the McCallum money-base targeting rule has undesirable stabilisation properties. Goodhart (1994) and Blinder (1994) provide arguments against the money base rule. Recently, Orphanides (1999) and McCallum (2000) suggested a nominal GDP rule (without money base) similar to the Taylor rule in that the policy instrument is the FFR, but instead of the output gap it has nominal GDP, inflation and an equilibrium real interest rate on the RHS. Rudebusch (2001a) argues that the stabilisation property of this rule is also poor. However, the Federal Reserve Board own research (Orphanides, Porter, Reifschneider, Tetlow and Finan, 2000) demonstrates that this type of rules dominates the Taylor rule when it is difficult to accurately assess the state of the economy in real time. By contrast, the Taylor rule appears to dominate if one assumes artificially low degrees of uncertainty. McCallum (2001) studies the same problem and reaches the same conclusion.
To test whether the Taylor rule is really different from the McCallum rule, this paper modifies the McCallum rule. I replace the money base as the policy instrument with the FFR by appealing to empirical evidence that base velocity is a stable function of the FFR. Then I compare this rule, which is no longer a money base rule, to the Taylor rule using quarterly data for the United States. I find that the modified McCallum rule, the Taylor rule and the FFR are cointegrated. Then I test its model-based stabilisation properties during the period 1980-2000 using the New Keynesian model in Rudebusch (2001a) and Dennis (2001).3

Provided that the two policy rules have similar degrees of “sufficiently high” interest rate smoothing, I find that the modified McCallum rule behaves just as well as the Taylor rule. I test for the equality of the unconditional variances of inflation, the output gap and the change in the interest rate across three variants of the New Keynesian model that differ in the degree of forwardness of expectations. Equality of unconditional variances cannot be rejected.

In summary, this paper offers two propositions. Let the Taylor rule be given by

$$i_t = \alpha f(x, \theta) + (1 - \alpha) \bar{y},$$

where $i_t$ is the interest rate, $f(x, \theta)$ is the rule’s argument ($x$ is a variable and $\theta$ is a parameter), and $0 < \alpha < 1$ is a smoothing parameter. And let another rule (eg, the McCallum rule) be given by

$$i_t = \alpha_2 g(y, \omega) + (1 - \alpha_2) \bar{y},$$

where $i_t$ is the interest rate, $g(y, \omega)$ is the rule’s argument ($y$ is a variable and $\omega$ is a parameter) and $0 < \alpha_2 < 1$ is a smoothing parameter, then it is trivial to show that given $f(x, \theta) \neq g(y, \omega)$, $i_t$ will be equal to $\bar{y}$ when $\alpha = \alpha_2 \rightarrow 0$. The second proposition is that if $g(y, \omega)$ is more variable than $f(x, \theta)$ then one can make $i_t \approx \bar{y}$ if both $\alpha$ and $\alpha_2 \rightarrow 0$, and $\alpha_2 < \alpha$, which means more interest rate smoothing.

The Taylor rule is briefly discussed in section 2. In section 3, the modified money-base rule is derived and the two rules are compared. Model-based evaluations of the rules are found in section 4. Conclusions are in section 5.

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3 The data source in this paper is the Federal Reserve Bank of St Louis Website.

## 2 Taylor rule

The Taylor rule is typically given by the following formula, though it could be written, and its variables may be defined, in many different ways.

$$i_t = \pi + \Delta \pi + \lambda_{\omega}(\Delta \pi - \Delta \bar{\pi}) + \lambda_{\bar{y}}(\bar{y})$$  (1)

The policy instrument in the Taylor rule is the FFR thus $i_t$ is the FFR implied by the rule (in per cent). The first variable on the RHS of the Taylor rule is the average real interest rate or the equilibrium real interest rate $\pi$. In annual terms, Taylor chooses 2 per cent.4 The second term in the Taylor rule, $\Delta \pi$, is either the contemporaneous inflation rate defined as $(\pi_t - \pi_{t-1}) \times 400$, where $\pi_t$ is a measure of the price level or its lagged value or the average inflation rate $\Delta \bar{\pi}$, for example, over the past four quarters $(\frac{1}{4} \sum_{t=1}^{4} \Delta \pi_{t-1})$. The third term in brackets is the deviation of the inflation rate from a specified target. The target value of the inflation rate $\Delta \pi$ is set to 2 per cent in Taylor’s original paper. The last term in brackets is the output gap defined as the deviation of real GDP from trend, which could be obtained by the HP filter or any other measure.5 Taylor also sets the coefficients $\lambda_{\omega}$ and $\lambda_{\bar{y}}$ equal to 0.5.

Figure 1 plots the Taylor rules and the actual FFR from 1960 to 2000. To compute equation (1), I use average CPI inflation over four quarters. The output gap is measured as the deviation of real GDP from potential output, which is measured using the

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4 The equilibrium real interest rate is unobservable. It is not clear whether, or not, the equilibrium real interest rate should be constant over time. Laubach and Williams (2001) argue that it will.

5 It could be measured in many different ways and it is a proxy for excess demand in the economy.

6 Many have estimated different values directly from the data.
Congressional Budget Office estimate of potential output, $y_t^*$, and $\pi$ and $\Delta \pi$ are set equal to 2. Both $\lambda_{\pi}$ and $\lambda_\gamma$ are set equal to 0.5.

The Taylor rule has the disadvantage that the real interest rate and the output gap are both unobservable. Orphanides (2000, 2001) show that under-predicting and over-predicting the output gap can lead to some serious policy errors. The output gap is also subject to data uncertainty, i.e., whether it is measured using real time data or data ex post.\footnote{Real time data is beyond the scope of this paper.}

The Taylor rule can also be written in the following form:

$$\Delta i_t = \alpha (i_t^* - i_{t-1})$$  
(2)

Substituting for $i_t^*$ from (1) into (2) gives:

$$\Delta i_t = \alpha \left( (\bar{r} + \Delta P_t^m + 0.5(\Delta P_t^m - \Delta \pi) + 0.5 \bar{y}_t - i_{t-1}\right)$$  
(3)

And in levels:

$$i_t = \alpha \left( \bar{r} + \Delta P_t^m + \lambda_{\pi} (\Delta P_t^m - \Delta \pi) + \lambda_\gamma \bar{y}_t + (1 - \alpha) i_{t-1}\right)$$  
(4)

This kind of formulation implies that the Fed moves its instrument (short-term interest rate) closer to the desired interest rate gradually. Economists call this “interest rate smoothing” or “policy inertia.” Rudebusch (2001b) provides a good discussion about these two descriptions, which will become a key issue in this paper.

There are many estimates of $\alpha$ in the literature, but two of them are well known. Judd and Rudebusch (1998) estimated the smoothing parameter $\alpha$ to be 0.27 using a single-equation estimation technique, and Clarida et al. (2000) estimated it to be 0.21 using GMM. Both studies use quarterly data for the sample that covers the Volcker-Greenspan chairmanship periods. The Taylor rule in equation (4) can be computed using the average of those two estimates, 0.24. Figure 2 plots equation (4).

Note that interest rate smoothing makes the interest rate path implied by the rule much more similar to the actual FFR. Compare figure 2 with figure 1.

3 Money base-nominal GDP targeting rule

Now consider the money base-nominal GDP targeting rule.

$$\Delta h_t = \Delta \pi - \Delta y_t^* + \lambda_{\pi} (\Delta \pi - \Delta x_t)$$  
(5)

The policy instrument is the growth rate of the money base $(\ln h_t - \ln h_{t-1}) * 400$. The first term on the RHS of equation (5), $\Delta \pi$, is the target value of the growth rate of nominal GDP – a constant, which McCallum assumes to be 4.5 per cent. The second term is average base velocity (McCallum uses a four year period) where base velocity is defined as the ratio of nominal GDP to the money base. The term $\Delta \pi - \Delta x_t$ is the deviation of nominal GDP growth rate from its (constant) targeted value. McCallum assumes that $\lambda_{\pi}$ is 0.5.
A major advantage this rule has over the Taylor rule is that it does not include unobservable variables such as the real interest rate and the output gap. It seems that the major disadvantage of this rule is that the instrument is the money base and not the FFR.\(^8\)

It is hard to compare the two rules in equations (1) and (5) directly because the monetary policy instruments are different. McCallum and Nelson (1999), Orphanides (1999) and McCallum (2000) suggest a nominal GDP rule such as 

\[ i_t = r + \Delta P_t + \lambda (\Delta x_t - \Delta \tau_t), \]

where the rule sets the short-term interest rate equal to the equilibrium real interest rate plus average inflation plus some fraction \(\lambda\) of the deviation of nominal GDP growth over four quarters from its target. This rule is not derived from equation (5) directly. Rudebusch (2001a) argues that the performance of such a rule is poor. Again, Orphanides, Porter, Reifschneider, Tetlow and Finan (2000) and McCallum (2001) reach a different conclusion. Interest rate rules of this type appears to dominate the Taylor rule when it is difficult to assess the state of the economy in real time. In contrast, the Taylor rule dominates if one assumes artificially low degrees of uncertainty.

To compare the Taylor rule with the McCallum rule directly I modify the McCallum rule in equation (5). In this paper, the rule in equation (5) is modified such that the FFR is the Fed’s policy instrument instead of the money base. The modification is based on the empirical evidence that the relationship between base velocity and the FFR is stable, which would enable us to substitute changes in the money base by changes in the FFR and nominal GDP in equation (5).

Let velocity be a function of the FFR, \(\Delta V_t = f(\Delta i_t) + \psi_t\), where \(f\) is an unknown function, either a linear or non-linear function, and \(\psi_t\) is a mean-zero white noise error term. It is crucial that the relationship between velocity and the FFR is stable before we make any substitution. An approximation of \(f\) is also required.

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\(^8\) For example, see Goodhart (1994) and Blinder (1994) for arguments against this rule.

First, I check the stability requirement. I provide two different tests for stability. The first is based on cointegration. If the two variables are cointegrated over a reasonably long period of time then the relationship between them is said to be stable.

To motivate the idea of cointegration, let us examine a plot of the levels of base velocity and the FFR from March 1957 to March 2000. Figure 3 may lend support to the “stability” argument.

To test formally, table 1 reports three regressions: A regression of the level of base velocity on a constant and the FFR, the Engle-Granger (ADF) test statistic for the null hypothesis that the residuals from the level regression above have a unit root, and an error correction regression. Results of the regressions indicate that there is a statistically significant cointegration relationship at the 10 per cent level, but what really matters the most is that the error correction term is statistically significant provided that the residuals are white noise with a zero mean. The stability of the relationship between base velocity and the FFR can also be investigated in another way.

In table 2, I take a second, completely different, approach. The second approach is to assume that we do not know anything about cointegration and we do not really trust the ADF - Engle-Granger test. How would we test for stability? I report the estimates from the regression \(\Delta V_t = a + b \Delta i_t + \epsilon_t\), which is a proxy for \(\Delta V_t = f(\Delta i_t) + \psi_t\), over three different sub-samples and test whether
the parameters, particularly $b$, remained constant over time. The first sub-sample is from March 1957 to December 1974 – the fixed exchange rate period – the second sub-sample is from March 1975 to December 1986 – the flexible exchange rate period. The last sub-sample is March 1987 to March 2000 – the Greenspan period. The coefficient $b$ is stable as indicated by the $F$ statistic and has a value of 0.1, which is not statistically different from the value of $\gamma$ in the error correction model reported in table 1.


Having established stability between base velocity and the FFR, the McCallum rule can be modified to include the FFR. Given that the relationship between velocity and interest rate is stable then there is a stable demand for the real money base:

$$\Delta h_t = -\Delta v_t + \Delta r_t$$  \hspace{1cm} (6)$$

Given the approximation:

$$\Delta v_t = \kappa \Delta i_t$$  \hspace{1cm} (7)$$

Equation (6) becomes:

$$\Delta h_t = -\kappa \Delta i_t + \Delta r_t$$ \hspace{1cm} (8)$$

Substitute equation (8) in the original McCallum rule in equation (5).

$$\Delta x_t = \kappa \Delta i_t = \Delta y - \Delta V^*_t + \lambda_v (\Delta y - \Delta x_t)$$  \hspace{1cm} (9)$$

Re-arranging terms yields:

$$\Delta i_t = \frac{\Delta V^*_t - (1 + \lambda_v)(\Delta y - \Delta x_t)}{\kappa}$$ \hspace{1cm} (10)$$

The hat on $i_t$ is to distinguish the interest rate implied by this rule from that implied by the Taylor rule. The modified McCallum rule in equation (10) differs from the original rule in equation (5) in three ways: First, the monetary policy instrument is the FFR and not the money base, which dropped out. Second, the response coefficient $\lambda_v (0.5)$ is larger, equals $1+\lambda_v (1.5)$. Third, the modified rule is scaled by the parameter $\kappa$, which measures the sensitivity of base velocity to changes in the FFR. Further, equation (10) suggests that, in addition to the usual argument of the rule, the level of the FFR implied by the rule at time $t$ also depends on the level of the FFR implied by the rule last period, ie, monetary policy inertia. Thus, the policymaker takes into account her past policy action when making decisions today. And finally, this rule is much closer to a nominal GDP rule than to a money base rule.

There is a major difference in the way the Taylor rule in equation (2) and the modified McCallum rule in equation (10) work. The Taylor rule suggests that the Fed raises the interest rate vigorously, but gradually in response to current deviations of output and inflation. This partial adjustment is determined by the size of $\alpha$. In the modified McCallum rule, equation (10), when nominal GDP growth exceeds its target rate, the Fed keeps raising the FFR until nominal GDP growth is equal to its target. This continuous response is an important feature of this rule that differentiates it from the Taylor rule. There is no smoothing or gradual adjustment in the modified McCallum rule. In this regard, equation (10) is similar to the rule in Levin et al (2001).

However, $\kappa$ in equation (10), which measures the sensitivity of base velocity to changes in the FFR acts as a smoother. A large value of $\kappa$ dampens the volatility of nominal GDP growth fluctuations and makes past FFR important in a similar way the smoothing parameter in the Taylor rule works. The way the rule works is not by vigorously responding to the deviations of nominal GDP growth from its target (ie, $(1 + \lambda_v)/\kappa$ is small in magnitude) but by
continuously hammering on these deviations. This is in essence similar to Bernanke and Woodford (1997).

I plot the Taylor rule, equation (2), and the modified McCallum rule, equation (10). The Taylor rule in equation (2) is calibrated using $r = 2$, $\Delta P = 2$, $\lambda_\pi = 0.5$, $\lambda_r = 0.5$ and $\alpha = 0.24$. For the McCallum rule, average velocity is calculated for as a four-year average, $(1/16)(x_{t-4} + x_{t-3} + x_{t-2} + x_{t-1})$, $\lambda_\pi = 0.5$ and $\Delta T = 4.5$. We don’t know $\kappa$. I estimate $\kappa$ from the data to be 50, which provides substantial smoothing. Equation (2) and (10) are plotted in figure 4. The two rules look similar, flat, in the 1960s and 1970s, but there is a little disagreement between them at the second half of the sample starting in the 1980s. Changes in the FFR implied by either the Taylor or the modified McCallum rules do not match the change in the FFR. The changes in the FFR implied by both rules are much smoother than the actual changes in FFR. The changes in the FFR implied by both rules are much smoother than the actual changes in FFR.

Of course if the magnitude of $\kappa$ gets smaller the changes in the FFR implied by the modified McCallum rule become more volatile and matches with the actual changes in FFR. However, this kind of volatility will not be so apparent if we plot the rule in the levels instead because of the high persistence of interest rate in the rule and the fact that $(1 + \lambda_\pi) / \kappa$ is small in magnitude. The first term in equation (11) dominates the second.

Although the modified McCallum rule response to the nominal GDP growth deviations from target is small in size, $1 + \lambda_\pi / \kappa = 1.5/50 = 0.03$, it is stable and there is no indeterminacy problem. The eigenvalues are fine and the response to past interest rate and nominal GDP growth indicates that the root is less than one.

**Note:**

9 They argued that a rule like $i_t = \rho i_{t-1} + \pi_t$, with a small $\tau$ and large $\rho$ has a similar stabilisation properties as a rule like $i_t = \pi_t$, with a large $\tau$ except that it is much smoother.

10 Equation (10) is estimated by non-linear least squares from March 1962 to March 2000. The $t$-statistic is 2.5, $F^2$ is 0.12 and $DW$ is 1.63. Estimating the same regression from 1980 to 2000 gives similar results.

11 The correlation coefficients between the change in FFR and each of the rules are –0.34 and 0.35 for the modified McCallum rule and the Taylor rule respectively.

Further, the two rules in the level and the actual FFR are cointegrated. The cointegration relationship is plotted in figure 6.
So far, the results suggest that these two different policy rules can be made similar if sufficient smoothing is applied to the rules such that past interest rate, interest rate smoothing or policy inertia dominate the rules’ arguments. Or, if the amount of smoothing chosen is such that the two rules are the same. This seems trivial. Also, it seems discomforting. These results suggest that the policymaker may be free to adopt any policy rule, even most volatile ones, provided that sufficient interest rate smoothing is applied.

Economists evaluate the stabilisation properties of policy rules by comparing the unconditional variances of inflation and output. These unconditional variances can be computed in different ways. Model-based simulation is a typical way. The results presented so far will be tested further using model-based simulations, which will be presented next. The results presented above are confirmed.

4 Model-based evaluations of the two rules

McCallum advocates robustness in the sense that a policy rule should perform well across different models. This is because economists do not agree on a particular macroeconomic model. Typically, researchers either estimate different models (eg, Rudebusch (2001), and McCallum and Nelson (1999b)) or calibrate different models. Then, these models are simulated along with the policy rules. The simulations are stochastic in the sense that a large number of shock realizations are generated randomly from a distribution that has the covariance properties of the historical shock estimates. Although one can compute the unconditional variances without conducting stochastic simulation, Bryant et al. (1993, p. 373-375) suggest that stochastic simulation are consistent with the robustness issue.

In this paper, I choose a well-known New Keynesian model that has recently been estimated by Rudebusch (2001) and Dennis (2001). Meyer (2001) calls the New Keynesian model “the consensus model.” The model is also endorsed by Svensson (1999b), Clarida et al. (2000), McCallum and Nelson (1999a, b), Rotemberg and Woodford (1999) and Taylor (1999) and it is shown to have a foundation in dynamic general equilibrium models. The model consists of two equations, a Phillips curve and an aggregate demand curve. The Phillips curve is given by:

$$\Delta P_t = a_{ii} + a_{i1}E_{t-1}\Delta P_{t-1} + a_{i2}[\sum_{i=1}^{4} \beta_i \Delta P_{t-i}] + a_{i3} \bar{y}_{t-1} + \eta_t$$  \hspace{1cm} (12)$$

The inflation rate $\Delta P_t$ is $(\ln P_t - \ln P_{t-1}) \times 400$ where $P_t$ is a price index. The inflation rate depends on expected inflation, which consists of two components: a forward looking expectation and a fully backward-looking component that consists of four lagged values of inflation. Inflation also depends on lagged value of the output gap. The output gap is $(\ln y_t - \ln y_{t-1}) \times 100$, where $y_t$ is real GDP and $y_{t-1}$ is the estimate of potential output published by the Congressional Budget Office. The restriction $a_{i1} + a_{i2} = 1$ is typically tested and imposed on the model.

Researchers used different measures of inflation. For example, Rudebusch (2001a) and Dennis (2001) use GDP chain-weighted price index. Researchers also used different ways to estimate the degree of forwardness of inflation expectations. Rudebusch (2001a) used the University of Michigan Survey data of a year ahead expected price changes, while Dennis (2001) assumes $a_{i1} = 0$ in the Phillips curve. Instead, he imposes the restriction that $\sum_{i=1}^{4} \beta_i = 1$. Others used ML estimator (Fuhrer, 1997).13

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13 Also, see Roberts (1995, 2001).
The second equation in the model is the output equation, which is the intertemporal Euler equation given by:

\[ \bar{y}_t = a_{11}E \bar{y}_{t+1} + a_{21}(i_t - E\Delta P_{t+1} - \bar{r}) + \eta_{t+1} \]  

(Rudebusch (2001a) shows that when this model is simulated along with a McCallum rule like

\[ \Delta P_t = \alpha_0 + \alpha_1 E \Delta \bar{P}_{t+1} + \alpha_2 (\sum_{i=1}^{s} \beta_i \Delta P_{t-1}) + \alpha_3 \bar{y}_{t+1} + \eta_{t+1} \]  

and allowing for model and output gap uncertainty it indicates that the rule has very poor stabilisation properties. In other words, the unconditional variances of inflation, output and changes in interest rates are large. Because the objective of this paper is to evaluate the stabilisation properties of the McCallum rule derived in equation (10), I will use the same model. Data uncertainty is beyond the scope of this paper.

The experimental design

The inflation rate is defined in terms of the CPI instead of the GDP chain-weighted price index. It will be shown that the choice of the price index has a crucial impact on the degree of forwardness of expected inflation in the Phillips curve. The output gap is similar to the output gap used in Rudebusch (2001a) and Dennis (2001), deviations of real GDP from potential output measured by the CBO. The forward-looking component is measured by the University of Michigan Survey data of changes in prices. The monthly survey data are averaged to get the quarterly figures. The empirical inflation and the output equations given by Rudebusch (2001a) are:

\[ \Delta P_t = \alpha_0 + \alpha_1 E \Delta \bar{P}_{t+1} + \alpha_2 (\sum_{i=1}^{s} \beta_i \Delta P_{t-1}) + \alpha_3 \bar{y}_{t+1} + \eta_{t+1} \]  

and

\[ \bar{y}_t = a_{20} + a_{21} \bar{y}_{t+1} + a_{22} \bar{y}_{t+2} + a_{23}(i_t - E \Delta \bar{P}_{t+1}) + \eta_{t+1} \]  

where \( a_{u+} \) is \( a_u \bar{r} \) in equation (13). I calibrate the model from March 1980 to March 2000. This sample period represents most of the Volcker-Greenspan period. I start the calibration from 1980 instead of March 1979 or earlier because some researchers suggested that there is a structural break in 1979 (Fuhrer (1997), Rotemberg and Woodford (1997), Clarida et al (1998)).

I calibrate the Phillips curve and the output gap equations (14) and (15) using values for the coefficients within the ballpark of coefficient values reported in Rudebusch (2001a) and Dennis (2001). The output gap equation is given by:

\[ \bar{y}_t = -0.24 + 1.28 \bar{y}_{t-1} -0.37(i_{t-1} - E\Delta \bar{P}_{t-1}) \]  

Three different versions of the Phillips curve are used. Two of them are calibrated. First, I set \( a_1 = 0 \) and \( a_1' = 1 \) and the restriction \( \sum_{i=1}^{s} \beta_i = 1 \). So the Phillips curve has only backward-looking expectations. In the second, I set \( a_{\alpha} = a_1' = 1 \) and for \( \beta \)’s I use coefficients similar to those reported in Rudebusch (2001a). The third version of the Phillips curve is an estimated one. The output gap equation is the same in all three cases.

The three Phillips equations are:

**Backward-looking expectations**

\[ \Delta P_t = 0.06 + 0.65 \Delta P_{t-1} - 0.10 \Delta P_{t-2} + 0.30 \Delta P_{t-1} + 0.15 \Delta P_{t-2} + 0.15 \bar{y}_{t-1} \]  

Note that \( \sum_{i=1}^{s} \beta_i = 1 \), Dennis (2001).

**Mixed expectations**

\[ \Delta P_t = 0.06 + 0.30 E \Delta \bar{P}_{t+1} + 0.70 \times [0.65 \Delta P_{t-1} - 0.10 \Delta P_{t-2} + 0.30 \Delta P_{t-1} + 0.15 \Delta P_{t-2} + 0.15 \bar{y}_{t-1} \]  

Note that \( a_{\alpha} = a_1' = 1 \), Rudebusch (2001a).

**Forward-looking expectations**

There is no consensus among researchers about the degree of forwardness (of inflation expectations) in the Phillips curve. Ball
Svensson (1999a) and Orphanides (1999) set \( a_{i} \) to zero. McCallum and Nelson (1999a, b) and Rotemberg and Woodford (1999) set \( a_{i} = 1 \). Estrella and Fuhrer (2001) argue that \( a_{i} = 0 \) and Rudebusch (2001a) estimates \( a_{i} \) to be 0.29. While the estimation techniques, sample sizes, and measurements vary from one paper to another the consensus conclusion is: expectations are more backward looking than forward looking in the Phillips curve.

Figure 7 is a scatter plot of the CPI inflation rate and the University of Michigan Survey of price changes from 1980:1-2000:1.

I estimate two regressions. The first regression is a single-equation Phillips curve from March 1980 to March 2000 by OLS:

\[
\Delta p_t = a_0 + a_1 E_{t-1} \Delta p_{t-1} + \sum_{i=1}^{I} \beta_i \Delta p_{t-i} + \alpha_2 \bar{y}_{t-i} \tag{19}
\]

The second regression is to estimate the Phillips curve and the output gap equation (14 & 15) simultaneously from March 1980 to March 2000 by FIML.

\[
\Delta p_t = a_0 + a_1 E_{t-1} \Delta p_{t-1} + \sum_{i=1}^{I} \beta_i \Delta p_{t-i} + \alpha_2 \bar{y}_{t-i} + \eta_t
\]

\[
\bar{y}_{t} = a_{20} + a_{21} \bar{y}_{t-1} + a_{22} \bar{y}_{t-2} + a_{23} (i_{t-1} - E_{t-1} \Delta p_{t-1}) + \eta_{t-1}
\]

The coefficient \( a_{20} \) in equation (21) is \( a_{20} \Delta P \) in equation (13). The results are reported in table 3 (a, b). The hypotheses that coefficient \( a_{i} = 1 \), and \( \sum_{i=1}^{I} \beta_j = 0 \) cannot be rejected. The restriction that \( a_{i} + \sum_{i=1}^{I} \beta_j = 1 \) does not hold either. The estimates of \( a_{20} \) (-0.24) and \( a_{23} \) (-0.08) imply that the equilibrium real interest rate is 0.019, which is approximately 2 per cent as Taylor suggested.

Thus, the third model I simulate is the following forward-looking model:

\[
\Delta P = -0.66 + 1.1 E_{t-1} \Delta p_{t-1} + 0.15 \Delta P_{t-2} + 0.17 \Delta P_{t-3} + 0.20 \Delta P_{t-4} - 0.05 \Delta P_{t-5} + 0.12 \bar{y}_{t-1}
\]

\[
\bar{y}_{t-1} = -0.24 + 1.28 \bar{y}_{t-2} - 0.37 \bar{y}_{t-3} - 0.08 (i_{t-1} - E_{t-1} \Delta p_{t-1})
\]

With each of the models, the backward-looking, mixed expectations and the forward-looking, I simulate the two policy rules, the Taylor rule in equation (2) and the modified McCallum rule in equation (10) separately (there is some evidence that the rules in differences are more robust than the rules in levels). The identity \( \Delta x_{t} = \Delta y_{t} + \Delta P_{t} \), and the fact that \( \bar{y}_{t} = y_{t} - y_{t}^{\pi} \) and \( \Delta \bar{y}_{t} = \Delta y_{t} - \Delta y_{t}^{\pi} \) are used in the simulations. I experimented with different degrees of smoothing in the Taylor rule.

Each model and the policy rule are solved simultaneously. First, I solve the model deterministically using static simulation for each observation from March 1980 to March 2000 and an iterative algorithm (Gauss-Seidel) to compute the values of the endogenous variables \( \Delta P, \bar{y}_{t} \) and \( \Delta i \). Then the model is solved stochastically. In this simulation, the model is solved repeatedly for different draws of the stochastic components of the model (the innovations to the Phillips curve and the output gap equations). The coefficients are fixed over the simulation. The innovations of the stochastic equations are generated by independently randomly drawing from the standard normal distribution. Each simulation consists of 1000 iterations.

The results suggest that the hypothesis, that the unconditional volatility of inflation and the output gap across all three models and
the two rules are equal, cannot be rejected. However, the unconditional volatility of $\Delta i$, across models and rules is sensitive to the degree of interest rate smoothing in the rules. The degree of interest rate smoothing in the modified McCallum rule (equation 10) is determined by $1/\kappa$, which is fixed to 1.5/50 (0.03). This provides much higher smoothing than $\alpha = 0.24$ provides to the Taylor rule.

One can always choose the degree of interest rate smoothing in the Taylor rule, $\alpha$, such that the two rules have similar effects on the variance of $\Delta i$. As the value of $\alpha$ in the Taylor rule increases (i.e., less interest rate smoothing) the unconditional variance of $\Delta i$, increases. I report the results of two experiments, one when $\alpha$ is 0.24 (i.e., modified McCallum rule smoothes interest rate more than the Taylor rule does), and the other when $\alpha$ is 0.02 and 0.01 (arbitrary). This implies that the modified McCallum rule and Taylor rule have equal degrees of interest rate smoothing. When $\alpha$ in the Taylor rule is 0.24 the modified McCallum rule has superior stabilisation properties, and when $\alpha$ is 0.02 and 0.01, the two rules produce the same volatility across all models.

Table 4a reports the unconditional standard deviations of the actual data, and of the simulated paths of inflation, the output gap and the change in FFR (when $\alpha$ is 0.24). It also reports the probability of the $F$ statistics, which test the hypothesis that these unconditional standard deviations are equal. Table 4b is similar to table 4a except that $\alpha$ in the Taylor rule is 0.02 and 0.01 (more interest rate smoothing than when $\alpha$ is 0.24).

The modified McCallum rule is stable even though the rule is presented in terms of $\Delta i$, as a function of $(\Delta V, \Delta x, \lambda, \kappa)$. Although the response to deviations of nominal GDP growth from its target is relatively small in magnitude (i.e., $(1 + \lambda)/\kappa = 0.03$) and the response of $i$ to its own past is large (i.e., 1), the rule achieves its objective by continuously responding to the deviations until they are eliminated. This is in essence similar to what Bernanke and Woodford (1997) have proposed. They argued for more interest rate smoothing in an interest rate rule that has a small response coefficient to inflation’s deviations and a large response coefficient to past interest rate. They showed that this rule has similar stabilisation properties as an interest rate rule that responds only to inflation’s deviation with a very large coefficient. The difference is that the former rule is smoother than the latter. The modified McCallum rule in this paper should not be confused with, and it is not the same as a constant interest rate rule or an interest rate peg rule. This rule has different stabilisation properties.

These results are not surprising and confirm the propositions stated in the introduction. They suggest that one can choose the smoothing parameter such that a similar path for $i$, can be computed using many different policy rules. Levin et al (2001) demonstrate that a policy rule that is nearly optimal is a rule with a high interest rate smoothing.

### 5 Conclusions

It is difficult to compare the Taylor rule with the money base-nominal GDP targeting rule because the instruments are different. The policy instrument in the Taylor rule is the Federal funds rate (FFR). In the money base-nominal GDP targeting rule, the instrument is the monetary base. The underlying theories of transmission mechanism of the two rules are also different.

To compare the two rules this paper modifies the money base-nominal GDP targeting rule by replacing the monetary base as the instrument with the FFR. This rule is referred to as the modified McCallum rule. To modify the rule, I relied on the empirical evidence that base velocity is, on average, a stable function of the FFR. In other words, base velocity is either cointegrated with the actual FFR or the coefficients of the regression of interest rate on velocity are stable over a long period of time. It was assumed that changes in velocity are proportional to changes in the FFR. This linear approximation proved to be a good proxy for the underlying relationship between base velocity and the FFR. This smoothed modified McCallum rule is in fact very similar to the Taylor rule in the level. The levels of the modified McCallum rule, the Taylor rule and the FFR are themselves cointegrated.

There is a disagreement on the stabilisation properties of the McCallum rule when the instrument is the nominal interest rate. Some stochastic simulation exercises using a “consensus” New...
Keynesian model suggest that the McCallum rule has inferior stabilisation properties. Also, it has been argued elsewhere that the rule would produce more volatility in output and inflation and it would typically introduce instrument instability or in other words, a highly volatile path for the interest rate. Other research demonstrates that the rule dominates the Taylor rule under certain conditions.

In this paper I tested the stabilisation properties of the two rules using the same New Keynesian model that is typically used in evaluation experiments. Three models were used. They differ in the degree of forwardness of inflation expectations (backward looking, forward-looking and a mixed expectations). I found that the Taylor rule and the modified McCallum rule can be made essentially equivalent in the sense that the unconditional volatility of inflation and output are similar across the models. For the two rules to have similar stabilisation properties they have to have similar “sufficiently high” degree of interest rate smoothing. The modified McCallum rule presented in this paper is stable. The modified McCallum rule achieves its objective not by responding so aggressively to deviations of nominal GDP growth from the target, but rather by continuously responding the deviations until they are eliminated.

References


Table 1: Relationship between velocity and the interest rate 1957:1 – 2000:1

<table>
<thead>
<tr>
<th>OLS $V_t = \phi_0 + \phi_1 ffr_t + \eta_t$</th>
<th>Coefficient</th>
<th>t-Statistics</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>13.4</td>
<td>51.89</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.57</td>
<td>15.84*</td>
<td>0.0001</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$V_t$ is the base velocity defined as $\bar{P}_t Y_t / b_t$, where $\bar{P}_t$ is GDP deflator, $Y_t$ is real GDP and $b_t$ is the money base. The FFR is given by $ffr_t$.

The Engle-Granger Test

<table>
<thead>
<tr>
<th>OLS $\Delta \eta_t = \delta_0 + \rho \eta_{t-1} + \sum_{i=1}^{\tau} \delta_i \Delta \eta_{t-i} + \delta_i$</th>
<th>Coefficient</th>
<th>t-Statistics</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>0.01</td>
<td>0.40</td>
<td>0.6854</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.08</td>
<td>-3.00*</td>
<td>0.0032</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.15</td>
<td>1.96</td>
<td>0.0520</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.12</td>
<td>-1.54</td>
<td>0.1238</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.06</td>
<td>0.85</td>
<td>0.3960</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>0.15</td>
<td>1.97</td>
<td>0.0494</td>
</tr>
</tbody>
</table>

The asymptotic critical value at the 95% level is -3.34 so the cointegration is significant at the 90% level.

An Error Correction Regression

<table>
<thead>
<tr>
<th>OLS $\Delta V_t = \theta + \gamma \Delta i_t + \rho \eta_{t-1} + \zeta_t$</th>
<th>Coefficient</th>
<th>t-Statistics</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.02</td>
<td>1.81</td>
<td>0.0720</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.09</td>
<td>6.61*</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.03</td>
<td>-3.53*</td>
<td>0.0005</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>1.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Asterisks mean significant at the 95% level.
### Table 2
OLS $\Delta V_t = a + b\Delta t_i + \varepsilon_t$

<table>
<thead>
<tr>
<th>Sub samples</th>
<th>57Q1-74Q4</th>
<th>75Q1-86Q4</th>
<th>87Q1-2000Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.07</td>
<td>0.10</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$F$</td>
<td>3.86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$F$ is to test the hypothesis that $b$ is equal across sub-samples. P values in parentheses.

### Table 3a
OLS, sample March 1980 – March 2000

$$\Delta P_t = a_{t0} + a_{t1}E_{t1-3} + \sum_{i=3}^4 \beta_i \Delta P_{t-i} + a_{t2} \tilde{y}_{t-3}$$

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{t0}$</td>
<td>-0.62</td>
<td>-1.07</td>
<td>0.2880</td>
</tr>
<tr>
<td>$a_{t1}$</td>
<td>1.15</td>
<td>2.84</td>
<td>0.0058</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.12</td>
<td>0.90</td>
<td>0.3670</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.18</td>
<td>-1.45</td>
<td>0.1491</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.23</td>
<td>1.92</td>
<td>0.0590</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-0.07</td>
<td>-0.70</td>
<td>0.4838</td>
</tr>
<tr>
<td>$a_{t2}$</td>
<td>0.12</td>
<td>1.43</td>
<td>0.1564</td>
</tr>
</tbody>
</table>

Wald $H_0: a_{t1} = 1$

The Wald statistic tests the hypothesis that $a_{t1} = 1$.

### Table 3b
FIML sample March 1980 – March 2000

$$\Delta P_t = a_{t0} + a_{t1}E_{t1-3} + \sum_{i=3}^4 \beta_i \Delta P_{t-i} + a_{t2} \tilde{y}_{t-3}$$

$$\tilde{y}_t = a_{t0} + a_{t2} \tilde{y}_{t-4} + a_{t3} \tilde{y}_{t-3} + a_{t4} (\tilde{y}_{t-2} - E_{t1-3} \Delta P_{t-3})$$

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>t-statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{t0}$</td>
<td>-0.64</td>
<td>-1.21</td>
<td>0.2258</td>
</tr>
<tr>
<td>$a_{t1}$</td>
<td>1.11</td>
<td>3.14</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.15</td>
<td>1.12</td>
<td>0.2633</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.17</td>
<td>-1.51</td>
<td>0.1307</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.20</td>
<td>1.82</td>
<td>0.0685</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-0.04</td>
<td>-0.41</td>
<td>0.6786</td>
</tr>
<tr>
<td>$a_{t2}$</td>
<td>0.12</td>
<td>1.85</td>
<td>0.0643</td>
</tr>
</tbody>
</table>

Wald $H_0: a_{t1} = 1$

Wald $\sum_i \beta_i = 0$

Wald $\sum_i \beta_i = 1$

$\hat{R}^2$

$DW$

$\hat{\sigma}$

$a_{t0}$

$a_{t1}$

$\beta_1$

$\beta_2$

$\beta_3$

$\beta_4$

$\sigma$

$R^2$

$DW$

$\hat{\sigma}$

The Wald statistic tests the hypothesis that $a_{t1} = 1$. 

$$\tilde{y}_t = a_{t0} + a_{t2} \tilde{y}_{t-4} + a_{t3} \tilde{y}_{t-3} + a_{t4} (\tilde{y}_{t-2} - E_{t1-3} \Delta P_{t-3})$$
### Table 4a

Unconditional Standard Deviation of the Endogenous Variables across the Three Models and the Two Policy Rules in equations (2) and (10)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Taylor Rule (α = 0.24)</td>
<td>Modified McCallum Rule</td>
<td></td>
</tr>
<tr>
<td>σ²F</td>
<td>2.762894</td>
<td>2.857274</td>
<td>2.503492</td>
</tr>
<tr>
<td>σ²y</td>
<td>2.325268</td>
<td>2.161327</td>
<td>2.176283</td>
</tr>
<tr>
<td>σ²ι</td>
<td>1.141051</td>
<td>0.701355</td>
<td>0.686967</td>
</tr>
<tr>
<td></td>
<td>F statistics ^a</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9930</td>
<td>0.9913</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9086</td>
<td>0.9126</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

### Table 4b

Unconditional Standard Deviation of the Endogenous Variables across the Three Models and the Two Policy Rules equations (2) and (10)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Taylor Rule (α = 0.025)</td>
<td>Modified McCallum Rule</td>
<td></td>
</tr>
<tr>
<td>σ²F</td>
<td>2.762894</td>
<td>2.853956</td>
<td>2.500325</td>
</tr>
<tr>
<td>σ²y</td>
<td>2.325268</td>
<td>2.137075</td>
<td>2.150952</td>
</tr>
<tr>
<td>σ²ι</td>
<td>1.141051</td>
<td>0.072356</td>
<td>0.028331</td>
</tr>
<tr>
<td></td>
<td>F statistics ^a</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9987</td>
<td>0.9997</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9887</td>
<td>0.9957</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5349</td>
<td>0.0409</td>
</tr>
</tbody>
</table>

α , F statistics test the hypothesis that the variances σ²F, σ²y and σ²ι corresponding to the model with the Taylor rule are equal to those corresponding to the model with the modified McCallum rule.