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**Is the Taylor rule really different
from the McCallum rule?**

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W A Razzak

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Is the Taylor rule really different from the McCallum rule?

Abstract²

When base velocity is a stable function of the Federal funds rate (FFR), the money base-nominal GDP targeting rule (McCallum rule) can be re-parameterised and presented in terms of FFR as the policy instrument. Comparison of this McCallum modified policy rule with the popular Taylor rule suggests that these two rules and the FFR are actually cointegrated. Model-based evaluations of the two rules' stabilisation properties indicate that the modified McCallum rule is similar to the Taylor rule. The key to this result is the degree of interest rate smoothing applied to the policy rules.

¹ The first version of this paper was written in October 2000.

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1 Introduction

The Taylor rule (Taylor, 1993) has become very popular in recent years. The rule represents an automatic response of the Federal funds rate (FFR) to any deviation of the inflation rate from a desired target value and to the output gap (deviation of real GDP from its potential). In essence, the rule involves changes of the *ex-ante* real interest rate, relative to its equilibrium value.

The money base-nominal GDP targeting rule (McCallum, 1988) is also an adaptive policy formula, but with a different policy instrument and a different underlying theory of transmission mechanism. The policy instrument is the money base instead of the FFR. With this rule, the money-base growth rate changes in response to deviation of the nominal GDP growth rate (or the level) from a desired target value that grows at a specified rate. The rule also allows for gradual changes in base velocity. Nominal GDP targeting and money base targeting are equivalent when changes in base velocity are not large.

Meltzer (1987), Gordon (1985), Hall and Mankiw (1994) and Feldstein and Stock (1994) recommended a nominal GDP targeting rule for monetary policy. However, many researchers suggest that the McCallum money-base targeting rule has undesirable stabilisation properties. Goodhart (1994) and Blinder (1994) provide arguments against the money base rule. Recently, Orphanides (1999) and McCallum (2000) suggested a nominal GDP rule (without money base) similar to the Taylor rule in that the policy instrument is the FFR, but instead of the output gap it has nominal GDP, inflation and an equilibrium real interest rate on the RHS. Rudebusch (2001a) argues that the stabilisation property of this rule is also poor. However, the Federal Reserve Board own research (Orphanides, Porter, Reifschneider, Tetlow and Finan, 2000) demonstrates that this type of rules dominates the Taylor rule when it is difficult to accurately assess the state of the economy in real time. By contrast, the Taylor rule appears to dominate if one assumes artificially low degrees of uncertainty. McCallum (2001) studies the same problem and reaches the same conclusion.

To test whether the Taylor rule is really different from the McCallum rule, this paper modifies the McCallum rule. I replace the money base as the policy instrument with the FFR by appealing to empirical evidence that base velocity is a stable function of the FFR. Then I compare this rule, which is no longer a money base rule, to the Taylor rule using quarterly data for the United States. I find that the modified McCallum rule, the Taylor rule and the FFR are cointegrated. Then I test its model-based stabilisation properties during the period 1980-2000 using the New Keynesian model in Rudebusch (2001a) and Dennis (2001).³

Provided that the two policy rules have similar degrees of “sufficiently high” interest rate smoothing, I find that the modified McCallum rule behaves just as well as the Taylor rule. I test for the equality of the unconditional variances of inflation, the output gap and the change in the interest rate across three variants of the New Keynesian model that differ in the degree of forwardness of expectations. Equality of unconditional variances cannot be rejected.

In summary, this paper offers two propositions. Let the Taylor rule be given by $i_{1t} = \alpha_1 f(x_t, \theta) + (1 - \alpha_1)i_{1t-1}$, where i_{1t} is the interest rate, $f(x_t, \theta)$ is the rule’s argument (x_t is a variable and θ is a parameter), and $0 < \alpha_1 < 1$ is a smoothing parameter. And let another rule (eg, the McCallum rule) be given by $i_{2t} = \alpha_2 g(y_t, \omega) + (1 - \alpha_2)i_{2t-1}$, where i_{2t} is the interest rate, $g(y_t, \omega)$ is the rule’s argument (y_t is a variable and ω is a parameter) and $0 < \alpha_2 < 1$ is a smoothing parameter, then it is trivial to show that that given $f(x_t, \theta) \neq g(y_t, \omega)$, i_{1t} will be equal to i_{2t} when $\alpha_1 = \alpha_2 \rightarrow 0$. The second proposition is that if $g(y_t, \omega)$ is more variable than $f(x_t, \theta)$ then one can make $i_{1t} \approx i_{2t}$ if both α_1 and $\alpha_2 \rightarrow 0$, and $\alpha_2 < \alpha_1$, which means more interest rate smoothing.

The Taylor rule is briefly discussed in section 2. In section 3, the modified money-base rule is derived and the two rules are compared. Model-based evaluations of the rules are found in section 4. Conclusions are in section 5.

³ The data source in this paper is the Federal Reserve Bank of St Louis Website.

2 Taylor rule

The Taylor rule is typically given by the following formula, though it could be written, and its variables may be defined, in many different ways.

$$i_t^* = \bar{r} + \Delta P_t + \lambda_{\Delta P} (\Delta P_t - \Delta \bar{P}) + \lambda_y (\tilde{y}_t) \quad (1)$$

The policy instrument in the Taylor rule is the FFR thus i_t^* is the FFR implied by the rule (in per cent). The first variable on the RHS of the Taylor rule is the average real interest rate or the equilibrium real interest rate \bar{r} . In annual terms, Taylor chooses 2 per cent.⁴ The second term in the Taylor rule, ΔP_t , is either the contemporaneous inflation rate defined as $(\ln P_t - \ln P_{t-1}) * 400$, where P_t is a measure of the price level or its lagged value or the average inflation rate ΔP_t^a , for example, over the past four quarters ($\frac{1}{4} \sum_{i=0}^3 \Delta P_{t-i}$). The third term in brackets is the deviation of the inflation rate from a specified target. The target value of the inflation rate $\Delta \bar{P}$ is set to 2 per cent in Taylor’s original paper. The last term in brackets is the output gap defined as the deviation of real GDP from trend, which could be obtained by the HP filter or any other measure.⁵ Taylor also sets the coefficients $\lambda_{\Delta P}$ and λ_y equal to 0.5.⁶

Figure 1 plots the Taylor rules and the actual FFR from 1960 to 2000. To compute equation (1), I use *average* CPI inflation over four quarters. The output gap is measured as the deviation of real GDP from potential output, which is measured using the

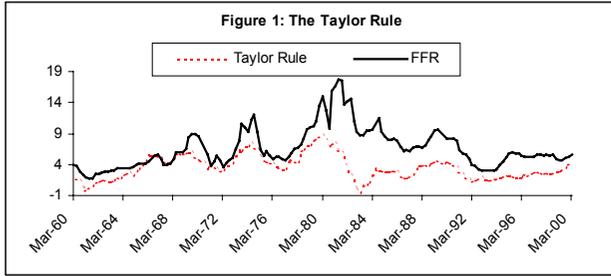
⁴ The equilibrium real interest rate is unobservable. It is not clear whether, or not, the equilibrium real interest rate should be constant over time. Laubach and Williams (2001) argue that it will.

⁵ It could be measured in many different ways and it is a proxy for excess demand in the economy.

⁶ Many have estimated different values directly from the data.

Congressional Budget Office estimate of potential output, y_t^p , and \bar{r} and $\Delta\bar{P}$ are set equal to 2. Both $\lambda_{\Delta P}$ and $\lambda_{\tilde{y}}$ are set equal to 0.5.

The Taylor rule has the disadvantage that the real interest rate and the output gap are both unobservable. Orphanides (2000, 2001) show that under-predicting and over-predicting the output gap can lead to some serious policy errors. The output gap is also subject to data uncertainty, ie, whether it is measured using real time data or data ex post.⁷



The Taylor rule can also be written in the following form:

$$\Delta i_t = \alpha(i_t^* - i_{t-1}) \quad (2)$$

Substituting for i_t^* from (1) into (2) gives:

$$\Delta i_t = \alpha (\{\bar{r} + \Delta P_t^a + 0.5(\Delta P_t^a - \Delta\bar{P}) + 0.5\tilde{y}_t\} - i_{t-1}) \quad (3)$$

And in levels:

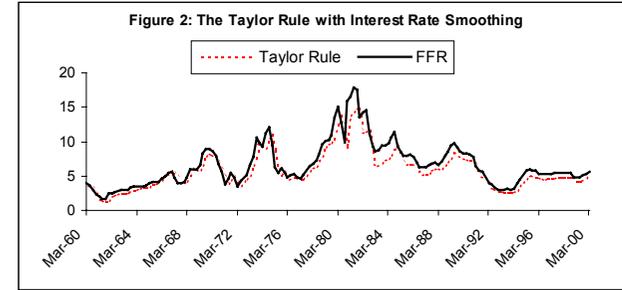
$$i_t = \alpha (\bar{r} + \Delta P_t^a + \lambda_{\Delta P}(\Delta P_t^a - \Delta\bar{P}) + \lambda_{\tilde{y}}\tilde{y}_t) + (1 - \alpha)i_{t-1} \quad (4)$$

This kind of formulation implies that the Fed moves its instrument (short-term interest rate) closer to the desired interest rate gradually. Economists call this “interest rate smoothing” or “policy inertia.” Rudebusch (2001b) provides a good discussion about these two descriptions, which will become a key issue in this paper.

⁷ Real time data is beyond the scope of this paper.

There are many estimates of α in the literature, but two of them are well known. Judd and Rudebusch (1998) estimated the smoothing parameter α to be 0.27 using a single-equation estimation technique, and Clarida et al. (2000) estimated it to be 0.21 using GMM. Both studies use quarterly data for the sample that covers the Volcker-Greenspan chairmanship periods. The Taylor rule in equation (4) can be computed using the average of those two estimates, 0.24. Figure 2 plots equation (4).

Note that interest rate smoothing makes the interest rate path implied by the rule much more similar to the actual FFR. Compare figure 2 with figure 1.



3 Money base-nominal GDP targeting rule

Now consider the money base-nominal GDP targeting rule.

$$\Delta b_t = \Delta\bar{x} - \Delta V_t^a + \lambda_{\Delta x}(\Delta\bar{x} - \Delta x_t) \quad (5)$$

The policy instrument is the growth rate of the money base $(\ln b_t - \ln b_{t-1}) * 400$. The first term on the RHS of equation (5), $\Delta\bar{x}$, is the target value of the growth rate of nominal GDP – a constant, which McCallum assumes to be 4.5 per cent. The second term is average base velocity (McCallum uses a four year period) where base velocity is defined as the ratio of nominal GDP to the money base. The term $\Delta\bar{x} - \Delta x_t$ is the deviation of nominal GDP growth rate from its (constant) targeted value. McCallum assumes that $\lambda_{\Delta x}$ is 0.5.

A major advantage this rule has over the Taylor rule is that it does not include unobservable variables such as the real interest rate and the output gap. It seems that the major disadvantage of this rule is that the instrument is the money base and not the FFR.⁸

It is hard to compare the two rules in equations (1) and (5) directly because the monetary policy instruments are different. McCallum and Nelson (1999), Orphanides (1999) and McCallum (2000) suggest a nominal GDP rule such as $i_t = \bar{r} + \Delta \bar{P} + \lambda_{\Delta x} (\Delta x_t - \Delta \bar{x})$, where the rule sets the short-term interest rate equal to the equilibrium real interest rate plus average inflation plus some fraction $\lambda_{\Delta x}$ of the deviation of nominal GDP growth over four quarters from its target. This rule is not derived from equation (5) directly. Rudebusch (2001a) argues that the performance of such a rule is poor. Again, Orphanides, Porter, Reifschneider, Tetlow and Finan (2000) and McCallum (2001) reach a different conclusion. Interest rate rules of this type appears to dominate the Taylor rule when it is difficult to assess the state of the economy in real time. In contrast, the Taylor rule dominates if one assumes artificially low degrees of uncertainty.

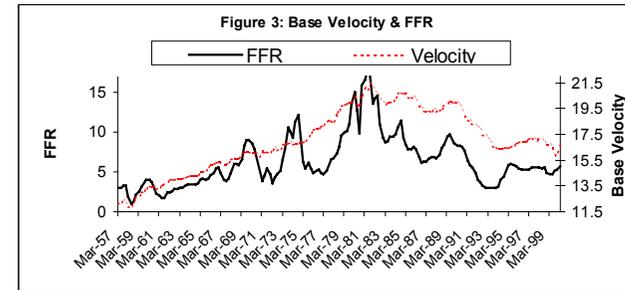
To compare the Taylor rule with the McCallum rule directly I modify the McCallum rule in equation (5). In this paper, the rule in equation (5) is modified such that the FFR is the Fed's policy instrument instead of the money base. The modification is based on the empirical evidence that the relationship between base velocity and the FFR is stable, which would enable us to substitute changes in the money base by changes in the FFR and nominal GDP in equation (5).

Let velocity be a function of the FFR, $\Delta V_t = f(\Delta i_t) + \psi_t$, where f is an unknown function, either a linear or non-linear function, and ψ_t is a mean-zero white noise error term. It is crucial that the relationship between velocity and the FFR is stable before we make any substitution. An approximation of f is also required.

⁸ For example, see Goodhart (1994) and Blinder (1994) for arguments against this rule.

First, I check the stability requirement. I provide two different tests for stability. The first is based on cointegration. If the two variables are cointegrated over a reasonably long period of time then the relationship between them is said to be stable.

To motivate the idea of cointegration, let us examine a plot of the levels of base velocity and the FFR from March 1957 to March 2000. Figure 3 may lend support to the "stability" argument.



To test formally, table 1 reports three regressions: A regression of the level of base velocity on a constant and the FFR, the Engle-Granger (ADF) test statistic for the null hypothesis that the residuals from the level regression above have a unit root, and an error correction regression. Results of the regressions indicate that there is a statistically significant cointegration relationship at the 10 per cent level, but what really matters the most is that the error correction term is statistically significant provided that the residuals are white noise with a zero mean. The error correction term is indeed significant and the error term is white noise. Keep in mind the size of the coefficient $\gamma = 0.09$. The stability of the relationship between base velocity and the FFR can also be investigated in another way.

In table 2, I take a second, completely different, approach. The second approach is to assume that we do not know anything about cointegration and we do not really trust the ADF - Engle-Granger test. How would we test for stability? I report the estimates from the regression $\Delta V_t = a + b\Delta i_t + e_t$, which is a proxy for $\Delta V_t = f(\Delta i_t) + \psi_t$, over three different sub-samples and test whether

the parameters, particularly b , remained constant over time. The first sub-sample is from March 1957 to December 1974 – the fixed exchange rate period – the second sub-sample is from March 1975 to December 1986 – the flexible exchange rate period. The last sub-sample is March 1987 to March 2000 – the Greenspan period. The coefficient b is stable as indicated by the F statistic and has a value of 0.1, which is not statistically different from the value of γ in the error correction model reported in table 1.

Meltzer (1998) estimates a similar regression using the long-term interest rate and reports the same coefficient for a century long data set. Lucas (1988) and Stock and Watson (1993) report the same coefficient for a very long data set using demand for money functions for the United States.

Having established stability between base velocity and the FFR, the McCallum rule can be modified to include the FFR. Given that the relationship between velocity and interest rate is stable then there is a stable demand for the real money base:

$$\Delta b_t = -\Delta v_t + \Delta x_t \quad (6)$$

Given the approximation:

$$\Delta v_t = \kappa \Delta i_t \quad (7)$$

Equation (6) becomes:

$$\Delta b_t = -\kappa \Delta i_t + \Delta x_t \quad (8)$$

Substitute equation (8) in the original McCallum rule in equation (5).

$$\Delta x_t - \kappa \Delta i_t = \Delta \bar{x} - \Delta V_t^a + \lambda_{\Delta x} (\Delta \bar{x} - \Delta x_t) \quad (9)$$

Re-arranging terms yields:

$$\Delta \hat{i}_t = \frac{\Delta V_t^a - (1 + \lambda_{\Delta x})(\Delta \bar{x} - \Delta x_t)}{\kappa} \quad (10)$$

The hat on i_t is to distinguish the interest rate implied by this rule from that implied by the Taylor rule. The modified McCallum rule in equation (10) differs from the original rule in equation (5) in three ways: First, the monetary policy instrument is the FFR and not the money base, which dropped out. Second, the response coefficient $\lambda_{\Delta x}$ (0.5) is larger, equals $1 + \lambda_{\Delta x}$ (1.5). Third, the modified rule is scaled by the parameter κ , which measures the sensitivity of base velocity to changes in the FFR. Further, equation (10) suggests that, in addition to the usual argument of the rule, the *level* of the FFR implied by the rule at time t also depends on the level of the FFR implied by the rule last period, ie, monetary policy inertia. Thus, the policymaker takes into account her past policy action when making decisions today. And finally, this rule is much closer to a nominal GDP rule than to a money base rule.

There is a major difference in the way the Taylor rule in equation (2) and the modified McCallum rule in equation (10) work. The Taylor rule suggests that the Fed raises the interest rate vigorously, but gradually in response to current deviations of output and inflation. This partial adjustment is determined by the size of α . In the modified McCallum rule, equation (10), when nominal GDP growth exceeds its target rate, the Fed keeps raising the FFR until nominal GDP growth is equal to its target. This continuous response is an important feature of this rule that differentiates it from the Taylor rule. There is no smoothing or gradual adjustment in the modified McCallum rule. In this regard, equation (10) is similar to the rule in Levin *et al* (2001).

However, κ in equation (10), which measures the sensitivity of base velocity to changes in the FFR acts as a smoother. A large value of κ dampens the volatility of nominal GDP growth fluctuations and makes past FFR important in a similar way the smoothing parameter in the Taylor rule works. The way the rule works is not by vigorously responding to the deviations of nominal GDP growth from its target (ie, $(1 + \lambda_{\Delta x})/\kappa$) is small in magnitude) but by

continuously hammering on these deviations. This is in essence similar to Bernanke and Woodford (1997).⁹

I plot the Taylor rule, equation (2), and the modified McCallum rule, equation (10). The Taylor rule in equation (2) is calibrated using $\bar{r} = 2$, $\Delta P_t^a = 2$, $\lambda_{\Delta P} = 0.5$, $\lambda_{\bar{y}} = 0.5$ and $\alpha = 0.24$. For the McCallum rule, average velocity is calculated for as a four-year average, $(1/16)(x_{t-1} - b_{t-1} + x_{t-17} + b_{t-17})$, $\lambda_{\Delta x} = 0.5$ and $\Delta \bar{x} = 4.5$. We don't know κ . I estimate κ from the data to be 50,¹⁰ which provides substantial smoothing. Equation (2) and (10) are plotted in figure 4. The two rules look similar, flat, in the 1960s and 1970s, but there is a little disagreement between them at the second half of the sample starting in the 1980s. Changes in the FFR implied by either the Taylor or the modified McCallum rules do not match the change in the FFR. The changes in the FFR implied by both rules are much smoother than the actual changes in FFR.¹¹

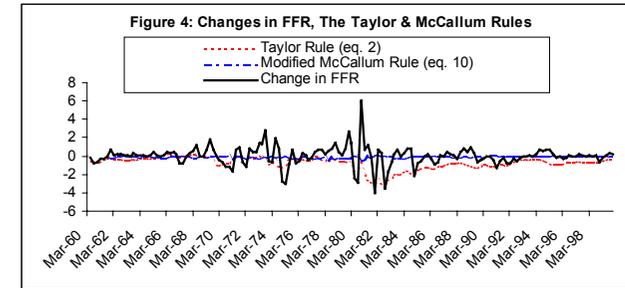
Of course if the magnitude of κ gets smaller the changes in the FFR implied by the modified McCallum rule become more volatile and matches with the actual changes in FFR. However, this kind of volatility will not be so apparent if we plot the rule in the levels instead because of the high persistence of interest rate in the rule and the fact that $((1 + \lambda_{\Delta x}) / \kappa)$ is small in magnitude. The first term in equation (11) dominates the second.

Although the modified McCallum rule response to the nominal GDP growth deviations from target is small in size, $1 + \lambda_{\Delta x} / \kappa = 1.5/50 = 0.03$, it is stable and there is no indeterminacy problem. The eigenvalues are fine and the response to past interest rate and nominal GDP growth indicates that the root is less than one.

⁹ They argued that a rule like $i_t = \rho i_{t-1} + \tau \pi_t$ with a small τ and large ρ has a similar stabilisation properties as a rule like $i_t = \tau \pi_t$ with a large τ except that it is much smoother.

¹⁰ Equation (10) is estimated by non-linear least squares from March 1962 to March 2000. The t_{κ} -statistic is 2.5, \bar{R}^2 is 0.12 and DW is 1.63. Estimating the same regression from 1980 to 2000 gives similar results.

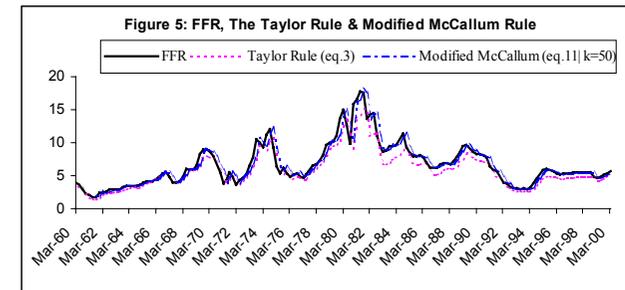
¹¹ The correlation coefficients between the change in FFR and each of the rules are -0.34 and 0.35 for the modified McCallum rule and the Taylor rule respectively.



Equation (10) in levels is given by:

$$\hat{i}_t = \hat{i}_{t-1} + \frac{\Delta V_t^a - (1 + \lambda_{\Delta x})(\Delta \bar{x} - \Delta x_t)}{\kappa} \quad (11)$$

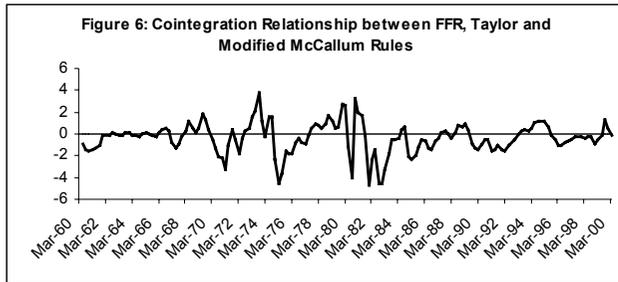
The level equations are plotted in figure 5. The FFR implied by the two rules and the actual FFR are indistinguishable. In general, figures 4 and 5 reveal that the linear relationship $\Delta V_t = \kappa \Delta i_t$ is a good proxy for $\Delta V_t = f(\Delta i_t) + \psi_t$.



Further, the two rules in the level and the actual FFR are cointegrated.¹² The cointegration relationship is plotted in figure 6.

¹² A Johansen system consists of the Federal funds rate, the interest rate implied by the Taylor rule i_t^* and the interest rate implied by the modified McCallum rule \hat{i}_t , a constant in the unrestricted model, each variable includes a linear trend that is not present in the cointegration relationship. The system has six lags. The trace and lambda max test statistics indicate that there is perhaps one cointegrating relationship. Results are not reported, but they are available upon request.

So far, the results suggest that these two different policy rules can be made similar if sufficient smoothing is applied to the rules such that past interest rate, interest rate smoothing or policy inertia dominate the rules' arguments. Or, if the amount of smoothing chosen is such that the two rules are the same. This seems trivial. Also, it seems discomfoting. These results suggest that the policymaker may be free to adopt any policy rule, even most volatile ones, provided that sufficient interest rate smoothing is applied.



Economists evaluate the stabilisation properties of policy rules by comparing the unconditional variances of inflation and output. These unconditional variances can be computed in different ways. Model-based simulation is a typical way. The results presented so far will be tested further using model-based simulations, which will be presented next. The results presented above are confirmed.

4 Model-based evaluations of the two rules

McCallum advocates robustness in the sense that a policy rule should perform well across *different* models. This is because economists do not agree on a particular macroeconomic model. Typically, researchers either estimate different models (eg, Rudebusch (2001), and McCallum and Nelson (1999b)) or calibrate different models. Then, these models are simulated along with the policy rules. The simulations are stochastic in the sense that a large number of shock realizations are generated randomly from a distribution that has the covariance properties of the historical shock estimates. Although one can compute the unconditional variances

without conducting stochastic simulation, Bryant et al. (1993, p. 373-375) suggest that stochastic simulation are consistent with the robustness issue.

In this paper, I choose a well-known New Keynesian model that has recently been estimated by Rudebusch (2001) and Dennis (2001). Meyer (2001) calls the New Keynesian model “The consensus model.” The model is also endorsed by Svensson (1999b), Clarida et al. (2000), McCallum and Nelson (1999a, b), Rotemberg and Woodford (1999) and Taylor (1999) and it is shown to have a foundation in dynamic general equilibrium models. The model consists of two equations, a Phillips curve and an aggregate demand curve. The Phillips curve is given by:

$$\Delta P_t = a_{10} + a_{11}E_t\Delta P_{t+1} + a'_{11}\left[\sum_{i=1}^4\beta_i\Delta P_{t-i}\right] + a_{12}\tilde{y}_{t-1} + \eta_{1t} \quad (12)$$

The inflation rate ΔP_t is $(\ln P_t - \ln P_{t-1}) * 400$ where P_t is a price index. The inflation rate depends on expected inflation, which consists of two components: a forward looking expectation and a fully backward-looking component that consists of four lagged values of inflation. Inflation also depends on lagged value of the output gap. The output gap is $(\ln y_t - \ln y_t^p) * 100$, where y_t is real GDP and y_t^p is the estimate of potential output published by the Congressional Budget Office. The restriction $a_{11} + a'_{11} = 1$ is typically tested and imposed on the model.

Researchers used different measures of inflation. For example, Rudebusch (2001a) and Dennis (2001) use GDP chain-weighted price index. Researchers also used different ways to estimate the degree of forwardness of inflation expectations. Rudebusch (2001a) used the University of Michigan Survey data of a year ahead expected price changes, while Dennis (2001) assumes $a_{11}=0$ in the Phillips curve. Instead, he imposes the restriction that $\sum_{i=1}^4\beta_i=1$. Others used ML estimator (Fuhrer, 1997).¹³

¹³ Also, see Roberts (1995, 2001).

The second equation in the model is the output equation, which is the intertemporal Euler equation given by:

$$\tilde{y}_t = a_{21}E_t\tilde{y}_{t+1} + a_{22}(i_t - E_t\Delta P_{t+1} - \bar{r}) + \eta_{2t} \quad (13)$$

Rudebusch (2001a) shows that when this model is simulated along with a McCallum rule like $i_t = \bar{r} + \Delta\bar{P}_t + \lambda_{\Delta x}(\Delta x_t - \Delta\bar{x})$ or $i_t = \lambda_{\Delta x}(\Delta x_t - \Delta\bar{x}) + \rho i_{t-1}$ and allowing for model and output gap uncertainty it indicates that the rule has very poor stabilisation properties. In other words, the unconditional variances of inflation, output and changes in interest rates are large. Because the objective of this paper is to evaluate the stabilisation properties of the McCallum rule derived in equation (10), I will use the same model. Data uncertainty is beyond the scope of this paper.

The experimental design

The inflation rate is defined in terms of the CPI instead of the GDP chain-weighted price index. It will be shown that the choice of the price index has a crucial impact on the degree of forwardness of expected inflation in the Phillips curve. The output gap is similar to the output gap used in Rudebusch (2001a) and Dennis (2001), deviations of real GDP from potential output measured by the CBO. The forward-looking component is measured by the University of Michigan Survey data of changes in prices, $E_{t-1}\Delta\bar{P}_{t+3}$. The monthly survey data are averaged to get the quarterly figures. The empirical inflation and the output equations given by Rudebusch (2001a) are:

$$\Delta P_t = a_{10} + a_{11}E_{t-1}\Delta\bar{P}_{t+3} + a'_{11}\left(\sum_{i=1}^4\beta_i\Delta P_{t-i}\right) + a_{12}\tilde{y}_{t-1} + \eta_{1t} \quad (14)$$

$$\tilde{y}_t = a_{20} + a_{21}\tilde{y}_{t-1} + a_{22}\tilde{y}_{t-2} + a_{23}(i_{t-1} - E_{t-1}\Delta\bar{P}_{t+3}) + \eta_{2t} \quad (15)$$

where a_{20} is $a_{22}\bar{r}$ in equation (13). I calibrate the model from March 1980 to March 2000. This sample period represents most of the Volcker-Greenspan period. I start the calibration from 1980 instead of March 1979 or earlier because some researchers suggested that there is a structural break in 1979 (Fuhrer (1997), Rotemberg and Woodford (1997), Clarida *et al* (1998)).

I calibrate the Phillips curve and the output gap equations (14) and (15) using values for the coefficients within the ballpark of coefficient values reported in Rudebusch (2001a) and Dennis (2001). The output gap equation is given by:

$$\tilde{y}_t = -0.24 + 1.28\tilde{y}_{t-1} - 0.37\tilde{y}_{t-2} - 0.08(i_{t-1} - E_t\Delta\bar{P}_{t+3}) \quad (16)$$

Three different versions of the Phillips curve are used. Two of them are calibrated. First, I set $a_{11} = 0$ and $a'_{11} = 1$ and the restriction $\sum_{i=1}^4\beta_i = 1$. So the Phillips curve has only backward-looking expectations. In the second, I set $a_{11} + a'_{11} = 1$ and for β 's I use coefficients similar to those reported in Rudebusch (2001a). The third version of the Phillips curve is an estimated one. The output gap equation is the same in all three cases.

The three Phillips equations are:

Backward-looking expectations

$$\Delta P_t = 0.06 + 0.65\Delta P_{t-1} - 0.10\Delta P_{t-2} + 0.30\Delta P_{t-3} + 0.15\Delta P_{t-4} + 0.15\tilde{y}_{t-1} \quad (17)$$

Note that $\sum_{i=1}^4\beta_i = 1$, Dennis (2001).

Mixed expectations

$$\Delta P_t = 0.06 + 0.30E_{t-1}\Delta\bar{P}_{t+3} + 0.70*[0.65\Delta P_{t-1} - 0.10\Delta P_{t-2} + 0.30\Delta P_{t-3} + 0.15\Delta P_{t-4}] + 0.15\tilde{y}_{t-1} \quad (18)$$

Note that $a_{11} + a'_{11} = 1$, Rudebusch (2001a).

Forward-looking expectations

There is no consensus among researchers about the degree of forwardness (of inflation expectations) in the Phillips curve. Ball

(1999), Svensson (1999a) and Orphanides (1999) set a_{11} to zero. McCallum and Nelson (1999a, b) and Rotemberg and Woodford (1999) set $a_{11}=1$. Estrella and Fuhrer (2001) argue that $a_{11}=0$ and Rudebusch (2001a) estimates a_{11} to be 0.29. While the estimation techniques, sample sizes, and measurements vary from one paper to another the consensus conclusion is: expectations are more backward looking than forward looking in the Phillips curve.

Figure 7 is a scatter plot of the CPI inflation rate and the University of Michigan Survey of price changes from 1980:1-2000:1.

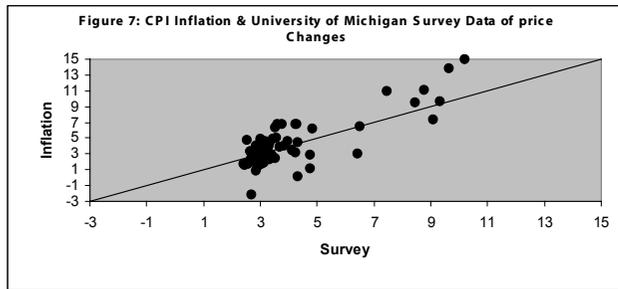
I estimate two regressions. The first regression is a single-equation Phillips curve from March 1980 to March 2000 by OLS:

$$\Delta P_t = a_{10} + a_{11}E_{t-1}\Delta\bar{P}_{t+3} + \sum_{j=1}^4 \beta_j \Delta P_{t-j} + a_{12}\tilde{y}_{t-1} \quad (19)$$

The second regression is to estimate the Phillips curve and the output gap equation (14 & 15) simultaneously from March 1980 to March 2000 by FIML.

$$\Delta P_t = a_{10} + a_{11}E_{t-1}\Delta\bar{P}_{t+3} + \sum_{j=1}^4 \beta_j \Delta P_{t-j} + a_{12}\tilde{y}_{t-1} + \eta_{1t} \quad (20)$$

$$\tilde{y}_t = a_{20} + a_{21}\tilde{y}_{t-1} + a_{22}\tilde{y}_{t-2} + a_{23}(i_{t-1} - E_{t-1}\Delta\bar{P}_{t+3}) + \eta_{2t} \quad (21)$$



The coefficient a_{20} in equation (21) is $a_{22}\bar{r}$ in equation (13). The results are reported in table 3 (a, b). The hypotheses that coefficient

$a_{11}=1$, and $\sum_{i=1}^4 \beta_i = 0$ cannot be rejected. The restriction that $a_{11} + \sum_{i=1}^4 \beta_i = 1$ does not hold either. The estimates of a_{20} (-0.24) and a_{23} (-0.08) imply that the equilibrium real interest rate is 0.019, which is approximately 2 per cent as Taylor suggested.

Thus, the third model I simulate is the following forward-looking model:

$$\Delta P_t = -0.66 + 1.1 E_{t-1}\Delta\bar{P}_{t+3} + 0.15 \Delta P_{t-1} - 0.17 \Delta P_{t-2} + 0.20 \Delta P_{t-3} - 0.05 \Delta P_{t-4} + 0.12 \tilde{y}_{t-1} \quad (22)$$

$$\tilde{y}_t = -0.24 + 1.28 \tilde{y}_{t-1} - 0.37 \tilde{y}_{t-2} - 0.08(i_{t-1} - E_t\Delta\bar{P}_{t+3}) \quad (23)$$

With each of the models, the backward-looking, mixed expectations and the forward-looking, I simulate the two policy rules, the Taylor rule in equation (2) and the modified McCallum rule in equation (10) separately (there is some evidence that the rules in differences are more robust than the rules in levels). The identity $\Delta x_t = \Delta y_t + \Delta P_t$, and the fact that $\tilde{y}_t = y_t - y_t^p$ and $\Delta\tilde{y}_t = \Delta y_t - \Delta y_t^p$ are used in the simulations. I experimented with different degrees of smoothing in the Taylor rule.

Each model and the policy rule are solved simultaneously. First, I solve the model deterministically using static simulation for each observation from March 1980 to March 2000 and an iterative algorithm (Gauss-Seidel) to compute the values of the endogenous variables ΔP , \tilde{y}_t and Δi_t . Then the model is solved stochastically. In this simulation, the model is solved repeatedly for different draws of the stochastic components of the model (the innovations to the Phillips curve and the output gap equations). The coefficients are fixed over the simulation. The innovations of the stochastic equations are generated by independently randomly drawing from the standard normal distribution. Each simulation consists of 1000 iterations.

The results suggest that the hypothesis, that the unconditional volatility of inflation and the output gap across all three models and

the two rules are equal, cannot be rejected. However, the unconditional volatility of Δi_t across models and rules is sensitive to the degree of interest rate smoothing in the rules. The degree of interest rate smoothing in the modified McCallum rule (equation 10) is determined by $1/\kappa$, which is fixed to 1.5/50 (0.03). This provides much higher smoothing than $\alpha = 0.24$ provides to the Taylor rule.

One can always choose the degree of interest rate smoothing in the Taylor rule, α , such that the two rules have similar effects on the variance of Δi_t . As the value of α in the Taylor rule increases (ie, less interest rate smoothing) the unconditional variance of Δi_t increases. I report the results of two experiments, one when α is 0.24 (ie, modified McCallum rule smoothes interest rate more than the Taylor rule does), and the other when α is 0.02 and 0.01 (arbitrary). This implies that the modified McCallum rule and Taylor rule have equal degrees of interest rate smoothing. When α in the Taylor rule is 0.24 the modified McCallum rule has superior stabilisation properties, and when α is 0.02 and 0.01, the two rules produce the same volatility across all models.

Table 4a reports the unconditional standard deviations of the actual data, and of the simulated paths of inflation, the output gap and the change in FFR (when α is 0.24). It also reports the probability of the F statistics, which test the hypothesis that these unconditional standard deviations are equal. Table 4b is similar to table 4a except that α in the Taylor rule is 0.02 and 0.01 (more interest rate smoothing than when α is 0.24).

The modified McCallum rule is stable even though the rule is presented in terms of Δi_t as a function of $(\Delta V_t^a, \Delta x_t, \lambda, \kappa)$. Although the response to deviations of nominal GDP growth from its target is relatively small in magnitude (ie, $(1 + \lambda)/\kappa = 0.03$) and the response of i_t to its own past is large (ie, 1), the rule achieves its objective by continuously responding to the deviations until they are eliminated. This is in essence similar to what Bernanke and Woodford (1997) have proposed. They argued for more interest rate smoothing in an interest rate rule that has a small response coefficient to inflation's deviations and a large response coefficient to past interest rate. They showed that this rule has similar stabilisation properties as an interest rate rule that responds only to inflation's deviation with a

very large coefficient. The difference is that the former rule is smoother than the later. The modified McCallum rule in this paper should not be confused with, and it is not the same as a constant interest rate rule or an interest rate peg rule. This rule has different stabilisation properties.

These results are not surprising and confirm the propositions stated in the introduction. They suggest that one can choose the smoothing parameter such that a similar path for i_t can be computed using many different policy rules. Levin *et al* (2001) demonstrate that a policy rule that is nearly optimal is a rule with a high interest rate smoothing

5 Conclusions

It is difficult to compare the Taylor rule with the money base-nominal GDP targeting rule because the instruments are different. The policy instrument in the Taylor rule is the Federal funds rate (FFR). In the money base-nominal GDP targeting rule, the instrument is the monetary base. The underlying theories of transmission mechanism of the two rules are also different.

To compare the two rules this paper modifies the money base-nominal GDP targeting rule by replacing the monetary base as the instrument with the FFR. This rule is referred to as the modified McCallum rule. To modify the rule, I relied on the empirical evidence that base velocity is, on average, a stable function of the FFR. In other words, base velocity is either cointegrated with the actual FFR or the coefficients of the regression of interest rate on velocity are stable over a long period of time. It was assumed that changes in velocity are proportional to changes in the FFR. This linear approximation proved to be a good proxy for the underlying relationship between base velocity and the FFR. This smoothed modified McCallum rule is in fact very similar to the Taylor rule in the level. The levels of the modified McCallum rule, the Taylor rule and the FFR are themselves cointegrated.

There is a disagreement on the stabilisation properties of the McCallum rule when the instrument is the nominal interest rate. Some stochastic simulation exercises using a "consensus" New

Keynesian model suggest that the McCallum rule has inferior stabilisation properties. Also, it has been argued elsewhere that the rule would produce more volatility in output and inflation and it would typically introduce instrument instability or in other words, a highly volatile path for the interest rate. Other research demonstrates that the rule dominates the Taylor rule under certain conditions.

In this paper I tested the stabilisation properties of the two rules using the same New Keynesian model that is typically used in evaluation experiments. Three models were used. They differ in the degree of forwardness of inflation expectations (backward looking, forward-looking and a mixed expectations). I found that the Taylor rule and the modified McCallum rule can be made essentially equivalent in the sense that the unconditional volatility of inflation and output are similar across the models. For the two rules to have similar stabilisation properties they have to have similar “sufficiently high” degree of interest rate smoothing. The modified McCallum rule presented in this paper is stable. The modified McCallum rule achieves its objective not by responding so aggressively to deviations of nominal GDP growth from the target, but rather by continuously responding the deviations until they are eliminated.

References

- Ball, L, (1999), “Efficient rules for monetary policy,” *International Finance*, Vol. 2, 63-83.
- Bernanke B and M Woodford, (1997), “Inflation forecasts and monetary policy,” *Journal of Money, Credit and Banking*, 653-684.
- Blinder, A S, (1994), “The rules-versus discretion debate in the light of recent experience,” *Weltwirtschaftliches Archiv*, CXXIII, 400-414.
- Bryant, R C, P Hooper and C L Mann, (1993), Evaluating Policy Regimes: New Research in Empirical Macroeconomics, Washington: *Brookings Institution*.
- Clarida, R, J Gali, and M Gertler, (2000), “Monetary policy rules and macroeconomic stability: Evidence and some theory,” *Quarterly Journal of Economics*, 147-180.
- Clarida, R, J, Gali and M Gertler, (1998), “Monetary policy rules in Practice: Some international evidence,” *European Economic Review* 42, 1033-67.
- Dennis, R, (2001), “The policy preferences of the US Federal Reserve,” Manuscript, *Federal Reserve Bank of San Francisco*.
- Estrella, A and J Fuhrer, (2001), “Dynamic inconsistencies: counterfactual implications of a class of rational expectations models,” Manuscript, *Federal Reserve Bank of Boston*. Forthcoming in the *American Economic Review*.
- Feldstein, M and J Stock, (1994), “The Use of a Monetary Aggregate to Target Nominal GDP,” in Monetary Policy (ed. G Mankiw), 7-62. Chicago: Chicago University Press.

- Fuhrer, J C, (1997), “The (Un) importance of forward-looking behavior in price specifications,” *Journal of Money, Credit and Banking*, vol 29, no 3, 338-350.
- Goodhart, C E A, (1994), “What should central banks do? What should be their macroeconomic objectives and operations?” *The Economic Journal*, vol 104, 1424-1436.
- Gordon R, (1985), “The Conduct Of Domestic Monetary Policy,” in Monetary Policy in Our Times (ed. A. Ando et al.) 45-81. Cambridge, MA: MIT Press.
- Hall, R and G Mankiw, (1994), “Nominal Income Targeting,” in Monetary Policy (ed. G Mankiw), 71-93. Chicago: Chicago University Press.
- Judd, J P and G Rudebusch, (1998), “Taylor’s Rule and the Fed: 1970-1997,” *Federal Reserve Bank of San Francisco Economic Review*, no 3, 3-16.
- Laubach, T and J C Williams, (2001), “Measuring the neutral rate of interest,” *Federal Reserve Board of Governors*. Washington DC.
- Levin, A, V Wieland and J C Williams, (2001), “The performance of forecast-based monetary policy rules under model uncertainty,” *Federal Reserve Board of Governors*. Washington DC.
- Lucas, R Jr, (1988), “Money demand in the United States: A quantitative review,” *Carnegie-Rochester Conference Series on Public Policy* 29, 137-68.
- McCallum, B T, (1988), “Robustness properties of a rule for monetary policy,” *Carnegie – Rochester Conference on Public Policy* 29, 173-204.

- McCallum, B T and E Nelson, (1999a), “Performance Of Operational Policy Rules In An Estimated Semi-Classical Structural Model,” in Monetary Policy Rules (ed. John Taylor), 15-54. Chicago: Chicago University Press.
- McCallum, B T and E Nelson, (1999b), “Nominal income targeting in an open-economy optimizing model,” *Journal of Monetary Economics*, vol 43, 553-78.
- Meltzer, A, (1998), “Monetarism: the issues and the outcome,” *Atlantic Economic Journal*, vol 26, no 1, 8-31.
- Meltzer A, (1987), “Limits of short-run stabilisation policy,” *Economic Inquiry* 25, 1-13.
- Meyer, L H, (2001), “Does money matter?” *Federal Reserve Bank of St. Louis*, the 2001 Homer Jones Memorial Lecture.
- Orphanides, A, (2000), “Activist stabilisation policy and inflation: The Taylor rule in the 1970s,” *Federal Reserve Board of Governors*, Washington, DC.
- Orphanides, A, (1999), “The quest for prosperity without inflation,” Manuscript, *Federal Reserve Board of Governors*. Washington, DC.
- Orphanides, A, (1997), “Monetary policy rules based on real-time data,” *Finance and Economics Discussion Paper Series*, 1998-03, Federal Reserve Board of Governors, Washington DC.
- Roberts, J, (1995), “New Keynesian economics and the Phillips Curve,” *Journal of Money Credit and Banking*, 975-84.
- Roberts, J, (2001), “How well does the Keynesian Sticky-Price Model fit the data?” *FEDS Working Paper* 2001-13, Federal Reserve Board, Washington, DC.
- Rotemberg, J and M Woodford, (1999), “Interest Rate Rules In An Estimated Sticky Price Model,” in Monetary Policy rules (ed. John Taylor), 57-119. Chicago: Chicago university Press.

- Rudebusch, G, (2001a), “Assessing nominal income rules for monetary policy with model and data uncertainty,” Forthcoming in the *Economic Journal*.
- Rudebusch, G, (2001b), “Term structure evidence on interest rate smoothing and monetary policy inertia,” Manuscript, *Federal Reserve Bank of San Francisco*.
- Stock, J and M Watson, (1993), “A simple estimator of cointegrating vectors in higher order cointegrated systems,” *Econometrica* 61, 783-820.
- Svensson, L, (1999a), “Inflation targeting: some extensions,” *Scandinavian Journal of Economics*, vol 101, 337-61.
- Svensson, L, (1999b), “Inflation targeting as a monetary policy rule,” *Journal of Monetary Economics*, vol 43, 607-54.
- Taylor, J, ”An historical analysis of monetary policy rules,” in Monetary Policy Rules (ed. John Taylor), (1999), 319-41. Chicago University Press, Chicago, IL.
- Taylor J B, (1993), Macroeconomic Policy in a World Economy, W W Norton, New York.

Table 1: Relationship between velocity and the interest rate 1957:1 – 2000:1

OLS $V_t = \phi_0 + \phi_1 ffr_t + \eta_t$			
	Coefficient	t- Statistics	P- Value
ϕ_0	13.4	51.89*	0.0001
ϕ_1	0.57	15.84*	0.0001
\bar{R}^2	0.60		
DW	0.11		

V_t is the base velocity defined as $P_t Y_t / b_t$, where P_t is GDP deflator, Y_t is real GDP and b_t is the money base. The FFR is given by frr_t .

The Engle-Granger Test

OLS $\Delta \eta_t = \delta_0 + \rho \eta_{t-1} + \sum_{i=1}^j \delta_i \Delta \eta_{t-i} + \vartheta_t$			
	Coefficient	t-Statistics	P-Value
δ_0	0.01	0.40	0.6854
ρ	-0.08	-3.00 [#]	0.0032
δ_1	0.15	1.96	0.0520
δ_2	-0.12	-1.54	0.1238
δ_3	0.06	0.85	0.3960
δ_4	0.15	1.97	0.0494

The asymptotic critical value at the 95% level is -3.34 so the cointegration is significant at the 90% level.

An Error Correction Regression

OLS $\Delta V_t = \theta + \gamma \Delta i_t + \rho \eta_{t-1} + \zeta_t$			
	Coefficient	t-Statistics	P-Value
θ	0.02	1.81	0.0720
γ	0.09	6.61*	0.0001
ρ	-0.03	-3.53*	0.0005
\bar{R}^2	0.20		
DW	1.74		

Asterisks mean significant at the 95% level.

Table 2OLS $\Delta V_t = a + b\Delta i_t + e_t$

Sub samples	57Q1-74Q4	75Q1-86Q4	87Q1-2000Q1
b	0.07 (0.0001)	0.10 (0.0001)	0.11 (0.0001)
F	3.86		

F is to test the hypothesis that b is equal across sub-samples.
P values in parentheses.

Table 3a

OLS, sample March 1980 – March 2000

$$\Delta P_t = a_{10} + a_{11}E_{t-1}\Delta\bar{P}_{t+3} + \sum_{i=1}^4 \beta_i \Delta P_{t-i} + a_{12}\tilde{y}_{t-1}$$

Coefficient	Estimate	t-statistic	Prob.
a_{10}	-0.62	-1.07	0.2880
a_{11}	1.15	2.84	0.0058
β_1	0.12	0.90	0.3670
β_2	-0.18	-1.45	0.1491
β_3	0.23	1.92	0.0590
β_4	-0.07	-0.70	0.4838
a_{12}	0.12	1.43	0.1564
Wald $H_0 : a_{11} = 1$	0.70		0.8875
\bar{R}^2	0.67		
DW	2.08		
$\hat{\sigma}$	1.56		

The Wald statistic tests the hypothesis that $a_{11} = 1$.

Table 3b

FIML sample March 1980 – March 2000

$$\Delta P_t = a_{10} + a_{11}E_{t-1}\Delta\bar{P}_{t+3} + \sum_{i=1}^4 \beta_i \Delta P_{t-i} + a_{12}\tilde{y}_{t-1}$$

$$\tilde{y}_t = a_{20} + a_{21}\tilde{y}_{t-1} + a_{22}\tilde{y}_{t-2} + a_{23}(i_{t-1} - E_{t-1}\Delta\bar{P}_{t+3})$$

Coefficient	Estimate	t-statistic	Prob.
a_{10}	-0.64	-1.21	0.2258
a_{11}	1.11	3.14	0.0017
β_1	0.15	1.12	0.2633
β_2	-0.17	-1.51	0.1307
β_3	0.20	1.82	0.0685
β_4	-0.04	-0.41	0.6786
a_{12}	0.12	1.85	0.0643
Wald $H_0 : a_{11} = 1$	0.09		0.7523
Wald $\sum_{i=1}^4 \beta_i = 0$	0.35		0.5506
Wald $\sum_{i=1}^4 \beta_i = 1$	14.58		0.0001
\bar{R}^2	0.67		
DW	2.10		
$\hat{\sigma}$	1.56		
a_{20}	-0.24	-1.52	0.1269
a_{21}	1.28	11.60	0.0001
a_{22}	-0.37	-3.56	0.0004
a_{23}	-0.08	-3.00	0.0027
\bar{R}^2	0.91		
DW	2.13		
$\hat{\sigma}$	0.69		

Table 4a

Unconditional Standard Deviation of the Endogenous Variables across the Three Models and the Two Policy Rules in equations (2) and (10)

	Actual	Simulation		
		New Keynesian Backward-Looking Model		
		Taylor Rule ($\alpha = 0.24$)	Modified McCallum Rule	Prob. F statistics ^a
$\sigma_{\Delta P}$	2.762894	2.857274	2.854476	0.9930
$\sigma_{\bar{y}}$	2.325268	2.161327	2.133673	0.9086
$\sigma_{\Delta i}$	1.141051	0.701355	0.077573	0.0000
		New Keynesian Mixed Expectations Model		
$\sigma_{\Delta P}$		2.503492	2.500447	0.9913
$\sigma_{\bar{y}}$		2.176283	2.149646	0.9126
$\sigma_{\Delta i}$		0.686967	0.022502	0.0000
		New Keynesian Forward-Looking Model		
$\sigma_{\Delta P}$		2.219793	2.188742	0.9000
$\sigma_{\bar{y}}$		2.216740	2.189973	0.9137
$\sigma_{\Delta i}$		0.680162	0.020061	0.0000

a , F statistics test the hypothesis that the variances $\sigma_{\Delta P}^2$, $\sigma_{\bar{y}}^2$ and $\sigma_{\Delta i}^2$ corresponding to the model with the Taylor rule are equal to those corresponding to the model with the modified McCallum rule

Table 4b

Unconditional Standard Deviation of the Endogenous Variables across the Three Models and the Two Policy Rules equations (2) and (10)

	Actual	Simulation		
		New Keynesian Backward-Looking Model		
		Taylor Rule ($\alpha = 0.025$)	Modified McCallum Rule	Prob. F statistics ^a
$\sigma_{\Delta P}$	2.762894	2.853956	2.854476	0.9987
$\sigma_{\bar{y}}$	2.325268	2.137075	2.133673	0.9887
$\sigma_{\Delta i}$	1.141051	0.072356	0.077573	0.5349
		New Keynesian Mixed Expectations Model ($\alpha = 0.01$)		
$\sigma_{\Delta P}$		2.500325	2.500447	0.9997
$\sigma_{\bar{y}}$		2.150952	2.149646	0.9957
$\sigma_{\Delta i}$		0.028331	0.022502	0.0409
		New Keynesian Forward-Looking Model ($\alpha = 0.01$)		
$\sigma_{\Delta P}$		2.217431	2.188742	0.9076
$\sigma_{\bar{y}}$		2.191263	2.189973	0.9958
$\sigma_{\Delta i}$		0.028056	0.020061	0.0030

a , F statistics test the hypothesis that the variances $\sigma_{\Delta P}^2$, $\sigma_{\bar{y}}^2$ and $\sigma_{\Delta i}^2$ corresponding to the model with the Taylor rule are equal to those corresponding to the model with the modified McCallum rule.