Inflation Targeting under Potential Output Uncertainty

Victor Gaiduch and Benjamin Hunt

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Abstract

The significant lags between policy actions and their subsequent impact on prices means that, to achieve their price stability objective, many monetary authorities must base their actions on indicators of future price pressure. One indicator of future price pressure that is often relied on is the output gap, defined as the difference between current output and potential output. However, an economy’s potential output is unobservable and estimates of its level must be extracted from observable data. A common feature that all the techniques for estimating potential output share is a high degree of uncertainty as to their

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1 Reserve Bank of New Zealand and International Monetary Fund Research Department respectively. The views expressed in this paper are those of the authors and may not reflect those of the International Monetary Fund or the Reserve Bank of New Zealand. The authors would like to thank, without implication, Aaron Drew, Peter Isard and Douglas Laxton for helpful comments. The authors alone are responsible for all errors and omissions.

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accuracy. Because of this uncertainty, policymakers are often criticised for relying on these estimates to guide their actions.

In this paper, measures of the uncertainty surrounding estimates of New Zealand’s potential output are used to consider whether the output gap is a useful concept for the monetary authority to base policy actions on. The analysis relies on stochastic simulations of the Reserve Bank of New Zealand’s Forecasting and Policy System macroeconomic model (FPS). The analysis shows that when the policymaker’s estimates of the output gap have large serially correlated errors that are positively correlated with the business cycle, both output and inflation become more variable. However, the output gap is still useful for stabilising output and inflation. Basing policy actions on the output gap directly and/or indirectly through forecasts of inflation leads to better macroeconomic stability than basing policy actions solely on currently available observable data such as inflation and output growth.
1 Introduction

The concept of sustainable productive capacity is playing an increasingly important role in monetary policy formulation throughout the world. Specifying price stability as a central objective of monetary policy has contributed to this increased importance. The long lags between policy actions and inflation outcomes mean that indicators of future inflation pressures must be relied on to guide current policy actions that are aimed at achieving price stability. The extent to which an economy’s productive resources are being utilised is considered to be a useful indicator of future price pressures. Whether productive resources are defined in terms of the goods market (potential output) or the labor market (trend unemployment), policymakers rely on estimates of these concepts to determine whether current levels of activity can be sustained without generating price pressures. If activity is deemed to be above a sustainable level, policymakers may suspect that upward pressure on inflation will emerge if they do not take actions to moderate activity. Conversely, if current activity is below the sustainable level this may lead policymakers to want to stimulate activity to avoid future downward pressure on inflation.

Although this framework for guiding monetary policy has received significant attention and accreditation in both the literature and in practice, it still has numerous critics. The most often heard criticism is that measures of an economy’s productive capacity are very uncertain. There are no directly observable measures of either potential output in the goods market or the trend rate of unemployment in the labor market. Policymakers must derive estimates of these theoretical concepts from observable economic data. The various techniques that are used to generate these estimates have one common feature; they provide very uncertain estimates of what the output gap or the labor market gap might be.

The objective of the analysis presented in this paper is twofold. The first is to consider whether there is some point where the policymaker’s errors about potential output and thus the output gap become large enough that it is no longer a useful guide for policy actions. The second objective is to consider how potential output uncertainty affects the performance of simple policy rules. The analysis relies on historical estimates of New Zealand’s output gap generated from three different estimation techniques to specify what output gap errors might look like. Given these errors, stochastic simulations of the Reserve Bank of New Zealand’s macroeconomic model are then used to examine their

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2 In Phillips (1958), the relationship between wage inflation and unemployment was first specified. The natural rate extensions to the theory were added in Friedman (1968) and Phelps (1970). It is in within this framework that extensive research has been conducted examining the relationship between the utilisation of an economy’s productive resources and inflation. Orphanides (1999) provides a recent perspective on the importance of potential output in the conduct of monetary policy.

3 For example see McCallum and Nelson (1999).

implications for the stabilisation properties of a range of simple rules for guiding monetary policy actions.\footnote{Only simple policy rules are considered for several reasons. First, Rudebusch and Svensson (1998) have shown that simple rules yield similar macroeconomic stability to optimal rules. Second, Levin, Wieland and Williams (1999) show that simple rules are more robust to model uncertainty. Finally, Clarida Gali and Gertler (1997) demonstrate that simple rules appear to explain how policymakers actually behave.}

The estimates from three alternative models of New Zealand’s potential output suggest that, in addition to statistical uncertainty, there are two other possible sources of output gap estimation error of concern to policymakers.\footnote{The three models of New Zealand’s output gap are presented in Conway and Hunt (1997), Scott (2000) and Claus (2000a).} Errors that could arise from using an incorrect model and errors from revisions to real-time estimates. The three estimates of New Zealand’s potential output that are considered suggest that these errors may be quite large and could have a high degree of serial correlation and correlation with the business cycle. If estimation models assign too much of the longer-term variability in output to the supply side then an important component of the policymaker’s output gap error may be highly serially correlated and positively correlated with the business cycle. Although considerable research has been conducted examining the implications for policy design of potential output errors arising from statistical uncertainty,\footnote{For example see Wieland (1998), Rudebusch (1998), Orphanides (1998), Estrella and Mishkin (1999), Smets (1999) and Orphanides et al (2000).} less has been done focussing on errors that exhibit both serial correlation and positive correlation with the business cycle.\footnote{See Isard et al (1998) and Drew and Hunt (2000)} Consequently, this paper focuses on the implications for simple monetary policy rules of output gap errors that exhibit these properties.

The results indicate that when the policymaker’s estimates of the output gap have large serially correlated errors that are positively correlated with the business cycle, relying on them to guide policy is still a sensible thing to do. Responding to an erroneous estimate of the output gap directly and/or indirectly through its implications for an inflation forecast results in lower inflation and output variability than responding only to the observable data on inflation and output growth. The results suggest that the optimal response to output gap uncertainty may not always be to respond less aggressively to the estimates of the output gap. Under the error process considered here, the attenuation in optimal policy responses occurred in terms of the policy response to actual or forecast inflation. In the absence of instrument variability constraints, attenuation in optimal responses is only consistently evident when the policymaker’s preferences are either equally weighted in terms of inflation and output variability control or more heavily weighted towards output variability control. If the policymaker’s preferences are more heavily weighted towards inflation control, no consistent attenuation in optimal response is evident. However, once constraints are imposed on instrument variability, the policy attenuation result is evident for the range of policymakers’ preferences considered.
The remainder of the paper is structured as follows. Section 2 contains a very brief outline of the macroeconomic model that is used for the analysis. The estimates of New Zealand’s historical output gap generated from three different techniques are presented in section 3. This section also contains some statistics describing what the time-series properties of certain dimensions of output gap errors might look like and presents the statistical properties of the errors used in the simulation analysis. The simulation results are presented in sections 4 and 5 and section 6 contains some conclusions.

2 The Forecasting and Policy System Model (FPS)

The Reserve Bank’s Forecasting and Policy System consists of a set of models that together form the framework for generating economic projections and conducting policy analysis. The system consists of the core macroeconomic model, indicator models and satellite models. To prepare economic projections, all the models in the system are used. To conduct policy analysis, like that presented in this paper, just the core macroeconomic model is used.

The core FPS model describes the interaction of five economic agents: households, firms, a foreign sector, the fiscal authority and the monetary authority. The model has a two-tiered structure. The first tier is an underlying steady-state structure that determines the long-run equilibrium to which the economy converges. The second tier is the dynamic adjustment structure that traces out how the economy converges towards that long-run equilibrium.

The long-run equilibrium is characterised by a neo-classical balanced growth path. Along that growth path, consumers maximise utility, firms maximise profits and the fiscal authority achieves exogenously-specified targets for debt and expenditures. The foreign sector trades in goods and assets with the domestic economy. Taken together, the actions of these agents determine expenditure flows that support the set of stock equilibrium conditions underlying the balanced growth path.

The dynamic adjustment process overlaid on the equilibrium structure embodies both “expectational” and “intrinsic” dynamics. Expectational dynamics arise through the interaction of exogenous disturbances, policy actions and private agents’ expectations. Policy actions are introduced to re-anchor expectations when exogenous disturbances move the economy away from equilibrium. Because policy actions do not immediately re-anchor private expectations, real variables in the economy must follow disequilibrium paths until expectations return to equilibrium. To capture this notion, expectations are modeled as a linear combination of a backward-looking autoregressive process and a forward-looking model-consistent process. Intrinsic dynamics arise because adjustment is

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9 One can only hypothesis what the error process might look like because actual potential output is never observed.

10 See Black, Cassino, Drew, Hansen, Hunt, Rose and Scott (1997) and Hunt, Rose and Scott (2000) for more detailed descriptions of the FPS core model and its properties.
The costs of adjustment are modeled using a polynomial adjustment cost framework. In addition to expectational and intrinsic dynamics, the behaviour of the fiscal authority also contributes to the overall dynamic adjustment process.

On the supply side, FPS is a single good model. That single good is differentiated in its use by a system of relative prices. Overlaid on this system of relative prices is an inflation process. While inflation can potentially arise from many sources in the model, it is fundamentally the difference between the economy’s supply capacity and the demand for goods and services that determines inflation in domestic prices. Further, the relationship between goods-markets disequilibrium and inflation is asymmetric. Excess demand generates more inflationary pressure than an identical amount of excess supply generates in deflationary pressure. Although direct exchange rate effects have a small impact on domestic prices and, consequently, on expectations, they enter consumer price inflation primarily as price-level effects.

The monetary authority effectively closes the model by enforcing a nominal anchor. In the model, the primary channel through which monetary policy achieves its objective is via its influence on the level of demand for goods and services relative to the economy’s productive capacity ie the output gap. The open economy dimension means that both interest rates and the exchange rate have important influences on the level of demand for goods and services. Interest rates reflect the relative cost of consuming and investing today versus tomorrow. Consequently, interest rates affect aggregate demand through their impact on the intertemporal consumption/savings decisions of households and the intertemporal investment decisions of firms. The exchange rate influences aggregate demand through its impact on the relative price of domestically- versus foreign-produced goods.

The model’s dynamic response to a temporary 100 basis point increase in the short-term nominal interest rate under its standard inflation-forecast-based monetary policy rule is presented in appendix A.

3 Errors associated with estimates of the output gap

Because potential output is unobservable, it is impossible to ever know the precise time-series properties of the errors associated with estimates of the output gap. However, looking across different estimation techniques provides some insights about some of the statistical properties that output gap estimation errors may process. In this section, three alternative techniques for estimating New Zealand’s output gap are examined to help specify the output-gap error process used in the simulation analysis of alternative policy rules.

The essence of estimating potential output is decomposing an economy’s real output series into two components. A permanent component, which represent the economy’s

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11 The direct exchange rate effect on domestic prices is assumed to arise through competitive pressures.
underlying productive capacity, and a cyclical component, which represents temporary fluctuations in the demand for goods and services around that permanent component. There are a large number of techniques that have been used to estimate the permanent and cyclical components of an economy’s output. A review of many of these techniques can be found in Laxton and Tetlow (1992) with additional techniques presented in Kuttner (1994) and Baxter and King (1995).

The technique currently used by the Reserve Bank of New Zealand within the Forecasting and Policy System is outlined in Conway and Hunt (1997). The technique is a multivariate filter (MVF) approach similar to that found in Laxton and Tetlow (1992). This technique adds conditioning information from simultaneously estimated economic relationships to improve the demand and supply identification properties of the Hodrick and Prescott (1997) univariate filter. The conditioning relationships are based on a natural-rate price Philips curve, Okun’s Law and capacity utilisation.

Because of the important role accorded to potential output and the output gap in the implementation of New Zealand’s inflation targeting framework, two other models of New Zealand’s potential output have also been estimated. Scott (2000) presents an estimated unobserved-components model (UCM) similar to that in Kuttner (1994). Using the Kalman filter and maximum likelihood estimation, the unobserved trend, cyclical and irregular components are simultaneously estimated for three related economic series: output, unemployment and capacity utilisation. The model imposes a common cyclical component across the three series but allows each to have a unique trend and irregular component. Claus (2000a) estimates a structural vector autoregression (SVAR) based on Blanchard and Quah (1989). This model includes output, employment and capacity utilisation. To decompose output into its cyclical and permanent components, the techniques relies on the restriction that only supply shocks have a permanent effect on output.

One important source of error in estimating the output gap is statistical uncertainty. All estimated models have a quantifiable degree of statistical uncertainty associated with their parameters and residuals. This is the uncertainty captured by the confidence intervals presented around estimates of the output gap derived from models like the UCM and the SVAR. The implications for operating monetary policy under this type of uncertainty are probably the best understood. Looking at its implication for simple policy rules, Smets (1999) and Wieland (1998) show that it yields the familiar Brainard (1967) attenuation result in terms of the response of the policy instrument to the noisy estimate of the state variable. Adding persistence to this uncertainty Rudebusch (1998) and Orphanides et al (2000) also show how it also leads to milder policy responses relative to the certainty case.

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12 Smets (1999) uses the estimated state vector error covariance matrix returned by the Kalman recursion which does not include the uncertainty associated with the parameter estimates. In this sense it underestimates the true uncertainty associated with the estimates of the state variables.
However, two remaining sources of possible output gap estimation error are the focus of this paper. One is the error that could arise from using the “wrong” model of potential output. The other is the revision to real-time estimates that arises from two-sided estimation techniques like the MVF and the UCM.

3.1 Errors from using the wrong model

The three alternative estimates of New Zealand’s output gap are presented in figure 1. The estimates of the output gap from the UCM and the SVAR are often larger than the MVF estimate of the output gap. How does the policymaker decide which estimate to base policy actions on? Looking at the properties of different output gap estimates in the frequency domain as in Conway and Frame (2000) provides some useful insights. The comparison of the inflation forecasting properties of the alternative estimates in Claus (2000b) also provides some valuable information. However, neither approach appears to be able to definitively determine which model might be considered “the truth.” Looking across the estimates from these three alternative models, which by no means is an exhaustive set of the possible alternatives, indicates that large output gap estimation errors could arise from the policymaker using the “wrong” model of potential output.

Figure 1: Alternative estimates of New Zealand’s output gap

In the New Zealand case, the MVF is the technique that is used to generate the “official” measure of the output gap. However, one of the alternative models could be closer to the “truth.” The statistical properties of the errors if either the UCM or the SVAR model were correct are presented in table 1. Although the magnitudes vary slightly when the pre-reform period is included in the sample period, the basic story remains the same. Three statistical properties are notable. First, the errors associated with using an incorrect model of the output gap could be quite large with the standard deviations usually in excess of 2 percentage points. Second, the errors arising from an incorrect model all have a high degree of persistence. The first-order serial correlation coefficient is generally around 0.9. Finally, in the particular error cases considered, the errors are positively correlated with the output gap that is assumed to be correct.
The frequency domain analysis of the properties of UCM and SVAR estimates of the output gap in Conway and Frame (2000) highlights that they both have considerably more power at very low frequencies than does the MVF estimate. The UCM and the SVAR estimates of the output gap allocate a considerable portion of shocks with relatively long duration (more than 32 quarters) to the cyclical component of output. Consequently, the resulting output gaps tend to be larger and longer lived. This behaviour of the estimated SVAR and UCM output gaps is the primary reason for the properties reported in table 1.

Table 1: Summary statistics of possible output gap errors arising from using the “wrong” model

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation of difference</th>
<th>First-order serial correlation of difference</th>
<th>Correlation of the difference with the “true” output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample period 197 to 1999</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truth (SVAR) – estimate (MVF)</td>
<td>2.58</td>
<td>0.96</td>
<td>0.76</td>
</tr>
<tr>
<td>Truth (UCM) – estimate (MVF)</td>
<td>1.77</td>
<td>0.87</td>
<td>0.67</td>
</tr>
<tr>
<td>Sample period 1985 to 1999</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truth (SVAR) – estimate (MVF)</td>
<td>2.94</td>
<td>0.96</td>
<td>0.91</td>
</tr>
<tr>
<td>Truth (UCM) – estimate (MVF)</td>
<td>2.10</td>
<td>0.91</td>
<td>0.86</td>
</tr>
</tbody>
</table>

### 3.2 Revisions to real-time estimates

Both the UCM and the MVF are based on two-sided filtering techniques. Essentially the decomposition of date \( t \) output into a trend and cyclical component relies not only on past and current realisations of observed data, but on future realisations as well. Using future realisations allows for better identification of innovations that are permanent and those that are temporary. When deriving estimates of the historical output gap it makes sense to use as much information as possible to derive the best possible estimates; however, this two-sided dimension has an unfortunate implication for policymakers.

For policymakers the most useful indicator of future inflation pressure is today’s estimate of the output gap. However, the estimate of today’s output gap, the real-time estimate,
produced by two-sided estimation techniques will undergo significant revision as time passes and new observations on output (and other filtered data used by the model) become available. The magnitude of these revisions to the real-time estimates can be very large. Some descriptive statistics for these revisions from the MVF and UCM are presented in table 2.

**Table 2: Summary statistics of revisions to real-time estimates**

<table>
<thead>
<tr>
<th>Estimation model</th>
<th>Standard deviation of revisions</th>
<th>First-order serial correlation in revisions</th>
<th>Correlation of revision with fully revised output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVF</td>
<td>1.30</td>
<td>0.97</td>
<td>-0.30</td>
</tr>
<tr>
<td>UCM</td>
<td>1.50</td>
<td>0.96</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Similar properties in the errors that arise from using the wrong model also appear evident in the errors arising from revisions to real-time estimates, they can be large and persistent and those from the UCM are positively correlated with the business cycle. Orphanides and Van Norden (1999) note these same properties in the revisions to real-time estimates of the output gap from a wide range of univariate estimation techniques. They note that the magnitude and persistence properties are evident in the revisions to official real-time estimates of the US output gap presented in Orphanides (1999).

As was the case with the differences between the full-sample estimates from the three alternative models, one of the reasons that the revisions to real-time estimates exhibit the statistical properties they do is because there is a tendency to initially assign too much of the variation in output at the end of the sample to the supply side. As more observations become available that suggest the variation is temporary, revisions occur.

### 3.3 Output gap errors for the simulation analysis

The estimates of New Zealand’s potential output that are considered suggest that in addition to the white noise associated with the estimation model’s statistical uncertainty, the policymaker’s output gap errors may also have an important component that has a high degree of serial correlation and is positively correlated with the business cycle. Using an estimation technique that assigns too much of the longer-term variability in output to the supply side can give rise to such errors. The estimation technique may in

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13 One of the reasons that the revisions to real-time estimates from the MVF do not exhibit positive correlation with the business cycle is because the filter places additional constraints on the growth rate of trend output at the end of sample. When there are no future observations on output, these constraints prevent the filter from allocating a large portion of the change in output to the supply side. Although this technique is an effective means of constraining the growth rate in the estimate of potential output, it increases the risk of missing shifts in the growth rate when they occur.
general have this property and/or it could be present in its real-time estimates. To focus the simulation analysis on the implications of the policymaker’s estimates of the output gap exhibiting errors with these two properties, the Hodrick Prescott filter and a base-case data set generated using the same set of seeded stochastic disturbances used in all the subsequent analysis are used to generate the errors. The details of how the errors are generated can be found in appendix B. The statistical properties of the errors for the three uncertainty cases examined are presented in table 3.

Table 3: Statistical properties of the output gap errors used in the stochastic simulations

<table>
<thead>
<tr>
<th>Case</th>
<th>Standard deviation of output gap errors</th>
<th>First-order serial correlation of output gap errors</th>
<th>Correlation of output gap errors with the true output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>0.90</td>
<td>0.94</td>
<td>0.83</td>
</tr>
<tr>
<td>Two</td>
<td>1.80</td>
<td>0.95</td>
<td>0.88</td>
</tr>
<tr>
<td>Three</td>
<td>3.50</td>
<td>0.95</td>
<td>0.88</td>
</tr>
</tbody>
</table>

4 The implications of potential output uncertainty

Stochastic simulations of FPS are conducted to examine the effects of output gap uncertainty on the efficiency and robustness of simple monetary policy rules. In these simulations, stochastic disturbances affect consumption, investment, the exchange rate, inflation, foreign demand, and the terms of trade. FPS’s rest-of-world sector is closed by a rest-of-world monetary authority that targets inflation. Consequently, the response of the rest-of-world sector and its monetary authority to the stochastic disturbances implies that the foreign interest rates and the commodity prices faced by the domestic economy also fluctuate in a systematic fashion over rest-of-world business cycles.14

In the output gap uncertainty simulations, the monetary authority can misperceive the lagged, current and future output gaps. This uncertainty enters the simulations as unforeseen and only slowly perceived (by the monetary authority) shocks to productivity. The errors do not persist indefinitely so that on average the monetary authority gets the level of potential output right. Households and firms correctly perceive the current level of the output gap; however, they cannot perfectly forecast future productivity and so their forecasts of future output gaps can be incorrect.

14 More details regarding the process for performing stochastic simulations with FPS can be found in Drew and Hunt (1998).
4.1 The monetary rules

The general form of the simple monetary policy rule used in the simulations is as follows:

\[ rs_t = rs_{eq} + \alpha \cdot \text{ygap}_t + \theta \cdot (\log(y/y_{t-1}) - g^*) + \beta \cdot (\pi_{t+i} - \pi^*), \tag{1} \]

where \( rs_t \) is the short-term nominal interest rate, \( rs_{eq} \) is its equilibrium level, \( \text{ygap}_t \) is the policymaker’s perceived output gap, \( \log(y/y_{t-1}) \) is output growth, \( g^* \) is a trend growth rate, \( \pi_{t+i} \) is the model-consistent forecast of inflation \( i \) quarters ahead, \( \pi^* \) is target inflation and \( \alpha, \theta, \) and \( \beta \) are response coefficients.

When \( \alpha \) and \( \beta \) are greater than zero and \( i \) and \( \theta \) equal zero then the policy rule becomes a generalized Taylor rule (GT). When \( \alpha \) and \( \theta \) are equal to zero and \( i \) and \( \beta \) are greater than zero the rule becomes an inflation-forecast-based rule (IFB). When \( \theta \) and \( \beta \) are greater than zero and \( \alpha \) and \( i \) are equal to zero the rule becomes an unconstrained nominal-income-growth rule (NIG).\(^{15}\) The target growth rate, \( g^* \), is set equal to the true long-run average growth rate in real output. When \( \alpha \) and \( \beta \) are greater than zero, \( \theta \) is equal to zero and \( i \) is greater than zero the rule become a hybrid of a generalized Taylor rule and an inflation-forecast-based rule (IFB+G). These are the four classes of simple policy rules searched over to trace out the implications of output gap uncertainty. Six values for \( \alpha \) and \( \theta \), seven values for \( \beta \) and 13 values for \( i \) are searched over. In total this comes to 588 different policy rules.\(^{16}\)

4.2 The efficient frontiers

The efficient frontiers, as defined in Taylor (1979) to be the locus of the lowest achievable combinations of inflation and output variability, are traced out in figure 2 for the four cases considered. The frontiers are traced out for what might be termed “reasonable” preferences over inflation and output variability. The quadratic loss function is given by:

\[ L = \sum_{t=1}^{\infty} (1-\lambda) \cdot (\text{ygap}_t)^2 + \lambda \cdot (\pi_t - \pi^*)^2, \tag{2} \]

\(^{15}\) This rule is unconstrained in the sense that the response to the deviations of real output growth from trend and inflation from its target level are not constrained to be the same (i.e. \( \theta \) and \( \beta \) are not constrained to be identical) as would be the case in a simpler nominal-income-growth rule.

\(^{16}\) This breaks down to 42 GT rules, 42 NIG rules, 84 IFB rules and 420 IFB+G rules. For each rule, 70 draws covering 100 time periods are done. Because FPS is non-linear rational expectations model, calculating the inflation and output stabilisation properties of policy rules requires the use of a forward-path solution technique. For the uncertainty cases, each time period requires 2 full forward-path model simulations. In total this exercise required just under 29 million full forward-path model simulations.
where $y_{gap}$ is the output gap, $\pi$ is year-over-year CPI inflation, $\pi^*$ is the target rate of inflation and $\lambda$ is the relative weight on inflation versus output variability. The frontiers are traced out for values of $\lambda$ between 0.25 and 0.75. The monetary authority for which $\lambda = 0.75$ could be thought of as an inflation hawk, while a monetary authority for which $\lambda = 0.25$ could be thought of as an inflation dove. The cases where the policymaker is either an inflation nutter, $\lambda = 1$, or an output nutter, $\lambda = 0$, are not graphed. It is worth noting that because the loss function uses the deterministic level of potential output and target inflation it penalizes policy rules for the deviations between these and the average levels of output and inflation that result in the simulations. Because FPS has a nonlinear inflation process, the average or stochastic-steady-state level of output will be below potential output and the stochastic-steady-state rate of inflation will be above the target rate. If the loss function used the simple variance of output and inflation, the impact of different policy rules on the two variables first moments would not be penalised.

**Figure 2: The efficient frontiers**

Not surprisingly, both inflation and output are more variable when the policymaker misperceives the true value of the output gap. The frontier closest to the origin in figure 3 is for the case when the policymaker is not misperceiving the output gap. The frontiers shift towards the north east as the standard deviation of the policymaker’s output gap error increases. Where the most deterioration shows up depends on preferences. Relative to the case of no uncertainty, the inflation hawk ($\lambda = 0.75$) takes relatively more of the deterioration in increased output variability while the inflation dove ($\lambda = 0.25$) takes relatively more of the deterioration in increased inflation variability. Table 4 contains the deterioration in inflation and output variability relative to the case of no output gap errors for $\lambda$ values of 0.75, 0.5 and 0.25.
Table 4: The deterioration in inflation and output variability

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\lambda = 0.75$ (Hawk)</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 0.25$ (Dove)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 (Standard deviation of error = 0.9)</td>
<td>$\Delta$RMSDCPI</td>
<td>0.05</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>$\Delta$RMSDYGAP</td>
<td>0.55</td>
<td>0.40</td>
</tr>
<tr>
<td>Case 2 (Standard deviation of error = 1.8)</td>
<td>$\Delta$RMSDCPI</td>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>$\Delta$RMSDYGAP</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>Case 3 (Standard deviation of error = 3.5)</td>
<td>$\Delta$RMSDCPI</td>
<td>0.60</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>$\Delta$RMSDYGAP</td>
<td>1.80</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Assuming that, given preferences, the policymaker can choose where on the efficient frontier to be implicitly assumes that the policymaker has the ability to re-optimise conditional on the errors being made about the output gap. In the short run, this is not an option for the policymaker. Consequently, an important question becomes ‘how does the nature of efficient policy rules change in the face of this output gap uncertainty?’ Table 5 contains the characteristics of the rules that lie on the frontiers graphed in figure 3. The columns of the table denote the four alternative cases. The rows correspond to different preferences over inflation and output variability. The weight on inflation variability, $\lambda$, varies from 0.25 to 0.75 in increments of 0.05. Each rule is characterised by three numbers. The first number is the optimal response coefficient on the output gap ($\alpha$). The second is the optimal response coefficient on the deviation of inflation from target ($\beta$) and the third is the optimal horizon (quarterly) for the forecast deviation of inflation from target ($i$).

The first point to note about the policy rule characteristics presented in table 5 is that the policy rules that respond only to contemporaneous, observable variables do not appear on any of the frontiers.\(^{17}\) The optimal value of the weight on the deviation of contemporaneous output growth from its long-run trend, $\theta$, is always zero and the optimal horizon, $i$, is always greater than zero. In other words, rules that adjust the policy instrument only in response to contemporaneous deviations of inflation or nominal income from target never achieve better macroeconomic stability than those that respond either directly or indirectly to the misperceived output gap. This result is consistent with

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\(^{17}\) This is true even under the extreme preferences of the inflation or output nutters ($\lambda = 0$ or 1).
that presented in Orphanides et al (2000). Rules responding only to contemporaneous inflation and/or output growth do not appear on the frontiers. Moreover in almost all cases these rules comprise the envelope that could be termed the least efficient frontier. They are the rules that lie the furthest from the efficient frontier. Appendix C contains graphs of all the inflation and output variability combinations achieved under all the policy rules searched over for each of the four cases. The outcomes under the contemporaneous inflation and nominal income targeting rules are indicated by the Xs, while all the other rules’ outcomes are denoted with dots.

The second point to note is that the efficient response coefficient on the output gap represent a corner solution at the largest output gap response coefficient searched over. Work with a smaller version of FPS outlined in Hargreaves (1999) indicates that response coefficients on the output gap larger than 2.25 are not feasible as they consistently violate the nonnegativity constraint on the short-term nominal interest rate. However, imposing this constraint explicitly in the full FPS model increases solution times sufficiently to make simulation tasks as large as the one conducted here infeasible. Consequently, to keep solution times manageable and at the same time rule out response coefficients previously shown to violate the nonnegativity constraint those values were eliminated from the grid search conducted here.

The third point to note about the characteristics of the rules is that the policy response attenuation result, reported in several other studies of potential output uncertainty, is certainly not as pervasive in these result. For the policymakers whose preferences can be represented by \( \lambda \in [0.25, 0.5] \), there is some mild reduction in the optimal response of the policy instrument to the inflation forecast. However, there is no reduction in the optimal response to the misperceived output gap itself.

The policy response attenuation to the inflation forecast is evident on two margins. First, the response coefficient on the model-consistent inflation forecast tends to decline as output gap uncertainty increases. Depending on preferences and the degree of uncertainty, response coefficients decline from 1.5 and 3 to 1 and 1.5. The second margin along which there is also some attenuation is the optimal response horizon. Holding the response coefficient fixed, the optimal response horizon lengthens as uncertainty increases. To see why this should be interpreted as attenuation, consider a single inflation forecast done at date \( t \). Given the dynamic properties of the model, the peak effect on inflation of shocks affecting the economy at date \( t \) will generally occur at approximately date \( t+6 \). Provided there is some level of policy response to the inflationary consequences of the disturbance, by dates \( t+9 \) through to date \( t+12 \), the forecasted deviations of inflation from target will be getting smaller and smaller. With an unchanged response coefficient on the output gap of 2.25, inflation in each individual forecast is getting closer and closer to target as the response horizon lengthens beyond about 6 quarters. Consequently, holding the response coefficient fixed the net response of the policy instrument is getting smaller and smaller as the response horizon is extended beyond 6 quarters.
For the policymakers whose preferences can be represented by \( \lambda , [0.55, 0.75] \) the changes in characteristics of efficient rules does not point as unambiguously to attenuation as output gap uncertainty increases. There is a case where an inflation forecast response coefficient declines and there are some case where response coefficients increase. However, response horizons do lengthen when the response coefficients increase so the net result on the degree of responsiveness of the instrument is unclear.

In total, the pervasiveness of the policy response attenuation effect of the output gap uncertainty in these results is milder than that found in Smets (1999), Wieland (1998), Rudebusch (1999) and Orphanides et al (2000). One source of the difference could be the nature of the errors considered here. Using statically similar errors, Isard et al (1998) and Drew and Hunt (2000) find less evidence that optimal policy responses become milder under this type of uncertainty. Another source of the difference could be the non-linearities in inflation process in the models used in Drew and Hunt (2000) and Isard et al (1998). However, another possible source could be the restrictions placed on instrument variability in some of the other studies. In Smets (1999) and Orphanides et al (2000) explicit constraints are placed on instrument variability within the optimization. Because output and inflation variability increase when the output gap is estimated with less precision, leaving response coefficients unchanged in the face of this increased variability would imply increases in instrument variability. Consequently, placing constraints on that variability could be one of the reasons for the decline in optimal response coefficients reported in the other studies.\(^{18}\)

\(^{18}\) The only restrictions imposed have been in terms of the magnitudes of the policy weights searched over. They have been selected based on earlier work to remove from consideration rules that would consistently violate the non-negativity constraint on nominal interest rates.
Table 5:
Efficient rules, presented output gap coefficient \((\alpha)\) inflation gap coefficient \((\beta)\) horizon \((i)\) \((\theta\) is always zero on the efficient frontier\)

<table>
<thead>
<tr>
<th>Preferences</th>
<th>No uncertainty</th>
<th>Case 1 Standard deviation of error = 0.9</th>
<th>Case 2 Standard deviation of error = 1.8</th>
<th>Case 3 Standard deviation of error = 3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda= 0.25) (Dove)</td>
<td>(\alpha)</td>
<td>(\beta)</td>
<td>(i)</td>
<td>(\alpha)</td>
</tr>
<tr>
<td>2.25</td>
<td>1.50</td>
<td>4.00</td>
<td>2.25</td>
<td>1.00</td>
</tr>
<tr>
<td>2.25</td>
<td>1.50</td>
<td>5.00</td>
<td>2.25</td>
<td>1.50</td>
</tr>
<tr>
<td>2.25</td>
<td>3.00</td>
<td>12.00</td>
<td>2.25</td>
<td>3.00</td>
</tr>
<tr>
<td>2.25</td>
<td>3.00</td>
<td>10.00</td>
<td>2.25</td>
<td>3.00</td>
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<tr>
<td>2.25</td>
<td>3.00</td>
<td>5.00</td>
<td>2.25</td>
<td>3.00</td>
</tr>
<tr>
<td>(\lambda = 0.5)</td>
<td>2.25</td>
<td>3.00</td>
<td>5.00</td>
<td>2.25</td>
</tr>
<tr>
<td>2.25</td>
<td>3.00</td>
<td>5.00</td>
<td>2.25</td>
<td>3.00</td>
</tr>
<tr>
<td>2.25</td>
<td>6.00</td>
<td>3.00</td>
<td>2.25</td>
<td>3.00</td>
</tr>
<tr>
<td>2.25</td>
<td>6.00</td>
<td>4.00</td>
<td>2.25</td>
<td>6.00</td>
</tr>
<tr>
<td>(\lambda = 0.75) (Hawk)</td>
<td>2.25</td>
<td>9.00</td>
<td>3.00</td>
<td>2.25</td>
</tr>
</tbody>
</table>

5  Implications of constraints on instrument variability

To consider the effect of instrument variability constraints, frontiers are recomputed placing two restrictions on instrument variability. The first considers only rules for which the standard deviation of the first difference in the policy instrument, the 90-day nominal interest rate, is less than 2 percentage points.\(^{19}\) The second eases the restriction to 2.5 percentage points. The rules that lie on these frontiers are presented in tables 6 and 7.

Imposing restrictions on instrument variability reduces the number of different rules that appear on the frontiers. Consequently, before placing too much weight on these results, a finer grid search in the neighborhood of the rules appearing on these two frontiers would probably be advisable. However, caveats in mind, three interesting results emerge from imposing restrictions on instrument variability. First, attenuation does still not occur in the response coefficient on the output gap. Second, the policy attenuation result in the

\(^{19}\) The historical average in New Zealand over the 1985 to 1999 period was just under 2 percentage points.
optimal response to inflation is consistently evident for the complete range of policymakers’ preferences. Third, imposing instrument constraints influences the type of policy rules that appear on the frontier.

Imposing the instrument constraint reduces the magnitude of the optimal coefficient on the output gap from 2.25 to 1.00 for all cases considered. However, the magnitude of the optimal response coefficient on the output gap does not vary as the degree of potential output uncertainty varies. In terms of the optimal policy response to inflation, under the 2 percentage point constraint, attenuation is evident in the magnitude of the response coefficient on inflation. Under the 2.5 percentage point constraint, the attenuation in optimal response is evident in both the response coefficient and the response horizon. In terms of the nature of efficient rules, under the 2 percentage point constraint, the optimal inflation-forecast horizon is always zero. The frontier is comprised totally of GT rules which contrasts with the fact that no GT rules appear on the frontier when no constraints are imposed on instrument variability. As the instrument constraint is eased to 2.5 percentage points, fewer GT rules appear on the frontier and rules with a wide range of inflation forecast horizons appear. Under the 2.5 percentage point constraint, GT rules appear only for inflation-hawkish preferences under no uncertainty and case 1.

Because so few rules appear on the frontier under the 2 percentage point constraint, this result about the optimal forecast horizon should be interpreted with caution. However, if the result was to hold up under a more detailed grid search in the neighborhood of the GT rules appearing on this frontier, it would help reconcile a result reported in Levine, Wieland and Williams (1999). The authors find that using model-consistent inflation forecasts of up to four quarters ahead does not materially improve upon the stabilisation properties of GT rules that respond only to contemporaneous inflation. In the analysis, explicit constraints are imposed on the quarterly change in interest rates within the optimization framework. Several other studies, which have not imposed explicit constraints on the quarterly change in instrument variability, have found that using forecasts of inflation rather than contemporaneous inflation does improve the stabilisation properties of Generalised Taylor rules.²⁰

One reason that imposing a constraint on the quarter-to-quarter change in the policy instrument could lead to this optimal forecast horizon result is that when the inflation argument appearing in the policy rule is forward looking versus contemporaneous, the policy instrument tends to behave more as a jumper. For any given disturbance, the initial movement in the instrument is larger because the future impact of the disturbance on inflation is being factored in. Consequently, when instrument constraints are imposed via the quarterly change in the policy instrument, they will tend to be more binding for IFB and IFB+G rules than for GT rule. This, among other things, may help explain why in Levine, Wieland and Williams (1999) using model-consistent inflation forecasts of up to four quarters ahead does not materially improve upon the stabilisation properties of GT rules that respond only contemporaneous inflation. Because explicit constraints are

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²⁰See for example Black, Macklem and Rose (1998), Haldane and Batini (1998) and Isard, Laxton and Eliasson (1999).
imposed on the quarterly change in interest rates within the optimization framework, the informational advantage arising from being forward looking may not be exploitable.

Table 6: Efficient rules when the standard deviation of the short-term nominal interest rate is less than 2 percentage points

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Certainty</th>
<th>Case 1 Standard deviation of error = 0.9</th>
<th>Case 2 Standard deviation of error = 1.8</th>
<th>Case 3 Standard deviation of error = 3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$i$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\lambda = 0.25$</td>
<td>1.0</td>
<td>3.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\lambda = 0.5$</td>
<td>1.0</td>
<td>6.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>6.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>6.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>6.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\lambda = 0.75$</td>
<td>1.0</td>
<td>6.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Table 7: Efficient rules when the standard deviation of the short-term nominal interest rate is less than 2.5 percentage points

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Certainty</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard deviation of error = 0.9</td>
<td>Standard deviation of error = 1.8</td>
<td>Standard deviation of error = 3.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda = 0.25$</td>
<td>$\lambda = 0.5$</td>
<td>$\lambda = 0.75$</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$i$</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
</tr>
</tbody>
</table>

6 Conclusions

Uncertainty about the level of potential output or the trend rate of unemployment will always be an important issue for monetary policymakers. However, the simulation analysis presented in this paper suggests that if uncertainty gives rise to output gap errors that have an important component that is serially correlated and positively correlated with the business cycle, the misperceived output gap is still useful for guiding monetary policy actions. Responding to the misperceived output gap either directly or indirectly through its implication for an inflation forecast can achieve better macroeconomic stability than responding to current inflation and/or output growth.

As has been found in other studies of output gap uncertainty, optimal policy responses to the uncertainty considered here also exhibit the standard Brainard (1967) attenuation result. However, if explicit instrument variability constraints are not imposed, the attenuation in optimal responses is less widespread across reasonable alternative preferences over inflation and output variability. The dimension along which the attenuation in optimal policy response is evident appears to be dependent on the statistical properties of the errors. In most other studies the optimal policy response to the noisy state variable itself declines. However, given the serial correlation in the errors and their correlation with the business cycle the decline in optimal policy response reported here shows up in the response to inflation. The response coefficient itself declines in magnitude and/or the optimal forecast horizon increases. These results suggest that if in
addition to white noise statistical uncertainty, the policymaker’s errors about potential output also have an important component that is serially correlated and positively correlated with the business cycle then the optimal response to the uncertain output gap may not decline as much as other studies have suggested.

The properties of the three techniques for estimating New Zealand’s potential output suggest that if the policymaker’s estimation technique erroneously assigns more of the longer-term variation in output to the supply side then an important component of its output gap errors will be serially correlated and positively correlated with the business cycle. A comparison of the full-sample estimates from the three alternative models for New Zealand illustrates that such an error can arise because the estimation model simply decomposes output into its trend and cyclical components that way. Further, erroneously assigning too much of the longer-term variability in real output to the supply side can also arise in the real-time estimates from models based on two-sided filtering techniques.

The result of the increase in the inflation-forecast horizon is interesting in the sense that some may find it a little counter-intuitive. Forecasting uncertainty is often the foundation for criticism leveled against policymakers who are forward-looking in the sense that they rely on forecasts of inflation to guide their actions. The results presented here certainly do not validate that criticism. In fact they suggest that the optimal response in the face of an important dimension of the output gap uncertainty faced by policymakers is to be even more forward looking than most say they are.
References


Appendix A

Impact of a four-quarter, one percentage-point increase in the short-term nominal interest rate

- **Nominal 90 day interest rate**

- **Inflation**

- **Output gap**

- **Real exchange rate**
Appendix B

Generating output gap errors

A multi-step procedure is used to generate the time-series paths for the output gap errors used in the simulation analysis. The model is first simulated under the set of seeded stochastic disturbances with monetary policy characterised by an inflation-forecast-based rule\textsuperscript{21} and potential output is known with certainty. To generate the errors, the resulting times-series paths for real output from each draw are de-trended using the HP filter in two ways. First, the HP filter is run as a true filter through data using only data up to time $t$ to extract the real-time estimate of potential up to and including time $t$. Next the HP filter is run as a smoother using the complete data set up to time $T$ to extract final estimates of potential output for all $t$. The monetary authority’s error about potential output is the difference between the real-time filter estimate and the full-sample smoother estimate.

To generate the case 1 errors, before the filter is applied to extract the date $t$ estimate, four quarters of estimated data points are added to the real output series using the correct average growth rate in potential output. This is done to reduce the standard deviation of the errors to roughly 1 percentage point. To generate case 2 errors no future data is used to generate date $t$ real-time estimates. This results in errors with a standard deviation of slightly less than 2 percentage points and slightly more serial correlation and correlation with the business cycle than in case 1. Case 3 errors are simple twice as large as case 2 errors.

\textsuperscript{21} This inflation-forecast-base rule is the one used with FPS to generate the economic forecast published by the Reserve Bank of New Zealand. The policy instrument responds to the model-consistent deviations of inflation from target six, seven and eight quarters ahead. The response coefficient is set at 1.4.
Appendix C

Inflation and output variability outcomes under all policy rules considered with no uncertainty about potential output

Case 1: Potential output uncertainty
Case 2: Potential output uncertainty

Case 3: Potential output uncertainty