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**SDS-FPS: A small demand-side version of the
Forecasting and Policy System Core Model**

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Abstract¹

This paper describes the development of SDS-FPS, which is a small demand-side model calibrated to match some of the dynamic properties of the Reserve Bank's Forecasting and Policy System (FPS) Core Model. SDS-FPS is capable of matching the dynamic properties of FPS for a wide range of disturbances, despite lacking relative prices, having no explicit supply side, and having the entire demand side described by a single IS equation. The calibration of SDS-FPS has also provided some insights into the features of FPS that cannot be replicated in a small demand-side model.

The size of SDS-FPS makes its use feasible in situations where the use of FPS may not be practical. For example, stochastic simulation experiments can be performed much faster with SDS-FPS than with FPS, making more complex experiments computationally feasible.

¹ This work has benefited greatly from discussions with all members of the modelling team, and participants in an internal seminar at the Reserve Bank in June 1998. The views here are the authors and they may not represent the views of the Reserve Bank of New Zealand.

1 Introduction

At the centre of the Reserve Bank's Forecasting and Policy System is a large-scale modern macroeconomic model, referred to as the Core model. The behaviour of agents within the model in many areas is forward looking and based on explicit utility maximisation. The model has been calibrated to reflect the dynamic properties of the New Zealand economy².

These characteristics make FPS very different to the small, stylised demand-side models frequently used in research, particularly research related to monetary policy (for example McCallum (1995), Svensson (1997), Ball (1996), and De Brouwer and Ellis (1998)). These models do not have the richness of a full-scale macroeconomic model. But the parsimony of these models makes them easy to solve (they can frequently be solved analytically), and makes the dynamic interactions between variables more transparent.

In late 1997, the Reserve Bank elected to develop a small structural model (SDS-FPS) similar to those in the papers noted above, with parameters calibrated to achieve dynamic behaviour similar to that of the full FPS model. One motivation behind this was to see how closely a small model of this sort could match the dynamic properties of a more fully-articulated macroeconomic model.

The structure of SDS-FPS and the calibration methodology used in its creation are discussed in sections two and three respectively. A key diagnostic test while developing SDS-FPS was assessing its ability to replicate the behaviour of FPS under a range of deterministic shocks. As shown in section four, the parameterisation of SDS-FPS eventually settled on closely matches FPS under most shocks.

SDS-FPS is also intended to be used to carry out stochastic simulation analysis. In recent research, the Reserve Bank has performed stochastic simulations using FPS and shocks from a VAR model estimated on New Zealand data³. As documented in section five of this paper, SDS-FPS is able to mimic this stochastic simulation method, and closely match the moments generated by FPS in a comparable experiment.

SDS-FPS can be used to produce stochastic simulation results much (approximately 8 to 10 times) faster than FPS. This is particularly important for computationally intensive stochastic experiments: for example, searching over a range of policy rules to find efficient ones (as in Drew and Hunt (1999)). Moreover, SDS-FPS can often be recalibrated much more simply than FPS. This, for example, makes it much easier to study *model uncertainty* by running experiments where the policymaker is not using the true model of the economy: these experiments require multiple recalibrations.

² For a full description of the FPS model, see Black et al. (1997).

³ Conway, Drew, Hunt and Scott (1998), Drew and Hunt (1999).

2 The structure of SDS-FPS

In this section, the model is introduced and discussed. At the core of the model is an IS curve, Phillips curve, exchange rate equation and monetary policy reaction. The IS curve represents the evolution of the real economy as a result of policy actions, and exogenous determinants like the foreign output gap. The Phillips curve relates inflation to inflation expectations and the evolution of the real economy. The exchange rate is determined by a ‘dampened’ uncovered interest parity relationship. Finally, interest rates are determined by the monetary authority to control inflation.

The remaining equations can be divided into 4 blocks.

- Those determining the relative price of consumption goods, which is important as it is the CPI the monetary authority seeks to stabilise.
- Equations determining agent’s inflation and exchange rate expectations.
- The interest rate block, which determines short-term and long-term interest rates consistent with the expectations hypothesis.
- The trade price block, which contains simple relationships between domestic and foreign export and import prices and an identity determining the terms of trade.

The equations are listed and discussed in more detail below. In the equations, (L) denotes a lag polynomial and (F) a lead polynomial. Coefficients are denoted by Greek letters. Variable names are in lower case roman letters, with ε terms representing exogenous disturbances to that equation, and $*$ terms representing the exogenous equilibrium value of a variable.

The IS curve (specified in output gap terms) is given by;

$$y = (A_1(L)y + A_2(F)y) + A_3(L)rsl + A_4(L)(z - z^*) + A_5(L)(pxrow - pxrow^*) + A_6(L)(pmrow - pmrow^*) + A_7(L)gaprow + \varepsilon_y \quad (1)$$

where y denotes the output gap, rsl the slope of the yield curve, z the real exchange rate, $pxrow$ and $pmrow$ export and import prices denominated in foreign currency, and $gaprow$ the foreign output gap.

The IS curve is designed to replicate the demand side of FPS. Innovations to the output gap are persistent, with both lags and leads of the gap appearing on the right hand side of the equation. Recent theoretical work has argued that an IS curve derived from an optimising framework gives a relation between the current output gap and its

expected lead rather than its lag⁴. However, there are many potential sources of output *inertia* in FPS which suggest lags will play an important role in creating persistence⁵. The specification allows the dynamic evolution of the output gap to be influenced by monetary policy (through the interest and exchange rate channels), the foreign sector (via foreign demand and the prices of traded goods) and by exogenous disturbances to domestic demand. This captures many of the demand side influences contained in FPS. However, SDS-FPS has no stock-flow relationships (for example, there are no wealth or capital stock variables). This absence is standard for smaller models of this sort but means that some of the influences on demand seen in the full FPS model (such as wealth effects) cannot be captured here. Moreover, unlike in FPS where potential output comes from a fully articulated production function, permanent shocks to productivity or the level of a factor input cannot be explicitly modelled.

Equation (1) is similar to the aggregate demand equation that Svensson (1998), and Rotemberg and Woodford (1997) derive from representative agent microfoundations. Svensson derives a structure in which aggregate demand is a function of foreign demand, current and expected future interest rates, and the real exchange rate. These are the same determinants as seen in (1) except that we also allow for an impact from the relative prices of traded goods, as we believe the openness of the New Zealand economy means that these relative price shifts can have large influences on aggregate demand. As in Svensson, partial adjustment is imposed (i.e. the lag of the left hand side variable is put on the right hand side without explicitly deriving that representation). To match the properties of FPS, it was also necessary to put lag terms into the relationship between output and the variables that influence it.

The Phillips curve equation is given by;

$$\begin{aligned} \pi = & B_1 * \pi^e + (1 - B_1) * B_2(L)\pi + B_3(L)y + B_4(L)[y+] \\ & + B_5\Delta pm + B_6\Delta px + B_7(L)(\pi^c - \pi) + \varepsilon_\pi \end{aligned} \quad (2)$$

where π denotes the rate of change in the price of domestically produced and consumed output, π^e its expected rate of change, pm and px domestically denominated export and import prices, and π^c the rate of change in the consumption deflator.

Expected inflation is given by;

$$\pi^e = C_1(L)\pi + C_2(F)\pi \quad (3)$$

The key price in FPS and SDS-FPS is the *price of domestically produced and consumed output*. The rate of change in this deflator (π) is given by the Phillips curve. As in FPS, the Phillips curve is partially autoregressive, with a lag polynomial in π representing inflation inertia. Inflation is also influenced by a term representing inflation expectations (π^e). Inflation expectations are partly forward and partly backward looking. The sum of the coefficients on the terms representing expectations

⁴ See McCallum (1997) and the references contained therein, also Sargent (1998).

⁵ Lags produce dynamics consistent with the dynamics generated by the costly adjustment optimising framework employed in the full FPS model.

and inflation inertia is one. This restriction, often described as the natural rate restriction, implies that there is no permanent trade-off between inflation and output.

The output gap enters via a distributed lag of the actual gap, as well the *positive* realisations. This creates an *asymmetry* in the relationship between the output gap and inflation, which has very important policy implications. For example, stabilising inflation after a positive demand shock will require a temporary negative output gap, as in a linear model. But the asymmetry implies there will be a net output loss as a result of the shock, and that loss will be minimised by prompt action to bring output back to potential. There would be no net output loss if the model was linear, no matter how slowly the monetary authority reacted to the shock.⁶

Increases in the price of imported intermediate goods, or the price at which finished goods can be sold overseas will tend to push up domestic prices through competitive pressures: these influences are captured here by the pm and px terms. Finally, the gap between π^c and π is a proxy for wage pressure, which is clearly much simpler than the wage bargaining process in the Core FPS model.

We interpret innovations in the Phillips curve as arising from exogenous temporary supply shocks like droughts: that is, shocks which alter prices while leaving the long run level of potential output unchanged.

In Svensson (1997), a similar Phillips curve is derived using an open-economy extension of Rotemberg and Woodford's (1997) representative consumer/producer model. In his derived Phillips curve, prices are determined by the (model-consistent) expectation of future prices, aggregate demand, and the real exchange rate (which impacts through the cost of imported intermediate imports). Partial adjustment is imposed to slow the response of prices to these underlying determinants. The differences between Svensson's specification and (2) are that the (domestically denominated) price of imports is used to capture intermediate input effects, and a different, somewhat ad-hoc measure of wage pressure is employed. Also, we impose an expectational process which is only partly rational (equation 3), and allow more lags and an asymmetry in the relationship between inflation and the output gap.

Consumer prices are determined in the following block;

$$\pi^c = \pi + D_1(L)\Delta pm + D_2(L)\Delta y \quad (4)$$

$$\pi^{cpi} = E_1(L)\pi^c + E_2(z - z(-3)) \quad (5)$$

$$\pi^{cpi4} = \sum_{-3}^0 \pi^{cpi} / 4 \quad (6)$$

where π^{cpi} denotes the quarterly rate of change in the consumer price index (excluding interest and GST) and π^{cpi4} the annual rate of change in the same index.

⁶ For a fuller discussion of these issues and the evidence for the asymmetric Phillips curve see Black et al. (1997, page 44) and the references contained in that paper.

Many small demand-side models⁷ contain just one price, determined by the Phillips curve, without explicitly modelling consumer prices. Because New Zealand's inflation target is generally specified in terms of consumer prices, it is necessary to model the CPI. The CPI is gradually built up from π . π^c represents (quarterly) inflation in the deflator of FPS model consumption: it is effectively π plus a term representing passthrough from domestic import prices to consumer prices, and a small weight on *changes* in the output gap (so that π^c is slightly faster to react to the business cycle than π). π^{cpi} represents quarterly inflation in the consumer price index (excluding interest and indirect taxes), and is a distributed lag of π^c with a term that slows the passthrough of exchange rate fluctuations from π^c to π^{cpi} . The slow passthrough replicates the stylised fact that exchange rate changes take more time to flow through to the consumer price index than into the consumption deflator. The monetary authority's target is the annual rate of CPI inflation, π^{cpi4} . This separate modelling of 'overall' domestic inflation and the CPI has important policy implications, as discussed in Conway, Drew, Hunt and Scott (1998).

The exchange rate and exchange rate expectations are determined in equations 7 and 8;

$$z = G_1 z(-1) + G_2 (ze + rf - r) + (1 - G_1 - G_2) z^* + \varepsilon_z \quad (7)$$

$$ze = H_1 z(1) + H_2 z(-1) + (1 - H_1 - H_2) z^* \quad (8)$$

where ze denotes the expected exchange rate next period, and r and rf the domestic and foreign real interest rates.

The exchange rate adjusts to changes in the differential between domestic and foreign interest rates. The structure of the basic relationship between interest and exchange rates is like that of the overshooting model of Dornbusch (1976). However, relative to Dornbusch's model, the dynamics of the relationship are substantially slowed here by three factors: the lag term in the z equation, the weight placed on the equilibrium exchange rate z^* , and the fact that exchange rate expectations (ze) combine forward and backward looking aspects (and also have a small weight on the equilibrium level). The equilibrium exchange rate z^* is exogenous in SDS-FPS⁸.

Interest rates are determined in the following block;

$$rsl = rsl^* + I_1(F)(\pi^{cpi4} - \pi^{tar}) \quad (9)$$

$$rn = (1 + rnl)/(1 + rsl) - 1 \quad (10)$$

⁷ Notable exceptions include McCallum (1995) and Svensson (1997).

⁸ This is a simplification relative to FPS, where the equilibrium rate is solved for to support the equilibrium NFA/GDP ratio.

$$\begin{aligned}
rnl = & [(J_1(1 + rn) + J_2(1 + \sigma(F)rn))(1 + rt5^*) + \\
& J_3(1 + rnlrow + rp(1 + rt5^*)(1 + \sigma(F)\pi^{row}))] \\
& (1 + \sigma(F)\pi^{row} / 1 + \sigma(F)\pi^{row}) + \\
& (1 - J_1 - J_2 - J_3)(1 + rnl^*) - 1
\end{aligned} \tag{11}$$

$$r = (1 + rn)/(1 + \pi(1)) - 1 \tag{12}$$

Where π^{tar} denotes the inflation target, rnl the nominal long rate, rn the nominal short rate, $rt5^*$ the term premium, $rnlrow$ the foreign nominal long rate, rp the risk premium on New Zealand assets and π^{row} the foreign inflation rate. The $\sigma(F)$ lead polynomial is a 20 quarter forward average.

Monetary policy reaction is described in terms of the slope of the yield curve (rsl). The monetary authority selects the desired slope based on model-consistent forecast of the deviations of inflation from the target in the future. In the ‘base-case’ formulation of this rule, which is intended to describe roughly how policy is currently formulated, the response coefficient is 1.4 on deviations from target 6, 7 and 8 quarters ahead.

After the desired yield curve slope has been set, short and long nominal rates are solved for simultaneously in equations 10 and 11. Long rates respond to contemporaneous short rates, and also respond to the 20 quarter forward average of short rates in accordance with the expectations theory of the yield curve⁹. They are also affected by movements in foreign long rates, adjusted by an exogenous risk premium on NZ assets, and scaling factors to capture differentials in expected domestic and foreign inflation over the period. Finally, a small weight is placed on the equilibrium nominal long interest rate. In effect, the model finds a nominal short rate and nominal long rate jointly consistent with equation 11 that gives the desired yield gap.

Finally, equations 13-15 determine export and import prices and the terms of trade;

$$px = px^* + L_1(L)(pxrow - pxrow^*) + L_2(L)(z - z^*) + \varepsilon_{px} \tag{13}$$

$$pm = pm^* + M_1(L)(pmrow - pmrow^*) + M_2(L)(z - z^*) + \varepsilon_{pm} \tag{14}$$

$$tot = px / pm \tag{15}$$

where tot denotes the terms of trade.

The relative price of exports and imports are determined by the (exogenous) world prices of New Zealand’s exports and imports, and exchange rate fluctuations.

⁹ Strictly, the 40 quarter forward average should be solved for to give the 10 year rate equivalent to the expectations hypothesis, but only solving the first 20 quarters makes the computational problem much simpler.

3 Calibration methodology

Once the structure discussed above was specified, we then parameterised the model to match FPS properties as far as possible¹⁰.

There is a growing literature on the methodology of model calibration. Although this work (for example King (1995), Kim and Pagan (1995), Kydland and Prescott (1996), Hanson and Heckman (1996), and Cooley (1997)) largely deals with the more common case where a model is being designed to match certain properties of the data rather than an existing model, the techniques and criterion used in this work are essentially an adapted form of the recommendations of that literature, namely:

- 1 Begin with a **“well-accepted theoretical structure.”**¹¹ It seems clear that for monetary policy analysis the small open economy model used here is a commonly used and accepted model design. With some simplification and adjustment, we employ structures derived from formal theory in work such as Svensson (1997), and Dornbusch (1976).
- 2 Proceed from formal theory to quantitative theory by parameterising the model. In the real business cycle literature, parameters are frequently drawn from microeconomic studies where possible. When calibrating a model to match another model, where a parameter has an obvious correspondence to a parameter in the existing model it seems appropriate to retain that parameter. This determined the parameterisation of equations that were very similar in FPS and SDS-FPS (the exchange rate and interest rate equations, for example).
- 3 Analysis of microeconomic studies and other parameter sources generally leaves a number of unknown parameters, which are chosen so that the model economy matches other known facts. In this study, parameters that could not be set by reference to FPS (such as those in the IS curve) were then set on the basis of the *deterministic simulation properties of the model*, as recommended by Masson (1986). This was an iterative step: an initial guess at the parameterisation was made and the dynamic properties compared to FPS, parameters were adjusted on the basis of this run and the simulations re-run, and the process continued until the properties of the two models were close.

¹⁰ When simplifying a model, it is sometimes feasible to explicitly *derive* a reduced specification (Masson (1986)). This involves isolating equations into blocks which will each be described by a single equation in the small model, and then deriving the appropriate functional form for the single equation by combining all the equations in the block. Once the appropriate functional form is derived, simulation of the block in the larger model can be used to calibrate the parameters in the small equation. However, because we wish to produce a radical simplification (completely removing aspects of FPS’s design) and imitate the typical small model structure found in the literature, it was not possible to derive an “exact” reduced SDS-FPS structure to parameterise in this case.

¹¹ See Kydland and Prescott (1996).

- 4 Evaluate the model by simulating it and assessing its ability to describe the phenomena it sets out to capture: in this case, the business cycle properties demonstrated by FPS. As in Drew and Hunt (1999), the model is simulated stochastically with auto and cross-correlated shocks specified using a VAR. This allows us to compare the moments of generated data with FPS as an additional test of how well the two models match. In Cooley's (1997, p58) terms, this allows us to "match the model to the measurements" along an important dimension, ensuring that the moments of generated data in SDS-FPS are similar to those in the actual data and those generated by the full FPS Core model.

Much of the recent literature on calibration has sought to compare, and perhaps ultimately reconcile, the calibration methodology with more traditional estimation methods. We note the ideas in this recent work provide some ideas for further work with SDS-FPS. For example, King (1995) and Sargent (1998) note that system-based estimation techniques such as the Hansen-Sargent procedure provide an alternative to both calibration and equation by equation model estimation. However, King expresses some scepticism about this approach, noting that full system estimation generally gives unreasonable results for a portion of model parameters, and recommends generalised methods of moment analysis like that of Christiano and Eichenbaum (1992), where a "subset of the model's empirical implications,"¹² or *moment conditions*, are used to estimate a vector of model parameters simultaneously. King notes that GMM analysis is similar to calibration in spirit, but provides a variance-covariance matrix for the parameters in the model, which gives the researcher information on the extent of parameter uncertainty. Either of these estimation techniques could be used with SDS-FPS to provide an alternative set of parameters, and a measure of the uncertainty surrounding them.¹³

We end this section by noting five important structural differences between SDS-FPS and FPS. Firstly, SDS-FPS attempts to capture the entire interaction between supply and demand via an output gap equation, while FPS has an explicit representation of each component of aggregate demand and the supply side. Secondly, FPS has a much more comprehensive system of price/factor income accounting: to give two examples, wages are ultimately determined by the marginal product of labour in the production function, and the relative prices of imported consumption, investment and government goods are separately modelled. These influences on prices are captured in SDS-FPS in a much simpler fashion. Thirdly, the SDS-FPS model is effectively written in deviations from a steady state which is assumed to be fixed: the implications of a change in the equilibrium level of interest rates, potential output growth, or private sector wealth cannot be analysed.¹⁴ Fourthly, the polynomial adjustment costs used in FPS are removed, and proxied by the lag structures in SDS-FPS. Finally, SDS-FPS does not contain any of the stock-flow consistency present in FPS. As shown in the

¹² See King (1995), page 88.

¹³ In forthcoming work, Drew and Weiss perform a Bayesian analysis where the calibrated parameters were viewed as the prior and evaluated against a posterior distribution partly based on actual data.

¹⁴ With one exception: the dynamics resulting from a shift in the inflation target are included.

next section, these last two differences limit the ability of SDS-FPS to replicate some of the properties of the full FPS model.

4 Matching properties: deterministic simulations of FPS and SDS-FPS

This section shows the responses of SDS-FPS to a standard set of shocks used to calibrate the model. These include permanent movements in the inflation target, and temporary fluctuations in aggregate supply and aggregate demand, foreign demand, the exchange rate, and the terms of trade. In each graph, the dotted line represents the results in the SDS-FPS model, while the solid line represents the results of an equivalent shock to the FPS Core model. All graphs are drawn in terms of deviations from equilibrium, with interest rates and inflation variables expressed at annual rates. Shocks occur in 2000q1, to an economy initially at equilibrium.¹⁵

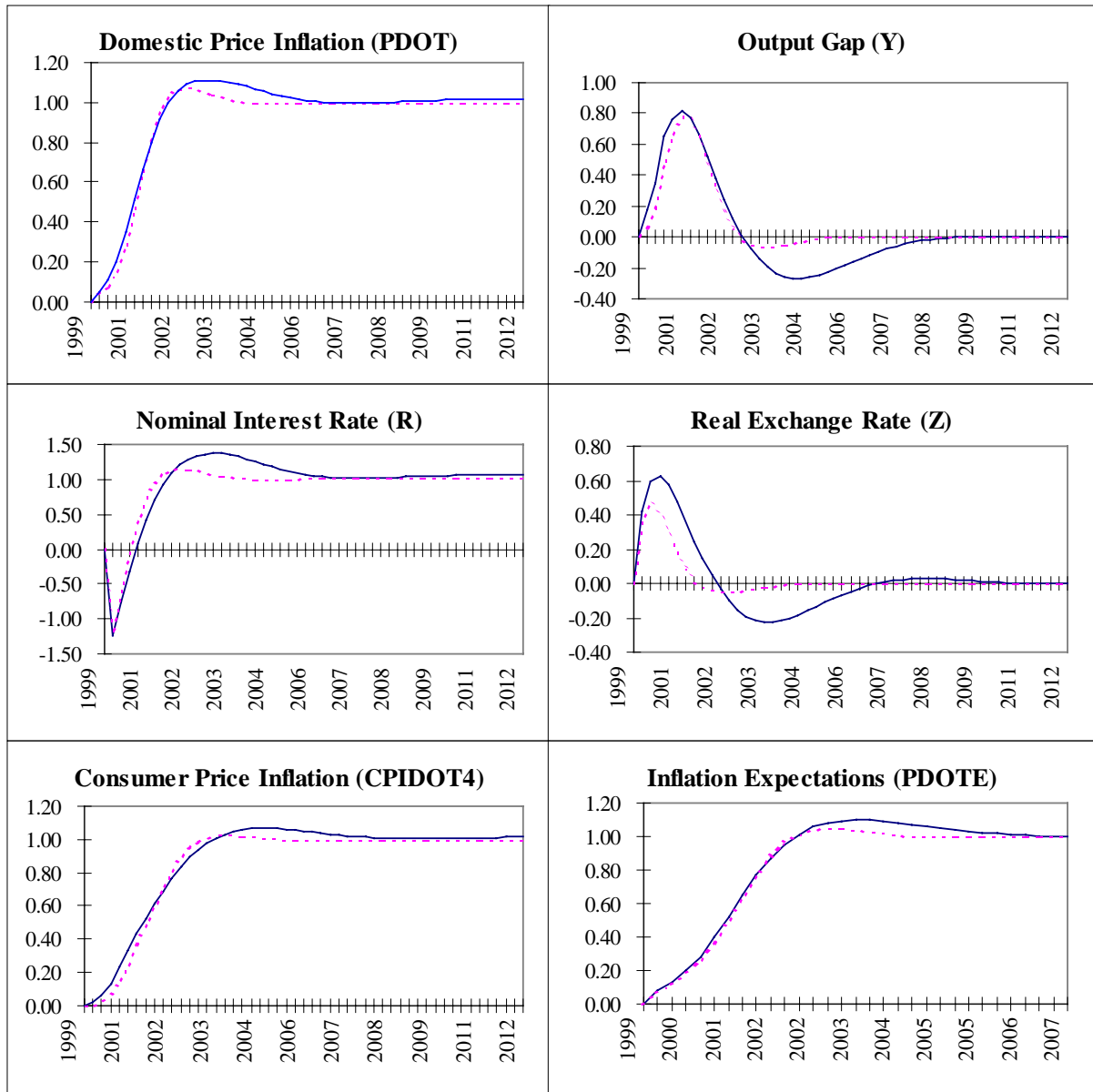
4.1 Raising or lowering the inflation target

In figure 1, the effects of increasing the inflation target permanently by one percent are shown. The opposite shock (lowering the inflation target by one percent) is shown in figure 3. These shocks produce paths for endogenous variables that are very similar to those seen in FPS¹⁶ - policy gradually pushes inflation to the new target, overshooting slightly and then reanchoring. The effect of the target change on the output gap is very like that in FPS, with the asymmetric output gap term in the Phillips curve making the output gain from raising the target considerably smaller than the output loss resulting from lowering the target.

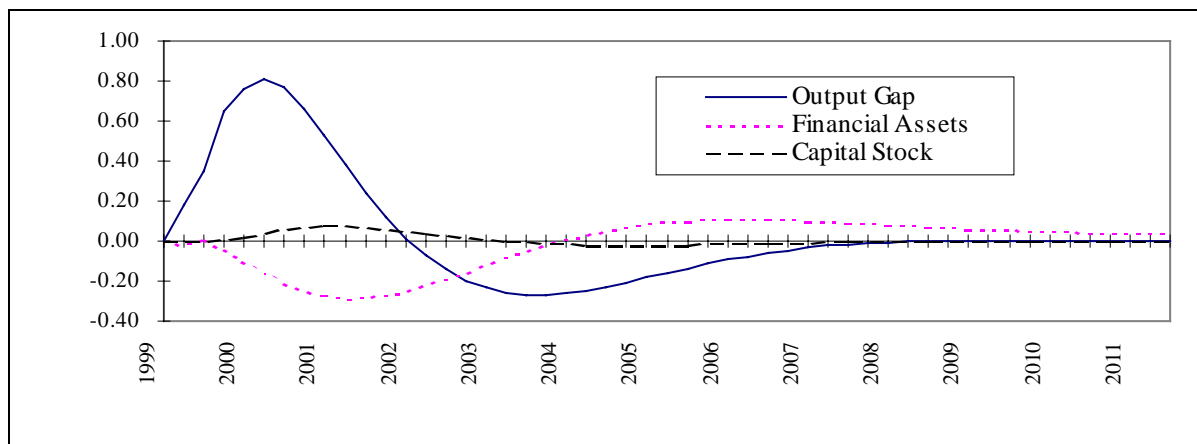
¹⁵ In the charts, π is labelled PDOT, π^e is labelled PDOTE, π^c is labelled PCDOT and π^{cpi4} is denoted CPIDOT4.

¹⁶ See for example 5.2.4 in Black et al (1997).

Figure 1: Raising the target



Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

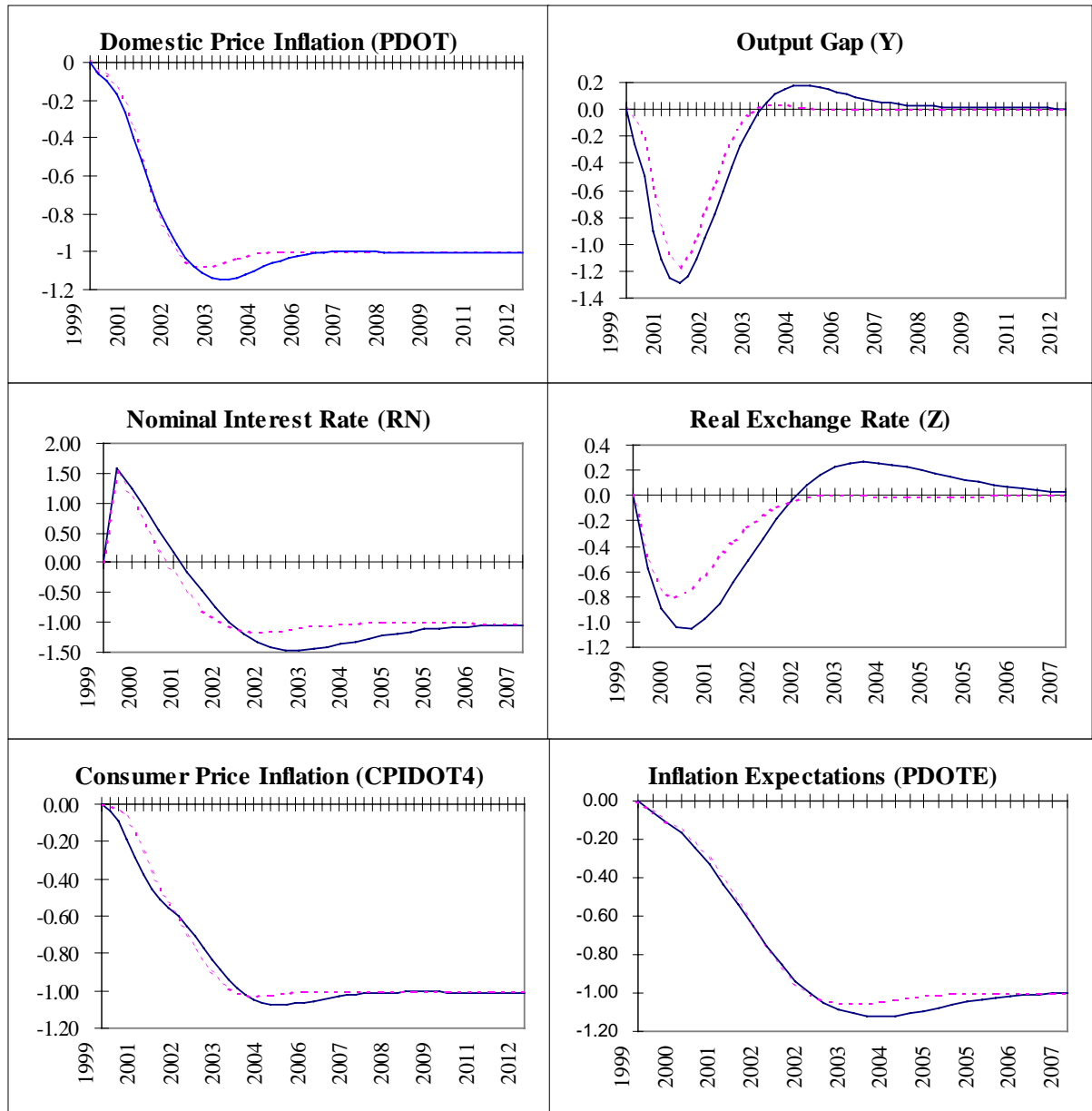
Figure 2: Stock adjustments in FPS when the inflation target rises

The properties of the models under these shocks (figures 1 and 3) match FPS fairly well, but there are two substantive differences, which are worth discussing at some length as they recur throughout the deterministic simulations. Firstly, real exchange rate volatility is somewhat lower in SDS-FPS. Secondly, cycles are of shorter duration in SDS-FPS, and secondary cycling is considerably less pronounced.

The longer cycles in FPS are partly caused by the polynomial adjustment costs built into that model.¹⁷ Cycles are also lengthened in FPS, and secondary cycling is induced, through the movement of stock variables. In the example of an increasing inflation target, the temporary reduction in interest rates boosts consumption and investment. The consumption spending causes an erosion in the wealth of forward-looking consumers, the latter moves the capital stock above equilibrium (figure 2). To return to equilibrium at the new inflation rate, households have to rebuild their financial assets, and investment has to fall below equilibrium temporarily to bring the capital stock back to equilibrium. In this shock, these effects reduce consumption and investment between roughly 2002 and 2005, exacerbating the secondary cycle.

Finally, the reduced exchange rate volatility in SDS-FPS is a direct result of the reduced duration of its cycles. The uncovered interest parity relation implies that the exchange rate moves in response to a shock according to the expected real interest rate differential, cumulated over the period that the differential persists. Shorter cycles mean that the expected differential is smaller and hence the exchange rate moves less in SDS-FPS.

¹⁷ See Black et al (1997).

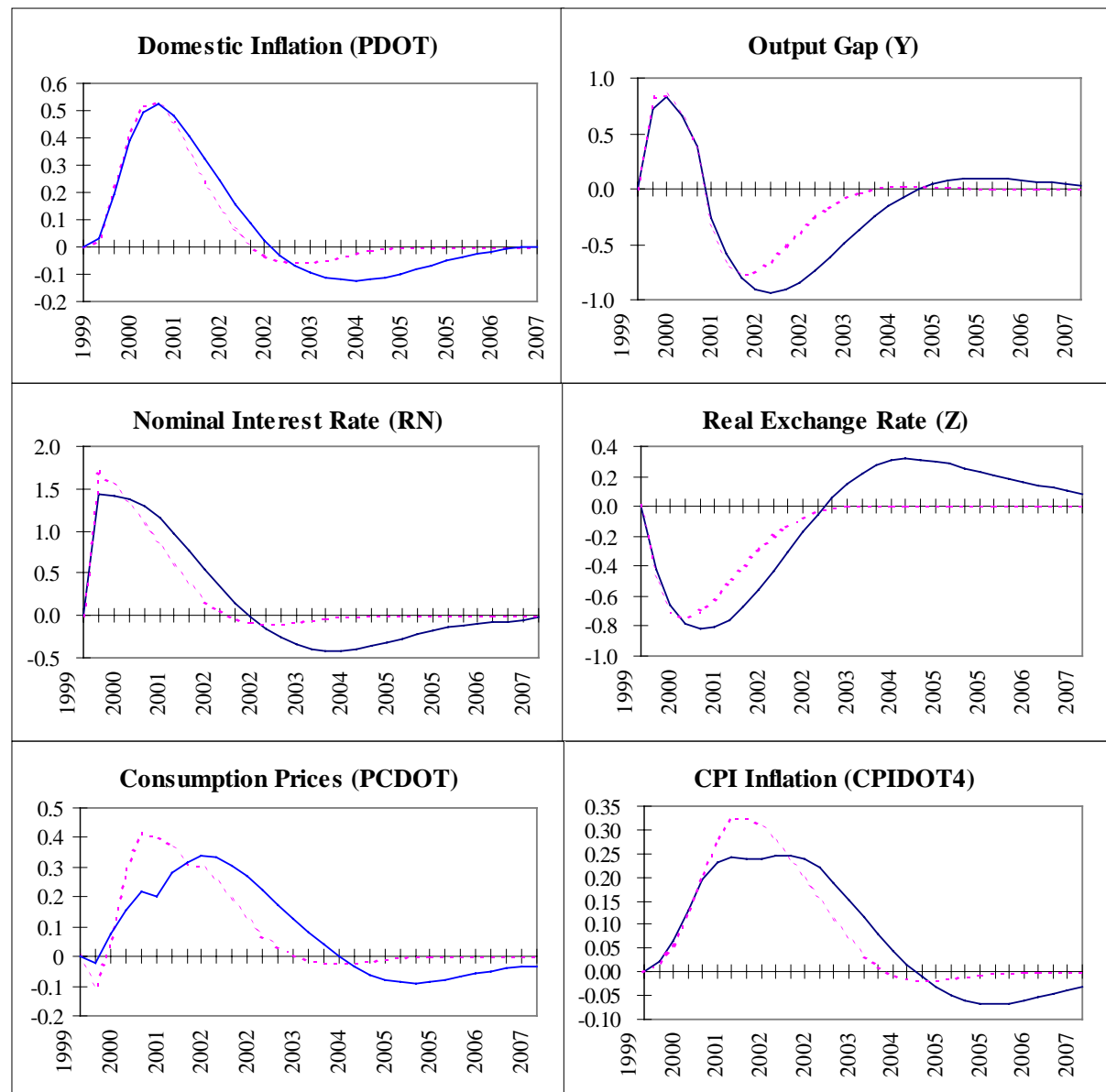
Figure 3: Lowering the target

Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

4.2 A positive demand shock

A demand shock yields a very similar response to FPS¹⁸. In both models, there is a strong secondary cycle in output, which is required to reanchor inflation expectations to the target rate of inflation. The length of the cycle, and the magnitude of the secondary cycle, is again slightly smaller in SDS-FPS. In this example, this is because the increased spending by consumers erodes their wealth in FPS, exacerbating the secondary cycle as consumers spend less in order to rebuild wealth.

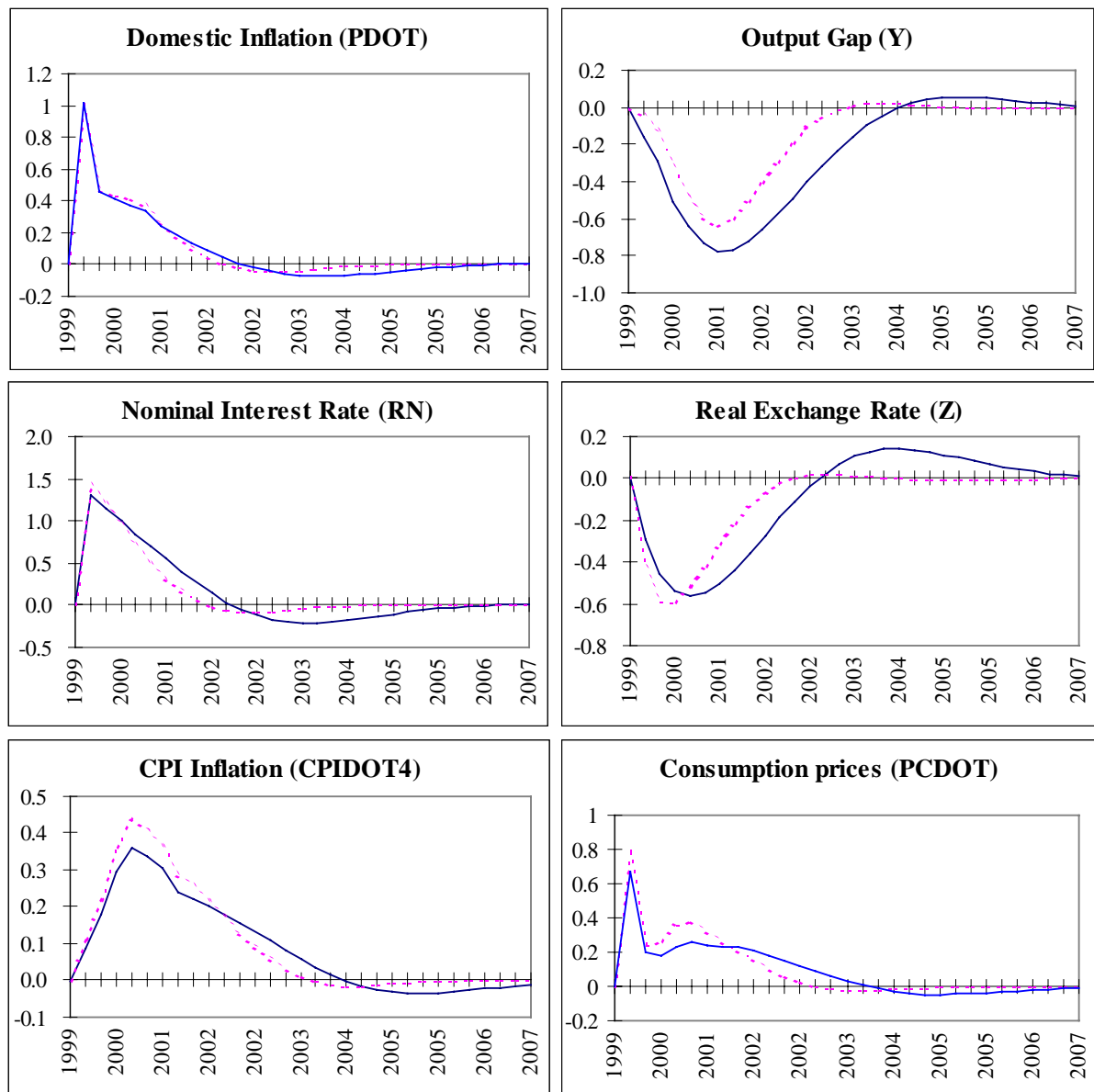
¹⁸ The shock in SDS-FPS is a positive value for the shock term in the IS equation for the output gap. The shock is set for 4 quarters at a size that ensures the magnitude of the output gap over those quarters is about the same as in FPS (because we are directly shocking the gap instead of components of demand, this is necessary to ensure the experiments are comparable).

Figure 4: A positive demand shock

Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

4.3 An increase in domestic inflation (a temporary supply shock)

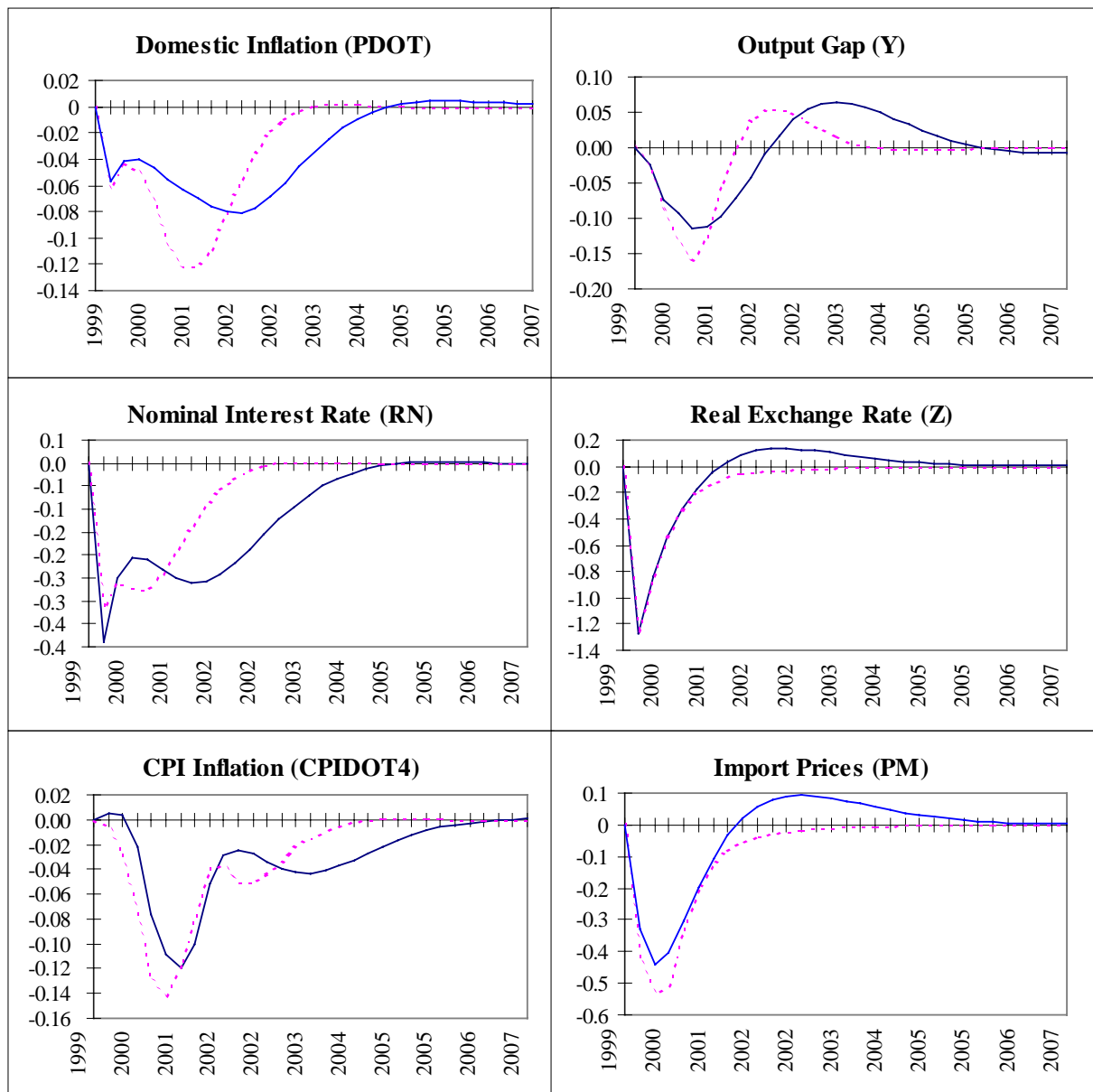
After an exogenous positive innovation in π , in either model, policy gradually returns π and π^{cpi} to equilibrium after a small secondary cycle. Policy instruments, inflation and the output gap all follow similar paths to those in FPS, although inflation is brought under control slightly faster in SDS-FPS. Again, this appears to be a result of stock-flow interactions. In FPS, the higher interest rates cause consumers to build up financial assets and firms to delay investment: as interest rates return to equilibrium consumers in FPS spend this wealth and firms begin to invest, creating additional inflationary pressure for the monetary authority to contend with.

Figure 5: A temporary supply shock

Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

4.4 A shock to the risk premium on NZD assets

This shock is modelled as an exogenous appreciation in the real exchange rate. This causes inflation to fall below control, and opens up a negative output gap by reducing demand for domestically produced goods. The effect on the CPI in both models is quicker than the effect (via the output gap) on π . Policy eases in response to the reduced inflationary pressure, leading to a positive output gap, which is larger in FPS. As with the other shocks, in FPS the initial cycle is somewhat longer, and secondary cycles somewhat stronger.

Figure 6: A risk premium shock¹⁹

Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

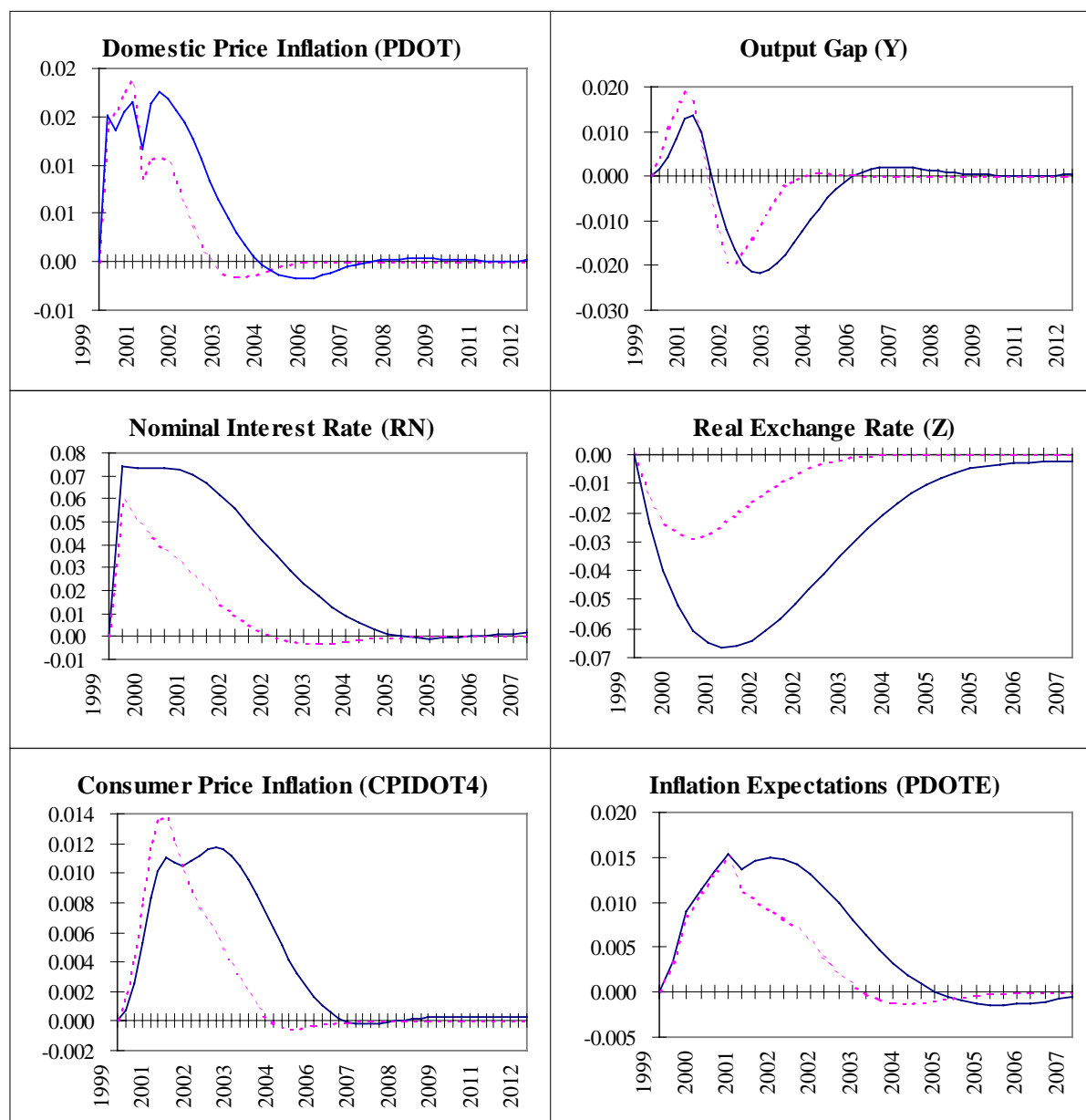
4.5 Temporary changes in the terms of trade

Increasing export prices temporarily had a similar effect in FPS and SDS-FPS: a positive impulse to the output gap (since the quantity of domestically produced output rises), creating inflationary pressure and leading to a tightening of policy. However, it was difficult to create *sustained* inflationary pressure in response to the shock in SDS-FPS. Again, this is because the small model does not contain asset positions. In FPS, the increased value of exports builds up the financial assets of forward-looking households (consumers), as shown in figure 9. Consumers gradually spend this

¹⁹ The real exchange rate is defined in units of domestic currency per unit of foreign currency, so a negative change is an *appreciation*.

wealth, which increases demand pressures on π further out and forces the monetary authority to tighten policy more and for longer.

Figure 7: An increase in export prices

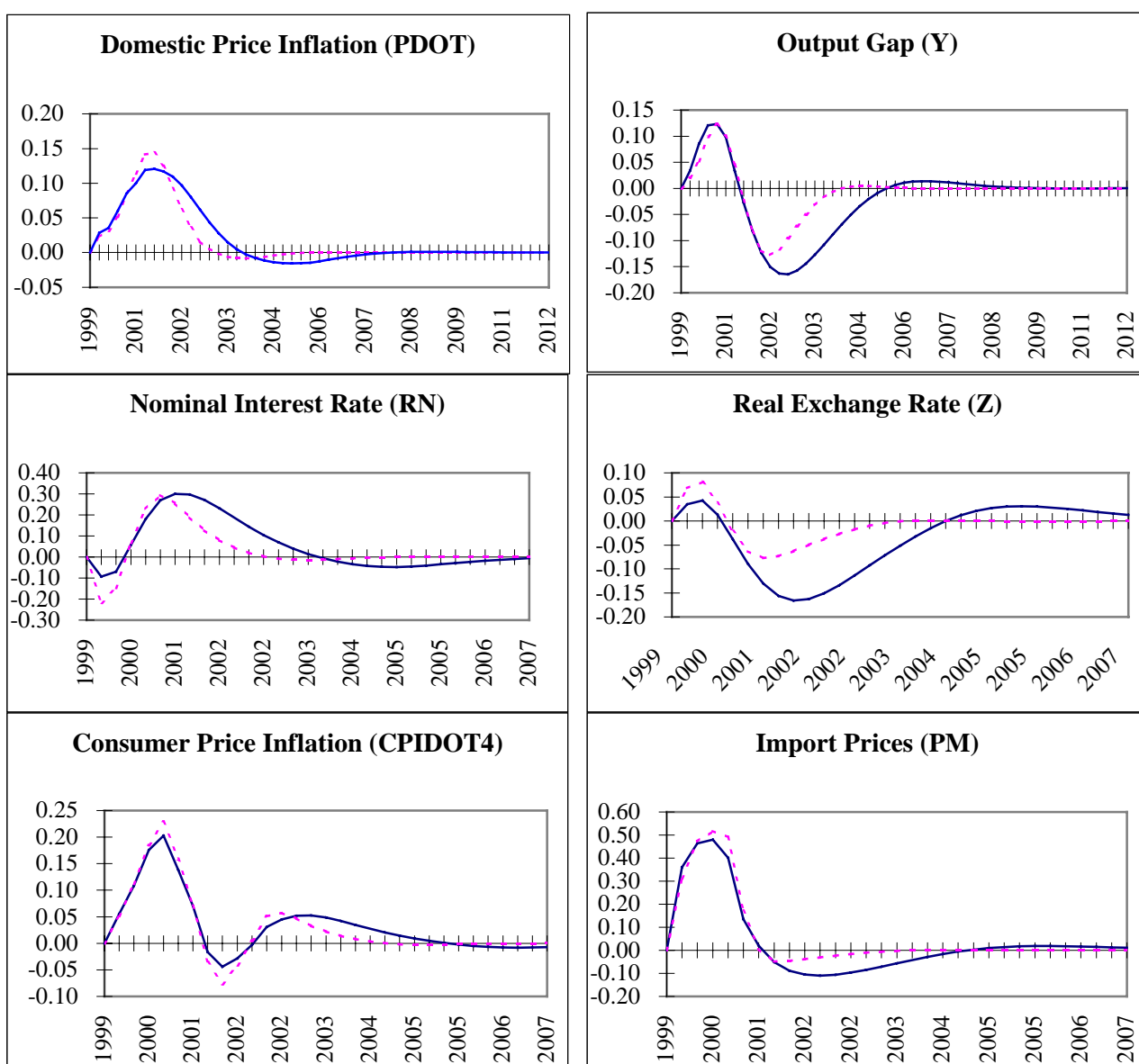


Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

A brief (4-quarter) shock to the price of imports in FPS leads to a curious response from the monetary authority. Recall that the reaction function in both models targets the forecasted deviation of π^{cpi4} from target 6,7 and 8 quarters out. Because π^{cpi4} dips below the target during 2002 (as consumer price inflation falls below control in response to import prices dropping back to equilibrium), this leads the monetary authority to initially ease in response to this shock. This doesn't seem like an optimal response: indeed, Conway, Drew, Hunt and Scott (1998) demonstrate that if the monetary authority targets core domestic inflation (π), which would eliminate this temporary easing, the variability in output, inflation and instruments can all be reduced.

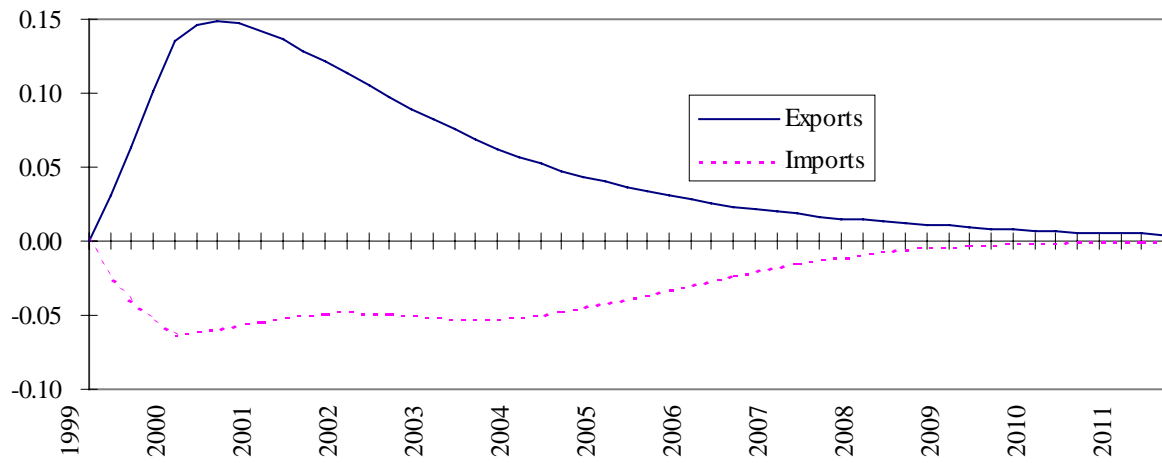
The behaviour of the two models is quite similar, although the initial decline in interest rates is more substantial in SDS-FPS than FPS. This is because the swings in the CPI in FPS are slowed slightly by adjustment costs, mitigating the effect in that model. Notice that in this shock, the secondary cycling in SDS-FPS is almost as pronounced as that in the larger model. This is because, as figure 9 shows, the magnitude of the shift in asset positions in the import price shock is very small relative to the export price shock: import prices rise but import volumes fall to leave nominal imports approximately unchanged.

Figure 8: An increase in the price of imports



Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

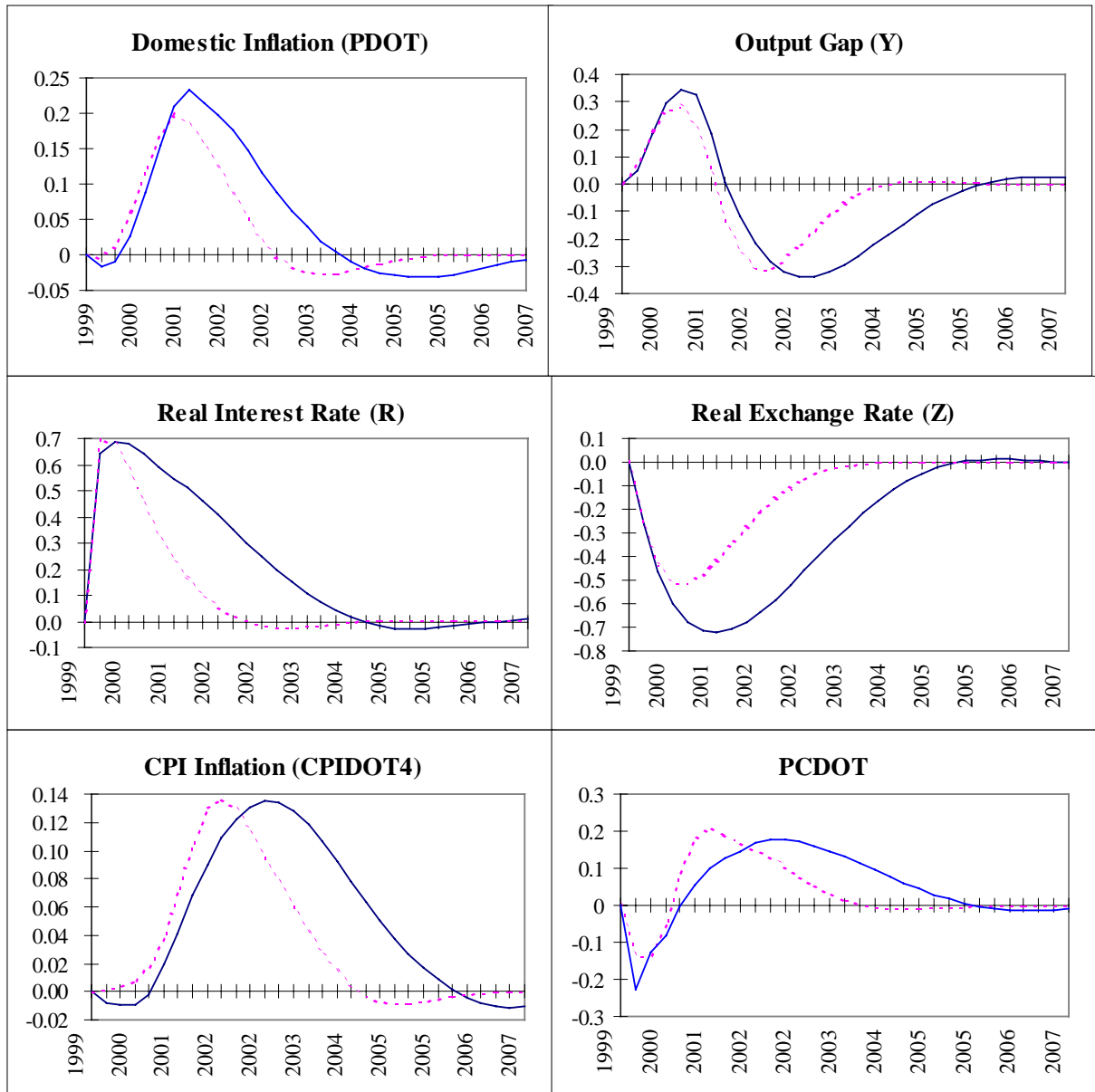
Figure 9: FPS net foreign asset positions after an export and import price shock



Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

4.6 A positive foreign demand shock

In FPS, the world output gap influences export volumes. In SDS-FPS, the increase in foreign demand affects the IS curve directly, causing an increase in output, with inflationary consequences that are countered by a tightening of monetary policy. Again, the absence of wealth variables allows the monetary authority to get on top of the problem quite a bit sooner in SDS-FPS (households build up wealth in FPS because domestic spending is temporarily reduced by the interest rate increases).

Figure 10: A shock to foreign demand

Dotted line denotes SDS-FPS results, Solid line denotes FPS Core model results.

Overall, these simulations indicate that SDS-FPS is capable of producing qualitatively similar behaviour to FPS in simple deterministic experiments. More importantly, the areas in which SDS-FPS cannot adequately mimic FPS behaviour can be directly traced to features of FPS which do not exist in small demand-side models like SDS-FPS, which demonstrates the importance of some of the richer features of FPS.

We note that some properties could be matched more closely through adjustment of some of the equations that were taken directly from FPS. For example, if the responsiveness of the real exchange rate to real interest rates was increased, real exchange rate volatility in SDS-FPS would rise and this would match FPS more closely under most shocks. But that change would involve arbitrarily adjusting a SDS-FPS parameter away from its FPS equivalent in order to compensate for the fact

that model structure forces properties to be different in another area. We do not consider this to be a desirable trade-off.

5 Matching moments: performing stochastic simulation with SDS-FPS

To run stochastic simulations, a judgement has to be made about the distribution of the shocks hitting the economy. Because SDS-FPS, like FPS, is not estimated, there is no set of historical residuals from which to draw ‘typical’ shocks. Instead, we elect to draw shocks from paths based on the first four quarters of the impulse response functions of a VAR.

The VAR includes domestic and foreign output, the terms of trade, domestic inflation, the real exchange rate and the yield gap. When we draw shocks in stochastic simulation, we effectively draw impulses to each of these variables²⁰. This methodology, which attempts to capture the cross and auto correlations typically present in the shocks hitting the economy, is described in detail in Drew and Hunt (1998).

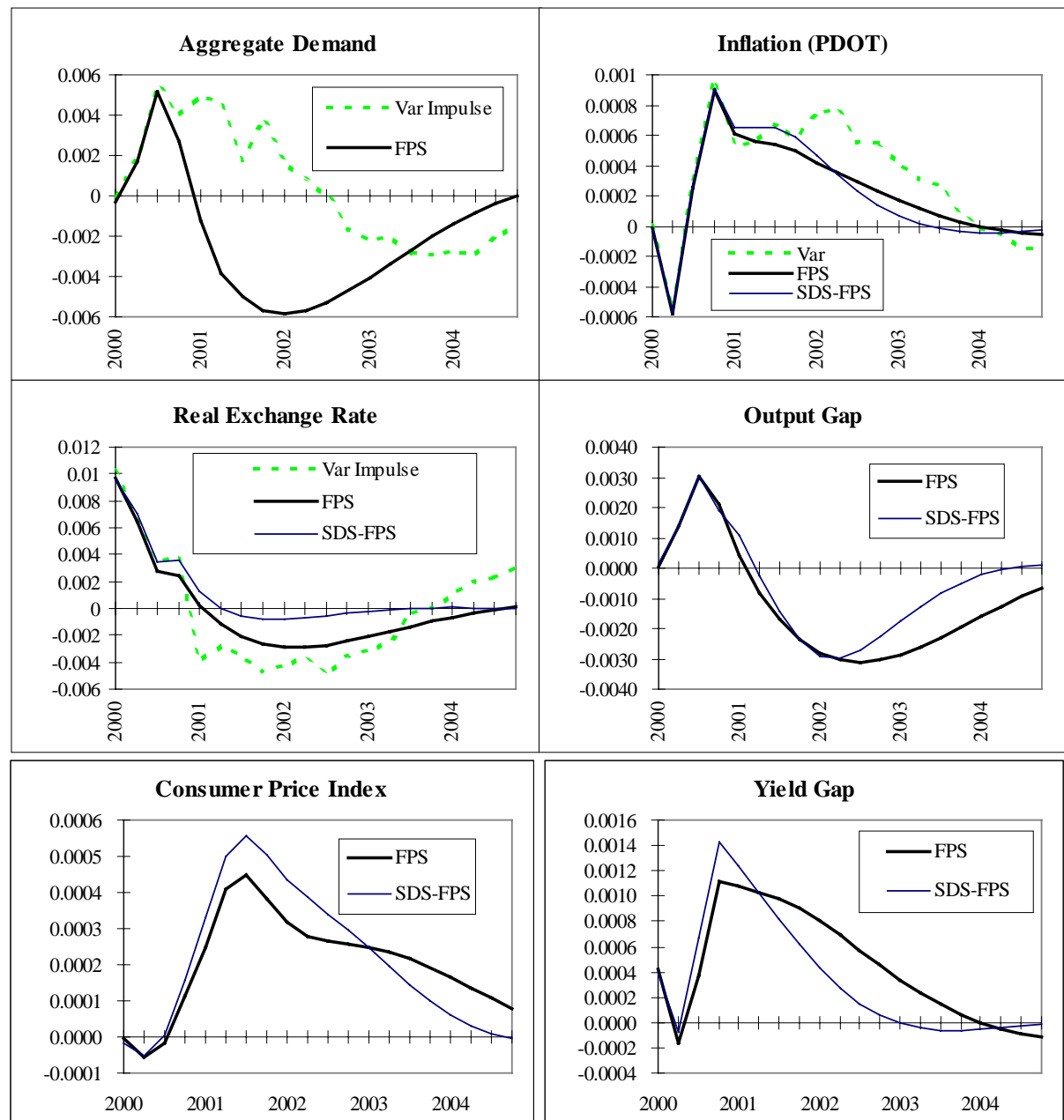
As an example of the methodology, consider an upward impulse to the real exchange rate. In the VAR, this is correlated with a temporary increase in aggregate demand, and an increase in the rate of inflation. The VAR results are used to calculate a matrix of shocks that produce the same response in FPS over four quarters, and the process is then repeated with SDS-FPS.

Next, to simulate a real exchange rate shock, the matrix of shocks calculated in the step above is run through the model. By construction, both models respond in a way that almost exactly matches the VAR for the first four quarters²¹, and then the resulting tightening in monetary policy causes inflation, the exchange rate and output to come back toward equilibrium. The SDS-FPS and FPS paths are very similar throughout the simulation, although the faster dynamics and reduced secondary cycle seen in the deterministic simulations are also present here.

²⁰ Except the yield gap: innovations to the gap are interpreted as representing historical monetary policy, which we don’t want to capture in simulating the effect of our current monetary policy rule.

²¹ For technical reasons explained in Drew and Hunt (1998), the shock matrix is calculated with monetary policy reaction turned off: when the shocks are run back through the model there is a policy reaction which makes the initial movements in the variables slightly different to those in the VAR.

Figure 11: Responses to an exchange rate impulse using SDS-FPS and FPS

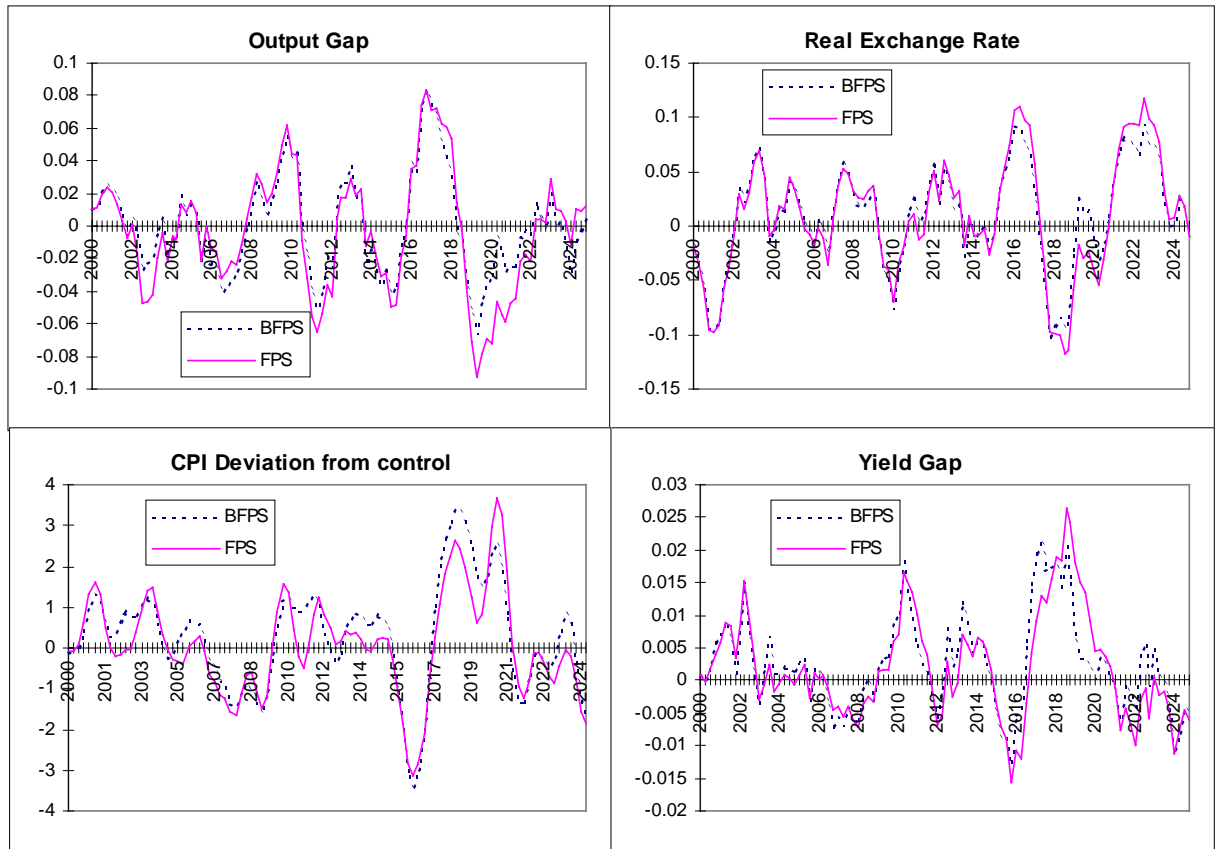


Assessing the ability of SDS-FPS to replicate FPS behaviour in stochastic simulation

One stochastic ‘draw’ is a 100 quarter simulation. In each quarter, the model is subjected to stochastic shocks and the monetary authority resets policy based on the inflation outlook. The next quarter, the model is solved again based on the lagged values of all variables, and a new set of shocks. The process is repeated for the 100 quarters.

An obvious test of SDS-FPS's ability to mimic FPS is to compare the two model's dynamic path for a given draw. If SDS-FPS and FPS have similar properties, the paths coming out of each model should be similar. Paths for output, the exchange rate, the CPI inflation rate and the policy instrument are shown in the figure below.

Figure 12: A stochastic simulation 'draw'



The two models clearly give similar results over this 25 year-period, but that may be a chance occurrence based on the shocks which the simulation method generated for that draw. To assess that possibility, the two models were simulated over 50 draws. For each draw, the root mean squared deviation of key variables was calculated and a t-test was performed on the null hypothesis that there was no difference on average (across draws) between the moments from the two models.

Root mean squared deviations over 50 draws

Variable	PDOT	CPIDOT4	Y	Z	RSL	RN
RMSD: SDS-FPS	1.88	1.18	2.81	4.77	2.55	3.39
RMSD: FPS	1.86	1.17	3.07	5.21	2.59	3.42
Difference (%)	1.1	1.4	-8.5	-8.4	-1.6	-1.0
<i>T-statistic on difference</i>	1.4	0.9	-7.9**	11.2**	-1.4	-0.9
Correlation across models	0.93	0.76	0.91	0.95	0.87	0.89
RMSD: History ¹	na	3.9(1.2)	1.7	4.9	2.0(1.0)	5.7(2.6)

(* denotes significance at the 5% level, ** significance at the 1% level)

¹ Moments shown are from 1985q2-1997q2. Moments in brackets are from 1990q1-1997q2.

The first two rows show the average root mean squared deviations obtained for FPS and SDS-FPS over the 50 draws. In the third row, the percentage difference between the results from the two models is shown, and the fourth row contains the t-statistic for that difference. No statistically significant difference was detectable between the average moments coming from the two models for either price measure. Similarly, nominal interest rates and the yield curve behave almost identically in the two models. However, exchange rate and output volatility are statistically significantly lower in SDS-FPS, which seems to reflect the differences highlighted in the deterministic simulations presented in the previous section. While these differences are statistically significant, they are sufficiently small that they would not be economically important in many stochastic simulation experiments.

The fifth row shows the correlation between the results for each draw across the two models. The correlations are strong, which implies that a set of shocks which lead to high variability in one variable in FPS also do so in SDS-FPS. If this wasn't the case, the fact that the moments from the two models were indistinguishable could merely be the result of a high variance in the differences between the two models.

Finally, the last row shows the historical (1985-1997) moments for the variables, as reported in Drew and Hunt (1998). Comparing these moments to those from the model generated data provides an indicator of the model's congruence with New Zealand's recent experience. However, in some cases the structural changes seen over the period can be expected to have altered the time-series properties of certain macroeconomic data: for example, short term interest rates and inflation are likely to behave differently in an inflation targeting regime. To partially alleviate this concern, we consider nominal data (interest rates and inflation) for the 1990-1997 period as well as 1985-1997, but we still interpret this comparison as a broad indicator of plausibility rather than a formal test of fit.

Like the FPS moments, the SDS-FPS moments are fairly similar to New Zealand's recent experience: particularly for inflation and the exchange rate. Inflation and nominal interest rate volatility do both appear to have fallen substantially in the second period when the disinflation process was largely complete, and the model matches these moments better than those for the full sample. The output gap is somewhat more volatile than the historical figure, which we suspect is a consequence of our characterisation of innovations in expenditure as arising from demand shocks. If some movements in consumption or investment were treated as resulting from supply side innovation, this would tend to reduce our simulated output gap volatility.

This is a potential area for further work. Lastly, the historical moments for the yield gap are lower than those generated by the model. This may suggest the monetary policy rule in SDS-FPS and FPS targets inflation more strictly than the average over the historical period.

6 Conclusions

At first glance, the small stylised models often used in monetary policy research and the more fully articulated macroeconomic models generally used by policy institutions appear very different. However, this paper suggests that a suitably calibrated small model can match the properties and moments of a richly structured model like FPS surprisingly well. Clearly, small models do not provide sufficient disaggregation for many forecasting or policy analysis applications, but this research suggests they can be usefully applied to questions where an aggregate picture is all that is required.

The structure of SDS-FPS is similar to that of the small demand-side macroeconomic models typically used in monetary policy research. Hence, SDS-FPS is likely to be of interest to researchers, and it will hopefully be possible to use existing techniques developed for smaller models on SDS-FPS. Also, SDS-FPS is small enough to make large scale stochastic simulation experiments feasible. For example, in Hargreaves (1998) we use SDS-FPS to analyse the effectiveness of various monetary policy rules under a range of alternative assumptions, and explore the idea of ‘learning algorithms.’

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