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The Alleged Instability of Nominal Income Targeting

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Abstract

Recently it has been argued that a monetary policy of nominal income targeting would result in dynamically unstable processes for output and inflation. That result holds in a theoretical model that includes backward-looking IS and Phillips curve relations, but these are rather special and theoretically unattractive. The present paper demonstrates that replacement of the special Phillips curve with one of several more plausible specifications overturns the instability result, whether or not the IS equation is replaced with a forward-looking version. Thus the instability result is quite fragile and therefore provides almost no basis for a negative judgment regarding nominal income targeting.

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I. Introduction

Recently it has been argued by Ball (1997) that a monetary policy of nominal income targeting would be destabilizing for output and inflation, even though it succeeded in stabilizing nominal income (in either a growth-rate or growing-level fashion). A similar result has also been presented by Svensson (1997b). While the latter's discussion is quite circumspect, Ball (1997, abstract) expresses the claim in strong language: "Finally, nominal income targets are not merely inefficient, but also disastrous: they imply that output and inflation have infinite variances." This seems a rather startling proposition, given that a number of economists have previously presented both theoretical[1] and empirical[2] analyses in which nominal income targeting yields stable behavior for output and inflation. Evidently the Ball-Svensson conclusions must stem from the use of a model specification that is somewhat unusual - but the model consists merely of a dynamic IS equation and an accelerationist Phillips curve relation, both of which give the appearance of being rather orthodox.

In this case, however, appearances are misleading. Although both of the model's behavioral relations are similar in some respects to specifications that have recently been given considerable theoretical support,[3] they differ in subtle but crucial ways.[4] The purpose of the present paper, accordingly, is to determine whether the instability result is robust to minor alterations in the specification of the Ball-Svensson model.[5] The findings indicate that replacement of the Ball-Svensson Phillips curve with the mentioned alternative results in a model in which both output and inflation are dynamically stable under nominal income targeting, whether or not the IS relations is respecified. Furthermore, consideration of additional Phillips curve specifications indicates that the instability

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1. See, for example, Bean (1983), Frankel and Chinn (1995), and Ratti (1997).
2. See, for example, Taylor (1985), McCallum (1990), Bryant, Hooper, and Mann (1993), Feldstein and Stock (1994), and Hall and Mankiw (1994).
3. This is recognized by Svensson (1997b, pp. 21-23) but not Ball (1997). References are provided below.
4. A major difference is that the Ball-Svensson specification is "backward looking" whereas the theoretically cleaner alternative relations are "forward looking." Svensson (1997b, pp. 22-23) also discusses an intermediate specification, mentioned below in footnote 11.
5. Svensson (1997b, p. 21) expresses some concern regarding the possibility of nonrobustness.

result is highly fragile, ie, it obtains only in quite special - and dubious - cases.

II. Specifications

The behavioral relations used by Ball (1997) and Svensson (1997a, 1997b) may be written as follows:

$$y_{t+1} = \lambda y_t - \beta r_t + \varepsilon_{t+1} \qquad \beta>0<\lambda<1 \qquad (1)$$

$$\pi_{t+1} = \pi_t + \alpha y_t + \eta_{t+1} \qquad \alpha>0 \qquad (2)$$

Here y_t is the log of output relative to some capacity measure (the output "gap"), r_t is a one-period real interest rate, $\pi_t = \Delta p_t$ with p_t being the log of the price level, and ε_t, η_t are white-noise shocks. Clearly, (1) is a backward-looking relation of the IS type, in which each period's output demand depends on its previous value and the previous period's real interest rate. Equation (2) is a backward-looking NAIRU type of Phillips curve in which inflation is determined by its own previous value and the previous period's output gap. The model is closed by supposing that the central bank conducts policy by manipulating r_t to achieve its objectives, whatever they be, with emphasis given to Taylor-style (1993) rules of the form

$$r_t = \mu_0 + \mu_1 y_t + \mu_2 \pi_t. \qquad (3)$$

Since the central bank is assumed to observe current values of y_t and π_t,[6] it can in effect observe current inflation expectations $E_t\pi_{t+1}$ and thereby use its control over the nominal interest rate $R_t = r_t + E_t\pi_{t+1}$ to set r_t at its desired level. Expectations are rational, with $E_t z_{t+j}$ denoting the conditional expectation of z_{t+j} formed in t on the basis of full information regarding periods t, t-1,….

6 This assumption is in my opinion quite unrealistic; actual central banks must set nominal rates R_t in response to data from t-1 and earlier. That fact increases the likelihood of instrument instability, which in the case at hand would mean instability of nominal income. But this is an entirely different kind of concern, one that is not considered in the Ball or Svensson analyses, so we shall abstract from it here. For a brief discussion, see McCallum (1997, p. 39).

The foregoing represents the Ball-Svensson model. But Roberts (1995), in a useful exposition and synthesis, has shown that currently the closest thing to a standard model of the Phillips curve is a relation of the form

$$\pi_t = E_t\pi_{t+1} + ay_t + u_t \qquad a>0 \qquad (4)$$

which can be thought of as representing (or closely approximating) well-known models developed by Rotemberg (1982), Calvo (1983), and Taylor (1980).[7] It will be noted that (4) is rather like a rearranged version of (2) with E_t attached to π_{t+1}, but there are two important differences. First, (4) says that π_t depends upon expectations of future inflation, not on the realization of past inflation. Second, a>0 in (4) would imply $\alpha<0$ under this arrangement.[8] Of course the Rotemberg, Calvo, and Taylor models have been criticized on some points, but they certainly represent more careful and explicit theoretical analysis than does equation (2).

With respect to the IS relation, recent papers by Kerr and King (1996), McCallum and Nelson (1997), and Woodford (1996), among others, have shown that an optimizing analysis in a stripped-down framework of the Sidrauski-Brock type gives rise to a relation of the form

$$y_t = E_ty_{t+1} - br_t + v_t \qquad b>0 \qquad (5)$$

The relationship of the latter to equation (1) is analogous to that of (4) to (2). And while the derivation of (5) requires some simplifying assumptions, its theoretical basis would seem to be much stronger than that of (1).

III. Analysis

Let us consider, then, the outcome of nominal income targeting in a model in which (4) and (5) replace the comparatively ad hoc relations (1)

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7 In (4), u_t is a stochastic behavioral shock that we take to be white noise for simplicity and for comparability with the Ball and Svensson analyses. The same statement applies to the v_t term in (5) below. In realistic models these shocks are probably serially correlated, and their properties may be important for comparisons of nominal income targeting to other schemes. But the assumption of white noise is acceptable for the issue of dynamic stability, which is the only concern in the present paper.

8 Roberts (1995) presents econometric evidence for the United States that is supportive of the hypothesis a>0.

and (2). Thus we suppose that the central bank sets r_t each period so as to make $\Delta x_t = 0$, where $x_t = y_t + p_t$, the object being to keep nominal income growth equal to a target value Δx^* which is, for simplicity, set to zero. Then substituting (4) and (5) into $\pi_t + y_t - y_{t-1} = 0$ yields

$$E_t\pi_{t+1} + ay_t + u_t + E_t y_{t+1} - br_t + v_t - y_{t-1} = 0 \tag{6}$$

and solving this gives a rule for r_t that will stabilize Δx_t at zero. Putting that expression back into (5) yields

$$(1+a)\, y_t = y_{t-1} - E_t\pi_{t+1} - u_t. \tag{7}$$

Thus our task is to determine how y_t and π_t behave in as system consisting of (7) and (4).

In this system the relevant state variables are y_{t-1} and u_t so it is apparent that the bubble-free solution obtained via the minimal-state-variable (MSV) procedure (McCallum, 1983) will be of the form

$$y_t = \phi_{11} y_{t-1} + \phi_{12} u_t \tag{8}$$

$$\pi_t = \phi_{21} y_{t-1} + \phi_{22} u_t \tag{9}$$

in which case $E_t\pi_{t+1} = \phi_{21}(\phi_{11} y_{t-1} + \phi_{12} u_t)$. Substituting these into (7) and (4) yields the two relations

$$(1+a)\,(\phi_{11} y_{t-1} + \phi_{12} u_t) = y_{t-1} - \phi_{21}(\phi_{11} y_{t-1} + \phi_{12} u_t) - u_t \tag{10}$$

$$\phi_{21} y_{t-1} + \phi_{22} u_t = (\phi_{21} + a)(\phi_{11} y_{t-1} + \phi_{12} u_t) + u_t \tag{11}$$

Together these imply the undetermined-coefficient requirements

$$\begin{aligned} (1+a)\,\phi_{11} &= 1 - \phi_{21}\phi_{11} \\ (1+a)\,\phi_{12} &= -\phi_{21}\phi_{12} - 1 \\ \phi_{21} &= (\phi_{21} + a)\phi_{11} \\ \phi_{22} &= (\phi_{21} + a)\phi_{12} + 1 \end{aligned} \tag{12}$$

The first and third of equations (12) give ϕ^2_{11} - (2 + a)ϕ_{11} + 1 = 0 so the MSV value of ϕ_{11} is[9]

$$\phi_{11} = (1/2)\,(2 + a - \sqrt{a^2 + 4a}\,). \qquad (13)$$

For any a>0, the foregoing implies $0<\phi_{11}<1$ so we see from (8) that the behavior of y_t is dynamically stable. Then putting ϕ_{11} back into the third of equations (12) shows that $\phi_{21} = a\phi_{11}/(1-\phi_{11})$ is positive and finite. Finally $\phi_{12} = -1/(1 + a + \phi_{21}) < 0$ and $\phi_{22} = -\phi_{12}$. From the second of equations (8), stability of y_t implies stability of π_t.

IV. Extensions

Let us now explore the Ball-Svensson result a bit, to understand its source, before going on to other model specifications. As it happens, the result is purely a consequence of the particular Phillips curve specification (2), ie, it is independent of aggregate demand behavior. What is relevant in that regard is merely the assumption that the central bank can - with whatever instrument it chooses! - keep nominal income equal to its target value plus a random control error. For present purposes, we suppose that the target is $\Delta x^* = 0$ and we neglect the random control error, which is irrelevant for the issue of dynamic stability. Then let us write $\Delta x_t = 0$ as

$$y_t - y_{t-1} + \pi_t = 0 \qquad (14)$$

and consider this equation together with the Ball-Svensson Phillips relation, which we write now as

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + \eta_t. \qquad (2')$$

Combining these we easily obtain

$$y_t - (2-\alpha)y_{t-1} + y_{t-2} = \eta_t. \qquad (15)$$

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9 Here we know that it is $-\sqrt{a^2 + 4a}$ that is relevant by the MSV criterion developed in McCallum (1983, pp. 146-7). The more common dynamic stability criterion of (e.g.) Blanchard and Kahn (1980) would give the same result but seems somewhat less appropriate given the issue at hand.

But it is well-known that for stability of a second-order linear difference equation, one necessary condition is that the coefficient on y_{t-2} must be less than 1.0 in absolute value—see, eg, Sargent (1987, p. 189). So with relation (2), stability of y_t cannot prevail if nominal income targeting is successful. That is Ball's result.

But relation (2′) is, to reiterate, quite special and rather problematic. First note that if we attach a coefficient γ to the lagged inflation term, then the counterpart of (15) would have γ attached to y_{t-2}. So if γ were 0.99, rather than 1.00, stability would prevail. This observation is relevant because typical Phillips-curve specifications include $E_{t-1}\pi_t$ rather than π_{t-1}, which we will do momentarily. Another common difference relative to (2′) is that the role of output involves y_t, or y_t and y_{t-1}, instead of y_{t-1} alone. Suppose, then that instead of (2′) we adopt the relation

$$\pi_t = \pi_{t-1} + \alpha_1 y_t + \alpha_2 y_{t-1} + \eta_t \qquad (16)$$

where $\alpha_1>0$ and $|\alpha_2|<\alpha_1$. Then combining (16) with (14) gives

$$y_t - [(2-\alpha_2)/(1+\alpha_1)]y_{t-1} + [1/(1+\alpha_1)]y_{t-2} = \eta_t, \qquad (17)$$

in which all the conditions sufficient for stability are satisfied.[10]

Continuing with our exploration, let us next, as promised above, replace π_{t-1} in (16) with $E_{t-1}\pi_t$. This results in a specification like that of Lucas (1973), one which is shown by Svensson (1996) also to represent a generalization of the P-bar model promoted by McCallum (1994).[11] Then instead of (16) we have

$$\pi_t = E_{t-1}\,\pi_t + \alpha_1 y_t + \alpha_2 y_{t-1} + \eta_t. \qquad (18)$$

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[10] Using c_1 and c_2 to denote the coefficients on y_{t-1} and y_{t-2} in (17), the conditions are $1 + c_1 + c_2 > 0$, $1 - c_1 + c_2 > 0$, and $1 - c_2 > 0$. In the case at hand, we multiply by $1 + \alpha_1$ and then the first of these is $(1 + \alpha_1) - (2 + \alpha_2) + 1 = \alpha_1 + \alpha_2 > 0$, the second is $(1 + \alpha_1) + (2 - \alpha_2) + 1 = 4 + \alpha_1 - \alpha_2 > 0$, and the third is $1 + \alpha_1 - 1 = \alpha_1 > 0$.

[11] Svensson's (1996) equation (A.1) represents a generalization in the sense that current-period conditions are permitted to modify the price—totally predetermined—given by the P-bar model. This result requires, however, a particularly simple specification of aggregate demand.

But since π_t - E_{t-1} π_t is a linear combination of η_t and any control error in Δx_t, (18) immediately implies

$$y_t = (-\alpha_2/\alpha_1)\, y_{t-1} + \text{white noise}, \qquad (19)$$

which with $|\alpha_2|<\alpha_1$ is unambiguously stable.

Next, in order to include one Phillips curve specification in which p_t is entirely predetermined (ie, $p_t = E_{t-1}\, p_t$), let us consider the basic P-bar model without Svensson's simplified specification of aggregate demand or his inclusion of within-period effects on p_t.[12] Thus we assume that

$$p_t - p_{t-1} = (1 - \psi)\,(\bar{p}_{t-1} - p_{t-1}) + E_{t-1}\,(\bar{p}_t - \bar{p}_{t-1}) \qquad (20)$$

with $0 < \psi < 1$, where $\bar{p}_t$ is the value of p_t that would prevail in t if there were no nominal stickiness. In a log-linear model we have $p_t - \bar{p}_t = -\theta\, y_t$ under the Ball-Svensson convention that y_t is the relevant gap measure of output. (Of course $\theta > 0$.) In this case we can rearrange (20) as $p_t - E_{t-1}\bar{p}_t = \psi(p_{t-1} - \bar{p}_{t-1})$ and infer that

$$E_{t-1}\, y_t = \psi\, y_{t-1}. \qquad (21)$$

But the latter implies that y_t is stable and with policy designed to yield $\pi_t + y_t - y_{t-1} = 0$, $\pi_t = (1 - \psi)y_{t-1}$ is also stable.[13]

Finally, mention should be made of the Fuhrer-Moore (1995) Phillips curve specification, which has not been given much in the way of theoretical support but which performs nicely from an empirical perspective. Analysis of its implications takes a bit more space than is appropriate here, but an adaptation of the result in Section VI of McCallum and Nelson (1997) indicates that stability of y_t and π_t prevail in a model that includes (5) and the Fuhrer-Moore relation when policy is designed to keep Δx_t constant.

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12 The P-bar model, incidentally, has the theoretically attractive property of satisfying the strict version of the natural rate hypothesis (McCallum, 1994, pp. 259-261). Of the seven Phillips curve specifications considered here or in Section III, only it and the Lucas (1973) model--i.e., (16) but with $E_{t-1}\pi_t$ replacing π_{t-1}--have that property.

13 In realistic applications, the P-bar model implies that y_t differs from $E_{t-1}\, y_t$, but the Ball-Svensson assumption that current y_t is observed by the central bank when setting r_t implies that the v_t shock in (5) is precisely offset.

V. Level Targets

As in Ball (1997), we can also consider nominal income targeting in which the level rather than the growth rate is controlled. In place of $\Delta x_t = \Delta x^*$, we would have the objective $x_t = x_t^*$, with $x_t^* = x_{t-1}^* + \Delta x^*$. Letting $\Delta x^* = 0$ as before, we have as the counterpart of (14) the relation

$$y_t + p_t = x^*. \tag{22}$$

Again we rewrite the Ball-Svensson Phillips curve, this time as

$$p_t = p_{t-1} + p_{t-1} - p_{t-2} + \alpha y_{t-1} + \eta_t. \tag{2''}$$

Combining (2″) and (20) yields

$$p_t - (2-\alpha)p_{t-1} + p_{t-2} = \alpha x^* + \eta_t. \tag{23}$$

Just as in the case of (15), this implies instability of p_t – and therefore, from (22), of y_t.

But suppose we replace (2″) with (18). Then we have $p_t - E_{t-1} p_t$ in place of $\pi_t - E_{t-1}\pi_t$ in (18), and results analogous to (19) obtain again. Using instead (16) with (22) we have

$$p_t - [(2-\alpha_2)/(1+\alpha_1)]\, p_{t-1} + [1/(1+\alpha_1)]\, p_{t-2} = (\alpha_1 + \alpha_2)x^* + \eta_t \tag{24}$$

and stability holds as with (17).

No new proof is needed for the P-bar case so, finally, let us consider the forward-looking specification (4), which we rewrite as

$$p_t - p_{t-1} = E_t p_{t+1} - p_t + a y_t + \eta_t \tag{25}$$

Using (22) this gives

$$(2 + a)p_t = p_{t-1} + E_t\, p_{t+1} + \alpha x^* + \eta_t \tag{26}$$

The MSV solution will be

$$p_t = \phi_0 + \phi_1 p_{t-1} + \phi_2 \eta_t \tag{27}$$

so $E_t p_{t+1} = \phi_0 + \phi_1 (\phi_0 + \phi_1 p_{t-1} + \phi_2 \eta_t)$ and substitution into (25) gives rise to undetermined-coefficient relations (analogous to (12)) including

$$\phi_1^2 - (2 - a)\, \phi_1 + 1 = 0 \tag{28}$$

The MSV solution for ϕ_1 is exactly the same as for ϕ_{11} in (13), so $0 < \phi_1 < 1$ and again stability prevails.[14]

VI. Conclusion

What the foregoing demonstrates is that the Ball-Svensson instability result stems principally from the Phillips curve specification used in their analyses. That specification is neither standard nor theoretically attractive, and its replacement with a number of more attractive alternatives leads to a reversal of the finding that nominal income targeting results in non-stationary processes for output and inflation.

This demonstration does not establish, of course, that nominal income targeting is preferable to inflation targeting or to other rules for monetary policy. To reach such a conclusion would require an extensive combination of theoretical and empirical analyses, conducted in a manner that gives due emphasis to the principle of robustness to model specification, plus attention to concerns involving policy transparency and communication with the public.[15] The point of the present paper is not to attempt any such ambitious undertaking, but merely to make it clear that there is very little basis for a negative judgment regarding nominal income targeting provided by the fragile results announced by Ball (1997).

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[14] Another specification mentioned by Svensson (1997b, p. 22) is (4) but with $E_t\pi_{t+1}$ replaced by a weighted average of $E_t\pi_{t+1}$ and π_{t-1}. Svensson then modifies that by replacing the right-hand-side terms by their expectations from two periods before. I can see little justification for this last step, but the previous specification is one that might merit consideration. Its analysis is more difficult than for the specifications considered in Section IV.

[15] Some of the difficulties of such a study are discussed in McCallum (1997).

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