Expectations and the term premium in New Zealand long-term interest rates

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NON-TECHNICAL SUMMARY¹

As a small, indebted economy, it is important to understand how financial market shocks in the rest of the world transmit to New Zealand. A key channel is long-term interest rates, which are highly correlated across countries. A sharp increase in international long-term bond yields would affect a range of New Zealand interest rates, including mortgage rates.

I use a term structure model to analyse the drivers of long-term interest rates in New Zealand. Movements in long-term interest rates can be decomposed into a component that reflects expectations about the future path of short-term policy rates, and changes in the term premium. The term premium is the compensation investors require for the risk of holding interest rate securities.

The term premium in New Zealand 10-year bond rates has trended down since the 1990s. Stable inflation, a strong domestic economy, and low global bond market volatility are likely to have contributed to a low term premium in recent years.

The New Zealand term premium is highly correlated with foreign yields, which may present some challenges for domestic monetary policy. Specifically, an increase in the term premium, even if driven from overseas, would be associated with a fall in domestic inflation and activity over the following year. Monetary policy may sometimes need to offset term premium shocks to achieve domestic macroeconomic objectives.

The model presented in this note provides estimates of the drivers of long-term yields that can be monitored at a high frequency, and a framework for thinking about movements in long-term interest rates and their implications for policymakers.

¹ The author would like to thank Leo Krippner, Michelle Lewis, Punnoose Jacob, Jamie Culling, and Severin Bernhard.
1. Introduction

As a small, indebted economy, it is important to understand how financial market shocks in the rest of the world transmit to New Zealand. A key transmission channel is long-term interest rates, which are highly correlated across countries. A sharp increase in long-term bond yields would affect a range of New Zealand interest rates, including mortgage rates.

Interpreting information and the effects from long-term bond rates can be aided by separating expectations of future policy rates from the term (i.e. risk) premium. The expectations theory states that long-term interest rates should be equivalent to compounding current and future short-term interest rates. Therefore, a bond yield of a given maturity should reflect the average of expected short rates from now to that maturity. The literature largely rejects this hypothesis, noting that long-term interest rates also incorporate a time-varying risk premium component. This term premium is the expected additional return (i.e. risk premium) that investors demand to compensate them for the risk associated with a long-term bond.

In this note, I apply a three-factor, no-arbitrage affine term structure model to the New Zealand yield curve, to decompose the 10-year interest rate into expectation and term premium components. The decomposition shows that movements in the term premium are an important driver of long-term yields. By extracting the term premium, the model also provides an estimate of the neutral interest rate, which closely follows the mean of the Bank’s neutral interest rate suite.

Three factors are noted as being important determinants of the New Zealand term premium, consistent with the existing international literature:

1. Inflation stability,
2. The business cycle, and
3. Global bond financial market volatility

New Zealand interest rates are also highly correlated with US yields, and the term premium components of yields have become increasingly correlated since the global financial crisis. I show that the sharp narrowing in the NZ-US 10-year yield differential since mid-2017 has largely been driven by relative expectations about future policy rate movements, rather than relative risk premiums.

Subsequently, using an estimated vector autoregressive model, I also find that movements in the term premium have important macroeconomic effects in New Zealand. I show, using a vector autoregressive model, that an increase in the term premium is associated with a fall in domestic inflation and activity over the following

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3 The term premium can be negative if investors perceive interest rate securities as safe assets, see Callaghan (2017).
year. This implies that monetary policy may sometimes need to offset term premium shocks, if the tightening in domestic monetary policy settings is unwarranted.

The term structure model presented in this note provides a formal framework for policymakers and market participants for thinking about movements in long-term interest rates and their implications. The model also provides estimates of the drivers of long-term yields that can be monitored at a high frequency.

The rest of this note is structured as follows. Section 2 discusses the term structure model methodology. Section 3 shows the results from the model. Section 4 provides support for the validity of the model estimates, by comparing the long-run expectations component to the Bank's neutral interest rate suite, and exploring some of the potential drivers of the New Zealand term premium. Section 5 examines the term premium further by discussing the interaction of global long-term yields and New Zealand yields, and estimating a VAR model with term premium shocks. Section 6 concludes.

### 2. Methodology

Long term interest rates can be decomposed into a component that reflects the average expected policy rate over the maturity of the bond, and a term premium component:

\[
y_{nt} = \frac{1}{n} \sum_{i=0}^{n-1} E_t (r_{t+i}^{OCR}) + TP_{nt}^y
\]

There are numerous models in the term structure literature that attempt to decompose interest rates into expectations and term premium components. In this note I use estimates from the no-arbitrage affine term structure model of Adrian, Crump and Moench (2013) to decompose the New Zealand yield curve into expectations and term premium components. The term structure model is detailed in Appendix A.

The term structure model uses New Zealand zero-coupon yield data. The data is available at a daily frequency and begins in April 1992. The model is constructed solely using yield curve data; it does not use macroeconomic or survey data.

The model describes and forecasts the yield curve through the evolution of three pricing factors; the level, slope and curvature. The term structure model uses principal components to extract these pricing factors from the yield curve data. These pricing factors, \(X_t\), are assumed to evolve over time according to a vector autoregressive (VAR) process:

\[
X_{t+1} = \mu + \phi X_t + v_{t+1},
\]

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4 In principal components analysis, a set of time series is used to generate a second set of series (principal components), which are not correlated with (i.e. orthogonal to) each other, which capture a large share of the variation in the original series.
Interest rates, $lnP_t^{(n)}$, are modelled as linear functions of the pricing factors, $X_t$:

$$lnP_t^{(n)} = A_n + B_n'X_t + u_t^{(n)}$$

The model parameters are estimated by using a three-step linear regression approach. In the first step, the pricing factors are regressed on the lagged factors to estimate the factor VAR(1) parameters and the factor shocks. In the second step, zero-coupon bond returns are regressed on these factor shocks to estimate their loadings. The parameters that determine the relationship between the pricing factors and yields are restricted to ensure the absence of arbitrage opportunities. In the third step, risk premium parameters are estimated by using a cross-sectional regression of excess returns on these factor loadings.

The expectations component is calculated by setting the risk premium parameters to zero. The term premium can then be calculated as the difference between the model-implied fitted yield and the model-implied average expected future short-term interest rate over the relevant horizon, which is computed by forecasting the VAR of pricing factors.

3. Results

Figure 1 plots the decomposition of the New Zealand 10-year bond rate into an expectations component and a term premium.

Figure 1: A decomposition of the New Zealand 10-year bond yield

Note: The expectations component is the estimated average expected policy rate over the next 10 years.

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5 Absence of arbitrage opportunity means that the pricing makes it impossible to form a portfolio consisting of bonds with different maturities that generates a riskless profit.
The term premium has declined over the sample, consistent with a global downward trend in term premiums (Cohen, Hordahl and Xia, 2018). A high, positive term premium in the early part of the sample period implies that long-term interest rates were higher than what would be expected based purely on expectations of the future short-term policy rate. In that environment, bond holders would expect to earn a risk premium over time as compensation for bearing the risk of holding the long-term bond.

There was a sharp increase (+120bp) in the estimated term premium during the global financial crisis in 2009. The term premium has also spiked higher at other times when global yields increased, including during the 2013 US taper tantrum⁶ (+50bp) and following the 2016 US presidential election (+60bp).

The expectations component (the estimated average expected policy rate over the next 10 years) has also tended lower over the sample. The drift lower in future expected policy rates, particularly since the global financial crisis, is consistent with the global decline in neutral interest rates (Holston, Laubach and Williams, 2017).

Movements in the expectations component of New Zealand interest rates are consistent with the Reserve Bank’s Official Cash Rate (OCR) cycles and forward guidance (Callaghan, Culling and Richardson, forthcoming). For example, expectations fell as the OCR was cut during the global financial crisis, but rose soon after as the Bank signalled a quick return to higher policy rates. The expectations component also rose through 2013-2014, as the Bank increased the OCR and signalled further increases.

4. Validating the model results

In this section, I explore the plausibility of the model results, and provide external validation of the term structure estimates, by a) comparing the long-run expectations component to the Bank’s neutral interest rate suite; and, b) exploring some of the potential influences on the New Zealand term premium.

4.1 The expectations component: A neutral interest rate estimate

The neutral interest rate is the interest rate that would prevail once all business cycle shocks have dissipated and inflation is expected to remain at target. This measure of neutral interest rates helps the Bank identify the degree of stimulus being provided to the economy by current and expected interest rate settings, at horizons relevant for the operation of monetary policy.

A nominal neutral rate for New Zealand can be estimated by taking the 5-year, 5-year forward rate, which abstracts from the business cycle effects. The 5-year, 5-year yield

⁶ An important channel of unconventional monetary policy is through lowering long term interest rates, via the term premium. The 2013 taper tantrum refers to the widespread market volatility that followed the earlier-than-expected suggestion that the Federal Reserve would taper its asset purchase programme.
is the five-year interest rate (plus a term premium) that is expected to prevail starting five years from now, implied by the current structure of the yield curve. The 5-year, 5-year forward interest rate needs to be adjusted for the term premium, to obtain an estimate of market expectations.

Figure 2 shows the estimated neutral rate alongside the Bank’s neutral interest rate suite (see Richardson and Williams, 2015). The estimated neutral rate, taken from the term premium-adjusted 5-year, 5-year forward yield, tracks the average of the indicator suite well. The neutral rate estimate from the term structure model as of December 2018 is about 3 percent, slightly lower than the indicator suite mean.

These results suggest that the term structure model produces reasonable estimates of interest rate expectations.

Figure 2: RBNZ neutral interest rate suite and 5-year, 5-year forward expectations

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4.2 The term premium estimate: Potential drivers of the term premium

The asset pricing literature suggests several possible drivers of the term premium in long-term bonds. In this section, I explore some of the possible drivers of the New Zealand term premium, as an external validation of the term structure model estimate. I focus on the domestic business cycle, inflation stability, and global financial market volatility.

Term premiums tend to rise in recessions. A positive relationship between the term premium and the unemployment rate in the US suggests that the term premium is countercyclical (Cochrane and Piazzesi, 2005). Popular asset pricing models attribute

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7 Other factors are likely important in influencing the term premium, such as the effect of global quantitative easing (Bonis, Ihrig, and Wei, 2017), the government debt structure (Chadha, Turner, and Zampolli, 2013), and more formally, the covariance of payoffs with future marginal consumption growth (Cochrane, 2009).
this variation to countercyclical changes in risk aversion (Campbell and Cochrane, 1999).

Volatile inflation may cause investors to require greater compensation for holding fixed-income securities. Wright (2011) finds that term premiums have declined internationally since the 1990s, especially in countries that reduced inflation uncertainty by making substantial changes in their monetary policy frameworks.

Adrian et al. (2013) show that their US term premium estimate is strongly correlated with the MOVE index. The MOVE index is a measure of implied volatilities from options on US Treasury futures. This suggests that their US term premium estimate reflects the risk of holding US Treasury securities.

To estimate the effect of potential drivers of the New Zealand term premium, I simultaneously regress the 10-year term premium on New Zealand unemployment rate, the standard deviation of New Zealand CPI inflation, the MOVE index, and a constant.

Table 1 shows the estimated coefficients for potential influences on the New Zealand 10-year term premium. All three variable – inflation volatility, the unemployment rate, and US bond market option-implied volatility – are statistically significant and explain over 70 percent of the variation in the estimated term premium over the sample.

Table 1: NZ 10-year term premium regression

| Term premium | Standard deviation of inflation | 0.29*** |
|              | Unemployment rate               | 0.08*** |
|              | MOVE index                      | 0.01*** |
|              | Constant                        | -0.84*** |
| R-squared    | 0.74                            |
| Adj. R-squared| 0.73                           |
| No. obs.     | 106                             |

Note: ‘Standard deviation of inflation’ is the 4-quarter rolling standard deviation of two-year headline CPI inflation. All independent variables are lagged two quarters, which provided the best fit. Heteroscedasticity and autocorrelation consistent standard errors in parentheses. Regressions are estimated using data from 1992Q2 to 2018Q3. *** p<0.01, ** p<0.05, * p<0.1.

8 For example, in bad times such as recessions, risk premiums tend to rise (prices for risky assets are low), because investors are less willing or able to bear risk. In good times, when the economy is strong and employment is high, investors demand less compensation for bearing risk and risk premiums tend to be low.
The regression results suggest that the term structure estimate is plausible, because it is correlated with external variables in a way that we would expect, based on the international literature. Specifically, business cycle fluctuations, the stability of domestic inflation, and international financial market volatility may be important determinants of the New Zealand term premium (figure 3):

- The positive correlation with the unemployment rate suggests that the New Zealand term premium may be countercyclical, similar to the findings in the US.
- Greater inflation volatility is also positively correlated with the New Zealand term premium. Inflation targeting through the 1990s and the movement to an Official Cash Rate (OCR) system in 1999 provided greater certainty to market participants, and this may have contributed to a lower term premium.
- US financial bond market volatility, as proxied by the MOVE index, is also positively correlated with the New Zealand term premium. This suggests that higher bond market volatility abroad tends to flow through to New Zealand yields, as investors require greater compensation for holding relatively risky New Zealand-dollar fixed-income assets.

**Figure 3: New Zealand 10-year term premium indicator**

**Figure 3.1: Term premium and indicator**

**Figure 3.2: Contributions (demeaned)**

Note: Indicator is based on the regression in table 1.

5. The term structure of interest rates and the macroeconomy

In this section, firstly I explore the influence of global factors on New Zealand 10-year yield components. In the next sub-section, I estimate an SVAR model to evaluate the effect of term premium and expectations shocks on the domestic economy.

5.1. NZ long-term interest rates and global yields

Long-term bond yields are highly correlated across countries, and the correlation between New Zealand and US bond yields has increased in recent decades (Callaghan, Cassino, Vehbi, and Wong, forthcoming). Yields may be correlated across
economies because expectations for policy rates are correlated (e.g. because monetary policy cycles tend to move together), or because risk premiums are correlated.

Figure 4 shows the correlation between New Zealand and US 10-year yield components. The model results suggest that term premium shocks have become relatively more important in explaining the correlation between long-term yields, particularly since the global financial crisis.

The high correlation of the term premium components is consistent with the existing literature, which suggests that risk premiums are highly correlated across economies (Cohen, Hordahl and Xia, 2018). Jotikasthira, Le and Lundblad (2015) find that world inflation and the US yield level together explain over two-thirds of the covariance of yields, and that these effects operate largely through the risk compensation (i.e. term premium) channel. Dahlquist and Hasseltoft (2013) note the importance of US bond risk and global business cycles in driving global yields. They find that correlations between international bond risk premia have risen over time, suggesting that financial markets have become increasingly interconnected.

Figure 4: Co-movements of estimated components of NZ and US 10-year yields

Note: 36-month centred rolling correlations, based on monthly changes in the variables. Grey bars represent New Zealand recessions.

The term structure model used in this note can also help understand the drivers of the spread between yields in different economies. As an example, I can compare the term structure model estimates from New Zealand and US yields, to understand the drivers of the recent fall in the NZ-US 10-year spread. The spread between New Zealand and US 10-year government bond rates fell sharply, below zero, over 2018 (figure 5).

9 Estimates from the US 10-year yield are from Adrian et al (2013), and available on Bloomberg (tickers ACMTP10, ACMTRY10, and ACMTY10) and online: https://www.newyorkfed.org/research/data_indicators/term_premia.html
Figure 5: NZ and US 10-year zero-coupon yields

Figure 6 shows that New Zealand 10-year yields decreased since the start of 2017. The expectations component declined, as the RBNZ reiterated that policy would remain expansionary “for a considerable period”. In contrast, US 10-year yields increased, led by the expectations component, as growth and inflation strengthened and the Federal Reserve continued its policy rate hiking cycle. The estimated term premium in both countries’ yields declined, a result that is consistent with the strong correlation of term premia internationally.

Taken together, the model suggests that most of the decline in the NZ-US 10-year spread since 2017 has been due to a divergence in monetary policy stance (largely from expectations for higher future US policy rates), rather than relative risk premium movements.

Figure 6: Decomposition of NZ-US 10-year yield changes

Figure 6.1: Change in 10-year yields

Figure 6.2: Change in NZ-US 10-year spread

Note: Figures show change from 2017m01 to 2018m10. The US 10-year yield decomposition data are from Adrian, Crump and Moench (2013), see footnote 8.
5.2 Potential macroeconomic effects of the term premium

The macrofinance literature suggests that movements in risk premium can have important macroeconomic implications, because movements in the term premium affect borrowing and lending rates in the economy, which, in turn, determine aggregate demand. For example, Rudebusch, Sack and Swanson (2006) find that a decline in the term premium has typically been associated with higher future GDP growth. An important channel in the implementation of unconventional monetary policy is through influencing the term premium in long term interest rates. Central bank policymakers have also noted the role of the term premium in influencing financial conditions and monetary policy considerations.

In this final section, I provide an initial assessment of the potential effect that a change in the term premium may have on the macroeconomy. For this, I use the expectations and term premium estimates as data in a structural vector autoregressive (SVAR) model. By estimating a vector autoregressive model, I can construct impulse response functions to assess the persistence and relative effects of various macroeconomic shocks.

I use a standard Cholesky decomposition that imposes a recursive ordering for the contemporaneous effects of shocks. The unemployment rate gap (inverted) is ordered first, followed by core inflation (the sectoral factor measure), the expectations component of 10-year yields, and the 10-year term premium.

Ordering the financial market variable last is standard in the literature, and justified by the results in 4.2. Specifically, the term premium is influenced by macroeconomic variables, so ordering it last ensures that term premium movements due to the unemployment rate and inflation are appropriately removed from the model. I can then apply a ‘clean’ exogenous shock to the term premium to gauge its macroeconomic effect.

Figure 7 displays the impulse responses from the VAR from an expected policy shock (top row), and a term premium shock (bottom row). Appendix B shows the full impulse response results and similar results from alternate Cholesky orderings.

My results indicate the effect of that higher long-term interest rates, due to an increase in expectations of higher future policy rates, is similar to a positive demand shock – inflation and the inverted unemployment gap both increase. This is consistent with the expectations component capturing forward-looking information about the outlook for

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10 See Drought, Perry, and Richardson (2018).

11 See Bernanke (2006): “To the extent that the decline in forward rates can be traced to a decline in the term premium, […] the effect is financially stimulative and argues for greater monetary policy restraint, all else being equal. Specifically, if spending depends on long-term interest rates, special factors that lower the spread between short-term and long-term rates will stimulate aggregate demand. Thus, when the term premium declines, a higher short-term rate is required to obtain the long-term rate and the overall mix of financial conditions consistent with maximum sustainable employment and stable prices.”
economic growth and inflation. A 100 basis point increase in expectations about future policy rates, tends to be followed by a 25 basis point increase in core inflation, peaking after 8 quarters. Economic activity also increases, with the unemployment rate gap falling 25 basis points over the next 4 to 8 quarters.

In contrast, an increase in the term premium is similar to a negative aggregate demand shock. A 100 basis point increase in the term premium leads to a 30 basis point decrease in core inflation, which troughs after 6 to 8 quarters. Economic activity also decreases, as the unemployment rate gap rises 45 basis points over the next 6 to 8 quarters. These results provide some evidence for the risk premia channel of uncertainty shocks in New Zealand.\(^\text{12}\) A term premium shock also tends to be far less persistent than an expectations shock.

The policy response to higher long-term interest rates may therefore depend on the driver. If interest rates are higher due to expectations of higher future policy rates, tighter monetary policy may be consistent with a stronger outlook for future economy activity and inflation. However, if interest rates are higher due to a higher term premium, policy rates may need to fall to offset the tightening in financial conditions.\(^\text{13}\)

Figure 7: Impulse response functions

Response to an expectations shock:

Response to a term premium shock:

Note: Expectations and term premium shocks have been standardised to 100 basis points. The sample period is 1995Q1 to 2018Q3.

\(^\text{12}\) See Rice, Vehbi and Wong (2018) for a discussion of the effects of uncertainty shocks in New Zealand.

\(^\text{13}\) Term premium shocks can be incorporated into the Bank’s forecasting framework in NZSIM by adjusting the mortgage rate spread assumption and world demand via our export price assumption. For example, see the scenario analysis in the May 2018 Monetary Policy Statement, which simulates a ‘tighter financial conditions’ shock.
6. Conclusion

Long-term interest rates are a key channel through which financial market shocks transmit to New Zealand, and risk premiums in long-term interest rates are highly correlated globally. The term structure model I estimate in this note suggests that the term premium in New Zealand 10-year bond rates has trended down since the 1990s, and is close to zero currently.

The term premium is influenced by global financial market volatility and domestic macroeconomic stability. In New Zealand, inflation volatility, the business cycle, and global financial market volatility can largely explain movements in the term premium. Stable inflation, a strong domestic economy, and low global bond market volatility has contributed to a low term premium in recent years.

Movements in the term premium have important macroeconomic implications. An increase in the term premium is associated with a fall in domestic inflation and activity over the following year. Monetary policy may sometimes need to offset term premium shocks to achieve domestic macroeconomic objectives.

The term structure model presented in this note provides a formal framework for policymakers and market participants for thinking about movements in long-term interest rates and their implications. The model also provides estimates of the drivers of New Zealand long-term yields that can be monitored at a high frequency.
References


Appendix A: Affine term structure model

The key equations of a standard Gaussian affine term structure model (ATSM) are detailed below. This follows the exposition from Adrian, Crump, and Moench (2013), and readers are referred to that paper for further details.

Assume that the dynamics of a K x 1 vector of pricing factors, $X_t$, evolve according to a Gaussian VAR(1):

$$X_{t+1} = \mu + \phi X_t + v_{t+1},$$

where the shocks $v_{t+1} \sim \mathcal{N}(0, \Sigma)$ are conditionally Gaussian, homoscedastic, and independent across time.
The assumption of no-arbitrage implies the existence of a pricing kernel $M_{t+1}$ which discounts the expected future price of a zero coupon bond $P_{t+1}^{(n-1)}$ to the price in the current period, that is:

$$p_t^{(n)} = E_t[M_{t+1}P_{t+1}^{(n-1)}]$$

The pricing kernel is assumed to be an exponential function of the risk-free rate, market price of risk and the shocks to the pricing factors, which links the risk-neutral and physical probability distributions:

$$M_{t+1} = \exp(-r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \Sigma^{1/2} v_{t+1})$$

Where $r_t = ln P_t^{(1)}$ denotes the continuously compounded one-period risk-free rate, which is assumed to be affine in the factors:

$$r_t = A_0 + B_1 X_t$$

and the market prices of risk ($\lambda_t$) are affine in the factors:

$$\lambda_t = \Sigma^{1/2}(\lambda_0 + \lambda_1 X_t).$$

The log excess one-period holding return of a zero coupon bond maturing in $n$ periods is defined as

$$rx_{t+1}^{(n-1)} = \ln P_{t+1}^{(n-1)} - ln P_t^{(n)} - r_t$$

Combining these elements, Adrian, Crump and Moench show that excess holding period returns can be expressed as a function of the bonds’ expected return, a convexity adjustment, the shocks to the pricing factors, and an orthogonal return pricing error:

$$rx_{t+1}^{(n-1)} = \beta^{(n-1)'}(\lambda_0 + \lambda_1 x_t) - \frac{1}{2}(\beta^{(n-1)'} \Sigma \beta^{(n-1)} + \sigma^2) + \beta^{(n-1)'}v_{t+1} + e_t^{(n-1)}$$

where

$$\beta^{(n-1)'} = Cov_t[rx_{t+1}^{(n-1)}(n-1)' v_{t+1}]\Sigma^{-1}$$

Stacked across maturities and time periods, this can be rewritten as

$$rx = \beta' (\lambda_0 i_T + \lambda_1 X_\tau) - \frac{1}{2}(B' \text{vec}(\Sigma) + \sigma^2 i_N) i_T' + B'V + E$$

### Estimation

**Step 1:** The VAR is estimated via ordinary least squares:

$$X_{t+1} = \mu + \phi X_t + v_{t+1},$$
This allows the decomposition of $X_t$ into a predictable component and an estimate of the innovation $v_{t+1}$. These innovations are stacked into the $K \times N$ matrix $\hat{V}$ and an estimator of the state variable variance-covariance matrix $\hat{\Sigma} = \hat{V} \hat{V}' / T$ is constructed.

**Step 2:** Regress excess returns on a constant $i_t'$, contemporaneous pricing factor innovations $\hat{V}$, and lagged pricing factors $X_\sim$, according to:

$$rx = ai_t' + \beta' \hat{V} + cX_\sim + E$$

where

$$a = \beta' \lambda_0 - \frac{1}{2} (B' \text{vec}(\Sigma) + \sigma^2 i_N)$$

$$c = \beta' \lambda_1$$

The residuals are collected from this regression into the $N \times T$ matrix $\hat{E}$. Estimate $\hat{\sigma}^2 = \text{trace}(\hat{\Sigma} \hat{\Sigma}') / NT$ and construct $\hat{B}^*$ from $\hat{\beta}$.

**Step 3:** Estimate the price of risk parameters $\lambda_0$ and $\lambda_1$ via cross-sectional regression.

$$\hat{\lambda}_0 = (\hat{\beta} \hat{\beta}')^{-1} \hat{\beta} (\hat{a} + \frac{1}{2} (\hat{B} \text{vec}(\hat{\Sigma}) + \hat{\sigma}^2 i_N))$$

$$\hat{\lambda}_1 = (\hat{\beta} \hat{\beta}')^{-1} \hat{\beta} \hat{c}$$

**Affine prices**

From the estimated model parameters, a zero-coupon bond curve can be generated:

$$\ln P_t^{(n)} = A_n + B_n' X_t + u_t^{(n)}$$

Excess returns can be written as:

$$rx_{t+1}^{(n-1)} = A_{n-1} + B_{n-1}' X_{t+1} + u_{t+1}^{(n-1)} - A_n - B_n' X_t - u_t^{(n)} + A_1 + B_1' X_t + u_t^{(1)}$$

where

$$-r_t = A_1 + B_1' X_t + u_t^{(1)}$$

Equating this expression for excess returns with the return generating expression:

$$A_{n-1} + B_{n-1}' X_{t+1} + u_{t+1}^{(n-1)} - A_n - B_n' X_t - u_t^{(n)} = \beta^{(n-1)'} (\lambda_0 + \lambda_1 x_t + v_{t+1}) - \frac{1}{2} (\beta^{(n-1)'} \Sigma \beta^{(n-1)} + \sigma^2) + e_{t+1}^{(n-1)}$$

Matching terms, the system of recursive linear restrictions for the bond pricing parameters are:

$$A_n = A_{n-1} + B_{n-1}' (\phi - \lambda_0) + \frac{1}{2} (B_{n-1}' \Sigma B_{n-1} + \sigma^2) + A_1$$

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\[ B_n' = B_{n-1}'(\phi - \lambda_1) + B_1 \]
\[ A_0 = 0, \quad B_0' = 0 \]

The equations are recalculated with the market prices of risk \( \lambda_0 \) and \( \lambda_1 \) set to zero to generate the risk-neutral curve. The risk premium is the difference between the model estimated bond price and the risk-neutral bond price.

Appendix B: Detailed impulse response functions

The full results of the estimated VAR discussed in the text are shown in figure B.1. An alternate ordering of the VAR, see figure B.2, shows similar results to that shown in the text.

In figure B.1, I use a standard Cholesky decomposition that imposes a recursive ordering for the contemporaneous effects of shocks. The unemployment rate gap (inverted) is ordered first, followed by core inflation (the sectoral factor measure), the expectations component of 10-year yields, and the 10-year term premium.

Figure B.1: Impulse response functions

In figure B.2, I use a standard Cholesky decomposition that imposes a recursive ordering for the contemporaneous effects of shocks. The unemployment rate gap
(inverted) is ordered first, followed by core inflation (the sectoral factor measure), the 10-year term premium, and the expectations component of 10-year yields.

Figure B.2: Impulse response functions